

1. Find the average rate of change of  $y$  with respect to  $x$  for  $y = f(x) = \frac{3}{x^4}$  over the interval

[1, 7].

A) 0.375    B) -0.500    C) -0.500    D) -17.993    E) 2.999

Ans: B

Difficulty: Easy

Section: 2.1

2. Find the average rate of change of  $y$  with respect to  $x$  over the interval [1, 5].  $y = f(x) = 3x^3$

A) 62    B) 95    C) 93    D) 74    E) 372

Ans: C

Difficulty: Easy

Section: 2.1

3. Find the instantaneous rate of change of  $y = 4x^2$  with respect to  $x$  at  $x_0 = 7$ .

A) 8    B) 56    C) 14    D) 28    E) 22

Ans: B

Difficulty: Easy

Section: 2.1

4. Find the instantaneous rate of change of  $y = \frac{9}{x}$  with respect to  $x$  at  $x_0 = 5$ .

A) -225    B) -8.9600    C) 0.3600    D) -0.3600    E) -0.0617

Ans: D

Difficulty: Medium

Section: 2.1

5. Find the instantaneous rate of change of  $y = -4x^7$  with respect to  $x$  at a general point  $x_0$ .

A)  $-28x_0^7$     B)  $-4x_0$     C)  $-4x_0^7$     D)  $-4x_0^6$     E)  $-28x_0^6$

Ans: E

Difficulty: Easy

Section: 2.1

6. Find the instantaneous rate of change of  $y = \frac{2}{x^3}$  with respect to  $x$  at a general point  $x_0$ .

A)  $\frac{-6}{x_0^3}$     B)  $\frac{2}{x_0^4}$     C)  $\frac{-6}{x_0^4}$     D)  $\frac{6}{x_0^4}$     E)  $\frac{6}{x_0^3}$

Ans: C

Difficulty: Medium

Section: 2.1

7. Find the slope of the tangent line to the graph of  $f(x) = 7x^4 - 9$  at a general point  $x_0$ .  
A)  $28x_0^3 - 9$    B)  $7x_0^3$    C)  $28x_0^3$    D)  $7x_0^3 - 1$    E)  $7x_0^3 - 9$

Ans: C

Difficulty: Easy

Section: 2.1

8. Answer true or false. The slope of the tangent line to the graph of  $f(x) = -2x^2 - 1$  at  $x_0 = 3$  is  $-13$ .

Ans: False

Difficulty: Easy

Section: 2.1

9. Answer true or false. Use a graphing utility to graph  $y = 3t^2$  on  $[0, 4]$ . If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is increasing over the graphed domain.

Ans: True

Difficulty: Easy

Section: 2.1

10. Use a graphing utility to graph  $y = t^2 - 7t + 10$  on  $[0, 10]$ . If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is zero at what time? Assume time is in seconds.

A) 6s   B) 3s   C) 3.5s   D) 1.5s   E) 7s

Ans: C

Difficulty: Medium

Section: 2.1

11. A rock is dropped from a height of 2,704 feet and falls toward earth in a straight line. In  $t$  seconds the rock drops a distance of  $16t^2$  feet. What is the instantaneous velocity downward when it hits the ground?

A) 116,985,856 feet/s                                      D) 32 feet/s  
B) 416 feet/s    E) 26 feet/s  
C) 208 feet/s

Ans: B

Difficulty: Easy

Section: 2.1

12. Answer true or false. The magnitude of the instantaneous velocity is always less than the magnitude of the average velocity.

Ans: False

Difficulty: Easy

Section: 2.1

13. Answer true or false. If a rock is thrown straight upward to a height of 26 feet from the ground, when it returns to earth its average velocity will be its initial velocity.

Ans: False

Difficulty: Easy

Section: 2.1

14. Answer true or false. If an object is thrown straight upward with an instantaneous velocity of 35 m/s, its instantaneous velocity at the point where it stops rising is 0.

Ans: True

Difficulty: Easy

Section: 2.1

15. An object moves in a straight line so that after  $t$  s its distance in mm from its original position is given by  $s = 7t^3 + 4t$ . Its instantaneous velocity at  $t = 4$  s is

A) 336 mm    B) 1,348 mm    C) 5,380 mm    D) 340 mm    E) 116 mm

Ans: D

Difficulty: Medium

Section: 2.1

16. Find the instantaneous rate of change of  $y$  with respect to  $x$  at  $x_0 = 4$ .  $y = 6x^2 - 2$

A) 48    B) 46    C) 24    D) 50    E) 96

Ans: A

Difficulty: Easy

Section: 2.1

17. Find the instantaneous rate of change of  $y$  with respect to  $x$  at  $x_0 = 81$ .  $y = \sqrt{x} - 2$

A)  $\frac{1}{18}$     B)  $\frac{1}{9}$     C)  $\frac{11}{9}$     D)  $\frac{18}{17}$     E)  $\frac{1}{81}$

Ans: A

Difficulty: Hard

Section: 2.1

18. Let  $f(x) = \frac{1}{x^2}$ . Find the average rate of change of  $y$  with respect to  $x$  over the interval  $[5, 6]$ .

Ans:  $-\frac{11}{900}$

Difficulty: Easy

Section: 2.1

19. Let  $f(x) = \frac{1}{x^2}$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = 2$ .

Ans:  $-\frac{1}{4}$

Difficulty: Easy

Section: 2.1

20. Let  $y = x^2 + 2$ . Find the average rate of change of  $y$  with respect to  $x$  over the interval  $[-5, -1]$ .

Ans:  $-6$

Difficulty: Easy

Section: 2.1

21. Let  $y = x^2 + 6$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = -5$ .

Ans:  $-10$

Difficulty: Easy

Section: 2.1

22. Let  $y = \frac{1}{x-1}$ . Find the average rate of change of  $y$  with respect to  $x$  over the interval  $[2,4]$ .

Ans:  $-\frac{1}{3}$

Difficulty: Medium

Section: 2.1

23. Let  $y = \frac{1}{x-3}$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = 5$ .

Ans:  $-\frac{1}{4}$

Difficulty: Medium

Section: 2.1

24. Let  $y = \frac{2}{x+2}$ . Find the average rate of change of  $y$  with respect to  $x$  over the given interval  $[3,6]$ .

Ans:  $-\frac{1}{20}$

Difficulty: Medium

Section: 2.1

25. Let  $y = \frac{1}{x+4}$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = 1$ .

$$\text{Ans: } -\frac{1}{25}$$

Difficulty: Medium

Section: 2.1

26. Let  $f(x) = \frac{1}{5-x}$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent line at  $x_0 = 4$

$$\text{Ans: } \lim_{x_1 \rightarrow x_0} \frac{1}{(5-x_1)(5-x_0)} = \frac{1}{(5-x_0)^2}$$

The slope of the tangent line at  $x_0 = 4$  is  $\frac{1}{1}$ .

Difficulty: Medium

Section: 2.1

27. Let  $f(x) = \frac{1}{x-4}$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent at  $x_0 = 5$ .

$$\text{Ans: } \lim_{x_1 \rightarrow x_0} \frac{-1}{(x_1-4)(x_0-4)} = \frac{-1}{(x_0-4)^2}$$

The slope of the tangent line at  $x_0 = 5$  is  $-\frac{1}{1}$ .

Difficulty: Medium

Section: 2.1

28. Let  $f(x) = \frac{4}{x^4}$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent at  $x_0 = -5$ .

$$\text{Ans: } \lim_{x_1 \rightarrow -5} \frac{\frac{4}{x_1^4} - \frac{4}{-5^4}}{x_1 + 5} = \lim_{x_1 \rightarrow -5} \frac{4(-5^4 - x_1^4)}{625x_1^4(x_1 + 5)} = -\frac{16}{x_0^5}$$

The slope of the tangent line at  $x_0 = -5$  is  $\frac{16}{3,125}$ .

Difficulty: Medium

Section: 2.1

29. Let  $f(x) = 4x^3$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent at  $x_0 = 2$ .

$$\text{Ans: } \lim_{x_1 \rightarrow x_0} 4(x_1^2 + x_0^2) = 12x_0^2$$

Slope of tangent at  $x_0 = 2$  is 48

Difficulty: Easy

Section: 2.1

30. A rock is dropped from a height of 144 feet and falls toward the earth in a straight line. In  $t$  seconds, the rock drops a distance of  $s = 16t^2$  feet. What is the average velocity of the rock while it is falling? Use limits to find the instantaneous velocity of the rock when it hits the ground.

Ans: Average velocity: 48 feet per second

Instantaneous velocity at ground = 96 feet per second

Difficulty: Medium

Section: 2.1

31. A particle moves in a straight line from its initial position so that after  $t$  seconds, its distance is given by  $s = t^2 + t$  feet from its initial position. Find the average velocity of the particle over the interval  $[3,6]$  seconds. Use limits to find the instantaneous velocity of the particle at  $t = 1$  second.

Ans: Average velocity = 10 feet per second

The instantaneous velocity at  $t = 1$  second is 3 feet per second.

Difficulty: Medium

Section: 2.1

32. A particle moves in a straight line from its initial position so that after  $t$  seconds, its distance is given by  $s = \frac{t}{t+1}$  feet from its initial position. Find the average velocity of the particle over the interval  $[4,8]$  seconds. Use limits to find the instantaneous velocity of the particle at  $t = 4$  seconds.

Ans: Average velocity =  $\frac{1}{45}$  feet per second.

The instantaneous velocity at  $t = 4$  seconds is  $\frac{1}{25}$  feet per second.

Difficulty: Medium

Section: 2.1

33. Let  $f(x) = ax^2 + b$ , where  $a$  and  $b$  are constant. Use the method of Section 3.1 to show that the slope of the tangent to the graph of  $f$  at  $x = x_0$  is  $2ax_0$ .

$$\text{Ans: } m_{\text{tan}} = \lim_{x_1 \rightarrow x_0} \frac{(ax_1^2 + b) - (ax_0^2 + b)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} \frac{a(x_1^2 - x_0^2)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} a(x_1 + x_0) = 2ax_0$$

Difficulty: Hard

Section: 2.1

34. Let  $f(x) = ax^3 + b$ , where  $a$  and  $b$  are constants. Use the method of Section 3.1 to show that the slope of the tangent to the graph of  $f$  at  $x = x_0$  is  $3ax_0^2$ .

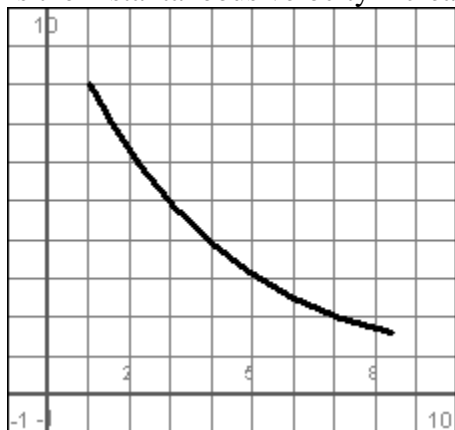
Ans:

$$m_{\text{tan}} = \lim_{x_1 \rightarrow x_0} \frac{(ax_1^3 + b) - (ax_0^3 + b)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} \frac{ax_1^3 - ax_0^3}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} a(x_1^2 + x_1x_0 + x_0^2) = 3ax_0^2$$

Difficulty: Medium

Section: 2.1

35. The graph shows the position versus time curve for a particle moving on a straight line. Is the instantaneous velocity increasing or decreasing with time?

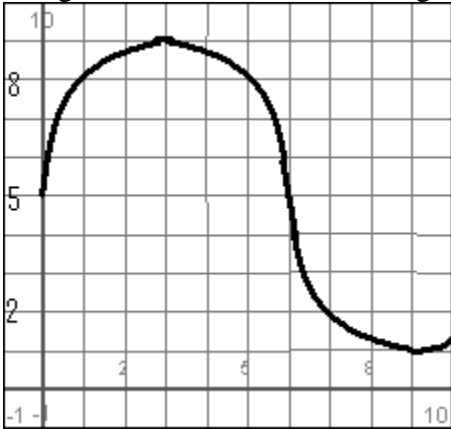


Ans: decreasing

Difficulty: Easy

Section: 2.1

36. The figure shows the position versus time curve for a certain particle moving along a straight line. Estimate, from the graph, the average velocity over the interval 3 to 9.



Ans:  $-4/3$

Difficulty: Easy

Section: 2.1

37. Given  $f(x) = x^3 - 1$ , find the slope of the graph of  $f$  at the  $x$ -value  $x_0 = 4$ .

Ans: 48

Difficulty: Medium

Section: 2.1

38. Given  $f(x) = 13\sqrt{x}$ , find the slope of the graph of  $f$  at  $x_0 = 1$ .

Ans:  $\frac{13}{2}$

Difficulty: Medium

Section: 2.1

39. Find the instantaneous rate of change of  $f(x) = \frac{2}{x^3}$  at  $x_0 = 5$ .

Ans:  $-\frac{6}{625}$

Difficulty: Medium

Section: 2.1

40. Find the instantaneous rate of change of  $f(x) = 5x^2 - 12$  at  $x_0 = 5$ .

Ans: 50

Difficulty: Medium

Section: 2.1



41. Find the instantaneous rate of change of  $f(x) = 5x^2 - 6x + 9$  at  $x_0 = 3$ .

Ans: 24

Difficulty: Medium

Section: 2.1