## CHAPTER 2 - HYDRAULICS

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## Solutions to Practice Problems

1. $\mathrm{P}=0.43 \times \mathrm{h}$ (Equation $2-2 \mathrm{~b}$ )
$\mathrm{P}=0.43 \times 50 \mathrm{ft}=22 \mathrm{psi}$ at the bottom of the reservoir $\mathrm{P}=0.43 \times(50-30)=0.43 \times 20 \mathrm{ft}=8.6 \mathrm{psi}$ above the bottom
2. $\mathrm{h}=0.1 \times \mathrm{P}=0.1 \times 50=5 \mathrm{~m}$ (Equation 2-3a)
3. Depth of water above the valve: $\mathrm{h}=(78 \mathrm{~m}-50 \mathrm{~m})+2 \mathrm{~m}=30 \mathrm{~m}$ $\mathrm{P}=9.8 \times \mathrm{h}=9.8 \times 30=294 \mathrm{kPa} \approx 290 \mathrm{kPa}$ (Equation 2-2a)
4. $\mathrm{h}=2.3 \times \mathrm{P}=2.3 \times 50=115 \mathrm{ft}$, in the water main
$\mathrm{h}=115-40=75 \mathrm{ft}$
$\mathrm{P}=0.43 \times 75=32 \mathrm{psi}, 40 \mathrm{ft}$ above the main (Equation $2-2 \mathrm{~b}$ )
5. Gage pressure $P=30+9.8 \times 1=39.8 \mathrm{kPa} \approx 40 \mathrm{kPa}$

Pressure head (in tube) $=0.1 \times 40 \mathrm{kPa}=4 \mathrm{~m}$
6. $\mathrm{Q}=\mathrm{A} \times \mathrm{V}($ Eq. 2-4), therefore $\mathrm{V}=\mathrm{Q} / \mathrm{A}$
$A=\pi D^{2} / 4=\pi(0.3)^{2} / 4=0.0707 \mathrm{~m}^{2}$
$100 \mathrm{~L} / \mathrm{s} \times 1 \mathrm{~m}^{3} / 1000 \mathrm{~L}=0.1 \mathrm{~m}^{3} / \mathrm{s}$
$\mathrm{V}=0.1 \mathrm{~m}^{3} / \mathrm{s} 0.707 \mathrm{~m}^{2}=1.4 \mathrm{~m} / \mathrm{s}$
7. $\mathrm{Q}=(500 \mathrm{gal} / \mathrm{min}) \times(1 \mathrm{~min} / 60 \mathrm{sec}) \times\left(1 \mathrm{ft}^{3} / 7.5 \mathrm{gal}\right)=1.11 \mathrm{cfs}$
$\mathrm{A}=\mathrm{Q} / \mathrm{V}$ (from Eq. 2-4)
$A=1.11 \mathrm{ft}^{3} / \mathrm{sec} / 1.4 \mathrm{ft} / \mathrm{sec}=0.794 \mathrm{ft}^{2}$
$A=\pi D^{2} / 4$, therefore $D=\sqrt{ } 4 A / \pi=\sqrt{ }(4)(0.794) / \pi=1 \mathrm{ft}=12 \mathrm{in}$.
8. Q=A1 x V1 = A2 x V2 (Eq.2-5)

Since $A=\pi D^{2} / 4$, we can write
$\mathrm{D}_{1}{ }^{2} \mathrm{x} \mathrm{V}_{1}=\mathrm{D}_{2}{ }^{2} \times \mathrm{V}_{2}$ and $\mathrm{V}_{2}=\mathrm{V}_{1} \times\left(\mathrm{D}_{1}^{2} / \mathrm{D}_{2}{ }^{2}\right)$
In the constriction, $\mathrm{V}_{2}=(2 \mathrm{~m} / \mathrm{s}) \times(4)=8 \mathrm{~m} / \mathrm{s}$
9. Area of pipe $A=\pi(0.3)^{2} / 4=0.0707 \mathrm{~m}^{2}$

Area of pipe $B=\pi(0.1)^{2} / 4=0.00785 \mathrm{~m}^{2}$
Area of pipe $C=\pi(0.2)^{2} / 4=0.03142 \mathrm{~m}^{2}$
$Q_{A}=Q_{B}+Q_{C}=0.00785 \mathrm{~m}^{2} \times 2 \mathrm{~m} / \mathrm{s}+0.03142 \mathrm{~m}^{2} \times 1 \mathrm{~m} / \mathrm{s}$
$=0.04712 \mathrm{~m}^{3} / \mathrm{s}$ (from continuity of flow: $\mathrm{Q}_{\text {IN }}=$ Qout)
$V_{A}=Q_{A} / A_{A}=0.4712 / 0.0707 \approx 0.67 \mathrm{~m} / \mathrm{s}$ (from Eq. 2-4)
10. $\mathrm{p}_{1} / \mathrm{w}+\mathrm{V}_{1}^{2} / 2 \mathrm{~g}=\mathrm{p}_{2} / \mathrm{W}+\mathrm{V}_{2}^{2} / 2 \mathrm{~g} \quad$ (Eq.2-8)

$$
\begin{array}{ll}
\mathrm{A}_{1}=\pi(1.33)^{2} / 4=1.4 \mathrm{ft}^{2} & \mathrm{~A}_{2}=\pi(0.67)^{2} / 4=0.349 \mathrm{ft}^{2} \\
\mathrm{~V}_{1}=6 / 1.4=4.29 \mathrm{ft} / \mathrm{sec} & \mathrm{~V}_{2}=6 / 0.349=17.2 \mathrm{ft} / \mathrm{sec} \\
\mathrm{w}=62.4 \mathrm{lb} / \mathrm{tt}^{3} \text { and } \mathrm{g}=32.2 \mathrm{ft} / \mathrm{sec}^{2} &
\end{array}
$$

From Eq. 2-8, and multiplying psix $144 \mathrm{in}^{2} / \mathrm{ft}^{2}$ to get lb/ft ${ }^{2}$

$$
50(144) / 62.4+4.29^{2} / 2(32.2)=p_{2}(144) / 62.4+17.2^{2} / 2(32.2)
$$

$$
115.38+0.28578=2.3076 p_{2}+4.5937
$$

$$
\mathrm{p}_{2}=111.07 / 2.307 \approx 48 \mathrm{psi}
$$

11. $p_{1} / w+v_{1}^{2} / 2 g=p_{2} / w+v_{2}^{2} / 2 g \quad$ (Eq.2-8)

$$
\mathrm{A}_{1}=\pi(0.300)^{2} / 4=0.0707 \mathrm{~m}^{2} \mathrm{~A}_{2}=\pi(0.100)^{2} / 4=0.00785 \mathrm{~m}^{2}
$$

$\mathrm{Q}=50 \mathrm{~L} / \mathrm{s} \times 1 \mathrm{~m}^{3} / 1000 \mathrm{~L}=0.05 \mathrm{~m}^{3} / \mathrm{s}$
$\mathrm{V}_{1}=0.05 / 0.0707=0.70721 \mathrm{~m} / \mathrm{sec} \mathrm{V}_{2}=0.05 / 0.00785=6.369 \mathrm{~m} / \mathrm{sec}$
$\mathrm{w}=9.81 \mathrm{kN} / \mathrm{m}^{3}$ and $\mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2}$; From Eq. 2-8,
$700 / 2(9.81)+0.70721^{2} / 2(9.81)=p_{2} / 2(9.81)+6.369^{2} / 2(9.81)$
$35.67789+0.02549=0.05097 \mathrm{p}_{2}+2.06775$ and $\mathrm{p}_{2}=660 \mathrm{kPa}$
12. From Figure 2.15 , with $Q=200 \mathrm{~L} / \mathrm{s}$ and $\mathrm{D}=600 \mathrm{~mm}$, read $\mathrm{S}=0.0013$. Therefore $h_{L}=S \times L=$ $0.0013 \times 1000 \mathrm{~m}=1.3 \mathrm{~m}$
Pressure drop p $=9.8 \times 1.3 \approx 12.7 \approx 13 \mathrm{kPa}$ per km
13. $\mathrm{h}_{\mathrm{L}}=2.3 \times 20=46 \mathrm{ft}$ and $\mathrm{S}=46 / 5280=0.0087$ (where $1 \mathrm{mi}=5280 \mathrm{ft}$ )

From Figure 2.15, with $Q=1000 \mathrm{gpm}$ and $\mathrm{S}=0.0087$, read $\mathrm{D}=10.3 \mathrm{in}$.
Use a 12 in . standard diameter pipe
14. $S=10 / 1000=0.01$

From the nomograph (Figure 2.15) read $\mathrm{Q} \approx 100 \mathrm{~L} / \mathrm{s}=0.1 \mathrm{~m}^{3} / \mathrm{s}$
Check with Eq. 2-9: $Q=0.28 \times 100 \times 0.3^{2.63} \times 0.01^{0.54} \approx 0.1 \mathrm{~m}^{3} / \mathrm{s}$ OK
15. Use (Eq. 2-10): $Q=C \times A_{2} \times\left\{\left(2 g\left(p_{1}-p_{2}\right) / w\right) /\left(1-\left(A_{2} / A_{1}\right)^{2}\right\}^{1 / 2}\right.$
where $A_{1}=\pi(6)^{2} / 4=28.27 \mathrm{in}^{2}$ and $\mathrm{A}_{2}=\pi(3)^{2} / 4=7.07 \mathrm{in}^{2}$
$\mathrm{g}=32.2 \mathrm{ft} / \mathrm{s}^{2}=386.4 \mathrm{in} / \mathrm{s}^{2}$
$\mathrm{w}=62.4 \mathrm{lb} / \mathrm{ft}^{3} \times 1 \mathrm{ft}^{3} / 12^{3} \mathrm{in}^{3}=0.0361 \mathrm{lb} / \mathrm{in}^{3}$

$$
\begin{aligned}
& \mathrm{Q}=0.98 \times 7.07 \times\left\{(2(386.4)(10) / 0.0361) 1\left(1-(7.07 / 28.27)^{2}\right)\right\}^{1 / 2} \\
& \mathrm{Q}=0.98 \times 7.07 \times \sqrt{ } 228,354=3311 \mathrm{in}^{3} / \mathrm{s}=1.9 \mathrm{cfs} \approx 2 \mathrm{cfs}
\end{aligned}
$$

16. Use (Eq. $2-10): Q=C \times A_{2} \times\left\{\left(2 g\left(p_{1}-p_{2}\right) / w\right) /\left(1-\left(A_{2} / A_{1}\right)^{2}\right)\right\}^{1 / 2}$
$A_{1}=\pi(0.15)^{2} / 4=0.01767 \mathrm{~m}^{2}$ and $\mathrm{A}_{2}=\pi(0.075)^{2} / 4=0.00442 \mathrm{~m}^{2}$
$\mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2} \quad \mathrm{w}=9.81 \mathrm{kN} / \mathrm{m}^{3}$
$1-\left(A_{2} / A_{1}\right)^{2}=1-(0.00442 / 0.01767)^{2}=0.93743$
$Q=0.98 \times 0.00442 \times\{(2(9.81)(100) / 9.81) / 0.93743)\}^{1 / 2}=0.063 \mathrm{~m}^{3} / \mathrm{s}$
(or, $Q=0.063 \mathrm{~m}^{3} / \mathrm{s} \times 1000 \mathrm{~L} / \mathrm{m}^{3}=63 \mathrm{~L} / \mathrm{s}$ )
17. Use Manning's nomograph (Figure 2.21): With $\mathrm{D}=800 \mathrm{~mm}=80 \mathrm{~cm}, \mathrm{n}=0.013$ and $\mathrm{S}=0.2 \%$ $=0.002$, read $Q=0.56 \mathrm{~m}^{3} / \mathrm{s}=560 \mathrm{~L} / \mathrm{s}$ and $\mathrm{V}=1.17 \mathrm{~m} / \mathrm{s}$
18. $S=1.5 / 1000=0.015$; from Fig. 2.21, $Q \approx 1800 \mathrm{gpm}$ and $V \approx 2.3 \mathrm{ft} / \mathrm{s}$
19. $\mathrm{Q}=200 \mathrm{~L} / \mathrm{s}=0.2 \mathrm{~m}^{3} / \mathrm{s}$; from Fig. 2.21, $\mathrm{D} \approx 42 \mathrm{~cm}$; Use 450 mm pipe
20. $Q=7 \mathrm{mgd}=7,000,000 \mathrm{gal} /$ day $\times 1$ day $/ 1440 \mathrm{~min} \approx 4900 \mathrm{gpm}$

From Fig. 2.21, with $n=0.013, D=36$ in and $Q=4900 \mathrm{gpm}: ~ S=0.00027, V=1.54 \mathrm{ft} / \mathrm{s}$ Since
$1.54 \mathrm{ft} / \mathrm{s}$ is less than the minimum self-cleansing velocity of $2 \mathrm{ft} / \mathrm{s}$, it is necessary to increase the slope of the 36 in pipe.
From Fig. 2.21, with 36 in and $2 \mathrm{ft} / \mathrm{s}$ : $\mathrm{S}=0.00047=0.047 \%=0.05 \%$
21. For full-flow conditions, with $D=300 \mathrm{~mm}$ and $\mathrm{S}=0.02$, read from

Fig. 2.21: $Q=0.135 \mathrm{~m}^{3} / \mathrm{s}=135 \mathrm{~L} / \mathrm{s}$ and $V=2 \mathrm{~m} / \mathrm{s}$
$q / Q=50 / 135=0.37$ From Fig. 2.22, $d / D=0.42$ and $v / V=0.92$
Depth at partial flow $\mathrm{d}=0.42 \times 300=126 \mathrm{~mm} \approx 130 \mathrm{~mm}$
Velocity at partial flow $v=0.92 \times 2 \approx 1.8 \mathrm{~m} / \mathrm{s}$
22. For full-flow conditions, from Fig. 2.21 read $Q=1800$ gpm. From Fig. 2.22, the maximum value of $q / Q=1.08$ when $d / D=0.93$. Therefore, the highest discharge capacity for the 18 " in pipe, $q_{\max }=1800 \times 1.08 \approx 1900 \mathrm{gpm}$, would occur at a depth of $d=18 \times 0.93 \approx 17 \mathrm{in}$.
23. For full-flow conditions, from Fig. 2.21 read $Q=0.55 \mathrm{~m}^{3} / \mathrm{s}=550 \mathrm{~L} / \mathrm{s}$. From Fig.2.22, the maximum value of $v / V=1.15$ when $\mathrm{d} / \mathrm{D}=0.82$. Therefore, the highest flow velocity for the 900 mm pipe, $\mathrm{v}_{\max }=0.9 \times 1.15 \approx 1 \mathrm{~m} / \mathrm{s}$, would occur at a depth of $\mathrm{d}=900 \times 0.82 \approx 740$ mm . When the flow occurs at that depth, $\mathrm{q} / \mathrm{Q}=1.05$ and the discharge $\mathrm{q}=580 \mathrm{~L} / \mathrm{s}$
24. $S=0.5 / 100=0.005$

For full-flow conditions, $\mathrm{Q}=0.44 \mathrm{~m}^{3} / \mathrm{s}=440 \mathrm{~L} / \mathrm{s}$ and $\mathrm{V}=1.6 \mathrm{~m} / \mathrm{s}$
Since $d / D=200 / 600=0.33$, from Fig. $2.22 q / Q=0.23$ and $v / V=0.8$ Therefore, $q=440 x$ $0.23 \approx 100 \mathrm{~L} / \mathrm{s}$ and $\mathrm{v}=1.6 \times 0.8 \approx 1.3 \mathrm{~m} / \mathrm{s}$
25. $Q=A \times V=2 \times 0.75 \times 25 / 75=0.5 \mathrm{~m}^{3} / \mathrm{s}=500 \mathrm{~L} / \mathrm{s}$
26. From Eq. $2-12, Q=2.5 \times(4 / 12)^{2.5}=0.16 \mathrm{cfs}$
27. $150 \mathrm{~mm} \times 1 \mathrm{in} / 25.4 \mathrm{~mm} \times 1 \mathrm{ft} / 12 \mathrm{in}=0.492 \mathrm{ft}$

From Eq. 2-12, $\mathrm{Q}=2.5 \times(0.492)^{2.5}=0.425 \mathrm{cfs} \times 28.32 \mathrm{~L} / \mathrm{ft}^{3} \approx 12 \mathrm{~L} / \mathrm{s}$
28. From Eq. 2-13, $Q=3.4 \times(20 / 12) \times(10 / 12)^{1.5}=4.3 \mathrm{cfs} \approx 120 \mathrm{~L} / \mathrm{s}$

