Chapter 2

Geometry

2.1 Lines and Angles

1.

$\angle ABE = 90^\circ$ because it is a vertically opposite angle to $\angle CBD$ which is also a right angle.

2.

Angles $\angle POR$ and $\angle QOR$ are complementary angles, so sum to 90°

$$\angle POR + \angle QOR = 90^\circ$$
$$32^\circ + \angle QOR = 90^\circ$$
$$\angle QOR = 90^\circ - 32^\circ$$
$$\angle QOR = 58^\circ$$

3.

4 pairs of adjacent angles:

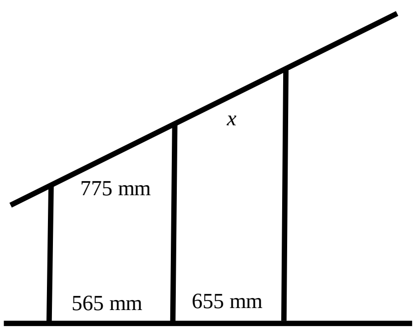
$\angle BOC$ and $\angle COA$ share common ray OC

$\angle COA$ and $\angle AOD$ share common ray OA

$\angle AOD$ and $\angle DOB$ share common ray OD

$\angle DOB$ and $\angle BOC$ share common ray OB

4.



(a) Using Eq. (2.1), we have

$$\frac{775}{565} = \frac{x}{655}$$
$$x = \frac{775(655)}{565}$$

$x = 898$ mm (the same answer as Example 5)

(b) More vertical, since the distance along the beam is longer for the same horizontal run, which can only be achieved if the angle increases from horizontal (see sketch).

5.
$\angle EBD$ and $\angle DBC$ are acute angles (i.e., $< 90°$) .

6.
$\angle ABE$ and $\angle CBE$ are right angles (i.e., $= 90°$) .

7.
$\angle ABC$ is a straight angle (i.e., $= 180°$) .

8.
$\angle ABD$ is an obtuse angle (i.e., between $90°$ and $180°$) .

9.
The complement of $\angle CBD = 65^\circ$ is $\angle DBE$

$$\begin{aligned}\angle CBD + \angle DBE &= 90^\circ \\ 65^\circ + \angle DBE &= 90^\circ \\ \angle DBE &= 90^\circ - 65^\circ \\ \angle DBE &= 25^\circ\end{aligned}$$

10.
The supplement of $\angle CBD = 65^\circ$ is $\angle ABD$

$$\begin{aligned}\angle CBD + \angle ABD &= 180^\circ \\ 65^\circ + \angle ABD &= 180^\circ \\ \angle ABD &= 180^\circ - 65^\circ \\ \angle ABD &= 115^\circ\end{aligned}$$

11.
Sides BD and BC are adjacent to $\angle DBC$.

12.
The angle adjacent to $\angle DBC$ is $\angle DBE$ since they share the common side BD,
and $\angle DBE$ is acute because it is less than 90°

13.
$$\angle AOB = \angle AOE + \angle EOB$$
but $\angle AOE = 90^\circ$ because it is vertically opposite to $\angle DOF$ a given right angle,
and $\angle EOB = 50^\circ$ because it is vertically opposite to $\angle COF$ a given angle of 50°,
so $\angle AOB = 90^\circ + 50^\circ = 140^\circ$

14.
$\angle AOC$ is complementary to $\angle COF$ a given angle of 50°,

$$\begin{aligned}\angle AOC + \angle COF &= 90^\circ \\ \angle AOC + 50^\circ &= 90^\circ \\ \angle AOC &= 90^\circ - 50^\circ \\ \angle AOC &= 40^\circ\end{aligned}$$

15.

$\angle BOD$ is vertically opposite to $\angle AOC$ a found angle of 40° (see Question 14), so

$\angle BOD = \angle AOC$

$\angle BOD = 40^\circ$

16.

$\angle 1$ is supplementary to 145°, so

$\angle 1 = 180^\circ - 145^\circ = 35^\circ$

$\angle 2 = \angle 1 = 35^\circ$

$\angle 3$ is supplementary to $\angle 2$, so

$\angle 3 = 180^\circ - \angle 2$

$\angle 3 = 180^\circ - 35^\circ$

$\angle 3 = 145^\circ$

17.

$\angle 1$ is supplementary to 145°, so

$\angle 1 = 180^\circ - 145^\circ = 35^\circ$

$\angle 2 = \angle 1 = 35^\circ$

$\angle 4$ is vertically opposite to $\angle 2$, so

$\angle 4 = \angle 2$

$\angle 4 = 35^\circ$

18.

$\angle 1$ is supplementary to 145°, so

$\angle 1 = 180^\circ - 145^\circ = 35^\circ$

$\angle 2 = \angle 1 = 35^\circ$

$\angle 5$ is supplementary to $\angle 2$, so

$\angle 5 = 180^\circ - \angle 2$

$\angle 5 = 180^\circ - 35^\circ$

$\angle 5 = 145^\circ$

19.

$\angle 1 = 62^\circ$ since they are vertically opposite

20.

$\angle 1 = 62^\circ$ since they are vertically opposite

$\angle 2$ is a corresponding angle to $\angle 5$, so

$\angle 2 = \angle 5$

since $\angle 1$ and $\angle 5$ are supplementary angles,

$\angle 5 + \angle 1 = 180^\circ$

$\angle 2 + \angle 1 = 180^\circ$

$\angle 2 = 180^\circ - \angle 1$

$\angle 2 = 180^\circ - 62^\circ$

$\angle 2 = 118^\circ$

21.

$\angle 6 = 90 - 62^{\circ}$ since they are complementary angles

$\angle 6 = 28^{\circ}$

$\angle 3$ is an alternate-interior angle to $\angle 6$, so

$\angle 3 = \angle 6$

$\angle 3 = 28^{\circ}$

22.

$\angle 3 = 28^{\circ}$ (see Question 21)

since $\angle 4$ and $\angle 3$ are supplementary angles,

$\angle 4 + \angle 3 = 180^{\circ}$

$\angle 4 = 180^{\circ} - \angle 3$

$\angle 4 = 180^{\circ} - 28^{\circ}$

$\angle 4 = 152^{\circ}$

23.

$\angle EDF = \angle BAD = 44^{\circ}$ because they are corresponding angles

$\angle BDE = 90^{\circ}$

$\angle BDF = \angle BDE + \angle EDF$

$\angle BDF = 90^{\circ} + 44^{\circ}$

$\angle BDF = 134^{\circ}$

24.

$\angle CBE = \angle BAD = 44^{\circ}$ because they are corresponding angles

$\angle DBE$ and $\angle CBE$ are complementary so

$\angle DBE + \angle CBE = 90^{\circ}$

$\angle DBE = 90^{\circ} - \angle CBE$

$\angle DBE = 90^{\circ} - 44^{\circ}$

$\angle DBE = 46^{\circ}$

and $\angle ABE = \angle ABD + \angle DBE$

$\angle ABE = 90^{\circ} + 46^{\circ}$

$\angle ABE = 136^{\circ}$

25.

$\angle CBE = \angle BAD = 44^{\circ}$ because they are corresponding angles

$\angle DEB$ and $\angle CBE$ are alternate interior angles, so

$\angle DEB = \angle CBE$

$\angle DEB = 44^{\circ}$

26.

$\angle CBE = \angle BAD = 44^\circ$ because they are corresponding angles

$\angle DBE$ and $\angle CBE$ are complementary so

$$\angle DBE + \angle CBE = 90^\circ$$
$$\angle DBE = 90^\circ - \angle CBE$$
$$\angle DBE = 90^\circ - 44^\circ$$
$$\angle DBE = 46^\circ$$

27.

$\angle EDF = \angle BAD = 44^\circ$ because they are corresponding angles

Angles $\angle ADB$, $\angle BDE$, and $\angle EDF$ make a straight angle

$$\angle ADB + \angle BDE + \angle EDF = 180^\circ$$
$$\angle ADB = 180^\circ - \angle BDE - \angle EDF$$
$$\angle ADB = 180^\circ - 90^\circ - 44^\circ$$
$$\angle ADB = 46^\circ$$

$\angle DFE = \angle ADB$ because they are corresponding angles

$$\angle DFE = 46^\circ$$

28.

$$\angle ADE = \angle ADB + \angle BDE$$

$\angle ADB = 46^\circ$ (see Question 27)

$$\angle BDE = 90^\circ$$
$$\angle ADE = 46^\circ + 90^\circ$$
$$\angle ADE = 136^\circ$$

29. Using Eq. (2.1),

$$\frac{a}{4.75} = \frac{3.05}{3.20}$$
$$a = 4.75 \cdot \frac{3.05}{3.20}$$
$$a = 4.53 \text{ m}$$

30. Using Eq. (2.1),

$$\frac{b}{6.25} = \frac{3.05}{3.20}$$
$$b = 6.25 \cdot \frac{3.05}{3.20}$$
$$b = 5.96 \text{ m}$$

31. Using Eq. (2.1),

$$\frac{c}{3.20} = \frac{5.05}{4.75}$$
$$c = \frac{(3.20)(5.05)}{4.75}$$
$$c = 3.40 \text{ m}$$

32. Using Eq. (2.1),

$$\frac{d}{6.25} = \frac{5.05}{4.75}$$

$$d = \frac{(6.25)(5.05)}{4.75}$$

$$d = 6.64 \text{ m}$$

33.

$\angle BCE = 47^\circ$ since those angles are alternate interior angles.

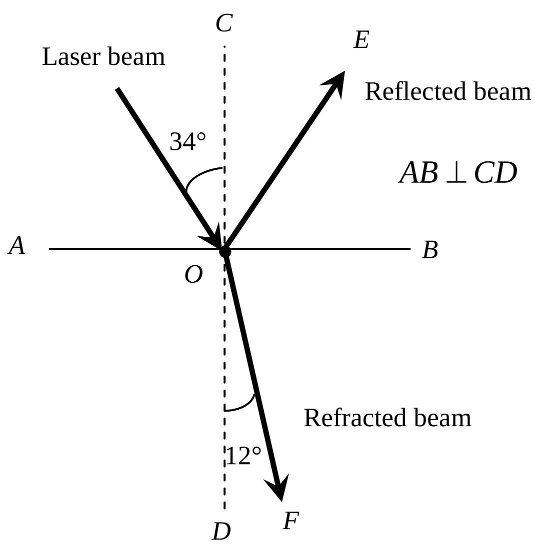
$\angle BCD$ and $\angle BCE$ are supplementary angles

$$\angle BCD + \angle BCE = 180^\circ$$

$$\angle BCD = 180^\circ - 47^\circ$$

$$\angle BCD = 133^\circ$$

34.



$$\angle COE = 34^\circ$$

$\angle COD$ is a straight angle, so the total angle between the reflected and refracted beams, $\angle EOF$ is

$$\angle COE + \angle EOF + \angle DOF = 180^\circ$$

$$34^\circ + \angle EOF + 12^\circ = 180^\circ$$

$$\angle EOF = 180^\circ - 46^\circ$$

$$\angle EOF = 134^\circ$$

This angle is a reflex angle.

35. Using Eq. (2.1),

$$\frac{x}{825} = \frac{555}{519}$$

$$x = \frac{(825)(555)}{519}$$

$$x = 882 \text{ m}$$

36. Using Eq. (2.1),

$$\frac{AB}{3} = \frac{BC}{2} =$$

$$AB = \frac{3(2.15)}{2}$$

$$AB = 3.225 \text{ cm}$$

$$AC = AB + BC$$

$$AC = 3.225 \text{ cm} + 2.15 \text{ cm}$$

$$AC = 5.375 \text{ cm}$$

$$AC = 5.38 \text{ cm}$$

37.

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$, because $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle.

38.

$\angle 1 = \angle 4$ since they are alternate interior angles

$\angle 3 = \angle 5$ since they are alternate interior angles

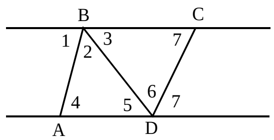
$\angle 1 + \angle 2 + \angle 3 = 180^\circ$, because $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle, so

$\angle 4 + \angle 2 + \angle 5 = 180^\circ$

39.

The sum of the angles with vertices at A, B, and D is 180°.
Since those angles are unknown quantities, the sum of
interior angles in a closed triangle is 180°.

40.



$\angle 1 + \angle 2 + \angle 3 = 180^\circ$ since those angles form a straight angle

$\angle 1 = \angle 4$ since they are alternate interior angles

$\angle 4 + \angle 2 + \angle 3 = 180^\circ$

$\angle 5 + \angle 6 + \angle 7 = 180^\circ$ since those angles form a straight angle

$\angle 7$ alternate interior angle is shown

For the interior angles of the closed geometric figure ABCD,

$$\text{sum} = \angle 4 + (\angle 2 + \angle 3) + \angle 7 + (\angle 5 + \angle 6)$$

$$\text{sum} = \angle 4 + \angle 2 + \angle 3 + \angle 5 + \angle 6 + \angle 7$$

$$\text{sum} = 180^\circ + 180^\circ$$

$$\text{sum} = 360^\circ$$

2.2 Triangles

1.

$\angle 5 = 45^{\circ}$

$\angle 3 = 45^{\circ}$ since $\angle 3$ and $\angle 5$ are alternate interior angles.

$\angle 1,\ \angle 2,$ and $\angle 3$ make a stright angle, so

$$\angle 1 + \angle 2 + \angle 3 = 180^{\circ}$$
$$70^{\circ} + \angle 2 + 45^{\circ} = 180^{\circ}$$
$$\angle 2 = 65^{\circ}$$

2.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(61.2)(5.75)$$
$$A = 176 \text{ cm}^2$$

3.

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 6.25^2 + 3.20^2$$
$$AC = \sqrt{6.25^2 + 3.20^2}$$
$$AC = 7.02 \text{ m}$$

4. Using Eq. (2.1),

$$\frac{h}{3.00} = \frac{24.0}{4.00}$$
$$h = \frac{(3.00)(24.0)}{4.00}$$
$$h = 18.0 \text{ m}$$

5.

$$\angle A + \angle B + \angle C = 180^{\circ}$$
$$\angle A + 40^{\circ} + 84^{\circ} = 180^{\circ}$$
$$\angle A = 56^{\circ}$$

6.

$$\angle A + \angle B + \angle C = 180^{\circ}$$
$$\angle A + 48^{\circ} + 90^{\circ} = 180^{\circ}$$
$$\angle A = 42^{\circ}$$

7.

This is an isosceles triangle, so the base angles are equal.

$$\angle B = \angle C = 66^\circ$$
$$\angle A + \angle B + \angle C = 180^\circ$$
$$\angle A + 66^\circ + 66^\circ = 180^\circ$$
$$\angle A = 180^\circ - \left(66^\circ + 66^\circ\right)$$
$$\angle A = 48^\circ$$

8.

This is an isosceles triangle, so the base angles are equal.

$$\angle A = \angle B$$
$$\angle A + \angle B + \angle C = 180^\circ$$
$$\angle A + \angle A + 110^\circ = 180^\circ$$
$$2\angle A = 70^\circ$$
$$\angle A = 35^\circ$$

9.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(7.6)(2.2)$$
$$A = 8.4 \text{ m}^2$$

10.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(16.0)(7.62)$$
$$A = 61.0 \text{ mm}^2$$

11. By Hero's formula,

$$p = 205 + 322 + 415 = 942 \text{ cm}$$
$$s = \frac{942}{2} = 471 \text{ cm}$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$A = \sqrt{471 \text{ cm} \cdot (471-205) \text{ cm} \cdot (471-322) \text{ cm} \cdot (471-415) \text{ cm}}$$
$$A = \sqrt{471(266)(149)(56)\text{cm}^4}$$
$$A = 32\ 300 \text{ cm}^2$$

12. By Hero's formula,
$p = 86.2 + 23.5 + 68.4 = 178.1 \text{ m}$

$s = \frac{178.1}{2} = 89.05 \text{ m}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{89.05(89.05-86.2)(89.05-23.5)(89.05-68.4)}$

$A = 586 \text{ m}^2$

13.
One leg can represent the base, the other leg the height.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(3.46)(2.55)$

$A = 4.41 \text{ cm}^2$

14.
One leg can represent the base, the other leg the height.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(234)(343)$

$A = 40\ 100 \text{ mm}^2$

15. By Hero's formula,
$s = \frac{p}{2} = \frac{0.986 + 0.986 + 0.884}{2} \text{ m} = 1.428 \text{ m}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{1.428(1.428-0.986)(1.428-0.986)(1.428-0.884)}$

$A = 0.390 \text{ m}^2$

16. By Hero's formula,
$s = \frac{3(322)}{2} = 483 \text{ dm}$

$A = \sqrt{s(s-a)^3}$

$A = \sqrt{483(483-322)^3}$

$A = 44\ 900 \text{ dm}^2$

17. We add the lengths of the sides to get
$p = 205 + 322 + 415$

$p = 942 \text{ cm}$

18. We add the lengths of the sides to get

$p = 23.5 + 86.2 + 68.4$

$p = 178.1 \text{ m}$

19. We add the lengths of the sides to get

$p = 3(21.5) = 64.5 \text{ cm}$

20. We add the lengths of the sides to get

$p = 2(2.45) + 3.22 = 8.12 \text{ mm}$

21.

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{13.8^2 + 22.7^2}$$
$$c = 26.6 \text{ mm}$$

22.

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{2.48^2 + 1.45^2}$$
$$c = 2.87 \text{ m}$$

23.

$$c^2 = a^2 + b^2$$
$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{551^2 - 175^2}$$
$$b = 522 \text{ cm}$$

24.

$$c^2 = a^2 + b^2$$
$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{0.836^2 - 0.474^2}$$
$$a = 0.689 \text{ km}$$

25.

All interior angles in a triangle add to 180°

$$23^\circ + \angle B + 90^\circ = 180^\circ$$
$$\angle B = 180^\circ - 90^\circ - 23^\circ$$
$$\angle B = 67^\circ$$

26.

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{38.4^2 + 90.5^2}$$
$$c = 98.3 \text{ cm}$$

27.

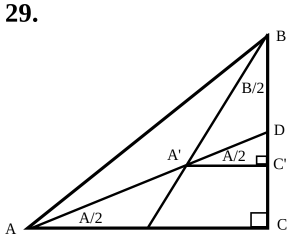
Length c is found in Question 26, $c = 98.309\ 77$ cm

$$p = 98.309\ 77 + 90.5 + 38.4 = 227.2 \text{ cm}$$

28.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(90.5)(38.4)$$
$$A = 1740 \text{ cm}^2$$

29.



$$\Delta ADC \sim \Delta A'DC'$$
$$\angle DA'C' = A/2$$
$$\angle BA'D = \angle \text{ between bisectors}$$

From $\Delta BA'C'$, and all angles in a triangle must sum to 180°

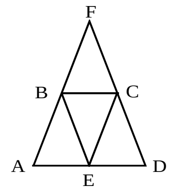
$$\frac{B}{2} + \left(\angle BA'D + A/2\right) + 90^\circ = 180^\circ$$
$$\angle BA'D = 90^\circ - \left(\frac{A}{2} + \frac{B}{2}\right)$$
$$\angle BA'D = 90^\circ - \left(\frac{A+B}{2}\right)$$

But ΔABC is a right triangle, and all angles in a triangle must sum to 180°,

so $A + B = 90^\circ$

$$\angle BA'D = 90^\circ - \frac{90^\circ}{2}$$
$$\angle BA'D = 45^\circ$$

30.



$\angle A = \angle D$ since ΔAFD is isosceles.

Since $AF = FD$ $(\Delta AFD$ is isosceles$)$ and since B and C are midpoints,

$AB = CD$

$AE = DE$ because E is a midpoint of AD,

so if two of the three sides are identical, the last side is the same too.

so $\Delta ABE = \Delta ECD$

Therefore, $BE = EC$ from which it follows that the inner ΔBCE is isosceles.

Also, since $AB = CD = FB = FC$

$\Delta ABE = \Delta ECD = \Delta BFC = \Delta BCE$

and all four triangles are similar triangles to the original ΔAFD

So, ΔBCE is also 1/4 of the area of the original ΔAFD.

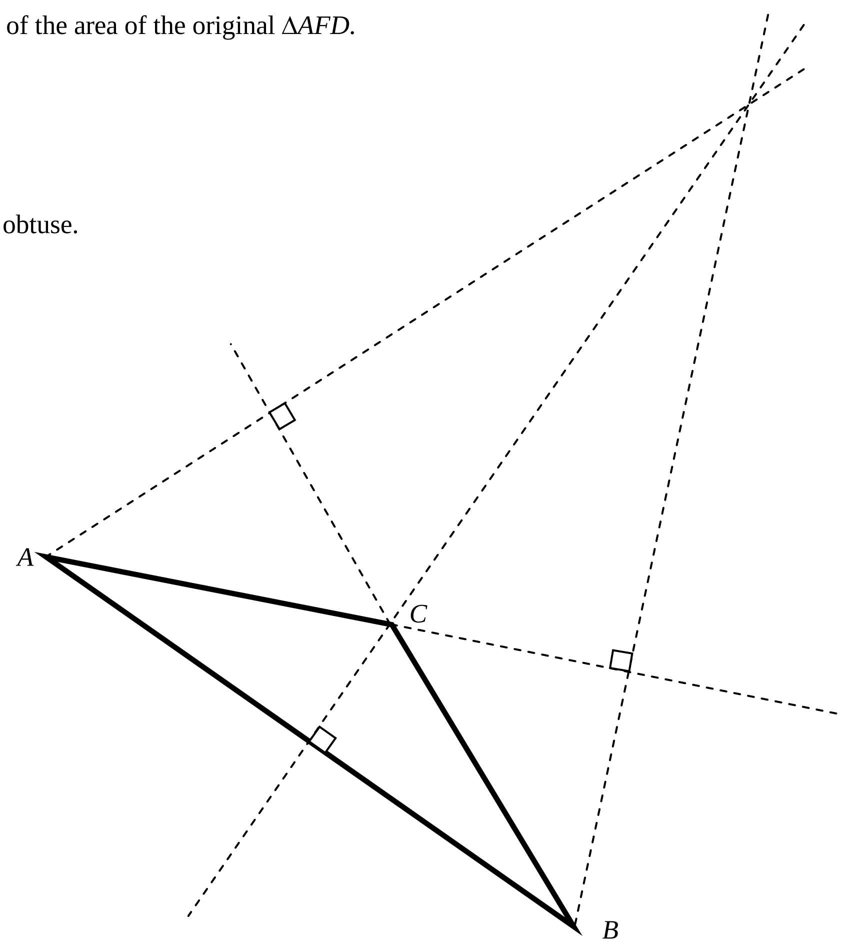
31.

An equilateral triangle.

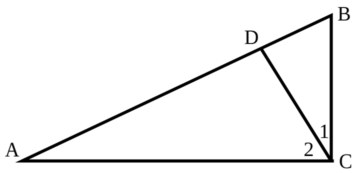
32.

Yes, if one of the angles of the triangle is obtuse.

For example, see ΔABC below.



33.

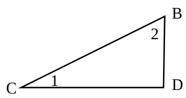


$\angle A + \angle B = 90^\circ$

$\angle 1 + \angle B = 90^\circ$

$\angle A = \angle 1$

redraw ΔBDC as

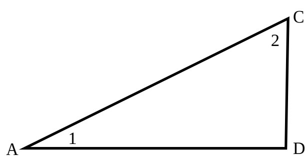


$\angle 1 + \angle 2 = 90^\circ$

$\angle 1 + \angle B = 90^\circ$

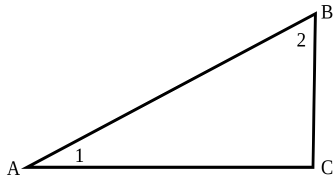
$\angle 2 = \angle B$

and ΔADC as



ΔBDC and ΔADC are similar.

34.



Comparing the original triangle to the two smaller triangles (see Question 33) shows that all three are similar.

35.

$\angle LMK$ and $\angle OMN$ are vertically opposite angles and thus equal.
Since each triangle has a right angle, the remaining angle in each triangle must be the same.
$\angle KLM = \angle MON$.
The triangles ΔMKL and ΔMNO have all the same angles, so therefore the triangles are similar:
$\Delta MKL \sim \Delta MNO$

36.

$\angle ACB = \angle ADC = 90^\circ$

$\angle DAC = \angle BAC$ since they share the common vertex A.

Since all angles in any triangle sum to 180°,

$\angle DCA = 180 - 90 - \angle BAC$,

$\angle ABC = 180 - 90 - \angle BAC$,

Therefore, all the angles in ΔACB and ΔADC are equivalent, so

$\Delta ACB \sim \Delta ADC$

37.

$$KM = KN - MN$$
$$KM = 15 - 9$$
$$KM = 6$$

Since $\Delta MKL \sim \Delta MNO$

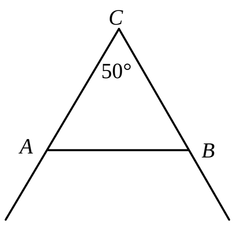
$$\frac{LM}{KM} = \frac{OM}{MN}$$
$$\frac{LM}{6} = \frac{12}{9}$$
$$LM = \frac{(6)(12)}{9}$$
$$LM = 8$$

38.

Since $\Delta ADC \sim \Delta ACB$

$$\frac{AB}{AC} = \frac{AC}{AD}$$
$$\frac{AB}{12} = \frac{12}{9}$$
$$AB = \frac{(12)(12)}{9}$$
$$AB = 16$$

39.



ΔABC is isosceles,

so $\angle CAB = \angle CBA$

But all interior angles in a triangle sum to 180°

$\angle CAB + \angle CBA + 50^\circ = 180^\circ$

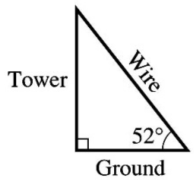
$2\angle CAB = 130^\circ$

$\angle CAB = 65^\circ$

40.

But all interior angles in a triangle sum to 180°

angle between tower and wire $= 180^\circ - 90^\circ - 52^\circ = 38^\circ$



41.

$$s = \frac{p}{2} = \frac{2(76.6)+30.6}{2} = 91.9 \text{ cm}$$

By Hero's formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{91.9(91.9-76.6)(91.9-76.6)(91.6-30.6)}$$

$$A = 1150 \text{ cm}^2$$

42.

$$s = \frac{p}{2} = \frac{3(1600)}{2} = 2400 \text{ km}$$

By Hero's formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{2400(2400-1600)^3}$$

$$A = 1,100,000 \text{ km}^2$$

43.

One leg can represent the base, the other leg the height.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(3.2)(6.0)$$

$$A = 9.6 \text{ m}^2$$

44.

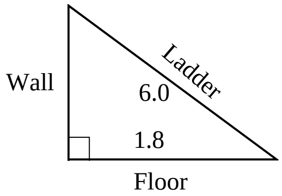
$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{750^2 + 550^2}$$

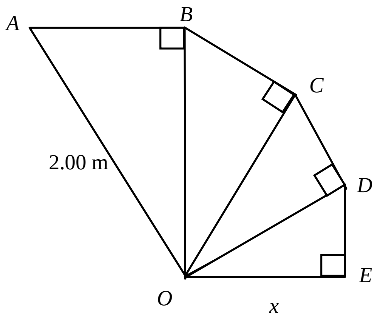
$$c = 930 \text{ m}$$

45.



$$c^2 = a^2 + b^2$$
$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{6.0^2 - 1.8^2}$$
$$b = 5.7 \text{ m}$$

46.



On ΔABO

the idea that the side opposite the 30° angle is half the hypotenuse gives

$$AB = 1.00 \text{ m}$$

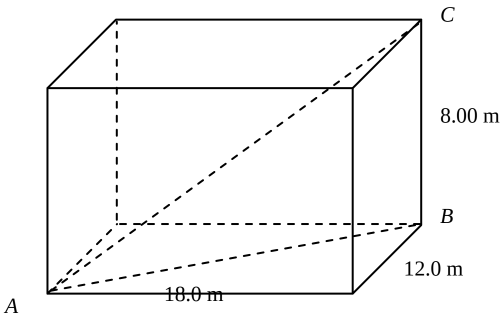
Using Pythagorean theorem gives

$$AO^2 = AB^2 + BO^2$$
$$BO = \sqrt{AO^2 - AB^2}$$
$$BO = \sqrt{2^2 - 1^2}$$
$$BO = \sqrt{3} \text{ m}$$

Using an identical technique on each successive triangle moving clockwise,

$$BC = \frac{\sqrt{3}}{2} \text{ m}$$
$$CO = \sqrt{3 - \tfrac{3}{4}}$$
$$CO = 1.50 \text{ m}$$
$$CD = 0.750 \text{ m}$$
$$DO = \sqrt{1.50^2 - (0.750)^2}$$
$$DO = 1.30 \text{ m}$$
$$DE = 0.650 \text{ m}$$
$$x = \sqrt{1.30^2 - (0.650)^2}$$
$$x = 1.125 \text{ m}$$
$$x = 1.12 \text{ m}$$

47.



Diagonal AB

$AB = \sqrt{18^2 + 12^2} = \sqrt{468}$ m

Diagonal AC

$AC = \sqrt{AB^2 + 8^2}$

$AC = \sqrt{468 + 64}$ m

$AC = \sqrt{532}$ m

$AC = 23.1$ m

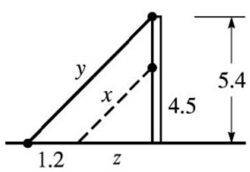
48. By Eq. (2.1),

$$\frac{45.6 \text{ cm}}{x} = \frac{1.20 \text{ cm}}{1.00 \text{ m}}$$

$$x = \frac{45.6 \text{ cm (1.00 m)}}{1.20 \text{ cm}}$$

$$x = 38.0 \text{ m}$$

49.



By Eq. (2.1),

$$\frac{z}{4.5} = \frac{1.2}{0.9}$$

$$z = \frac{(4.5)(1.2)}{0.9}$$

$$z = 6.0 \text{ m}$$

$$x^2 = z^2 + 4.5^2$$

$$x = \sqrt{56.25} \text{ m}$$

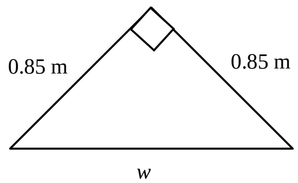
$$x = 7.5 \text{ m}$$

$$y^2 = (1.2 + 6)^2 + 5.4^2$$

$$x = \sqrt{81.0} \text{ m}$$

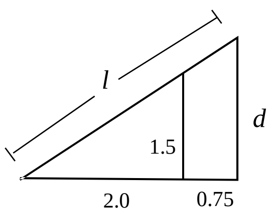
$$y = 9.0 \text{ m}$$

50.



$$w^2 = 0.85^2 + 0.85^2$$
$$w = \sqrt{1.445} \text{ m}$$
$$w = 1.2 \text{ m}$$

51.



By Eq. (2.1),

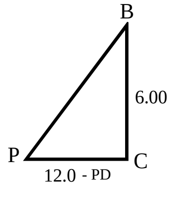
$$\frac{d}{2.75} = \frac{1.5}{2.0}$$
$$d = \frac{2.75(1.5)}{2.0}$$
$$d = 2.0625 \text{ m}$$
$$l^2 = 2.75^2 + d^2$$
$$l = \sqrt{2.75^2 + 2.0625^2}$$
$$l = 3.4 \text{ m}$$

52.

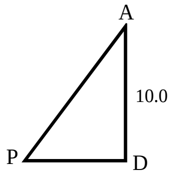
$$\frac{ED}{AB} = \frac{DC}{BC}$$
$$\frac{ED}{80.0} = \frac{312}{50.0}$$
$$ED = \frac{(80.0)(312)}{50.0}$$
$$ED = 499 \text{ m}$$

53.

Redraw ΔBCP as



ΔAPD is



since $\Delta BCP \sim \Delta ADP$

$$\frac{6.00}{12.0-PD}=\frac{10.0}{PD}$$
$$6PD=120-10PD$$
$$16PD=120$$
$$PD=7.50\text{ km}$$
$$PC=12.0-PD$$
$$PC=4.50\text{ km}$$
$$l=PB+PA$$
$$l=\sqrt{4.50^2+6.00^2}+\sqrt{7.50^2+10.0^2}$$
$$l=7.50+12.5$$
$$l=20.0\text{ km}$$

54.

Original area:

$$\mathrm{A}_{\mathrm{o}}=\frac{1}{2}bh$$
$$\mathrm{A}_{\mathrm{o}}=\frac{1}{2}x(x-12)$$

New area:

$$\mathrm{A}_{n}=\frac{1}{2}bh$$
$$\mathrm{A}_{n}=\frac{1}{2}x(x-12+16)$$
$$\mathrm{A}_{n}=\frac{1}{2}x(x+4)$$

If the new area is 160 cm^2 larger than the original,

$$\mathrm{A}_{n}-\mathrm{A}_{\mathrm{o}}=160$$
$$\frac{1}{2}x(x+4)-\frac{1}{2}x(x-12)=160$$
$$\frac{1}{2}x^2+2x-\frac{1}{2}x^2+6x=160$$
$$8x=160$$

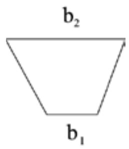
$x=20$ cm is the orignal width

$d=x-12$

$d=8$ cm is the original depth

2.3 Quadrilaterals

1.



2.

$L = 4s + 2w + 2l$

$L = 4(540) + 2(540) + 2(920)$

$L = 5080 \text{ mm}$

3.

$A_1 = \frac{1}{2}bh = \frac{1}{2}(72)(55) = 1980 = 2000 \text{ m}^2$

$A_2 = bh = 72(55) = 3960 = 4000 \text{ m}^2$

$A_3 = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(55)(72 + 35)$

$A_3 = 2942.5 = 2900 \text{ m}^2$

$A_{tot} = 1980 + 3960 + 2942.5 = 8900 \text{ m}^2$

4.

$2(w + 3.0) + 2w = 26.4$

$2w + 6.0 + 2w = 26.4$

$4w = 20.4$

$w = 5.1 \text{ mm}$

$w + 3.0 = 8.1 \text{ mm}$

5.

$p = 4s = 4(65) = 260 \text{ m}$

6.

$p = 4(2.46) = 9.84 \text{ km}$

7.

$p = 2(0.920) + 2(0.742) = 3.324 \text{ mm}$

8.

$p = 2(142) + 2(126) = 536 \text{ cm}$

9.

$p = 2l + 2w = 2(3.7) + 2(2.7) = 12.8 \text{ m}$

10.

$p = 2(27.3) + 2(14.2) = 83.0 \text{ mm}$

11.
$p = 36.2 + 73.0 + 44.0 + 61.2 = 214.4 \text{ dm}$

12.
$p = 272 + 392 + 223 + 672 = 1559 \text{ cm}$

13.
$A = s^2 = 2.7^2 = 7.3 \text{ mm}^2$

14.
$A = 15.6^2 = 243 \text{ m}^2$

15.
$A = lw = 0.920(0.742) = 0.683 \text{ km}^2$

16.
$A = lw = 142(126) = 17\ 900 \text{ cm}^2$

17.
$A = bh = 3.7(2.5) = 9.2 \text{ m}^2$

18.
$A = bh = 27.3(12.6) = 344 \text{ mm}^2$

19.
$$\begin{aligned} A &= \tfrac{1}{2}h(b_1 + b_2) \\ &= \tfrac{1}{2}(29.8)(61.2 + 73.0) \\ &= 2.00 \times 10^3 \text{ dm}^2 \end{aligned}$$

20.
$$\begin{aligned} A &= \tfrac{1}{2}h(b_1 + b_2) \\ &= \tfrac{1}{2}(201)(392 + 672) \\ &= 107\ 000 \text{ cm}^2 \end{aligned}$$

21.
$p = 2b + 4a$

22.
$$\begin{aligned} p &= a + b + b + a + (b - a) + (b - a) \\ p &= 2a + 2b + 2b - 2a \\ p &= 4b \end{aligned}$$

23.
$A = bh + a^2$

24.

$$A = \tfrac{1}{2}a\left[b + b - a\right] + \tfrac{1}{2}a\left[b + b - a\right]$$
$$A = ab - \tfrac{1}{2}a^2 + ab - \tfrac{1}{2}a^2$$
$$A = 2ab - a^2$$

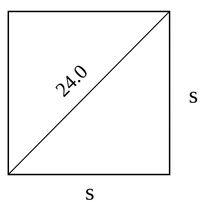
25.

The parallelogram is a rectangle.

26.

The triangles are congruent. Corresponding sides and angles are equal.

27.



$$\begin{aligned} s^2 + s^2 &= 24.0^2 \\ 2s^2 &= 576 \\ s^2 &= \frac{576}{2} \\ A = s^2 &= 288 \text{ cm}^2 \end{aligned}$$

28.

A *D*

B

F *C* *E*

G

$\angle CAD$ and $\angle FCA$ are alternate interior angles, and so

$$\angle CAD = \angle FCA$$

$\angle CAB = \frac{1}{2}\angle CAD$ because of the angle bisector AE

$$\angle CAB = \frac{1}{2}\angle FCA$$

$\angle ACE$ and $\angle FCA$ are supplementary angles, so

$$\angle ACE = 180^\circ - \angle FCA$$

$\angle ACB = \frac{1}{2}\angle ACE$ because of the angle bisector CD

$$\angle ACB = 90^\circ - \frac{1}{2}\angle FCA$$

Analysing ΔABC, all interior angles should sum to 180°

$$\angle CAB + \angle ABC + \angle ACB = 180^\circ$$

$$\frac{1}{2}\angle FCA + \angle ABC + \left(90^\circ - \frac{1}{2}\angle FCA\right) = 180^\circ$$

$$\angle ABC = 90^\circ$$

29.

The diagonal always divides the rhombus into two congruent triangles. All outer sides are always equal.

30.

The hypotenuse of the right triangle is

$$c^2 = a^2 + b^2$$

$$c = \sqrt{16^2 + 12^2}$$

$$c = \sqrt{400}$$

$$c = 20$$

In a rhombus, all four sides are equivalent, so

$$p = 4(20) = 80 \text{ mm}$$

31.

For the courtyard

$$s = \frac{p}{4} = \frac{324}{4} = 81.0 \text{ m}$$

For the outer edge of the walkway, each side will be

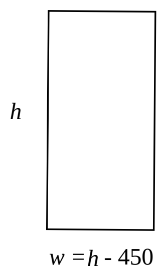
$$x = 81.0 + 6.00 = 87.0 \text{ m}$$

$$p = 4x$$

$$p = 4(87.0)$$

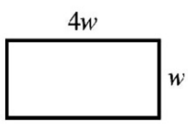
$$p = 348 \text{ m}$$

32.



$$\begin{aligned} p &= 2h + 2(h - 450) \\ 4500 &= 2h + 2h - 900 \\ 5400 &= 4h \\ h &= 1350 \text{ mm} \\ w &= h - 450 \\ w &= 900 \text{ mm} \end{aligned}$$

33.



If width increases by 1500 mm and length decreases by 4500 mm the dimensions will be equal (a square).

$$\begin{aligned} w + 1500 &= 4w - 4500 \\ 6000 &= 3w \\ w &= 2000 \text{ mm} \\ 4w &= 8000 \text{ mm} \end{aligned}$$

34.

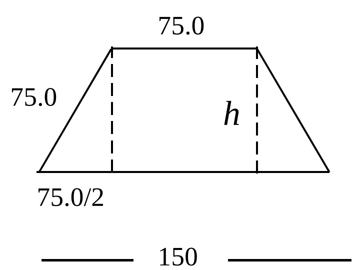
$$\begin{aligned} A &= bh \\ A &= 1.80(3.50) \\ A &= 6.30 \text{ m}^2 \end{aligned}$$

35.

The trapezoid has lower base 9300 mm and upper base 5300 mm, making the lower side 4000 mm longer than the upper side.
This means that a right triangle in each corner can be built with hypotenuse c of 3300 mm and horizontal leg (base b) of 2000 mm

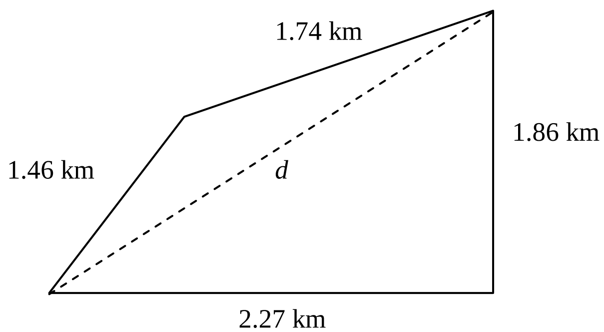
$$c^2 = b^2 + h^2$$
$$h = \sqrt{c^2 - b^2}$$
$$h = \sqrt{3300^2 - 2000^2}$$
$$h = \sqrt{6\ 890\ 000}$$
$$A_{\text{paint}} = 2\left(\text{area of trapezoid} - \text{area of window}\right)$$
$$A_{\text{paint}} = 2\left(\tfrac{1}{2}h(b_1 + b_2) - lw\right)$$
$$A_{\text{paint}} = 2\left(\tfrac{1}{2}\sqrt{6\ 890\ 000}\cdot\left(9300 + 5300\right) - 1200\left(4200\right)\right)$$
$$A_{\text{paint}} = 28\ 243\ 262\ \text{mm}^2$$
$$V_{\text{paint}} = 28\ 243\ 262\ \text{mm}^2 \times \left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^2 \times \frac{1\ \text{L}}{12\ \text{m}^2}$$
$$V_{\text{paint}} = 2.4\ \text{L of paint (to two significant digits)}$$

36.



$$75^2 = 37.5^2 + h^2$$
$$h = \sqrt{75.0^2 - \left(37.5\right)^2}$$
$$h = 64.9519\ \text{cm}$$
$$A = \text{area of 6 identical trapezoids}$$
$$A = 6\left[\tfrac{1}{2}h(b_1 + b_2)\right]$$
$$A = 3(64.9519\ \text{cm})\left(75.0 + 150\right)\text{cm}$$
$$A = 43\ 800\ \text{cm}^2$$

37.



$d = \sqrt{2.27^2 + 1.86^2}$

$d = 2.934706$ km

For the right triangle,

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(2.27)(1.86)$

$A = 2.1111 \text{ km}^2$

For obtuse triangle,

$s = \dfrac{1.46 + 1.74 + d}{2}$

$s = \dfrac{1.46 + 1.74 + 2.934706}{2}$

$s = 3.06735$ km

$A = \sqrt{s(s - 1.46)(s - d)(s - 1.74)}$

$A = \sqrt{3.06735(3.06735 - 1.46)(3.06735 - 2.934706)(3.06735 - 1.74)}$

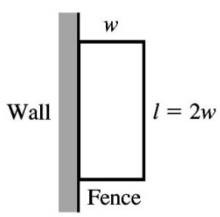
$A = 0.931707 \text{ km}^2$

$A_{\text{quadrilateral}}$ = Sum of areas of two triangles

$A = 2.1111 \text{ km}^2 + 0.931707 \text{ km}^2$

$A = 3.04 \text{ km}^2$

38.



$Cost$ = cost of wall + cost of fence

$13200 = 50(2w) + 5(2w) + 5w + 5w$

$13200 = 120w$

$w = 110$ m

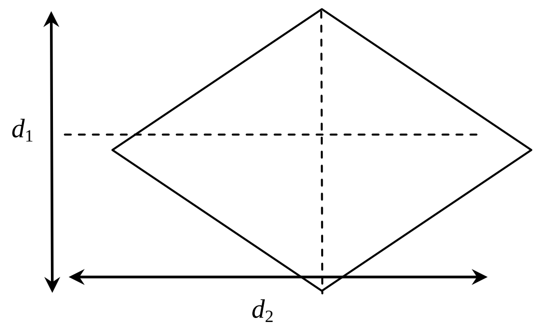
$l = 2w$

$l = 220$ m

39.

360°. A diagonal divides a quadrilateral into two triangles, and the sum of the interior angles of each triangle is 180°.

40.



The rhombus consists of four triangles, the areas of which are equal since the sides are all consistently $\frac{1}{2}d_1$ and $\frac{1}{2}d_2$

$A = 4\left(\frac{1}{2}bh\right)$

$A = 4\left(\frac{1}{2}\left(\frac{1}{2}d_2\right)\left(\frac{1}{2}d_1\right)\right)$

$A = \frac{1}{2}d_1 d_2$

2.4 Circles

1. $\angle OAB + OBA + \angle AOB = 180^\circ$

$$\angle OAB + 90^\circ + 72^\circ = 180^\circ$$

$$\angle OAB = 18^\circ$$

2. $A = \pi r^2 = \pi(2.4)^2$

$A = 18 \text{ km}^2$

3. $p = 2s + \dfrac{2\pi s}{4} = 2s + \dfrac{\pi s}{2}$

$p = 2(3.25) + \dfrac{\pi(3.25)}{2}$

$p = 11.6$ in.

$A = \dfrac{\pi s^2}{4} = \dfrac{\pi(3.25)}{4}$

$A = 8.30 \text{ in}^2$

4. $\overset{\frown}{AC} = 2 \cdot \angle ABC$

$= 2(25^\circ)$

$= 50^\circ$

5. (a) AD is a secant line.
(b) AF is a tangent line.

6. (a) EC and BC are chords.
(b) $\angle ECO$ is an inscribed angle.

7. (a) $AF \perp OE$.
(b) $\triangle OCE$ is isosceles.

8. (a) EC and $\overset{\frown}{EC}$ enclose a segment.
(b) Radii OE and OB enclose a sector with an acute central angle.

9. $c = 2\pi r = 2\pi(275) = 1730$ cm

10. $c = 2\pi r = 2\pi(0.563) = 3.54$ m

11. $d = 2r$; $c = \pi d = \pi(23.1) = 72.6$ mm

12. $c = \pi d = \pi(8.2) = 26$ dm

13. $A = \pi r^2 = \pi(0.0952)^2 = 0.0285 \text{ km}^2$

14. $A = \pi r^2 = \pi(45.8)^2 = 6590 \text{ cm}^2$

15. $A = \pi(d/2)^2 = \pi(2.33/2)^2 = 4.26 \text{ m}^2$

16. $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(1256)^2 = 1\,239\,000 \text{ mm}^2$

17. $\angle CBT = 90^\circ - \angle ABC = 90^\circ - 65^\circ = 25^\circ$

18. $\angle BCT = 90^\circ$, any angle such as $\angle BCA$ inscribed in a semicircle is a right angle and $\angle BCT$ is supplementary to $\angle BCA$.

19. A tangent to a circle is perpendicular to the radius drawn to the point of contact. Therefore,
$\angle ABT = 90^\circ$
$\angle CBT = \angle ABT - \angle ABC = 90^\circ - 65^\circ = 25^\circ$;
$\angle CAB = 25^\circ$

20. $\angle BTC = 65^\circ$; $\angle CBT = 35^\circ$ since it is complementary to $\angle ABC = 65^\circ$.
$(\angle CBT = 35^\circ) + \angle BTC = 90^\circ$
Therefore $\angle BTC = 65^\circ$

21. $\overset{\frown}{BC} = 2(60^\circ) = 120^\circ$

22.
$$\overset{\frown}{BC} = 2(60^\circ) = 120^\circ$$
$$\overset{\frown}{AB} + 80^\circ + 120^\circ = 360^\circ$$
$$\overset{\frown}{AB} = 160^\circ$$

23. $\angle ABC = (1/2)(80^\circ) = 40^\circ$ since the measure of an inscribed angle is one-half its intercepted arc.

24. $\angle ACB = \dfrac{1}{2}(160^\circ) = 80^\circ$

25. $022.5^\circ = 022.5^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 0.393 \text{ rad}$

26. $60.0^\circ = 60.0^\circ \cdot \dfrac{\pi \text{ rad}}{180^\circ} = 1.05 \text{ rad}$

27. $125.2^\circ = 125.2\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 2.185 \text{ rad}$

28. $323.0^\circ = 323.0^\circ \cdot \dfrac{\pi \text{ rad}}{180^\circ} = 5.64 \text{ rad}$

29. $\text{Perimeter} = \dfrac{1}{4}(2\pi r) + 2r = \dfrac{\pi r}{2} + 2r$

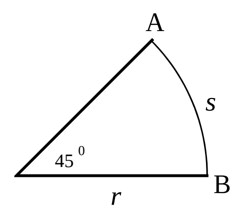
30. $\text{Perimeter} = a + b + \dfrac{1}{4} \cdot 2\pi r + r$

31. $\text{Area} = \dfrac{1}{4}\pi r^2 - \dfrac{1}{2}r^2$

32. $\text{Area} = \dfrac{1}{2}(ar) + \dfrac{1}{4}\pi r^2$

33. All are on the same diameter.

34.

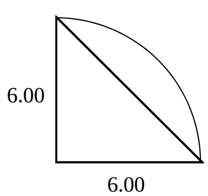


$\overset{\frown}{AB} = 45^\circ$

$\dfrac{s}{2\pi r} = \dfrac{45^\circ}{360^\circ}$

$s = \dfrac{\pi}{4} \cdot r$

35.



A of sector = A of quarter circle – A of triangle

$A = \dfrac{1}{4} \cdot \pi (6.00)^2 - \dfrac{1}{2}(6.00)(6.00)$

$A = 10.3 \text{ cm}^2$

36. $\angle ACB = \angle DCE$ (vertical angles)
$\angle BAC = \angle DEC$ and
$\angle ABC = \angle CDE$ (alternate interior angles)
The triangles are similar since
corresponding angles are equal.

37. $C = 2\pi r = 2\pi(6375) = 40\ 060 \text{ km}$

38. $11(2\pi r) = 109$

$$r = 1.58 \text{ mm}$$

39. $\dfrac{A_{\text{basketball}}}{A_{\text{hoop}}} = \dfrac{\pi\left(\dfrac{30.5}{2}\right)^2}{\pi\left(\dfrac{45.7}{2}\right)^2} = \dfrac{0.445}{1}$

40.
$$\text{flow rate} = \frac{\text{volume}}{\text{time}} = \frac{\pi r_1^2 L}{t}$$
$$2 \cdot \text{flow rate} = \frac{\pi r_2^2}{t} = \frac{2\pi r_1^2}{t}$$
$$r_2^2 = 2 \cdot r_1^2$$
$$r_2 = \sqrt{2} r_1$$

41.
$$\begin{aligned} c &= 112 \\ c &= \pi d \\ d &= c / \pi \\ &= 112 / \pi \\ &= 35.7 \text{ cm} \end{aligned}$$

42. $A = \pi\left(\dfrac{15.8}{2}\right)^2 = 196 \text{ cm}^2$

43. $A = \dfrac{\pi}{2}\left(90^2 - 45^2\right)$

$$A = 9500 \text{ cm}^2$$

44.
Let D = diameter of large conduit, then

$D = 3d$, where d = diameter of smaller conduit

$$\begin{aligned} F &= \frac{\text{area large conduit}}{\text{area 7 small conduits}} \\ &= \frac{7\pi \dfrac{d^2}{4}}{\pi \dfrac{D^2}{4}} \\ &= \frac{7d^2}{D^2} = \frac{7d^2}{(3d)^2} = \frac{7d^2}{9d^2} \\ F &= \frac{7}{9} \end{aligned}$$

The smaller conduits occupy $\dfrac{7}{9}$ of the larger conduits.

45. A of room = A of rectangle $+\frac{3}{4}A$ of circle

$$A = 8100(12\ 000) + \frac{3}{4}\pi(320)^2$$
$$A = 9.7 \times 10^7\ \text{mm}^2$$

46. Length $= (2)\frac{3}{4}(2\pi)(5.5) + (4)(5.5) = 73.8$ cm

47. Horizontally and opposite to original direction

48. Let A be the left end point at which the dashed lines intersect and C be the center of the gear. Draw a line from C bisecting the 20° angle. Call the intersection of this line and the extension of the upper dashed line B, then

$$\frac{360^\circ}{24\text{ teeth}} = \frac{15^\circ}{\text{tooth}} \Rightarrow \angle ACB = 7.5^\circ$$
$$\angle ABC = 180^\circ - \frac{20^\circ}{2} = 170^\circ$$
$$\angle\frac{1}{2}x + \angle ABC + \angle ACB = 180^\circ$$
$$\angle\frac{1}{2}x + 170^\circ + 7.5^\circ = 180^\circ$$
$$\angle\frac{1}{2}x = 2.5^\circ$$
$$x = 5^\circ$$

49. $s = \theta r$

$$s = (2.8)\left(\frac{450}{2}\text{km}\right)$$
$$s = 630\text{ km}$$

2.5 Measurement of Irregular Areas

1.
The use of smaller intervals improves the approximation since the total omitted area or the total extra area is smaller. Also, since the number of intervals would be 10 (an even number) Simpson's Rule could be employed to achieve a more accurate estimate.

2.
Using data from the south end as stated gives only five intervals. Therefore, the trapezoidal rule must be used since Simpson's rule cannot be used for an odd number of intervals.

3.
Simpson's rule should be more accurate in that it accounts better for the arcs between points on the curve, and since the number of intervals (6) is even, Simpson's Rule can be used.

4.
The calculated area would be too high since each trapezoid would include more area than under the curve. The shape of the curve is such that a straight line approximation for the curve will always overestimate the area below the curve (the curve dips below the straight line approximation).

5.

$$A_{trap} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n\right]$$

$$A_{trap} = \frac{2.0}{2}\left[0.0 + 2(6.4) + 2(7.4) + 2(7.0) + 2(6.1) + 2(5.2) + 2(5.0) + 2(5.1) + 0.0\right]$$

$$A_{trap} = 84.4 = 84 \text{ m}^2 \text{ to two significant digits}$$

6.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{2}{3}\left[0 + 4(6.4) + 2(7.4) + 4(7.0) + 2(6.1) + 4(5.2) + 2(5.0) + 4(5.1) + 0\right]$$

$$A_{\text{simp}} = 87.8667 \text{ m}^2 = 88 \text{ m}^2 \text{(to two significant digits)}$$

7.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{0.30}{3}\left[0 + 4(0.16) + 2(0.23) + 4(0.32) + 2(0.35) + 4(0.30) + 0.20\right]$$

$$A_{\text{simp}} = 0.448 \text{ m}^2 = 0.45 \text{ m}^2 \text{(rounded to 2 significant digits)}$$

8.

$$A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$$

$$A_{\text{trap}} = \frac{0.3}{2}\left[0 + 2(0.16) + 2(0.23) + 2(0.32) + 2(0.35) + 2(0.30) + 0.20\right]$$

$$A_{\text{trap}} = 0.438 \text{ m}^2 = 0.44 \text{ m}^2 \text{ (rounded to 2 significant digits)}$$

9.

$$A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$$

$$A_{\text{trap}} = \frac{0.5}{2}\left[0.6 + 2(2.2) + 2(4.7) + 2(3.1) + 2(3.6) + 2(1.6) + 2(2.2) + 2(1.5) + 0.8\right]$$

$$A_{\text{trap}} = 9.8 \text{ km}^2$$

10.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{0.5}{3}\left[0.6 + 4(2.2) + 2(4.7) + 4(3.1) + 2(3.6) + 4(1.6) + 2(2.2) + 4(1.5) + 0.8\right]$$

$$A_{\text{simp}} = 9.3333 \text{ km}^2 = 9.3 \text{ km}^2 \text{ (rounded to 2 significant digits)}$$

11.

$$A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$$

$$A_{\text{trap}} = \frac{6.0}{2}\left[7 + 2(15) + 2(7) + 2(11) + 2(13) + 2(10) + 2(9) + 2(12) + 2(8) + 3\right]$$

$$A_{\text{trap}} = \left(540 \text{ mm}^2\right)\left(\frac{6 \text{ km}}{1 \text{ mm}}\right)^2$$

$$A_{\text{trap}} = 19\,000 \text{ km}^2$$

12.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{1.5}{3}\left[0 + 4(5.0) + 2(7.2) + 4(8.3) + 2(8.6) + 4(8.3) + 2(7.2) + 4(5.0) + 0.0\right]$$

$$A_{\text{simp}} = 76.2 \text{ m}^2 = 76 \text{ m}^2$$

13.

$$A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$$

$$A_{\text{trap}} = \frac{2.0}{2}\Big[0 + 2(5.2) + 2(14.1) + 2(19.9) + 2(22.0) + 2(23.4) + 2(23.6) + 2(22.5)$$
$$+ 2(17.9) + 2(16.5) + 2(13.5) + 2(9.1) + 0\Big]$$

$$A_{\text{trap}} = 375.4 \text{ km}^2 = 380 \text{ km}^2$$

14.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{2}{3}\left[0 + 4(5.2) + 2(14.1) + 4(19.9) + 2(22.0) + 4(23.4) + 2(23.6) + 4(22.5)\right.$$
$$\left. + 2(17.9) + 4(16.5) + 2(13.5) + 4(9.1) + 0\right]$$

$$A_{\text{simp}} = 379.07 \text{ km}^2 = 380 \text{ km}^2$$

15.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{50}{3}\left[5 + 4(12) + 2(17) + 4(21) + 2(22) + 4(25) + 2(26) + 4(16) + 2(10) + 4(8) + 0\right]$$

$$A_{\text{simp}} = 8050 \text{ m}^2 = 8.0\times10^3 \text{ m}^2$$

16.

$$A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n\right]$$

$$A_{\text{trap}} = \frac{2.0}{2}\left[3.5 + 2(6.0) + 2(7.6) + 2(10.8) + 2(16.2) + 2(18.2) + 2(19.0) + 2(17.8) + 2(12.5) + 8.2\right]$$

$$A_{\text{trap}} = 228.7 \text{ cm}^2$$

$$A_{\text{circles}} = 2\left(\frac{\pi d^2}{4}\right)$$

$$A_{\text{circles}} = \frac{\pi(2.50 \text{ cm})^2}{2} = 9.817477 \text{ cm}^2$$

$$A_{\text{total}} = 228.7 \text{ cm}^2 - 9.817477 \text{ cm}^2$$

$$A_{\text{total}} = 218.88 \text{ cm}^2 = 220 \text{ cm}^2$$

17.

$$A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n\right]$$

$$A_{\text{trap}} = \frac{0.500}{2}\left[0.0 + 2(1.732) + 2(2.000) + 2(1.732) + 0.0\right]$$

$$A_{\text{trap}} = 2.73 \text{ cm}^2$$

This value is less than 3.14 cm^2 because all of the trapezoids are inscribed.

18.

$$A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n\right]$$

$$A_{\text{trap}} = \frac{0.250}{2}\begin{bmatrix} 0.000 + 2(1.323) + 2(1.732) + 2(1.936) + 2(2.000) \\ +2(1.936) + 2(1.732) + 2(1.323) + 0.000 \end{bmatrix}$$

$$A_{\text{trap}} = 3.00 \text{ cm}^2$$

The trapezoids are smaller so they can get closer to the boundary, and less area is missed from the calculation.

19.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{0.500}{3}\left[0.000 + 4(1.732) + 2(2.000) + 4(1.732) + 0.000\right]$$

$$A_{\text{simp}} = 2.98 \text{ cm}^2$$

The ends of the areas are curved so they can get closer to the boundary, including more area in the calculation.

20.

$$A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$A_{\text{simp}} = \frac{0.250}{3}\left[\begin{array}{l} 0.000 + 4(1.323) + 2(1.732) + 4(1.936) + 2(2.000) \\ +4(1.936) + 2(1.732) + 4(1.323) + 0.000 \end{array}\right]$$

$$A_{\text{simp}} = 3.08 \text{ cm}^2$$

The areas are smaller so they can get closer to the boundary.

2.6 Solid Geometric Figures

1.

$V_1 = lwh$

$V_2 = (2l)(w)(2h)$

$V_2 = 4lwh$

$V_2 = 4V_1$

The volume increases by a factor of 4.

2.

$s^2 = r^2 + h^2$

$h = \sqrt{s^2 - r^2}$

$h = \sqrt{17.5^2 - 11.9^2}$

$h = 12.8 \text{ cm}$

3.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{11.9 \text{ cm}}{2}\right)^2 \left(2(10.4 \text{ cm})\right)$$

$$V = 771 \text{ cm}^3$$

4.

$$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \pi(12.0)^2\left(\frac{40}{2}\right) + \frac{2}{3}\pi(12.0)^3$$

$$V = 12666.902 \text{ m}^3$$

$$V = 12700 \text{ m}^3$$

5.

$V = s^3$

$V = (7.15 \text{ cm})^3$

$V = 366 \text{ cm}^3$

6.

$V = \pi r^2 h$

$V = \pi(23.5 \text{ cm})^2(48.4 \text{ cm})$

$V = 83971.3 \text{ cm}^3$

$V = 8.40 \times 10^4 \text{ cm}^3$

7.

$A = 2\pi r^2 + 2\pi rh$

$A = 2\pi(689)^2 + 2\pi(689)(233)$

$A = 3\ 991\ 444\ \text{m}^2$

$A = 3.99\times 10^6\ \text{m}^2$

8.

$A = 4\pi r^2$

$A = 4\pi(0.067\ \text{mm})^2$

$A = 0.056\ \text{mm}^2$

9.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(0.877\ \text{m})^3$

$V = 2.83\ \text{m}^3$

10.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(25.1\ \text{mm})^2(5.66\ \text{mm})$

$V = 3730\ \text{mm}^3$

11.

$S = \pi rs$

$S = \pi(78.0\ \text{cm})(83.8\ \text{cm})$

$S = 20\ 534.71\ \text{cm}^2$

$S = 20\ 500\ \text{cm}^2$

12.

$S = \frac{1}{2}ps$

$S = \frac{1}{2}(345\ \text{m})(272\ \text{m})$

$S = 46\ 900\ \text{m}^2$

13.

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(76\ \text{cm})^2(130\ \text{cm})$

$V = 250\ 293\ \text{cm}^3$

$V = 2.5\times 10^5\ \text{cm}^3$

14.

$V = Bh$

$V = (29.0\text{ cm})^2(11.2\text{ cm})$

$V = 9419.2\text{ cm}^3$

$V = 9420\text{ cm}^3$

15.

$S = ph$

$S = (3\times 1.092\text{ m})(1.025\text{ m})$

$S = 3.358\text{ m}^2$

16.

$S = 2\pi rh$

$S = 2\pi\left(\frac{d}{2}\right)h$

$S = \pi(250\text{ mm})(347\text{ mm})$

$S = 272\ 533\text{ mm}^2$

$S = 270\ 000\text{ mm}^2$

$S = 2.7\times 10^5\text{ mm}^2$

17.

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{2\pi}{3}\left(\frac{d}{2}\right)^3$

$V = \frac{2\pi}{3}\left(\frac{0.83\text{ cm}}{2}\right)^3$

$V = 0.14969\text{ cm}^3$

$V = 0.15\text{ cm}^3$

18.

To analyze the right triangle formed by the center of the pyramid base, the top of the pyramid, and any lateral facelength s, notice that the bottom of that triangle has width of half the square base side length.

$$b = \frac{22.4}{2} = 11.2$$

$$s^2 = h^2 + b^2$$

$$h = \sqrt{s^2 - b^2}$$

$$h = \sqrt{14.2^2 - 11.2^2}$$

$$h = 8.72926 \text{ m}$$

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}(22.4 \text{ m})^2 (8.72926 \text{ m})$$

$$V = 1459.998 \text{ m}^3$$

$$V = 1460 \text{ m}^3$$

19.

$$s^2 = h^2 + r^2$$

$$s = \sqrt{h^2 + r^2}$$

$$s = \sqrt{0.274^2 + 3.39^2}$$

$$s = 3.401055 \text{ cm}$$

$$A = \pi r^2 + \pi r s$$

$$A = \pi(3.39 \text{ cm})^2 + \pi(3.39 \text{ cm})(3.401055 \text{ cm})$$

$$A = 72.3 \text{ cm}^2$$

20.

There are four triangles in this shape, all having the same area.
Using Hero's formula for each triangle:

$s = \frac{1}{2}(a+b+c)$

$s = \frac{1}{2}(3\times 3.67 \text{ dm})$

$s = 5.505 \text{ dm}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{5.505(1.835)^3}$

$A = \sqrt{5.505(1.835)^3}$

$A = 5.832205 \text{ dm}^2$

The total surface area A consists of four of these triangles,

$A = 4\times 5.832205 \text{ dm}^2$

$A = 23.3 \text{ dm}^2$

Or, we could determine the lateral side length *h* (triangle heights) from the Pythagorean Theorem

$a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$h = \sqrt{3.67^2 - \left(\frac{3.67}{2}\right)^2}$

$h = 3.17831 \text{ dm}$

There are four triangles of the same area, so total surface area is:

$A = 4\times \frac{1}{2}bh$

$A = 2(3.67 \text{ dm})(3.17831 \text{ dm})$

$A = 23.3 \text{ dm}^2$

21.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi \left(\frac{d}{2}\right)^3$

$V = \frac{4}{3}\pi \frac{d^3}{8}$

$V = \frac{1}{6}\pi d^3$

22.

$A = A_{\text{flat}} + A_{\text{curved}}$

$A = \pi r^2 + \frac{1}{2}\cdot 4\pi r^2$

$A = \pi r^2 + 2\pi r^2$

$A = 3\pi r^3$

23.

Let r = radius of cone,

Let h = height of the cone

$$\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = \frac{\pi(2r)^2\,\frac{h}{2}}{\frac{1}{3}\pi r^2 h}$$

$$\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = \frac{2\pi r^2 h}{\frac{1}{3}\pi r^2 h}$$

$$\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = 6$$

24.

$$A_{conebase} = \frac{1}{4}A$$

$$\pi r^2 = \frac{1}{4}\left(\pi r^2 + \pi rs\right)$$

$$4\pi r^2 = \pi r^2 + \pi rs$$

$$3\pi r^2 = \pi rs$$

$$\frac{r}{s} = \frac{1}{3}$$

25.

$$\frac{A_{final}}{A_{original}} = \frac{4\pi(2r)^2}{4\pi r^2}$$

$$\frac{A_{final}}{A_{original}} = \frac{16\pi r^2}{4\pi r^2}$$

$$\frac{A_{final}}{A_{original}} = 4$$

26.

$$w = \text{weight density} \times \text{volume}$$

$$w = \gamma V$$

$$w = 9800\frac{\text{N}}{\text{m}^3}(3.00\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(1.00\text{ km}^2\right)\left(\frac{1000\text{ m}}{\text{km}}\right)^2$$

$$w = 2.94 \times 10^8\text{ N}$$

27.

$$A = A_{base} + A_{ends} + A_{sides}$$

$$A = 2lw + 2wh + 2lh$$

$$A = 2(12.0)(9.50) + 2(9.50)(8.75) + 2(12.0)(8.75)$$

$$A = 604\text{ cm}^2$$

28.

The volume of pool can be represented by a trapezoidal right prism

$$V = A_{trapezoid} \times width$$

$$V = \frac{1}{2}h\left(b_1 + b_2\right)\cdot w$$

$$V = \frac{1}{2}(24.0)\left(2.60 + 1.00\right)\cdot(15.0)$$

$$V = 648 \text{ m}^3$$

29.

$$V = \pi r^2 h$$

$$V = \pi\left(\tfrac{d}{2}\right)^2 h$$

$$V = \frac{\pi}{4}\left(0.76 \text{ m}\right)^2\left(540\ 000 \text{ m}\right)$$

$$V = 244\ 969 \text{ m}^3$$

$$V = 2.4\times 10^5 \text{ m}^3$$

30.

There are three rectangles and two triangles in this shape.

The triangles have hypotenuse

$$c^2 = a^2 + b^2$$

$$c = \sqrt{3^2 + 4^2}$$

$$c = 5.00 \text{ cm}$$

$$A = A_{rec\tan gles} + A_{triangles}$$

$$A = (8.50)(5.00) + (8.50)(3.00) + (8.50)(4.00) + 2\left(\frac{1}{2}\right)(4.00)(3.00)$$

$$A = 114 \text{ cm}^2$$

31.

$$V = \frac{1}{3}BH$$

$$V = \frac{1}{3}\left(230^2\right)\left(150\right)$$

$$V = 2\ 645\ 000 \text{ m}^3$$

$$V = 2.6\times 10^6 \text{ m}^3$$

32.

Use the Pythagorean Thoerem

$s^2 = h^2 + r^2$

$s = \sqrt{h^2 + r^2}$

$s = \sqrt{8.90^2 + 4.60^2}$

$s = 10.0185 \text{ cm}$

$S = \pi rs$

$S = \pi\left(4.60 \text{ cm}\right)\left(10.0185 \text{ cm}\right)$

$S = 145 \text{ cm}^2$

33.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$V = \frac{4}{3}\pi\left(\frac{50.3}{2}\right)^3$

$V = 66\ 635 \text{ m}^3$

$V = 66\ 600 \text{ m}^3$

34.

$V = \frac{4}{3}\pi r^3 + \pi r^2 h$

$V = \frac{4}{3}\pi\left(0.61\right)^3 + \pi\left(0.61\right)^2\left(1.98\right)$

$V = 3.27 \text{ m}^3$

35.

The lateral side length can be determined from the Pythagorean Theorem

$s^2 = 8.0^2 + h^2$

$s = \sqrt{8.0^2 + 40.0^2}$

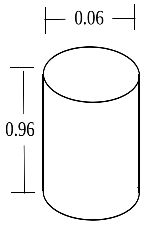
$s = 40.792 \text{ mm}$

$A = x^2 + \frac{1}{2}ps$

$A = 16^2 + \frac{1}{2}(4\times 16)(40.792)$

$A = 1560 \text{ mm}^2$

36.



Let n = number of revolutions of the lateral surface area S

$$n \cdot S = 76$$
$$n \cdot 2\pi rh = 76$$
$$n = \frac{76}{2\pi\left(\frac{d}{2}\right)h}$$
$$n = \frac{76}{\pi dh}$$
$$n = \frac{76 \text{ m}^2}{\pi(0.60 \text{ m})(0.96 \text{ m})}$$
$$n = 42 \text{ revolutions}$$

37.

$$c = 2\pi r$$
$$75.7 = 2\pi r$$
$$r = \frac{75.7}{2\pi}$$
$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi\left(\frac{75.7}{2\pi}\right)^3$$
$$V = 7330 \text{ cm}^3$$

38.

$$S = 2\pi rh$$
$$S = 2\pi\left(\frac{d}{2}\right)h$$
$$S = \pi dh$$

Assuming the label overlaps on both ends of the can by 0.50 cm

$$S = \pi(8.50 \text{ cm})(11.5 + 0.5 + 0.5)$$
$$S = 334 \text{ cm}^2$$

39.

$V = V_{cylinder} + V_{cone}$

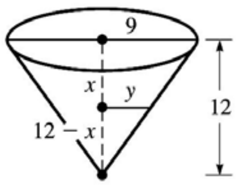
$V = \pi r_{cylinder}{}^2 h_{cylinder} + \frac{1}{3}\pi r_{cone}{}^2 h_{cone}$

$V = \pi (0.625/2)^2 (2.75) + \frac{1}{3}\pi (1.25/2)^2 (0.625)$

$V = 1.09935 \text{ cm}^3$

$V = 1.10 \text{ cm}^3$

40.



$$\frac{9}{12} = \frac{y}{12 - x}$$

$$y = \frac{3}{4}(12 - x)$$

To achieve half the volume of the cone

$$\frac{V_{cone}}{2} = V_{\text{fluid}}$$

$$\frac{\frac{1}{3}\pi r_{cone}{}^2 h_{cone}}{2} = \tfrac{1}{3}\pi r_{fluid}{}^2 h_{fluid}$$

$$\frac{\frac{1}{3}\pi (9^2)12}{2} = \frac{1}{3}\pi \left(\frac{3}{4}(12 - x)\right)^2 (12 - x)$$

$$\frac{9^2 \cdot 12}{2} = \left(\frac{3}{4}(12 - x)\right)^2 \cdot (12 - x)$$

$$486 = 0.5625(12 - x)^3$$

$$864 = (12 - x)^3$$

$$\sqrt[3]{864} = 12 - x$$

$$x = 12 - \sqrt[3]{864}$$

$$x = 2.48 \text{ cm}$$

Review Exercises

1.
$\angle CGH$ and given angle 148° are corresponding angles, so
$\angle CGH = 148^\circ$
$\angle CGE$ and $\angle CGH$ are supplementary angles so
$\angle CGE + \angle CGH = 180^\circ$
$\angle CGE = 180^\circ - 148^\circ$
$\angle CGE = 32^\circ$

2.
$\angle CGE = 32^\circ$ from Question 1
$\angle CGE$ and $\angle EGF$ are complementary angles so
$\angle CGE + \angle EGF = 90^\circ$
$\angle EGF = 90^\circ - 32^\circ$
$\angle EGF = 58^\circ$

3.
$\angle CGE = 32^\circ$ from Question 1
$\angle CGE$ and $\angle DGH$ are vertically opposite angles
$\angle DGH = \angle CGE$
$\angle DGH = 32^\circ$

4.
$\angle CGE = 32^\circ$ from Question 1
$\angle EGI = \angle CGE + 90^\circ$
$\angle EGI = 32^\circ + 90^\circ$
$\angle EGI = 122^\circ$

5.
$c^2 = a^2 + b^2$
$c = \sqrt{9^2 + 40^2}$
$c = \sqrt{1681}$
$c = 41$

6.
$c^2 = a^2 + b^2$
$c = \sqrt{14^2 + 48^2}$
$c = \sqrt{2500}$
$c = 50$

7.

$$c^2 = a^2 + b^2$$
$$c = \sqrt{400^2 + 580^2}$$
$$c = \sqrt{496\ 400}$$
$$c = 704.55659815$$
$$c = 700$$

8.

$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$a = \sqrt{65^2 - 56^2}$$
$$a = \sqrt{1089}$$
$$a = 33$$

9.

$$c^2 = a^2 + b^2$$
$$c = \sqrt{6.30^2 + 3.80^2}$$
$$c = \sqrt{54.13}$$
$$c = 7.357309291$$
$$c = 7.36$$

10.

$$c^2 = a^2 + b^2$$
$$c = \sqrt{126^2 + 25.1^2}$$
$$c = \sqrt{16506.01}$$
$$c = 128.4757175$$
$$c = 128$$

11.

$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$a = \sqrt{36.1^2 - 29.3^2}$$
$$a = \sqrt{444.72}$$
$$a = 21.088839$$
$$a = 21.1$$

12.

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$b = \sqrt{0.885^2 - 0.782^2}$

$b = \sqrt{0.171701}$

$b = 0.41436819$

$b = 0.414$

13.

$p = 3s$

$p = 3(8.5 \text{ mm})$

$p = 25.5 \text{ mm}$

14.

$p = 4s$

$p = 4(15.2 \text{ cm})$

$p = 60.8 \text{ cm}$

15.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(3.25 \text{ m})(1.88 \text{ m})$

$A = 3.06 \text{ m}^2$

16.

$s = \frac{1}{2}(a+b+c)$

$s = \frac{1}{2}(175+138+119)$

$s = 216 \text{ cm}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{216(216-175)(216-138)(216-119)}$

$A = \sqrt{216(41)(78)(97)}$

$A = \sqrt{67\ 004\ 496}$

$A = 8185.627404 \text{ cm}^2$

$A = 8190 \text{ cm}^2$

17.

$c = 2\pi r$

$c = \pi d$

$c = \pi(98.4 \text{ mm})$

$c = 309.1327171 \text{ mm}$

$c = 309 \text{ mm}$

18.

$p = 2l + 2w$

$p = 2(2.98\text{ dm}) + 2(1.86\text{ dm})$

$p = 9.68\text{ dm}$

19.

$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(34.2\text{ cm})(67.2\text{ cm} + 126.7\text{ cm})$

$A = 3315.69\text{ cm}^2$

$A = 3320\text{ cm}^2$

20.

$A = \pi r^2$

$A = \pi\left(\frac{d}{2}\right)^2$

$A = \frac{\pi d^2}{4}$

$A = \frac{\pi(32.8\text{ m})^2}{4}$

$A = 844.9627601\text{ m}^2$

$A = 845\text{ m}^2$

21.

$V = Bh$

$V = \frac{1}{2}bl \cdot h$

$V = \frac{1}{2}(26.0\text{ cm} \times 34.0\text{ cm})(14.0\text{ cm})$

$V = 6188\text{ cm}^3$

$V = 6190\text{ cm}^3$

22.

$V = \pi r^2 h$

$V = \pi(36.0\text{ cm})^2(2.40\text{ cm})$

$V = 9771.60979\text{ cm}^3$

$V = 9770\text{ cm}^3$

23.

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}\left(3850\ \text{m}^2\right)\left(125\ \text{m}\right)$

$V = 160416.6667\ \text{m}^3$

$V = 1.60 \times 10^5\ \text{m}^3$

24.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi\left(\frac{22.1\ \text{mm}}{2}\right)^3$

$V = 5651.652404\ \text{mm}^3$

$V = 5650\ \text{mm}^3$

25.

$A = 6s^2$

$A = 6\left(0.520\ \text{m}\right)^2$

$A = 1.6224\ \text{m}^2$

$A = 1.62\ \text{m}^2$

26.

$A = 2\pi r^2 + 2\pi rh$

$A = 2\pi\left(\frac{d}{2}\right)^2 + 2\pi\left(\frac{d}{2}\right)h$

$A = \frac{\pi d^2}{2} + \pi dh$

$A = \frac{\pi(1.20\ \text{cm})^2}{2} + \pi(1.20\ \text{cm})(5.80\ \text{cm})$

$A = 24.12743158\ \text{cm}^2$

$A = 24.1\ \text{cm}^2$

27.

$s^2 = r^2 + h^2$

$s = \sqrt{1.82^2 + 11.5^2}$

$s = \sqrt{135.5624}$

$s = 11.64312673\ \text{mm}$

$S = \pi rs$

$S = \pi\left(1.82\ \text{mm}\right)(11.64312673\ \text{mm})$

$S = 66.57188974\ \text{mm}^2$

$S = 66.6\ \text{mm}^2$

28.

$A = 4\pi r^2$

$A = 4\pi\left(\dfrac{12\ 760\ \text{km}}{2}\right)^2$

$A = 511\ 506\ 576\ \text{km}^2$

$A = 5.115\times10^8\ \text{km}^2$

29.

$\angle BTA = \dfrac{50^\circ}{2} = 25^\circ$

30.

$\angle TBA = 90^\circ$ since an angle inscribed in a semicircle is 90°

$\angle BTA = 25^\circ$ from Question 29

All angles in ΔBTA must sum to 180°

$\angle TAB + \angle BTA + \angle TBA = 180^\circ$

$\angle TAB = 180^\circ - 90^\circ - 25^\circ$

$\angle TAB = 65^\circ$

31.

$\angle BTC$ is a complementary angle to $\angle BTA$

$\angle BTA = 25^\circ$ from Question 29

$\angle BTC + \angle BTA = 90^\circ$

$\angle BTC = 90^\circ - 25^\circ$

$\angle BTC = 65^\circ$

32.

$\angle ABT = 90^\circ$ since any angle inscribed in a semi-circle is 90°

33.

$\angle ABE$ and $\angle ADC$ are corresponding angles since $\Delta ABE \sim \Delta ADC$

$\angle ABE = \angle ADC$

$\angle ABE = 53^\circ$

34.

$AD^2 = AC^2 + CD^2$

$AD = \sqrt{(4+4)^2 + 6^2}$

$AD = \sqrt{100}$

$AD = 10$

35.

since $\Delta ABE \sim \Delta ADC$

$$\frac{BE}{CD} = \frac{AB}{AD}$$
$$\frac{BE}{6} = \frac{4}{10}$$
$$BE = \frac{6(4)}{10}$$
$$BE = 2.4$$

36.

since $\Delta ABE \sim \Delta ADC$

$$\frac{AE}{AC} = \frac{AB}{AD}$$
$$\frac{AE}{8} = \frac{4}{10}$$
$$AE = \frac{4(8)}{10}$$
$$AE = 3.2$$

37.

$p =$ base of triangle + hypotenuse of triangle + semicircle perimeter

$$p = b + \sqrt{b^2 + (2a)^2} + \frac{1}{2}\pi(2a)$$
$$p = b + \sqrt{b^2 + 4a^2} + \pi a$$

38.

$p =$ perimeter of semicircle + 4 square lengths

$$p = \frac{1}{2}(2\pi s) + 4s$$
$$p = \pi s + 4s$$

39.

$A =$ area of triangle + area of semicircle

$$A = \frac{1}{2}b(2a) + \frac{1}{2} \cdot \pi(a)^2$$
$$A = ab + \frac{1}{2}\pi a^2$$

40.

$A =$ area of semicircle + area square

$$A = \frac{1}{2}(\pi s^2) + s^2$$

41.

A square is a rectangle with four equal sides.

A rectangle is a parallelogram with perpendicular intersecting sides so a square is a parallelogram.

A rhombus is a parallelogram with four equal sides and since a square is a parallelogram, a square is a rhombus.

42.

If two triangles share two angles that are the same, then the third angle must also be the same in both triangles.

The triangles are similar to each other because they all have the same angles, and the sides must be proportional.

43.

$A = \pi r^2$

If the radius of the circle is multplied by n, then the area of the new circle is:

$A = \pi\left(nr\right)^2$

$A = \pi(n^2 r^2)$

$A = n^2\left(\pi r^2\right)$

The area of the circle is multiplied by n^2, when the radius is multplied by *n*.

Any plane geometric figure scaled by n in each dimension will increase its area by n^2.

44.

$V = s^3$

If the length of a cube's side is multplied by n, then the volume of the new cube is:

$V = \left(ns\right)^3$

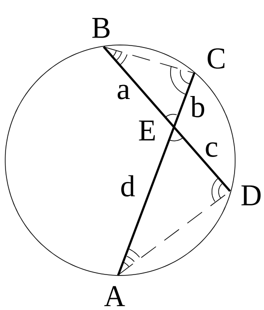
$V = (n^3 s^3)$

$V = n^3\left(s^3\right)$

The volume of the cube is multiplied by n^3, when the length of the side is multplied by *n*.

This will be true of any geometric figure scaled by n in all dimensions.

45.



$\angle BEC = \angle AED$, since they are vertically opposite angles

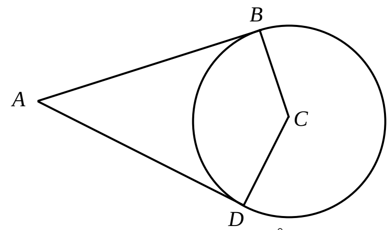
$\angle BCA = \angle ADB$, both are inscribed in $\overset{\frown}{AB}$

$\angle CBD = \angle CAD$, both are inscribed in $\overset{\frown}{CD}$

which shows $\Delta AED \sim \Delta BEC$

$\dfrac{a}{d} = \dfrac{b}{c}$

46.



We are given $\angle BAD = 36^\circ$.

The two angles $\angle ABC$ and $\angle ADC$ of the quadrilateral at the point where the tangents touch the circle are each 90°.

The four angles of the quadrilateral will add up to 360°.

$$\angle ABC + \angle ADC + \angle BAD + \angle BCD = 360^\circ$$
$$90^\circ + 90^\circ + 36^\circ + \angle BCD = 360^\circ$$
$$\angle BCD = 144^\circ$$

47.

The three angles of the triangle will add up to 180°.

If the tip of the isosceles triangle is 38°, find the other two equal angles.

$$2(\text{base angle}) + 38^\circ = 180^\circ$$
$$2(\text{base angle}) = 142^\circ$$
$$\text{base angle} = 71^\circ$$

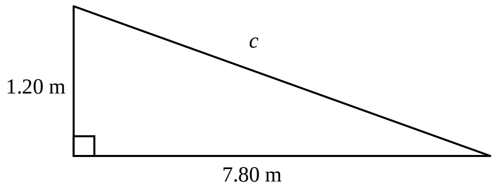
48.

The two volumes are equal

$$V_{sphere} = V_{sheet}$$
$$\frac{4}{3}\pi r_{sphere}{}^3 = \pi r_{sheet}{}^2 t$$
$$\frac{4}{3}\left(\frac{d_{sphere}}{2}\right)^3 = \left(\frac{d_{sheet}}{2}\right)^2 t$$
$$\frac{d_{sphere}{}^3}{6} = \frac{d_{sheet}{}^2}{4} t$$
$$t = \frac{2}{3} \cdot \frac{d_{sphere}{}^3}{d_{sheet}{}^2}$$
$$t = \frac{2}{3} \cdot \frac{(1.50\text{ cm})^3}{(14.0\text{ cm})^2}$$
$$t = 0.011479591\text{ cm}$$
$$t = 0.0115\text{ cm}$$

The flattened sphere is 0.0115 cm thick.

49.



$c^2 = a^2 + b^2$

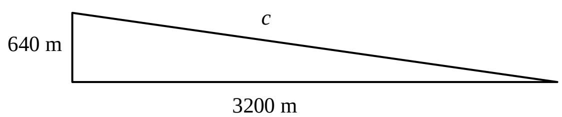
$c = \sqrt{(1.20\text{ m})^2 + (7.80\text{ m})^2}$

$c = \sqrt{62.28\text{ m}^2}$

$c = 7.891767863\text{ m}$

$c = 7.89\text{ m}$

50.



$c^2 = a^2 + b^2$

$c = \sqrt{(640\text{ m})^2 + (3200\text{ m})^2}$

$c^2 = \sqrt{10\ 649\ 600\text{ m}^2}$

$c = 3263.372489\text{ m}$

$c = 3300\text{ m}$

51.

An equilateral triangle has 3 equal sides,
so all edges of the triangle and square are 2 cm

$p = 6(2\text{ cm}) = 12\text{ cm}$

52.

A = Area of square + Area of 4 semi-circles

$A = s^2 + 4\left(\dfrac{\pi r^2}{2}\right)$

$A = s^2 + 2\pi r^2$

$A = s^2 + 2\pi\left(\dfrac{s}{2}\right)^2$

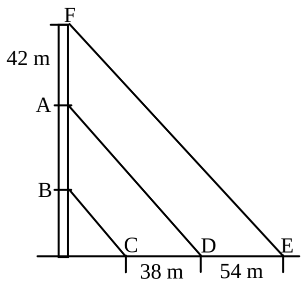
$A = s^2 + \dfrac{\pi s^2}{2}$

$A = (4.50\text{ m})^2 + \dfrac{\pi(4.50\text{ m})^2}{2}$

$A = 52.05862562\text{ m}^2$

$A = 52.1\text{ m}^2$

53.



Since line segments BC, AD, and EF are parallel,
the segments AB and CD are proportional to AF and DE

$$\frac{AB}{CD} = \frac{AF}{DE}$$

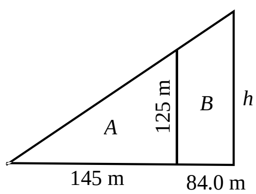
$$\frac{AB}{38\text{ m}} = \frac{42\text{ m}}{54\text{ m}}$$

$$AB = \frac{38\text{ m}(42\text{ m})}{54\text{ m}}$$

$$AB = 29.55555556\text{ m}$$

$$AB = 30\text{ m}$$

54.



Since the triangles are similar, their sides are proportional.

$$\frac{h}{145+84} = \frac{125}{145}$$

$$h = \frac{125(229)}{145}$$

$$h = 197.41379\text{ m}$$

Lot A is a triangle

$$A_A = \frac{1}{2}(145\text{ m})(125\text{ m}) = 9060\text{ m}^2$$

Lot B is a trapezoid

$$A_B = \frac{1}{2}(125\text{ m} + 197.41379\text{ m})(84.0\text{ m}) = 13\,500\text{ m}^2$$

55.

Since the triangles are proportional

$$\frac{BF}{AE}=\frac{MB}{AM}$$
$$\frac{BF}{1.6\text{ m}}=\frac{4.5\text{ m}}{1.2\text{ m}}$$
$$BF=\frac{4.5\text{ m}(1.6\text{ m})}{1.2\text{ m}}$$
$$BF=6.0\text{ m}$$

56.

The triangles are proportional so,

$$\frac{DE}{BC}=\frac{AD}{AB}$$
$$\frac{DE}{33.0\text{ cm}}=\frac{16.0\text{ cm}}{24.0\text{ cm}}$$
$$DE=\frac{16.0\text{ cm}(33.0\text{ cm})}{24.0\text{ cm}}$$
$$DE=22.0\text{ cm}$$

57.

The longest distance between points on the photograph is

$$c^2=a^2+b^2$$
$$c=\sqrt{(20.0\text{ cm})^2+(25.0\text{ cm})^2}$$
$$c=\sqrt{1025\text{ cm}^2}$$
$$c=32.01562119\text{ cm}$$

Find the distance in km represented by the longest measure on the map

$$x=(32.01562119\text{ cm})\left(\frac{18\ 450}{1}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(\frac{1\text{ km}}{1000\text{ m}}\right)$$
$$x=5.906882\text{ km}$$
$$x=5.91\text{ km}$$

58.

$$MA = \frac{\pi {r_L}^2}{\pi {r_S}^2}$$

d_L = diameter of large piston in cm

d_S = diameter of small piston in cm

$$MA = \frac{\pi \left(\frac{d_L}{2}\right)^2}{\pi \left(\frac{d_S}{2}\right)^2}$$

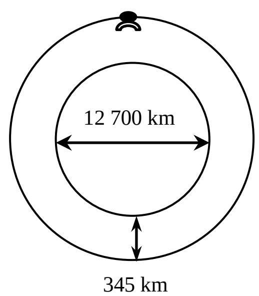
$$MA = \left(\frac{d_L}{d_s}\right)^2$$

$$MA = \left(\frac{3.10}{2.25}\right)^2$$

$$MA = 1.898271605$$

$$MA = 1.90$$

59.



The diameter of the satellite's orbit is Earth diameter plus two times its distance from the surface of Earth.

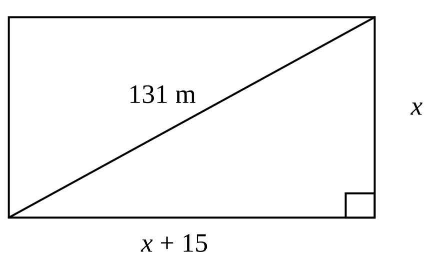
$c = \pi D$

$c = \pi\left(12\ 700\text{ km} + 2\left(345\text{ km}\right)\right)$

$c = \pi(12\ 700\text{ km} + 690\text{ km})$

$c = 42\ 100\text{ km}$

60.



$$A = lw$$
$$A = x(x+15)$$
$$A = x^2 + 15x$$

The diagonal is given, so

$$a^2 + b^2 = c^2$$
$$(x+15)^2 + x^2 = 131^2$$
$$x^2 + 30x + 225 + x^2 = 17161$$
$$2x^2 + 30x = 16936$$

The left side is twice the area!

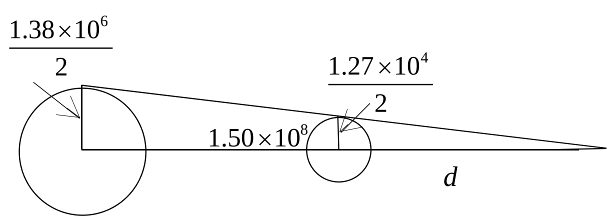
$$2(x^2 + 15x) = 16936$$
$$2A = 16936$$
$$A = \frac{16936}{2}$$
$$A = 8470 \text{ m}^2$$

61.

Area of the drywall is the area of the rectangle subtract the two circular cutouts.

$$A = lw - 2\left(\pi r^2\right)$$
$$A = lw - 2\left(\frac{\pi d^2}{4}\right)$$
$$A = lw - \frac{\pi d^2}{2}$$
$$A = \left(1200 \text{ mm}\right)\left(2400 \text{ mm}\right) - \frac{\pi (350 \text{ mm})^2}{2}$$
$$A = 2\ 687\ 577.45 \text{ mm}^2$$
$$A = 2.7 \times 10^6 \text{ mm}^2$$

62.



The triangles are similar so,

$$\frac{d}{\frac{12\ 700}{2}} = \frac{d + 150\ 000\ 000}{\frac{1\ 380\ 000}{2}}$$

$$\frac{d}{6350} = \frac{d + 150\ 000\ 000}{690\ 000}$$

$$690\ 000\ d = 6350\ \left(d + 150\ 000\ 000\right)$$

$$690\ 000\ d = 6350\ d + 952\ 500\ 000\ 000$$

$$683\ 650\ d = 952\ 500\ 000\ 000$$

$$d = \frac{952\ 500\ 000\ 000}{683\ 650}$$

$$d = 1\ 393\ 256.783 \text{ km}$$

$$d = 1.39 \times 10^6 \text{ km}$$

63.

$$A = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right]$$

$$A = \frac{250}{3}\begin{bmatrix} 220 + 4(530) + 2(480) + 4(320 + 190 + 260) \\ +2(510) + 4(350) + 2(730) + 4(560) + 240 \end{bmatrix}$$

$$A = \frac{250}{3}(12\ 740)$$

$$A = 1\ 061\ 666 \text{ m}^2$$

$$A = 1.1 \times 10^{6} \text{ m}^2$$

64.

$$V = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$$

$$V = \frac{250}{2}\left[560 + 2(1780) + 2(4650) + 2(6730) + 2(5600) + 2(6280) + 2(2260) + 230\right]$$

$$V = \frac{250}{2}(55\ 390)$$

$$V = 6\ 923\ 750 \text{ m}^3$$

$$V = 6.92 \times 10^6 \text{ m}^3$$

65.

$$V = \pi r^2 h$$

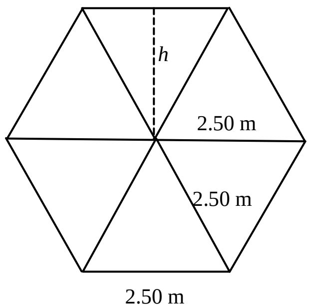
$$V = \frac{\pi d^2}{4} h$$

$$V = \frac{\pi (4.32 \text{ m})^2}{4}\left(13.2 \text{ m}\right)$$

$$V = 193.47787 \text{ m}^3$$

$$V = 193 \text{ m}^3$$

66.



Area of cross-section is the area of six equilateral triangles with sides of 2.50 m each

Using Hero's formula,

$s = \frac{1}{2}(a+b+c)$

$s = \frac{1}{2}(2.5+2.5+2.5)$

$s = 3.75 \text{ m}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{3.75(3.75-2.5)^3}$

$A = 2.70633 \text{ m}^2$

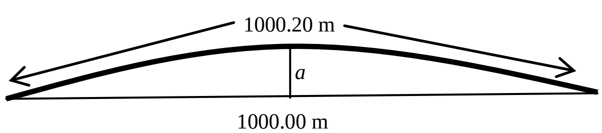
$V = \text{area of cross section} \times \text{height}$

$V = 6\left[2.70633 \text{ m}^2\right](6.75 \text{ m})$

$V = 109.6063402 \text{ m}^3$

$V = 1.10 \times 10^2 \text{ m}^3$

67.



$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

$a = \sqrt{(500.10 \text{ m})^2 - (500.00 \text{ m})^2}$

$a = \sqrt{100.01 \text{ m}^2}$

$a = 10.000 \text{ m}$

68.

$c = \text{distance apart in km}$

$c^2 = a^2 + b^2$

$c = \sqrt{(2.4 \text{ km})^2 + (3.7 \text{ km})^2}$

$c = \sqrt{19.45 \text{ km}^2}$

$c = 4.410215414 \text{ km}$

$c = 4.4 \text{ km}$

69.

$V = V_{cylinder} + V_{dome}$

$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

Note the height if the cylinder is the total height less the radius of the hemisphere.

$V = \pi(0.380\text{ m})^2(2.05\text{ m} - 0.380\text{ m}) + \frac{2}{3}\pi(0.380\text{ m})^3$

$V = 0.872512433\text{ m}^3$

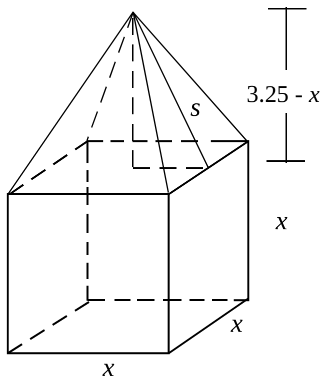
Convert m^3 to L,

$V = 0.872512433\text{ m}^3\left(\frac{1000\text{ L}}{\text{m}^3}\right)$

$V = 872.512433\text{ L}$

$V = 873\text{ L}$

70.



Given $x = 2.50$ m

Find the lateral height s of the pyramid's triangles

$s^2 = a^2 + b^2$

$s^2 = (3.25 - x)^2 + \left(\frac{x}{2}\right)^2$

$s^2 = (0.75\text{ m})^2 + (1.25\text{ m})^2$

$s = \sqrt{2.125\text{ m}^2}$

$s = 1.457737974\text{ m}$

tent surface area = surface area of pyramid + surface area of cube

tent surface area = 4 triangles + 4 squares

$A = 4\left(\frac{1}{2}xs\right) + 4x^2$

$A = 2(2.50\text{ m})(1.457738\text{ m}) + 4(2.50\text{ m})^2$

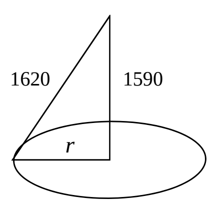
$A = 32.2887\text{ m}^2$

$A = 32.3\text{ m}^2$

71.

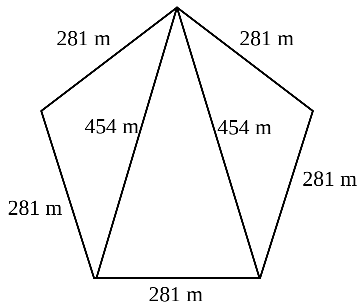
$$\frac{w}{h} = \frac{16}{9}$$
$$w = \frac{16h}{9}$$
$$107^2 = w^2 + h^2$$
$$11\ 449 = \left(\frac{16h}{9}\right)^2 + h^2$$
$$11\ 449 = \frac{256}{81}h^2 + h^2$$
$$11\ 449 = \frac{337}{81}h^2$$
$$h^2 = \frac{81(11449)}{337}$$
$$h = \sqrt{2751.836795\text{ cm}^2}$$
$$h = 52.457952645\text{ cm}$$
$$h = 52.5\text{ cm}$$
$$w = \frac{16h}{9}$$
$$w = \frac{16(52.457952645\text{ cm})}{9}$$
$$w = 93.25858247\text{ cm}$$
$$w = 93.3\text{ cm}$$

72.



$$r^2 = 1620^2 - 1590^2$$
$$r^2 = 96\ 300\text{ km}^2$$
$$A = \pi r^2$$
$$A = \pi(96\ 300\text{ km}^2)$$
$$A = 302\ 535.3725\text{ km}^2$$
$$A = 303\ 000\text{ km}^2$$

73.



The area is the sum of the areas of three triangles, one with sides 454, 454, and 281 and two with sides 281, 281, and 454. The semi-perimeters are given by

$$s_1 = \frac{281+281+454}{2} = 508$$

$$s_2 = \frac{454+454+281}{2} = 594.5$$

$$A = 2\sqrt{508(508-281)(508-281)(508-454)} + \sqrt{594.5(594.5-454)(594.5-454)(594.5-281)}$$

$$A = 136\ 000\ \text{m}^2$$