Please note : No solutions for Chapter 1 and Chapter 2

CHAPTER 3—Probability

3.1 [LO1]

Experiment—Any process of observation that has an uncertain outcome.

Event—A set of sample space outcomes.

Probability—The probability of an event is the sum of the probabilities of the sample space outcomes that correspond to the event.

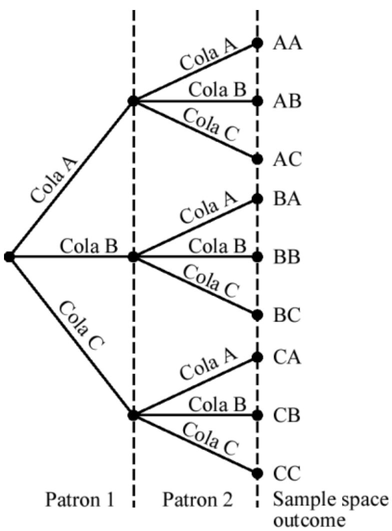
Sample Space—The set of all possible experimental outcomes.

3.2 [LO1]

The probability of an outcome must be between 0 and 1. The probabilities of all the experimental outcomes must sum to 1.

3.3 [LO2]

a.



b. (1) *AA*
(2) *AA*, *BB*, *CC*
(3) *AB*, *AC*, *BA*, *BC*, *CA*, *CB*
(4) *AA*, *AB*, *AC*, *BA*, *CA*
(5) *AA*, *AB*, *BA*, *BB*

c. Each outcome has probability $\frac{1}{9}$.

(1) $\frac{1}{9}$

(2) $3\left(\frac{1}{9}\right)=\frac{1}{3}$

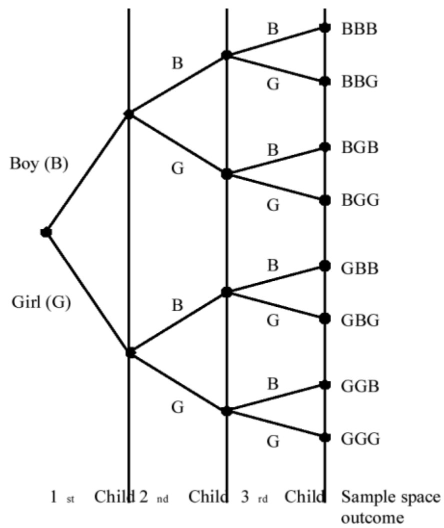
(3) $6\left(\frac{1}{9}\right)=\frac{2}{3}$

(4) $5\left(\frac{1}{9}\right)=\frac{5}{9}$

(5) $4\left(\frac{1}{9}\right)=\frac{4}{9}$

3.4 [LO2]

a.



b. (1) *BBB*, *GGG*
(2) *BGG*, *GBG*, *GGB*
(3) *GBB*, *BGB*, *BBG*
(4) *BBB*

c. Each outcome has probability $\frac{1}{8}$.

(1) $2\left(\frac{1}{8}\right)=\frac{1}{4}$

(2) $3\left(\frac{1}{8}\right)=\frac{3}{8}$

(3) $3\left(\frac{1}{8}\right)=\frac{3}{8}$

(4) $\frac{1}{8}$

3.5 [LO2]

a. (tree diagram not shown)
Sample space outcomes:
PPPP, *PPPN*, *PPNP*, *PPNN*, *PNPP*, *PNPN*, *PNNP*, *PNNN*, *NPPP*, *NPPN*, *NPNP*, *NPNN*, *NNPP*, *NNPN*, *NNNP*, *NNNN*

b. (1) *PPPN*, *PPNP*, *PNPP*, *NPPP*
(2) *PPNN*, *PNPN*, *PNNP*, *PNNN*, *NPPN*, *NPNP*, *NPNN*, *NNPP*, *NNPN*, *NNNP*, *NNNN*
(3) All outcomes except *NNNN*
(4) *PPPP*, *NNNN*

c. Each outcome has probability $\frac{1}{16}$

(1) $4\left(\frac{1}{16}\right)=\frac{1}{4}$

(2) $11\left(\frac{1}{16}\right)=\frac{11}{16}$

(3) $15\left(\frac{1}{16}\right)=\frac{15}{16}$

(4) $2\left(\frac{1}{16}\right)=\frac{1}{8}$

3.6 [LO2]

a. *PPPN*, *PPNP*, *PNPP*, *NPPP*
= 4(.1)(.1)(.1)(.9) = .0036

b. 1 – (*PPPP*, *PPPN*, *PPNP*, *PNPP*, *NPPP*)
= 1 – [(.1)(.1)(.1)(.1) + 4(.1)(.1)(.1)(.9)] = .9963

c. 1 – *NNNN*
= 1 – (.9)(.9)(.9)(.9) = .3439

d. *PPPP*, *NNNN*
= (.1)(.1)(.1)(.1) + (.9)(.9)(.9)(.9) = .6562

3.7 [LO2]

a. $\frac{119,200}{1,249,000} = .095$

b. $1 - \frac{137,700}{1,249,000} = .89$

c. $\frac{98,500 + 125,600 + 146,400}{1,249,000} = .30$

d. 1,125,000 * .084 = 94,500

3.8 [LO2]

The sum of the probabilities of the individual outcomes sum to 1.
$P(E)$ = 1 – (.2 + .15 + .3 + .2) = .15

3.9 [LO2]

1,748(.05) = 87.4 *(note, however, that you would almost certainly round this number to 87 or 88, as you cannot have ".4" of a loan)*

3.10 [LO2]

DDDDN, DDDND, DDNDD, DNDDD, NDDDD
= 5(.05)(.05)(.05)(.05)(.95) = 0.00003

3.11 [LO2]

The probability is equal for each throw, as the dice is said to be fair. Thus, the probability of winning a fifth time is .50.

3.12 [LO2]

WWWWW = (.5)(.5)(.5)(.5)(.5) = .03

3.13 [LO2]

There are 6 possible outcomes on the first die, and 6 possible outcomes on the second die – for a total of (6)(6) = 36 possible outcomes.

There are 6 ways to roll a seven:

1 and 6 or 6 and 1
2 and 5 or 5 and 2
3 and 4 or 4 and 3

There are 2 ways to roll an eleven:

5 and 6 or 6 and 5

There is 1 way to roll a two (1 and 1), or a twelve (6 and 6), and 2 ways to roll a three (1 and 2 or 2 and 1).

The probability of rolling a "natural", therefore, is:

$$\frac{6+2}{36} = .22$$

…and the probability of rolling "craps" is:

$$\frac{1+1+2}{36} = .11$$

3.14 [LO3]

Events are mutually exclusive if they have no sample space outcomes in common. The two events cannot occur simultaneously.

3.15 [LO3]

Complement of *A*: Event *A* does not occur

$A \cup B$: the union of events *A* and *B* (*A* <u>or</u> *B*)

$A \cap B$: the intersection of events *A* and *B* (*A* <u>and</u> *B*)

$A \cap B$: Event *A* does not occur and Event *B* does not occur

3.16 [LO2, LO3]

a. *R* = all diamonds and hearts
B = all clubs and spades
A = there are 4 aces, one of each suit
N = there are 4 nines, one of each suit
D = all diamonds, 13 cards
C = all clubs, 13 cards

b. (1) *R* and *A* are not mutually exclusive because there is an ace of diamonds and an ace of hearts.
(2) *R* and *C* are mutually exclusive because clubs are black.
(3) *A* and *N* are mutually exclusive because there are no aces that are also nines.
(4) *N* and *C* are not mutually exclusive because there is a nine of clubs.
(5) *D* and *C* are mutually exclusive because there are no diamonds that are also clubs.

3.17 [LO2, LO3]

a. (1) $P(M)=\dfrac{2{,}500}{10{,}000}=.25$

(2) $P(V)=\dfrac{4{,}000}{10{,}000}=.40$

(3) $P(M\cap V)=\dfrac{1{,}000}{10{,}000}=.10$

b.

	M	$\overline{M}$	Total
V	1,000	3,000	4,000
$\overline{V}$	1,500	4,500	6,000
Total	2,500	7,500	10,000

c. (1) $P(M\cup V)=P(M)+P(V)-P(M\cap V)=.25+.40-.10=.55$

(2) $P(\overline{M}\cap\overline{V})=\dfrac{4{,}500}{10{,}000}=.45$

(3) $P(M\cap\overline{V})+P(\overline{M}\cap V)=.15+.30=.45$

3.18 [LO2, LO3]

There are 24 total cards.

a. $P(J)=\dfrac{4}{24}=\dfrac{1}{6}$

b. $P(S)=\dfrac{6}{24}=\dfrac{1}{4}$

c. $P(J\cup A)=P(J)+P(A)-P(J\cap A)=\dfrac{1}{6}+\dfrac{1}{6}-0=\dfrac{1}{3}$

d. $P(J\cup S)=P(J)+P(S)-P(J\cap S)=\dfrac{1}{6}+\dfrac{1}{4}-\dfrac{1}{24}=\dfrac{3}{8}$

e. Yes, no; A jack and an ace cannot occur in a single draw, where a jack and a spade can occur simultaneously.

3.19 [LO2, LO3]

a. $\dfrac{25}{40}=\dfrac{5}{8}$

b. $\dfrac{6+15}{40}=\dfrac{21}{40}$

c. $\frac{9+10}{40}=\frac{19}{40}$

d. $\frac{15}{40}=\frac{3}{8}$

e. $\frac{15+10}{40}+\frac{6+15}{40}-\frac{15}{40}=\frac{31}{40}$

3.20 [LO2, LO3]

a. $\frac{151}{300}=.503$

b. $\frac{36+91+151}{300}=.927$

c. $\frac{48+36+9+4+4}{300}=.337, \frac{66+36+12+6+3}{300}=.410, \frac{37+19+15+3+2}{300}=.253$

d. $\frac{51}{300}=.170$

e. $\frac{2}{300}=.007$

3.21 [LO3]

a. .104 + .103 + .084 = .291

b. 1 – (.104 + .103 + .084 + .073 + .071) = .565

c. 1 – (.104 + .103 + .084 + .073 + .071 + .070 + .060) = .435

d. .073 + .071 + .070 + .060 + .058 + .053 + .048 = .433

3.22 [LO2, LO3]

Students will want to fill in a contingency table for this exercise. 40 out of the 100 purchases will result in the purchase of an accessory, and 85 of the 100 purchases will be accompanied by upselling. This gives us the marginal totals for "accessory purchase" and "upselling", respectively. Furthermore, 87.5% of the 40 accessory purchases were the result of upselling, and so 40*.875 = 35 purchases satisfied the conditions of "accessory purchase" AND "upselling". Thus, the contingency table would look like this:

	Accessory	No Accessory	
Upselling	35	50	85

No Upselling	5	10	15
	40	60	100

The probability of selling an accessory without upselling is, therefore, 5/100 = .05.

3.23 [LO2, LO3]

a. $20+1+8+1=30\%$

b. $40+1+0+1=42\%$

c. $\frac{8}{20+1+8+1}=\frac{8}{30}=.27=27\%$

d. $\frac{4}{4+40+7+9}=\frac{4}{60}=.067=6.7\%$

3.24 [LO4]

Explanations will vary.

3.25 [LO4]

Events are independent if the outcome of one does not affect the outcome of the other.

3.26 [LO4]

a. $P(V|M)=\frac{1{,}000}{2{,}500}=.40$

b. $P(M|V)=\frac{1{,}000}{4{,}000}=.25$

c. Yes, *P*(M | V) = *P*(M)

3.27 [LO4]

a. $\frac{9}{15}=.6$

b. $\frac{10}{25}=.4$

c. Dependent. For two events to be independent, *P*(Aero | Low) = *P*(Aero)

P(Aero | Low) = 2/7 but the *P*(Aero) = 3/8. They are not equal.

3.28 [LO4]

a. $P(John \cap Jane) = P(Jane)P(John \mid Jane) = (.5)(.7) = .35$

b. $P(\text{Jane} \mid \text{John}) = \frac{P(\text{John} \cap \text{Jane})}{P(\text{John})} = \frac{.35}{.4} = .875$

c. No $P(\text{John} \mid \text{Jane}) \neq P(\text{John})$

3.29 [LO3, LO4]

$P(\text{John}) + P(\text{Jane}) - P(\text{John} \cap \text{Jane}) = .4 + .5 - .35 = .55$

3.30 [LO4]

$P(\text{iTunes}) = .40$, $P(\text{Use} \mid \text{iTunes}) = .17$

a. $P(iTunes \cap Use) = P(iTunes)P(Use \mid iTunes)$

$= (.40)(.17) = .068$

b. .50 is much higher than .068.

3.31 [LO4]

$P(\text{Aware}) = .47$, $P(\text{Prog/Aware}) = .36$

In order to program the filter, the parent must be aware that it is possible.

$P(\text{Prog} \cap \text{Aware}) = P(\text{Aware})P(\text{Prog} \mid \text{Aware}) = (.47)(.36) = .1692$

3.32 [LO4]

a. $P(Manager \cap MBA) = P(Manager)P(MBA \mid Manager) = (.15)(.6) = .09$

b. $P(Manager \mid MBA) = \frac{P(Manager \cap MBA)}{P(MBA)} = \frac{.09}{.25} = .36$

c. No, $P(Manager \mid MBA) \neq P(Manager)$

3.33 [LO3, LO4]

$P(MBA \cup Manager) = P(MBA) + P(Manager) - P(MBA \cap Manager)$
$= .25 + .15 - .09 = .31$

3.34 [LO4]

a. $P(4 \text{ or } 5 | \text{Male}) = \frac{68+51}{150} = .793$

$P(4 \text{ or } 5 | \text{Female}) = \frac{83+40}{150} = .820$

.820 – .793 = .027 difference

b. $P(4 \text{ or } 5 | \text{age } 21-24) = \frac{48+36}{101} = .832$

$P(4 \text{ or } 5 | \text{age } 25-34) = \frac{66+36}{123} = .829$

$P(4 \text{ or } 5 | \text{age } 35-49) = \frac{37+19}{76} = .737$

Most appealing: age 21–24
Least appealing: age 35–49

3.35 [LO3, LO4]

a.

	FRAUD	$\overline{\text{FRAUD}}$	
FIRE	16	24	40
$\overline{\text{FIRE}}$	24	36	60
	40	60	100

b. $P(FRAUD | FIRE) = \frac{16}{40} = .4$

c. Yes; $P(FRAUD | FIRE) = \frac{16}{40} = .4 = P(\text{FRAUD})$

3.36 [LO3, LO4]

a. $P(AA) = (.8)(.8) = .64$
$P(AB) = (.8)(.1) = .08$
$P(AC) = (.8)(.1) = .08$
$P(BA) = (.1)(.8) = .08$
$P(BB) = (.1)(.1) = .01$
$P(BC) = (.1)(.1) = .01$
$P(CA) = (.1)(.8) = .08$
$P(CB) = (.1)(.1) = .01$
$P(CC) = (.1)(.1) = .01$

b. $P(BB) + P(BC) + P(CB) + P(CC) = 4(.01) = .04$

c. Probably not valid since the probability in part b is small.

3.37 [LO3, LO4]

a. $P(D1 \cap D2) = P(D1)P(D2) = (.95)(.92) = .874$

b. $P(D1 \cup D2) = P(D1) + P(D2) - P(D1 \cap D2) = .95 + .92 - .874 = .996$

c. $1 - P(D1 \cup D2) = 1 - .996 = .004$

3.38 [LO4]

a. $(.9973)^{50} = .874$

b. $x^{50} = .9973$
$x = (.9973)^{.02} = .999946$

3.39 [LO3, LO4]

a. $P(\text{none saw cut}) = (.9)^{22} \cdot (.5)^{1} \cdot (.6)^{1} = .0295$

b. $1 - P(\text{none saw cut}) = 1 - .0295 = .9705$

c. Probably not

d. Explanations will vary.

3.40 [LO5]

A prior probability is the probability an event will occur. A posterior probability is a revision to the prior probability based on new information.

3.41 [LO5]

To update probabilities based on new information.

3.42 [LO5]

$$P(A_1|B) = \frac{(.8)(.1)}{(.8)(.1)+(.2)(.3)} = .571, \; P(A_2|B) = \frac{(.2)(.3)}{(.8)(.1)+(.2)(.3)} = .429$$

3.43 [LO5]

$$P(A_1|B) = \frac{(.2)(.02)}{(.2)(.02)+(.5)(.05)+(.3)(.04)} = .098, \; P(A_2|B)$$

$$= \frac{(.5)(.05)}{(.2)(.02)+(.5)(.05)+(.3)(.04)} = .610, \; P(A_3|B) = \frac{(.3)(.04)}{(.2)(.02)+(.5)(.05)+(.3)(.04)} = .293$$

3.44 [LO5]

$$P(\text{AIDS} \mid \text{POS}) = \frac{(.01)(.999)}{(.01)(.999)+(.99)(.01)} = .502$$

3.45 [LO5]

a. $P(\text{default} \mid \text{late}) = \frac{(.03)(.95)}{(.03)(.95)+(.97)(.3)} = .089$

b. No, because the probability that a customer with at least two late payments will default (in part a) is small.

3.46 [LO5]

a. $P(\text{success} \mid \text{pass}) = \frac{(.6)(.85)}{(.6)(.85)+(.4)(.1)} = .927$

b. Yes, because the probability that an applicant who passes the exam will succeed in a management position (in part a) is high.

3.47 [LO5]

$$P(\text{specialist 1} \mid \text{incorrect}) = \frac{(.3)(.03)}{(.3)(.03)+(.45)(.05)+(.25)(.02)} = .247$$

$$P(\text{specialist 2} \mid \text{incorrect}) = \frac{(.45)(.05)}{(.3)(.03)+(.45)(.05)+(.25)(.02)} = .616$$

$$P(\text{specialist 3} \mid \text{incorrect}) = \frac{(.25)(.02)}{(.3)(.03)+(.45)(.05)+(.25)(.02)} = .137$$

3.48 [LO5]

$$P(\text{innocent} \mid \text{serum says guilty}) = \frac{(.95)(.01)}{(.95)(.01)+(.05)(.9)} = .174$$

3.49 [LO5]

Let P represent the event "predicted that the stock will increase"
Let I represent the event "stock increased"

a. $P(Correct) = P(P \mid I) + P(\sim P \mid \sim I) = .72 + .03 = .75$

Yes, the data supports his claim of 75% accuracy

b. $P(I \mid P) = \frac{72}{86} = .84$

or

$$P(I \mid P) = \frac{P(I)P(P \mid I)}{P(P)} = \frac{\left(\frac{83}{100}\right)\left(\frac{72}{83}\right)}{\frac{86}{100}} = \frac{72}{86} = .84$$

c. $P(\sim I \mid \sim P) = \frac{3}{14} = .21$

or

$$P(\sim I \mid \sim P) = \frac{P(\sim I)P(\sim P \mid \sim I)}{P(\sim P)} = \frac{\left(\frac{17}{100}\right)\left(\frac{3}{17}\right)}{\frac{14}{100}} = \frac{3}{14} = .21$$

3.50 [LO5]

$$P(\text{none} \mid \text{high}) = \frac{P(\text{none} \cap \text{high})}{P(\text{high})} = \frac{P(\text{none})P(\text{high} \mid \text{none})}{P(\text{high})} = \frac{.7(.04)}{.128} = .21875$$

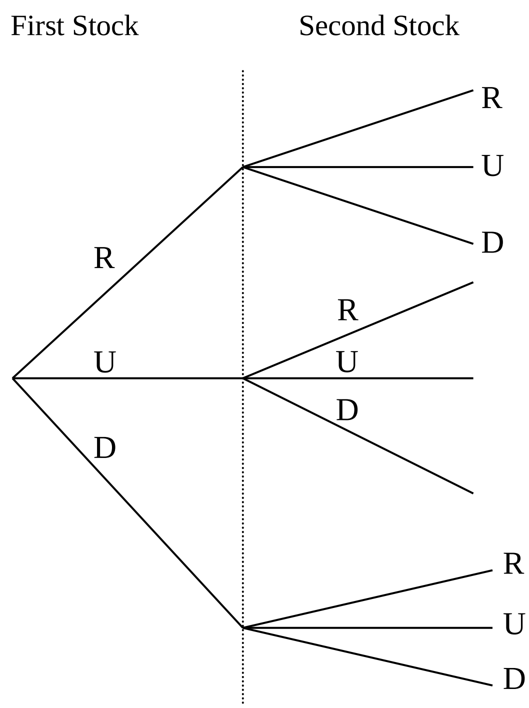
$$P(\text{some} \mid \text{high}) = \frac{P(\text{some} \cap \text{high})}{P(\text{high})} = \frac{P(\text{some})P(\text{high} \mid \text{some})}{P(\text{high})} = \frac{.2(.02)}{.128} = .03125$$

3.51 [LO5]

$$P(C \mid D) = \frac{P(C)P(D \mid C)}{P(D)} = \frac{(.333)(.171)}{.147} = .387$$

$$P(TS \mid D) = \frac{P(TS)P(D \mid TS)}{P(D)} = \frac{(.100)(.294)}{.147} = .200$$

3.52 [LO2]



3.53 [LO2, LO3]

Both stocks rise: *P*(A rises)**P*(B rises) = (1/3)(1/3) = 1/9

Both stocks decline: *P*(A decline)**P*(B decline) = (1/3)(1/3) = 1/9

Exactly one declines:

P(A+ ∩ B–) + *P*(A unchanged ∩ B–) + *P*(A– ∩ B+) + *P*(A– ∩ B unchanged) =
1/9 + 1/9 + 1/9 + 1/9 = 4/9

3.54 [LO2, LO3]

Both stocks rise: *P*(A rises)**P*(B rises) = (.6)(.6) = .36

Both stocks decline: *P*(A decline)**P*(B decline) = (.3)(.3) = .09

Exactly one declines:

P(A+ ∩ B–) + *P*(A unchanged ∩ B–) + *P*(A– ∩ B+) + *P*(A– ∩ B unchanged) =
[(.6)(.3) + (.1)(.3) + (.3)(.6) + (.3)(.1)] = .42

3.55 [LO3, LO4]

Given the table to the right, the probability that:

Both decline:
(.4)(.1) = .04

Exactly one rises:
.04 + .04 +.16 + .32 = .56

Exactly one unchanged:
.04 + .16 + .02 + .04 = .26

Both rise:
.32

P(A+)P(B+) = (.4)(.8) = .32
P(A+)P(Bu) = (.4)(.1) = .04
P(A+)P(B-) = (.4)(.1) = .04

P(Au)P(B+) = (.2)(.8) = .16
P(Au)P(Bu) = (.2)(.1) = .02
P(Au)P(B-) = (.2)(.1) = .02

P(A-)P(B+) = (.4)(.8) = .32
P(A-)P(Bu) = (.4)(.1) = .04
P(A-)P(B-) = (.4)(.1) = .04
Total = 1.00

3.56 [LO2, LO3]

$$\frac{2,507}{3,511} = .71$$

3.57 [LO2, LO3]

$$Male: \frac{2,023}{7,175} = .28$$

$$Female: \frac{2,217}{7,196} = .31$$

3.58 [LO2, LO3]

$$\frac{69}{272} = .25$$

3.59 [LO2, LO3]

$$\frac{268 + 1,704 + 1,216}{1,219 + 5,127 + 3,607} = \frac{3,188}{9,953} = .32$$

3.60 [LO4]

The probability of being in a union is different, according to the level of educational attainment achieved. Although we don't know how to test these differences (yet), this suggests that union membership depends (in some way) on educational attainment.

3.61 [LO2]

P(violence increased) = 721/1000 = .721

3.62 [LO2]

P(quality declined) = 454/1000 = .454

3.63 [LO2, LO3]

P(violence increased $\cap$ quality declined) = 362/1000 = .362

3.64 [LO2, LO3]

P(violence increased $\cup$ quality declined) = .721 + .454 – .362 = .813

3.65 [LO4]

P(quality declined | violence increased) = .362 /.721 = .502

3.66 [LO4]

P(violence increased | quality declined) = .362 / .454 = .797

3.67 [LO4]

Slight dependence: P(violence increased) = .721 vs. P(violence increased | quality declined) = .797 They are close but not equal.

3.68 [LO4]

a. P(purchased | recalled seeing ad) = .13 / .41 = .317

b. Yes, since the P(purchased) is less than the P(purchased | recalled seeing ad). The ad did work. If the ad had been ineffective, these two probabilities would have been equal.

3.69 [LO2, LO3, LO4]

a. P(bonus) = 83 / 400 = .2075
b. P(attended meeting) = 100 / 400 = .25
c. P(attended meeting $\cap$ bonus) = 42 / 400 = .105
d. P(bonus | attended meeting) = 42 / 100 = .42
e. Yes. P(bonus) is less than P(bonus | attended meeting). Also, the probabilities P(bonus) and P(bonus | attended meeting) are not equal; thus they are not independent events.

3.70 [LO3, LO4]

a. $P(A \cap B) = 0$
b. $P(A)*P(B)>0$
c. No, $P(A|B) = 0$ but $P(A) > 0$

3.71 **[LO5]**

$$P(\text{polluted} \mid \text{“device says polluted”}) = \frac{(.10)(.8)}{(.10)(.8)+(.2)(.9)} = .3077$$

3.72 **[LO5]**

$$P(\text{offered internship} \mid \text{good interview}) = \frac{(.4)(.9)}{(.4)(.9)+(.6)(.5)} = .5455$$

3.73 **[LO5]**

a. $P(\text{schizophrenia} \mid \text{brain atrophy}) = \dfrac{(.015)(.3)}{(.015)(.3)+(.985)(.02)} = .186$

b. Explanations will vary, but it doesn't help the case.

c. $P(\text{schizophrenia} \mid \text{brain atrophy}) = \dfrac{(.1)(.3)}{(.1)(.3)+(.9)(.02)} = .625$

d. More support than before

e. $P(\text{schizophrenia} \mid \text{Hinckley's scan shows atrophy}) = \dfrac{(.25)(.3)}{(.25)(.3)+(.75)(.02)} = .833$

Very strong case

Internet Exercise

3.74 January is the weakest month for auto sales.

The probability of a randomly selected vehicle being purchased in May is:

$$\frac{69,403}{32,183+39,579+\ldots+45,646}$$

$$= \frac{69,403}{691,079} = .10$$

The probability of a randomly selected vehicle being purchased in January is:

$$\frac{32,183}{691,079} = .047$$