## Chapter 3

3.1. What is a flowshop plant?

A flowshop plant is a plant in which several batch products are produced using all or a subset of the same equipment and in which the operations for each batch follow the same sequence. Thus the flow of any batch through equipment $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D} \ldots$ is always $\mathrm{A} \rightarrow \mathrm{B} \rightarrow \mathrm{C} \rightarrow \mathrm{D} \rightarrow \ldots$. Omissions of equipment are possible but no reversal in direction is allowed.
3.2. What is a jobshop plant?

A flowshop plant is a plant in which several batch products are produced using all or a subset of the same equipment but for which the operations of at least one batch product do not follow the same sequence, e.g., $\mathrm{A} \rightarrow \mathrm{C} \rightarrow \mathrm{D} \rightarrow \mathrm{B}$
3.3. What are the two main methods for sequencing multiproduct processes?

Either use multi-product campaigns or multiple single-product campaigns.
3.4. Give one advantage and one disadvantage of using single-product campaigns in a multiproduct plant.

Advantage - sequencing of single-product campaigns is relatively simple and repetitive and probably less prone to operator error since the batch recipe remains the same over the entire campaign.

Disadvantage - significant final product storage will be required since all products will not be made all the time and in order to even out supply some inventory of products will have to be maintained in storage. Single-product campaigns may be less efficient than multiproduct campaigns.
3.5. What is the difference between a zero-wait and a uis process?

A zero-wait process is one in which the batch is transferred immediately from the current piece of equipment to the next piece of equipment in the recipe sequence. This type of process eliminates the need for intermediate storage (storage of unfinished products or intermediates).

A uis (unlimited intermediate storage) process is one in which any amount of any intermediate product may be stored. Such a process maximizes the use of the processing equipment but obviously requires an unlimited amount of storage.
3.6 Number of batches of A is twice that for B or C - repeat Example 3.3 with this restriction using a 500 h cycle time.

Table E3.3: Equipment times needed to produce A, B, and C

| Product | Time in Mixer | Time in <br> Reactor | Time in <br> Separator | Time in <br> Packaging |
| :---: | :---: | :---: | :---: | :---: |
| A | 1.5 | 1.5 | 2.5 | 2.5 |
| B | 1.0 | 2.5 | 4.5 | 1.5 |
| C | 1.0 | 4.5 | 3.5 | 2.0 |

Using Equation (3.6) with $t_{\text {cycle }, A}=2.5, t_{\text {cycle }, B}=4.5$, and $t_{\text {cycle }, C}=4.5$
If $x$ is the number of batches of Products $B$ and $C$, then $2 x$ is the number of batches of Product A

$$
T=500=2 x(2.5)+x(4.5+4.5) \Rightarrow x=\frac{500}{14}=35.7
$$

Number of batches for each product are $A=70, B=35, C=35$
3.7 For Examples 3.3 and 3.4, determine the number of batches that can be produced in a month ( 500 h ) using a multi-product campaign strategy with the sequence ACBACBACB. Are there any other sequences for this problem other than the one used in Example 3.4 and the one used here?


The multi-product cycle time $=2.5+2.0+3.5+4.5=12.5 \mathrm{~h}$
Number of batches per month $=(500) /(12.5)=40$ each of A, B, and C
The only sequences that can be used for multi-product campaigns are ABCABCABC (Example 3.4) and ACBACBACB as used here.
3.8 Consider the multi-product batch plant described in Table P3.8

Table P3.8: Equipment Processing Times for Processes A, B, and C

| Process | Mixer | Reactor | Separator |
| :---: | :---: | :---: | :---: |
| $A$ | 2.0 h | 5.0 h | 4.0 h |
| B | 3.0 h | 4.0 h | 3.5 h |
| C | 1.0 h | 3.0 h | 4.5 h |

It is required to produce the same number of batches of each product. Determine the number of batches that can be produced in a 500 h operating period using the following strategies:
(a) using single-product campaigns for each product

Using Equation (3.6) with $t_{\text {cycle }, A}=5.0, t_{\text {cycle }, B}=4.0$, and $t_{c y c l e, C}=4.5$

$$
T=500=x(5.0+4.0+4.5) \Rightarrow x=\frac{500}{13.5}=37.0
$$

$$
x=37 \text { batches }
$$

(b) using a multi-product campaign using the sequence $\mathrm{ABCABCABC} \ldots$


From this diagram we see that the cycle time for the multi-product campaign using the sequence ABC is 12.5 h .
Therefore, the number of batches, $x$, of each product that can be made during a 500 h period is given by:

$$
T=500=12.5 x \Rightarrow x=\frac{500}{12.5}=40
$$

$x=40$ batches
(c) using a multi-product campaign using the sequence CBACBACBA...


From this diagram we see that the cycle time for the multi-product campaign using the sequence ABC is 13.5 h .
Therefore, the number of batches, $x$, of product that can be made during a 500 h period is given by:

$$
T=500=13.5 x \Rightarrow x=\frac{500}{13.5}=37.0
$$

$x=37$ batches
3.9 Consider the process given in Problem 3.8. Assuming that a single-product campaign strategy is repeated every 500 h operating period and further assuming that the production rate (for a year $=6,000 \mathrm{~h}$ ) for products $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are $18,000 \mathrm{~kg} / \mathrm{y}, 24,000 \mathrm{~kg} / \mathrm{y}$, and $30,000 \mathrm{~kg} / \mathrm{y}$, respectively, determine the minimum volume of product storage required. Assume that the product densities of A, B, and C are 1100, 1200, and 1000 $\mathrm{kg} / \mathrm{m}^{3}$, respectively

The tables below shows the results using data given from Problem 8

| Rate | Product A | Product B | Product C |
| :--- | :---: | :---: | :---: |
| Volume $\left(\mathrm{m}^{3}\right)$ of product <br> required per month | $18,000 / 12 / 1,100$ <br> $=1.36$ | $24,000 / 12 / 1,20$ <br> 0 <br> $=1.67$ | $30,000 / 12 / 1,000$ <br> $=2.5$ |
| Cycle time (h) | 5.0 | 4.0 | 4.5 |


| Product | Campaign time, <br> $t_{c a m p}$ <br> $(\mathrm{~h})$ | $r_{p}-r_{d}$ <br> $\left(\mathrm{~m}^{3} / \mathrm{h}\right)$ | Minimum volume of <br> product storage, $V_{s}$ <br> $\left(\mathrm{~m}^{3}\right)$ |
| :---: | :---: | :---: | :---: |
| A | $(37)(5)=185$ | $0.007371-0.00273=$ <br> 0.004644 | $(0.004644)(185)=$ <br> $\mathbf{0 . 8 5 9}$ |
| B | $(37)(4)=148$ | $0.01126-0.003333=$ | $(0.007928)(148)=$ |
|  |  | 0.007928 | $\mathbf{1 . 1 7 3}$ |
| C | $(37)(4.5)=166.5$ | $0.015015-0.005=$ | 0.010015 |

Table P3.10A: Production rates for A, B, and C

| Product | Yearly <br> production | Production <br> in 500 h |
| :---: | :---: | :---: |
| A | $150,000 \mathrm{~kg}$ | $12,500 \mathrm{~kg}$ |
| B | $210,000 \mathrm{~kg}$ | $17,500 \mathrm{~kg}$ |
| C | $360,000 \mathrm{~kg}$ | $30,000 \mathrm{~kg}$ |

Table P3.10B: Specific Reactor/Mixer Volumes for Processes A, B, and C

| Process | A | B | C |
| :---: | :---: | :---: | :---: |
| $v_{\text {react }}\left(\mathrm{m}^{3} /\right.$ kg-product $)$ | 0.0073 | 0.0095 | 0.0047 |
| $t_{\text {cycle }}(\mathrm{h})$ | 6.0 | 9.5 | 18.5 |

Let the single-product campaign times for the three products be $t_{A}, t_{B}$, and $t_{C}$, respectively. Applying Equation (3.6), the following relationship is obtained:

$$
\begin{equation*}
t_{A}+t_{B}+t_{C}=500 \tag{3.9}
\end{equation*}
$$

The number of campaigns per product is then given by $t_{x} / t_{\text {cycle }, x}$ and

$$
\begin{equation*}
\text { batch size }(\mathrm{kg} / \text { batch })=\frac{\text { production of } \mathrm{x}}{t_{\chi} / t_{\text {cycle }, x}} \tag{3.10}
\end{equation*}
$$

Furthermore, the volume of a batch is found by multiplying Equation (3.10) by $v_{\text {reac, },}$, and equating batch volumes for the different products yields:

$$
\begin{gather*}
\text { Volume of batch }=\frac{(\text { production of } \mathrm{x})\left(v_{\text {react }, x}\right)}{t_{x} / t_{\text {cycle }, x}}  \tag{3.11}\\
\frac{(12,500)(.0073)}{t_{A} / 6.0}=\frac{(17,500)(.0095)}{t_{B} / 9.5}=\frac{(30,000)(.0047)}{t_{C} / 18.5} \tag{3.12}
\end{gather*}
$$

Solving Equations (3.9) and (3.12), yields:

$$
\begin{gathered}
t_{A}=57.8 \mathrm{~h} \\
t_{B}=166.8 \mathrm{~h} \\
t_{C}=275.4 \mathrm{~h} \\
v_{\text {react }, A}=v_{\text {react }, B}=v_{\text {react }, C}=9.47 \mathrm{~m}^{3}
\end{gathered}
$$

> \#batches per campaign for product $\mathrm{A}=t_{A} / 6.0=9.6$ \#batches per campaign for product $\mathrm{B}=t_{B} / 9.5=17.6$
> \#batches per campaign for product $\mathrm{C}=t_{C} / 18.5=14.9$

Clearly the number of batches should be an integer value. Rounding these numbers yields
For product A
Number of batches $=10$
$t_{A}=(10)(6.0)=60 \mathrm{~h}$
$V_{A}=(12,500)(0.0073) /(10)=9.13 \mathrm{~m}^{3}$
For product B
Number of batches $=17$
$t_{B}=(17)(9.5)=161.5 \mathrm{~h}$
$V_{B}=(17,500)(0.0095) /(17)=9.78 \mathrm{~m}^{3}$
For product C
Number of batches $=15$
$t_{C}=(15)(18.5)=277.5 \mathrm{~h}$
$V_{C}=(30,000)(0.0047) /(15)=9.40 \mathrm{~m}^{3}$
Total time for production cycle $=499 \mathrm{~h} \sim 500 \mathrm{~h}$
Volume of reactor $=9.78 \mathrm{~m}^{3}$ (limiting condition for Product B)
3.11

Table P3.11: Batch step times (in hours) for Reactor and Bacteria Filter for Project 8 in Appendix B

| Product | Reactor* | Precoating of <br> Bacteria <br> Filter | Filtration <br> of <br> Bacteria | Mass <br> produced <br> per batch, <br> kg | Ratio of <br> product,s |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L-aspartic Acid | 40 | 25 | 5 | 1020 | 1 |
| L-phenylalanine | 70 | 25 | 5 | 716 | 1.25 |

*includes 5 h for filling, cleaning, and heating plus 5 hours for emptying
(a) let
$t_{A}=$ campaign time for L-aspartic acid
$t_{P}=$ campaign time for L-phenylalanine
Assuming equal recovery ratios for each amino acid we have

$$
\begin{aligned}
& t_{A}+t_{P}=8000 \\
& \frac{(716)\left(t_{P}\right)}{(70)}=1.25 \frac{(1020)\left(t_{A}\right)}{(40)}
\end{aligned}
$$

Solving we get
$t_{A}=1944 \mathrm{~h}$
$t_{P}=6056 \mathrm{~h}$
yearly production of L-aspartic acid $=(1944)(1020) /(40)=49,560 \mathrm{~kg}$
yearly production of L-phenylalanine $=(6056)(716) /(70)=61,950 \mathrm{~kg}$

Number of batches per year for L-aspartic acid $=(1944) /(40)=48$
Number of batches per year for L-phenylalanine $=(6056) /(70)=86$
(b) For each product calculate the average yearly demand and production rate in $\mathrm{m}^{3} / \mathrm{h}$ and then find the storage needed for each product

| Rate | L-aspartic acid | L-phenylalanine |
| :--- | :---: | :---: |
| Volume $\left(\mathrm{m}^{3}\right)$ of product <br> required per year | $(49,560)(0.9) /(1,200)=$ <br> 37.17 | $(61,950)(0.9) /(1,200)=$ <br> 46.46 |
| Cycle time $(\mathrm{h})$ | 40 | 70 |
| Campaign time $(\mathrm{h})$ | 1944 | 6056 |
| Production rate, $r_{p}\left(\mathrm{~m}^{3} / \mathrm{h}\right)$ | $(37.17) /(1944)=0.019125$ | $(46.46) /(6056)=0.0076714$ |
| Demand rate, $r_{d}\left(\mathrm{~m}^{3} / \mathrm{h}\right)$ | $(37.17) /(8,000)=0.004646$ | $(46.46) /(8000)=0.005808$ |
| $r_{p}-r_{d}\left(\mathrm{~m}^{3} / \mathrm{h}\right)$ | 0.014479 | 0.001864 |
| Storage Volume $\left(\mathrm{m}^{3}\right)$ | $(0.014479)(1944)=\mathbf{2 8 . 1 4}$ | $(0.001864)(6056)=\mathbf{1 1 . 2 9}$ |

(c) Rework part (b) using a 1 month cycle time $=8,000 / 12=666.67 \mathrm{~h}$

Assuming equal recovery ratios for each amino acid we have

$$
\begin{aligned}
& t_{A}+t_{P}=666.67 \\
& \frac{(716)\left(t_{P}\right)}{(70)}=1.25 \frac{(1020)\left(t_{A}\right)}{(40)}
\end{aligned}
$$

Solving we get
$t_{A}=162 \mathrm{~h}$ and $t_{P}=504.7 \mathrm{~h}$
monthly production of L-aspartic acid $=(4)(1020)=4,080 \mathrm{~kg}$
monthly production of L-phenylalanine $=(7)(716)=5,012 \mathrm{~kg}$

Number of batches per month for L-aspartic acid $=(162) /(40)=4$
Number of batches per month for L-phenylalanine $=(504.7) /(70)=7$

Note that these are rounded down so that integer numbers are given per month this gives rise to a slightly lower production rate per year than before.

| Rate | L-aspartic acid | L-phenylalanine |
| :--- | :---: | :---: |
| Volume $\left(\mathrm{m}^{3}\right)$ of product <br> required per month | $(4,080)(0.9) /(1,200)=3.06$ | $(5,012)(0.9) /(1,200)=3.76$ |
| Cycle time $(\mathrm{h})$ | 40 | 70 |
| Campaign time $(\mathrm{h})$ | 160 | 490 |
| Production rate, $r_{p}\left(\mathrm{~m}^{3} / \mathrm{h}\right)$ | $(3.06) /(160)=0.019125$ | $(3.76) /(490)=0.0076714$ |
| Demand rate, $r_{d}\left(\mathrm{~m}^{3} / \mathrm{h}\right)$ | $(3.06) /(666.7)=0.00459$ | $(3.76) /(666.7)=0.005638$ |
| $r_{p}-r_{d}\left(\mathrm{~m}^{3} / \mathrm{h}\right)$ | 0.014535 | 0.002033 |
| Storage Volume $\left(\mathrm{m}^{3}\right)$ | $(0.014535)(160)=\mathbf{2 . 3 3}$ | $(0.002033)(490)=\mathbf{1 . 0 0}$ |

These values are (not surprisingly) approximately $1 / 12$ of the previous results
3.12
(a) Referring to Project B.8, Figures B.8.2 and B.8.3 and using batch reaction times for Laspartic acid and L-phenylalanine of 25 and 55 h , respectively. We get the following information:

Conversion of L-aspartic acid $=42 \%$ ( $84 \%$ of equilibrium) (base case $=45 \%$ ) Exit concentration of L-phenylalanine $=18.5 \mathrm{~kg} / \mathrm{m}^{3}($ base case $=21 \%)$

| Product | Reactor* | Precoating <br> of Bacteria <br> Filter | Filtration <br> of Bacteria | Mass produced <br> per batch, kg | Ratio of <br> products |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L-aspartic Acid | 35 | 25 | 5 | $(42 / 45)(1020)=$ <br> 952 | 1 |
| L-phenylalanine | 65 | 25 | 5 | $(18.5 / 21)(716)=$ <br> 630.8 | 1.25 |

let
$t_{A}=$ campaign time for L-aspartic acid
$t_{P}=$ campaign time for L-phenylalanine
Assuming equal recovery ratios for each amino acid we have

$$
\begin{aligned}
& t_{A}+t_{P}=8000 \\
& \frac{(630.8)\left(t_{P}\right)}{(65)}=1.25 \frac{(952)\left(t_{A}\right)}{(35)}
\end{aligned}
$$

Solving we get
$t_{A}=1776 \mathrm{~h}$
$t_{P}=6224 \mathrm{~h}$
yearly production of L-aspartic acid $=(1776)(952) /(35)=48,316 \mathrm{~kg}$
yearly production of L-phenylalanine $=(6224)(630.8) /(65)=60,395 \mathrm{~kg}$

Number of batches per year for L-aspartic acid $=(1776) /(35)=\mathbf{5 0}$ or 51
Number of batches per year for L-phenylalanine $=(6224) /(65)=\mathbf{9 5}$ or 96

Therefore, the number of batches increases but the yearly production decreases
(b) Referring to Project B.8, Figures B.8.2 and B.8.3 and using batch reaction times for Laspartic acid and L-phenylalanine of 35 and 65 h , respectively. We get the following information:

Conversion of L-aspartic acid $=47 \%(94 \%$ of equilibrium $)($ base case $=45 \%)$ Exit concentration of L-phenylalanine $=21.5 \mathrm{~kg} / \mathrm{m}^{3}$ (base case $=21 \%$ )

| Product | Reactor* | Precoating <br> of Bacteria <br> Filter | Filtration <br> of Bacteria | Mass produced <br> per batch, kg | Ratio of <br> products |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L-aspartic Acid | 45 | 25 | 5 | $(47 / 45)(1020)=$ <br> 1065 | 1 |
| L-phenylalanine | 75 | 25 | 5 | $(21.5 / 21)(716)=$ <br> 733 | 1.25 |

let
$t_{A}=$ campaign time for L-aspartic acid
$t_{P}=$ campaign time for L-phenylalanine
Assuming equal recovery ratios for each amino acid we have

$$
\begin{aligned}
& t_{A}+t_{P}=8000 \\
& \frac{(733)\left(t_{P}\right)}{(75)}=1.25 \frac{(1065)\left(t_{A}\right)}{(45)}
\end{aligned}
$$

Solving we get
$t_{A}=1986 \mathrm{~h}$
$t_{P}=6014 \mathrm{~h}$
yearly production of L-aspartic acid $=(1986)(1065) /(45)=47,002 \mathrm{~kg}$
yearly production of L-phenylalanine $=(6014)(733) /(75)=58,777 \mathrm{~kg}$

Number of batches per year for L-aspartic acid $=(1986) /(45)=44$
Number of batches per year for L-phenylalanine $=(6014) /(75)=\mathbf{8 0}$

Therefore, the number of batches decreases and the yearly production decreases - plot the results from problems 11 and 12


This shows that the base case conditions are close to optimal.
3.13
a. Let $x$ be the number of batches of A in a $600-\mathrm{h}$ period.

$$
\begin{aligned}
& t_{c y c l e, A}=5.0 \mathrm{~h} \\
& t_{c y c l e, B}=5.5 \mathrm{~h} \\
& t_{\text {cycle }, C}=5.0 \mathrm{~h} \\
& 5.0 x+5.5(2 x)+5.0(3 x)=600 \mathrm{~h} \\
& \Rightarrow x=600 /(5+11+15)=600 / 31=19.4 \text { batches } \\
& x=19-\text { rounded down (but one could round up) }
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{A}=19 \text { batches } \\
& \mathrm{B}=38 \text { batches } \\
& \mathrm{C}=57 \text { batches }
\end{aligned}
$$

b. Demand in one month of $C$ is $180,000 / 12=15,000 \mathrm{~kg} /$ month

$$
\# \text { batches }=57, t_{\text {cycle }, C}=5.0 \mathrm{~h} \Rightarrow(5)(57)=285 \mathrm{~h} / \text { month operating time }
$$

$$
\text { production rate }=15,000 / 285=52.63 \mathrm{~kg} / \mathrm{h} \quad(52.63 \mathrm{~kg} / \mathrm{h}) /\left(850 \mathrm{~kg} / \mathrm{m}^{3}\right)=0.06192 \mathrm{~m}^{3} / \mathrm{h}
$$

$$
\text { demand rate }=15,000 / 850 / 600=0.02941 \mathrm{~m}^{3} / \mathrm{h}
$$

$$
V_{\text {storage }}=\left(0.06192-0.02941 \mathrm{~m}^{3} / \mathrm{h}\right)(285 \mathrm{~h})=9.27 \mathrm{~m}^{3}
$$

c. Crystallizer is limiting piece of equipment

$$
\begin{aligned}
& \Rightarrow t_{\text {cycle }}=4.5+5.5+5.5+5.0+5.0+5.0=30.5 \mathrm{~h} \\
& \# \text { repeats }=600 / 30.5=19.7 \\
& \# \text { batches same as part a }
\end{aligned}
$$

3.14
a. $n_{B}=n_{C}=2 n_{A}=n$

$$
\begin{aligned}
& t_{\text {cycle }, A}=3.5 \mathrm{~h} \\
& t_{\text {cycle }, B}=4.0 \mathrm{~h} \\
& t_{\text {cycle }, C}=3.0 \mathrm{~h} \\
& \Rightarrow 0.5 n(3.5)+4 n+3 n=600 \mathrm{~h} \\
& n(1.75+4+3)=600 \\
& n=68.6 \text { batches }- \text { round to } 69 \\
& n_{B}=n_{C}=69 \text { batches } \\
& n_{A}=34 \text { batches }
\end{aligned}
$$

b. $\mathrm{ABBCCABBCC}-$ as shown on figure - crystallizer is limiting


```
\(t_{\text {cycle }}=\mathrm{ABBCC}=3.5+(4.0+4.0)+(3.0+3.0)=17.5\)
\(n(17.5)=600\)
\(n=34.3\)
```

$$
\begin{aligned}
& n_{B}=n_{C}=69 \text { batches } \\
& n_{A}=34 \text { batches } \\
& \text { same as above }
\end{aligned}
$$

c. $2 n(3.5)+4 n+3 n=600$

$$
n=600 / 14=42.9
$$

$$
n_{B}=n_{C}=43 \text { batches }
$$

$$
n_{A}=85 \text { or } 86 \text { batches }
$$

d. $\quad r_{d}=17.5 / 600=0.02917 \mathrm{~m}^{3} / \mathrm{h}$

$$
r_{p}=17.5 /[(4)(69)]=0.06341 \mathrm{~m}^{3} / \mathrm{h}
$$

$$
t_{\text {campaign }}=4(69)=276 \mathrm{~h}
$$

$$
V_{S, B}=\left(r_{p}-r_{d}\right) t_{\text {campaign }}=(0.06341-0.02917)(276)
$$

$$
V_{S, B}=9.45 \mathrm{~m}^{3}
$$

