

## Mini-Lecture 1.1

### Study Skill Tips for Success in Mathematics

#### Learning Objectives:

1. Get ready for this course.
2. Understand some general tips for success.
3. Know how to use the text.
4. Know how to use text resources.
5. Get help as soon as you need it.
6. Learn how to prepare for and take an exam.
7. Develop good time management.

#### Examples:

1. Getting ready for this course.
  - a) Positive attitude
  - b) Allow adequate time for class arrival
  - c) Bring all required materials
2. General tips for success.
  - d) Find a contact person
  - e) Choose to attend all classes
  - f) Do your homework
  - g) Check your work
  - h) Learn from mistakes
  - i) Seek help when needed
  - j) Stay organized
  - k) Read textbook before class
  - l) Ask questions
  - m) Hand in all assignments on time
3. Understand how to use the text.
  - a) Practice Exercises – an exercise the student can try to match each example in each section.
  - b) Review the meaning of icons used in text.
  - c) At beginning of each section, a list of icons shows availability of support materials.
  - d) Integrated Reviews – found in the middle of each chapter for students to practice the concepts previous learned in a chapter
  - e) Each chapter ends with Chapter Highlights, Reviews, and Practice Tests.
4. Know how to use text resources.
  - a) Chapter Test Prep Video CD – (CD inside book) each Chapter test exercise worked out by the author.
  - b) Lecture Video CDs – (not included with book) the author explains each section and works specified example problems within each section
5. Learn to take exams.
  - a) Review previous homework assignments, class notes, quizzes, etc.
  - b) Read Chapter Highlights to review concepts and definitions.
  - c) Practice working out exercises in the end-of-the-chapter Review and Test.
  - d) When taking a test, read directions and problems carefully.
  - e) Pace yourself. Use all available time. Check your work and answers.
6. Good time management.
  - a) Make a list of all weekly commitments with estimated time needed.
  - b) Be sure to schedule study time. Don't forget eating, sleeping, and relaxing!

#### Teaching Notes:

- Most developmental students have a high anxiety level with mathematics.
- Many developmental students are hesitant to ask questions and seek extra help.
- Be sure to include your individual expectations. Keep your expectations clear and concise.

## Mini-Lecture 1.2

### Place Value, Names for Numbers, and Reading Tables

#### Learning Objectives:

1. Find the place value of a digit in a whole number.
2. Write a whole number in words and in standard form.
3. Write a whole number in expanded form.
4. Read tables.

**Key Vocabulary:** *whole numbers, place value, standard form, period, expanded form, tables*

#### Examples:

1. Find the place value of the digit 7 in each whole number.

a) 7,352                                      b) 607                                      c) 702,433                                      d) 17,009,321

2. Write each whole number in words.

a) 62    b) 698    c) 17,403    d) 1,067,599

Write each number in standard form.

- e) nine hundred fifty-two  
f) three hundred sixty-two thousand, five hundred eighty-six  
g) three million, four hundred thousand, one hundred two

3. Write each number in expanded form.

a) 398    b) 2,907    c) 4,089,347

4. Use the following table of Number of Students Enrolled to answer the questions.

Subject	Section 1	Section 2	Section 3	Total
Basic Mathematics	23	27	19	69
Statistics	20	25	22	67

- a) How many total students are enrolled in Basic Mathematics?
- b) How many students are enrolled in Section 3 of Statistics?

#### Teaching Notes:

- Students who do not have English as their first language will need additional assistance learning place value vocabulary.
- Students who do not have English as their first language may use periods instead of commas in writing numbers.

*Answers:* 1a) thousands, b) ones, c) hundred thousands, d) million; 2a) sixty-two, b) six hundred ninety-eight, c) seventeen thousand, four hundred three, d) one million, sixty-seven thousand, five hundred ninety-nine, e) 952, f) 362,586, g) 3,400,102; 3a)  $300+90+8$ , b)  $2000+900+7$ , c)  $4,000,000+80,000+9,000+300+40+7$ ; 4a) 69, b) 22

## Mini-Lecture 1.3

### Adding and Subtracting Whole Numbers, and Perimeter

#### Learning Objectives:

1. Add whole numbers.
2. Subtract whole numbers.
3. Find the perimeter of a polygon.
4. Solve problems by adding or subtracting whole numbers.

**Key Vocabulary:** *sum, addend, perimeter, minuend, subtrahend and difference*

#### Examples:

1. Add.

a)  $3 + 9$

b)  $40 + 70$

c)  $1900 + 17$

d)  $5703 + 0$

e) 
$$\begin{array}{r} 51 \\ +27 \\ \hline \end{array}$$

f) 
$$\begin{array}{r} 7329 \\ + 683 \\ \hline \end{array}$$

g)  $93 + 145 + 69$

h) 
$$\begin{array}{r} 6,403 \\ + 793 \\ \hline 17,187 \end{array}$$

2. Subtract. Check by adding.

a)  $11 - 7$

b)  $15 - 8$

c)  $22 - 22$

d)  $31 - 0$

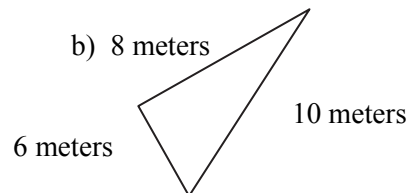
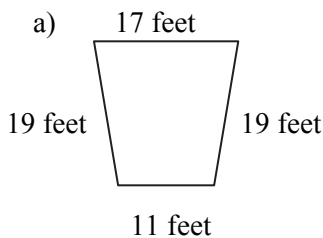
e) 
$$\begin{array}{r} 27 \\ - 13 \\ \hline \end{array}$$

f) 
$$\begin{array}{r} 198 \\ - 94 \\ \hline \end{array}$$

g) 
$$\begin{array}{r} 3004 \\ -2965 \\ \hline \end{array}$$

h) 
$$\begin{array}{r} 20,003 \\ -16,867 \\ \hline \end{array}$$

3. Find the perimeter of each figure



4. Solve the following word problems.

a) What is the sum of 8,932 and 14,799?

b) Subtract 376 from 803.

c) The Library Renovation Project has set a goal of \$75,000 to fundraise. To date, \$47,908 has been fundraised. How much more money does the Library Renovation Project need to fundraise?

d) On Monday, Karen drove 57 miles; on Tuesday, she drove 39 miles; and on Wednesday, Karen drove 92 miles. How many total miles did Karen drive?

#### Teaching Notes:

- Some students need additional practice with basic addition and subtraction facts.
- Remind students that it is acceptable to write the carry digit in order to obtain the correct answer.
- Most students will find this section easy but may need assistance with word problems.
- Many students need to write the borrowing/regrouping step to maintain accuracy.
- Many students are challenged when borrowing with zeros.

*Answers:* 1a) 12, b) 110, c) 1917, d) 5703, e) 78, f) 8012, g) 307, h) 24,383; 2a) 4, b) 7, c) 0, d) 31, e) 14, f) 104, g) 39, h) 3,136; 3a) 66 ft., b) 24 m.; 4a) 23,731, b) 427, c) \$27,092, d) 188 miles

# Mini-Lecture 1.4

## Rounding and Estimating

### Learning Objectives:

1. Round whole numbers.
2. Use rounding to estimate sums and differences.
3. Solve problems by estimating.

**Key Vocabulary:** *rounding, graph, estimating, and exact*

### Examples:

1. Round to the nearest ten.

a) 31                      b) 57                      c) 346                      d) 2,795

Round to the nearest hundred.

e) 312                      f) 6,658                      g) 8,672                      h) 1,899

2. Round to the nearest thousand to find the estimated sum or difference.

a) 
$$\begin{array}{r} 4892 \\ -2305 \\ \hline \end{array}$$
                      b) 
$$\begin{array}{r} 2731 \\ + 3020 \\ \hline \end{array}$$
                      c) 
$$\begin{array}{r} 17,032 \\ - 12,513 \\ \hline \end{array}$$
                      d) 
$$\begin{array}{r} 24,803 \\ + 14,587 \\ \hline \end{array}$$

3. Solve.

a) At the last 3 dances, attendance was 657 students, 403 students, and 559 students. Estimate the total attendance by rounding each to the nearest hundred.

b) Enrollment figures at the Town of Johnson's School Department increased from 6,721 students to 7,653 students. Round each number to the nearest hundred to estimate the increase.

c) The Carlisle family needs to buy a refrigerator for \$999, a stove for \$459, and a dishwasher for \$449. Round each cost to the nearest hundred to estimate the total cost.

### Teaching Notes:

- Some students need to be repeatedly reminded to look at the digit to the right of the rounding position. Have students draw a line after the digit in the rounding position.
- A common error students make is to leave the digits to the right of the rounding position the same instead of changing them to zeros after rounding.
- Stress the importance of rounding and estimating with applications.

*Answers:* 1a) 30, b) 60, c) 350, d) 2,800, e) 300, f) 6,700, g) 8,700, h) 1,900; 2a) 3000, b) 6000, c) 4000, d) 40,000; 3a) 1700, b) 1,000, c) \$1,900

## Mini-Lecture 1.5

### Multiplying Whole Numbers and Area

#### Learning Objectives:

1. Use the properties of multiplication.
2. Multiply whole numbers.
3. Find the area of a rectangle.
4. Solve problems by multiplying whole numbers.

**Key Vocabulary:** *factor, product, distribute, and area*

#### Examples:

1. Multiply.

a)  $37 \cdot 1$

b)  $1 \cdot 22$

c)  $0 \cdot 183$

d)  $9 \cdot 5 \cdot 0$

Use the distributive property to rewrite each expression.

e)  $2(5 + 4)$

f)  $5(1 + 9)$

g)  $10(9 + 6)$

h)  $15(0 + 14)$

2. Multiply.

a) 
$$\begin{array}{r} 37 \\ \times 6 \\ \hline \end{array}$$

b)  $412 \times 4$

c) 
$$\begin{array}{r} 1708 \\ \times 9 \\ \hline \end{array}$$

d) 
$$\begin{array}{r} 337 \\ \times 25 \\ \hline \end{array}$$

e) 
$$\begin{array}{r} 643 \\ \times 27 \\ \hline \end{array}$$

f) 
$$\begin{array}{r} 309 \\ \times 800 \\ \hline \end{array}$$

g)  $825 \times 1,000$

h) 
$$\begin{array}{r} 2,477 \\ \times 963 \\ \hline \end{array}$$

3. Find the area of a rectangle with length 14 feet and width 8 feet.
4. At a recent football game, 413 adult tickets were sold at a price of \$5 each. There were 127 child tickets sold at a price of \$3 each. How much total amount of money in ticket sales for the game?

#### Teaching Notes:

- Some students need additional practice with basic multiplication facts.
- Some students do not know the different types of symbols used for multiplication.
- When using distributive property, many students forget to distribute over both terms.
- When multiplying, remind students to carefully line up the ones, tens, hundreds, etc.

*Answers:* 1a) 37, b) 22, c) 0, d) 0, e) 18, f) 50, g) 150, h) 210; 2a) 222, b) 1648, c) 15,372, d) 8,425, e) 17,361, f) 247,200, g) 825,000, h) 2,385,351; 3) 112 sq. ft.; 4) \$2,446

## Mini-Lecture 1.6

### Dividing Whole Numbers

#### Learning Objectives:

1. Divide whole numbers
2. Perform long division.
3. Solve problems that require dividing by whole numbers.
4. Find the average of a list of numbers

**Key Vocabulary:** *dividend, divisor, quotient, and average*

#### Examples:

1. Find each quotient. Check by multiplying.

a)  $3\overline{)12}$       b)  $13 \div 1$       c)  $\frac{5}{5}$       d)  $15 \div 15$       e)  $0\overline{)5}$

2. Divide. Check by multiplying.

a)  $228 \div 4$       b)  $\frac{572}{7}$       c)  $1570 \div 3$   
d)  $14\overline{)7070}$       e)  $97\overline{)41,270}$       f)  $603\overline{)604,911}$

3. a) Find the quotient of 94 and 5.  
b) Recently, Amy earned \$1,722 selling calendars. If each calendar cost \$14, how many calendars did Amy sell?
4. a) During the semester, Kyle's test scores were: 87, 93, 62, 83 and 100. What was Kyle's average for the semester?

#### Teaching Notes:

- Some students need additional practice with basic division facts.
- Many students confuse division by zero (undefined) and zero divided by any non-zero number ( $\neq 0$ ).
- Many students need to be cautious with placement of digits in quotient and dividend. Be sure appropriate place values are lined up. Stress organization!

*Answers:* 1a) 4, b) 13, c) 1, d) 1, e) undefined; 2a) 57, b)  $81r5$ , c)  $523r1$ ; d) 505, e)  $425r45$ , f)  $1003r102$ ; 3a)  $18r4$ , b) 123; 4a) 85

## Mini-Lecture 1.7

### Exponents and Order of Operations

#### Learning Objectives:

1. Write repeated factors using exponential notation.
2. Evaluate expressions containing exponents.
3. Use the order of operations.
4. Find the area of a square.

**Key Vocabulary:** *exponential notation, exponent, base, order of operations, and area of a square*

#### Examples:

1. Write using exponential notation.

a)  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$     b)  $(7)(7)(7)$     c)  $4 \cdot 4 \cdot 3 \cdot 3 \cdot 3$     d)  $5 \cdot 5 \cdot 8 \cdot 8 \cdot 5 \cdot 5$

2. Evaluate.

a)  $5^2$                       b)  $7^3$                       c)  $3^6$                       d)  $10^4$

3. Using order of operations, simplify.

a)  $3 \cdot 4 - 10 \div 2$                       b)  $6^2 \div 3 \cdot 2$                       c)  $8 \cdot 4 + \{27 \div [8 - (3 + 2)]\}$

4. a) Find the area of a square whose side measures 6 feet.

b) Find the area of a square whose side measures 23 miles.

#### Teaching Notes:

- Students may confuse exponent and base.
- Many students have trouble with order of operations.
- Avoid “PEMDAS” as many students will multiply before dividing and add before subtracting.
- Stress to students that all multiplication/division must be performed in order from left to right.
- Stress to students that addition/subtraction is performed in order from left to right.
- Stress to students you can only add/subtract after all multiplication/division is complete.

*Answers: 1a)  $2^6$ , b)  $7^3$ , c)  $3^3 \cdot 4^2$ , d)  $5^4 \cdot 8^2$ ; 2a) 25, b) 343, c) 729, d) 10,000; 3a) 7, b) 24, c) 41; 4a) 36 sq. ft, b) 529 sq. ft*

## Mini-Lecture 1.8

### Introduction to Variables, Algebraic Expressions, and Equations

#### Learning Objectives:

1. Evaluate algebraic expressions given replacement values.
2. Identify solutions of equations.
3. Translate phrases into variable expressions.

Addition (+)	Subtraction (-)	Multiplication (•)	Division (÷)
Sum, plus, added to, more than, increased by, total	Difference, minus, subtract, less than, decreased by, less	Product, times, multiply, multiply by, of, double, triple	Quotient, divide, shared equally among, divided by, divided into

**Key Vocabulary:** *variable, algebraic expression, equation, and solution.*

#### Examples:

1. Evaluate each expression for  $x = 12$ ,  $y = 4$ , and  $z = 3$

a)  $x - y + z$                       b)  $x - (y + z)$                       c)  $5(3x + 7)$                       d)  $2xy - 3z$

e)  $x^2 - 4y$                       f)  $y^3 - 2x$                       g)  $\frac{3x}{4} - \frac{yz}{3}$                       h)  $\left(\frac{2yz - x}{2}\right)^3$

2. Determine whether the given number is a solution of the given equation.

- a) Is 10 a solution of  $n - 3 = 7$ ?
- b) Is 3 a solution of  $2n = 12$ ?

Determine which numbers in each set are solutions to the corresponding equations.

- c)  $n - 3 = 12$ ; {11, 12, 15}
- d)  $4n = 24$ ; {3, 6, 20}

3. Write each phrase as a variable expression. Use  $x$  to represent “a number.”

- a) The sum of a number and eleven
- b) Fifteen added to a number
- c) The difference between a number and three hundred
- d) A number subtracted from forty-two
- e) The product of sixteen and a number
- f) A number times thirteen
- g) The quotient of thirty and a number
- h) Seven divided by a number
- i) The quotient of eighteen and a number, decreased by two

#### Teaching Notes:

- Remind students that order of operations apply with variables.
- Stress to students that an equation has an equal sign and an expression does not.
- Many students will have difficulty translating a phrase into an algebraic expression.
- Refer students to textbook for ***Translating Phrases into Variable Expressions Chart***.

*Answers:* 1a) 11, b) 5, c) 215, d) 87, e) 128, f) 40, g) 5, h) 216; 2a) yes, b) no, c) 15, d) 6; 3a)  $x + 11$ , b)  $15 + x$ , c)  $x - 300$ , d)  $42 - x$ , e)  $16x$ , f)  $13x$ , g)  $30/x$ , h)  $7/x$ , i)  $\frac{18}{x} - 2$



# Mini-Lecture 2.1

## Introduction to Integers

### Learning Objectives:

1. Represent real-life situations with integers.
2. Graph integers on a number line.
3. Compare integers.
4. Find the absolute value of a number.
5. Find the opposite of a number.
6. Read bar graphs containing integers.

**Key Vocabulary:** *positive numbers, negative numbers, signed numbers, integers, is less than, is greater than, opposite, absolute value*

### Examples:

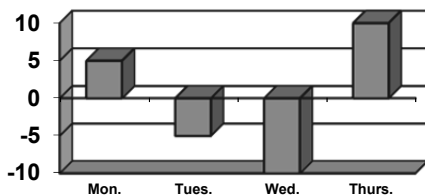
1. Represent each quantity by an integer.
  - a) A scuba diver is swimming 25 feet below sea level.
  - b) The record high temperature for the town is 113°F.
  - c) The number of televisions sold reflected a 35 percent loss from the previous year.
2. Graph each integer in the list on the same number line.
 

a) 1, 3, 5, 6	b) 2, -2, 3, -3	c) 4, 0, -2, -5	d) 0, -1, -2, -5
---------------	-----------------	-----------------	------------------
3. Insert  $<$  or  $>$  between each pair of integers to make a true statement.
 

a) 5 ___ 10	b) 0 ___ -3	c) -42 ___ -38	d) -22 ___ 22
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4. Simplify.
 

a) $ 2 $	b) $ -12 $	c) $ -3 $	d) $- 14 $
e) $- 45 $	f) $- -103 $	g) $ -x $ if $x = -25$	h) $ x $ if $x = -8$
5. Find the opposite of each integer.
 

a) 9	b) -15	c) 0	d) -16
------	--------	------	--------
6. The bar graph shows the January temperatures for four days in Boston.



- a) Which day was the coldest?
- b) Which day was the warmest?

### Teaching Notes:

- Many students will confuse absolute value and opposite.
- Encourage students to list everyday situations where negative numbers are used.

*Answers:* 1a) -25; b) +113; c) -35; 2a) b) c) d)

d) ; 3a)  $<$ ; b)  $>$ ; c)  $<$ ; d)  $<$ ; 4a) 2, b) 12; c) 3; d) -14; e) -45; f) -103;

g) 25; h) 8; 5a) -9; b) 15; c) 0; d) 16; 6a) Wed.; b) Thurs.

## Mini-Lecture 2.2

### Adding Integers

#### Learning Objectives:

1. Add integers.
2. Evaluate an algebraic expression by adding.
3. Solve problems by adding integers.

#### Examples:

1. Add.

a)  $23+12$

b)  $-23+(-17)$

c)  $-11+(-2)$

d)  $-21+(-13)$

e)  $6+(-8)$

f)  $-3+5$

g)  $-74+27$

h)  $-51+(24)$

i)  $-8+(-13)$

j)  $-79+97$

k)  $46+(-54)$

l)  $-4+(-24)$

m)  $23+(-19)+(-8)$

n)  $14+25+(-16)$

o)  $-25+(-4)+(-2)+(-6)$

2. Evaluate  $x+y$  for the given replacement values.

a)  $x=-5$  and  $y=14$

b)  $x=-33$  and  $y=-27$

c)  $x=-43$  and  $y=38$

3. Solve.

a) Find the sum of  $-7$  and  $25$ .

b) Find the sum of  $-52$ ,  $13$ , and  $-82$

c) During a storm in Anchorage Alaska, the temperature was  $10^{\circ}\text{F}$  at Noon. At 1 p.m., the temperature had dropped  $7^{\circ}$ . At 2 p.m., the temperature dropped another  $5^{\circ}$ ; and finally, at 3 p.m., the temperature had dropped an additional  $9^{\circ}$ . Use positive and negative numbers to represent his situation. Then find the present temperature.

#### Teaching Notes:

- Some students need to see adding integers done on a number line first.
- Many students have a better understanding if they think of depositing and withdrawing money from a bank account.
- Refer students to the rules for adding signed numbers in the textbook.

*Answers:* 1a) 35; b)  $-40$ ; c)  $-13$ ; d)  $-34$ ; e)  $-2$ ; f) 2; g)  $-47$ ; h)  $-27$ ; i)  $-21$ ; j) 18; k)  $-8$ ; l)  $-28$ ; m)  $-4$ ; n) 23; o)  $-37$ ; 2a) 9; b)  $-60$ ; c)  $-5$ ; 3a) 18; b)  $-121$ ; c)  $-11^{\circ}$

## Mini-Lecture 2.3

### Subtracting Integers

#### Learning Objectives:

1. Subtract integers.
2. Add and subtract integers.
3. Evaluate an algebraic expression by subtracting.
4. Solve problems by subtracting integers.

#### Key Vocabulary: *additive inverse*

#### Examples:

1. Subtract.

a)  $-9 - (-2)$

b)  $-14 - (-2)$

c)  $4 - (-3)$

d)  $20 - 20$

e)  $2 - 5$

f)  $-2 - 12$

g)  $-150 - 410$

h)  $-147 - (-85)$

2. Simplify.

a)  $6 + 20 - 15$

b)  $-1 - 11 - 12$

c)  $-1 - 20 + 10$

d)  $-16 + 11 - 18 + (-4)$

3. Evaluate  $x - y$  for the given replacement values.

a)  $x = -2$  and  $y = -8$

b)  $x = 8$  and  $y = -32$

c)  $x = -9$  and  $y = -9$

d)  $x = 3$  and  $y = -15$

4. Solve.

a) Amy has \$545 in her checking account. She writes a check for \$257, makes a deposit of \$75, and then writes another check for \$409. Find the balance in her account. (Write the amount as an integer.)

b) The city of Manchester has an elevation of 13,005 feet above sea level while the city of Catherine has an elevation of 17,532 feet below sea level. Find the difference in elevation between those two cities.

c) The temperature on a January morning in Worcester is  $-5^{\circ}F$  at 2 a.m. If the temperature drops  $4^{\circ}$  by 3 a.m., rises  $6^{\circ}$  by 4 a.m., and then drops  $8^{\circ}$  by 5 a.m., find the temperature by 8 a.m.

#### Teaching Notes:

- Many students find subtracting signed numbers difficult at first.
- Some students like to see subtracting signed numbers on a number line.
- Many students make errors when evaluating  $x - y$  when  $y$  is a negative number. Encourage students to make a direct substitution first so they do not forget to write the subtraction symbol.

*Answers:* 1a)  $-7$ ; b)  $-12$ ; c)  $7$ ; d)  $0$ ; e)  $-3$ ; f)  $-14$ ; g)  $-560$ ; h)  $-62$ ; 2a)  $11$ ; b)  $-24$ ; c)  $-11$ ; d)  $-27$ ;  
3a)  $6$ ; b)  $40$ ; c)  $0$ ; d)  $18$ ; 4a)  $-\$46$ ; b)  $4527$  ft.; c)  $-11^{\circ}F$

## Mini-Lecture 2.4

### Multiplying and Dividing Integers

#### Learning Objectives:

1. Multiply integers.
2. Divide integers.
3. Evaluate an algebraic expression by multiplying or dividing.
4. Solve problems by multiplying or dividing integers.

#### Examples:

1. Multiply.

a)  $7(-6)$

b)  $-4(10)$

c)  $-20(13)$

d)  $-10(-19)$

e)  $(-4)(-3)(6)$

f)  $(-50)(0)(-5)(8)$

g)  $(-4)(-5)(-4)(-3)$

h)  $(-2)(3)(-1)(-4)(2)$

i)  $-4^2$

j)  $(-3)^3$

k)  $-3^3$

l)  $(-8)^2$

2. Find each quotient.

a)  $21 \div 7$

b)  $36 \div (-6)$

c)  $\frac{-48}{6}$

d)  $\frac{-17}{0}$

3. Evaluate  $xy$  and also  $\frac{x}{y}$  for the given replacement values.

a)  $x = 8$  and  $y = -4$

b)  $x = -30$  and  $y = -10$

c)  $x = 0$  and  $y = -16$

4. Solve.

a) Find the product of  $-13$  and  $-5$ .

b) Find the quotient of  $63$  and  $-9$ .

c) Better Electric Co. marked \$15 off the price of each microwave in stock. If there are 57 microwaves in stock, write the total reduction in price of all microwaves as an integer.

d) During a cold front in Canada the temperature dropped  $4^\circ\text{F}$  each hour for 7 hours. Express the total drop in temperature as an integer.

#### Teaching Notes:

- Some students need a review of basic multiplication and division facts before they begin working with integers.
- Some students mix up the rules for addition of integers and the rules for multiplication/division of integers.
- Many students have a hard time understanding the difference between  $-3^2$  and  $(-3)^2$

*Answers:* 1a)  $-42$ ; b)  $-40$ ; c)  $-260$ ; d)  $190$ ; e)  $72$ ; f)  $0$ ; g)  $240$ ; h)  $-48$ ; i)  $-16$ ; j)  $-27$ ; k)  $-27$ ; l)  $64$ ; 2a)  $3$ ; b)  $-6$ ; c)  $-8$ ; d) *undefined*; 3a)  $-2$ ; b)  $3$ ; c)  $0$ ; 4a)  $65$ ; b)  $-7$ ; c)  $-\$855$ ; d)  $-28^\circ\text{F}$

## Mini-Lecture 2.5

### Order of Operations

#### Learning Objectives:

1. Simplify expressions by using the order of operations.
2. Evaluate an algebraic expression.
3. Find the average of a list of numbers.

#### Examples:

1. Simplify.

a)  $-2 + 5 \cdot 6$

b)  $-2 - 5(5 - 8)$

c)  $2(-5)(7 - 3) - 7$

d)  $80 \div (-8) - 15$

e)  $3^3 - 8(2)$

f)  $8 - 2(7 - 2^2) + 3$

g)  $8^2 - 2(6) + 45 \div 5$

h)  $3(-2) + (8 - 10)^2$

i)  $21 \div [7 \cdot (-15 \div (-5))]$

j)  $\frac{8(-2) - 4 + 3}{-85 \div 5}$

k)  $\frac{[-36 \div (-4) - 1]}{[2 - (-2)]}$

l)  $\frac{20(-1) - (-5)(-2)}{3[-12 \div (-3 - 3)]}$

2. Evaluate each expression for  $x = -3$ ,  $y = 6$ , and  $z = -1$ .

a)  $x + y + z$

b)  $2y - 3z + x$

c)  $x^2 - y + z$

d)  $\frac{8x}{2y}$

e)  $5y - x^2$

f)  $x^3 + yz$

3. Find the average of each list of numbers.

a)  $-20, -9, -1, 0, 4, 6, 6$

b)  $-50, -30, -15, -5$

#### Teaching Notes:

- Many students confuse the addition/subtraction rules with the multiplication/division rules when working with many operations in one expression.
- Encourage students to perform one operation at a time.
- Refer students to ***Order of Operations*** in the textbook.

*Answers:* 1a) 2, b) 13, c) -47, d) -25, e) 11, f) 5, g) 61, h) -2, i) 1, j) 1, k) 2, l) -5; 2a) 2, b) 12, c) 2, d) -2, e) 21, f) -31; 3a) -2, b) -25

## Mini-Lecture 2.6

### Solving Equations: The Addition and Multiplication Properties

#### Learning Objectives:

1. Identify solutions of equations.
2. Use the addition property of equality to solve equations.
3. Use the multiplication property of equality to solve equations.

**Key Vocabulary:** *equation, expression, multiplication, solution, addition and equivalent*

#### Examples:

1. Decide whether the given number is a solution of the given equation.
  - a) Is 12 a solution of  $x + 3 = 15$
  - b) Is 8 a solution of  $z - 15 = 23$
  - c) Is  $-2$  a solution of  $4k = k - 6$
  - d) Is 5 a solution of  $6(x - 2) = 3x + 1$
  - e) Is  $\frac{1}{2}$  a solution of  $-3x = 5x + 1$
  - f) Is  $-2$  a solution of  $-2x + 5 = 6x - 5x + 7$
2. Solve. Check each solution.
  - a)  $a + 7 = 25$
  - b)  $d - 4 = -19$
  - c)  $10z = 9z - 13$
  - d)  $-14 = 15 + x$
3. Solve. Check each solution.
  - a)  $3x = 18$
  - b)  $\frac{x}{-5} = 5$
  - c)  $-5y = 0$
  - d)  $-20x = -20$

#### Teaching Notes:

- Encourage students to write down all steps in a neat, organized manner. This habit will help students as equations increase in difficulty.
- Encourage students to use the addition property in such a way that the variable ends up with a positive coefficient.
- Mention to students that it does not matter on which side of the equation you isolate the variable.
- Remind students to always check their final answer by substituting it back into the original equation.

Answers: 1a) yes, b) no, c) yes, d) no, e) no, f) no; 2a) 18, b)-15, c) -13, d)-29; 3a) 6, b)-25, c) 0, d) 1

## Mini-Lecture 3.1

### Simplifying Algebraic Expressions

#### Learning Objectives:

1. Use properties of numbers to combine like terms.
2. Use properties of numbers to multiply expressions.
3. Simplify expressions by multiplying and then combining like terms.
4. Find the perimeter and area of figures.

**Key Vocabulary:** *algebraic expression, constant, variable, numerical coefficient, like terms, distributive property, simplify*

#### Examples:

1. Simplify each expression by combining like terms.

a)  $3x + 5x$

b)  $11y - 9y$

c)  $5a - 19a$

d)  $6z + 15z - 5z + 7$

e)  $4.2 + 8.7x - 1.9 - 3.3x$

f)  $\frac{4}{5}x - \frac{2}{3} + \frac{1}{3}x - \frac{1}{5}$

2. Multiply.

a)  $7(4x)$

b)  $-12(6a)$

c)  $\frac{2}{5}(-15x)$

d)  $-5(3y - 2)$

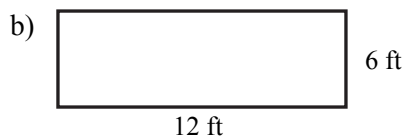
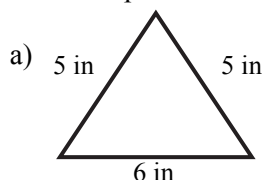
3. Simplify each expression. Use the distributive property to remove parentheses.

a)  $5(y + 3) - 6$

b)  $2(7 - 3a) + a$

c)  $-3(x + 1) + 4(8 - x) - 18$

4. Find the perimeter of the figures.



#### Teaching Notes:

- Some students need to practice identifying “like terms”.
- Some students do not know that a variable without a numerical coefficient actually has a coefficient of 1.
- Many students tend to make careless errors associated with the distributive property.
- Remind students that perimeter is the distance around an object.

*Answers:* 1a)  $8x$ , b)  $2y$ , c)  $-14a$ , d)  $16z + 7$ , e)  $5.4x + 2.3$ , f)  $\frac{17}{15}x - \frac{13}{15}$ ; 2a)  $28x$ , b)  $-72a$ , c)  $-6x$ , d)  $-15y + 10$ ; 3a)  $5y + 9$ , b)  $14 - 5a$ , c)  $-7x + 11$ ; 4a)  $16\text{in}$ , b)  $36\text{ft}$

## Mini-Lecture 3.2

### Solving Equations: Review of the Addition and Multiplication Properties

#### Learning Objectives:

1. Use the addition property or the multiplication property to solve equations.
2. Use both properties to solve equations.
3. Translate word phrases to mathematical expressions.

#### Key Vocabulary: *expression, equation*

#### Examples:

1. Solve. Check each solution.

a)  $-14 = 15 + x$

b)  $3y - 7y = 12$

c)  $\frac{x}{4} = 11 - 5$

d)  $5x + 2 - 4x = 7 - 19$

e)  $3(3x - 5) = 10x$

f)  $13x = 4(3x - 1)$

2. Solve. Both properties.

a)  $5y + 2 = 17$

b)  $3x - 5 = 10$

c)  $-4(x + 2) - 60 = -8$

d)  $9 - 14 = \frac{x}{-12}$

3. Write each phrase as a variable expression. Use  $x$  to represent "a number."

- a) Eight subtracted from a number
- b) The product of a number and 5
- c) The quotient of a number and negative 7
- d) The total of twice a number and 3

#### Teaching Notes:

- Encourage students to write down all steps in a neat, organized manner. This habit will help students as equations increase in difficulty.
- Encourage students to use the addition property in such a way that the variable ends up with a positive coefficient.
- Mention to students that it does not matter on which side of the equation you isolate the variable.
- Remind students to always check their final answer by substituting it back into the original equation.

*Answers:* 1a) -29, b)-3, c)24, d)-14, e)-15, f)-4; 2a)3, b)152, c) -15, d)60; 3a) $x - 8$ , b) $5x$ , c) $x/-7$ , d) $2x + 3$



## Mini-Lecture 3.3

### Solving Linear Equations in One Variable

#### Learning Objectives:

1. Solve linear equations using the addition and multiplication properties.
2. Solve linear equations containing parentheses.
3. Write numerical sentences as equations.

#### Examples:

1. Solve each equation. Remember to check your answer by substitution.

a)  $2x - 20 = 0$

b)  $3p + 5 = 4p + 11$

c)  $6y + 21 = 5y + 9$

d)  $10z = 7z + 10 + 2z$

e)  $-2a + 24 = -8a - 6a$

f)  $40 - 5y + 5 = -2y - 10 - 4y$

2. Solve each equation. Remember to check your answer by substitution.

a)  $5(y + 5) = 6(y - 8)$

b)  $3(y + 5) = 4(y - 4)$

c)  $4(2x - 4) = 7(x + 5)$

d)  $6(2a - 3) = 9(a + 4)$

e)  $-7y + 6(-3y - 7) = -64 - 3y$

f)  $6b + 5(-3b - 2) = -12 - 7b$

g)  $-2(8y - 6) - 2(-7y - 3) = -8$

h)  $5(2z - 2) = 9(z + 5)$

3. Write each sentence as an equation.

a) The sum of  $-57$  and  $49$  is  $-8$ .

b) The difference of negative  $31$  and  $15$  is negative  $46$ .

c) The quotient of  $-10$  and  $2$  amounts to  $-5$ .

#### Teaching Notes:

- Encourage students to write out each step rather than doing it in their head.
- Remind students that it does not matter which side you isolate the variable.
- Caution students to take their time using the distributive property.
- Refer students to the textbook for **Steps for Solving an Equation**.
- Refer students to the textbook for **Key Words or Phrases** chart.
- Remind students to always check their final answer by substituting it back into the original equation.

*Answers:* 1a) 10, b)  $-6$ , c)  $-12$ , d) 10, e)  $-2$ , f)  $-55$ ; 2a) 73, b) 31, c) 51, d) 18, e) 1, f) 1, g) 13, h) 55;  
3a)  $-57 + 49 = -8$ , b)  $-31 - 15 = -46$ , c)  $-10/2 = -5$

## Mini-Lecture 3.4

### Linear Equations in One Variable and Problem Solving

#### **Learning Objectives:**

1. Write sentences as equations.
2. Use problem-solving steps to solve problems.

**Key Vocabulary:** *sentence* → *equation*

#### **Examples:**

1. Write each sentence as an equation. Use  $x$  to represent “a number”. Do not solve.
  - a) A number added to  $-12$  equals  $15$ .
  - b) Two subtracted from a number amounts to  $55$ .
  - c) Ten subtracted from ten times a number is equal to  $150$ .
  - d) The product of a number and  $-4$  is twice the sum of the number and  $2$ .
  - e) The quotient of  $10$  and a number is  $130$ .
2. Translate each to an equation and solve the resulting equation.
  - a) Six times a number yields  $36$ . Find the number.
  - b) A number subtracted from  $16$  amounts to the quotient of  $42$  and  $6$ . Find the number.
  - c) The difference of  $-6$  times some number and  $12$  gives  $-8$  times the sum of the number and  $-8$ . Find the number.
  - d) A Ford Taurus is traveling three times as fast as a Honda CRV. If their combined speed is  $96$  miles per hour, find the speed of each car.

#### **Teaching Notes:**

- Refer students to **Key Words and Phrases** chart.
- Refer students to **Problem-Solving Steps** chart.
- Remind students that a phrase is translated into an expression; a sentence is translated into an equation.
- Many students have difficulty translating words into mathematical symbols. This section will be a challenge to most students.

*Answers:* 1a)  $x + (-12) = 15$ , b)  $x - 2 = 55$ , c)  $10x - 10 = 150$ , d)  $-4x = 2(x+2)$ , e)  $10x = 130$ ; 2a)  $6x + 36, x = 6$ , b)  $16 - x = 42/6, x = 9$ , c)  $-6x - 12 = -8[x + (-8)], x = 38$ , d)  $x + 3x = 96$ ; Ford =  $72$  mph, Honda =  $24$  mph

# Mini-Lecture 4.1

## Introduction to Fractions and Mixed Numbers

### Learning Objectives:

1. Identify the numerator and the denominator of a fraction.
2. Write a fraction to represent parts of figures or real-life data.
3. Graph fractions on a number line.
4. Review division properties for 0 and 1.
5. Write mixed numbers as improper fractions.
6. Write improper fractions as mixed numbers or whole numbers.

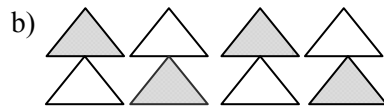
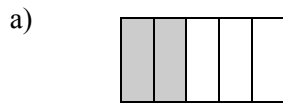
**Key Vocabulary:** *fractions, numerator, denominator, proper fraction, improper fraction, mixed number*

### Examples:

1. Identify the numerator and the denominator of a fraction.

a)  $\frac{3}{7}$                       b)  $\frac{12}{13}$                       c)  $\frac{10}{7}$                       d)  $\frac{13}{13}$

2. Represent the shaded part and unshaded part of each figure as a fraction..



Draw and shade a part of a diagram to represent each fraction.

c)  $\frac{1}{6}$  of a diagram

d)  $\frac{5}{9}$  of a diagram.

Write a fraction to represent the following information.

- e) Of the 207 students taking Basic Mathematics, 143 are freshman. What fraction of the class is freshman?
3. Graph each fraction on a number line.

a)  $\frac{1}{4}$                       b)  $\frac{8}{3}$                       c)  $\frac{9}{7}$                       d)  $\frac{3}{5}$

4. Simplify.

a)  $\frac{4}{4}$                       b)  $\frac{-7}{1}$                       c)  $\frac{0}{2}$                       d)  $\frac{-12}{0}$

5. Write each mixed number as an improper fraction.

a)  $3\frac{1}{2}$                       b)  $2\frac{8}{9}$                       c)  $13\frac{2}{9}$                       d)  $103\frac{3}{11}$

6. Write each improper fraction as a mixed number or a whole number.

a)  $\frac{16}{3}$                       b)  $\frac{38}{5}$                       c)  $\frac{156}{12}$                       d)  $\frac{159}{143}$

### Teaching Notes:

- Students need to have a firm grasp of fraction vocabulary before continuing.
- Many students confuse  $\frac{0}{x}$  with  $\frac{x}{0}$ . Be sure to stress the difference.
- Many students can write a fraction to represent a real-life situation, but they do not truly understand the meaning.

*Answers:* 1a)  $n=3, d=7$ ; b)  $n=12, d=13$ ; c)  $n=10, d=7$ ; d)  $n=13, d=13$ ; 2a)  $\frac{2}{5}, \frac{3}{5}$ ; b)  $\frac{4}{8}, \frac{4}{8}$ ; 2c – 2d) diagrams will vary; e)  $\frac{143}{207}$ ; 3a)–3d) see mini-lecture graphing answers 4a) 1; 4b)  $-7$ ; 4c) 0; 4d) undefined; 5a)  $\frac{7}{2}$ ; 5b)  $\frac{26}{9}$ ; 5c)  $\frac{119}{9}$ ; 5d)  $\frac{1136}{11}$ ; 6a)  $5\frac{1}{3}$ ; 6b)  $7\frac{3}{5}$ ; 6c) 13; 6d)  $1\frac{16}{143}$

## Mini-Lecture 4.2

### Factors and Simplest Form

#### Learning Objectives:

1. Write a number as a product of prime numbers.
2. Write a fraction in simplest form.
3. Determine whether two fractions are equivalent.
4. Solve problems by writing fractions in simplest form.

**Key Vocabulary:** *factor, prime factorization, prime numbers, composite number, simplest form, lowest terms*

#### Examples:

1. Write the prime factorization of each number.

a) 30                      b) 75                      c) 170                      d) 360

2. Write each fraction in simplest form.

a)  $\frac{10}{16}$                       b)  $\frac{36}{63}$                       c)  $\frac{77}{88}$                       d)  $-\frac{12}{42}$

e)  $\frac{11}{34}$                       f)  $\frac{-27}{36}$                       g)  $\frac{30}{80}$                       h)  $\frac{6}{-105}$

3. Determine whether each pair of fractions is equivalent.

a)  $\frac{5}{10}$  and  $\frac{11}{22}$                       b)  $\frac{7}{21}$  and  $\frac{8}{24}$                       c)  $\frac{2}{7}$  and  $\frac{8}{15}$                       d)  $\frac{6}{0}$  and  $\frac{0}{6}$

4. Solve. Write each fraction in simplest form.

a) Alicia was scheduled to work 6 hours at the tanning salon. What fraction of Alicia's shift is represented by 4 hours?

b) There are 36 inches in a yard. What fraction of a yard is represent by 9 inches?

c) There are 140 students in a freshman lecture class. If 16 students are absent, what fraction of the students are absent?

#### Teaching Notes:

- Many students will understand equivalent fractions if they are shown drawings.
- Some students will confuse cross products and simplifying. Stress that cross products is only a check to determine equality of fractions.
- Some students prefer to reduce fractions by factoring the numerator and denominator as products of prime numbers, then canceling all common factors. Others prefer to repeatedly divide the numerator and the denominator by a common factor.

*Answers:* 1a)  $2 \cdot 3 \cdot 5$ , b)  $3 \cdot 5^2$ , c)  $2 \cdot 5 \cdot 17$ , d)  $2^3 \cdot 3^2 \cdot 5$ ; 2a)  $5/8$ , b)  $4/7$ , c)  $7/8$ , d)  $-2/7$ , e) cannot be simplified, f)  $-3/4$ , g)  $3/8$ , d)  $-2/35$ ; 3a) yes, b) yes, c) no, d) no; 4a)  $2/3$ ; b)  $1/4$ ; c)  $4/35$

## Mini-Lecture 4.3

### Multiplying and Dividing Fractions

#### Learning Objectives:

1. Multiply fractions.
2. Evaluate exponential expressions with fractional bases.
3. Divide fractions.
4. Multiply and divide given fractional replacement values.
5. Solve applications that require multiplication of fractions.

#### Key Vocabulary: *reciprocal, "of"*

#### Examples:

1. Multiply. Write the product in simplest form.

a)  $\frac{1}{9} \cdot \frac{1}{7}$       b)  $\frac{2}{3} \cdot \frac{1}{4}$       c)  $-\frac{5}{6} \cdot -\frac{2}{3}$       d)  $-\frac{7}{2} \cdot \frac{6}{3}$   
e)  $\frac{5}{2} \cdot \frac{18}{15}$       f)  $\frac{7}{8} \cdot 0$       g)  $\frac{1}{2} \cdot -\frac{3}{5} \cdot \frac{1}{5}$       h)  $-\frac{12}{14} \cdot -\frac{3}{9} \cdot -\frac{2}{10}$

2. Evaluate.

a)  $\left(\frac{1}{2}\right)^2$       b)  $\left(-\frac{1}{3}\right)^4$       c)  $\left(-\frac{2}{5}\right)^3$       d)  $\left(\frac{2}{7}\right)^2 \cdot \frac{1}{4}$

3. Divide. Write all quotients in simplest form.

a)  $\frac{3}{5} \div \frac{4}{7}$       b)  $\frac{1}{4} \div \frac{1}{4}$       c)  $-\frac{1}{5} \div \frac{9}{19}$       d)  $\frac{8}{17} \div -\frac{12}{15}$   
e)  $-\frac{2}{17} \div -\frac{3}{17}$       f)  $\frac{1}{14} \div 0$       g)  $\frac{27}{-7} \div \frac{4}{7}$       h)  $0 \div -\frac{3}{11}$

4. Given the following replacement values, evaluate (a)  $xy$  and (b)  $x \div y$ .

a)  $x = -\frac{1}{3}$  and  $y = \frac{4}{9}$       b)  $x = \frac{5}{7}$  and  $y = -\frac{5}{9}$

5. Solve. Write each answer in simplest form.

a) Find  $\frac{1}{3}$  of 48.      b) Find  $\frac{3}{7}$  of  $-63$   
c) A bike trail is 27 miles long. Michelle bikes  $\frac{2}{3}$  of the trail. How many miles did Michelle bike?

#### Teaching Notes:

- Encourage students to divide out common factors in the numerator and denominator before multiplying.
- When dividing, encourage students take the time and rewrite the problem by changing the division symbol to multiplication and multiply by the reciprocal. Many students begin "simplifying" and forget to multiply by the reciprocal.

*Answers:* 1a)  $1/63$ , b)  $1/6$ , c)  $5/9$ , d)  $-7$ , e)  $3$ , f)  $0$ , g)  $-3/50$ , h)  $-2/35$ ; 2a)  $1/4$ , b)  $1/81$ , c)  $-8/125$ , d)  $1/49$ , 3a)  $21/20$ , b)  $1$ , c)  $-19/45$ , d)  $10/17$ , e)  $2/3$ , f) *undefined*, g)  $-27/4$ , h)  $0$ ; 4a)  $-4/27$ ,  $-3/4$ ; b)  $-25/63$ ,  $-9/7$ ; 5a)  $16$ , b)  $-27$ , c)  $18$

## Mini-Lecture 4.4

### Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions

#### Learning Objectives:

1. Add or subtract like fractions.
2. Add or subtract given fractional replacement values.
3. Solve problems by adding or subtracting like fractions.
4. Find the least common denominator of a list of fractions.
5. Write equivalent fractions.

**Key Vocabulary:** *like fractions, unlike fractions, (LCD) least common denominator, (LCM) least common multiple, equivalent fractions*

#### Examples:

1. Add and simplify.

a)  $\frac{1}{9} + \frac{4}{9}$       b)  $\frac{1}{8} + \frac{5}{8}$       c)  $-\frac{1}{10} + \frac{9}{10}$       d)  $-\frac{3}{14} + \left(-\frac{5}{14}\right)$

Subtract and simplify.

e)  $\frac{6}{8} - \frac{3}{8}$       f)  $-\frac{6}{21} - \frac{5}{21}$       g)  $\frac{25}{42} - \left(-\frac{7}{42}\right)$       h)  $-\frac{28}{13} - \left(-\frac{5}{13}\right)$

2. Evaluate each expression if  $x = \frac{3}{5}$  and  $y = -\frac{1}{5}$ .

a)  $x + y$       b)  $x \cdot y$       c)  $x - y$       d)  $x \div y$

3. Solve.

a) Find the perimeter of a triangle with sides:  $\frac{6}{25}$  inch,  $\frac{9}{25}$  inch, and  $\frac{5}{25}$  inch.

b) Cori read  $\frac{2}{11}$  of her book on Friday,  $\frac{3}{11}$  of her book on Saturday, and  $\frac{4}{11}$  of her book on Sunday.

What part of her book has she read?

4. Find the LCM of each list of numbers.

a) 10, 12      b) 12, 14      c) 30, 35      d) 30, 20, 50

5. Write each fraction as an equivalent fraction with the given denominator.

a)  $\frac{4}{9} = \frac{\quad}{18}$       b)  $\frac{7}{11} = \frac{\quad}{55}$       c)  $\frac{5}{2} = \frac{\quad}{6}$       d)  $\frac{2}{3} = \frac{\quad}{24}$

#### Teaching Notes:

- Many students add or subtract both numerator and denominator.
- When subtracting, some students may need to take the intermediate step of writing out the operations performed on the numerators. For example:  $-\frac{1}{7} - \left(-\frac{3}{7}\right) = -\frac{1}{7} + \left(\frac{3}{7}\right) = \frac{-1+3}{7} = \frac{2}{7}$ .
- Some students forget to multiply the numerator when building equivalent fractions.

*Answers:* 1a)  $\frac{5}{9}$ , b)  $\frac{3}{4}$ , c)  $\frac{4}{5}$ , d)  $-\frac{4}{7}$ , e)  $\frac{3}{8}$ , f)  $-\frac{11}{21}$ , g)  $\frac{16}{21}$ , h)  $-\frac{23}{13}$ ; 2a)  $\frac{2}{5}$ , b)  $-\frac{3}{25}$ , c)  $\frac{4}{5}$ , d)  $-3$ ; 3a)  $\frac{4}{5}$  inch, b)  $\frac{9}{11}$ , 4a) 60, b) 84, c) 210, d) 300; 5a) 8, b) 35, c) 15, d) 16

## Mini-Lecture 4.5

### Adding and Subtracting Unlike Fractions

#### Learning Objectives:

1. Add or subtract unlike fractions.
2. Write fractions in order.
3. Evaluate expressions given fractional replacement values.
4. Solve problems by adding or subtracting unlike fractions.

#### Key Vocabulary: *least common denominator (LCD)*

#### Examples:

1. Add or subtract as indicated.

a)  $\frac{1}{10} + \frac{2}{5}$

b)  $-\frac{1}{5} + \left(-\frac{2}{25}\right)$

c)  $\frac{1}{7} + \left(-\frac{9}{10}\right)$

d)  $\frac{3}{5} + \frac{1}{20}$

e)  $\frac{4}{5} - \frac{3}{20}$

f)  $\frac{7}{9} - \left(-\frac{1}{12}\right)$

g)  $-\frac{5}{7} - \left(-\frac{1}{2}\right)$

h)  $\frac{1}{20} - \frac{8}{15}$

i)  $-\frac{7}{5} + \frac{8}{16} + \frac{4}{20}$

2. Insert  $<$  or  $>$  to form a true sentence.

a)  $\frac{2}{3}$  \_\_\_\_\_  $\frac{1}{9}$

b)  $\frac{5}{12}$  \_\_\_\_\_  $\frac{1}{2}$

c)  $-\frac{5}{6}$  \_\_\_\_\_  $-\frac{4}{5}$

3. Evaluate each expression if  $x = \frac{1}{4}$  and  $y = -\frac{3}{5}$ .

a)  $x + y$

b)  $x \cdot y$

c)  $x - y$

d)  $x \div y$

4. Solve.

a) Find the perimeter of a rectangle with width  $\frac{3}{4}$  feet and length  $\frac{3}{14}$  feet.

b) Sharon is making matching holiday outfits for her three children. Each outfit required  $\frac{7}{8}$  yards.

How many yards of material will be needed to make the three outfits?

#### Teaching Notes:

- Refer students back to Section 3.4: *Method 1: Finding the LCM of a List of numbers Using Multiples of the Largest Number* and *Method 2: Finding the LCM of a List of Numbers Using Prime Factorization*.
- Some students try to cross-cancel when adding or subtracting.
- Some students add and subtract both the numerator and denominator.
- Some students forget to multiply the numerator when building equivalent fractions.

*Answers:* 1a)  $\frac{1}{2}$ , b)  $-\frac{7}{25}$ , c)  $-\frac{53}{70}$ , d)  $\frac{13}{20}$ , e)  $\frac{13}{20}$ , f)  $\frac{31}{36}$ , g)  $-\frac{3}{14}$ , h)  $-\frac{29}{60}$ , i)  $-\frac{7}{10}$ ; 2a)  $>$ , b)  $<$ , c)  $<$ ; 3a)  $-\frac{7}{20}$ , b)  $-\frac{3}{20}$ , c)  $\frac{17}{20}$ , d)  $-\frac{5}{12}$ ; 4a)  $\frac{27}{14}$  feet, b)  $\frac{21}{8}$  yd

## Mini-Lecture 4.6

### Complex Fractions and Review of Order of Operations

#### Learning Objectives:

1. Simplify complex fractions.
2. Review the order of operations.
3. Evaluate expressions given replacement values.

#### Key Vocabulary: *complex fraction*

#### Examples:

1. Simplify each complex fraction.

a)  $\frac{\frac{1}{6}}{\frac{2}{3}}$

b)  $\frac{\frac{16}{7}}{\frac{8}{7}}$

c)  $\frac{\frac{1}{6} + \frac{1}{2}}{\frac{1}{3} + \frac{3}{4}}$

2. Use the order of operations to simplify each expression.

a)  $\frac{1}{4} + \frac{1}{4} \cdot \frac{1}{3}$

b)  $\frac{3}{2} \div \left( \frac{7}{8} + \frac{7}{16} \right)$

c)  $\left( \frac{2}{7} + \frac{3}{14} \right) \left( \frac{2}{7} - \frac{3}{14} \right)$

d)  $\left( \frac{2}{5} \right)^2 \cdot \frac{1}{2}$

e)  $\frac{1}{2} + \left( \frac{2}{3} \right)^2 - \frac{1}{3}$

f)  $\frac{2}{3} \cdot \left( \frac{1}{4} + \frac{1}{2} \right) \cdot 6$

3. Evaluate each expression if  $x = -\frac{1}{2}$ ,  $y = \frac{3}{5}$ , and  $z = \frac{7}{10}$ .

a)  $3x - z$

b)  $\frac{y}{z}$

c)  $x^2 + 2y$

d)  $\frac{x+y}{z}$

#### Teaching Notes:

- Many students make careless errors when using Method 2 for simplifying complex fractions. If this is the case, encourage students to use Method 1 (rewrite as a division problem).
- Remind students that when dividing fractions, you must change division to multiplication and multiply by the reciprocal.
- Some students will try to apply procedures for simplifying complex fractions to adding and subtracting fractions.

Answers: 1a)  $\frac{1}{4}$ , b) 2, c)  $\frac{8}{13}$ ; 2a)  $\frac{1}{3}$ , b)  $\frac{8}{7}$ , c)  $\frac{1}{28}$ , d)  $\frac{2}{25}$ , e)  $\frac{11}{18}$ , f) 3; 3a)  $-\frac{11}{5}$ , b)  $\frac{6}{7}$ , c)  $\frac{29}{20}$ , d)  $\frac{1}{7}$



## Mini-Lecture 4.7

### Operations On Mixed Numbers

#### Learning Objectives:

1. Graph positive and negative fractions and mixed numbers.
2. Multiply or divide mixed or whole numbers.
3. Add or subtract mixed numbers.
4. Solve problems containing mixed numbers.
5. Perform operations on negative mixed numbers.

#### Examples:

1. Graph each list of numbers on a number line.

a)  $-3, -3\frac{1}{2}, -1, \frac{3}{4}, 2$

b)  $4, -3\frac{3}{4}, 0, 1\frac{1}{5}, -\frac{1}{2}$

2. Multiply or divide.

a)  $2\frac{2}{3} \cdot \frac{1}{2}$

b)  $3\frac{3}{4} \cdot 1\frac{3}{5}$

c)  $4\frac{1}{5} \div \frac{1}{5}$

d)  $3\frac{1}{3} \div 2\frac{3}{5}$

3. Add or subtract.

a)  $10\frac{1}{2} + 7\frac{1}{9}$

b)  $6\frac{1}{3} + 13\frac{4}{9}$

c)  $4\frac{2}{3} + 9\frac{7}{9}$

d)  $8\frac{1}{3} + 2\frac{2}{3} + 3\frac{2}{9}$

e)  $15\frac{8}{9} - 6\frac{2}{9}$

f)  $19\frac{1}{25} - 7\frac{1}{5}$

g)  $17\frac{1}{6} - 5\frac{13}{24}$

h)  $13 - 6\frac{5}{9}$

4. Solve. Write answer in simplest form.

a) Amy rode her bicycle  $9\frac{4}{15}$  miles on each of 9 days. What is the total distance Amy rode?

b) John cuts a board  $13\frac{3}{7}$  feet long from one 20 feet long. How long is the remaining piece?

5. Perform the indicated operation.

a)  $-5\frac{7}{8} \div 5\frac{1}{4}$

b)  $-7\frac{7}{9} + \left(-4\frac{2}{9}\right)$

c)  $-8\frac{1}{3} \cdot \left(-1\frac{1}{5}\right)$

d)  $5\frac{5}{8} \div (-9)$

e)  $19\frac{3}{5} - \left(-5\frac{18}{20}\right)$

f)  $\left(-15\frac{3}{7}\right) + 14\frac{1}{5}$

g)  $10\frac{1}{9} + \left(-5\frac{5}{9}\right)$

h)  $-9 \cdot \left(5\frac{7}{12}\right)$

#### Teaching Notes:

- Most students forget that mixed numbers must be changed to improper fractions before multiplying. Some try to multiply the whole number parts together, and then multiply the fractional parts together.
- Many students confuse the rules for multiplication with adding/subtracting rules.
- Many students are challenged by word problems. Students have trouble deciding which operation to use for the word problems.

*Answers:* 1a)  1b)  2a) 4/3, b) 6, c) 21, d) 1 11/3;

3a) 17 11/18; 3b) 19 7/9; 3c) 14 4/9; 3d) 14 2/9; 3e) 9 2/3; 3f) 11 21/25; 3g) 11 5/8; 3h) 6 4/9; 4a) 83 2/5; 4b) 6 4/7; 5a) -47/42; 5b) -12; 5c) 10; 5d) -5/8; 5e) 25 1/2; 5f) -1 8/35; 5g) 4 5/9; 5h) -50 1/4

## Mini-Lecture 4.8

### Solving Equations Containing Fractions

#### **Learning Objectives:**

1. Solve equations containing fractions.
2. Solve equations by multiplying by the LCD.
3. Review adding and subtracting fractions.

#### **Examples:**

1. Solve each equation. Check your proposed solution.

a.  $x + \frac{1}{2} = -\frac{1}{2}$       b.  $z - \frac{4}{15} = \frac{3}{15}$       c.  $-\frac{3}{8} = x - \frac{5}{6}$

d.  $8x - \frac{3}{5} - 7x = \frac{11}{15}$

2. Solve each equation.

a.  $9x = 4$       b.  $\frac{1}{3}x = 5$       c.  $-\frac{3}{8}x = -\frac{4}{5}$

d.  $-8a = \frac{16}{25}$

3. Solve each equation.

a.  $\frac{3}{5}y = -\frac{7}{25}$       b.  $\frac{x}{7} - x = -5$       c.  $\frac{b}{3} = \frac{b}{5} + \frac{7}{3}$

4. Add or subtract as indicated.

a.  $\frac{n}{2} + \frac{4}{7}$       b.  $\frac{5c}{8} - \frac{c}{4}$

#### **Teaching Notes:**

- Emphasize checking proposed solutions.
- When adding or subtracting fractions, the denominators need to be the same.
- Review properties:      Addition Property of Equality  
  Multiplication Property of Equality
- Make sure students understand the difference between solving an equation containing fractions (multiply both sides of the equation by the LCD of the fractions) and adding or subtracting two fractions (create equivalent fractions).

Answers: 1a)-1, b)  $\frac{7}{15}$ , c)  $\frac{11}{24}$ , d)  $\frac{4}{3}$ ; 2a)  $\frac{4}{9}$ , b) 15, c)  $\frac{32}{15}$ , d)  $-\frac{2}{25}$ ; 3a)  $-\frac{7}{15}$ , b)  $\frac{35}{6}$ , c)  $\frac{35}{2}$ ; 4a)  $\frac{7n+8}{14}$ ,

b)  $\frac{3c}{8}$

# Mini-Lecture 1.1

## Study Skill Tips for Success in Mathematics

### **Learning Objectives:**

1. Get ready for this course.
2. Understand some general tips for success.
3. Know how to use this text.
4. Know how to use video and notebook organizer resources.
5. Get help as soon as you need it.
6. Learn how to prepare for and take an exam.
7. Develop good time management.

### **Examples:**

1. Get ready for this course.
  - a) Positive attitude
  - b) Be familiar with course structure
  - c) Avoid schedule conflicts
  - d) Allow adequate time for class arrival
  - e) Bring all required materials
2. Understand some general tips for success.
  - a) Organize materials
  - b) Make contact with other students
  - c) Choose to attend all classes
  - d) Do your homework
  - e) Check your work
  - f) Learn from mistakes
  - g) Ask questions
  - h) Hand in assignments on time
3. Know how to use this text.
  - a) Each example in every section has a Practice exercise associated with it.
  - b) At beginning of each section, a list of icons shows availability of support materials.
  - c) Each chapter ends with Chapter Highlights, Reviews, and Practice Tests.
4. Know how to use video and notebook organizer resources.
  - a) Video resources include interactive lectures, test prep, and student success tips.
  - b) Notebook organizer resources include video and student organizers
5. Get help as soon as you need it.
6. Learn how to prepare for and take an exam.
  - a) Review previous homework assignments, class notes, quizzes, etc.
  - b) Read Chapter Highlights to review concepts and definitions.
  - c) Practice working out exercises in the end-of-the-chapter Review and Test.
  - d) When taking a test, read directions and problems carefully.
7. Develop good time management.
  - a) Make a list of all weekly commitments with estimated time needed.
  - b) Be sure to schedule study time. Don't forget eating, sleeping, and relaxing!

### **Teaching Notes:**

- Many developmental students are hesitant to ask questions and seek extra help.
- Be sure to include your expectations. Keep your expectations clear and concise.

## Mini-Lecture 1.2

### Place Value, Names for Numbers, and Reading Tables

#### Learning Objectives:

1. Find the place value of a digit in a whole number.
2. Write a whole number in words and in standard form.
3. Write a whole number in expanded form.
4. Read tables.
5. Key Vocabulary: *whole numbers, place value, standard form, period, expanded form, tables*

#### Examples:

1. Find the place value of the digit 7 in each whole number.  
a) 7,352            b) 607            c) 702,433            d) 17,009,321
2. Write each whole number in words.  
a) 62            b) 698            c) 17,403            d) 1,067,599

Write each number in standard form.

- e) nine hundred fifty-two
  - f) three hundred sixty-two thousand, five hundred eighty-six
  - g) three million, four hundred thousand, one hundred two
3. Write each number in expanded form.  
a) 398            b) 2,907            c) 4,089,347
  4. Use the following table of Number of Students Enrolled to answer the questions.

Subject	Section 1	Section 2	Section 3	Total
Basic Mathematics	23	27	19	69
Statistics	20	25	22	67

- a) How many total students are enrolled in Basic Mathematics?
- b) How many students are enrolled in Section 3 of Statistics?

#### Teaching Notes:

- Students who do not have English as their first language will need additional assistance learning place value vocabulary.
- Students who do not have English as their first language may use periods instead of commas in writing numbers.

*Answers: 1a) thousands, b) ones, c) hundred thousands, d) ten million; 2a) sixty-two, b) six hundred ninety-eight, c) seventeen thousand, four hundred three, d) one million, sixty-seven thousand, five hundred ninety-nine, e) 952, f) 362,586, g) 3,400,102; 3a)  $300+90+8$ , b)  $2000+900+7$ , c)  $4,000,000+80,000+9,000+300+40+7$ ; 4a) 69, b) 22*

## Mini-Lecture 1.3

Adding Whole Numbers and Perimeter

### Learning Objectives:

1. Add whole numbers.
2. Find the perimeter of a polygon.
3. Solve problems by adding whole numbers.
4. Key Vocabulary: *sum, addend, commutative property of addition, associate property of addition, polygon, perimeter*

### Examples:

1. Add.

a)  $3 + 9$

b)  $40 + 70$

c)  $1900 + 17$

d)  $5703 + 0$

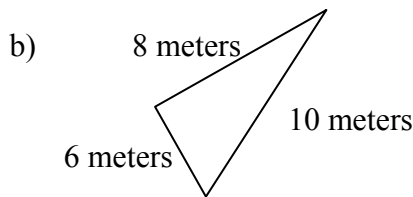
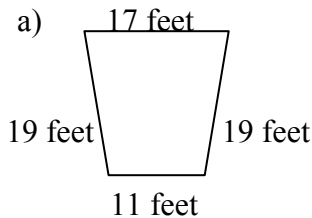
e) 
$$\begin{array}{r} 51 \\ +27 \\ \hline \end{array}$$

f) 
$$\begin{array}{r} 7329 \\ + 683 \\ \hline \end{array}$$

g)  $93 + 145 + 69$

h) 
$$\begin{array}{r} 6,403 \\ + 793 \\ \hline 17,187 \end{array}$$

2. Find the perimeter of each figure.



3. Solve the following word problems.

a) What is the sum of 8,932 and 14,799?

b) What is 830 plus 4,562 plus 88?

c) On Monday, Karen drove 57 miles; on Tuesday, she drove 39 miles; and on Wednesday, Karen drove 92 miles. How many total miles did Karen drive?

### Teaching Notes:

- Some students need additional practice with basic addition facts.
- Remind students that it is acceptable to write the carry digit in order to obtain the correct answer.
- Most students will find this section easy but may need assistance with word problems.

*Answers: 1a) 12, b) 110, c) 1917, d) 5703, e) 78, f) 8012, g) 307, h) 24,383; 2a) 66 ft., b) 24 m.; 3a) 23,731, b) 5,480, c) 188 miles*

## Mini-Lecture 1.4

### Subtracting Whole Numbers

#### Learning Objectives:

1. Subtract whole numbers.
2. Solve problems by subtracting whole numbers.
3. Key Vocabulary: *subtraction, minuend, subtrahend, difference, borrowing, regrouping, less than, take away, decreased by, subtracted from*

#### Examples:

1. Subtract. Check by adding.

a)  $11 - 7$

b)  $15 - 8$

c)  $22 - 22$

d)  $31 - 0$

e) 
$$\begin{array}{r} 27 \\ - 13 \\ \hline \end{array}$$

f) 
$$\begin{array}{r} 198 \\ - 94 \\ \hline \end{array}$$

g) 
$$\begin{array}{r} 5936 \\ - 803 \\ \hline \end{array}$$

h) 
$$\begin{array}{r} 16,395 \\ - 5,132 \\ \hline \end{array}$$

i) 
$$\begin{array}{r} 71 \\ - 32 \\ \hline \end{array}$$

j) 
$$\begin{array}{r} 712 \\ - 285 \\ \hline \end{array}$$

k) 
$$\begin{array}{r} 3004 \\ - 2965 \\ \hline \end{array}$$

l) 
$$\begin{array}{r} 20,003 \\ - 16,867 \\ \hline \end{array}$$

m)  $53,425 - 7977$

n)  $71,217 - 56,839$

o)  $60,000 - 26,386$

2. a) Find the difference of 93 and 27.  
b) Subtract 376 from 803.  
c) The Library Renovation Project has set a goal of \$75,000 to fundraise. To date, \$47,908 has been fundraised. How much more money does the Library Renovation Project need to fundraise?

#### Teaching Notes:

- Some students need additional practice with basic subtraction facts.
- Most students find subtraction without borrowing easy.
- Many students need to write the borrowing/regrouping step to maintain accuracy.
- Many students are challenged when borrowing with zeros.

Answers: 1a) 4, b) 7, c) 0, d) 31, e) 14, f) 104, g) 5,133, h) 11,263, i) 39, j) 427, k) 39, l) 3,136, m) 45,448, n) 14,378, o) 33,614; 2a) 66, b) 427, c) \$27,092

## Mini-Lecture 1.5

### Rounding and Estimating

#### Learning Objectives:

1. Round whole numbers.
2. Use rounding to estimate sums and differences.
3. Solve problems by estimating.
4. Key Vocabulary: *rounding, estimating*

#### Examples:

1. Round to the nearest ten.

a) 31                      b) 57                      c) 346                      d) 2,795

Round to the nearest hundred.

e) 312                      f) 6,658                      g) 8,672                      h) 1,899

2. Round to the nearest thousand to find the estimated sum or difference.

a)  $\begin{array}{r} 4892 \\ - 2305 \\ \hline \end{array}$                       b)  $\begin{array}{r} 2731 \\ + 3020 \\ \hline \end{array}$                       c)  $\begin{array}{r} 17,032 \\ - 12,513 \\ \hline \end{array}$                       d)  $\begin{array}{r} 24,803 \\ + 14,587 \\ \hline \end{array}$

3. a) At the last 3 dances, attendance was 657 students, 403 students, and 559 students. Estimate the total attendance by rounding each to the nearest hundred.

b) Enrollment figures at the Town of Johnson's School Department increased from 6,721 students to 7,653 students. Round each number to the nearest hundred to estimate the increase.

c) The Carlisle family needs to buy a refrigerator for \$999, a stove for \$459, and a dishwasher for \$449. Round each cost to the nearest hundred to estimate the total cost.

#### Teaching Notes:

- Some students need to be repeatedly reminded to look at the digit to the right of the rounding position. Have students draw a line after the digit in the rounding position.
- A common error students make is to leave the digits to the right of the rounding position the same instead of changing them to zeros after rounding.
- Stress the importance of rounding and estimating with applications.

*Answers: 1a) 30, b) 60, c) 350, d) 2,800, e) 300, f) 6,700, g) 8,700, h) 1,900; 2a) 3000, b) 6000, c) 4000, d) 40,000; 3a) 177, b) 1,000, c) \$1,900*

## Mini-Lecture 1.6

### Multiplying Whole Numbers and Area

#### Learning Objectives:

1. Use the properties of multiplication.
2. Multiply whole numbers.
3. Multiply by whole numbers ending in zero(s).
4. Find the area of a rectangle.
5. Solve problems by multiplying whole numbers.
6. Key Vocabulary: *multiplication sign, product, multiplication property of 0, multiplication property of 1, commutative property of multiplication, associative property of multiplication, distributive property, partial products, area*

#### Examples:

1. Multiply.

a)  $37 \cdot 1$                       b)  $1 \cdot 22$                       c)  $0 \cdot 183$                       d)  $9 \cdot 5 \cdot 0$

Use the distributive property to rewrite each expression.

e)  $2(5 + 4)$                       f)  $5(1 + 9)$                       g)  $10(9 + 6)$                       h)  $15(0 + 14)$

2. Multiply.

a) 
$$\begin{array}{r} 37 \\ \times 6 \\ \hline \end{array}$$
                      b)  $412 \times 4$                       c) 
$$\begin{array}{r} 1708 \\ \times 9 \\ \hline \end{array}$$
                      d) 
$$\begin{array}{r} 337 \\ \times 25 \\ \hline \end{array}$$

e) 
$$\begin{array}{r} 643 \\ \times 27 \\ \hline \end{array}$$
                      f) 
$$\begin{array}{r} 2,477 \\ \times 963 \\ \hline \end{array}$$

3. Multiply.

a) 
$$\begin{array}{r} 309 \\ \times 800 \\ \hline \end{array}$$
                      b)  $825 \times 1,000$

4. Find the area of a rectangle with length 14 feet and width 8 feet.
5. At a recent football game, 413 adult tickets were sold at a price of \$5 each. There were 127 child tickets sold at a price of \$3 each. How much total amount of money in ticket sales for the game?

#### Teaching Notes:

- Some students need additional practice with basic multiplication facts.
- Some students do not know the different types of symbols used for multiplication.
- When using distributive property, many students forget to distribute over both terms.
- When multiplying, remind students to carefully line up the ones, tens, hundreds, etc.

*Answers: 1a) 37, b) 22, c) 0, d) 0, e) 18, f) 50, g) 150, h) 210; 2a) 222, b) 1648, c) 15,372, d) 8,425, e) 17,361, f) 2,385,351; 3a) 247,200, b) 825,000; 4) 112 sq. ft.; 5) \$2,446*



## Mini-Lecture 1.7

### Dividing Whole Numbers

#### Learning Objectives:

1. Divide whole numbers.
2. Perform long division.
3. Solve problems that require dividing by whole numbers.
4. Find the average of a list of numbers.
5. Key Vocabulary: *division, fraction bar, quotient, divisor, dividend, division by zero, undefined, long division, remainders, divide, quotient, divided by, divided or shared equally among, average*

#### Examples:

1. Find each quotient. Check by multiplying.

a)  $3\overline{)12}$       b)  $13 \div 1$       c)  $\frac{5}{5}$       d)  $15 \div 15$       e)  $0\overline{)5}$

2. Divide. Check by multiplying.

a)  $228 \div 4$       b)  $\frac{572}{7}$       c)  $1570 \div 3$

d)  $14\overline{)7070}$       e)  $97\overline{)41,270}$       f)  $603\overline{)604,911}$

3. a) Find the quotient of 94 and 5.  
b) Recently, Amy earned \$1,722 selling calendars. If each calendar cost \$14, how many calendars did Amy sell?
4. a) During the semester, Kyle's test scores were: 87, 93, 62, 83 and 100. What was Kyle's average for the semester?

#### Teaching Notes:

- Some students need additional practice with basic division facts.
- Many students confuse division by zero (undefined) and zero divided by any non-zero number ( $= 0$ ).
- Many students need to be cautious with placement of digits in quotient and dividend. Be sure appropriate place values are lined up. Stress organization!

Answers: 1a) 4, b) 13, c) 1, d) 1, e) undefined; 2a) 57, b)  $81r5$ , c)  $523 r1$ , d) 505, e)  $425 r45$ , f)  $1003 r102$ ;  
3a)  $18 r4$ , b) 123; 4a) 85

# Mini-Lecture 1.8

## An Introduction to Problem Solving

### Learning Objectives:

1. Solve problems by adding, subtracting, multiplying, or dividing whole numbers.
2. Solve problems that require more than one operation.
3. Key Vocabulary: *understand, translate, solve, interpret*

### Examples:

1. Using Problem-Solving Steps, solve problems involving one type of operation.
  - a) Alicia owns a home that is 4,500 square feet. Anastasia owns a home that is 2,300 square feet. How much larger is Alicia's house than Anastasia's?
  - b) How many 16-ounce cans of tomato sauce can be made from a vat of tomato sauce containing 361 ounces? Will there be any sauce leftover?
  - c) Recently, Jason purchased a DVD for \$18, a CD for \$12, a set of headphones for \$19, and a magazine for \$6. How much money did Jason spend?
  - d) Last week, Tyler worked 38 hours at his job and is paid \$17/hour. How much money did Tyler earn last week?
2. Using Problem-Solving Steps, solve problems involving more than one operation.
  - a) Find the total cost of 5 calculators at \$69 each and 5 protective cases at \$8 each.
  - b) Andrea has \$1040 in her checking account. She spent \$130 at Matt's Sports Supplies, \$170 at The Pool Center, and \$80 at the grocery store. Finally, she stopped at her bank and deposited her paycheck for \$270. How much money is in Andrea's checking account?
  - c) In preparation for school, Kayla purchased two pairs of pants at \$59 each, four shirts at \$26 each, two pairs of shoes at \$69 each, and three pairs of socks at \$6 each. What was the total cost of these items?

### Teaching Notes:

- Many students have a very difficult time with word problems.
- Refer students to **Problem-Solving Steps** in the textbook.
- Encourage students to use estimation to check whether their answers are reasonable.

Answers: 1a) 2,200, b) 22, 9 oz., c) \$55, d) \$646; 2a) \$385, b) \$930, c) \$378

## Mini-Lecture 1.9

### Exponents, Square Roots, and Order of Operations

#### Learning Objectives:

1. Write repeated factors using exponential notation.
2. Evaluate expressions containing exponents.
3. Evaluate the square root of a perfect square.
4. Use the order of operations.
5. Find the area of a square.
6. Key Vocabulary: *exponential notation, exponent, base, order of operations, square root, perfect square, area of a square*

#### Examples:

1. Write using exponential notation.

a)  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$     b)  $(7)(7)(7)$     c)  $4 \cdot 4 \cdot 3 \cdot 3 \cdot 3$     d)  $5 \cdot 5 \cdot 8 \cdot 8 \cdot 5 \cdot 5$

2. Evaluate.

a)  $5^2$     b)  $7^3$     c)  $3^6$     d)  $10^4$

3. Find each square root.

a)  $\sqrt{9}$     b)  $\sqrt{49}$     c)  $\sqrt{1}$     d)  $\sqrt{225}$

4. Using order of operations, simplify.

a)  $3 \cdot 4 - 10 \div 2$     b)  $6^2 \div 3 \cdot 2$     c)  $8 \cdot 4 + \{27 \div [8 - (3 + 2)]\}$

5. a) Find the area of a square whose side measures 6 feet.

b) A square floor measures 12 feet on each side. If 1 can of floor finish can cover 16 square feet, how many cans are needed to finish this floor?

#### Teaching Notes:

- Students may confuse exponent and base.
- Many students have trouble with order of operations.
- Avoid “PEMDAS” as many students will multiply before dividing and add before subtracting.

Answers: 1a)  $2^6$ , b)  $7^3$ , c)  $3^3 \cdot 4^2$ , d)  $5^4 \cdot 8^2$ ; 2a) 25, b) 343, c) 729, d) 10,000; 3a) 3, b) 7, c) 1, d) 15; 4a) 7, b) 24, c) 41; 5a) 36 sq. ft.; b) 9 cans

## Mini-Lecture 2.1

### Introduction to Fractions and Mixed Numbers

#### Learning Objectives:

1. Identify the numerator and the denominator of a fraction and review division properties of 0 and 1.
2. Write a fraction to represent parts of figures or real-life data.
3. Identify proper fractions, improper fractions, and mixed numbers.
4. Write mixed numbers as improper fractions.
5. Write improper fractions as mixed numbers or whole numbers.
6. Key Vocabulary: *numerator, denominator, proper fraction, improper fraction, mixed number*

#### Examples:

1. Identify the numerator and the denominator of each fraction.

a)  $\frac{1}{7}$

b)  $\frac{11}{17}$

c)  $\frac{27}{13}$

d)  $\frac{12}{12}$


Simplify.

e)  $\frac{5}{1}$

f)  $\frac{8}{0}$

g)  $\frac{14}{14}$

h)  $\frac{19}{0}$

2. a) Write a fraction to represent the shaded part:   
b) Of the 18 employees at a restaurant, 11 are women. What fraction of the employees are women?
3. Determine whether each fraction is proper, improper, or a mixed number.  
a)  $\frac{4}{9}$       b)  $7\frac{1}{3}$       c)  $\frac{12}{5}$       d)  $\frac{9}{9}$
4. Write each mixed number as an improper fraction.  
a)  $4\frac{1}{2}$       b)  $10\frac{5}{8}$       c)  $14\frac{3}{4}$       d)  $121\frac{6}{11}$
5. Write each improper fraction as a mixed number.  
a)  $\frac{13}{2}$       b)  $\frac{57}{12}$       c)  $\frac{161}{7}$       d)  $\frac{207}{17}$

#### Teaching Notes:

- Most students find this material easy. Remind students that division by zero is undefined.
- As students progress in their study of fractions, they will need to be reminded how to convert between improper fractions and mixed numbers, and vice-versa.

Answers: 1a)  $n=1, d=7$ , b)  $n=11, d=17$ , c)  $n=27, d=13$ , d)  $n=12, d=12$ , e) 5, f) undefined, g) 1, h) undefined;  
2a)  $\frac{3}{7}$ , b)  $\frac{11}{18}$ ; 3a) proper, b) mixed, c) improper, d) improper; 4a)  $9\frac{1}{2}$ , b)  $85\frac{5}{8}$ , c)  $59\frac{3}{4}$ , d)  $1337\frac{6}{11}$ ; 5a)  $6\frac{1}{2}$ , b)  $4\frac{9}{12}$ , c) 23, d)  $12\frac{3}{17}$

## Mini-Lecture 2.2

### Factors and Prime Factorization

#### **Learning Objectives:**

1. Find the factors of a number.
2. Identify prime and composite numbers.
3. Find the prime factorization of a number.
4. Key Vocabulary: *natural numbers, factors, factorization, prime numbers, composite numbers, prime factorization, divisibility tests (2, 3, 4, 5, 6, 9)*

#### **Examples:**

1. List all the factors of each number.  
a) 15                      b) 64                      c) 43                      d) 144
2. Identify each number as prime, composite or neither.  
a) 6                      b) 47                      c) 123                      d) 1
3. Find the prime factorization of each number. Use exponents with any repeated factors.  
a) 16                      b) 50                      c) 72                      d) 132  
e) 76                      f) 200                      g) 480                      h) 12,600

#### **Teaching Notes:**

- Review the first 10 or so primes and have students be able to recognize them.
- Many students will not factor completely. For example:  $72 = 2^3 \times 9$
- Prime factorization is an important skill as students progress into the study of fractions.

*Answers: 1a) 1,3,5,15, b) 1,2,4,8,32,64 c) 1,43 d) 1,2,3,4,6,8,9,12,16,18,24,36,48,72,144; 2a) composite, b) prime, c) composite, d) neither; 3a)  $2^4$ , b)  $2 \cdot 5^2$ , c)  $2^3 \cdot 3^2$ , d)  $2^2 \cdot 3 \cdot 11$ , e)  $2^2 \cdot 19$ , f)  $2^3 \cdot 5^2$ , g)  $2^5 \cdot 3 \cdot 5$ , h)  $2^3 \cdot 3^2 \cdot 5^2 \cdot 7$*

## Mini-Lecture 2.3

### Simplest Form of a Fraction

#### Learning Objectives:

1. Write a fraction in simplest form or lowest terms.
2. Determine whether two fractions are equivalent.
3. Solve problems by writing fractions in simplest form.
4. Key Vocabulary: *equivalent fractions, simplest form, lowest form, simplifying, cross products, equality of fractions*

#### Examples:

1. Write each fraction in simplest form or lowest terms.

a)  $\frac{8}{8}$

b)  $\frac{2}{4}$

c)  $\frac{18}{45}$

d)  $\frac{24}{150}$

e)  $\frac{14}{84}$

f)  $\frac{35}{20}$

g)  $\frac{98}{14}$

h)  $\frac{138}{42}$

2. Determine whether each pair of fractions is equivalent.

a)  $\frac{3}{6}$  and  $\frac{11}{22}$

b)  $\frac{12}{20}$  and  $\frac{21}{35}$

c)  $\frac{14}{21}$  and  $\frac{5}{15}$

d)  $\frac{3}{9}$  and  $\frac{7}{21}$

e)  $\frac{30}{46}$  and  $\frac{15}{24}$

f)  $\frac{9}{27}$  and  $\frac{12}{36}$

3. a) There are 36 inches in a yard. What fraction of a yard is represented by 9 inches?  
b) Twenty-eight students are enrolled in Math Class. On Monday, only 21 students attended class. What fraction of the class attended on Monday?

#### Teaching Notes:

- Remind students to check their final answer to be sure it is completely reduced.
- Many students prefer to simplify rather than use cross products.
- Refer students to Section 1.8 for a review of **Problem-Solving Steps**.

Answers: 1a) 1, b)  $\frac{1}{2}$ , c)  $\frac{2}{5}$ , d)  $\frac{4}{25}$ , e)  $\frac{1}{6}$ , f)  $\frac{7}{4}$ , g) 7, h)  $\frac{23}{7}$ ; 2a) yes, b) yes, c) no, d) yes, e) no, f) yes; 3a)  $\frac{1}{4}$ , b)  $\frac{3}{4}$

## Mini-Lecture 2.4

### Multiplying Fractions and Mixed Numbers

#### Learning Objectives:

1. Multiply fractions.
2. Multiply fractions and mixed numbers or whole numbers.
3. Solve problems by multiplying fractions.
4. Key Vocabulary: *of, is*

#### Examples:

1. Multiply. Write each answer in simplest form.

a)  $\frac{1}{2} \cdot \frac{3}{7}$       b)  $\frac{2}{5} \cdot \frac{2}{7}$       c)  $\frac{2}{9} \cdot \frac{3}{8}$       d)  $\frac{7}{3} \cdot \frac{9}{14} \cdot \frac{8}{15}$

2. Multiply. Write each answer in simplest form.

a)  $2\frac{1}{2} \cdot \frac{1}{10}$       b)  $5\frac{1}{4} \cdot \frac{3}{7}$       c)  $\frac{2}{3} \cdot 4\frac{1}{2}$       d)  $7\frac{3}{4} \cdot 2\frac{2}{3}$

e)  $6 \cdot \frac{2}{3}$       f)  $\frac{7}{10} \cdot 5$       g)  $\frac{7}{7} \cdot 28$       h)  $\frac{1}{6} \cdot 0$

3. a) Find  $\frac{2}{5}$  of 75

b) A tiled wall is built 6 tiles wide. If the side of the square tile measures  $2\frac{5}{8}$ " , what is the width of the wall?

#### Teaching Notes:

- Suggest to students that they always write their fractions with a horizontal bar ( $\frac{2}{3}$ ) instead of a vertical bar (2/3).
- Students will need a review of converting mixed numbers to improper fractions.
- Most students will need to be shown how to convert a whole number to a fraction.
- Students need to be reminded to always convert mixed numbers to improper fractions before beginning multiplication.

*Answers: 1a) 3/14, b) 4/35, c) 1/12, d) 4/5; 2a) 1/4, b) 2 1/4, c) 3, d) 20 2/3, e) 4, f) 3 1/2, g) 28, h) 0; 3a) 30, b) 15 3/4"*

## Mini-Lecture 2.5

Dividing Fractions and Mixed Numbers

### Learning Objectives

1. Find the reciprocal of a fraction.
2. Divide fractions.
3. Divide fractions and mixed numbers or whole numbers.
4. Solve problems by dividing fractions.
5. Key Vocabulary: *reciprocal*

### Examples:

1. Find the reciprocal of each number.

a)  $\frac{2}{7}$

b)  $\frac{15}{8}$

c) 6

d)  $\frac{1}{10}$

2. Divide and simplify.

a)  $\frac{1}{3} \div \frac{5}{6}$

b)  $\frac{7}{8} \div \frac{3}{4}$

c)  $\frac{5}{9} \div \frac{10}{9}$

d)  $\frac{6}{17} \div \frac{6}{17}$

3. Divide and simplify.

a)  $3\frac{2}{9} \div \frac{3}{2}$

b)  $1\frac{3}{7} \div \frac{2}{7}$

c)  $2\frac{3}{5} \div 1\frac{3}{5}$

d)  $4\frac{1}{6} \div 3\frac{3}{4}$

e)  $27 \div 4\frac{1}{2}$

f)  $2\frac{8}{9} \div 13$

g)  $5\frac{5}{9} \div 10$

h)  $6\frac{1}{3} \div 0$

4. a) How many  $\frac{5}{8}$ -pound boxes of pasta can be made from 7880 pounds of pasta?  
b) Ally drove 262 miles on  $8\frac{1}{6}$  gallons of gasoline. How many miles per gallon did she average?

### Teaching Notes:

- Remind students that mixed numbers must be changed to improper fractions.
- Some students simplify before taking the reciprocal of the second fraction.
- Some students change division to multiplication but do not multiply by reciprocal.
- Many students have trouble deciding which number does the dividing in word problems.

*Answers: 1a)  $\frac{7}{2}$ , b)  $\frac{8}{15}$ , c)  $\frac{1}{6}$ , d)  $\frac{10}{1}$ ; 2a)  $\frac{2}{5}$ , b)  $1\frac{1}{6}$ , c)  $\frac{1}{2}$ , d) 1; 3a)  $2\frac{4}{27}$ , b) 5, c)  $1\frac{5}{8}$ , d)  $1\frac{1}{9}$ , e) 6, f)  $\frac{2}{9}$ , g)  $1\frac{5}{9}$ , h) undefined; 4a) 12,608 boxes, b)  $32\frac{4}{49}$  miles per gallon*



## Mini-Lecture 3.1

### Adding and Subtracting Like Fractions

#### Learning Objectives:

1. Add like fractions.
2. Subtract like fractions.
3. Solve problems by adding or subtracting like fractions.
4. Key Vocabulary: *like fractions, unlike fractions*

#### Examples:

1. Add and simplify.

a) $\frac{1}{9} + \frac{4}{9}$	b) $\frac{1}{12} + \frac{5}{12}$	c) $\frac{11}{17} + \frac{6}{17}$	d) $\frac{5}{32} + \frac{3}{32} + \frac{4}{32}$
e) $\frac{3}{15} + \frac{5}{15}$	f) $\frac{1}{10} + \frac{3}{10}$	g) $\frac{8}{11} + \frac{7}{11}$	h) $\frac{1}{7} + \frac{5}{7} + \frac{3}{7}$

2. Subtract and simplify.

a) $\frac{13}{21} - \frac{5}{21}$	b) $\frac{11}{15} - \frac{6}{15}$	c) $\frac{27}{35} - \frac{20}{35}$	d) $\frac{47}{54} - \frac{19}{54}$
e) $\frac{11}{12} - \frac{5}{12}$	f) $\frac{25}{20} - \frac{13}{20}$	g) $\frac{14}{43} - \frac{13}{43}$	h) $\frac{47}{50} - \frac{47}{50}$

3. a) A recipe calls for  $\frac{1}{8}$  cup of a sugar and later  $\frac{3}{8}$  cup of sugar. How much sugar is needed for the entire recipe?

b) How much greater is  $\frac{49}{100}$  than  $\frac{38}{100}$  ?

#### Teaching Notes:

- Even though many students can determine like and unlike fractions, they tend to add or subtract both the numerator and denominator.
- Remind students to always check to be sure the final answer is completely simplified.

*Answers: 1a) 5/9, b) 1/2, c) 1, d) 3/8, e) 8/15, f) 2/5, g) 15/11 or 1 4/11, h) 9/7 or 1 2/7; 2a) 8/21, b) 1/3, c) 1/5, d) 14/27, e) 1/2, f) 3/5, g) 1/43, h) 0; 3a) 1/2, b) 11/100*

## Mini-Lecture 3.2

### Least Common Multiple

#### Learning Objectives:

1. Find the least common multiple (LCM) using multiples.
2. Find the least common multiple (LCM) using prime factorization.
3. Write equivalent fractions.
4. Key Vocabulary: *least common multiple (LCM)*

#### Examples:

1. Find the least common multiple using multiples.

a) 3, 5

b) 4, 6

c) 6, 42

d) 15, 28

e) 16, 36

f) 10, 15, 35

2. Find the least common multiple using prime factorization.

a) 6, 15

b) 12, 32

c) 28, 20

d) 54, 42

e) 7, 14, 21, 36

f) 34, 36, 48

3. Write each fraction as an equivalent fraction with the given denominator.

a)  $\frac{1}{2} = \frac{\quad}{8}$

b)  $\frac{2}{3} = \frac{\quad}{12}$

c)  $\frac{2}{7} = \frac{\quad}{21}$

d)  $\frac{3}{10} = \frac{\quad}{60}$

e)  $\frac{11}{15} = \frac{\quad}{165}$

f)  $\frac{12}{7} = \frac{\quad}{105}$

#### Teaching Notes:

- Most students prefer to find the LCD by listing multiples until a common multiple appears.
- Most students will need a review of prime factorization.
- A common mistake in prime factorization is to not factor completely.
- Some students find the product of all the numbers rather than the LCM.

Answers: 1a) 15, b) 12, c) 42, d) 420, e) 144, f) 210; 2a) 30, b) 96, c) 140, 3) 378, e) 255, f) 2448; 3a) 4, b) 8, c) 6, d) 18, e) 121, f) 180

## Mini-Lecture 3.3

### Adding and Subtracting Unlike Fractions

#### Learning Objectives:

1. Add unlike fractions.
2. Subtract unlike fractions.
3. Solve problems by adding or subtracting unlike fractions.
4. Key Vocabulary: *least common denominator (LCD)*

#### Examples:

1. Add and simplify.

a)  $\frac{3}{4} + \frac{1}{8}$

b)  $\frac{1}{5} + \frac{18}{35}$

c)  $\frac{1}{9} + \frac{11}{27}$

d)  $\frac{2}{7} + \frac{3}{19}$

e)  $\frac{1}{2} + \frac{15}{42} + \frac{2}{3}$

f)  $\frac{13}{14} + \frac{3}{7} + \frac{11}{28}$

2. Subtract and simplify.

a)  $\frac{5}{8} - \frac{1}{16}$

b)  $\frac{6}{11} - \frac{1}{2}$

c)  $\frac{5}{6} - \frac{3}{8}$

d)  $\frac{1}{30} - \frac{1}{300}$

e)  $\frac{7}{9} - \frac{1}{12}$

f)  $\frac{1}{6} - \frac{1}{11}$

3. a) Raymond read  $\frac{1}{15}$  of the report on Monday and  $\frac{1}{6}$  of the report on Wednesday.  
How much of the report has Raymond read?

- b) Kari had  $\frac{8}{9}$  bag of pretzels. She ate  $\frac{3}{5}$  of the bag of pretzels. What fractional part of the bag of pretzels is left?

#### Teaching Notes:

- Some students will try to cross cancel instead of finding the LCD.
- Some students will add/subtract the denominators instead of finding the LCD.
- Some students will find the common denominator but forget to multiply the numerator when building equivalent fractions.

*Answers: 1a)  $\frac{7}{8}$ , b)  $\frac{5}{7}$ , c)  $\frac{14}{27}$ , d)  $\frac{59}{133}$ , e)  $1\frac{4}{21}$ , f)  $1\frac{3}{4}$ ; 2a)  $\frac{9}{16}$ , b)  $\frac{1}{22}$ , c)  $\frac{11}{24}$ , d)  $\frac{3}{100}$ , e)  $\frac{25}{36}$ , f)  $\frac{5}{66}$ ; 3a)  $\frac{7}{30}$ , b)  $\frac{13}{45}$*

## Mini-Lecture 3.4

### Adding and Subtracting Mixed Numbers

#### Learning Objectives:

1. Add mixed numbers.
2. Subtract mixed numbers.
3. Solve problems by adding or subtracting mixed numbers.

#### Examples:

1. Add.

a)  $14\frac{1}{4} + 17\frac{2}{5}$

b)  $9\frac{1}{2} + 18\frac{1}{3}$

c)  $14\frac{6}{7} + 19\frac{2}{3}$

d)  $16\frac{5}{6} + 6\frac{5}{9} + \frac{4}{9}$

2. Subtract.

a)  $14\frac{5}{6} - 9\frac{2}{3}$

b)  $11 - 7\frac{1}{7}$

c)  $16\frac{2}{15} - 7\frac{2}{9}$

d)  $17\frac{7}{25} - 8\frac{7}{15}$

3. a) Ray played hockey for  $3\frac{1}{2}$  hours on Monday and played hockey on Wednesday for  $4\frac{5}{9}$  hours. What was the total amount of time Ray played hockey?
- b) Chris cut a board  $13\frac{3}{5}$  feet long from a board that was 20 feet long. How long is the remaining piece?

#### Teaching Notes:

- Many students are challenged by this section.
- Some students need to be shown the following approach for borrowing or re-grouping:

$$11\frac{2}{9} \longrightarrow 10\overset{10}{\cancel{11}}\frac{2}{9} \longrightarrow 10\frac{11}{9}$$

*Answers: 1a)  $31\frac{13}{20}$ , b)  $27\frac{5}{6}$ , c)  $34\frac{11}{21}$ , d)  $23\frac{5}{6}$ ; 2a)  $5\frac{1}{6}$ , b)  $3\frac{6}{7}$ , c)  $8\frac{41}{45}$ , d)  $8\frac{61}{75}$ ; 3a)  $8\frac{1}{18}$  hours, b)  $6\frac{2}{5}$  feet*

## Mini-Lecture 3.5

### Order, Exponents, and the Order of Operations

#### Learning Objectives:

1. Compare fractions.
2. Evaluate fractions raised to powers.
3. Review operations on fractions.
4. Use the order of operations.
5. Key Vocabulary: *less than (<), greater than (>), order of operations*

#### Examples:

1. Insert < or > to form a true statement.

a)  $\frac{1}{18} > \frac{1}{3}$       b)  $\frac{2}{3} < \frac{4}{14}$       c)  $\frac{19}{20} < \frac{9}{10}$       d)  $1\frac{3}{12} < 1\frac{4}{7}$

2. Evaluate each expression.

a)  $\left(\frac{1}{8}\right)^2$       b)  $\left(\frac{3}{7}\right)^3$       c)  $\left(\frac{5}{6}\right)^3$       d)  $\left(\frac{2}{5}\right)^3\left(\frac{3}{4}\right)^2$

3. Perform each indicated operation.

a)  $\frac{1}{8} + \frac{5}{16}$       b)  $\frac{3}{5} \div \frac{14}{15}$       c)  $\frac{9}{10} - \frac{1}{3}$       d)  $\frac{2}{3} \cdot \frac{5}{12}$

4. Use the order of operations to simplify each expression.

a)  $\left(\frac{1}{3} + \frac{1}{6}\right) \cdot \frac{3}{2}$       b)  $\frac{1}{2} \cdot \frac{3}{4} + \frac{1}{8} \cdot \frac{1}{6}$

c)  $\frac{12}{7} \div \frac{9}{4} \cdot \frac{11}{3}$       d)  $\left(\frac{3}{7}\right)^2 \div \left(\frac{3}{7} - \frac{1}{13}\right)$

e) Find the average of the list of numbers:  $\frac{1}{3}$ ,  $\frac{11}{18}$ , and  $\frac{5}{6}$

#### Teaching Notes:

- Many students will make the following mistake:  $\left(\frac{1}{3}\right)^2 = \frac{1 \times 2}{3 \times 2} = \frac{3}{6}$
- Review operations on fractions.
- Remind students to multiply or divide from left to right; add or subtract from left to right.

Answers: 1a) <, b) >, c) >, d) <; 2a) 1/64, b) 27/343, c) 125/216, d) 9/250; 3a) 7/16, b) 9/14, c) 17/30, d) 5/18; 4a) 3/4, b) 19/48, c) 2 50/63, d) 117/224, e) 16/27

## Mini-Lecture 3.6

### Fractions and Problem Solving

#### Learning Objectives:

1. Solve problems by performing operations on fractions or mixed numbers.
2. Key Vocabulary: *perimeter, units, area, square units, volume, cubic units*

#### Examples:

1. Translate each to an expression. Then simplify the expression.
  - a) Find the quotient of 49 and 8.
  - b) Find 89 increased by 27.
  - c) Find the product of 27 and 3.
  - d) Subtract 43 from 102.
2. A recipe calls for  $\frac{3}{8}$  cup of sugar. If you triple the recipe, how much sugar is needed?
3. Find the perimeter of a rectangular room that measures  $14\frac{1}{3}$  feet long and  $12\frac{1}{2}$  feet wide.
4. Find the volume of a cereal box that measures  $11\frac{1}{4}$  inches by  $7\frac{1}{2}$  inches by 2 inches.
5. A contractor purchased 48 acres of land to develop. How many houses can be built if each lot will be  $1\frac{1}{2}$  acres.
6. Find the area of a picture frame  $13\frac{3}{4}$  inches by  $6\frac{1}{5}$  inches.
7. A factory produces bags of sand. Three bags of sand were randomly selected and they weighed  $15\frac{1}{3}$ ,  $16\frac{1}{6}$ , and  $14\frac{5}{6}$  pounds. What is the average weight of the bags of sand?

#### Teaching Notes:

- Many students have difficulty with word problems.
- Encourage students to estimate their answers before solving.
- Refer students to textbook for Problem-Solving Steps: Understand, Translate, Solve, Interpret.

*Answers: 1a)  $49 \div 8 = 6r1$ , b)  $89 + 27 = 116$ , c)  $27 \cdot 3 = 81$ , d)  $102 - 43 = 59$ ; 2)  $1\frac{1}{8}$  cup; 3)  $53\frac{2}{3}$  feet; 4)  $168\frac{3}{4}$  inches; 5) 32 houses; 6)  $85\frac{1}{4}$  inches; 7)  $46\frac{1}{3}$  pounds*

## Mini-Lecture 4.1

### Introduction to Decimals

#### Learning Objectives:

1. Know the meaning of place value for a decimal number, and write decimals in words.
2. Write decimals in standard form.
3. Write decimals as fractions.
4. Write fractions as decimals.
5. Key Vocabulary: *decimal notation*

#### Examples:

1. Determine the place value for the digit 4 in each number.

a) 40                      b) 0.4                      c) 5.014                      d) 3.145

Write each decimal number in words.

e) 3.98                      f) 0.78                      g) 541.184                      h) 20.0004

2. Write each decimal number in standard form.

a) Eight and four tenths                      b) Eleven and two hundredths  
c) Forty-seven thousandths                      d) one and thirty-two ten thousandths

3. Write each decimal as a fraction or a mixed number. Write your answer in simplest form.

a) 0.27                      b) 0.2                      c) 5.255                      d) 11.004

4. Write each fraction as a decimal.

a)  $\frac{7}{10}$                       b)  $\frac{55}{100}$                       c)  $\frac{25}{1000}$                       d)  $\frac{302}{10}$

#### Teaching Notes:

- Most students find examples 1 and 2 easy.
- Some students have difficulty with examples 3 and 4 when a whole number is involved.
- Refer students to the **Place Value Chart** in the textbook, beginning of Section 4.1.

*Answers: 1a) tens, b) tenths, c) thousandths, d) hundredths, e) three and ninety-eight hundredths, f) seventy-eight hundredths, g) five hundred forty-one and one hundred eighty-four thousandths, h) twenty and four ten-thousandths; 2a) 8.4, b) 11.02, c) 0.047, d) 1.0032; 3a)  $\frac{27}{100}$ , b)  $\frac{1}{5}$ , c)  $5\frac{51}{200}$ , d)  $11\frac{1}{250}$ ; 4a) 0.7, b) 0.55, c) 0.025, d) 30.2*

## Mini-Lecture 4.2

### Order and Rounding

#### Learning Objectives:

1. Compare decimals.
2. Round a decimal number to a given place value.
3. Key Vocabulary:  $<$  (*less than*),  $>$  (*greater than*),  $=$  (*equal to*), *round the decimal part*

#### Examples:

1. Insert  $<$ ,  $>$ , or  $=$  to form a true statement.

a)  $0.2$  \_\_\_\_  $0.5$

b)  $0.14$  \_\_\_\_  $0.14000$

c)  $0.6401$  \_\_\_\_  $0.6410$

d)  $15.0037$  \_\_\_\_  $15.00037$

Write the decimals in order from smallest to largest.

e)  $0.0071$ ,  $0.070$ ,  $0.007$

f)  $26.601$ ,  $26.631$ ,  $27.040$ ,  $26.630$

2. Round each decimal to the given place value.

a)  $0.39$  to the nearest tenth

b)  $0.174$  to the nearest hundredth

c)  $0.585$  to the nearest hundredth

d)  $1.4782$  to the nearest thousandth

e)  $22.099$  to the nearest hundredth

f)  $835.006$  to the nearest ten

Round each monetary amount to the nearest cent or dollar as indicated.

g)  $\$0.058$  to the nearest cent

h)  $\$17.88$  to the nearest dollar

#### Teaching Notes:

- Some students find a number line helpful to compare decimals.
- Most students find rounding relatively easy.
- Some students become confused when rounding monetary values. When rounding to the nearest cent, it is important to remind them that this is the hundredths position (one-hundredths-of-a-dollar position).

*Answers: 1a)  $<$ , b)  $=$ , c)  $<$ , d)  $>$ , e)  $0.070$ ,  $0.007$ ,  $0.0071$ , f)  $26.601$ ,  $26.630$ ,  $26.631$ ,  $27.040$ ; 2a)  $0.4$ , b)  $0.17$ , c)  $0.59$ , d)  $1.478$ , e)  $22.10$ , f)  $840.0$ , g)  $\$0.06$ , h)  $\$18$*



## Mini-Lecture 4.3

### Adding and Subtracting Decimals

#### Learning Objectives:

1. Add decimals.
2. Subtract decimals.
3. Estimate when adding or subtracting decimals.
4. Solve problems that involve adding and subtracting decimals.

#### Examples:

1. Add.

a)  $0.5 + 0.1$

b)  $2.7 + 3.2$

c)  $7.2 + 3.27$

d)  $372 + 9.302$

e)  $43.097 + 289.3887$

f)  $5.03 + 16.988 + 0.006$

2. Subtract.

a)  $0.8 - 0.2$

b)  $7.5 - 2.3$

c)  $187.5 - 8.39$

d)  $8.2 - 5.006$

e)  $632.021 - 295.9$

f)  $1000 - 3.0947$

3. Add or subtract. Be sure to estimate to see if the answer is reasonable.

a) Find the sum of 8.937 and 16.095.

b) Subtract 9.6802 from 22.3.

4. a) Recently, Allison went shopping and spent \$18.92 at the bookstore, \$68.03 at the grocery store, and \$129.76 at a department store. What is the total amount of money Allison spent?

b) Find the perimeter of a rectangular lawn that measures 40.93 feet long by 27.09 feet wide.

#### Teaching Notes:

- Remind students to work in a vertical format and line-up the decimal point and corresponding place values.
- Some students need to be shown how to add extra zeros to the ends of the decimal part of the numbers and where to place the decimal point with whole numbers.
- Some students must be reminded of how to borrow across zeros when subtracting.

*Answers: 1a) 0.6, b) 5.9, c) 10.47, d) 381.302, e) 332.4857, f) 22.024; 2a) 0.6, b) 5.2, c) 179.11, d) 3.194, e) 336.121, f) 996.9053; 3a) 25.032, b) 12.6198; 4a) \$216.71, b) 136.04 feet*

## Mini-Lecture 4.4

### Multiplying Decimals and Circumference of a Circle

#### Learning Objectives:

1. Multiply decimals.
2. Estimate when multiplying decimals.
3. Multiply by powers of 10.
4. Find the circumference of a circle.
5. Solve problems by multiplying decimals.
6. Key Vocabulary:  $\pi$  (*pi*), *perimeter*, *circumference*, *diameter*, *radius*

#### Examples:

1. Multiply.

a)  $0.7 \times 0.2$

b)  $1.33 \times 0.5$

c)  $7.2 \times 5.8$

2. Multiply. Check by estimating.

a) 
$$\begin{array}{r} 0.856 \\ \times 3.1 \\ \hline \end{array}$$

b) 
$$\begin{array}{r} 2.00033 \\ \times 6.9 \\ \hline \end{array}$$

c) 
$$\begin{array}{r} 0.0896 \\ \times 0.345 \\ \hline \end{array}$$

3. Multiply.

a)  $4.3 \times 10$

b)  $17.693 \times 100$

c)  $0.0027 \times 1000$

d)  $0.07 \times 0.1$

e)  $9.07 \times 0.01$

f)  $2.908 \times 0.001$

4. Find the circumference of a circle with the given information. Use  $\pi = 3.14$ .

a) radius: 7 feet

b) diameter: 16 inches

c) radius: 10.3 meters

5. a) Write 57.6 thousand in standard form.

b) A 1-ounce serving of hot cocoa contains 0.375 grams of fat. How many grams of fat are in an 8 oz. mug of hot cocoa?

#### Teaching Notes:

- Many students do not see the pattern that develops when multiplying by powers of 10; they must be shown.
- Many students prefer to multiply numbers by a power of ten the long way.
- Some students will attempt to line up the decimal point (like adding) when multiplying.
- Refer students to  $C = 2\pi r$ .

*Answers: 1a) 0.14, b) 0.665, c) 41.76, 2a) 2.6536, b) 13.802277, c) 0.030912; 3a) 43, b) 1769.3, c) 2.7, d) 0.007, e) 0.09707, f) 0.002908; 4a) 43.96 ft, b) 50.24 in., c) 64.684 m; 5a) 57,600, b) 3 g*

## Mini-Lecture 4.5

Dividing Decimals and Order of Operations

### Learning Objectives:

1. Divide decimals.
2. Estimate when dividing decimals.
3. Divide decimals by powers of 10.
4. Solve problems by dividing decimals.
5. Review order of operations by simplifying expressions containing decimals.

### Examples:

1. Divide.

a)  $1.5 \div 5$                                   b)  $26 \overline{)7.826}$                                   c)  $518 \overline{)0.9324}$

2. Divide. Use estimation to check answers.

a)  $8.9 \overline{)22.25}$                                   b)  $1411.51 \div 36.1$                                   c)  $0.02 \overline{)0.8}$

3. Divide decimals by powers of 10.

a)  $\frac{7.74}{10}$                                   b)  $1000 \overline{)887.73}$                                   c)  $1.047 \div 100$

4. a) Divide 0.894 by 0.041 and round the quotient to the nearest hundredth.

b) Preparing for a picnic, Carol went to the Deli and purchased: 0.52 pounds of salami at \$3.29/pound; 0.48 pounds of sliced turkey at \$8.99/pound; 1.04 pounds of ham at \$3.99/pound; and 0.98 pounds of cheese at \$4.29/pound. What was the total amount Carol spent at the deli? (Round your answer to the nearest cent.)

5. Simplify each expression.

a)  $0.1558 \div 0.02 \cdot 4.45$                                   b)  $(3.6)^2 + 4.5 - 8.7$   
c)  $41.6 - 8.71 \div 6.7 \cdot 20$                                   d)  $1.3 + 0.3 \div 15 \cdot 4.03 + 5.43$

### Teaching Notes:

- Most students have forgotten the mechanics of long division with decimals.
- Most students need a review of order of operations.
- Remind students when rounding the quotient to a specific place, you need to carry your division one more place than the rounding place.

*Answers: 1a) 0.3, b) 0.301, c) 0.0018, 2a) 2.5, b) 39.1, c) 40; 3a) 0.774, b) 0.88773, c) 0.01047; 4a) 21.80, b) \$14.38; 5a) 34.6655, b) 8.76, c) 15.6, d) 6.8106*

## Mini-Lecture 4.6

### Fractions and Decimals

#### Learning Objectives:

1. Write fractions as decimals.
2. Compare fractions and decimals.
3. Solve area problems containing fractions and decimals.
4. Key Vocabulary: *area of a triangle*

#### Examples:

1. Write each fraction as a decimal.

a)  $\frac{3}{5}$

b)  $\frac{3}{20}$

c)  $\frac{1}{3}$

d)  $\frac{27}{100}$

e)  $\frac{5}{16}$

f)  $\frac{13}{11}$

g)  $1\frac{7}{8}$

h)  $2\frac{2}{3}$

i)  $11\frac{3}{1000}$

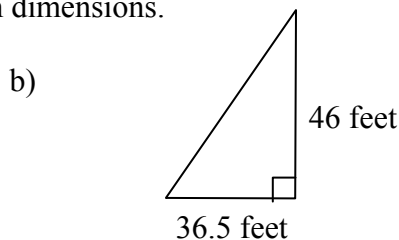
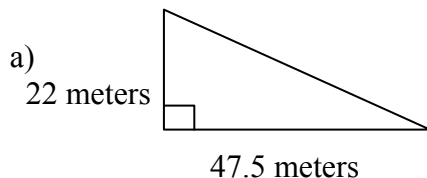
2. Write the numbers in order from smallest to largest.

a) 0.331,  $\frac{1}{3}$ , 0.330

b) 2.15, 2.142,  $\frac{15}{7}$

c) 1.5833,  $1\frac{21}{36}$ ,  $\frac{38}{36}$

3. Find the area of a triangle with the given dimensions.



#### Teaching Notes:

- Most students, once taught how to convert from fraction to decimal, have little problems.
- Some students have difficulty ordering numbers when they are mixed. Suggest that after converting to decimal, line the decimal points up vertically and compare corresponding place value.

Answers: 1a) 0.6, b) 0.15, c)  $0.\bar{3}$ , d) 0.27, e) 0.3125, f)  $1.1\bar{18}$ , g) 1.875, h)  $2.\bar{6}$ , i) 11.003; 2a) 0.330, 0.331,  $\frac{1}{3}$ , b) 2.142,  $\frac{15}{7}$ , 2.15, c)  $\frac{38}{36}$ , 1.5833,  $1\frac{21}{36}$ ; 3a)  $522.5\text{ m}^2$ , b)  $839.5\text{ ft}^2$

## Chapter 2

### Section 2.1 Introduction to Fractions and Mixed Numbers

#### Objective A

To refer to part of a whole, we can use fractions.

The number on top is the numerator.

The number on bottom is the denominator.

#### Video Exercises

1. 1 is the numerator. 2 is the denominator.

The fraction bar means division.

#### Video Exercises

- The numerator is 10. The denominator is 3.
- 1
- 13
- 0
- The fraction is undefined.

#### Objective B

#### Video Exercises

- $\frac{3}{7}$
- a.  $\frac{21}{50}$   
b. 29  
c.  $\frac{29}{50}$

#### Objective C

#### Video Exercises

- The fraction is proper.
- The fraction is improper.
- $\frac{4}{3}$ ;  $1\frac{1}{3}$

#### Objective D

A mixed number has a whole number part and a fraction part.

### Writing a Number as an Improper Fraction

**Step 1:** Multiply the denominator of the fraction by the whole number.

**Step 2:** Add the numerator of the fraction to the product from step 1.

**Step 3:** Write the sum from step 2 as the numerator of the improper fraction over the original denominator.

#### Video Exercises

- $\frac{7}{3}$
- $\frac{18}{5}$
- $\frac{187}{20}$

#### Objective E

### Writing an Improper Fraction as a Mixed Number or a Whole Number

**Step 1:** Divide the denominator into the numerator.

**Step 2:** The whole number part of the mixed number is the quotient. The fraction part of the mixed number is the remainder over the original denominator.

#### Video Exercises

- $3\frac{2}{5}$
- $4\frac{5}{8}$

### Section 2.2 Factors and Prime Factorization

#### Objective A

$2 \cdot 5$  is a factorization of 10.

#### Video Exercises

- 1, 5, 25
- 1, 2, 3, 4, 6, 12

## Objective B

### Video Exercises

3. 4 is composite.
4. 10 is composite.
5. 67 is prime.

## Objective C

### Video Exercises

6.  $15 = 3 \cdot 5$
7.  $36 = 2 \cdot 2 \cdot 3 \cdot 3$
8.  $240 = 2^4 \cdot 3 \cdot 5$

## Section 2.3 Simplest Form of a Fraction

### Objective A

Fractions that represent the same portion of a whole are equivalent fractions.

### Video Exercises

1.  $\frac{7}{8}$
2.  $\frac{3}{5}$
3.  $\frac{5}{14}$

### Objective B

### Video Exercises

4.  $\frac{7}{11} \neq \frac{5}{8}$
5.  $\frac{3}{9} = \frac{6}{18}$

### Objective C

### Video Exercises

6.  $\frac{1}{2}$  of a mi
7.  $\frac{5}{12}$  of the wall

## Section 2.4 Multiplying Fractions and Mixed Numbers

### Objective A

### Video Exercises

1.  $\frac{6}{35}$
2.  $\frac{5}{28}$
3.  $\frac{1}{56}$

### Objective B

### Video Exercises

4.  $\frac{5}{2}$  or  $2\frac{1}{2}$
5.  $\frac{77}{10}$  or  $7\frac{7}{10}$

### Objective C

### Video Exercises

6. 868 miles

Of usually translates to multiplication.

### Video Exercises

7.  $\frac{3}{16}$  inch

## Section 2.5 Dividing Fractions and Mixed Numbers

### Objective A

### Video Exercises

1.  $\frac{7}{4}$
2.  $\frac{1}{15}$

### Objective B

### Video Exercises

3.  $\frac{4}{5}$

4.  $\frac{1}{100}$

5.  $\frac{8}{45}$

6. undefined

**0 has no reciprocal.**

**Objective C**

**Video Exercises**

7.  $\frac{1}{6}$

8.  $\frac{36}{35}$  or  $1\frac{1}{35}$

**Objective D**

**Video Exercises**

9.  $\frac{5}{6}$  T