Chapter 2

GEOMETRY

2.1 Lines and Angles

1. $\angle ABE = 90^\circ$

2. $\angle QOR = 90^\circ - 32^\circ = 58^\circ$

3. 4; $\angle BOC$ and $\angle COA$,
$\angle COA$ and $\angle AOD$,
$\angle AOD$ and $\angle DOB$,
and, $\angle DOB$ and $\angle BOC$.

4. (a) $\frac{7.75}{5.65} = \frac{x}{6.50}$

$$x = \frac{7.75(6.50)}{5.65}$$

$$x = 8.92 \text{ ft}$$

(b) More vertical

5. $\angle EBD$ and $\angle DBC$ are acute angles.

6. $\angle ABE$ and $\angle CBE$ are right angles.

7. $\angle ABC$ is a straight angle.

8. $\angle ABD$ is an obtuse angle.

9. The complement of $\angle CBD = 65^\circ$ is 25°.

10. The supplement of $\angle CBD = 65^\circ$ is $180^\circ - 65^\circ = 115^\circ$.

11. Sides BD and BC are adjacent to $< DBC$

12. The angle adjacent to $\angle DBC$ is $\angle DBE$

13. $\angle AOB = 90^\circ + 50^\circ = 140^\circ$

14. $\angle AOC = 90^\circ - 50^\circ = 40^\circ$

15. $\angle BOD = 180^\circ - 90^\circ - 50^\circ = 40^\circ$

16. $\angle 3 = 180^\circ - 35^\circ = 145^\circ$

17. $\angle 1 = 180^\circ - 145^\circ = 35^\circ = \angle 2 = \angle 4$

18. $\angle 5 = \angle 3 = 145^\circ$ **19.** $\angle 1^\circ = 62^\circ$

20. $\angle 2 + \angle 1 = 180^\circ \Rightarrow \angle 2 = 180^\circ - \angle 1$

$$= 180^\circ - 62^\circ = 118^\circ$$

21. $\angle 3 = 90^\circ - 62^\circ = 28^\circ$

22. $\angle 1 + \angle 3 = 90^\circ$
$\angle 3 = 90^\circ - 62^\circ$
$\angle 3 = 28^\circ$

$\angle 3 + \angle 4 = 180^\circ$
$\angle 4 = 180^\circ$
$= 180^\circ$
$\angle 4 = 152^\circ$

23. $\angle BDE = 90^\circ - 44^\circ = 46^\circ$
$\angle BDF = 180^\circ - 46^\circ = 134^\circ$

24. $\angle BDE = 90^\circ - 44^\circ = 46^\circ$
$\angle ABE = 90^\circ + 46^\circ = 136^\circ$

25. $\angle DEB = 44^\circ$

26. $\angle DBE = 46^\circ$

27. $\measuredangle DFE = 90^\circ - \measuredangle FDE$

$$= 90^\circ - 44^\circ$$

$$= 46^\circ$$

28. $\measuredangle ADE = \measuredangle ADB + 90^\circ$

$$= (90^\circ - 44^\circ) + 90^\circ$$

$$= 136^\circ$$

29. $\frac{a}{4.75} = \frac{3.05}{3.20} \Rightarrow a = 4.75 \cdot \frac{3.05}{3.20} = 4.53 \text{ m}$

30. $\frac{3.20}{3.05} = \frac{6.25}{b}$
$b = 5.96$ m

31. $\frac{c}{3.05} = \frac{5.50}{a} = \frac{5.50}{4.53}$
$4.53c = 15.4025$
$c = 3.40$ m

32. $\frac{4.75}{5.05} = \frac{6.25}{d}$
$d = 6.64$ m

33. $\measuredangle BHC = \measuredangle CGD = 25^\circ$

34. $\measuredangle AHC = \measuredangle CGE = 45^\circ$

35. $\measuredangle BCH = \measuredangle CHG = \measuredangle HGC = \measuredangle GCD = 65^\circ$

36. $\measuredangle HAB = \measuredangle JHA = \measuredangle FGE = \measuredangle DEG = 70^\circ$

37. $\measuredangle GHA = \measuredangle HGE = 110^\circ$

38. $\measuredangle CGF = \measuredangle CHJ = 115^\circ$

39. $\measuredangle A = (x+10)^\circ$, $\measuredangle B = (4x-5)^\circ$
(a) $x + 10 = 4x - 5$
$x = 5^\circ$
(b) $x + 10 + 4x - 5 = 180$
$x = 35^\circ$

40. $\measuredangle A = (x+20)^\circ$, $\measuredangle B = (3x-2)^\circ$
(a) $x + 20 + 3x - 2 = 90^\circ$
$x = 18^\circ$
(b) $x + 20 = 3x - 2$
$x = 11^\circ$

41. $\angle BCD = 180^\circ - 47^\circ$
$= 133^\circ$

42. $? = 90^\circ - 28^\circ$
$= 62^\circ$

43. $\frac{2}{2.15} = \frac{3}{AB} \Rightarrow AB = 3.225$
$AC = 2.15 + 3.23 = 5.38$ cm

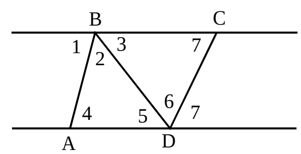
44. $\frac{x}{860} = \frac{590}{550}$
$x = 920$ m

45. $\measuredangle 1 + \measuredangle 2 + \measuredangle 3 = 180^\circ$,
$(\measuredangle 1, \measuredangle 2,$ and $\measuredangle 3$ form a straight line$)$

46. $\measuredangle 4 + \measuredangle 2 + \measuredangle 5 = 180^\circ$, $(\measuredangle 1 = \measuredangle 4, \measuredangle 3 = \measuredangle 5)$

47. The sum of the angles of *ABD* is 180°.

48.



$\measuredangle 1 + \measuredangle 2 + \measuredangle 3 = 180^\circ$, $\measuredangle 1 = \measuredangle 4$
$\measuredangle 4 + \measuredangle 2 + \measuredangle 3 = 180^\circ$
$\measuredangle 5 + \measuredangle 6 + \measuredangle 7 = 180^\circ$
$\measuredangle 4 + (\measuredangle 2 + \measuredangle 3) + (\measuredangle 5 + \measuredangle 6) + \measuredangle 7 = 180^\circ + 180^\circ$
$= 360^\circ$
The sum of the angles of ABCD = 360°

2.2 Triangles

1. $\angle 5 = 45^\circ \Rightarrow \angle 3 = 45^\circ$

$\angle 2 = 180^\circ - 70^\circ - 45^\circ = 65^\circ$

2. $A = \frac{1}{2}bh = \frac{1}{2}(61.2)(5.75)$

$A = 176 \text{ in.}^2$

3. $AC^2 = AB^2 + BC^2$

$= 6.25^2 + 3.2^2$

$AC = \sqrt{6.25^2 + 3.2^2}$

$AC = 7.02 \text{ m}$

4. $\frac{h}{3.0} = \frac{24}{4.0}$

$h = 18 \text{ ft}$

5. $\angle A = 180^\circ - 84^\circ - 40^\circ = 56^\circ$

6. $\measuredangle A = 90^\circ - 48^\circ = 42^\circ$

7. This is an isosceles triangle, so the base angles are equal. $\angle A = 180^\circ - (66^\circ + 66^\circ) = 48^\circ$

8. $\angle A = \frac{1}{2}(180^\circ - 110^\circ) = 35^\circ$

9. $A = \frac{1}{2}bh = \frac{1}{2}(7.6)(2.2) = 8.4 \text{ ft}^2$

10. $A = \frac{1}{2}bh = \frac{1}{2}(16.0)(7.62) = 61.0 \text{ mm}^2$

11. Area $= \sqrt{471(471-205)(471-415)(471-322)}$

$= 32,300 \text{ cm}^2$

12. $p = 0.862 + 0.235 + 0.684 = 1.781 \text{ in.}$

$s = \frac{1.781}{2} = 0.8905$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{.8905(.8905-.862)(.8905-.235)(.8905-.684)}$

$A = 0.586 \text{ in.}^2$

13. $A = \frac{1}{2}bh = \frac{1}{2}(3.46)(2.55) = 4.41 \text{ ft}^2$

14. $A = \frac{1}{2}bh = \frac{1}{2}(234)(342) = 40,000 \text{ mm}^2$

15. Area

$= \sqrt{1.428(1.428-0.9860)(1.428-0.986)}$

$\sqrt{(1.428-0.884)}$

$= 0.390 \text{ m}^2$

16. $s = \frac{3(320)}{2} = 480$

$A = \sqrt{s(s-a)^3}$

$= \sqrt{480(480-320)^3}$

$= 44,000 \text{ yd}^2$

17. $p = 205 + 322 + 415$

$p = 942 \text{ cm}$

18. $p = 23.5 + 86.2 + 68.4$

$p = 178 \text{ in.}$

19. $3(21.5) = 64.5 \text{ cm}$

20. Perimeter $= 2(2.45) + 3.22 = 8.12 \text{ in.}$

21. $c = \sqrt{13.8^2 + 22.7^2} = 26.6 \text{ ft}$

22. $c^2 = a^2 + b^2$

$= 2.48^2 + 1.45^2$

$c = 2.87 \text{ m}$

23. $b = \sqrt{551^2 - 175^2} = 522 \text{ cm}$

24. $c^2 = a^2 + b^2$

$0.836^2 = a^2 + 0.474^2$

$a = 0.689$ in.

25. $\angle B = 90^\circ - 23^\circ = 67^\circ$

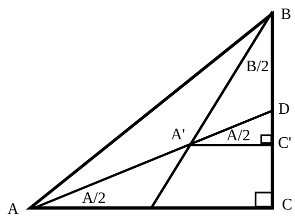
26. $c^2 = 90.5^2 + 38.4^2$

$c = 98.3$ cm

27. Perimeter $= 98.3 + 90.5 + 38.4 = 227.2$ cm

28. $A = \frac{1}{2}(90.5)(38.4) = 1740\ \text{cm}^2$

29.



$\Delta ADC \sim \Delta A'DC' \Rightarrow \measuredangle DA'C' = A/2$

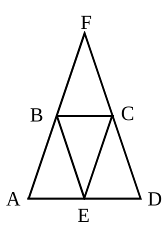
$\measuredangle$ between bisectors = $\measuredangle BA'D$

$\Delta BA'C',\ \frac{B}{2} + \left(\measuredangle BA'D + A/2\right) = 90^\circ$

from which $\measuredangle BA'D = 90^\circ - \left(\frac{A}{2} + \frac{B}{2}\right)$

or $\measuredangle BA'D = 90^\circ - \left(\frac{A+B}{2}\right) = 90^\circ - \frac{90^\circ}{2} = 45^\circ$

30.

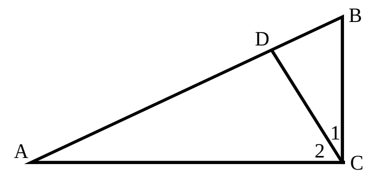


$\measuredangle A = \measuredangle D$ since ΔAFD is isosceles. Since $AF = FD$ $(\Delta AFD$ is isosceles$)$ and since B and C are mid-points, $AB = CD$ which means ΔBAE and ΔCED are the same size and shape. Therefore, $BE = EC$ from which it follows that the inner ΔBCE is isosceles.

31. An equilateral triangle.

32. Yes, if one of the angles of the triangle is obtuse.

33.

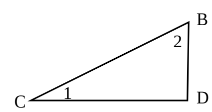


$\measuredangle A + \measuredangle B = 90^\circ$

$\measuredangle 1 + \measuredangle B = 90^\circ$

$\Rightarrow \quad \measuredangle A = \measuredangle 1$

redraw ΔBDC as

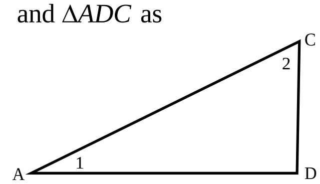


$\measuredangle 1 + \measuredangle 2 = 90^\circ$

$\measuredangle 1 + \measuredangle B = 90^\circ$

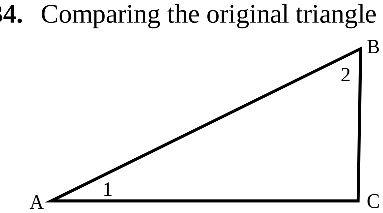
$\Rightarrow \quad \measuredangle 2 = \measuredangle B$

and ΔADC as



ΔBDC and ΔADC are similar.

34. Comparing the original triangle



to the two smaller triangles shows that all three are similar.

35. $\angle LMK$ and $\angle OMN$ are vertical angles and thus equal $\Rightarrow \angle KLM = \angle MON$. The corresponding angles are equal and the triangles are similar.

36. $\angle ACB = \angle ADC = 90^\circ$; $\angle A = \angle A$; $\angle DCA = \angle CBA$, therefore $\triangle ACB \sim \triangle ADC$

37. Since $\triangle MKL \sim \triangle MNO$; $KN = KM - MN$; $15 - 9 = 6$ $= KM$; $\frac{KM}{MN} = \frac{LM}{MO}$; $\frac{6}{9} = \frac{LM}{12}$; $9LM = 72$; $LM = 8$

38. $\frac{AB}{12} = \frac{12}{9}$

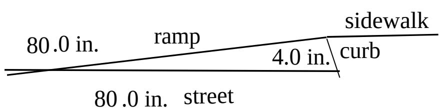
$AB = 16$

39. $p = 6 + 25 + 29 = 60$

$A = \sqrt{30(30-6)(30-25)(30-29)} = 60$

Yes, the triangle is perfect.

40.



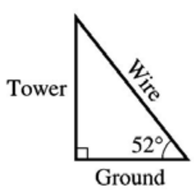
$\frac{\text{street}}{4.0} = \frac{20.0}{1} \Rightarrow \text{street} = 4.0(20.0)$

$\text{ramp} = \sqrt{(4.0(20.0))^2 + 4.0^2} = 80.09993758,$ calculator

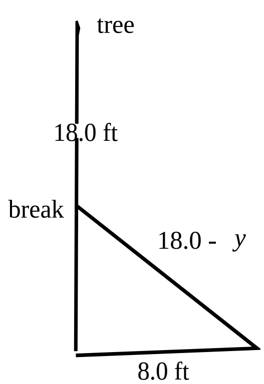
$\text{ramp} = 80$ in. (two significant digits)

41. $\angle = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

42. angle between tower and wire $= 90^\circ - 52^\circ = 38^\circ$



43.



$(18.0 - y)^2 = y^2 + 8.0^2$

$18.0^2 - 2(18.0)y + y^2 = y^2 + 8.0^2$

$y = 7.2$ ft (two significant digits)

44. $\frac{3(1600)}{2} = 2400$

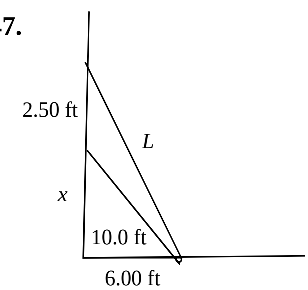
$A = \sqrt{2400(2400-1600)^3}$

$A = 1{,}100{,}000 \text{ km}^2$

45. $A = \frac{1}{2}bh = \frac{1}{2}(8.0)(15) = 60 \text{ ft}^2$

46. $d = \sqrt{750^2 + 550^2} = 930$ m

47.



$10.0^2 = x^2 + 6.00^2$

$x = 8.00$

$L = \sqrt{(8.00 + 2.50)^2 + 6.00^2}$

$L = 12.09338662$, calculator

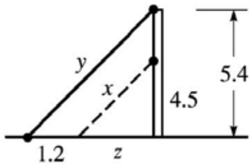
$L = 12.1$ ft (three significant digits)

48. Taking the triangles in clockwise order and using Pythagorean Theorem together with side opposite 30° angle is half the hypotenuse gives side opposite 30° angle and third side, respectively.

49. $d = \sqrt{18^2 + 12^2 + 8^2} = 23$ ft

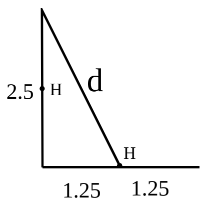
50. $\frac{x}{45.6} = \frac{1}{1.12}$, $x = 38$ m

51.



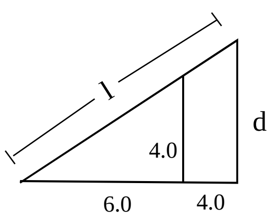
$$\frac{4.5}{z} = \frac{5.4}{1.2+z}$$
$$z = 6.0 \text{ m}$$
$$x^2 = z^2 + 4.5^2$$
$$x = 7.5 \text{ m}$$
$$y^2 = (1.2+6)^2 + 5.4^2$$
$$y = 9.0 \text{ m}$$

52.



$$d^2 = 1.25^2 + 5.0^2$$
$$d = 5.6 \text{ ft}$$

53.

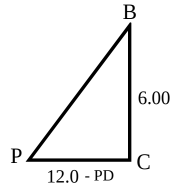


$$\frac{d}{8.0} = \frac{4.0}{6.0} \Rightarrow d = \frac{4.0(8.0)}{6.0}$$
$$l^2 = 8.0^2 + d^2 = 8.0^2 + \left(\frac{4.0(8.0)}{6.0}\right)^2$$
$$l = 9.6 \text{ ft}$$

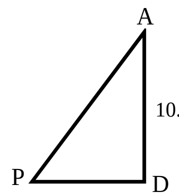
54. $\frac{ED}{80} = \frac{312}{50}$

$ED = 499$ ft

55. Redraw ΔBCP as



ΔAPD is



from which $\Delta BCP \sim \Delta ADP$, so $\frac{6.00}{12.0 - PD} = \frac{10.0}{PD}$

$\Rightarrow PD = 7.50$ and $PC = 12.0 - PD = 4.50$

$l = PB + PA = \sqrt{4.50^2 + 6.00^2} + \sqrt{7.50^2 + 10.0^2}$

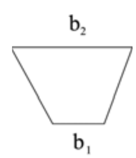
$l = 20.0$ mi

56. $\frac{1}{2}wd + 160 = \frac{1}{2}w(d+16)$

$$\frac{1}{2}wd + 160 = \frac{1}{2}wd + 8w$$
$$8w = 160$$
$$w = 20 \text{ cm}$$
$$d = w - 12 = 8 \text{ cm}$$

2.3 Quadrilaterals

1.



trapezoid

2. $L = 4s + 2w + 2l$

$= 4(21) + 2(21) + 2(36)$

$= 198$ in.

3. $A_1 = \frac{1}{2}bh = \frac{1}{2}(72)(55) = 2000 \text{ ft}^2$

$A_2 = bh = 72(55) = 4000 \text{ ft}^2$

$A_3 = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(55)(72+35)$

$= 2900 \text{ ft}^2$

The total lawn area is about 8900 ft^2.

4. $2(w+3.0) + 2w = 26.4$

$2w + 6.0 + 2w = 26.4$

$4w = 20.4$

$w = 5.1 \text{ mm}$

$w + 3.0 = 8.1 \text{ mm}$

5. $p = 4s = 4(65) = 260 \text{ m}$

6. $p = 4(2.46) = 9.84 \text{ ft}$

7. $p = 2(0.920) + 2(0.742) = 3.324 \text{ in.}$

8. $p = 2(142) + 2(126) = 536 \text{ cm}$

9. $p = 2l + 2w = 2(3.7) + 2(2.7) = 12.8 \text{ m}$

10. $p = 2(27.3) + 2(14.2) = 83.0 \text{ in.}$

11. $p = 0.362 + 0.730 + 0.440 + 0.612 = 2.144 \text{ ft}$

12. $p = 272 + 392 + 223 + 672 = 1559 \text{ cm}$

13. $A = s^2 = 2.7^2 = 7.3 \text{ mm}^2$

14. $A = 15.6^2 = 243 \text{ ft}^2$

15. $A = 0.920(0.742) = 0.683 \text{ in.}^2$

16. $A = 142(126) = 17,900 \text{ cm}^2$

17. $A = bh = 3.7(2.5) = 9.3 \text{ m}^2$

18. $A = 27.3(12.6) = 344 \text{ in.}^2$

19. $A = (1/2)(29.8)(61.2 + 73.0) = 2000 \text{ ft}^2$

20. $A = \frac{1}{2}(392 + 672)(201) = 107,000 \text{ cm}^2$

21. $p = 2b + 4a$

22. $p = a + b + b + a + (b-a) + (b-a) = 4b$

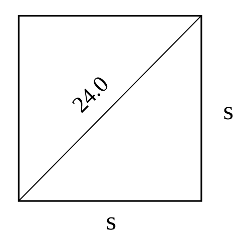
23. $A = b \times h + a^2 = bh + a^2$

24. $A = ab + a(b-a) = 2ab - a^2$

25. The parallelogram is a rectangle.

26. The triangles are congruent. Corresponding sides and angles are equal.

27.



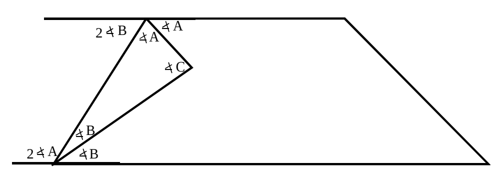
$s^2 + s^2 = 24.0^2$

$2s^2 = 24.0^2$

$s^2 = \frac{24.0^2}{2}$

$A = s^2 = 288 \text{ cm}^2$

28.



At top $2\measuredangle B + 2\measuredangle A = 180^\circ$

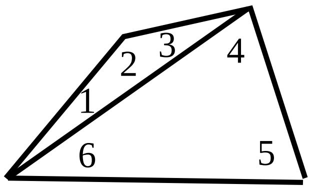
$\measuredangle B + \measuredangle A = 90^\circ$

In triangle $\measuredangle A + \measuredangle B + \measuredangle C = 180^\circ$

$90^\circ + \measuredangle C = 180^\circ$

$\measuredangle C = 90^\circ$

29.



sum of interior angles
$= \measuredangle 1 + \measuredangle 2 + \measuredangle 3 + \measuredangle 4 + \measuredangle 5 + \measuredangle 6$
$= 180^\circ + 180^\circ$
$= 360^\circ$

30. $S = 180(n-2)$

(a) $n = \dfrac{S}{180} + 2$

(b) $n = \dfrac{3600^\circ}{180^\circ} + 2$
$n = 22$

31. $A =$ area of left rectangle + area of right rectangle
$A = ab + ac$
$A =$ area of entire rectangle
$A = a(b+c)$ which illustrates the distributive property.

32. $A = (a+b)(a+b) = (a+b)^2$
$A = ab + ab + a^2 + b^2$
$A = a^2 + 2ab + b^2$ which illustrates that the square of the sum is the square of the first term plus twice the product of the two terms plus the square of the second term.

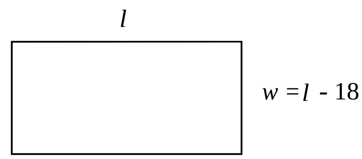
33. The diagonal always divides the rhombus into two congruent triangles. All outer sides are always equal.

34. $\sqrt{16^2 + 12^2} = \sqrt{400} = 20$

35. (a) For the courtyard: $s = \dfrac{p}{4} = \dfrac{320}{4} = 80$. For the outer edge of the walkway:
$p = 4(80+6) = 344$ m.

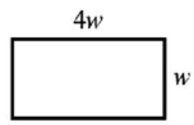
(b) $A = 86^2 - 80^2 = 996$
$A = 1000\ \text{m}^2$ (2 significant digits)

36.



$p = 2l + 2(l - 18) = 180$ from which
$l = 54$ in.
$w = l - 18 = 54 - 18$
$w = 36$ in.

37.



$w + 2.5 = 4w - 4.7$
$w = 2.4$ ft
$4w = 9.6$ ft

38. $A = 1.80 \times 3.50 = 6.30\ \text{ft}^2$

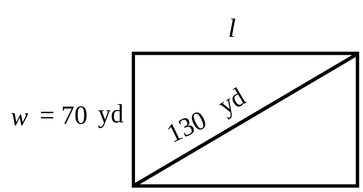
39. $A = 2(\text{area of trapezoid} - \text{area of window})$
$= 2\left(\dfrac{1}{2}(28+16)\cdot 8 - 12(3.5)\right)$
$= 268\ \text{ft}^2$

$\dfrac{1\ \text{gal}}{320\ \text{ft}^2} = \dfrac{x}{268\ \text{ft}^2}$

$x = 0.84$ gal

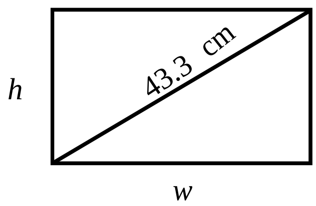
ht of trapezoid
$= \sqrt{10^2 - \left(\dfrac{28-16}{2}\right)^2}$
$= 8.0$ ft

40,



$l = \sqrt{130^2 - 70^2}$
$p = 2l + 2w$
$p = 2\sqrt{130^2 - 70^2} + 2(70)$
$p = 360$ yd

41.



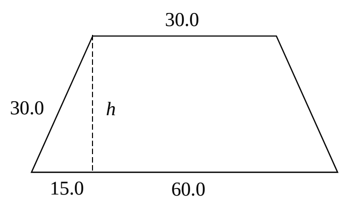
$$\frac{w}{h} = 1.60 \Rightarrow w = 1.60h$$

$$43.3^2 = h^2 + w^2 = h^2 + (1.6h)^2$$

$$h = 22.9 \text{ cm}$$

$$w = 1.60h = 36.7 \text{ cm}$$

42.

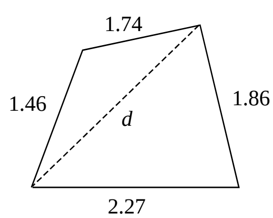


$$h = \sqrt{30.0^2 + 15.0^2}$$

$$A = 6 \cdot \frac{1}{2}(30.0 + 60.0) \cdot \sqrt{30.0^2 + 15.0^2}$$

$$A = 9060 \text{ in.}^2$$

43.



$$d = \sqrt{2.27^2 + 1.86^2}$$

For right triangle, $A = \frac{1}{2}(2.27)(1.86)$

For obtuse triangle, $s = \frac{1.46 + 1.74 + d}{2}$

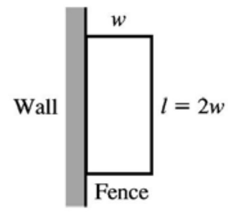
and $A = \sqrt{s(s - 1.46)(s - d)(s - 1.74)}$

A of quadrilateral = Sum of areas of two triangles,

$$A = \frac{1}{2}(2.27)(1.86) + \sqrt{s(s - 1.46)(s - d)(s - 1.74)}$$

$$A = 3.04 \text{ km}^2$$

44.



$$50(2w) + 5(2w) + 5w + 5w = 13,200$$

$$w = 110 \text{ m}$$

$$l = 2w = 220 \text{ m}$$

45. 360°. A diagonal divides a quadrilateral into two triangles, and the sum of the interior angles of each triangle is 180°.

46. $A = \frac{1}{2}d_1\left(\frac{d_2}{2}\right) + \frac{1}{2}d_1\left(\frac{d_2}{2}\right)$

$$A = \frac{1}{2}d_1 d_2$$

2.4 Circles

1. $\angle OAB + OBA + \angle AOB = 180^\circ$

$$\angle OAB + 90^\circ + 72^\circ = 180^\circ$$

$$\angle OAB = 18^\circ$$

2. $A = \pi r^2 = \pi(2.4)^2$

$$A = 18 \text{ km}^2$$

3. $p = 2s + \frac{2\pi s}{4} = 2s + \frac{\pi s}{2}$

$$p = 2(3.25) + \frac{\pi(3.25)}{2}$$

$$p = 11.6 \text{ in.}$$

$$A = \frac{\pi s^2}{4} = \frac{\pi(3.25)}{4}$$

$$A = 8.30 \text{ in.}^2$$

4. $\overset{\frown}{AC} = 2 \cdot \angle ABC$
$= 2(25^\circ)$
$= 50^\circ$

5. (a) AD is a secant line.
(b) AF is a tangent line.

6. (a) EC and BC are chords.
(b) $\angle ECO$ is an inscribed angle.

7. (a) $AF \perp OE$.
(b) $\triangle OCE$ is isosceles.

8. (a) EC and $\overset{\frown}{EC}$ enclose a segment.
(b) radii OE and OB enclose a sector with an acute central angle.

9. $c = 2\pi r = 2\pi(275) = 1730$ ft

10. $c = 2\pi r = 2\pi(0.563) = 3.54$ m

11. $d = 2r$; $c = \pi d = \pi(23.1) = 72.6$ mm

12. $c = \pi d = \pi(8.2) = 26$ in.

13. $A = \pi r^2 = \pi(0.0952^2) = 0.0285 \text{ yd}^2$

14. $A = \pi r^2 = \pi(45.8)^2 = 6590 \text{ cm}^2$

15. $A = \pi(d/2)^2 = \pi(2.33/2)^2 = 4.26 \text{ m}^2$

16. $A = \frac{1}{4}\pi d^2 = \frac{1}{4}\pi(1256)^2 = 1{,}239{,}000 \text{ ft}^2$

17. $\angle CBT = 90^\circ - \angle ABC = 90^\circ - 65^\circ = 25^\circ$

18. $\angle BCT = 90^\circ$, any angle such as $\angle BCA$ inscribed in a semicircle is a right angle and $\angle BCT$ is supplementary to $\angle BCA$.

19. A tangent to a circle is perpendicular to the radius drawn to the point of contact. Therefore,
$\angle ABT = 90^\circ$
$\angle CBT = \angle ABT - \angle ABC = 90^\circ - 65^\circ = 25^\circ$;
$\angle CAB = 25^\circ$

20. $\angle BTC = 65^\circ$; $\angle CBT = 35^\circ$ since it is complementary to $\angle ABC = 65^\circ$.

21. ARC BC $= 2(60^\circ) = 120^\circ$

22. $\overset{\frown}{BC} = 2(60^\circ) = 120^\circ$
$\overset{\frown}{AB} + 80^\circ + 120^\circ = 360^\circ$
$\overset{\frown}{AB} = 160^\circ$

23. $\angle ABC = (1/2)(80^\circ) = 40^\circ$ since the measure of an inscribed angle is one-half its intercepted arc.

24. $\angle ACB = \frac{1}{2}(160^\circ) = 80^\circ$

25. $022.5^\circ\left(\frac{\pi}{180^\circ}\right) = 0.393$ rad

26. $60.0^\circ = 60.0^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = 1.05$ rad

27. $125.2^\circ = 125.2\pi \text{ rad}/180^\circ = 2.185$ rad

28. $3230^\circ = 3230^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = 56.4$ rad

29. $P = \frac{1}{4}(2\pi r) + 2r = \frac{\pi r}{2} + 2r$

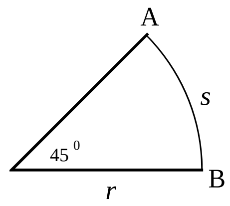
30. Perimeter $= a + b + \frac{1}{4} \cdot 2\pi r + r$

31. Area $= \frac{1}{4}\pi r^2 - \frac{1}{2}r^2$

32. Area $= \frac{1}{2}(ar) + \frac{1}{4}\pi r^2$

33. All are on the same diameter.

34.

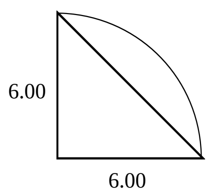


$$\widehat{AB} = 45^\circ$$

$$\frac{s}{2\pi r} = \frac{45^\circ}{360^\circ}$$

$$s = \frac{\pi}{4}\cdot r$$

35.



A of sector = A of quarter circle – A of triangle

$$A = \frac{1}{4}\cdot\pi(6.00)^2 - \frac{1}{2}(6.00)(6.00)$$

$$A = 10.3 \text{ in.}^2$$

36. $\angle ACB = \angle DCE$ (vertical angles)
$\angle BAC = \angle DEC$ and
$\angle ABC = \angle CDE$ (alternate interior angles)
Therefore, the triangles are similar since corresponding angles are equal.

37. $c = 2\pi r \Rightarrow \pi = \frac{c}{2r};\ d = 2r \Rightarrow r = \frac{d}{2}$ from which

$$\pi = \frac{c}{2\cdot\frac{d}{2}}$$

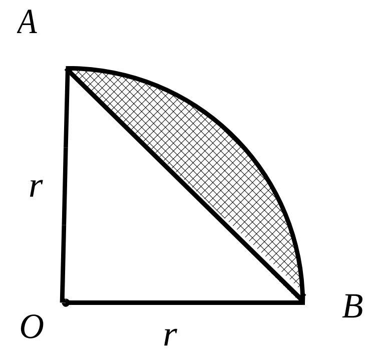
$\pi = \frac{c}{d}\cdot\pi$ is the ratio of the circumference to the diameter.

38. $\frac{d}{c} = \frac{\frac{5}{4}}{4} = \frac{5}{4}\cdot\frac{1}{4} = \frac{5}{16}$

$c = \frac{16}{5}d \Rightarrow \pi = \frac{16}{5} = 3.2$

which is incorrect since $\pi = 3.14159\cdots$

39.

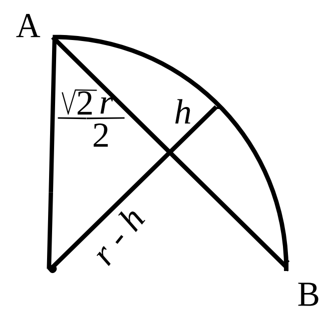


$$A_{\text{segment}} = \text{area of quarter circle} - \text{area of triangle}$$

$$A_{\text{segment}} = \frac{1}{4}\pi r^2 - \frac{1}{2}\cdot r\cdot r\cdot\frac{2}{2}$$

$$A_{\text{segment}} = \frac{r^2(\pi-2)}{4}$$

40.



$$\overline{AB}^2 = r^2 + r^2$$

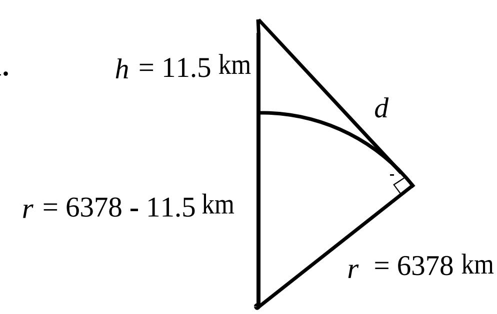
$$\overline{AB} = \sqrt{2}r$$

$\overline{AC} = \frac{1}{2}AB = \frac{\sqrt{2}r}{2}$ in right triangle OAC

$r^2 = \left(\frac{\sqrt{2}r}{2}\right)^2 + (r-h)^2$ from which

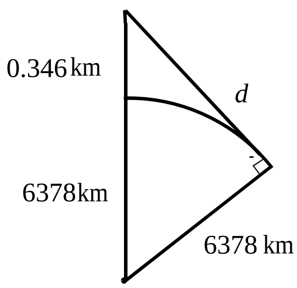
$$h = \frac{\left(2-\sqrt{2}\right)r}{2}$$

41.



$$(6378+11.5)^2 = d^2 + 6378^2$$
$$d = 383 \text{ km}$$

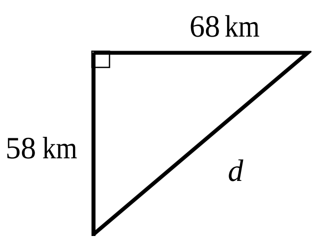
42.



$$(6378+0.346)^2 = d^2 + 6378^2$$
$$d = 66.4 \text{ km}$$

43. $d = \sqrt{68^2 + 58^2}$

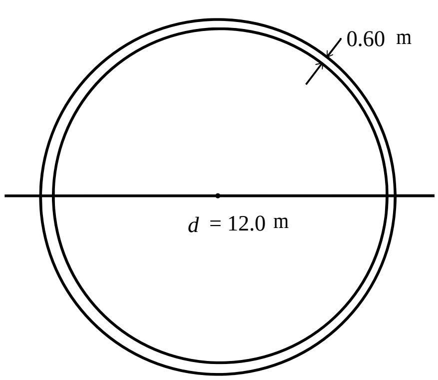
$d = 89 > 85$



Signal cannot be received.

44. Using $A = \dfrac{\pi d^2}{4}$,

$$A = \frac{\pi(12.0 + 2(0.60))^2}{4} - \frac{\pi(12.0)^2}{4}$$
$$A = 23.8 \text{ m}^2$$



using $A = \dfrac{\pi d^2}{4}$,

$$A = \frac{\pi(12.0 + 2(0.60))^2}{4} - \frac{\pi(12.0)^2}{4}$$
$$A = 23.8 \text{ m}^2$$

45. $C = 2\pi r = 2\pi(3960) = 24{,}900 \text{ mi}$

46. $11(2\pi r) = 109$

$r = 1.58 \text{ mm}$

47. $\dfrac{A_{\text{basketball}}}{A_{\text{hoop}}} = \dfrac{\pi\left(\dfrac{12.0}{2}\right)^2}{\pi\left(\dfrac{18.0}{2}\right)^2} = \dfrac{4}{9}$

48. $\text{flow rate} = \dfrac{\text{volume}}{\text{time}} = \dfrac{\pi r_1^2 L}{t}$

$2 \cdot \text{flow rate} = \dfrac{\pi r_2^2}{t} = \dfrac{2\pi r_1^2}{t}$

$r_2^2 = 2 \cdot r_1^2$

$r_2 = \sqrt{2} r_1$

49. $c = 112;\ c = \pi d;\ d = c/\pi = 112/\pi = 35.7$ in.

50. $A = \dfrac{\pi}{2}\left(90^2 - 45^2\right)$

$A = 9500 \text{ cm}^2$

51. Let D = diameter of large conduit, then

$D = 3d$ where d = diameter of smaller conduit

$F = \dfrac{\pi}{4} D^2 = 7 \cdot \dfrac{\pi}{4} \cdot d^2$

$F = \dfrac{7d^2}{D^2} = \dfrac{7d^2}{(3d)^2} = \dfrac{7d^2}{9d^2}$

$F = \dfrac{7}{9}$

The smaller conduits occupy $\dfrac{7}{9}$ of the larger conduits.

52. A of room = A of rectangle $+ \frac{3}{4} A$ of circle

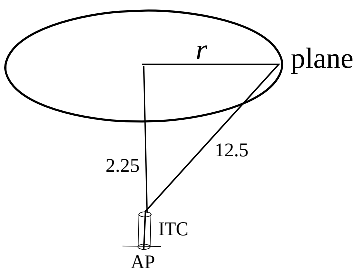
$$A = 24(35) + \frac{3}{4}\pi(9.0)^2 = 1030.85174$$

$$A = 1000 \text{ ft}^2, \text{two significant digits}$$

53. Length $= (2)\frac{3}{4}(2\pi)(5.5) + (4)(5.5) = 73.8$ in.

54. $d = 4 \cdot 2\pi\sqrt{12.5^2 - 2.25^2}$

$d = 309$ km



55. Horizontally and opposite to original direction

56. Let A be the left end point at which the dashed lines intersect and C be the center of the gear. Draw a line from C bisecting the 20° angle. Call the intersection of this line and the extension of the upper dashed line B, then

$$\frac{360^\circ}{24 \text{ teeth}} = \frac{15^\circ}{\text{tooth}} \Rightarrow \angle ACB = 7.5^\circ$$

$$\angle ABC = 180^\circ - \frac{20^\circ}{2} = 170^\circ$$

$$\angle \frac{1}{2}x + \angle ABC + \angle ACB = 180^\circ$$

$$\angle \frac{1}{2}x + 170^\circ + 7.5^\circ = 180^\circ$$

$$\angle \frac{1}{2}x = 2.5^\circ$$

$$x = 5^\circ$$

2.5 Measurement of Irregular Areas

1. The use of smaller intervals improves the approximation since the total omitted area or the total extra area is smaller.

2. Using data from the south end gives five intervals. Therefore, the trapezoidal rule must be used since Simpson's rule cannot be used for an odd number of intervals.

3. Simpson's rule should be more accurate in that it accounts better for the arcs between points on the upper curve.

4. The calculated area would be too high since each trapezoid would include more area than that under the curve.

5. $A_{\text{trap}} = \frac{2.0}{2}[0.0 + 2(6.4) + 2(7.4) + 2(7.0) + 2(6.1)]$

$[+2(5.2) + 2(5.0) + 2(5.1) + 0.0]$

$A_{\text{trap}} = 84.4 = 84 \text{ m}^2$ to two significant digits

6. A_{simp}

$$= \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$$

$$= \frac{20}{3}(0 + 4(6.4) + 2(7.4) + 4(7.0) + 2(6.1) + 4(5.2)$$

$$+ 2(5.0) + 4(5.1) + 0) = 88 \text{ m}^2$$

7. $A_{\text{simp}} = \frac{1.00}{3}(0 + 4(0.52) + 2(0.75) + 4(1.05)$

$+ 2(1.15) + 4(1.00) + 0.62) = 4.9 \text{ ft}^2$

8. $A_{\text{trap}} = \frac{1}{2}(0 + 2(0.52) + 2(0.75) + 2(1.05) + 2(1.15)$

$+ 2(1.00) + 0.62) = 4.8 \text{ ft}^2$

9. $A_{\text{trap}} = \frac{0.5}{2}[0.6 + 2(2.2) + 2(4.7) + 2(3.1) + 2(3.6)]$

$[+2(1.6) + 2(2.2) + 2(1.5) + 0.8]$

$A_{\text{trap}} = 9.8 \text{ m}^2$

10. $A_{\text{simp}} = \frac{0.5}{3}(0.6 + 4(2.2) + 2(4.7) + 4(3.1) + 2(3.6) + 4(1.6) + 2(2.2) + 4(1.5) + 0.8)$

$= 9.3 \text{ mi}^2$

11. $A = \frac{10}{2}(38 + 2(24) + 2(25) + 2(17) + 2(34) + 2(29) + 2(36) + 2(34) + 30)$

$A = 2330 \text{ mm}^2 \left(\frac{23^2 \text{ km}^2}{10^2 \text{ mm}^2}\right)$

$A = 12,000 \text{ km}^2$

12. $A = 2 \cdot \frac{4.0}{2}\left[2(55.0) + 2(2(54.8)) + 2(2(54.0)) + 2(2(53.6)) + 2(2(51.2)) + 2(2(49.0)) + 2(2(45.8)) + 2(2(42.0)) + 2(2(37.2)) + 2(2(31.1)) + 2(2(21.7)) + 2(0.0)\right]$

$A = 7500 \text{ m}^2$

13. $A_{\text{trap}} = \frac{45}{2}[170 + 2(360) + 2(420) + 2(410) + 2(390) + 2(350) + 2(330) + 2(290) + 230]$

$A_{\text{trap}} = 120,000 \text{ ft}^2$

14. $A_{\text{simp}} = \frac{45}{3}(230 + 4(290) + 2(330) + 4(340) + 2(390) + 4(410) + 2(420) + 4(360) + 170)$

$= 120,000 \text{ ft}^2$

15. $A_{\text{simp}} = \frac{50}{3}(5 + 4(12) + 2(17) + 4(21) + 2(22) + 4(25) + 2(26) + 4(16) + 2(10) + 4(8) + 0)$

$= 8100 \text{ ft}^2$

16. $A_{\text{trap}} = \frac{2.0}{2}(3.5 + 2(6.0) + 2(7.6) + 2(10.8) + 2(16.2) + 2(18.2) + 2(19.0) + 2(17.8) + 2(12.5) + 8.2)$

$= 229 \text{ in.}^2$

$A_{\text{circle}} = \pi r^2 = \pi(d/2)^2 = \pi(2.5/2)^2 = 4.9 \text{in.}^2$

$A_{\text{total}} = 229 - 2(4.9) = 219 \text{ in.}^2$

17. A_{trap}

$= \frac{0.500}{2}\left[0.0 + 2(1.732) + 2(2.000) + 2(1.732) + 0.0\right] = 2.73 \text{ in.}^2$

This value is less than 3.14 in.^{2} because all of the trapezoids are inscribed.

18. $A_{\text{trap}} = \frac{0.250}{2}(0.000 + 2(1.323) + 2(1.732) + 2(1.936) + 2(2.000) + 2(1.936) + 2(1.732) + 2(1.323) + 0.000)$

$= 3.00 \text{ in.}^2$

The trapezoids are small so they can get closer to the boundary.

19. $A_{\text{simp}} = \frac{0.500}{3}(0.000 + 4(1.732) + 2(2.000) + 4(1.732) + 0.000)$

$= 2.98 \text{ in.}^2$

The ends of the areas are curved so they can get closer to the boundary.

20. $A_{\text{simp}} = \frac{0.250}{3}(0.000 + 4(1.323) + 2(1.732) + 4(1.936) + 2(2.000) + 4(1.936) + 2(1.732) + 4(1.323) + 0.000)$

$= 3.08 \text{ in.}^2$

The areas are smaller so they can get closer to the boundary.

2.6 Solid Geometric Figures

1. $V_1 = lwh_1,\ V_2 = (2l)(w)(2h) = 4lwh = 4V_1$
The volume is four times as much.

2. $$s^2 = r^2 + h^2$$
$$17.5^2 = 11.9^2 + h^2$$
$$h = 12.8 \text{ cm}$$

3. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{11.9}{2}\right)^2 (2(10.4))$
$V = 771 \text{ cm}^3$

4. $V = \pi(40.0)^2\left(\frac{122}{2}\right) + \frac{2}{3}\pi(40.0)^3$
$V = 441{,}000 \text{ ft}^3$

5. $V = e^3 = 7.15^3 = 366 \text{ ft}^3$

6. $V = \pi r^2 h = \pi(23.5)^2(48.4) = 84{,}000 \text{ cm}^3$

7. $A = 2\pi r^2 + 2\pi rh = 2\pi(689)^2 + 2\pi(689)(233)$
$= 3{,}990{,}000 \text{ mm}^2$

8. $A = 4\pi r^2 = 4\pi(0.067)^2 = 0.056 \text{ in.}^2$

9. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.877^3) = 2.83 \text{ yd}^3$

10. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(25.1)^2(5.66) = 3730 \text{ m}^3$

11. $S = \pi rs = \pi(78.0)(83.8) = 20{,}500 \text{ cm}^2$

12. $S = \frac{1}{2}ps = \frac{1}{2}(345)(272) = 46{,}900 \text{ ft}^2$

13. $V = \frac{1}{3}Bh = \frac{1}{3}(0.76^2)(1.30) = 0.25 \text{ in.}^3$

14. $V = Bh = (29.0)^2(11.2) = 9420 \text{ cm}^3$

15. $S = \frac{1}{2}ps = \frac{1}{2}(3\times 1.092)(1.025) = 3.358 \text{ m}^2$

16. $S = 2\pi rh = 2\pi(d/2)h = 2\pi(250/2)(347)$
$= 273{,}000 \text{ ft}^2$

17. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi\left(\frac{0.83}{2}\right)^3 = 0.15 \text{ yd}^3$

18. $b = \frac{22.4}{2} = 11.2;\ h = \sqrt{s^2 - b^2} = \sqrt{14.2^2 - 11.2^2}$
$= 8.73 \text{ m}$
$V = \frac{1}{3}Bh = \frac{1}{3}(22.4)^2(8.73) = 1460 \text{ m}^3$

19. $s = \sqrt{h^2 + r^2} = \sqrt{0.274^2 + 3.39^2} = 3.40 \text{ cm}$
$A = \pi r^2 + \pi rs = \pi(3.39)^2 + \pi(3.39)(3.40)$
$= 72.3 \text{ cm}^2$

20. There are four triangles in this shape.
$s = \sqrt{3.67^2 - \left(\frac{3.67}{2}\right)^2} = 3.18,\ A = \frac{1}{2}ps$
$= \frac{1}{2}(4\times 3.67)(3.18) = 23.3 \text{ in.}^2$

21. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\frac{d^3}{8}$
$V = \frac{1}{6}\pi d^3$

22. $A = A_{\text{flat}} + A_{\text{curved}}$
$= \pi r^2 + \frac{1}{2}\cdot 4\pi r^2$
$= 3\pi r^3$

23. $\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = \frac{\pi(2r)^2\frac{h}{2}}{\frac{1}{3}\pi r^2 h} = \frac{6}{1}$

24. $\pi r^2 = \frac{1}{4}\left(\pi r^2 + \pi rs\right)$

$4r^2 = r^2 + rs$

$3r^2 = rs$

$\frac{r}{s} = \frac{1}{3}$

25. $\frac{\text{final surface area}}{\text{original surface area}} = \frac{4\pi(2r)^2}{4\pi r^2} = \frac{4}{1}$

26. $62.4\frac{\text{lb}}{\text{ft}^3}\cdot 1\text{ mi}^2\cdot\frac{5280\text{ ft}^2}{\text{mi}^2}\cdot\frac{\text{ft}}{12\text{ in.}}\cdot 1\text{ in.}$

$=1.45\times10^8\text{ lb}\cdot\frac{\text{ton}}{2000\text{ lb}}$

$= 72{,}500\text{ ton}$

27. $A = 2lh + 2lw + 2wh$

$= 2(12.0)(8.75) + 2(12.0)(9.50) + 2(9.50)(8.75)$

$A = 604\text{ in.}^2$

28. $V_1 = 50.0\times78.0\times3.50 = 13{,}650\text{ ft}^3$

$V_2 = \frac{1}{2}\times78.0\times5.00\times50.0 = 9750\text{ ft}^3$

$V = V_1 + V_2 = 13{,}650 + 9750 = 23{,}400\text{ ft}^3$

29. $V = \pi r^2 h = \pi(d/2)^2 h = \pi(4.0/2)^2(3{,}960{,}000)$

$= 5.0\times10^7\text{ ft}^3$ or 0.00034 mi^3

30. $V = \frac{1}{3}h\left(a^2 + ab + b^2\right)$

$= \frac{1}{3}(0.750)\left(2.50^2 + 2.50(3.25) + 3.25^2\right)$

$= 6.23\text{ m}^3$

31. $V = 1.80\sqrt{3.93^2 - 1.80^2}(1.50) = 9.43\text{ ft}^3$

32. There are three rectangles and two triangles in this shape.

$A = 2\left(\frac{1}{2}\right)(3.00)(4.00) + 3.00(8.50) + 4.00(8.50)$

$+8.50\sqrt{3.00^2 + 4.00^2} = 114\text{ cm}^2$

33. $V = \frac{1}{3}BH = \frac{1}{3}\left(250^2\right)(160) = 3{,}300{,}000\text{ yd}^3$

34. $s = \sqrt{h^2 + r^2} = \sqrt{3.50^2 + 1.80^2} = 3.94\text{ in.}$

$S = \pi rs = \pi(1.80)(3.94) = 22.3\text{ in.}^2$

35. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(d/2)^3$

$= \frac{4}{3}\pi(165/2)^3$

$= 2.35\times10^6\text{ ft}^3$

36. $V = \frac{4}{3}\pi r^3 + \pi r^2 h$

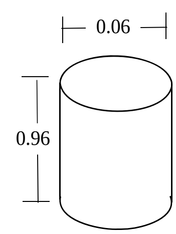
$= \frac{4}{3}\pi(2.00)^3 + \pi(2.00)^2(6.5)$

$= 115\text{ ft}^3$

37. $A = l^2 + \frac{1}{2}ps = 16^2 + \frac{1}{2}(4)(16)\sqrt{8^2 + 40^2}$

$= 1560\text{ mm}^2$

38.



$N\cdot\pi\left(\frac{0.06}{2}\right)^2(0.96) = 76$

$N = 280$ revolutions

39. $c = 2\pi r = 29.8 \Rightarrow r = \frac{29.8}{2\pi}$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{29.8}{2\pi}\right)^3$

$V = 447\text{ in.}^3$

40. Area $= (\pi\cdot3 + 0.25)(4.25) = 41\text{ in.}^2$

41. $V = \text{cylinder + cone (top of rivet)}$

$$= \pi r^2 h + \frac{1}{3}\pi r^2 h$$

$$= \pi\left(0.625/2\right)^2\left(2.75\right) + \frac{1}{3}\pi\left(1.25/2\right)^2\left(0.625\right)$$

$$= 1.10 \text{ in.}^3$$

42. $p = 18 = 2r + \pi r$

$$r = \frac{18}{\pi + 2}$$

$$V = \frac{1}{2}\cdot\pi\left(\frac{18}{\pi+2}\right)^2\left(0.075\right)$$

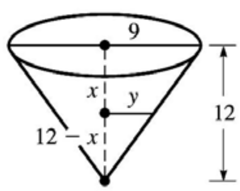
$$V = 1.4 \text{ m}^2$$

43. $V_2 = \frac{4}{3}\pi r_2^3 = 0.92V_1 = 0.92\left(\frac{4}{3}\pi r_1^3\right)$

$$r^2 = 0.97r_1$$

radius decreased by 3%

44.



$$\frac{9}{12} = \frac{y}{12-x}$$

$$y = \frac{3}{4}\left(12-x\right)$$

$$\frac{\frac{1}{3}\cdot\pi\cdot 9^2\cdot 12}{2} = \frac{1}{3}\cdot\pi\cdot\left(\frac{3}{4}\left(12-x\right)\right)^2\cdot\left(12-x\right)$$

$$x = 12 - \sqrt[3]{864}$$

$$x = 2.50 \text{ cm}$$

Chapter 2 Review Exercises

1. $\angle CGE = 180^\circ - 148^\circ = 32^\circ$

2. $\angle EGF = 180^\circ - 148^\circ - 90^\circ = 58^\circ$

3. $\angle DGH = 180^\circ - 148^\circ = 32^\circ$

4. $\angle EGI = 180 - 148^\circ + 90^\circ = 122^\circ$

5. $c = \sqrt{9^2 + 40^2} = 41$

6. $c^2 = a^2 + b^2 = 14^2 + 48^2 \Rightarrow c = 50$

7. $c^2 = a^2 + b^2 = 400^2 + 580^2 \Rightarrow c = 700$

8. $c^2 = a^2 + b^2 \Rightarrow 6500^2 = a^2 + 5600^2 \Rightarrow a = 3300$

9. $a = \sqrt{0.736^2 - 0.380^2} = 0.630$

10. $a = \sqrt{128^2 - 25.1^2} = 126$

11. $c^2 = a^2 + b^2 \Rightarrow 36.1^2 = a^2 + 29.3^2 \Rightarrow a = 21.1$

12. $c^2 = a^2 + b^2 \Rightarrow 0.885^2 = 0.782^2 + b^2 \Rightarrow b = 0.414$

13. $P = 3s = 3\left(8.5\right) = 25.5 \text{ mm}$

14. $p = 45 = 4\left(15.2\right) = 60.8 \text{ in.}$

15. $A = \frac{1}{2}bh = \frac{1}{2}\left(0.125\right)\left(0.188\right) = 0.0118 \text{ ft}^2$

16. $s = \frac{1}{2}\left(a+b+c\right) = \frac{1}{2}\left(175+138+119\right) = 216$

$$A = \sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)}$$

$$= \sqrt{216\left(216-175\right)\left(216-138\right)\left(216-119\right)}$$

$$A = 8190 \text{ ft}^2$$

17. $C = \pi d = \pi\left(98.4\right) = 309 \text{ mm}$

18. $p = 2l + 2w = 2\left(2980\right) + 2\left(1860\right) = 9680 \text{ yd}$

19. $A=\frac{1}{2}h(b_1+b_2)=\frac{1}{2}(34.2)(67.2+126.7)=3320\text{ in.}^2$

20. $A=\pi r^2=\pi\left(\frac{32.8}{2}\right)^2=845\text{ m}^2$

21. $V=Bh=\frac{1}{2}(26.0)(34.0)(14.0)=6190\text{ cm}^3$

22. $V=\pi r^2h=\pi(36.0)^2(2.40)=9770\text{ in.}^3$

23. $V=\frac{1}{3}Bh=\frac{1}{3}(3850)(125)=160{,}000\text{ ft}^3$

24. $V=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{2.21}{2}\right)^3=5.65\text{ mm}^3$

25. $A=6e^2=6(0.520)=1.62\text{ m}^2$

26. $A=2\pi r^2+2\pi rh=2\pi\left(\left(\frac{12.0}{2}\right)^2+\left(\frac{12.0}{2}\right)(58.0)\right)$

$A=2410\text{ ft}^2$

27. $s^2=r^2+h^2=1.82^2+11.5^2\Rightarrow s=\sqrt{1.82^2+11.5^2}$

$S=\pi rs=\pi(1.82)\sqrt{1.82^2+11.5^2}$

$S=66.6\text{ in.}^2$

28. $A=4\pi r^2=4\pi\left(\frac{12{,}760}{2}\right)^2=5.115\times10^8\text{ km}^2$

29. $\angle BTA=\frac{50^\circ}{2}=25^\circ$

30. $\angle TBA=90^\circ,\ \angle BTA=25^\circ\Rightarrow\angle TAB=90^\circ-25^\circ$

$=65^\circ$

31. $\angle BTC=90^\circ-\angle BTA=90^\circ-25^\circ=65^\circ$

32. $\angle ABT=90^\circ$

(any angle inscribed in a semi-circle is 90°)

33. $\angle ABE=90^\circ-37^\circ=53^\circ$

34. $AD=\sqrt{6^2+(4+4)^2}=10$

35. $\frac{BE}{4}=\frac{6}{10}\Rightarrow BE=2.4$

36. $\frac{AE}{4}=\frac{8}{10}\Rightarrow AE=3.2$

37. $P=b+\sqrt{b^2+(2a^2)^2}+\frac{1}{2}\pi(2a)=b+\sqrt{b^2+4a^2}+\pi a$

38. $p=\frac{1}{2}(2\pi s)+4s=\pi s+4s$

39. $A=\frac{1}{2}b(2a)+\frac{1}{2}\cdot\pi(a)^2=ab+\frac{1}{2}\pi a^2$

40. $A=\frac{1}{2}(\pi s^2)+s^2$

41. A square is a rectangle with four equal sides and a rectangle is a parallelogram with perpendicular intersecting sides so a square is a parallelogram. A rhombus is a parallelogram with four equal sides and since a square is a parallelogram, a square is a rhombus.

42. If two angles are equal then so is the third and the triangles are similar.

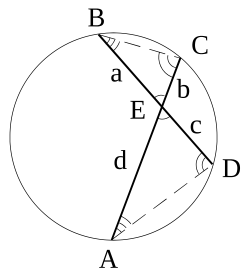
43. $A=\pi r^2,\ r\Rightarrow nr\Rightarrow A=\pi(nr)^2=n^2(\pi r^2)$

The area is multiplied by n^2.

44. $V=e^3;\ e\Rightarrow ne\Rightarrow V=(ne)^3=n^3e^3$

The volume is multiplied by n^3.

45.



$\measuredangle BEC = \measuredangle AED$, vertical $\measuredangle$'s.

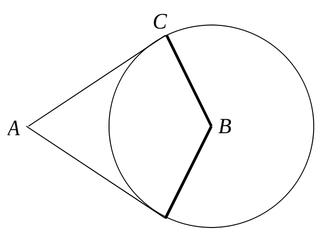
$\measuredangle BCA = \measuredangle ADB$, both are inscribed in $\overparen{AB}$

$\measuredangle CBE = \measuredangle CAD$, both are inscribed in $\overparen{CD}$

which shows $\Delta AED \sim \Delta BEC \Rightarrow \dfrac{a}{d} = \dfrac{b}{c}$

46. $\measuredangle B + 2\left(90^\circ\right) + 36^\circ = 180^\circ$

$\measuredangle B = 144^\circ$



47. $2(\text{base angle}) + 38^\circ = 180^\circ$

$\text{base angle} = 71^\circ$

48. The two volumes are equal.

$$\frac{4}{3}\pi\left(\frac{1.50}{2}\right)^3 = \pi\left(\frac{14.0}{2}\right)^2 \cdot t$$

$$t = 0.0115 \text{ in.}$$

49. $L = \sqrt{0.48^2 + 7.8^2} = 7.8$ m

50. $c = \sqrt{2100^2 + 9500^2} = 9700$ ft

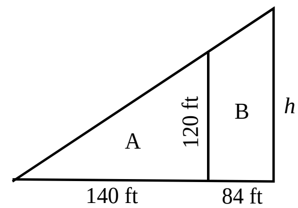
51. $p = 6\left(\dfrac{2.4}{\sqrt{2}}\right) = 10$ cm

52. $A = \left(\dfrac{18.0}{4}\right)^2 + 2\pi\left(\dfrac{18.0}{8}\right)^2 = 52.1 \text{ cm}^2$

53. $\dfrac{AB}{13} = \dfrac{14}{18}$

$AB = 10$ m

54.



$$\frac{h}{140+84} = \frac{120}{140} \Rightarrow h = 192$$

$$\text{area of } A = \frac{1}{2}(140)(120) = 8400 \text{ ft}^2$$

$$\text{area of } B = \frac{1}{2}(120+192)(84) = 13{,}000 \text{ ft}^2$$

55. $\dfrac{FB}{4.5} = \dfrac{1.60}{1.20} \Rightarrow FB = 6.0$ m

56. $\dfrac{DE}{16} = \dfrac{33}{24} \Rightarrow DE = 22$ in.

57. The longest distance in inches between points on the photograph is,

$\sqrt{8.00^2 + 10.0^2} = 12.8$ in. from which

$$\frac{x}{12.8} = \frac{18{,}450}{1}$$

$$x = (12.8)(18{,}450) \text{ in.} \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)\left(\frac{\text{mi}}{5280 \text{ ft}}\right)$$

$$x = 3.73 \text{ mi}$$

58. $MA = \dfrac{\pi\left(\dfrac{3.10}{2}\right)^2}{\pi\left(\dfrac{2.25}{2}\right)^2} = 1.90$

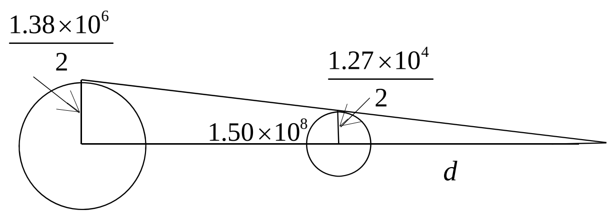
59. $c = \pi D = \pi\left(7920 + 2(210)\right) = 26{,}200$ mi

60. $c = 2\pi r = 651 \Rightarrow r = \dfrac{651}{2\pi}$

$$A = \pi r^2 = \pi\left(\frac{651}{2\pi}\right)^2 = 33,700 \text{ m}^2$$

61. $A = (4.0)(8.0) - 2\pi \cdot \dfrac{1.0^2}{4} = 30 \text{ ft}^2$

62.



$$\frac{d}{\frac{1.27\times10^4}{2}} = \frac{d + 1.50\times10^8}{\frac{1.38\times10^6}{2}}$$

$$d = 1.39\times10^6$$

63. $A = \dfrac{250}{3}\big[220 + 4(530) + 2(480)$

$$+4(320+190+260) + 2(510) + 4(350)$$

$$+2(730) + 4(560) + 240\big]$$

$$A = 1,000,000 \text{ m}^2$$

64. $V = \dfrac{250}{2}\big[560 + 2(1780) + 2(4650) + 2(6730)$

$$+2(5600) + 2(6280) + 2(2260) + 230\big]$$

$$V = 6,920,000 \text{ ft}^3$$

65. $V = \pi r^2 h = \pi\left(\dfrac{4.3}{2}\right)^2(13) = 190 \text{ m}^3$

66. Area of cross section = area of six equilateral triangles with sides of 2.50 each triangle has

semi-perimeter $= \dfrac{2.50(3)}{2} = 3.75$

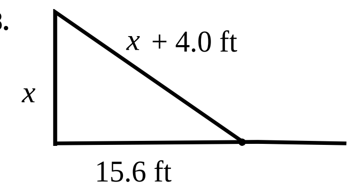
V = area of cross section $\times 6.75$

$$= 6\sqrt{3.75(3.75-2.50)^3} \times 6.75$$

$$= 110 \text{ m}^3$$

67. $\sqrt{(500.10)^2 - 500^2} = 10 \text{ ft}$

68.

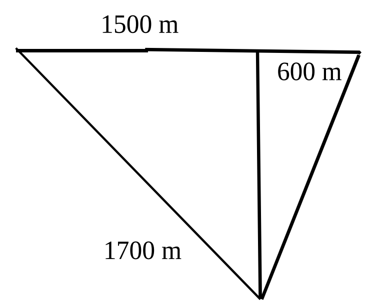


$$(x+4.0)^2 = x^2 + 15.6^2$$

$$x = 28.4$$

$x + 4 = 32.4$ ft, length of guy wire

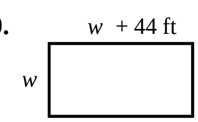
69.



$$d^2 = 1700^2 - 1500^2 + 600^2$$

$$d = 1000 \text{ m}$$

70.



$$p = 2l + 2w$$

$$288 = 2(w+44) + 2w$$

$$w = 50 \text{ ft}$$

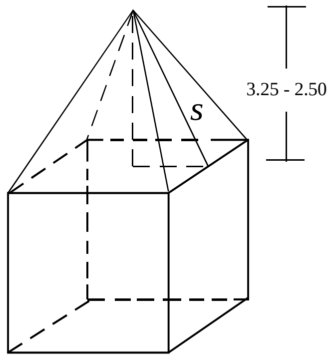
$$l = w + 44 = 94 \text{ ft}$$

71. $V = \pi r^2 h + \dfrac{1}{2}\cdot\dfrac{4}{3}\pi r^3$

$$= \left[\pi\left(\frac{2.50}{2}\right)^2\left(4.75 - \frac{2.50}{2}\right) + \frac{1}{2}\cdot\frac{4}{3}\cdot\pi\left(\frac{2.50}{2}\right)^3\right]\left(\frac{7.48 \text{ gal}}{\text{ft}^3}\right)$$

$$= 159 \text{ gal}$$

72.



tent surface area

= surface area of pyramid + surface area of cube

$$=\frac{1}{2}ps+4e^2$$

$$=\frac{1}{2}(4)(2.50)\sqrt{(3.25-2.50)^2+\left(\frac{2.50}{2}\right)^2}+4(2.50)^2$$

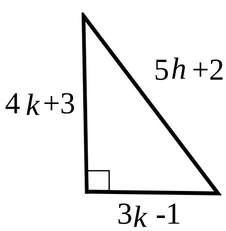
$$=32.3\text{ m}^2$$

73. $\frac{w}{h}=\frac{16}{9}\Rightarrow w=\frac{16h}{9}$

$$152^2=w^2+h^2=\left(\frac{16h}{9}\right)^2+h^2\Rightarrow h=74.5\text{ cm}$$

$$w=\frac{16h}{9}=132\text{ cm}$$

74.



$$(5k+2)^2=(4k+3)^2+(3k-1)^2$$

$$k=3$$

$$4k+3=15,\ 3k-1=8$$

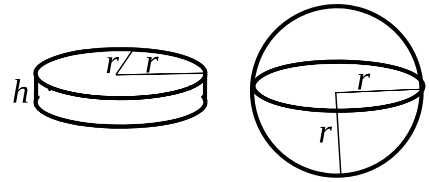
$$A=\frac{1}{2}(8)(15)=60\text{ ft}^2$$

Note: 1) $3k-1>0\Rightarrow k>\frac{1}{3}$

2) There is a solution for $\frac{1}{3}<k<1$.

3) For $k=1$ the triangle solution is an isosceles, but not right triangle.

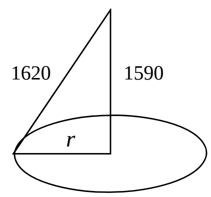
75.



$$V_{cyl}=\pi r^2h=V_{hemisphere}=\frac{1}{2}\cdot\frac{4}{3}\pi r^3$$

$$r=\frac{3h}{2}$$

76.



$$A=\pi r^2\left(1620\overset{?}{-}1590^2\right)=303{,}000\text{ km}^2$$

77. Label the vertices of the pentagon ABCDE. The area is the sum of the areas of three triangles, one with sides 921, 1490, and 1490 and two with sides 921, 921, and 1490. The semi-perimeters are given by

$$s_1=\frac{921+921+1490}{2}=1666\text{ and}$$

$$s_2=\frac{921+1490+1490}{2}=1950.5.$$

$$A=2\sqrt{1666(1666-921)(1666-921)(1666-1490)}$$
$$+\sqrt{1950.5(1950.5-1490)(1950.5-1490)}$$
$$+\sqrt{(1950.5-921)}$$
$$=1{,}460{,}000\text{ ft}^2$$