CHAPTER P
Preparation for Calculus

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Section P.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

x-intercept: (2, 0)

y-intercept: (0, 3)

Matches graph (b).

2. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: (0, 3)

Matches graph (d).

3. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: (0, 3)

Matches graph (a).

4. $y = x^3 - x$

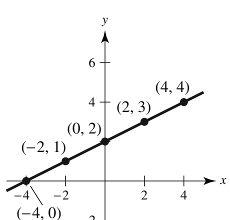
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: (0, 0)

Matches graph (c).

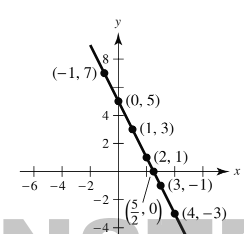
5. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



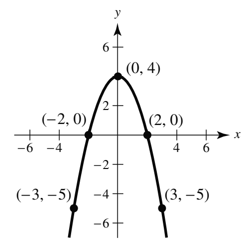
6. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



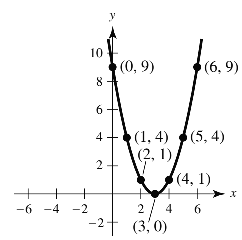
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



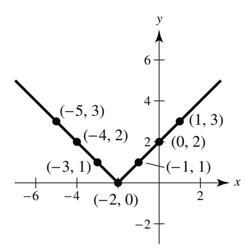
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



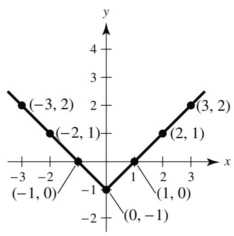
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



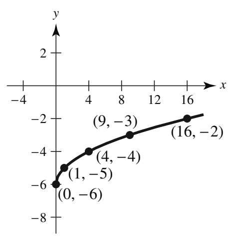
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



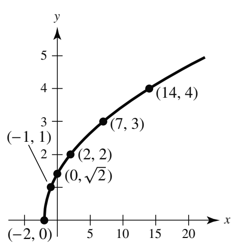
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



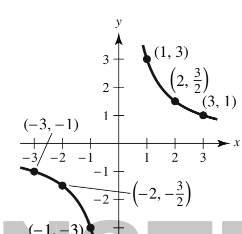
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



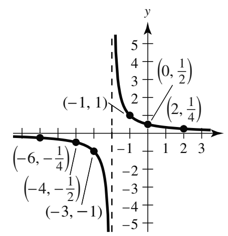
13. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

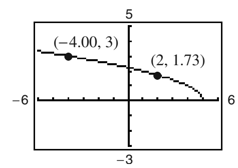


14. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



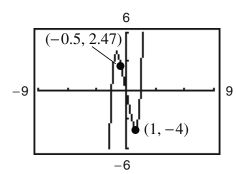
15. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73) \quad \left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3) \quad \left(3 = \sqrt{5 - (-4)}\right)$

16. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

17. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5$; $(0, -5)$

x-intercept:
$$0 = 2x - 5$$
$$5 = 2x$$
$$x = \tfrac{5}{2};\ \left(\tfrac{5}{2}, 0\right)$$

18. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3$; $(0, 3)$

x-intercept:
$$0 = 4x^2 + 3$$
$$-3 = 4x^2$$

None. y cannot equal 0.

19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2; (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1; (-2, 0), (1, 0)$

20. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0; (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2; (0, 0), (\pm 2, 0)$

21. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0; (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4; (0, 0), (4, 0), (-4, 0)$

22. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1; (1, 0)$

23. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2\ ;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4\ ;\ (4, 0)$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0; (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3; (0, 0), (-3, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0; (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0; (0, 0)$

26. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3}; \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

27. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6.$

28. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

29. Symmetric with respect to the x-axis because $(-y)^2 = y^2 = x^3 - 8x.$

30. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

31. Symmetric with respect to the origin because $(-x)(-y) = xy = 4.$

32. Symmetric with respect to the x-axis because $x(-y)^2 = xy^2 = -10.$

33. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

34. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

35. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

36. $y = \dfrac{x^2}{x^2 + 1}$ is symmetric with respect to the y-axis because $y = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1}.$

37. $y = \left|x^3 + x\right|$ is symmetric with respect to the y-axis because $y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|.$

38. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

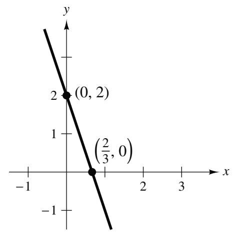
39. $y = 2 - 3x$

$y = 2 - 3(0) = 2,$ y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3},$ x-intercept

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

Symmetry: none



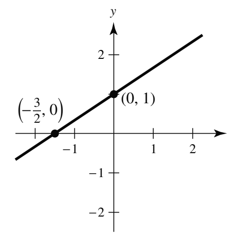
40. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1,$ y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2},$ x-intercept

Intercepts: $(0, 1), \left(-\frac{3}{2}, 0\right)$

Symmetry: none



41. $y = 9 - x^2$

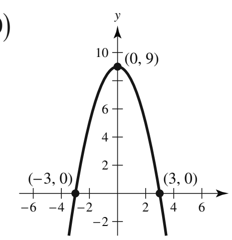
$y = 9 - (0)^2 = 9,$ y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3,$ x-intercepts

Intercepts: $(0, 9), (3, 0), (-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



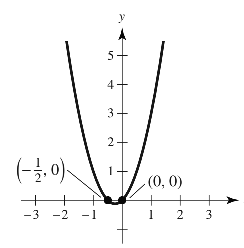
42. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0,$ y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2},$ x-intercepts

Intercepts: $(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none



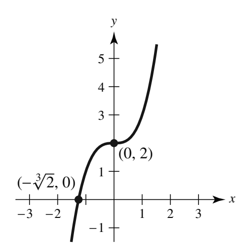
43. $y = x^3 + 2$

$y = 0^3 + 2 = 2,$ y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2},$ x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



44. $y = x^3 - 4x$

$y = 0^3 - 4(0) = 0$, y-intercept

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

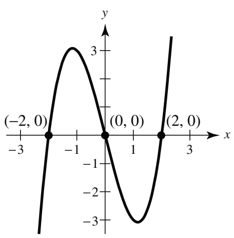
$x(x + 2)(x - 2) = 0$

$x = 0, \pm 2$, x-intercepts

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



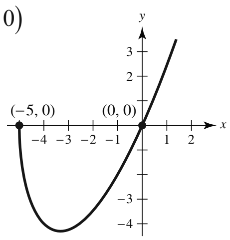
45. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



46. $y = \sqrt{25 - x^2}$

$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$\sqrt{25 - x^2} = 0$

$25 - x^2 = 0$

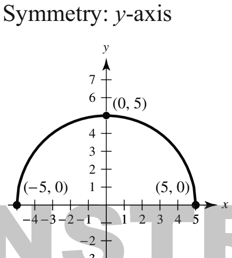
$(5 + x)(5 - x) = 0$

$x = \pm 5$, x-intercept

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



47. $x = y^3$

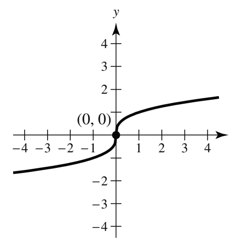
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: (0, 0)

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



48. $x = y^2 - 4$

$y^2 - 4 = 0$

$(y + 2)(y - 2) = 0$

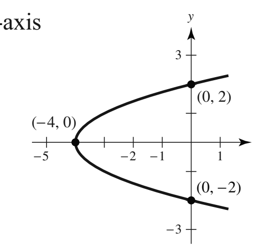
$y = \pm 2$, y-intercepts

$x = 0^2 - 4 = -4$, x-intercept

Intercepts: $(0, 2), (0, -2), (-4, 0)$

$x = (-y)^2 - 4 = y^2 - 4$

Symmetry: x-axis



49. $y = \dfrac{8}{x}$

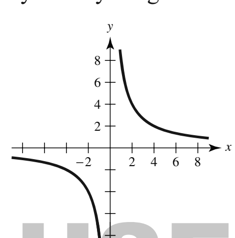
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



50. $y = \dfrac{10}{x^2 + 1}$

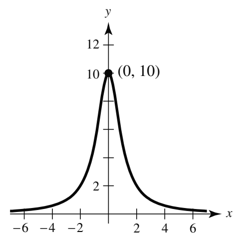
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: (0, 10)

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



51. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

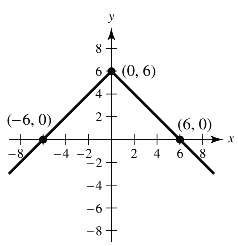
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6), (-6, 0), (6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



52. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

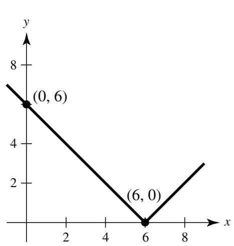
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: (0, 6), (6, 0)

Symmetry: none



53. $y^2 - x = 9$

$y^2 = x + 9$

$y = \pm\sqrt{x + 9}$

$y = \pm\sqrt{0 + 9} = \pm\sqrt{9} = \pm 3$, y-intercepts

$\pm\sqrt{x + 9} = 0$

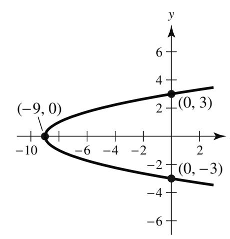
$x + 9 = 0$

$x = -9$, x-intercept

Intercepts: $(0, 3), (0, -3), (-9, 0)$

$(-y)^2 - x = 9 \Rightarrow y^2 - x = 9$

Symmetry: x-axis



54. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$x^2 + 4(0)^2 = 4$

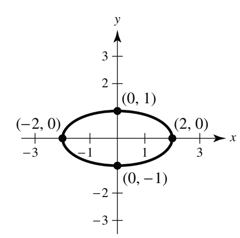
$x^2 = 4$

$x = \pm 2$, x-intercepts

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



55. $x + 3y^2 = 6$

$3y^2 = 6 - x$

$y = \pm\sqrt{\dfrac{6 - x}{3}}$

$y = \pm\sqrt{\dfrac{6 - 0}{3}} = \pm\sqrt{2}$, y-intercepts

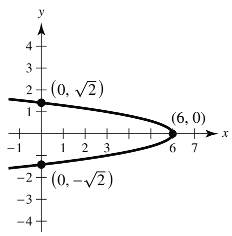
$x + 3(0)^2 = 6$

$x = 6$, x-intercept

Intercepts: $(6, 0), \left(0, \sqrt{2}\right), \left(0, -\sqrt{2}\right)$

$x + 3(-y)^2 = 6 \Rightarrow x + 3y^2 = 6$

Symmetry: x-axis



56. $3x - 4y^2 = 8$

$4y^2 = 3x - 8$

$y = \pm\sqrt{\tfrac{3}{4}x - 2}$

$y = \pm\sqrt{\dfrac{3}{4}(0) - 2} = \pm\sqrt{-2}$

$\Rightarrow$ no solution $\Rightarrow$ no y-intercepts

$3x - 4(0)^2 = 8$

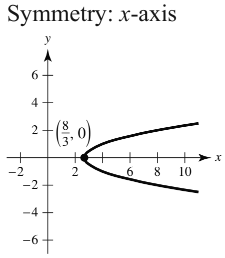
$3x = 8$

$x = \dfrac{8}{3}$, x-intercept

Intercept: $\left(\tfrac{8}{3}, 0\right)$

$3x - 4(-y)^2 = 8 \Rightarrow 3x - 4y^2 = 8$

Symmetry: x-axis



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$8 - x = 4x - 7$

$15 = 5x$

$3 = x$

The corresponding y-value is $y = 5$.

Point of intersection: (3, 5)

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$\dfrac{3x + 4}{2} = \dfrac{-4x - 10}{2}$

$3x + 4 = -4x - 10$

$7x = -14$

$x = -2$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$6 - x^2 = 4 - x$

$0 = x^2 - x - 2$

$0 = (x - 2)(x + 1)$

$x = 2, -1$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2), (-1, 5)$

60. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$3 - x = (x - 1)^2$

$3 - x = x^2 - 2x + 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$5 - x^2 = (x - 1)^2$

$5 - x^2 = x^2 - 2x + 1$

$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

62. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$-3x + y = 15 \Rightarrow y = 3x + 15$

$25 - x^2 = (3x + 15)^2$

$25 - x^2 = 9x^2 + 90x + 225$

$0 = 10x^2 + 90x + 200$

$0 = x^2 + 9x + 20$

$0 = (x + 5)(x + 4)$

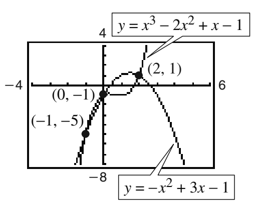
$x = -4$ or $x = -5$

The corresponding y-values are $y = 3$ (for $x = -4$) and $y = 0$ (for $x = -5$).

Points of intersection: $(-4, 3), (-5, 0)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$



Points of intersection: $(-1, -5), (0, -1), (2, 1)$

Analytically, $x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$

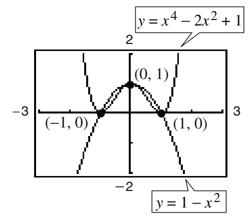
$x^3 - x^2 - 2x = 0$

$x(x - 2)(x + 1) = 0$

$x = -1, 0, 2.$

64. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$



Points of intersection: $(-1, 0), (0, 1), (1, 0)$

Analytically, $1 - x^2 = x^4 - 2x^2 + 1$

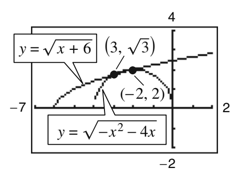
$0 = x^4 - x^2$

$0 = x^2(x + 1)(x - 1)$

$x = -1, 0, 1.$

65. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



Points of intersection: $(-2, 2), (-3, \sqrt{3}) \approx (-3, 1.732)$

Analytically, $\sqrt{x + 6} = \sqrt{-x^2 - 4x}$

$x + 6 = -x^2 - 4x$

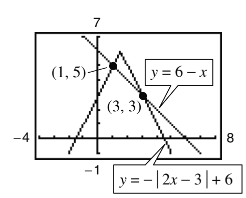
$x^2 + 5x + 6 = 0$

$(x + 3)(x + 2) = 0$

$x = -3, -2.$

66. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: $(3, 3), (1, 5)$

Analytically, $-|2x - 3| + 6 = 6 - x$

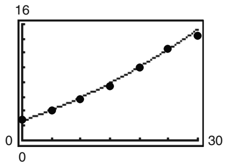
$|2x - 3| = x$

$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $x = 1.$

67. (a) Using a graphing utility, you obtain $y = 0.005t^2 + 0.27t + 2.7$.

(b)



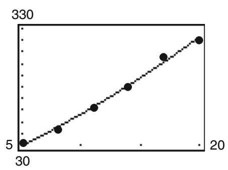
(c) For 2020, $t = 40$.

$$y = 0.005(40)^2 + 0.27(40) + 2.7 = 21.5$$

The GDP in 2020 will be \$21.5 trillion.

68. (a) Using a graphing utility, you obtain $y = 0.24t^2 + 12.6t - 40$.

(b)



The model is a good fit for the data.

(c) For 2020, $t = 30$.

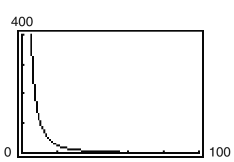
$$y = 0.24(30)^2 + 12.6(30) - 40 = 554$$

The number of cellular phone subscribers in 2020 will be 554 million.

69.
$$\begin{aligned} C &= R \\ 2.04x + 5600 &= 3.29x \\ 5600 &= 3.29x - 2.04x \\ 5600 &= 1.25x \\ x &= \frac{5600}{1.25} = 4480 \end{aligned}$$

To break even, 4480 units must be sold.

70. $y = \dfrac{10{,}770}{x^2} - 0.37$



If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

71. $y = kx^3$

(a) $(1, 4)$: $4 = k(1)^3 \Rightarrow k = 4$

(b) $(-2, 1)$: $1 = k(-2)^3 = -8k \Rightarrow k = -\frac{1}{8}$

(c) $(0, 0)$: $0 = k(0)^3 \Rightarrow k$ can be any real number.

(d) $(-1, -1)$: $-1 = k(-1)^3 = -k \Rightarrow k = 1$

72. $y^2 = 4kx$

(a) $(1, 1)$: $1^2 = 4k(1)$; $1 = 4k$; $k = \frac{1}{4}$

(b) $(2, 4)$: $(4)^2 = 4k(2)$; $16 = 8k$; $k = 2$

(c) $(0, 0)$: $0^2 = 4k(0)$; k can be any real number.

(d) $(3, 3)$: $(3)^2 = 4k(3)$; $9 = 12k$; $k = \frac{9}{12} = \frac{3}{4}$

73. Answers may vary. *Sample answer*: $y = (x + 4)(x - 3)(x - 8)$ has intercepts at $x = -4$, $x = 3$, and $x = 8$.

74. Answers may vary. *Sample answer*: $y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

75. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

76. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because $-y = (-x)^3 - (-x) = -x^3 + x$.

Symmetry with respect to the y-axis for $y = x^2 + 2$ because $y = (-x)^2 + 2 = x^2 + 2$.

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \Rightarrow y = 6$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

77. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

78. True. $f(4) = f(-4)$.

79. True. The x-intercepts are $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

80. True. The x-intercept is $\left(-\frac{b}{2a}, 0\right)$.

Section P.2 Linear Models and Rates of Change

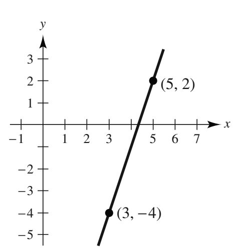
1. $m = 2$

2. $m = 0$

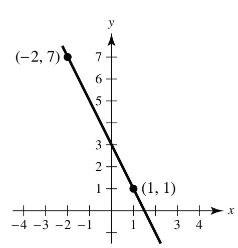
3. $m = -1$

4. $m = -12$

5. $m = \frac{2 - (-4)}{5 - 3} = \frac{6}{2} = 3$

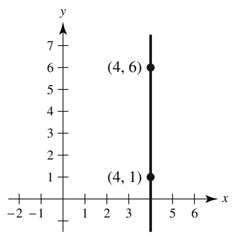


6. $m = \frac{7 - 1}{-2 - 1} = \frac{6}{-3} = -2$



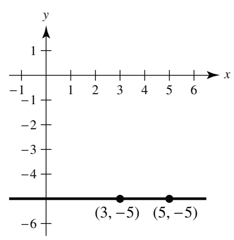
7. $m = \dfrac{1-6}{4-4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

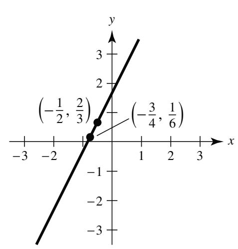


8. $m = \dfrac{-5-(-5)}{5-3} = \dfrac{0}{2} = 0$

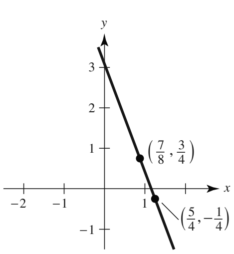
The line is horizontal.



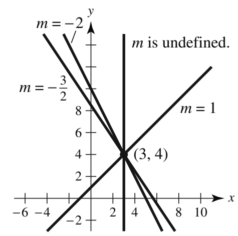
9. $m = \dfrac{\frac{2}{3}-\frac{1}{6}}{-\frac{1}{2}-\left(-\frac{3}{4}\right)} = \dfrac{\frac{1}{2}}{\frac{1}{4}} = 2$



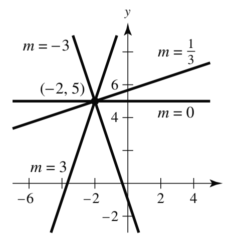
10. $m = \dfrac{\left(\frac{3}{4}\right)-\left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right)-\left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$



11.



12.



13. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2), (1, 2), (5, 2)$.

14. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0), (-4, 1), (-4, 2)$.

15. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

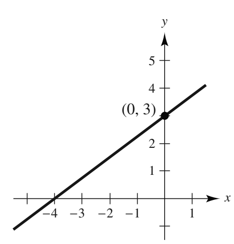
Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

16. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

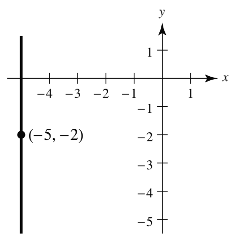
Therefore, three additional points are $(-3, -4), (-1, 0)$, and (0, 2).

17.
$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$

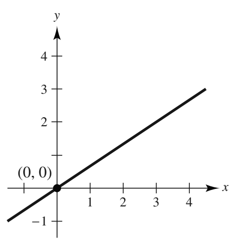


18. The slope is undefined so the line is vertical.

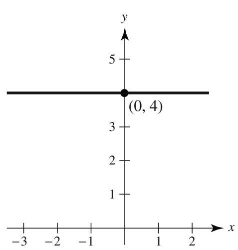
$$x = -5$$
$$x + 5 = 0$$



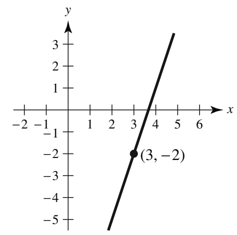
19.
$$y = \tfrac{2}{3}x$$
$$3y = 2x$$
$$0 = 2x - 3y$$



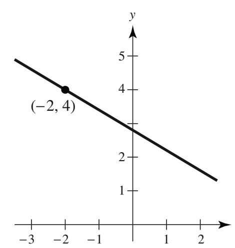
20.
$$y = 4$$
$$y - 4 = 0$$



21.
$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$0 = 3x - y - 11$$

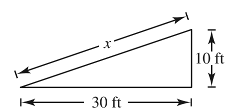


22.
$$y - 4 = -\tfrac{3}{5}(x + 2)$$
$$5y - 20 = -3x - 6$$
$$3x + 5y - 14 = 0$$



23. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

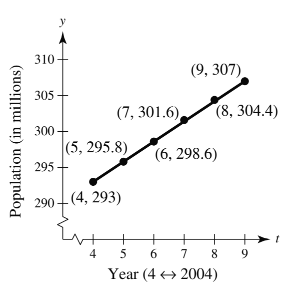
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

24. (a)



(b) The slopes are:
$$\frac{295.8 - 293.0}{5 - 4} = 2.8$$
$$\frac{298.6 - 295.8}{6 - 5} = 2.8$$
$$\frac{301.6 - 298.6}{7 - 6} = 3.0$$
$$\frac{304.4 - 301.6}{8 - 7} = 2.8$$
$$\frac{307.0 - 304.4}{9 - 8} = 2.6$$

The population increased least rapidly from 2008 to 2009.

(c) Average rate of change from 2004 to 2009:

$$\frac{307.0 - 293.0}{9 - 4} = \frac{14}{5}$$
$$= 2.8 \text{ million per yr}$$

(d) For 2020, $t = 20$ and $y \approx 16(2.8) + 293.0 = 337.8$ million.

$\left[\text{Equivalently, } y \approx 11(2.8) + 307.0 = 337.8.\right]$

25. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

26. $-x + y = 1$

$y = x + 1$

The slope is $m = 1$ and the y-intercept is $(0, 1)$.

27. $x + 5y = 20$

$y = -\frac{1}{5}x + 4$

Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is $(0, 4)$.

28. $6x - 5y = 15$

$y = \frac{6}{5}x - 3$

Therefore, the slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

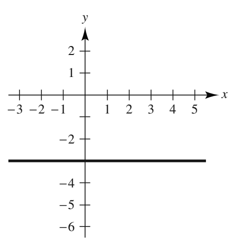
29. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

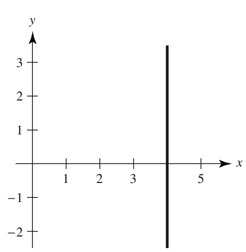
30. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

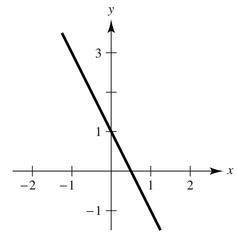
31. $y = -3$



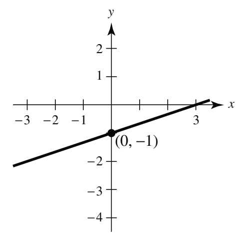
32. $x = 4$



33. $y = -2x + 1$

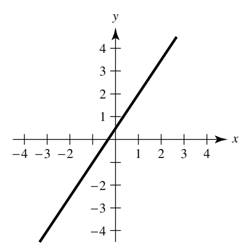


34. $y = \frac{1}{3}x - 1$



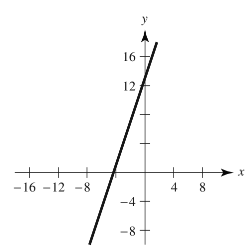
35. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



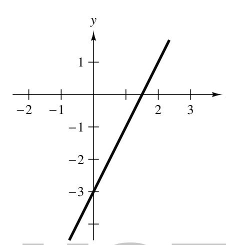
36. $y - 1 = 3(x + 4)$

$y = 3x + 13$



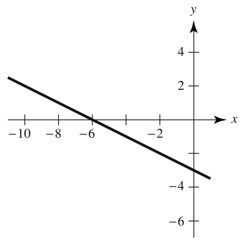
37. $2x - y - 3 = 0$

$y = 2x - 3$



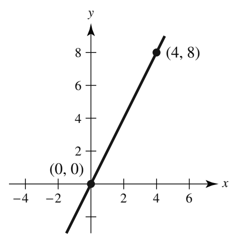
38. $x + 2y + 6 = 0$

$$y = -\tfrac{1}{2}x - 3$$



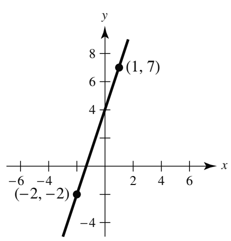
39. $$m = \frac{8-0}{4-0} = 2$$

$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$0 = 2x - y$$



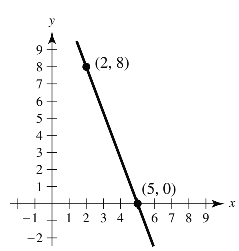
40. $$m = \frac{7-(-2)}{1-(-2)} = \frac{9}{3} = 3$$

$$y - (-2) = 3(x - (-2))$$
$$y + 2 = 3(x + 2)$$
$$y = 3x + 4$$
$$0 = 3x - y + 4$$



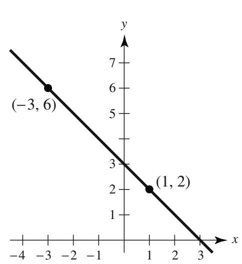
41. $$m = \frac{8-0}{2-5} = -\frac{8}{3}$$

$$y - 0 = -\frac{8}{3}(x - 5)$$
$$y = -\frac{8}{3}x + \frac{40}{3}$$
$$8x + 3y - 40 = 0$$



42. $$m = \frac{6-2}{-3-1} = \frac{4}{-4} = -1$$

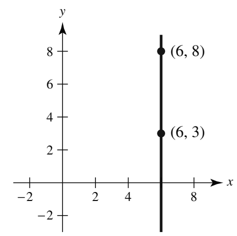
$$y - 2 = -1(x - 1)$$
$$y - 2 = -x + 1$$
$$x + y - 3 = 0$$



43. $m = \dfrac{8-3}{6-6} = \dfrac{5}{0}$, undefined

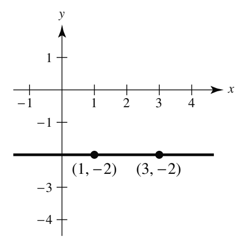
The line is horizontal.

$$x = 6$$
$$x - 6 = 0$$



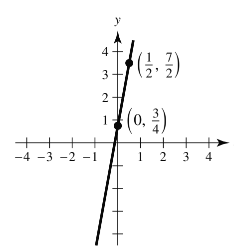
44. $$m = \frac{-2-(-2)}{3-1} = \frac{0}{2} = 0$$

$$y = -2$$
$$y + 2 = 0$$



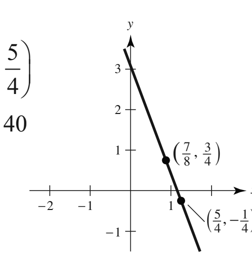
45. $$m = \frac{\frac{7}{2} - \frac{3}{4}}{\frac{1}{2} - 0} = \frac{\frac{11}{4}}{\frac{1}{2}} = \frac{11}{2}$$

$$y - \frac{3}{4} = \frac{11}{2}(x - 0)$$
$$y = \frac{11}{2}x + \frac{3}{4}$$
$$0 = 22x - 4y + 3$$



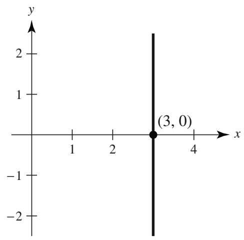
46. $$m = \frac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \frac{1}{-\frac{3}{8}} = -\frac{8}{3}$$

$$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$$
$$12y + 3 = -32x + 40$$
$$32x + 12y - 37 = 0$$



47. $x = 3$

$x - 3 = 0$

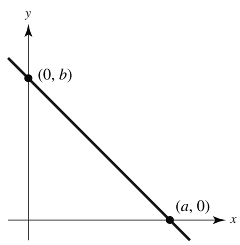


48. $m = -\dfrac{b}{a}$

$y = \dfrac{-b}{a}x + b$

$\dfrac{b}{a}x + y = b$

$\dfrac{x}{a} + \dfrac{y}{b} = 1$



49. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

50. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$\dfrac{-3x}{2} - \dfrac{y}{2} = 1$

$3x + y = -2$

$3x + y + 2 = 0$

51. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{1}{a} + \dfrac{2}{a} = 1$

$\dfrac{3}{a} = 1$

$a = 3 \Rightarrow x + y = 3$

$x + y - 3 = 0$

52. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{-3}{a} + \dfrac{4}{a} = 1$

$\dfrac{1}{a} = 1$

$a = 1 \Rightarrow x + y = 1$

$x + y - 1 = 0$

53. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$\dfrac{9}{2a} + \dfrac{-2}{a} = 1$

$\dfrac{9 - 4}{2a} = 1$

$5 = 2a$

$a = \dfrac{5}{2}$

$\dfrac{x}{2\left(\frac{5}{2}\right)} + \dfrac{y}{\left(\frac{5}{2}\right)} = 1$

$\dfrac{x}{5} + \dfrac{2y}{5} = 1$

$x + 2y = 5$

$x + 2y - 5 = 0$

54. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$\dfrac{\left(-\frac{2}{3}\right)}{a} + \dfrac{(-2)}{-a} = 1$

$-\dfrac{2}{3} + 2 = a$

$a = \dfrac{4}{3}$

$\dfrac{x}{\left(\frac{4}{3}\right)} + \dfrac{y}{\left(-\frac{4}{3}\right)} = 1$

$x - y = \dfrac{4}{3}$

$3x - 3y - 4 = 0$

55. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

56. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

57. $x - y = -2$
$y = x + 2$
$m = 1$

(a) $y - 5 = 1(x - 2)$
$y - 5 = x - 2$
$x - y + 3 = 0$

(b) $y - 5 = -1(x - 2)$
$y - 5 = -x + 2$
$x + y - 7 = 0$

58. $x + y = 7$
$y = -x + 7$
$m = -1$

(a) $y - 2 = -1(x + 3)$
$y - 2 = -x - 3$
$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$
$y - 2 = x + 3$
$0 = x - y + 5$

59. $4x - 2y = 3$
$y = 2x - \frac{3}{2}$
$m = 2$

(a) $y - 1 = 2(x - 2)$
$y - 1 = 2x - 4$
$0 = 2x - y - 3$

(b) $y - 1 = -\frac{1}{2}(x - 2)$
$2y - 2 = -x + 2$
$x + 2y - 4 = 0$

60. $7x + 4y = 8$
$4y = -7x + 8$
$y = \frac{-7}{4}x + 2$
$m = -\frac{7}{4}$

(a) $y + \frac{1}{2} = \frac{-7}{4}\left(x - \frac{5}{6}\right)$
$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$
$24y + 12 = -42x + 35$
$42x + 24y - 23 = 0$

(b) $y + \frac{1}{2} = \frac{4}{7}\left(x - \frac{5}{6}\right)$
$42y + 21 = 24x - 20$
$24x - 42y - 41 = 0$

61. $5x - 3y = 0$
$y = \frac{5}{3}x$
$m = \frac{5}{3}$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$
$24y - 21 = 40x - 30$
$0 = 40x - 24y - 9$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$
$40y - 35 = -24x + 18$
$24x + 40y - 53 = 0$

62. $3x + 4y = 7$
$4y = -3x + 7$
$y = -\frac{3}{4}x + \frac{7}{4}$
$m = -\frac{3}{4}$

(a) $y - (-5) = -\frac{3}{4}(x - 4)$
$y + 5 = -\frac{3}{4}x + 3$
$4y + 20 = -3x + 12$
$3x + 4y + 8 = 0$

(b) $y - (-5) = \frac{4}{3}(x - 4)$
$y + 5 = \frac{4}{3}x - \frac{16}{3}$
$3y + 15 = 4x - 16$
$0 = 4x - 3y - 31$

63. The slope is 250.
$V = 1850$ when $t = 2$.
$V = 250(t - 2) + 1850$
$= 250t + 1350$

64. The slope is 4.50.
$V = 156$ when $t = 2$.
$V = 4.5(t - 2) + 156$
$= 4.5t + 147$

65. The slope is −1600.
$V = 17{,}200$ when $t = 2$.
$V = -1600(t - 2) + 17{,}200$
$= -1600t + 20{,}400$

66. The slope is −5600.
$V = 245{,}000$ when $t = 2$.
$V = -5600(t - 2) + 245{,}000$
$= -5600t + 256{,}200$

67. $m_1 = \dfrac{1-0}{-2-(-1)} = -1$

$m_2 = \dfrac{-2-0}{2-(-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

68. $m_1 = \dfrac{-6-4}{7-0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11-4}{-5-0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

69. Equations of perpendicular bisectors:

$y - \dfrac{c}{2} = \dfrac{a-b}{c}\left(x - \dfrac{a+b}{2}\right)$

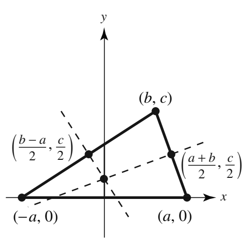
$y - \dfrac{c}{2} = \dfrac{a+b}{-c}\left(x - \dfrac{b-a}{2}\right)$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$\left(0, \dfrac{-a^2+b^2+c^2}{2c}\right).$

This point lies on the third perpendicular bisector, $x = 0$.

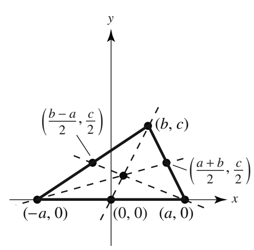


70. Equations of medians:

$y = \dfrac{c}{b}x$

$y = \dfrac{c}{3a+b}(x+a)$

$y = \dfrac{c}{-3a+b}(x-a)$



Solving simultaneously, the point of intersection is $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.

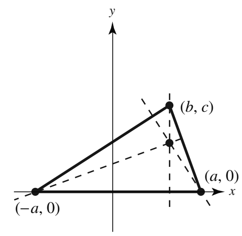
71. Equations of altitudes:

$y = \dfrac{a-b}{c}(x+a)$

$x = b$

$y = -\dfrac{a+b}{c}(x-a)$

Solving simultaneously, the point of intersection is $\left(b, \dfrac{a^2-b^2}{c}\right)$.



72. The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(b, \dfrac{a^2-b^2}{c}\right)$ is:

$m_1 = \dfrac{\left[(a^2-b^2)/c\right] - (c/3)}{b - (b/3)}$

$= \dfrac{(3a^2-3b^2-c^2)/(3c)}{(2b)/3} = \dfrac{3a^2-3b^2-c^2}{2bc}$

The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(0, \dfrac{-a^2+b^2+c^2}{2c}\right)$ is:

$m_2 = \dfrac{\left[(-a^2+b^2+c^2)/(2c)\right] - (c/3)}{0 - (b/3)}$

$= \dfrac{(-3a^2+3b^2+3c^2-2c^2)/(6c)}{-b/3} = \dfrac{3a^2-3b^2-c^2}{2bc}$

$m_1 = m_2$

Therefore, the points are collinear.

73. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$$-5x + 8y = 4$$
$$y = \frac{1}{8}(5x + 4) = \frac{5}{8}x + \frac{1}{2}$$

(d) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$$5x + 2y = 4$$
$$y = \frac{1}{2}(-5x + 4) = -\frac{5}{2}x + 2$$

(e) $a = \frac{5}{2}$ and $b = 3$.

$$\frac{5}{2}x + 3y = 4$$
$$5x + 6y = 8$$

74. (a) Lines c, d, e and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel.
Lines d and f appear parallel.

(d) Lines b and f appear perpendicular.
Lines b and d appear perpendicular.

75. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \frac{180}{100} = \frac{9}{5}$$
$$F - 32 = \frac{9}{5}(C - 0)$$
$$F = \frac{9}{5}C + 32$$

or

$$C = \frac{1}{9}(5F - 160)$$
$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

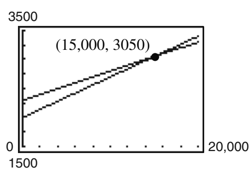
76. $C = 0.51x + 200$

For $x = 137$, $C = 0.51(137) + 200 = \$269.87$.

77. (a) Current job: $W_1 = 0.07s + 2000$

New job offer: $W_2 = 0.05s + 2300$

(b)



Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$
$$0.02s = 300$$
$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050$.

When sales exceed \$15,000, the current job pays more.

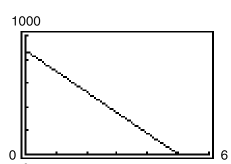
(c) No, if you can sell \$20,000 worth of goods, then $W_1 > W_2$.

(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

78. (a) Depreciation per year:

$$\frac{875}{5} = \$175$$
$$y = 875 - 175x$$

where $0 \leq x \leq 5$.



(b) $y = 875 - 175(2) = \$525$

(c)
$$200 = 875 - 175x$$
$$175x = 675$$
$$x \approx 3.86 \text{ years}$$

79. (a) Two points are (50, 780) and (47, 825).
The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

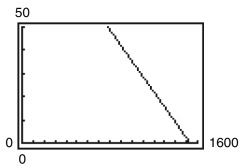
$$p - 780 = -15(x - 50)$$

$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)



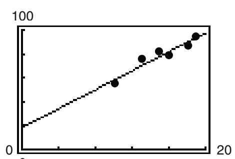
If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units

80. (a) $y = 18.91 + 3.97x$

$(x = \text{quiz score}, \; y = \text{test score})$

(b)

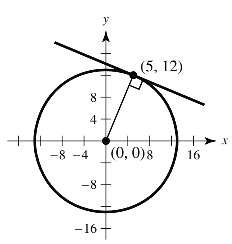


(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

81. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).



Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

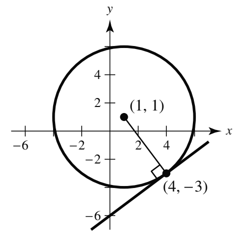
The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$

$$y = \frac{-5}{12}x + \frac{169}{12}$$

$$5x + 12y - 169 = 0.$$

82. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$$\frac{1 + 3}{1 - 4} = \frac{-4}{3}.$$

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x - 6$$

$$0 = 3x - 4y - 24$$

83. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

84. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

85. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

86. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|3(-1) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

87. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$
$$Ay - Ay_1 = Bx - Bx_1$$
$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Rightarrow A^2x + ABy = -AC \qquad (1)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2)$$
$$(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$
$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Rightarrow \quad ABx + B^2y = -BC \qquad (3)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4)$$
$$(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$
$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

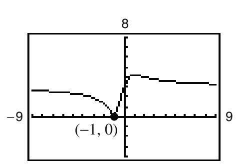
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$d = \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2}$$
$$= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2}$$
$$= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

88. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).

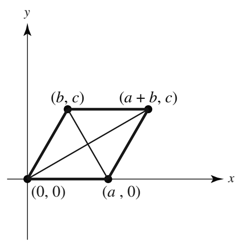


89. For simplicity, let the vertices of the rhombus be (0, 0), $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \dfrac{c}{a + b}$ and $m_2 = \dfrac{c}{b - a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



90. For simplicity, let the vertices of the quadrilateral be (0, 0), (a, 0), (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

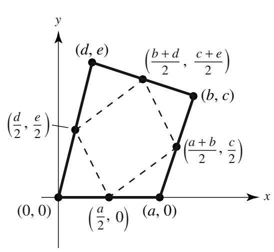
$$\left(\frac{a}{2}, 0\right), \left(\frac{a + b}{2}, \frac{c}{2}\right), \left(\frac{b + d}{2}, \frac{c + e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a + b}{2} - \frac{a}{2}} = \frac{\frac{c + e}{2} - \frac{e}{2}}{\frac{b + d}{2} - \frac{d}{2}} = \frac{c}{b}$$

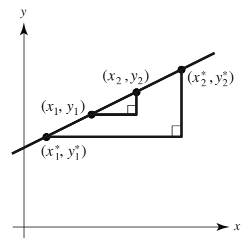
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c + e}{2}}{\frac{a + b}{2} - \frac{b + d}{2}} = -\frac{e}{a - d}$$

Therefore, the figure is a parallelogram.



91. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



92. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

93. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

94. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

95. True. The slope must be positive.

96. True. The general form $Ax + By + C = 0$ includes both horizontal and vertical lines.

Section P.3 Functions and Their Graphs

1. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

2. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(4) = \sqrt{4 + 5} = \sqrt{9} = 3$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$

3. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t-1) = 5 - (t-1)^2 = 5 - (t^2 - 2t + 1)$

$= 4 + 2t - t^2$

4. (a) $g(4) = 4^2(4-4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c-4) = c^3 - 4c^2$

(d) $g(t+4) = (t+4)^2(t+4-4)$

$= (t+4)^2 t = t^3 + 8t^2 + 16t$

5. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

(d) $f(\pi) = \cos(2(\pi)) = 1$

6. (a) $f(\pi) = \sin \pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(d) $f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

7. $$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2,\ \Delta x \neq 0$$

8. $$\frac{f(x) - f(1)}{x-1} = \frac{3x - 1 - (3-1)}{x-1} = \frac{3(x-1)}{x-1} = 3,\ x \neq 1$$

9. $$\frac{f(x) - f(2)}{x-2} = \frac{\left(1/\sqrt{x-1} - 1\right)}{x-2}$$

$$= \frac{1 - \sqrt{x-1}}{(x-2)\sqrt{x-1}} \cdot \frac{1 + \sqrt{x-1}}{1 + \sqrt{x-1}} = \frac{2-x}{(x-2)\sqrt{x-1}\left(1 + \sqrt{x-1}\right)} = \frac{-1}{\sqrt{x-1}\left(1 + \sqrt{x-1}\right)},\ x \neq 2$$

10. $$\frac{f(x) - f(1)}{x-1} = \frac{x^3 - x - 0}{x-1} = \frac{x(x+1)(x-1)}{x-1} = x(x+1),\ x \neq 1$$

11. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

12. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

13. $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

14. $h(x) = 4 - x^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

15. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

16. $h(x) = -\sqrt{x+3}$

Domain: $x + 3 \geq 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

17. $f(x) = \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 \leq 16$

Domain: $[-4, 4]$

Range: $[0, 4]$

Note: $y = \sqrt{16 - x^2}$ is a semicircle of radius 4.

18. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

19. $f(t) = \sec\dfrac{\pi t}{4}$

$\dfrac{\pi t}{4} \neq \dfrac{(2n + 1)\pi}{2} \Rightarrow t \neq 4n + 2$

Domain: all $t \neq 4n + 2$, n an integer

Range: $(-\infty, -1] \cup [1, \infty)$

20. $h(t) = \cot t$

Domain: all $t = n\pi$, n an integer

Range: $(-\infty, \infty)$

21. $f(x) = \dfrac{3}{x}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

22. $f(x) = \dfrac{x - 2}{x + 4}$

Domain: all $x \neq -4$

Range: all $y \neq 1$

[**Note:** You can see that the range is all $y \neq 1$ by graphing f.]

23. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

24. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$

$(x - 2)(x - 1) \geq 0$

Domain: $x \geq 2$ or $x \leq 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

25. $g(x) = \dfrac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n an integer

26. $h(x) = \dfrac{1}{\sin x - (1/2)}$

$\sin x - \dfrac{1}{2} \neq 0$

$\sin x \neq \dfrac{1}{2}$

Domain: all $x \neq \dfrac{\pi}{6} + 2n\pi, \dfrac{5\pi}{6} + 2n\pi$, n integer

27. $f(x) = \dfrac{1}{|x + 3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

28. $g(x) = \dfrac{1}{|x^2 - 4|}$

$|x^2 - 4| \neq 0$

$(x - 2)(x + 2) \neq 0$

Domain: all $x \neq \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

30. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all s.)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

31. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

32. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

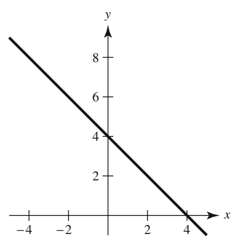
Domain: $[-4, \infty)$

Range: $[0, \infty)$

33. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

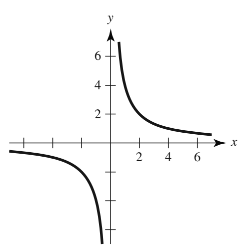
Range: $(-\infty, \infty)$



34. $g(x) = \dfrac{4}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$



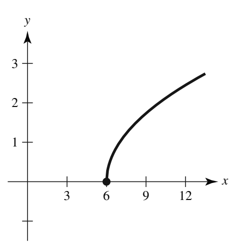
35. $h(x) = \sqrt{x - 6}$

Domain:

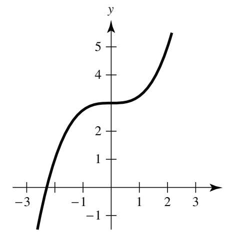
$x - 6 \geq 0$

$x \geq 6 \Rightarrow [6, \infty)$

Range: $[0, \infty)$



36. $f(x) = \frac{1}{4}x^3 + 3$



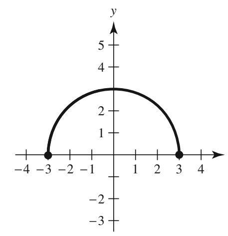
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

37. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



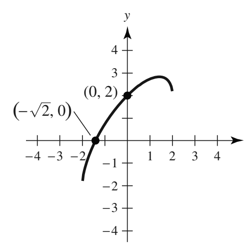
38. $f(x) = x + \sqrt{4 - x^2}$

Domain: $[-2, 2]$

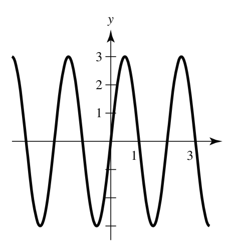
Range: $[-2, 2\sqrt{2}] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

x-intercept: $(-\sqrt{2}, 0)$



39. $g(t) = 3 \sin \pi t$



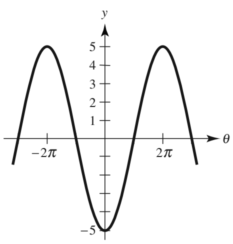
Domain: $(-\infty, \infty)$

Range: $[-3, 3]$

40. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

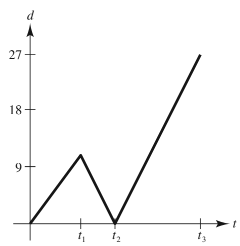
Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



41. The student travels $\dfrac{2 - 0}{4 - 0} = \dfrac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the next 2 minutes. Finally, the student travels $\dfrac{6 - 2}{10 - 6} = 1$ mi/min during the final 4 minutes.

42.



43. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

44. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

45. y is a function of x. Vertical lines intersect the graph at most once.

46. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

47. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

48. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

49. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

50. $x^2y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2 + 4}$

y is a function of x because there is one value of y for each x.

51. The transformation is a horizontal shift two units to the right.

Shifted function: $y = \sqrt{x - 2}$

52. The transformation is a vertical shift 4 units upward.

Shifted function: $y = \sin x + 4$

53. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward.

Shifted function: $y = (x - 2)^2 - 1$

54. The transformation is a horizontal shift 1 unit to the left and a vertical shift 2 units upward.

Shifted function: $y = (x + 1)^3 + 2$

55. $y = f(x + 5)$ is a horizontal shift 5 units to the left. Matches d.

56. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches b.

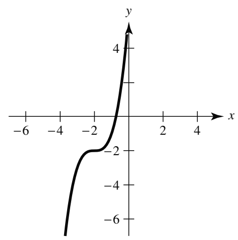
57. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

58. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

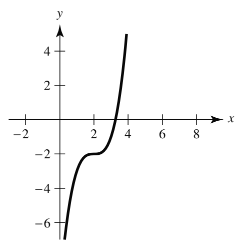
59. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

60. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

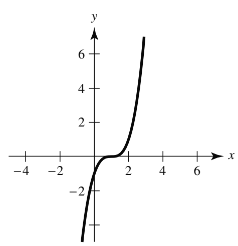
61. (a) The graph is shifted 3 units to the left.



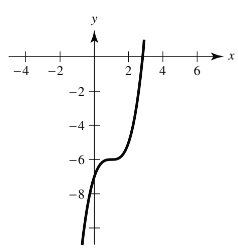
(b) The graph is shifted 1 unit to the right.



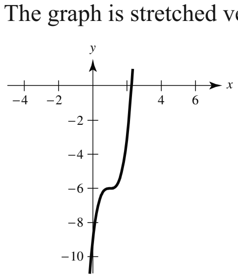
(c) The graph is shifted 2 units upward.



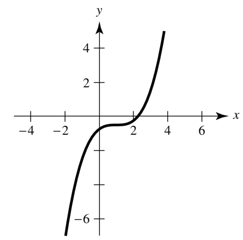
(d) The graph is shifted 4 units downward.



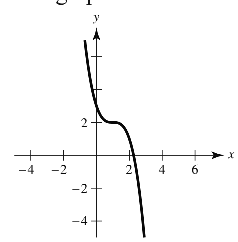
(e) The graph is stretched vertically by a factor of 3.



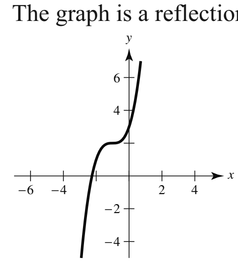
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



(g) The graph is a reflection in the *x*-axis.



(h) The graph is a reflection about the origin.

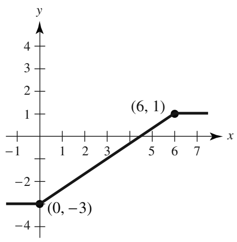


62. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

$g(0) = f(-4) = -3$

The graph is shifted 4 units to the right.

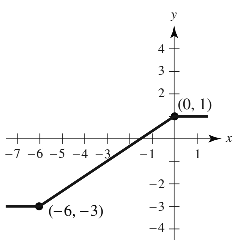


(b) $g(x) = f(x + 2)$

$g(0) = f(2) = 1$

$g(-6) = f(-4) = -3$

The graph is shifted 2 units to the left.

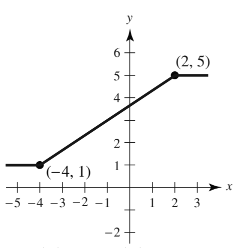


(c) $g(x) = f(x) + 4$

$g(2) = f(2) + 4 = 5$

$g(-4) = f(-4) + 4 = 1$

The graph is shifted 4 units upward.

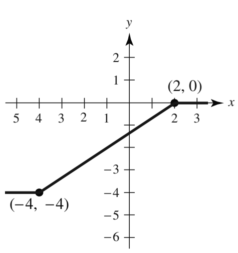


(d) $g(x) = f(x) - 1$

$g(2) = f(2) - 1 = 0$

$g(-4) = f(-4) - 1 = -4$

The graph is shifted 1 unit downward.

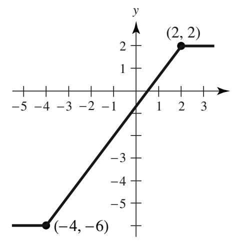


(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

$g(-4) = 2f(-4) = -6$

The graph is stretched vertically by a factor of 2.

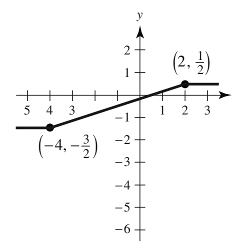


(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

The graph is stretched vertically by a factor of $\frac{1}{2}$.

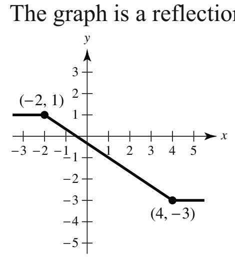


(g) $g(x) = f(-x)$

$g(-2) = f(2) = 1$

$g(4) = f(-4) = -3$

The graph is a reflection in the *y*-axis.

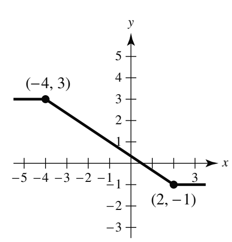


(h) $g(x) = -f(x)$

$g(2) = f(2) = -1$

$g(-4) = f(-4) = 3$

The graph is a reflection in the *x*-axis.



63. $f(x) = 3x - 4, \quad g(x) = 4$

(a) $f(x) + g(x) = (3x - 4) + 4 = 3x$

(b) $f(x) - g(x) = (3x - 4) - 4 = 3x - 8$

(c) $f(x) \cdot g(x) = (3x - 4)(4) = 12x - 16$

(d) $f(x)/g(x) = \dfrac{3x - 4}{4} = \dfrac{3}{4}x - 1$

64. $f(x) = x^2 + 5x + 4, \quad g(x) = x + 1$

(a) $f(x) + g(x) = (x^2 + 5x + 4) + (x + 1) = x^2 + 6x + 5$

(b) $f(x) - g(x) = (x^2 + 5x + 4) - (x + 1) = x^2 + 4x + 3$

(c) $f(x) \cdot g(x) = (x^2 + 5x + 4)(x + 1)$

$= x^3 + 5x^2 + 4x + x^2 + 5x + 4$

$= x^3 + 6x^2 + 9x + 4$

(d) $f(x)/g(x) = \dfrac{x^2 + 5x + 4}{x + 1} = \dfrac{(x + 4)(x + 1)}{x + 1} = x + 4, x \neq -1$

65. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ (x \geq 0)$

66. $f(x) = \sin x, g(x) = \pi x$

(a) $f(g(2)) = f(2\pi) = \sin(2\pi) = 0$

(b) $f\left(g\left(\dfrac{1}{2}\right)\right) = f\left(\dfrac{\pi}{2}\right) = \sin\left(\dfrac{\pi}{2}\right) = 1$

(c) $g(f(0)) = g(0) = 0$

(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right) = g\left(\sin\left(\dfrac{\pi}{4}\right)\right)$

$= g\left(\dfrac{\sqrt{2}}{2}\right) = \pi\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi\sqrt{2}}{2}$

(e) $f(g(x)) = f(\pi x) = \sin(\pi x)$

(f) $g(f(x)) = g(\sin x) = \pi \sin x$

67. $f(x) = x^2, g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x}) = (\sqrt{x})^2 = x, x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

68. $f(x) = x^2 - 1, g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

No, $f \circ g \neq g \circ f$.

69. $f(x) = \dfrac{3}{x},\ g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x))$

$= g\left(\dfrac{3}{x}\right) = \left(\dfrac{3}{x}\right)^2 - 1 = \dfrac{9}{x^2} - 1 = \dfrac{9 - x^2}{x^2}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

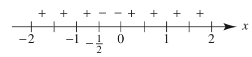
No, $f \circ g \neq g \circ f$.

70. $(f \circ g)(x) = f\left(\sqrt{x + 2}\right) = \dfrac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

71. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

72. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

73. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x,\ g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

[Other answers possible]

74. $F(x) = -4 \sin(1 - x)$

Let $f(x) = -4x,\ g(x) = \sin x$ and $h(x) = 1 - x$. Then,

$(f \circ g \circ h)(x) = f(g(1 - x)) = f(\sin(1 - x)) = -4 \sin(1 - x) = F(x)$.

[Other answers possible]

75. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

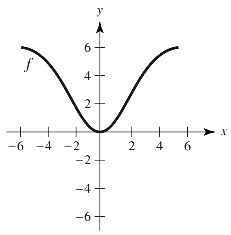
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

76. (a) If f is even, then $(-4, 9)$ is on the graph.

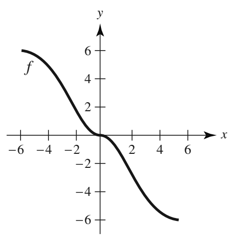
(b) If f is odd, then $(-4, -9)$ is on the graph.

77. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

78. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



79. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = 0, -2, 2$

80. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x} = -f(x)$

f is odd.

$f(x) = \sqrt[3]{x} = 0 \Rightarrow x = 0$ is the zero.

81. $f(x) = x \cos x$

$f(-x) = (-x)\cos(-x) = -x \cos x = -f(x)$

f is odd.

$f(x) = x \cos x = 0$

Zeros: $x = 0, \dfrac{\pi}{2} + n\pi$, where n is an integer

82. $f(x) = \sin^2 x$

$f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

f is even.

$\sin^2 x = 0 \Rightarrow \sin x = 0$

Zeros: $x = n\pi$, where n is an integer

83. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

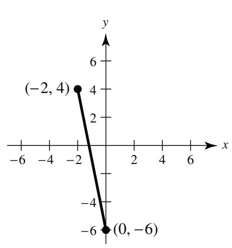
$y - 4 = -5(x - (-2))$

$y - 4 = -5x - 10$

$y = -5x - 6$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, -2 \le x \le 0$



84. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

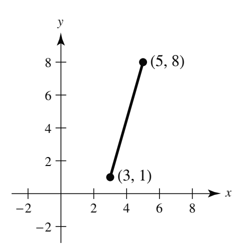
$y - 1 = \dfrac{7}{2}(x - 3)$

$y - 1 = \dfrac{7}{2}x - \dfrac{21}{2}$

$y = \dfrac{7}{2}x - \dfrac{19}{2}$

For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, 3 \le x \le 5$

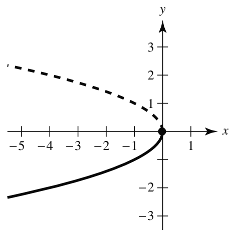


85. $x + y^2 = 0$

$$y^2 = -x$$

$$y = -\sqrt{-x}$$

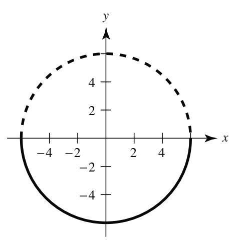
$$f(x) = -\sqrt{-x},\ x \le 0$$



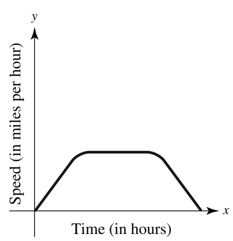
86. $x^2 + y^2 = 36$

$$y^2 = 36 - x^2$$

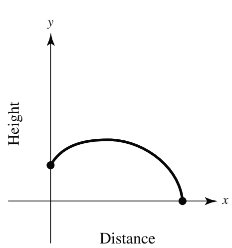
$$y = -\sqrt{36 - x^2},\ -6 \le x \le 6$$



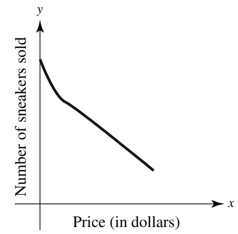
87. Answers will vary. *Sample answer*: Speed begins and ends at 0. The speed might be constant in the middle:



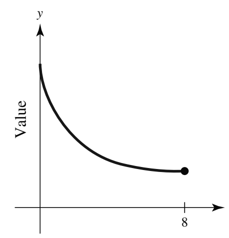
88. Answers will vary. *Sample answer*: Height begins a few feet above 0, and ends at 0.



89. Answers will vary. *Sample answer*: In general, as the price decreases, the store will sell more.



90. Answers will vary. *Sample answer*: As time goes on, the value of the car will decrease



91.

$$y = \sqrt{c - x^2}$$

$$y^2 = c - x^2$$

$$x^2 + y^2 = c, \text{ a circle.}$$

For the domain to be $[-5, 5]$, $c = 25$.

92. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \ne 0$. So, the discriminant must be less than zero:

$$(3c)^2 - 4(6) < 0$$

$$9c^2 < 24$$

$$c^2 < \tfrac{8}{3}$$

$$-\sqrt{\tfrac{8}{3}} < c < \sqrt{\tfrac{8}{3}}$$

$$-\tfrac{2}{3}\sqrt{6} < c < \tfrac{2}{3}\sqrt{6}$$

93. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

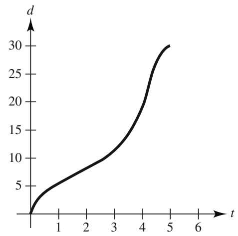
(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

94. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

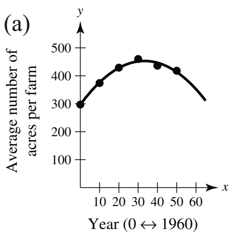
Range: $0 \le d \le 30$

(c)



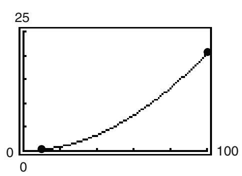
(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 cm.

95. (a)



(b) $A(25) \approx 445$ (Answers will vary.)

96. (a)



(b) $H\left(\dfrac{x}{1.6}\right) = 0.002\left(\dfrac{x}{1.6}\right)^2 + 0.005\left(\dfrac{x}{1.6}\right) - 0.029$

$= 0.00078125x^2 + 0.003125x - 0.029$

97. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2$.

If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2$.

So,

$$f(x) = \begin{cases} -2x + 2, & x \le 0 \\ 2, & 0 < x < 2. \\ 2x - 2, & x \ge 2 \end{cases}$$

98. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

99. $f(-x) = a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x)$

$= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right]$

$= -f(x)$

Odd

100. $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$

$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$= f(x)$

Even

101. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$.

So, $F(x)$ is even.

102. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

$F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x)$.

So, $F(x)$ is odd.

103. By equating slopes, $\dfrac{y-2}{0-3} = \dfrac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

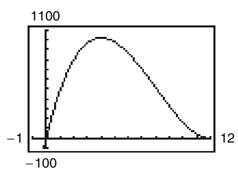
$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

104. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

(c)

x	*length and width*	*volume*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions of the box that yield a maximum volume appear to be $4 \times 16 \times 16$ cm.

105. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

106. True

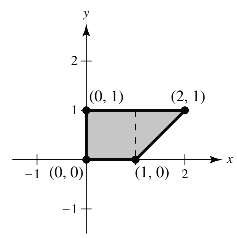
107. True. The function is even.

108. False. If $f(x) = x^2$ then, $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

109. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

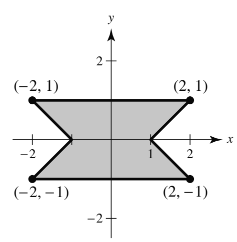
110. True. If the domain is $\{a\}$, then the range is $\{f(a)\}$.

111. First consider the portion of R in the first quadrant: $x \geq 0$, $0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

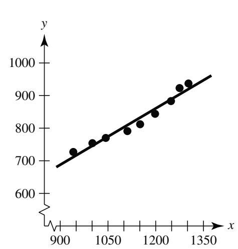
112. Let $g(x) = c$ be constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

So, $f(c) = c$. Because this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section P.4 Fitting Models to Data

1. (a) and (b)

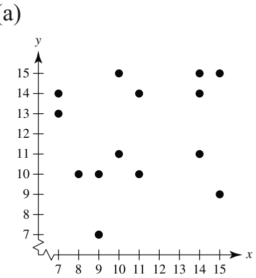


Yes, the data appear to be approximately linear.

The data can be modeled by equation $y = 0.6x + 150$. (Answers will vary).

(c) When $x = 1075$, $y = 0.6(1075) + 150 = 795$.

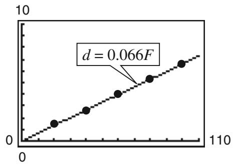
2. (a)



The data do not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, and so on. These variables may change from one quiz to the next.

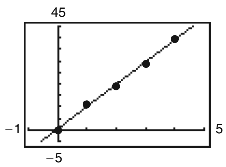
3. (a) $d = 0.066F$

(b)


The model fits the data well.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ cm.

4. (a) $s = 9.7t + 0.4$

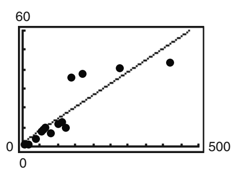
(b)


The model fits the data well.

(c) If $t = 2.5$, $s = 24.65$ meters/second.

5. (a) Using a graphing utility, $y = 0.122x + 2.07$

The correlation coefficient is $r \approx 0.87$.

(b)


(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national income. The three countries that most differ from the linear model are Canada, Japan, and Italy.

(d) Using a graphing utility, the new model is $y = 0.142x - 1.66$.

The correlation coefficient is $r \approx 0.97$.

6. (a) Trigonometric function

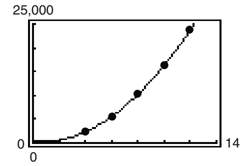
(b) Quadratic function

(c) No relationship

(d) Linear function

7. (a) Using graphing utility,

$S = 180.89x^2 - 205.79x + 272.$

(b)


(c) When $x = 2$, $S \approx 583.98$ pounds.

(d) $\dfrac{2370}{584} \approx 4.06$

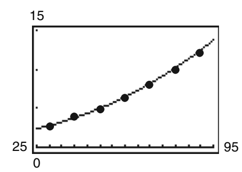
The breaking strength is approximately 4 times greater.

(e) $\dfrac{23{,}860}{5460} \approx 4.37$

When the height is doubled, the breaking strength increases approximately by a factor of 4.

8. (a) Using a graphing utility

$t = 0.0013s^2 + 0.005s + 1.48.$

(b)


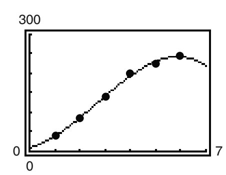
(c) According to the model, the times required to attain speeds of less than 20 miles per hour are all about the same. Furthermore, it takes 1.48 seconds to reach 0 miles per hour, which does not make sense.

(d) Adding $(0, 0)$ to the data produces

$t = 0.0009s^2 + 0.053s + 0.10.$

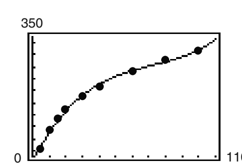
(e) Yes. Now the car starts at rest.

9. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

(b)


(c) If $x = 4.5$, $y \approx 214$ horsepower.

10. (a) $T = 2.9856 \times 10^{-4} p^3 - 0.0641p^2 + 5.282p + 143.1$

(b)


(c) For $T = 300°F$, $p \approx 68.29$ lb/in.^{2}.

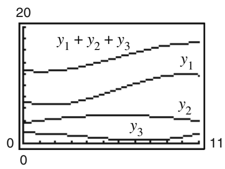
(d) The model is based on data up to 100 pounds per square inch.

11. (a) $y_1 = -0.0172t^3 + 0.305t^2 - 0.87t + 7.3$

$y_2 = -0.038t^2 + 0.45t + 3.5$

$y_3 = 0.0063t^3 + -0.072t^2 + 0.02t + 1.8$

(b)



$y_1 + y_2 + y_3 = -0.0109t^3 + 0.195t^2 - 0.40t + 12.6$

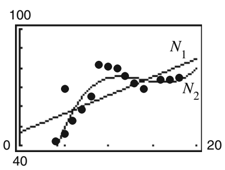
For 2014, $t = 14$. So,

$$y_1 + y_2 + y_3 = -0.0109(14)^3 + 0.195(14)^2 - 0.40(14) + 12.6$$
$$\approx 15.31 \text{ cents/mile}$$

12. (a) $N_1 = 1.89t + 46.8$ Linear model

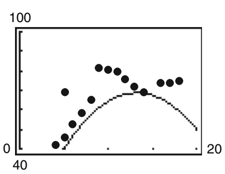
$N_2 = 0.0485t^3 - 2.015t^2 + 27.00t - 42.3$ Cubic model

(b)



(c) The cubic model is the better model.

(d) $N_3 = -0.414t^2 + 11.00t + 4.4$ Quadratic model



The model does not fit the data well.

(e) For 2014, $t = 24$ and

$N_1 \approx 92.16$ million

$N_2 \approx 115.524$ million

The linear model seems too high. The cubic model is better.

(f) Answers will vary.

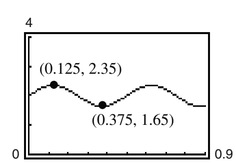
13. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately $(2.35 - 1.65)/2 = 0.35$.

The period is approximately $2(0.375 - 0.125) = 0.5$.

(c) One model is $y = 0.35 \sin(4\pi t) + 2$.

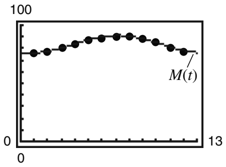
(d)



The model appears to fit the data.

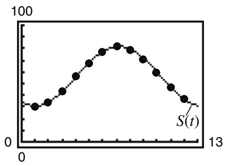
14. (a) $S(t) = 56.37 + 25.47\sin(0.5080t - 2.07)$

(b)



The model is a good fit.

(c)



The model is a good fit.

(d) The average is the constant term in each model. 83.70°F for Miami and 56.37°F for Syracuse.

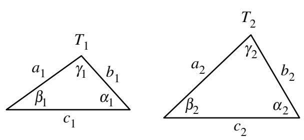
(e) The period for Miami is $2\pi/0.4912 \approx 12.8$. The period for Syracuse is $2\pi/0.5080 \approx 12.4$. In both cases the period is approximately 12, or one year.

(f) Syracuse has greater variability because $25.47 > 7.46$.

15. Answers will vary.

16. Answers will vary.

17. Yes, $A_1 \le A_2$. To see this, consider the two triangles of areas A_1 and A_2:



For $i = 1, 2$, the angles satisfy $\alpha i + \beta i + \gamma i = \pi$. At least one of $\alpha_1 \le \alpha_2$, $\beta_1 \le \beta_2$, $\gamma_1 \le \gamma_2$ must hold. Assume $\alpha_1 \le \alpha_2$. Because $\alpha_2 \le \pi/2$ (acute triangle), and the sine function increases on $[0, \pi/2]$, you have

$$A_1 = \tfrac{1}{2}b_1c_1\sin\alpha_1 \le \tfrac{1}{2}b_2c_2\sin\alpha_1$$
$$\le \tfrac{1}{2}b_2c_2\sin\alpha_2 = A_2$$

Review Exercises for Chapter P

1. $y = 5x - 8$

$x = 0$: $y = 5(0) - 8 = -8 \Rightarrow (0, -8)$, y-intercept

$y = 0$: $0 = 5x - 8 \Rightarrow x = \frac{8}{5} \Rightarrow \left(\frac{8}{5}, 0\right)$, x-intercept

2. $y = x^2 - 8x + 12$

$x = 0$: $y = (0)^2 - 8(0) + 12 = 12 \Rightarrow (0, 12)$, y-intercept

$y = 0$: $x^2 - 8x + 12 = (x - 6)(x - 2) = 0 \Rightarrow x = 2, 6 \Rightarrow (2, 0), (6, 0)$, x-intercepts

3. $y = \dfrac{x - 3}{x - 4}$

$x = 0$: $y = \dfrac{0 - 3}{0 - 4} = \dfrac{3}{4} \Rightarrow \left(0, \dfrac{3}{4}\right)$, y-intercept

$y = 0$: $0 = \dfrac{x - 3}{x - 4} \Rightarrow x = 3 \Rightarrow (3, 0)$, x-intercept

4. $y = (x - 3)\sqrt{x + 4}$

$x = 0$: $y = (0 - 3)\sqrt{0 + 4} = -3\sqrt{4} = -3(2) = -6 \Rightarrow (0, -6)$, y-intercept

$y = 0$: $(x - 3)\sqrt{x + 4} = 0 \Rightarrow x = 3, -4 \Rightarrow (3, 0), (-4, 0)$, x-intercepts

5. $y = x^2 + 4x$ does not have symmetry with respect to either axis or the origin.

6. Symmetric with respect to y-axis because

$$y = (-x)^4 - (-x)^2 + 3$$
$$y = x^4 - x^2 + 3.$$

7. Symmetric with respect to both axes and the origin because:

$$y^2 = (-x^2) - 5 \qquad (-y)^2 = x^2 - 5 \qquad (-y)^2 = (-x)^2 - 5$$
$$y^2 = x^2 - 5 \qquad y^2 = x^2 - 5 \qquad y^2 = x^2 - 5$$

8. Symmetric with respect to the origin because:

$$(-x)(-y) = -2$$
$$xy = -2.$$

9. $y = -\frac{1}{2}x + 3$

y-intercept: $y = -\frac{1}{2}(0) + 3 = 3$

$(0, 3)$

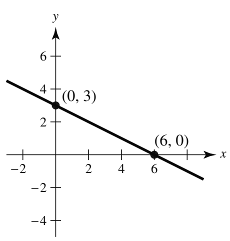
x-intercept: $-\frac{1}{2}x + 3 = 0$

$-\frac{1}{2}x = -3$

$x = 6$

$(6, 0)$

Symmetry: none



10. $y = -x^2 + 4$

y-intercept: $y = -(0)^2 + 4 = 4$

$(0, 4)$

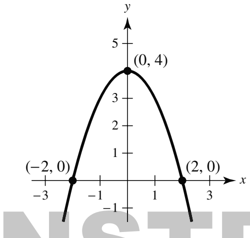
x-intercepts: $-x^2 + 4 = 0$

$(2 - x)(2 + x) = 0$

$x = \pm 2$

$(2, 0), (-2, 0)$

Symmetric with respect to the y-axis because $-(-x)^2 + 4 = -x^2 + 4$.



11. $y = x^3 - 4x$

y-intercept: $y = 0^3 - 4(0) = 0$

$(0, 0)$

x-intercepts: $x^3 - 4x = 0$

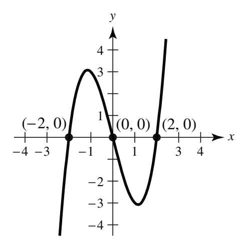
$x(x^2 - 4) = 0$

$x(x - 2)(x + 2) = 0$

$x = 0, 2, -2$

$(0, 0), (2, 0), (-2, 0)$

Symmetric with respect to the origin because $(-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$.



12. $y^2 = 9 - x$

$y^2 + x - 9 = 0$

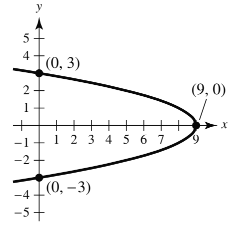
y-intercept: $y^2 = 9 - 0 = 9 \Rightarrow y = \pm 3$

$(0, 3), (0, -3)$

x-intercept: $0^2 = 9 - x \Rightarrow x = 9$

$(9, 0)$

Symmetric with respect to the x-axis because $(-y)^2 + x - 9 = y^2 + x - 9 = 0$.



13. $y = 2\sqrt{4 - x}$

y-intercept: $y = 2\sqrt{4 - 0} = 2\sqrt{4} = 4$

$(0, 4)$

x-intercept: $2\sqrt{4 - x} = 0$

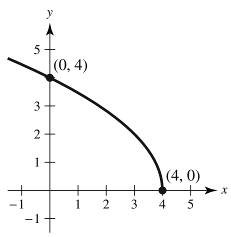
$\sqrt{4 - x} = 0$

$4 - x = 0$

$x = 4$

$(4, 0)$

Symmetry: none



14. $y = |x - 4| - 4$

y-intercept: $y = |0 - 4| - 4 = |-4| - 4 = 4 - 4 = 0$

$(0, 0)$

x-intercepts: $|x - 4| - 4 = 0$

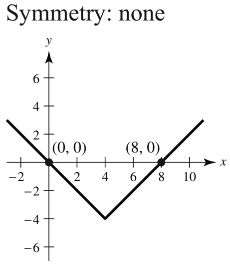
$|x - 4| = 4$

$x - 4 = 4$ or $x - 4 = -4$

$x = 8 \quad x = 0$

$(0, 0), (8, 0)$

Symmetry: none



15. $5x + 3y = -1 \Rightarrow y = \frac{1}{3}(-5x - 1)$

$x - y = -5 \Rightarrow y = x + 5$

$\frac{1}{3}(-5x - 1) = x + 5$

$-5x - 1 = 3x + 15$

$-16 = 8x$

$-2 = x$

For $x = -2$, $y = x + 5 = -2 + 5 = 3$.

Point of intersection is: $(-2, 3)$

16. $2x + 4y = 9 \Rightarrow y = \dfrac{-2x + 9}{4}$

$6x - 4y = 7 \Rightarrow y = \dfrac{6x - 7}{4}$

$\dfrac{-2x + 9}{4} = \dfrac{6x - 7}{4}$

$-2x + 9 = 6x - 7$

$-8x = -16$

$x = 2$

For $x = 2$, $y = \dfrac{6(2) - 7}{4} = \dfrac{5}{4}$

Point of intersection: $\left(2, \dfrac{5}{4}\right)$

17. $x - y = -5 \Rightarrow y = x + 5$

$x^2 - y = 1 \Rightarrow y = x^2 - 1$

$x + 5 = x^2 - 1$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$x = 3$ or $x = -2$

For $x = 3$, $y = 3 + 5 = 8$.

For $x = -2$, $y = -2 + 5 = 3$.

Points of intersection: $(3, 8), (-2, 3)$

18. $x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$-x + y = 1 \Rightarrow y = x + 1$

$1 - x^2 = (x + 1)^2$

$1 - x^2 = x^2 + 2x + 1$

$0 = 2x^2 + 2x$

$0 = 2x(x + 1)$

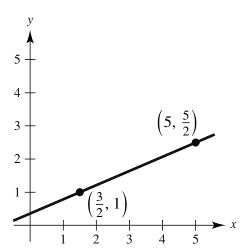
$x = 0$ or $x = -1$

For $x = 0$, $y = 0 + 1 = 1$.

For $x = -1$, $y = -1 + 1 = 0$.

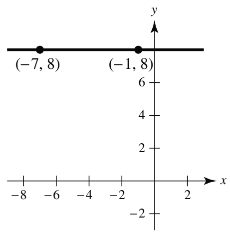
Points of intersection: $(0, 1), (-1, 0)$

19.

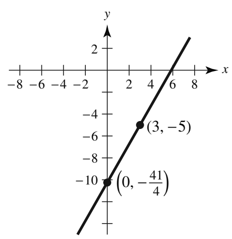


$$\text{Slope} = \frac{\left(\frac{5}{2}\right) - 1}{5 - \left(\frac{3}{2}\right)} = \frac{\frac{3}{2}}{\frac{7}{2}} = \frac{3}{7}$$

20. The line is horizontal and has slope 0.

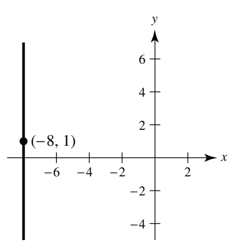


21.
$$y - (-5) = \tfrac{7}{4}(x - 3)$$
$$y + 5 = \tfrac{7}{4}x - \tfrac{21}{4}$$
$$4y + 20 = 7x - 21$$
$$0 = 7x - 4y - 41$$

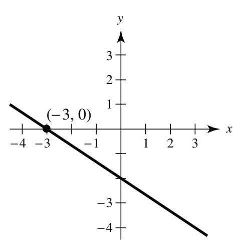


22. Because m is undefined the line is vertical.

$x = -8$ or $x + 8 = 0$



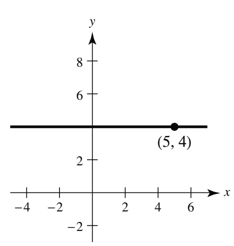
23.
$$y - 0 = -\tfrac{2}{3}(x - (-3))$$
$$y = -\tfrac{2}{3}x - 2$$
$$2x + 3y + 6 = 0$$



24. Because $m = 0$, the line is horizontal.

$y - 4 = 0(x - 5)$

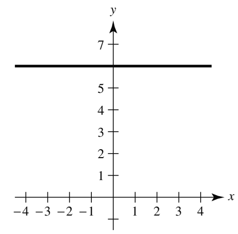
$y = 4$ or $y - 4 = 0$



25. $y = 6$

Slope: 0

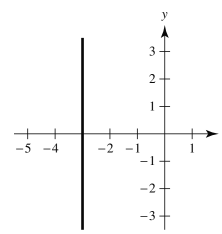
y-intercept: $(0, 6)$



26. $x = -3$

Slope: undefined

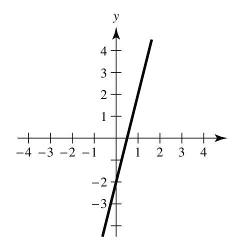
Line is vertical.



27. $y = 4x - 2$

Slope: 4

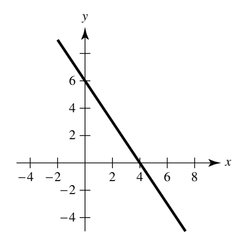
y-intercept: $(0, -2)$



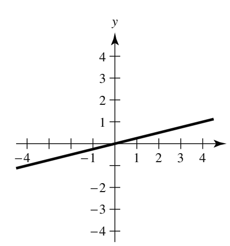
28.
$$3x + 2y = 12$$
$$2y = -3x + 12$$
$$y = \frac{-3}{2}x + 6$$

Slope : $-\dfrac{3}{2}$

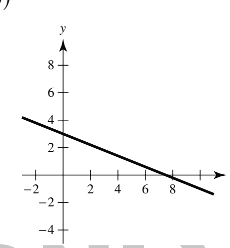
y-intercept: $(0, 6)$



29.
$$m = \frac{2 - 0}{8 - 0} = \frac{1}{4}$$
$$y - 0 = \frac{1}{4}(x - 0)$$
$$y = \frac{1}{4}x$$
$$4y - x = 0$$



30.
$$m = \frac{-1 - 5}{10 - (-5)} = \frac{-6}{15} = -\frac{2}{5}$$
$$y - 5 = \frac{-2}{5}(x - (-5))$$
$$5y - 25 = -2x - 10$$
$$5y + 2x - 15 = 0$$



31. (a) $y - 5 = \frac{7}{16}(x + 3)$

$16y - 80 = 7x + 21$

$0 = 7x - 16y + 101$

(b) $5x - 3y = 3$ has slope $\frac{5}{3}$.

$y - 5 = \frac{5}{3}(x + 3)$

$3y - 15 = 5x + 15$

$0 = 5x - 3y + 30$

(c) $3x + 4y = 8$

$4y = -3x + 8$

$y = \frac{-3}{4}x + 2$

Perpendicular line has slope $\frac{4}{3}$.

$y - 5 = \frac{4}{3}(x - (-3))$

$3y - 15 = 4x + 12$

$4x - 3y + 27 = 0$ or $y = \frac{4}{3}x + 9$

(d) Slope is undefined so the line is vertical.

$x = -3$

$x + 3 = 0$

32. (a) $y - 4 = -\frac{2}{3}(x - 2)$

$3y - 12 = -2x + 4$

$2x + 3y - 16 = 0$

(b) $x + y = 0$ has slope -1. Slope of the perpendicular line is 1.

$y - 4 = 1(x - 2)$

$y = x + 2$

$0 = x - y + 2$

(c) $m = \frac{4 - 1}{2 - 6} = -\frac{3}{4}$

$y - 4 = -\frac{3}{4}(x - 2)$

$4y - 16 = -3x + 6$

$3x + 4y - 22 = 0$

(d) Because the line is horizontal the slope is 0.

$y = 4$

$y - 4 = 0$

33. The slope is -850.

$V = -850t + 12{,}500$.

$V(3) = -850(3) + 12{,}500 = \$9950$

34. (a) $C = 9.25t + 13.50t + 36{,}500 = 22.75t + 36{,}500$

(b) $R = 30t$

(c) $30t = 22.75t + 36{,}500$

$7.25t = 36{,}500$

$t \approx 5034.48$ hours to break even

35. $f(x) = 5x + 4$

(a) $f(0) = 5(0) + 4 = 4$

(b) $f(5) = 5(5) + 4 = 29$

(c) $f(-3) = 5(-3) + 4 = -11$

(d) $f(t + 1) = 5(t + 1) + 4 = 5t + 9$

36. $f(x) = x^3 - 2x$

(a) $f(-3) = (-3)^3 - 2(-3) = -27 + 6 = -21$

(b) $f(2) = 2^3 - 2(2) = 8 - 4 = 4$

(c) $f(-1) = (-1)^3 - 2(-1) = -1 + 2 = 1$

(d) $f(c - 1) = (c - 1)^3 - 2(c - 1)$

$= c^3 - 3c^2 + 3c - 1 - 2c + 2$

$= c^3 - 3c^2 + c + 1$

37. $f(x) = 4x^2$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{4(x + \Delta x)^2 - 4x^2}{\Delta x}$$

$$= \frac{4\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 4x^2}{\Delta x}$$

$$= \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 4x^2}{\Delta x}$$

$$= \frac{8x\Delta x + 4(\Delta x)^2}{\Delta x}$$

$$= 8x + 4\Delta x, \quad \Delta x \neq 0$$

38. $f(x) = 2x - 6$

$f(1) = 2(1) - 6 = -4$

$$\frac{f(x) - f(-1)}{x - 1} = \frac{(2x - 6) - (-4)}{x - 1}$$

$$= \frac{2x - 6 + 4}{x - 1}$$

$$= \frac{2x - 2}{x - 1}$$

$$= \frac{2(x - 1)}{x - 1}$$

$$= 2, \quad x \neq 1$$

39. $f(x) = x^2 + 3$

Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

40. $g(x) = \sqrt{6 - x}$

Domain: $6 - x \geq 0$

$6 \geq x$

$(-\infty, 6]$

Range: $[0, \infty)$

41. $f(x) = -|x + 1|$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

42. $h(x) = \dfrac{2}{x + 1}$

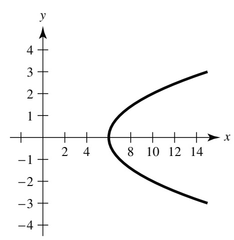
Domain: all $x \neq -1$; $(-\infty, -1) \cup (-1, \infty)$

Range: all $y \neq 0$; $(-\infty, 0) \cup (0, \infty)$

43. $x - y^2 = 6$

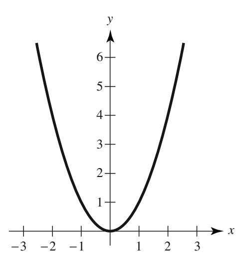
$y = \pm\sqrt{x - 6}$

Not a function because there are two values of y for some x.



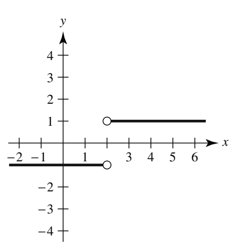
44. $x^2 - y = 0$

Function of x because there is one value for y for each x.



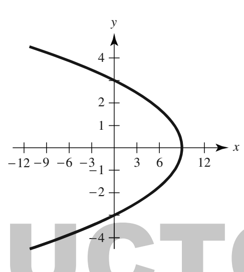
45. $y = \dfrac{|x - 2|}{x - 2}$

y is a function of x because there is one value of y for each x.

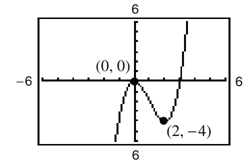


46. $x = 9 - y^2$

Not a function of x since there are two values of y for some x.



47. $f(x) = x^3 - 3x^2$



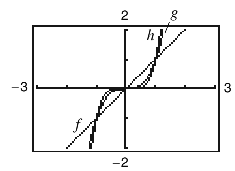
(a) The graph of g is obtained from f by a vertical shift down 1 unit, followed by a reflection in the x-axis:

$g(x) = -[f(x) - 1] = -x^3 + 3x^2 + 1$

(b) The graph of g is obtained from f by a vertical shift upwards of 1 and a horizontal shift of 2 to the right.

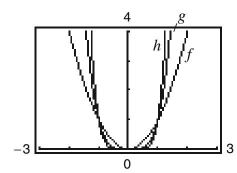
$g(x) = f(x - 2) + 1 = (x - 2)^3 - 3(x - 2)^2 + 1$

48. (a) Odd powers: $f(x) = x$, $g(x) = x^3$, $h(x) = x^5$



The graphs of f, g, and h all rise to the right and fall to the left. As the degree increases, the graph rises and falls more steeply. All three graphs pass through the points (0, 0), (1, 1), and $(-1, -1)$ and are symmetric with respect to the origin.

Even powers: $f(x) = x^2$, $g(x) = x^4$, $h(x) = x^6$



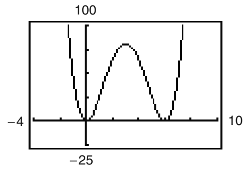
The graphs of f, g, and h all rise to the left and to the right. As the degree increases, the graph rises more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$ and are symmetric with respect to the y-axis.

All of the graphs, even and odd, pass through the origin. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

(b) $y = x^7$ will look like $h(x) = x^5$, but rise and fall even more steeply. $y = x^8$ will look like $h(x) = x^6$, but rise even more steeply.

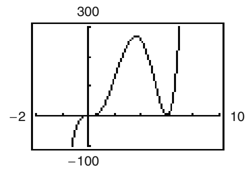
49. (a) $f(x) = x^2(x - 6)^2$

The leading coefficient is positive and the degree is even so the graph will rise to the left and to the right.



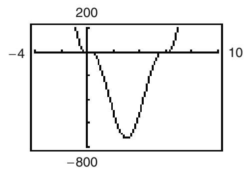
(b) $g(x) = x^3(x - 6)^2$

The leading coefficient is positive and the degree is odd so the graph will rise to the right and fall to the left.



(c) $h(x) = x^3(x - 6)^3$

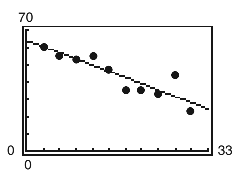
The leading coefficient is positive and the degree is even so the graph will rise to the left and to the right.



50. (a) 3 (cubic), negative leading coefficient
(b) 4 (quartic), positive leading coefficient
(c) 2 (quadratic), negative leading coefficient
(d) 5, positive leading coefficient

51. (a) $y = -1.204x + 64.2667$

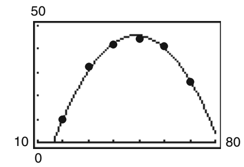
(b)



(c) The data point (27, 44) is probably an error. Without this point, the new model is
$y = -1.4344x + 66.4387$.

52. (a) Using a graphing utility, you obtain
$y = -0.043x^2 + 4.19x - 56.2$.

(b)



(c) For $x = 26$:

$$y = -0.043(26)^2 + 4.19(26) - 56.2$$
$$\approx \$23.7 \text{ thousand}$$

(d) For $x = 34$:

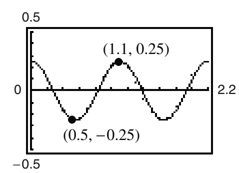
$$y = -0.043(34)^2 + 4.19(34) - 56.2$$
$$\approx \$36.6 \text{ thousand}$$

53. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately $\left(0.25 - (-0.25)\right)/2 = 0.25$. The period is approximately 1.1.

(c) One model is $y = \frac{1}{4}\cos\left(\frac{2\pi}{1.1}t\right) \approx \frac{1}{4}\cos(5.7t)$

(d)



The model appears to fit the data.

Problem Solving for Chapter P

1. (a)

$$x^2 - 6x + y^2 - 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: (3, 4); Radius: 5

(b) Slope of line from (0, 0) to (3, 4) is $\frac{4}{3}$.

Slope of tangent line is $-\frac{3}{4}$. So,

$y - 0 = -\frac{3}{4}(x - 0) \Rightarrow y = -\frac{3}{4}x$, Tangent line

(c) Slope of line from (6, 0) to (3, 4) is $\frac{4 - 0}{3 - 6} = -\frac{4}{3}$.

Slope of tangent line is $\frac{3}{4}$. So,

$y - 0 = \frac{3}{4}(x - 6) \Rightarrow y = \frac{3}{4}x - \frac{9}{2}$, Tangent line

(d)
$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

2. Let $y = mx + 1$ be a tangent line to the circle from the point (0, 1). Because the center of the circle is at $(0, -1)$ and the radius is 1 you have the following.

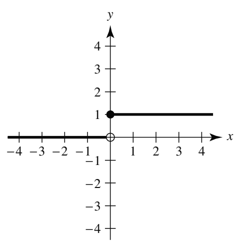
$$x^2 + (y + 1)^2 = 1$$
$$x^2 + (mx + 1 + 1)^2 = 1$$
$$(m^2 + 1)x^2 + 4mx + 3 = 0$$

Setting the discriminant $b^2 - 4ac$ equal to zero,

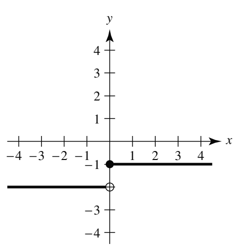
$$16m^2 - 4(m^2 + 1)(3) = 0$$
$$16m^2 - 12m^2 = 12$$
$$4m^2 = 12$$
$$m = \pm\sqrt{3}$$

Tangent lines: $y = \sqrt{3}x + 1$ and $y = -\sqrt{3}x + 1$.

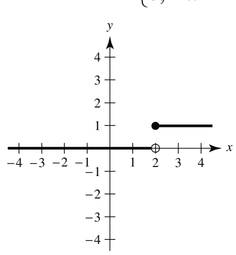
3. $H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



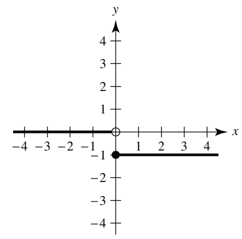
(a) $H(x) - 2 = \begin{cases} -1, & x \ge 0 \\ -2, & x < 0 \end{cases}$



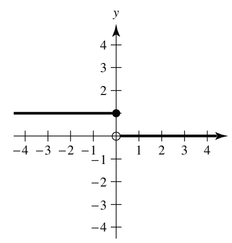
(b) $H(x - 2) = \begin{cases} 1, & x \ge 2 \\ 0, & x < 2 \end{cases}$



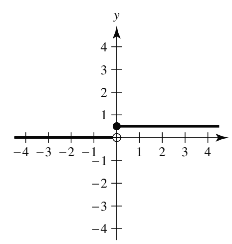
(c) $-H(x) = \begin{cases} -1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



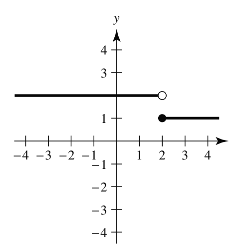
(d) $H(-x) = \begin{cases} 1, & x \le 0 \\ 0, & x > 0 \end{cases}$



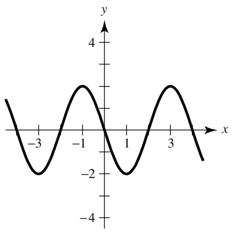
(e) $\frac{1}{2}H(x) = \begin{cases} \frac{1}{2}, & x \ge 0 \\ 0, & x < 0 \end{cases}$



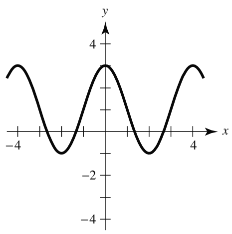
(f) $-H(x - 2) + 2 = \begin{cases} 1, & x \ge 2 \\ 2, & x < 2 \end{cases}$



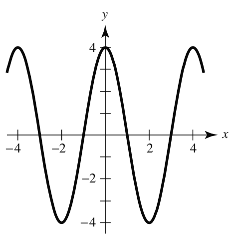
4. (a) $f(x+1)$



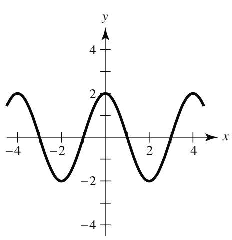
(b) $f(x) + 1$



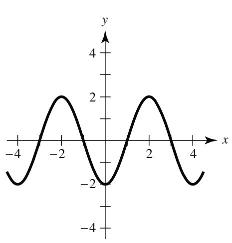
(c) $2f(x)$



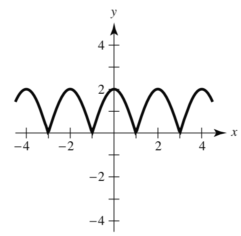
(d) $f(-x)$



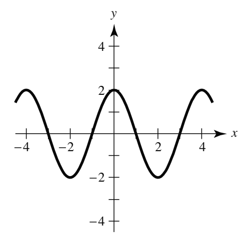
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$

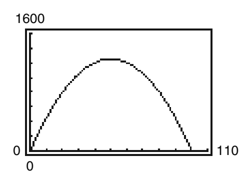


5. (a) $x + 2y = 100 \Rightarrow y = \dfrac{100 - x}{2}$

$$A(x) = xy = x\left(\frac{100 - x}{2}\right) = -\frac{x^2}{2} + 50x$$

Domain: $0 < x < 100$ or $(0, 100)$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c) $A(x) = -\frac{1}{2}(x^2 - 100x)$

$= -\frac{1}{2}(x^2 - 100x + 2500) + 1250$

$= -\frac{1}{2}(x - 50)^2 + 1250$

$A(50) = 1250$ m^2 is the maximum.

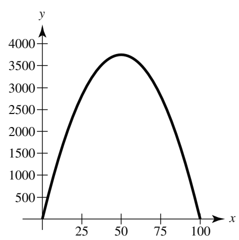
$x = 50$ m, $y = 25$ m

6. (a) $4y + 3x = 300 \Rightarrow y = \dfrac{300 - 3x}{4}$

$$A(x) = x(2y) = x\left(\frac{300 - 3x}{2}\right) = \frac{-3x^2 + 300x}{2}$$

Domain: $0 < x < 100$

(b)



Maximum of 3750 ft^2 at $x = 50$ ft, $y = 37.5$ ft.

(c) $A(x) = -\frac{3}{2}(x^2 - 100x)$

$= -\frac{3}{2}(x^2 - 100x + 2500) + 3750$

$= -\frac{3}{2}(x - 50)^2 + 3750$

$A(50) = 3750$ square feet is the maximum area, where $x = 50$ ft and $y = 37.5$ ft.

7. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. So, the total time is $T = \dfrac{\sqrt{4 + x^2}}{2} + \dfrac{\sqrt{1 + (3 - x)^2}}{4}$ hours.

8. Let d be the distance from the starting point to the beach.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{2d}{\frac{d}{120} + \frac{d}{60}} = \frac{2}{\frac{1}{120} + \frac{1}{60}} = 80 \text{ km/h}$$

9. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$. Slope of tangent line is less than 5.

(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$. Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$. Slope of tangent line is less than 4.1.

(d) Slope $= \dfrac{f(2 + h) - f(2)}{(2 + h) - 2}$

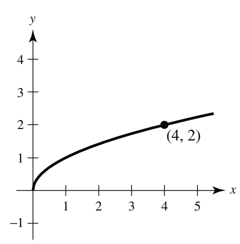
$= \dfrac{(2 + h)^2 - 4}{h}$

$= \dfrac{4h + h^2}{h}$

$= 4 + h, h \neq 0$

(e) Letting h get closer and closer to 0, the slope approaches 4. So, the slope at (2, 4) is 4.

10.



(a) Slope $= \dfrac{3 - 2}{9 - 4} = \dfrac{1}{5}$. Slope of tangent line is greater than $\dfrac{1}{5}$.

(b) Slope $= \dfrac{2 - 1}{4 - 1} = \dfrac{1}{3}$. Slope of tangent line is less than $\dfrac{1}{3}$.

(c) Slope $= \dfrac{2.1 - 2}{4.41 - 4} = \dfrac{10}{41}$. Slope of tangent line is greater than $\dfrac{10}{41}$.

(d) Slope $= \dfrac{f(4 + h) - f(4)}{(4 + h) - 4} = \dfrac{\sqrt{4 + h} - 2}{h}$

(e) $\dfrac{\sqrt{4 + h} - 2}{h} = \dfrac{\sqrt{4 + h} - 2}{h} \cdot \dfrac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2} = \dfrac{(4 + h) - 4}{h\left(\sqrt{4 + h} + 2\right)} = \dfrac{1}{\sqrt{4 + h} + 2}, h \neq 0$

As h gets closer to 0, the slope gets closer to $\dfrac{1}{4}$. The slope is $\dfrac{1}{4}$ at the point (4, 2).

11. $f(x) = y = \dfrac{1}{1 - x}$

(a) Domain: all $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

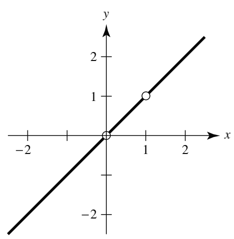
(b) $f(f(x)) = f\left(\dfrac{1}{1 - x}\right) = \dfrac{1}{1 - \left(\dfrac{1}{1 - x}\right)} = \dfrac{1}{\dfrac{1 - x - 1}{1 - x}} = \dfrac{1 - x}{-x} = \dfrac{x - 1}{x}$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $f(f(f(x))) = f\left(\dfrac{x - 1}{x}\right) = \dfrac{1}{1 - \left(\dfrac{x - 1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(d) The graph is not a line. It has holes at (0, 0) and (1, 1).



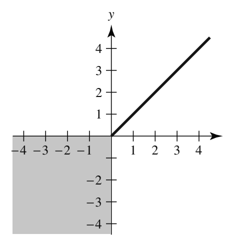
12. Using the definition of absolute value, you can rewrite the equation.

$y + |y| = x + |x|$

$$\begin{cases} 2y, & y > 0 \\ 0, & y \leq 0 \end{cases} = \begin{cases} 2x, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

For $x > 0$ and $y > 0$, you have $2y = 2x \Rightarrow y = x$.

For any $x \leq 0$, y is any $y \leq 0$. So, the graph of $y + |y| = x + |x|$ is as follows.



13. (a) $\dfrac{I}{x^2} = \dfrac{2I}{(x - 3)^2}$

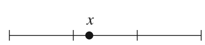
$x^2 - 6x + 9 = 2x^2$

$x^2 + 6x - 9 = 0$

$x = \dfrac{-6 \pm \sqrt{36 + 36}}{2}$

$= -3 \pm \sqrt{18}$

$\approx 1.2426, -7.2426$



(b) $\dfrac{I}{x^2 + y^2} = \dfrac{2I}{(x - 3)^2 + y^2}$

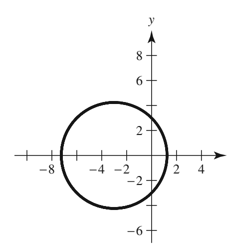
$(x - 3)^2 + y^2 = 2(x^2 + y^2)$

$x^2 - 6x + 9 + y^2 = 2x^2 + 2y^2$

$x^2 + y^2 + 6x - 9 = 0$

$(x + 3)^2 + y^2 = 18$

Circle of radius $\sqrt{18}$ and center $(-3, 0)$.



14. (a) $\dfrac{I}{x^2 + y^2} = \dfrac{kI}{(x-4)^2 + y^2}$

$$(x-4)^2 + y^2 = k(x^2 + y^2)$$

$$(k-1)x^2 + 8x + (k-1)y^2 = 16$$

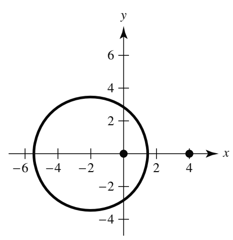
If $k = 1$, then $x = 2$ is a vertical line. Assume $k \neq 1$.

$$x^2 + \frac{8x}{k-1} + y^2 = \frac{16}{k-1}$$

$$x^2 + \frac{8x}{k-1} + \frac{16}{(k-1)^2} + y^2 = \frac{16}{k-1} + \frac{16}{(k-1)^2}$$

$$\left(x + \frac{4}{k-1}\right)^2 + y^2 = \frac{16k}{(k-1)^2}, \text{ Circle}$$

(b) If $k = 3$, $(x+2)^2 + y^2 = 12$



(c) As k becomes very large, $\dfrac{4}{k-1} \to 0$ and $\dfrac{16k}{(k-1)^2} \to 0$.

The center of the circle gets closer to (0, 0), and its radius approaches 0.

15.

$$d_1 d_2 = 1$$

$$\left[(x+1)^2 + y^2\right]\left[(x-1)^2 + y^2\right] = 1$$

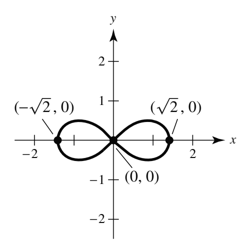
$$(x+1)^2(x-1)^2 + y^2\left[(x+1)^2 + (x-1)^2\right] + y^4 = 1$$

$$(x^2 - 1)^2 + y^2\left[2x^2 + 2\right] + y^4 = 1$$

$$x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 = 1$$

$$(x^4 + 2x^2y^2 + y^4) - 2x^2 + 2y^2 = 0$$

$$(x^2 + y^2)^2 = 2(x^2 - y^2)$$



Let $y = 0$. Then $x^4 = 2x^2 \Rightarrow x = 0$ or $x^2 = 2$.

So, $(0, 0)$, $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$ are on the curve.

CHAPTER 1
Limits and Their Properties

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Limits and Their Properties

Section 1.1 A Preview of Calculus

1. Precalculus: $(20 \text{ ft/sec})(15 \text{ sec}) = 300 \text{ ft}$

2. Calculus required: Velocity is not constant.
Distance $\approx (20 \text{ ft/sec})(15 \text{ sec}) = 300 \text{ ft}$

3. Calculus required: Slope of the tangent line at $x = 2$ is the rate of change, and equals about 0.16.

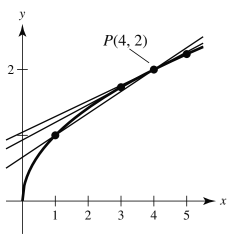
4. Precalculus: rate of change $=$ slope $= 0.08$

5. (a) Precalculus: Area $= \frac{1}{2}bh = \frac{1}{2}(5)(4) = 10$ sq. units

(b) Calculus required: Area $= bh$
$\approx 2(2.5)$
$= 5$ sq. units

6. $f(x) = \sqrt{x}$

(a)



(b) $$\text{slope} = m = \frac{\sqrt{x} - 2}{x - 4} = \frac{\sqrt{x} - 2}{(\sqrt{x} + 2)(\sqrt{x} - 2)} = \frac{1}{\sqrt{x} + 2}, x \neq 4$$

$$x = 1: m = \frac{1}{\sqrt{1} + 2} = \frac{1}{3}$$

$$x = 3: m = \frac{1}{\sqrt{3} + 2} \approx 0.2679$$

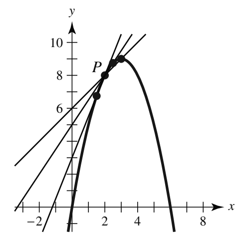
$$x = 5: m = \frac{1}{\sqrt{5} + 2} \approx 0.2361$$

(c) At $P(4, 2)$ the slope is $\frac{1}{\sqrt{4} + 2} = \frac{1}{4} = 0.25$.

You can improve your approximation of the slope at $x = 4$ by considering x-values very close to 4.

7. $f(x) = 6x - x^2$

(a)



(b) $$\text{slope} = m = \frac{(6x - x^2) - 8}{x - 2} = \frac{(x - 2)(4 - x)}{x - 2} = (4 - x), x \neq 2$$

For $x = 3, m = 4 - 3 = 1$

For $x = 2.5, m = 4 - 2.5 = 1.5 = \frac{3}{2}$

For $x = 1.5, m = 4 - 1.5 = 2.5 = \frac{5}{2}$

(c) At $P(2, 8)$, the slope is 2. You can improve your approximation by considering values of x close to 2.

8. Answers will vary. *Sample answer*:
The instantaneous rate of change of an automobile's position is the velocity of the automobile, and can be determined by the speedometer.

9. (a) Area $\approx 5 + \frac{5}{2} + \frac{5}{3} + \frac{5}{4} \approx 10.417$

Area $\approx \frac{1}{2}\left(5 + \frac{5}{1.5} + \frac{5}{2} + \frac{5}{2.5} + \frac{5}{3} + \frac{5}{3.5} + \frac{5}{4} + \frac{5}{4.5}\right) \approx 9.145$

(b) You could improve the approximation by using more rectangles.

10. (a) $D_1 = \sqrt{(5-1)^2 + (1-5)^2} = \sqrt{16+16} \approx 5.66$

(b) $D_2 = \sqrt{1 + \left(\frac{5}{2}\right)^2} + \sqrt{1 + \left(\frac{5}{2} - \frac{5}{3}\right)^2} + \sqrt{1 + \left(\frac{5}{3} - \frac{5}{4}\right)^2} + \sqrt{1 + \left(\frac{5}{4} - 1\right)^2}$

$\approx 2.693 + 1.302 + 1.083 + 1.031 \approx 6.11$

(c) Increase the number of line segments.

Section 1.2 Finding Limits Graphically and Numerically

1.

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	0.2041	0.2004	0.2000	0.2000	0.1996	0.1961

$\lim_{x \to 4} \frac{x-4}{x^2 - 3x - 4} \approx 0.2000$ $\left(\text{Actual limit is } \frac{1}{5}.\right)$

2.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	0.1695	0.1669	0.1667	?	0.1666	0.1664	0.1639

$\lim_{x \to 3} \frac{x-3}{x^2 - 9} \approx 0.1667$ $\left(\text{Actual limit is } \frac{1}{6}.\right)$

3.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.5132	0.5013	0.5001	?	0.4999	0.4988	0.4881

$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} \approx 0.5000$ $\left(\text{Actual limit is } \frac{1}{2}.\right)$

4.

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	–0.0641	–0.0627	–0.0625	–0.0625	–0.0623	–0.0610

$\lim_{x \to 3} \frac{[1/(x+1)] - (1/4)}{x - 3} \approx -0.0625$ $\left(\text{Actual limit is } -\frac{1}{16}.\right)$

5.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.9983	0.99998	1.0000	1.0000	0.99998	0.9983

$\lim_{x \to 0} \frac{\sin x}{x} \approx 1.0000$ (Actual limit is 1.) (Make sure you use radian mode.)

6.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.0500	0.0050	0.0005	–0.0005	–0.0050	–0.0500

$\lim_{x \to 0} \frac{\cos x - 1}{x} \approx 0.0000$ (Actual limit is 0.) (Make sure you use radian mode.)

7.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.2564	0.2506	0.2501	0.2499	0.2494	0.2439

$\lim_{x\to 1} \frac{x-2}{x^2+x-6} \approx 0.2500$ $\left(\text{Actual limit is } \frac{1}{4}.\right)$

8.

x	–4.1	–4.01	–4.001	–4	–3.999	–3.99	–3.9
$f(x)$	1.1111	1.0101	1.0010	?	0.9990	0.9901	0.9091

$\lim_{x\to -4} \frac{x+4}{x^2+9x+20} \approx 1.0000$ (Actual limit is 1.)

9.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.7340	0.6733	0.6673	0.6660	0.6600	0.6015

$\lim_{x\to 1} \frac{x^4-1}{x^6-1} \approx 0.6666$ $\left(\text{Actual limit is } \frac{2}{3}.\right)$

10.

x	–3.1	–3.01	–3.001	–3	–2.999	–2.99	–2.9
$f(x)$	27.91	27.0901	27.0090	?	26.9910	26.9101	26.11

$\lim_{x\to -3} \frac{x^3+27}{x+3} \approx 27.0000$ (Actual limit is 27.)

11.

x	–6.1	–6.01	–6.001	–6	–5.999	–5.99	–5.9
$f(x)$	–0.1248	–0.1250	–0.1250	?	–0.1250	–0.1250	–0.1252

$\lim_{x\to -6} \frac{\sqrt{10-x}-4}{x+6} \approx -0.1250$ $\left(\text{Actual limit is } -\frac{1}{8}.\right)$

12.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.1149	0.115	0.1111	?	0.1111	0.1107	0.1075

$\lim_{x\to 2} \frac{x/(x+1)-2/3}{x-2} \approx 0.1111$ $\left(\text{Actual limit is } \frac{1}{9}.\right)$

13.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	1.9867	1.9999	2.0000	2.0000	1.9999	1.9867

$\lim_{x\to 0} \frac{\sin 2x}{x} \approx 2.0000$ (Actual limit is 2.) (Make sure you use radian mode.)

14.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.4950	0.5000	0.5000	0.5000	0.5000	0.4950

$\lim_{x\to 0} \frac{\tan x}{\tan 2x} \approx 0.5000$ $\left(\text{Actual limit is } \frac{1}{2}.\right)$

15. $\lim_{x \to 3} (4 - x) = 1$

16. $\lim_{x \to 0} \sec x = 1$

17. $\lim_{x \to 2} f(x) = \lim_{x \to 2} (4 - x) = 2$

18. $\lim_{x \to 1} f(x) = \lim_{x \to 1} (x^2 + 3) = 4$

19. $\lim_{x \to 2} \frac{|x - 2|}{x - 2}$ does not exist.

For values of x to the left of 2, $\frac{|x - 2|}{(x - 2)} = -1$, whereas for values of x to the right of 2, $\frac{|x - 2|}{(x - 2)} = 1$.

20. $\lim_{x \to 5} \frac{2}{x - 5}$ does not exist because the function increases and decreases without bound as x approaches 5.

21. $\lim_{x \to 0} \cos(1/x)$ does not exist because the function oscillates between –1 and 1 as x approaches 0.

22. $\lim_{x \to \pi/2} \tan x$ does not exist because the function increases without bound as x approaches $\frac{\pi}{2}$ from the left and decreases without bound as x approaches $\frac{\pi}{2}$ from the right.

23. (a) $f(1)$ exists. The black dot at (1, 2) indicates that $f(1) = 2$.

(b) $\lim_{x \to 1} f(x)$ does not exist. As x approaches 1 from the left, $f(x)$ approaches 3.5, whereas as x approaches 1 from the right, $f(x)$ approaches 1.

(c) $f(4)$ does not exist. The hollow circle at $(4, 2)$ indicates that f is not defined at 4.

(d) $\lim_{x \to 4} f(x)$ exists. As x approaches 4, $f(x)$ approaches 2: $\lim_{x \to 4} f(x) = 2$.

24. (a) $f(-2)$ does not exist. The vertical dotted line indicates that f is not defined at –2.

(b) $\lim_{x \to -2} f(x)$ does not exist. As x approaches –2, the values of $f(x)$ do not approach a specific number.

(c) $f(0)$ exists. The black dot at $(0, 4)$ indicates that $f(0) = 4$.

(d) $\lim_{x \to 0} f(x)$ does not exist. As x approaches 0 from the left, $f(x)$ approaches $\frac{1}{2}$, whereas as x approaches 0 from the right, $f(x)$ approaches 4.

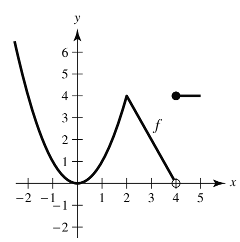
(e) $f(2)$ does not exist. The hollow circle at $\left(2, \frac{1}{2}\right)$ indicates that $f(2)$ is not defined.

(f) $\lim_{x \to 2} f(x)$ exists. As x approaches 2, $f(x)$ approaches $\frac{1}{2}$: $\lim_{x \to 2} f(x) = \frac{1}{2}$.

(g) $f(4)$ exists. The black dot at $(4, 2)$ indicates that $f(4) = 2$.

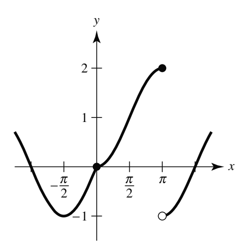
(h) $\lim_{x \to 4} f(x)$ does not exist. As x approaches 4, the values of $f(x)$ do not approach a specific number.

25.



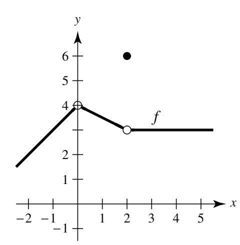
$\lim_{x \to c} f(x)$ exists for all values of $c \neq 4$.

26.

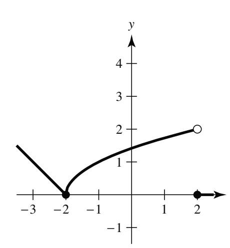


$\lim_{x \to c} f(x)$ exists for all values of $c \neq \pi$.

27. One possible answer is



28. One possible answer is



29. You need $|f(x) - 3| = |(x + 1) - 3| = |x - 2| < 0.4$. So, take $\delta = 0.4$. If $0 < |x - 2| < 0.4$, then $|x - 2| = |(x + 1) - 3| = |f(x) - 3| < 0.4$, as desired.

30. You need $|f(x) - 1| = \left|\frac{1}{x - 1} - 1\right| = \left|\frac{2 - x}{x - 1}\right| < 0.01$. Let $\delta = \frac{1}{101}$. If $0 < |x - 2| < \frac{1}{101}$, then

$$-\frac{1}{101} < x - 2 < \frac{1}{101} \Rightarrow 1 - \frac{1}{101} < x - 1 < 1 + \frac{1}{101}$$
$$\Rightarrow \frac{100}{101} < x - 1 < \frac{102}{101}$$
$$\Rightarrow |x - 1| > \frac{100}{101}$$

and you have

$$|f(x) - 1| = \left|\frac{1}{x - 1} - 1\right| = \left|\frac{2 - x}{x - 1}\right| < \frac{1/101}{100/101} = \frac{1}{100} = 0.01.$$

31. You need to find δ such that $0 < |x - 1| < \delta$ implies $|f(x) - 1| = \left|\frac{1}{x} - 1\right| < 0.1$. That is,

$$-0.1 < \frac{1}{x} - 1 < 0.1$$
$$1 - 0.1 < \frac{1}{x} < 1 + 0.1$$
$$\frac{9}{10} < \frac{1}{x} < \frac{11}{10}$$
$$\frac{10}{9} > x > \frac{10}{11}$$
$$\frac{10}{9} - 1 > x - 1 > \frac{10}{11} - 1$$
$$\frac{1}{9} > x - 1 > -\frac{1}{11}.$$

So take $\delta = \frac{1}{11}$. Then $0 < |x - 1| < \delta$ implies

$$-\frac{1}{11} < x - 1 < \frac{1}{11}$$
$$-\frac{1}{11} < x - 1 < \frac{1}{9}.$$

Using the first series of equivalent inequalities, you obtain

$$|f(x) - 1| = \left|\frac{1}{x} - 1\right| < 0.1.$$

32. You need to find δ such that $0 < |x - 2| < \delta$ implies $|f(x) - 3| = |x^2 - 1 - 3| = |x^2 - 4| < 0.2$. That is,

$$-0.2 < x^2 - 4 < 0.2$$
$$4 - 0.2 < x^2 < 4 + 0.2$$
$$3.8 < x^2 < 4.2$$
$$\sqrt{3.8} < x < \sqrt{4.2}$$
$$\sqrt{3.8} - 2 < x - 2 < \sqrt{4.2} - 2$$

So take $\delta = \sqrt{4.2} - 2 \approx 0.0494$.

Then $0 < |x - 2| < \delta$ implies

$$-\left(\sqrt{4.2} - 2\right) < x - 2 < \sqrt{4.2} - 2$$
$$\sqrt{3.8} - 2 < x - 2 < \sqrt{4.2} - 2.$$

Using the first series of equivalent inequalities, you obtain

$$|f(x) - 3| = |x^2 - 4| < 0.2.$$

33. $\lim_{x\to 2}(3x + 2) = 3(2) + 2 = 8 = L$

$$|(3x + 2) - 8| < 0.01$$
$$|3x - 6| < 0.01$$
$$3|x - 2| < 0.01$$
$$0 < |x - 2| < \tfrac{0.01}{3} \approx 0.0033 = \delta$$

So, if $0 < |x - 2| < \delta = \frac{0.01}{3}$, you have

$$3|x - 2| < 0.01$$
$$|3x - 6| < 0.01$$
$$|(3x + 2) - 8| < 0.01$$
$$|f(x) - L| < 0.01.$$

34. $\lim_{x\to 6}\left(6 - \frac{x}{3}\right) = 6 - \frac{6}{3} = 4 = L$

$$\left|\left(6 - \frac{x}{3}\right) - 4\right| < 0.01$$
$$\left|2 - \frac{x}{3}\right| < 0.01$$
$$\left|-\tfrac{1}{3}(x - 6)\right| < 0.01$$
$$|x - 6| < 0.03$$
$$0 < |x - 6| < 0.03 = \delta$$

So, if $0 < |x - 6| < \delta = 0.03$, you have

$$\left|-\tfrac{1}{3}(x - 6)\right| < 0.01$$
$$\left|2 - \frac{x}{3}\right| < 0.01$$
$$\left|\left(6 - \frac{x}{3}\right) - 4\right| < 0.01$$
$$|f(x) - L| < 0.01.$$

35. $\lim_{x\to 2}\left(x^2 - 3\right) = 2^2 - 3 = 1 = L$

$$\left|\left(x^2 - 3\right) - 1\right| < 0.01$$
$$\left|x^2 - 4\right| < 0.01$$
$$|(x + 2)(x - 2)| < 0.01$$
$$|x + 2||x - 2| < 0.01$$
$$|x - 2| < \frac{0.01}{|x + 2|}$$

If you assume $1 < x < 3$, then $\delta \approx 0.01/5 = 0.002$.

So, if $0 < |x - 2| < \delta \approx 0.002$, you have

$$|x - 2| < 0.002 = \frac{1}{5}(0.01) < \frac{1}{|x + 2|}(0.01)$$
$$|x + 2||x - 2| < 0.01$$
$$\left|x^2 - 4\right| < 0.01$$
$$\left|\left(x^2 - 3\right) - 1\right| < 0.01$$
$$|f(x) - L| < 0.01.$$

36. $\lim_{x\to 4}\left(x^2 + 6\right) = 4^2 + 6 = 22 = L$

$$\left|\left(x^2 + 6\right) - 22\right| < 0.01$$
$$\left|x^2 - 16\right| < 0.01$$
$$|(x + 4)(x - 4)| < 0.01$$
$$|x - 4| < \frac{0.01}{|x + 4|}$$

If you assume $3 < x < 5$, then $\delta = \frac{0.01}{9} \approx 0.00111$.

So, if $0 < |x - 4| < \delta \approx \frac{0.01}{9}$, you have

$$|x - 4| < \frac{0.01}{9} < \frac{0.01}{|x + 4|}$$
$$|(x + 4)(x - 4)| < 0.01$$
$$\left|x^2 - 16\right| < 0.01$$
$$\left|\left(x^2 + 6\right) - 22\right| < 0.01$$
$$|f(x) - L| < 0.01.$$

37. $\lim_{x\to 4}(x + 2) = 4 + 2 = 6$

Given $\varepsilon > 0$:

$$|(x + 2) - 6| < \varepsilon$$
$$|x - 4| < \varepsilon = \delta$$

So, let $\delta = \varepsilon$. So, if $0 < |x - 4| < \delta = \varepsilon$, you have

$$|x - 4| < \varepsilon$$
$$|(x + 2) - 6| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

38. $\lim_{x\to -2}(4x + 5) = 4(-2) + 5 = -3$

Given $\varepsilon > 0$:

$$|(4x + 5) - (-3)| < \varepsilon$$
$$|4x + 8| < \varepsilon$$
$$4|x + 2| < \varepsilon$$
$$|x + 2| < \frac{\varepsilon}{4} = \delta$$

So, let $\delta = \frac{\varepsilon}{4}$.

So, if $0 < |x + 2| < \delta = \frac{\varepsilon}{4}$, you have

$$|x + 2| < \frac{\varepsilon}{4}$$
$$|4x + 8| < \varepsilon$$
$$|(4x + 5) - (-3)| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

39. $\lim_{x\to -4}\left(\frac{1}{2}x - 1\right) = \frac{1}{2}(-4) - 1 = -3$

Given $\varepsilon > 0$:

$$\left|\left(\tfrac{1}{2}x - 1\right) - (-3)\right| < \varepsilon$$
$$\left|\tfrac{1}{2}x + 2\right| < \varepsilon$$
$$\tfrac{1}{2}\left|x - (-4)\right| < \varepsilon$$
$$\left|x - (-4)\right| < 2\varepsilon$$

So, let $\delta = 2\varepsilon$.

So, if $0 < \left|x - (-4)\right| < \delta = 2\varepsilon$, you have

$$\left|x - (-4)\right| < 2\varepsilon$$
$$\left|\tfrac{1}{2}x + 2\right| < \varepsilon$$
$$\left|\left(\tfrac{1}{2}x - 1\right) + 3\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

40. $\lim_{x\to 3}\left(\frac{3}{4}x + 1\right) = \frac{3}{4}(3) + 1 = \frac{13}{4}$

Given $\varepsilon > 0$:

$$\left|\left(\tfrac{3}{4}x + 1\right) - \tfrac{13}{4}\right| < \varepsilon$$
$$\left|\tfrac{3}{4}x - \tfrac{9}{4}\right| < \varepsilon$$
$$\tfrac{3}{4}\left|x - 3\right| < \varepsilon$$
$$\left|x - 3\right| < \tfrac{4}{3}\varepsilon$$

So, let $\delta = \frac{4}{3}\varepsilon$.

So, if $0 < \left|x - 3\right| < \delta = \frac{4}{3}\varepsilon$, you have

$$\left|x - 3\right| < \tfrac{4}{3}\varepsilon$$
$$\tfrac{3}{4}\left|x - 3\right| < \varepsilon$$
$$\left|\tfrac{3}{4}x - \tfrac{9}{4}\right| < \varepsilon$$
$$\left|\left(\tfrac{3}{4}x + 1\right) - \tfrac{13}{4}\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

41. $\lim_{x\to 6} 3 = 3$

Given $\varepsilon > 0$:

$$\left|3 - 3\right| < \varepsilon$$
$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$\left|3 - 3\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

42. $\lim_{x\to 2}(-1) = -1$

Given $\varepsilon > 0$: $\left|-1 - (-1)\right| < \varepsilon$

$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$\left|(-1) - (-1)\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

43. $\lim_{x\to 0}\sqrt[3]{x} = 0$

Given $\varepsilon > 0$: $\left|\sqrt[3]{x} - 0\right| < \varepsilon$

$$\left|\sqrt[3]{x}\right| < \varepsilon$$
$$\left|x\right| < \varepsilon^3 = \delta$$

So, let $\delta = \varepsilon^3$.

So, for $0\left|x - 0\right|\delta = \varepsilon^3$, you have

$$\left|x\right| < \varepsilon^3$$
$$\left|\sqrt[3]{x}\right| < \varepsilon$$
$$\left|\sqrt[3]{x} - 0\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

44. $\lim_{x\to 4}\sqrt{x} = \sqrt{4} = 2$

Given $\varepsilon > 0$: $\left|\sqrt{x} - 2\right| < \varepsilon$

$$\left|\sqrt{x} - 2\right|\left|\sqrt{x} + 2\right| < \varepsilon\left|\sqrt{x} + 2\right|$$
$$\left|x - 4\right| < \varepsilon\left|\sqrt{x} + 2\right|$$

Assuming $1 < x < 9$, you can choose $\delta = 3\varepsilon$. Then,

$$0 < \left|x - 4\right| < \delta = 3\varepsilon \Rightarrow \left|x - 4\right| < \varepsilon\left|\sqrt{x} + 2\right|$$
$$\Rightarrow \left|\sqrt{x} - 2\right| < \varepsilon.$$

45. $\lim_{x\to -5}|x - 5| = |(-5) - 5| = |-10| = 10$

Given $\varepsilon > 0$: $||x - 5| - 10| < \varepsilon$

$$|-(x - 5) - 10| < \varepsilon \quad (x - 5 < 0)$$
$$|-x - 5| < \varepsilon$$
$$|x - (-5)| < \varepsilon$$

So, let $\delta = \varepsilon$.

So for $|x - (-5)| < \delta = \varepsilon$, you have

$$|-(x + 5)| < \varepsilon$$
$$|-(x - 5) - 10| < \varepsilon$$
$$||x - 5| - 10| < \varepsilon \quad (\text{because } x - 5 < 0)$$
$$|f(x) - L| < \varepsilon.$$

46. $\lim_{x\to 3}|x - 3| = |3 - 3| = 0$

Given $\varepsilon > 0$: $||x - 3| - 0| < \varepsilon$

$$|x - 3| < \varepsilon$$

So, let $\delta = \varepsilon$.

So, for $0 < |x - 3| < \delta = \varepsilon$, you have

$$|x - 3| < \varepsilon$$
$$||x - 3| - 0| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

47. $\lim_{x\to 1}(x^2 + 1) = 1^2 + 1 = 2$

Given $\varepsilon > 0$:

$$|(x^2 + 1) - 2| < \varepsilon$$
$$|x^2 - 1| < \varepsilon$$
$$|(x + 1)(x - 1)| < \varepsilon$$
$$|x - 1| < \frac{\varepsilon}{|x + 1|}$$

If you assume $0 < x < 2$, then $\delta = \varepsilon/3$.

So for $0 < |x - 1| < \delta = \dfrac{\varepsilon}{3}$, you have

$$|x - 1| < \frac{1}{3}\varepsilon < \frac{1}{|x + 1|}\varepsilon$$
$$|x^2 - 1| < \varepsilon$$
$$|(x^2 + 1) - 2| < \varepsilon$$
$$|f(x) - 2| < \varepsilon.$$

48. $\lim_{x\to -4}(x^2 + 4x) = (-4)^2 + 4(-4) = 0$

Given $\varepsilon > 0$:

$$|(x^2 + 4x) - 0| < \varepsilon$$
$$|x(x + 4)| < \varepsilon$$
$$|x + 4| < \frac{\varepsilon}{|x|}$$

If you assume $-5 < x < -3$, then $\delta = \dfrac{\varepsilon}{5}$.

So for $0 < |x - (-4)| < \delta = \dfrac{\varepsilon}{5}$, you have

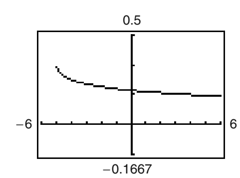
$$|x + 4| < \frac{\varepsilon}{5} < \frac{1}{|x|}\varepsilon$$
$$|x(x + 4)| < \varepsilon$$
$$|(x^2 + 4x) - 0| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

49. $\lim_{x\to\pi} f(x) = \lim_{x\to\pi} 4 = 4$

50. $\lim_{x\to\pi} f(x) = \lim_{x\to\pi} x = \pi$

51. $f(x) = \dfrac{\sqrt{x + 5} - 3}{x - 4}$

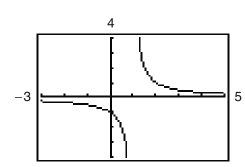
$\lim_{x\to 4} f(x) = \dfrac{1}{6}$



The domain is $[-5, 4) \cup (4, \infty)$. The graphing utility does not show the hole at $\left(4, \frac{1}{6}\right)$.

52. $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

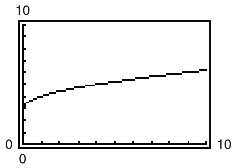
$\lim_{x\to 3} f(x) = \dfrac{1}{2}$



The domain is all $x \neq 1, 3$. The graphing utility does not show the hole at $\left(3, \frac{1}{2}\right)$.

53. $f(x) = \dfrac{x - 9}{\sqrt{x} - 3}$

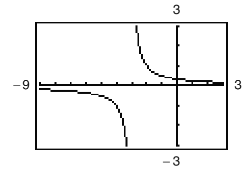
$\lim_{x \to 9} f(x) = 6$



The domain is all $x \geq 0$ except $x = 9$. The graphing utility does not show the hole at $(9, 6)$.

54. $f(x) = \dfrac{x - 3}{x^2 - 9}$

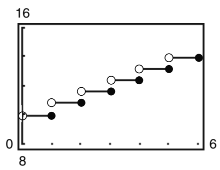
$\lim_{x \to 3} f(x) = \dfrac{1}{6}$



The domain is all $x \neq \pm 3$. The graphing utility does not show the hole at $\left(3, \dfrac{1}{6}\right)$.

55. $C(t) = 9.99 - 0.79[[-(t - 1)]]$

(a)



(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	11.57	12.36	12.36	12.36	12.36	12.36	12.36

$\lim_{t \to 3.5} C(t) = 12.36$

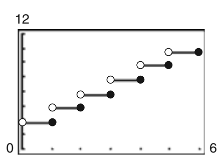
(c)

t	2	2.5	2.9	3	3.1	3.5	4
C	10.78	11.57	11.57	11.57	12.36	12.36	12.36

The $\lim_{t \to 3} C(t)$ does not exist because the values of C approach different values as t approaches 3 from both sides.

56. $C(t) = 5.79 - 0.99[[-(t - 1)]]$

(a)



(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	7.77	8.76	8.76	8.76	8.76	8.76	8.76

$\lim_{t \to 3.5} C(t) = 8.76$

(c)

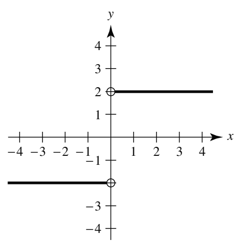
t	2	2.5	2.9	3	3.1	3.5	4
C	6.78	7.77	7.77	7.77	8.76	8.76	8.76

The limit $\lim_{t \to 3} C(t)$ does not exist because the values of C approach different values as t approaches 3 from both sides.

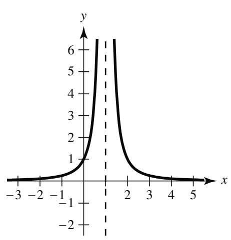
57. $\lim_{x\to 8} f(x) = 25$ means that the values of f approach 25 as x gets closer and closer to 8.

58. In the definition of $\lim_{x\to c} f(x)$, f must be defined on both sides of c, but does not have to be defined at c itself. The value of f at c has no bearing on the limit as x approaches c.

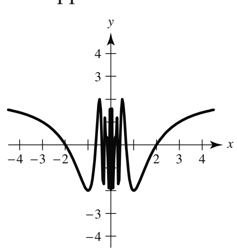
59. (i) The values of f approach different numbers as x approaches c from different sides of c:



(ii) The values of f increase without bound as x approaches c:



(iii) The values of f oscillate between two fixed numbers as x approaches c:



60. (a) No. The fact that $f(2) = 4$ has no bearing on the existence of the limit of $f(x)$ as x approaches 2.

(b) No. The fact that $\lim_{x\to 2} f(x) = 4$ has no bearing on the value of f at 2.

61. (a) $C = 2\pi r$

$$r = \frac{C}{2\pi} = \frac{6}{2\pi} = \frac{3}{\pi} \approx 0.9549 \text{ cm}$$

(b) When $C = 5.5$: $r = \dfrac{5.5}{2\pi} \approx 0.87535$ cm

When $C = 6.5$: $r = \dfrac{6.5}{2\pi} \approx 1.03451$ cm

So $0.87535 < r < 1.03451$.

(c) $\lim_{x\to 3/\pi} (2\pi r) = 6$; $\varepsilon = 0.5$; $\delta \approx 0.0796$

62. $V = \frac{4}{3}\pi r^3$, $V = 2.48$

(a) $2.48 = \frac{4}{3}\pi r^3$

$$r^3 = \frac{1.86}{\pi}$$

$$r \approx 0.8397 \text{ in.}$$

(b) $2.45 \le V \le 2.51$

$$2.45 \le \frac{4}{3}\pi r^3 \le 2.51$$

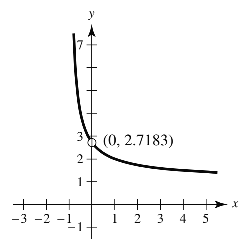
$$0.5849 \le r^3 \le 0.5992$$

$$0.8363 \le r \le 0.8431$$

(c) For $\varepsilon = 2.51 - 2.48 = 0.03$, $\delta \approx 0.003$

63. $f(x) = (1 + x)^{1/x}$

$$\lim_{x\to 0}(1 + x)^{1/x} = e \approx 2.71828$$



x	$f(x)$
–0.1	2.867972
–0.01	2.731999
–0.001	2.719642
–0.0001	2.718418
–0.00001	2.718295
–0.000001	2.718283

x	$f(x)$
0.1	2.593742
0.01	2.704814
0.001	2.716942
0.0001	2.718146
0.00001	2.718268
0.000001	2.718280

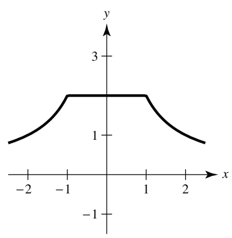
64. $f(x) = \dfrac{|x+1| - |x-1|}{x}$

x	–1	–0.5	–0.1	0	0.1	0.5	1.0
f(x)	2	2	2	Undef.	2	2	2

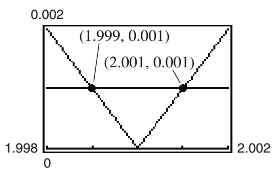
$\lim_{x\to 0} f(x) = 2$

Note that for

$-1 < x < 1, x \neq 0, f(x) = \dfrac{(x+1)+(x-1)}{x} = 2.$



65.



Using the zoom and trace feature, $\delta = 0.001$. So $(2-\delta, 2+\delta) = (1.999, 2.001)$.

Note: $\dfrac{x^2-4}{x-2} = x+2$ for $x \neq 2$.

66. (a) $\lim_{x\to c} f(x)$ exists for all $c \neq -3$.

(b) $\lim_{x\to c} f(x)$ exists for all $c \neq -2, 0$.

67. False. The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as $x \to c$.

68. True

69. False. Let

$f(x) = \begin{cases} x-4, & x \neq 2 \\ 0, & x = 2 \end{cases}$

$f(2) = 0$

$\lim_{x\to 2} f(x) = \lim_{x\to 2}(x-4) = 2 \neq 0$

70. False. Let

$f(x) = \begin{cases} x-4, & x \neq 2 \\ 0, & x = 2 \end{cases}$

$\lim_{x\to 2} f(x) = \lim_{x\to 2}(x-4) = 2$ and $f(2) = 0 \neq 2$

71. $f(x) = \sqrt{x}$

$\lim_{x\to 0.25} \sqrt{x} = 0.5$ is true.

As x approaches $0.25 = \frac{1}{4}$ from either side,

$f(x) = \sqrt{x}$ approaches $\frac{1}{2} = 0.5$.

72. $f(x) = \sqrt{x}$

$\lim_{x\to 0} \sqrt{x} = 0$ is false.

$f(x) = \sqrt{x}$ is not defined on an open interval containing 0 because the domain of f is $x \geq 0$.

73. Using a graphing utility, you see that

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

$\lim_{x\to 0} \dfrac{\sin 2x}{x} = 2$, etc.

So, $\lim_{x\to 0} \dfrac{\sin nx}{x} = n$.

74. Using a graphing utility, you see that

$\lim_{x\to 0} \dfrac{\tan x}{x} = 1$

$\lim_{x\to 0} \dfrac{\tan 2x}{x} = 2$, etc.

So, $\lim_{x\to 0} \dfrac{\tan(nx)}{x} = n$.

75. If $\lim_{x\to c} f(x) = L_1$ and $\lim_{x\to c} f(x) = L_2$, then for every $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$ such that $|x-c| < \delta_1 \Rightarrow |f(x) - L_1| < \varepsilon$ and $|x-c| < \delta_2 \Rightarrow |f(x) - L_2| < \varepsilon$. Let δ equal the smaller of δ_1 and δ_2. Then for $|x-c| < \delta$, you have $|L_1 - L_2| = |L_1 - f(x) + f(x) - L_2| \leq |L_1 - f(x)| + |f(x) - L_2| < \varepsilon + \varepsilon$. Therefore, $|L_1 - L_2| < 2\varepsilon$. Since $\varepsilon > 0$ is arbitrary, it follows that $L_1 = L_2$.

76. $f(x) = mx + b, m \neq 0$. Let $\varepsilon > 0$ be given. Take

$\delta = \dfrac{\varepsilon}{|m|}$.

If $0 < |x - c| < \delta = \dfrac{\varepsilon}{|m|}$, then

$$|m||x - c| < \varepsilon$$
$$|mx - mc| < \varepsilon$$
$$|(mx + b) - (mc + b)| < \varepsilon$$

which shows that $\lim_{x \to c} (mx + b) = mc + b$.

77. $\lim_{x \to c} [f(x) - L] = 0$ means that for every $\varepsilon > 0$ there exists $\delta > 0$ such that if

$0 < |x - c| < \delta$,

then

$|(f(x) - L) - 0| < \varepsilon$.

This means the same as $|f(x) - L| < \varepsilon$ when

$0 < |x - c| < \delta$.

So, $\lim_{x \to c} f(x) = L$.

78. (a) $$(3x + 1)(3x - 1)x^2 + 0.01 = (9x^2 - 1)x^2 + \frac{1}{100}$$
$$= 9x^4 - x^2 + \frac{1}{100}$$
$$= \frac{1}{100}(10x^2 - 1)(90x^2 - 1)$$

So, $(3x + 1)(3x - 1)x^2 + 0.01 > 0$ if

$10x^2 - 1 < 0$ and $90x^2 - 1 < 0$.

Let $(a, b) = \left(-\dfrac{1}{\sqrt{90}}, \dfrac{1}{\sqrt{90}}\right)$.

For all $x \neq 0$ in (a, b), the graph is positive.

You can verify this with a graphing utility.

(b) You are given $\lim_{x \to c} g(x) = L > 0$. Let

$\varepsilon = \dfrac{1}{2}L$. There exists $\delta > 0$ such that

$0 < |x - c| < \delta$ implies that

$|g(x) - L| < \varepsilon = \dfrac{L}{2}$. That is,

$$-\frac{L}{2} < g(x) - L < \frac{L}{2}$$
$$\frac{L}{2} < g(x) < \frac{3L}{2}$$

For x in the interval $(c - \delta, c + \delta)$, $x \neq c$, you

have $g(x) > \dfrac{L}{2} > 0$, as desired.

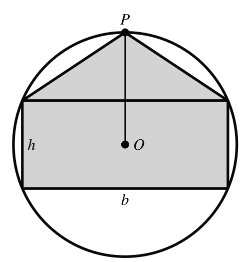
79. The radius OP has a length equal to the altitude z of the triangle plus $\dfrac{h}{2}$. So, $z = 1 - \dfrac{h}{2}$.

Area triangle $= \dfrac{1}{2}b\left(1 - \dfrac{h}{2}\right)$

Area rectangle $= bh$

Because these are equal, $\dfrac{1}{2}b\left(1 - \dfrac{h}{2}\right) = bh$

$$1 - \frac{h}{2} = 2h$$
$$\frac{5}{2}h = 1$$
$$h = \frac{2}{5}.$$



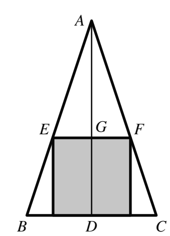
80. Consider a cross section of the cone, where EF is a diagonal of the inscribed cube. $AD = 3$, $BC = 2$.

Let x be the length of a side of the cube.

Then $EF = x\sqrt{2}$.

By similar triangles,

$$\frac{EF}{BC} = \frac{AG}{AD}$$
$$\frac{x\sqrt{2}}{2} = \frac{3 - x}{3}$$

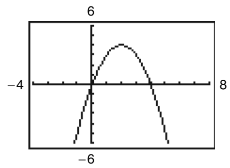


Solving for x,

$$3\sqrt{2}x = 6 - 2x$$
$$(3\sqrt{2} + 2)x = 6$$
$$x = \frac{6}{3\sqrt{2} + 2} = \frac{9\sqrt{2} - 6}{7} \approx 0.96.$$

Section 1.3 Evaluating Limits Analytically

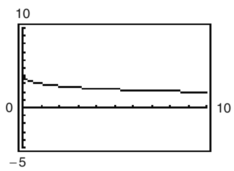
1.



(a) $\lim_{x \to 4} h(x) = 0$

(b) $\lim_{x \to -1} h(x) = -5$

2.

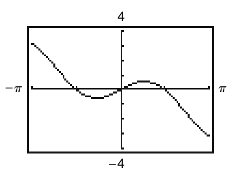


$g(x) = \dfrac{12(\sqrt{x} - 3)}{x - 9}$

(a) $\lim_{x \to 4} g(x) = 2.4$

(b) $\lim_{x \to 0} g(x) = 4$

3.



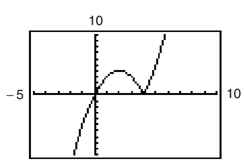
$f(x) = x \cos x$

(a) $\lim_{x \to 0} f(x) = 0$

(b) $\lim_{x \to \pi/3} f(x) \approx 0.524$

$\left(= \dfrac{\pi}{6}\right)$

4.



$f(t) = t|t - 4|$

(a) $\lim_{t \to 4} f(t) = 0$

(b) $\lim_{t \to -1} f(t) = -5$

5. $\lim_{x \to 2} x^3 = 2^3 = 8$

6. $\lim_{x \to -3} x^4 = (-3)^4 = 81$

7. $\lim_{x \to 0} (2x - 1) = 2(0) - 1 = -1$

8. $\lim_{x \to -4} (2x + 3) = 2(-4) + 3 = -8 + 3 = -5$

9. $\lim_{x \to -3} (x^2 + 3x) = (-3)^2 + 3(-3) = 9 - 9 = 0$

10. $\lim_{x \to 2} (-x^3 + 1) = (-2)^3 + 1 = -8 + 1 = -7$

11. $\lim_{x \to -3} (2x^2 + 4x + 1) = 2(-3)^2 + 4(-3) + 1$

$= 18 - 12 + 1 = 7$

12. $\lim_{x \to 1} (2x^3 - 6x + 5) = 2(1)^3 - 6(1) + 5$

$= 2 - 6 + 5 = 1$

13. $\lim_{x \to 3} \sqrt{x + 1} = \sqrt{3 + 1} = 2$

14. $\lim_{x \to 2} \sqrt[3]{12x + 3} = \sqrt[3]{12(2) + 3}$

$= \sqrt[3]{24 + 3} = \sqrt[3]{27} = 3$

15. $\lim_{x \to -4} (x + 3)^2 = (-4 + 3)^2 = 1$

16. $\lim_{x \to 0} (3x - 2)^4 = (3(0) - 2)^4 = (-2)^4 = 16$

17. $\lim_{x \to 2} \dfrac{1}{x} = \dfrac{1}{2}$

18. $\lim_{x \to -5} \dfrac{5}{x + 3} = \dfrac{5}{-5 + 3} = -\dfrac{5}{2}$

19. $\lim_{x \to 1} \dfrac{x}{x^2 + 4} = \dfrac{1}{1^2 + 4} = \dfrac{1}{5}$

20. $\lim_{x \to 1} \dfrac{3x + 5}{x + 1} = \dfrac{3(1) + 5}{1 + 1} = \dfrac{3 + 5}{2} = \dfrac{8}{2} = 4$

21. $\lim_{x \to 7} \dfrac{3x}{\sqrt{x + 2}} = \dfrac{3(7)}{\sqrt{7 + 2}} = \dfrac{21}{3} = 7$

22. $\lim_{x \to 3} \dfrac{\sqrt{x + 6}}{x + 2} = \dfrac{\sqrt{3 + 6}}{3 + 2} = \dfrac{\sqrt{9}}{5} = \dfrac{3}{5}$

23. (a) $\lim_{x \to 1} f(x) = 5 - 1 = 4$

(b) $\lim_{x \to 4} g(x) = 4^3 = 64$

(c) $\lim_{x \to 1} g(f(x)) = g(f(1)) = g(4) = 64$

24. (a) $\lim_{x \to -3} f(x) = (-3) + 7 = 4$

(b) $\lim_{x \to 4} g(x) = 4^2 = 16$

(c) $\lim_{x \to -3} g(f(x)) = g(4) = 16$

25. (a) $\lim_{x\to 1} f(x) = 4 - 1 = 3$

(b) $\lim_{x\to 3} g(x) = \sqrt{3 + 1} = 2$

(c) $\lim_{x\to 1} g(f(x)) = g(3) = 2$

26. (a) $\lim_{x\to 4} f(x) = 2(4^2) - 3(4) + 1 = 21$

(b) $\lim_{x\to 21} g(x) = \sqrt[3]{21 + 6} = 3$

(c) $\lim_{x\to 4} g(f(x)) = g(21) = 3$

27. $\lim_{x\to \pi/2} \sin x = \sin \frac{\pi}{2} = 1$

28. $\lim_{x\to \pi} \tan x = \tan \pi = 0$

29. $\lim_{x\to 1} \cos \frac{\pi x}{3} = \cos \frac{\pi}{3} = \frac{1}{2}$

30. $\lim_{x\to 2} \sin \frac{\pi x}{2} = \sin \frac{\pi(2)}{2} = 0$

31. $\lim_{x\to 0} \sec 2x = \sec 0 = 1$

32. $\lim_{x\to \pi} \cos 3x = \cos 3\pi = -1$

33. $\lim_{x\to 5\pi/6} \sin x = \sin \frac{5\pi}{6} = \frac{1}{2}$

34. $\lim_{x\to 5\pi/3} \cos x = \cos \frac{5\pi}{3} = \frac{1}{2}$

35. $\lim_{x\to 3} \tan\left(\frac{\pi x}{4}\right) = \tan \frac{3\pi}{4} = -1$

36. $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right) = \sec \frac{7\pi}{6} = \frac{-2\sqrt{3}}{3}$

37. (a) $\lim_{x\to c}\left[5g(x)\right] = 5 \lim_{x\to c} g(x) = 5(2) = 10$

(b) $\lim_{x\to c}\left[f(x) + g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = 3 + 2 = 5$

(c) $\lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right] = (3)(2) = 6$

(d) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{3}{2}$

38. (a) $\lim_{x\to c}\left[4f(x)\right] = 4 \lim_{x\to c} f(x) = 4(2) = 8$

(b) $\lim_{x\to c}\left[f(x) + g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = 2 + \frac{3}{4} = \frac{11}{4}$

(c) $\lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right] = 2\left(\frac{3}{4}\right) = \frac{3}{2}$

(d) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{2}{(3/4)} = \frac{8}{3}$

39. (a) $\lim_{x\to c}\left[f(x)\right]^3 = \left[\lim_{x\to c} f(x)\right]^3 = (4)^3 = 64$

(b) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{\lim_{x\to c} f(x)} = \sqrt{4} = 2$

(c) $\lim_{x\to c}\left[3f(x)\right] = 3 \lim_{x\to c} f(x) = 3(4) = 12$

(d) $\lim_{x\to c}\left[f(x)\right]^{3/2} = \left[\lim_{x\to c} f(x)\right]^{3/2} = (4)^{3/2} = 8$

40. (a) $\lim_{x\to c}\sqrt[3]{f(x)} = \sqrt[3]{\lim_{x\to c} f(x)} = \sqrt[3]{27} = 3$

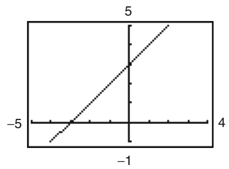
(b) $\lim_{x\to c}\frac{f(x)}{18} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} 18} = \frac{27}{18} = \frac{3}{2}$

(c) $\lim_{x\to c}\left[f(x)\right]^2 = \left[\lim_{x\to c} f(x)\right]^2 = (27)^2 = 729$

(d) $\lim_{x\to c}\left[f(x)\right]^{2/3} = \left[\lim_{x\to c} f(x)\right]^{2/3} = (27)^{2/3} = 9$

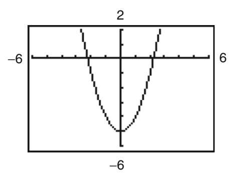
41. $f(x) = \dfrac{x^2 + 3x}{x} = \dfrac{x(x + 3)}{x}$ and $g(x) = x + 3$ agree except at $x = 0$.

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = \lim_{x\to 0} (x + 3) = 0 + 3 = 3$$



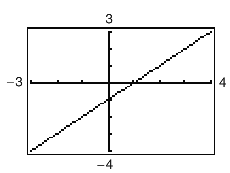
42. $f(x) = \dfrac{x^4 - 5x^2}{x^2} = \dfrac{x^2(x^2 - 5)}{x^2}$ and $g(x) = x^2 - 5$ agree except at $x = 0$.

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = \lim_{x\to 0} (x^2 - 5) = 0^2 - 5 = -5$$



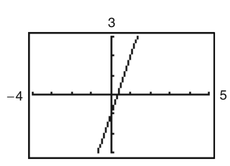
43. $f(x) = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x + 1)(x - 1)}{x + 1}$ and $g(x) = x - 1$ agree except at $x = -1$.

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = \lim_{x\to -1} (x - 1) = -1 - 1 = -2$$



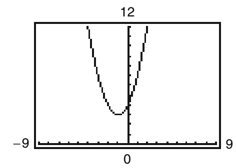
44. $f(x) = \dfrac{3x^2 + 5x - 2}{x + 2} = \dfrac{(x + 2)(3x - 1)}{x + 2}$ and $g(x) = 3x - 1$ agree except at $x = -2$.

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} g(x) = \lim_{x\to -2} (3x - 1) = 3(-2) - 1 = -7$$



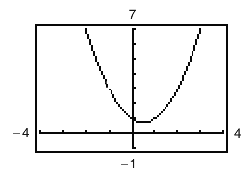
45. $f(x) = \dfrac{x^3 - 8}{x - 2}$ and $g(x) = x^2 + 2x + 4$ agree except at $x = 2$.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x) = \lim_{x\to 2} (x^2 + 2x + 4) = 2^2 + 2(2) + 4 = 12$$



46. $f(x) = \dfrac{x^3 + 1}{x + 1}$ and $g(x) = x^2 - x + 1$ agree except at $x = -1$.

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = \lim_{x\to -1} (x^2 - x + 1) = (-1)^2 - (-1) + 1 = 3$$



47. $\displaystyle\lim_{x\to 0} \frac{x}{x^2 - x} = \lim_{x\to 0} \frac{x}{x(x - 1)} = \lim_{x\to 0} \frac{1}{x - 1} = \frac{1}{0 - 1} = -1$

48. $\displaystyle\lim_{x\to 0} \frac{2x}{x^2 + 4x} = \lim_{x\to 0} \frac{2x}{x(x + 4)} = \lim_{x\to 0} \frac{2}{x + 4} = \frac{2}{0 + 4} = \frac{2}{4} = \frac{1}{2}$

49. $\displaystyle\lim_{x\to 4} \frac{x - 4}{x^2 - 16} = \lim_{x\to 4} \frac{x - 4}{(x + 4)(x - 4)} = \lim_{x\to 4} \frac{1}{x + 4} = \frac{1}{4 + 4} = \frac{1}{8}$

50. $\displaystyle\lim_{x\to 5} \frac{5 - x}{x^2 - 25} = \lim_{x\to 5} \frac{-(x - 5)}{(x - 5)(x + 5)} = \lim_{x\to 5} \frac{-1}{x + 5} = \frac{-1}{5 + 5} = -\frac{1}{10}$

51. $\displaystyle\lim_{x\to -3} \frac{x^2 + x - 6}{x^2 - 9} = \lim_{x\to -3} \frac{(x + 3)(x - 2)}{(x + 3)(x - 3)} = \lim_{x\to -3} \frac{x - 2}{x - 3} = \frac{-3 - 2}{-3 - 3} = \frac{-5}{-6} = \frac{5}{6}$

52. $\lim_{x\to 2}\dfrac{x^2+2x-8}{x^2-x-2} = \lim_{x\to 2}\dfrac{(x-2)(x+4)}{(x-2)(x+1)}$

$= \lim_{x\to 2}\dfrac{x+4}{x+1} = \dfrac{2+4}{2+1} = \dfrac{6}{3} = 2$

53. $\lim_{x\to 4}\dfrac{\sqrt{x+5}-3}{x-4} = \lim_{x\to 4}\dfrac{\sqrt{x+5}-3}{x-4}\cdot\dfrac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$= \lim_{x\to 4}\dfrac{(x+5)-9}{(x-4)\left(\sqrt{x+5}+3\right)} = \lim_{x\to 4}\dfrac{1}{\sqrt{x+5}+3} = \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}$

54. $\lim_{x\to 3}\dfrac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3}\dfrac{\sqrt{x+1}-2}{x-3}\cdot\dfrac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3}\dfrac{x-3}{(x-3)\left[\sqrt{x+1}+2\right]}$

$= \lim_{x\to 3}\dfrac{1}{\sqrt{x+1}+2} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$

55. $\lim_{x\to 0}\dfrac{\sqrt{x+5}-\sqrt{5}}{x} = \lim_{x\to 0}\dfrac{\sqrt{x+5}-\sqrt{5}}{x}\cdot\dfrac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$

$= \lim_{x\to 0}\dfrac{(x+5)-5}{x\left(\sqrt{x+5}+\sqrt{5}\right)} = \lim_{x\to 0}\dfrac{1}{\sqrt{x+5}+\sqrt{5}} = \dfrac{1}{\sqrt{5}+\sqrt{5}} = \dfrac{1}{2\sqrt{5}} = \dfrac{\sqrt{5}}{10}$

56. $\lim_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x} = \lim_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}\cdot\dfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$= \lim_{x\to 0}\dfrac{2+x-2}{\left(\sqrt{2+x}+\sqrt{2}\right)x} = \lim_{x\to 0}\dfrac{1}{\sqrt{2+x}+\sqrt{2}} = \dfrac{1}{\sqrt{2}+\sqrt{2}} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

57. $\lim_{x\to 0}\dfrac{\dfrac{1}{3+x}-\dfrac{1}{3}}{x} = \lim_{x\to 0}\dfrac{3-(3+x)}{(3+x)3(x)} = \lim_{x\to 0}\dfrac{-x}{(3+x)(3)(x)} = \lim_{x\to 0}\dfrac{-1}{(3+x)3} = \dfrac{-1}{(3)3} = -\dfrac{1}{9}$

58. $\lim_{x\to 0}\dfrac{\dfrac{1}{x+4}-\dfrac{1}{4}}{x} = \lim_{x\to 0}\dfrac{\dfrac{4-(x+4)}{4(x+4)}}{x}$

$= \lim_{x\to 0}\dfrac{-1}{4(x+4)} = \dfrac{-1}{4(4)} = -\dfrac{1}{16}$

59. $\lim_{\Delta x\to 0}\dfrac{2(x+\Delta x)-2x}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{2x+2\Delta x-2x}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{2\Delta x}{\Delta x} = \lim_{\Delta x\to 0} 2 = 2$

60. $\lim_{\Delta x\to 0}\dfrac{(x+\Delta x)^2-x^2}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{x^2+2x\Delta x+(\Delta x)^2-x^2}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{\Delta x(2x+\Delta x)}{\Delta x} = \lim_{\Delta x\to 0}(2x+\Delta x) = 2x$

61. $\lim_{\Delta x\to 0}\dfrac{(x+\Delta x)^2-2(x+\Delta x)+1-\left(x^2-2x+1\right)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{x^2+2x\Delta x+(\Delta x)^2-2x-2\Delta x+1-x^2+2x-1}{\Delta x}$

$= \lim_{\Delta x\to 0}(2x+\Delta x-2) = 2x-2$

62. $\lim_{\Delta x\to 0}\dfrac{(x+\Delta x)^3-x^3}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3-x^3}{\Delta x}$

$= \lim_{\Delta x\to 0}\dfrac{\Delta x\left(3x^2+3x\Delta x+(\Delta x)^2\right)}{\Delta x} = \lim_{\Delta x\to 0}\left(3x^2+3x\Delta x+(\Delta x)^2\right) = 3x^2$

63. $\lim\limits_{x\to 0}\dfrac{\sin x}{5x} = \lim\limits_{x\to 0}\left[\left(\dfrac{\sin x}{x}\right)\left(\dfrac{1}{5}\right)\right] = (1)\left(\dfrac{1}{5}\right) = \dfrac{1}{5}$

64. $\lim\limits_{x\to 0}\dfrac{3(1-\cos x)}{x} = \lim\limits_{x\to 0}\left[3\left(\dfrac{(1-\cos x)}{x}\right)\right] = (3)(0) = 0$

65. $\lim\limits_{x\to 0}\dfrac{\sin x(1-\cos x)}{x^2} = \lim\limits_{x\to 0}\left[\dfrac{\sin x}{x}\cdot\dfrac{1-\cos x}{x}\right]$

$= (1)(0) = 0$

66. $\lim\limits_{\theta\to 0}\dfrac{\cos\theta\tan\theta}{\theta} = \lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta} = 1$

67. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x} = \lim\limits_{x\to 0}\left[\dfrac{\sin x}{x}\sin x\right] = (1)\sin 0 = 0$

68. $\lim\limits_{x\to 0}\dfrac{\tan^2 x}{x} = \lim\limits_{x\to 0}\dfrac{\sin^2 x}{x\cos^2 x} = \lim\limits_{x\to 0}\left[\dfrac{\sin x}{x}\cdot\dfrac{\sin x}{\cos^2 x}\right]$

$= (1)(0) = 0$

69. $\lim\limits_{h\to 0}\dfrac{(1-\cos h)^2}{h} = \lim\limits_{h\to 0}\left[\dfrac{1-\cos h}{h}(1-\cos h)\right]$

$= (0)(0) = 0$

70. $\lim\limits_{\phi\to\pi}\phi\sec\phi = \pi(-1) = -\pi$

71. $\lim\limits_{x\to\pi/2}\dfrac{\cos x}{\cot x} = \lim\limits_{x\to\pi/2}\sin x = 1$

72. $\lim\limits_{x\to\pi/4}\dfrac{1-\tan x}{\sin x-\cos x} = \lim\limits_{x\to\pi/4}\dfrac{\cos x-\sin x}{\sin x\cos x-\cos^2 x}$

$= \lim\limits_{x\to\pi/4}\dfrac{-(\sin x-\cos x)}{\cos x(\sin x-\cos x)}$

$= \lim\limits_{x\to\pi/4}\dfrac{-1}{\cos x}$

$= \lim\limits_{x\to\pi/4}(-\sec x)$

$= -\sqrt{2}$

73. $\lim\limits_{t\to 0}\dfrac{\sin 3t}{2t} = \lim\limits_{t\to 0}\left(\dfrac{\sin 3t}{3t}\right)\left(\dfrac{3}{2}\right) = (1)\left(\dfrac{3}{2}\right) = \dfrac{3}{2}$

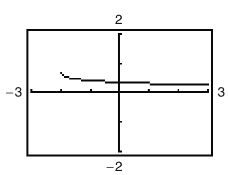
74. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 3x} = \lim\limits_{x\to 0}\left[2\left(\dfrac{\sin 2x}{2x}\right)\left(\dfrac{1}{3}\right)\left(\dfrac{3x}{\sin 3x}\right)\right]$

$= 2(1)\left(\dfrac{1}{3}\right)(1) = \dfrac{2}{3}$

75. $f(x) = \dfrac{\sqrt{x+2}-\sqrt{2}}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.358	0.354	0.354	?	0.354	0.353	0.349

It appears that the limit is 0.354.



The graph has a hole at $x = 0$.

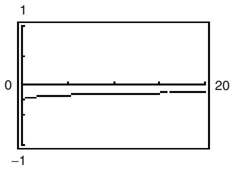
Analytically, $\lim\limits_{x\to 0}\dfrac{\sqrt{x+2}-\sqrt{2}}{x} = \lim\limits_{x\to 0}\dfrac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$= \lim\limits_{x\to 0}\dfrac{x+2-2}{x\left(\sqrt{x+2}+\sqrt{2}\right)} = \lim\limits_{x\to 0}\dfrac{1}{\sqrt{x+2}+\sqrt{2}} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4} \approx 0.354.$

76. $f(x) = \dfrac{4 - \sqrt{x}}{x - 16}$

x	15.9	15.99	15.999	16	16.001	16.01	16.1
$f(x)$	–0.1252	–0.125	–0.125	?	–0.125	–0.125	–0.1248

It appears that the limit is –0.125.



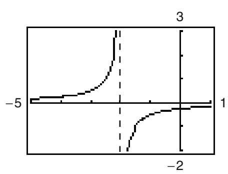
The graph has a hole at $x = 16$.

Analytically, $\displaystyle\lim_{x \to 16} \frac{4 - \sqrt{x}}{x - 16} = \lim_{x \to 16} \frac{\left(4 - \sqrt{x}\right)}{\left(\sqrt{x} + 4\right)\left(\sqrt{x} - 4\right)} = \lim_{x \to 16} \frac{-1}{\sqrt{x} + 4} = -\frac{1}{8}.$

77. $f(x) = \dfrac{\dfrac{1}{2 + x} - \dfrac{1}{2}}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	–0.263	–0.251	–0.250	?	–0.250	–0.249	–0.238

It appears that the limit is –0.250.



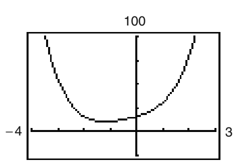
The graph has a hole at $x = 0$.

Analytically, $\displaystyle\lim_{x \to 0} \frac{\dfrac{1}{2 + x} - \dfrac{1}{2}}{x} = \lim_{x \to 0} \frac{2 - (2 + x)}{2(2 + x)} \cdot \frac{1}{x} = \lim_{x \to 0} \frac{-x}{2(2 + x)} \cdot \frac{1}{x} = \lim_{x \to 0} \frac{-1}{2(2 + x)} = -\frac{1}{4}.$

78. $f(x) = \dfrac{x^5 - 32}{x - 2}$

x	1.9	1.99	1.999	1.9999	2.0	2.0001	2.001	2.01	2.1
$f(x)$	72.39	79.20	79.92	79.99	?	80.01	80.08	80.80	88.41

It appears that the limit is 80.



The graph has a hole at $x = 2$.

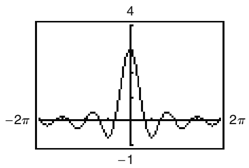
Analytically, $\displaystyle\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = \lim_{x \to 2} \frac{(x - 2)\left(x^4 + 2x^3 + 4x^2 + 8x + 16\right)}{x - 2} = \lim_{x \to 2}\left(x^4 + 2x^3 + 4x^2 + 8x + 16\right) = 80.$

(*Hint:* Use long division to factor $x^5 - 32$.)

79. $f(t) = \dfrac{\sin 3t}{t}$

t	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(t)$	2.96	2.9996	3	?	3	2.9996	2.96

It appears that the limit is 3.



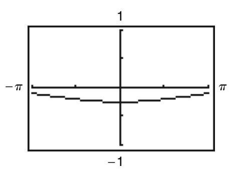
The graph has a hole at $t = 0$.

Analytically, $\lim_{t\to 0}\dfrac{\sin 3t}{t} = \lim_{t\to 0} 3\left(\dfrac{\sin 3t}{3t}\right) = 3(1) = 3.$

80. $f(x) = \dfrac{\cos x - 1}{2x^2}$

x	–1	–0.1	–0.01	0.01	0.1	1
$f(x)$	–0.2298	–0.2498	–0.25	–0.25	–0.2498	–0.2298

It appears that the limit is –0.25.



The graph has a hole at $x = 0$.

Analytically, $\dfrac{\cos x - 1}{2x^2} \cdot \dfrac{\cos x + 1}{\cos x + 1} = \dfrac{\cos^2 x - 1}{2x^2(\cos x + 1)}$

$$= \frac{-\sin^2 x}{2x^2(\cos x + 1)}$$

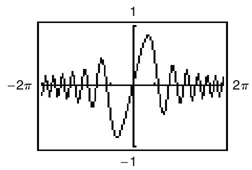
$$= \frac{\sin^2 x}{x^2} \cdot \frac{-1}{2(\cos x + 1)}$$

$$\lim_{x\to 0}\left[\frac{\sin^2 x}{x^2} \cdot \frac{-1}{2(\cos x + 1)}\right] = 1\left(\frac{-1}{4}\right) = -\frac{1}{4} = -0.25$$

81. $f(x) = \dfrac{\sin x^2}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	–0.099998	–0.01	–0.001	?	0.001	0.01	0.099998

It appears that the limit is 0.



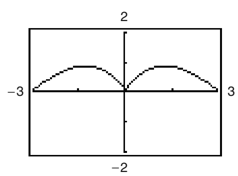
The graph has a hole at $x = 0$.

Analytically, $\lim_{x\to 0}\dfrac{\sin x^2}{x} = \lim_{x\to 0} x\left(\dfrac{\sin x^2}{x}\right) = 0(1) = 0.$

82. $f(x) = \dfrac{\sin x}{\sqrt[3]{x}}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.215	0.0464	0.01	?	0.01	0.0464	0.215

It appears that the limit is 0.



The graph has a hole at $x = 0$.

Analytically, $\lim_{x\to 0}\dfrac{\sin x}{\sqrt[3]{x}} = \lim_{x\to 0}\sqrt[3]{x^2}\left(\dfrac{\sin x}{x}\right) = (0)(1) = 0.$

83. $\lim_{\Delta x\to 0}\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{3(x+\Delta x) - 2 - (3x - 2)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{3x + 3\Delta x - 2 - 3x + 2}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{3\Delta x}{\Delta x} = 3$

84. $\lim_{\Delta x\to 0}\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{[-6(x+\Delta x) + 3] - [-6x + 3]}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{-6x - 6\Delta x + 3 + 6x - 3}{\Delta x}$

$= \lim_{\Delta x\to 0}\dfrac{-6\Delta x}{\Delta x} = \lim_{\Delta x\to 0}(-6) = -6$

85. $\lim_{\Delta x\to 0}\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{(x+\Delta x)^2 - 4(x+\Delta x) - (x^2 - 4x)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{x^2 + 2x\Delta x + \Delta x^2 - 4x - 4\Delta x - x^2 + 4x}{\Delta x}$

$= \lim_{\Delta x\to 0}\dfrac{\Delta x(2x + \Delta x - 4)}{\Delta x} = \lim_{\Delta x\to 0}(2x + \Delta x - 4) = 2x - 4$

86. $\lim_{\Delta x\to 0}\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}\cdot\dfrac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$

$= \lim_{\Delta x\to 0}\dfrac{x + \Delta x - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \lim_{\Delta x\to 0}\dfrac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \dfrac{1}{2\sqrt{x}}$

87. $$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{x + \Delta x + 3} - \frac{1}{x + 3}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x + 3 - (x + \Delta x + 3)}{(x + \Delta x + 3)(x + 3)} \cdot \frac{1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{(x + \Delta x + 3)(x + 3)\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x + 3)(x + 3)} = \frac{-1}{(x + 3)^2}$$

88. $$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{(x + \Delta x)^2} - \frac{1}{x^2}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 - (x + \Delta x)^2}{x^2(x + \Delta x)^2 \Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 - \left[x^2 + 2x\Delta x + (\Delta x)^2\right]}{x^2(x + \Delta x)^2 \Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-2x\Delta x - (\Delta x)^2}{x^2(x + \Delta x)^2 \Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{x^2(x + \Delta x)^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}$$

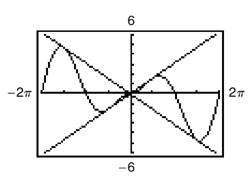
89. $$\lim_{x \to 0}(4 - x^2) \le \lim_{x \to 0} f(x) \le \lim_{x \to 0}(4 + x^2)$$
$$4 \le \lim_{x \to 0} f(x) \le 4$$

Therefore, $\lim_{x \to 0} f(x) = 4$.

90. $$\lim_{x \to a}\left[b - |x - a|\right] \le \lim_{x \to a} f(x) \le \lim_{x \to a}\left[b + |x - a|\right]$$
$$b \le \lim_{x \to a} f(x) \le b$$

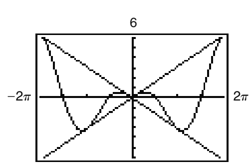
Therefore, $\lim_{x \to a} f(x) = b$.

91. $f(x) = |x|\sin x$



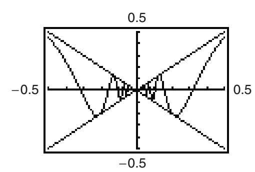
$\lim_{x \to 0} |x|\sin x = 0$

92. $f(x) = |x|\cos x$



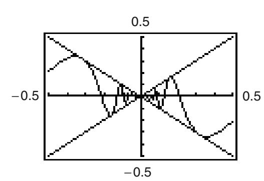
$\lim_{x \to 0} |x|\cos x = 0$

93. $f(x) = x \sin \frac{1}{x}$



$\lim_{x \to 0}\left(x \sin \frac{1}{x}\right) = 0$

94. $h(x) = x \cos \frac{1}{x}$



$\lim_{x \to 0}\left(x \cos \frac{1}{x}\right) = 0$

95. (a) Two functions *f* and *g* agree at all but one point (on an open interval) if $f(x) = g(x)$ for all *x* in the interval except for $x = c$, where *c* is in the interval.

(b) $f(x) = \frac{x^2 - 1}{x - 1} = \frac{(x + 1)(x - 1)}{x - 1}$ and $g(x) = x + 1$ agree at all points except $x = 1$.

(Other answers possible.)

96. An indeterminant form is obtained when evaluating a limit using direct substitution produces a meaningless fractional expression such as $0/0$. That is,

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

for which $\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0$

97. If a function f is squeezed between two functions h and g, $h(x) \le f(x) \le g(x)$, and h and g have the same limit L as $x \to c$, then $\lim_{x \to c} f(x)$ exists and equals L

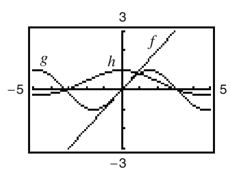
98. (a) Use the dividing out technique because the numerator and denominator have a common factor.

$$\lim_{x \to -2} \frac{x^2 + x - 2}{x + 2} = \lim_{x \to -2} \frac{(x + 2)(x - 1)}{x + 2}$$
$$= \lim_{x \to -2} (x - 1) = -2 - 1 = -3$$

(b) Use the rationalizing technique because the numerator involves a radical expression.

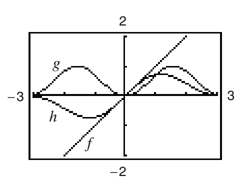
$$\lim_{x \to 0} \frac{\sqrt{x + 4} - 2}{x} = \lim_{x \to 0} \frac{\sqrt{x + 4} - 2}{x} \cdot \frac{\sqrt{x + 4} + 2}{\sqrt{x + 4} + 2}$$
$$= \lim_{x \to 0} \frac{(x + 4) - 4}{x\left(\sqrt{x + 4} + 2\right)}$$
$$= \lim_{x \to 0} \frac{1}{\sqrt{x + 4} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$$

99. $f(x) = x,\ g(x) = \sin x,\ h(x) = \dfrac{\sin x}{x}$



When the x-values are "close to" 0 the magnitude of f is approximately equal to the magnitude of g. So, $|g|/|f| \approx 1$ when x is "close to" 0.

100. $f(x) = x,\ g(x) = \sin^2 x,\ h(x) = \dfrac{\sin^2 x}{x}$



When the x-values are "close to" 0 the magnitude of g is "smaller" than the magnitude of f and the magnitude of g is approaching zero "faster" than the magnitude of f. So, $|g|/|f| \approx 0$ when x is "close to" 0.

101. $s(t) = -16t^2 + 500$

$$\lim_{t \to 2} \frac{s(2) - s(t)}{2 - t} = \lim_{t \to 2} \frac{-16(2)^2 + 500 - \left(-16t^2 + 500\right)}{2 - t}$$
$$= \lim_{t \to 2} \frac{436 + 16t^2 - 500}{2 - t}$$
$$= \lim_{t \to 2} \frac{16\left(t^2 - 4\right)}{2 - t}$$
$$= \lim_{t \to 2} \frac{16(t - 2)(t + 2)}{2 - t}$$
$$= \lim_{t \to 2} -16(t + 2) = -64 \text{ ft/sec}$$

The paint can is falling at about 64 feet/second.

102. $s(t) = -16t^2 + 500 = 0$ when $t = \sqrt{\frac{500}{16}} = \frac{5\sqrt{5}}{2}$ sec. The velocity at time $a = \frac{5\sqrt{5}}{2}$ is

$$\lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)} \frac{s\left(\frac{5\sqrt{5}}{2}\right) - s(t)}{\frac{5\sqrt{5}}{2} - t} = \lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)} \frac{0 - \left(-16t^2 + 500\right)}{\frac{5\sqrt{5}}{2} - t}$$

$$= \lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)} \frac{16\left(t^2 - \frac{125}{4}\right)}{\frac{5\sqrt{5}}{2} - t}$$

$$= \lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)} \frac{16\left(t + \frac{5\sqrt{5}}{2}\right)\left(t - \frac{5\sqrt{5}}{2}\right)}{\frac{5\sqrt{5}}{2} - t}$$

$$= \lim_{t\to\frac{5\sqrt{5}}{2}} \left[-16\left(t + \frac{5\sqrt{5}}{2}\right)\right] = -80\sqrt{5} \text{ ft/sec}$$

$$\approx -178.9 \text{ ft/sec.}$$

The velocity of the paint can when it hits the ground is about 178.9 ft/sec.

103. $s(t) = -4.9t^2 + 200$

$$\lim_{t\to 3} \frac{s(3) - s(t)}{3 - t} = \lim_{t\to 3} \frac{-4.9(3)^2 + 200 - \left(-4.9t^2 + 200\right)}{3 - t}$$

$$= \lim_{t\to 3} \frac{4.9\left(t^2 - 9\right)}{3 - t}$$

$$= \lim_{t\to 3} \frac{4.9(t - 3)(t + 3)}{3 - t}$$

$$= \lim_{t\to 3} \left[-4.9(t + 3)\right]$$

$$= -29.4 \text{ m/sec}$$

The object is falling about 29.4 m/sec.

104. $-4.9t^2 + 200 = 0$ when $t = \sqrt{\frac{200}{4.9}} = \frac{20\sqrt{5}}{7}$ sec. The velocity at time $a = \frac{20\sqrt{5}}{7}$ is

$$\lim_{t\to a} \frac{s(a) - s(t)}{a - t} = \lim_{t\to a} \frac{0 - \left[-4.9t^2 + 200\right]}{a - t}$$

$$= \lim_{t\to a} \frac{4.9(t + a)(t - a)}{a - t}$$

$$= \lim_{t\to\frac{20\sqrt{5}}{7}} \left[-4.9\left(t + \frac{20\sqrt{5}}{7}\right)\right] = -28\sqrt{5} \text{ m/sec}$$

$$\approx -62.6 \text{ m/sec.}$$

The velocity of the object when it hits the ground is about 62.6 m/sec.

105. Let $f(x) = 1/x$ and $g(x) = -1/x$. $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist. However,

$$\lim_{x\to 0}\left[f(x) + g(x)\right] = \lim_{x\to 0}\left[\frac{1}{x} + \left(-\frac{1}{x}\right)\right] = \lim_{x\to 0}[0] = 0$$

and therefore does not exist.

106. Suppose, on the contrary, that $\lim_{x\to c} g(x)$ exists. Then, because $\lim_{x\to c} f(x)$ exists, so would $\lim_{x\to c}[f(x) + g(x)]$, which is a contradiction. So, $\lim_{x\to c} g(x)$ does not exist.

107. Given $f(x) = b$, show that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - b| < \varepsilon$ whenever $|x - c| < \delta$. Because $|f(x) - b| = |b - b| = 0 < \varepsilon$ for every $\varepsilon > 0$, any value of $\delta > 0$ will work.

108. Given $f(x) = x^n$, n is a positive integer, then

$$\begin{aligned}\lim_{x\to c} x^n &= \lim_{x\to c}(xx^{n-1})\\ &= \left[\lim_{x\to c} x\right]\left[\lim_{x\to c} x^{n-1}\right] = c\left[\lim_{x\to c}(xx^{n-2})\right]\\ &= c\left[\lim_{x\to c} x\right]\left[\lim_{x\to c} x^{n-2}\right] = c(c)\lim_{x\to c}(xx^{n-3})\\ &= \cdots = c^n.\end{aligned}$$

109. If $b = 0$, the property is true because both sides are equal to 0. If $b \neq 0$, let $\varepsilon > 0$ be given. Because $\lim_{x\to c} f(x) = L$, there exists $\delta > 0$ such that $|f(x) - L| < \varepsilon/|b|$ whenever $0 < |x - c| < \delta$. So, whenever $0 < |x - c| < \delta$, we have

$$|b||f(x) - L| < \varepsilon \quad \text{or} \quad |bf(x) - bL| < \varepsilon$$

which implies that $\lim_{x\to c}[bf(x)] = bL$.

110. Given $\lim_{x\to c} f(x) = 0$:

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $|f(x) - 0| < \varepsilon$ whenever $0 < |x - c| < \delta$.

Now $|f(x) - 0| = |f(x)| = \big||f(x)| - 0\big| < \varepsilon$ for $|x - c| < \delta$. Therefore, $\lim_{x\to c}|f(x)| = 0$.

111.

$$\begin{aligned}-M|f(x)| \le f(x)&g(x) \le M|f(x)|\\ \lim_{x\to c}(-M|f(x)|) \le \lim_{x\to c} f(x)&g(x) \le \lim_{x\to c}(M|f(x)|)\\ -M(0) \le \lim_{x\to c} f(x)&g(x) \le M(0)\\ 0 \le \lim_{x\to c} f(x)&g(x) \le 0\end{aligned}$$

Therefore, $\lim_{x\to c} f(x)g(x) = 0$.

112. (a) If $\lim_{x\to c}|f(x)| = 0$, then $\lim_{x\to c}[-|f(x)|] = 0$.

$$\begin{aligned}-|f(x)| \le f(x) &\le |f(x)|\\ \lim_{x\to c}[-|f(x)|] \le \lim_{x\to c} f(x) &\le \lim_{x\to c}|f(x)|\\ 0 \le \lim_{x\to c} f(x) &\le 0\end{aligned}$$

Therefore, $\lim_{x\to c} f(x) = 0$.

(b) Given $\lim_{x\to c} f(x) = L$:

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$. Since $\big||f(x)| - |L|\big| \le |f(x) - L| < \varepsilon$ for $|x - c| < \delta$, then $\lim_{x\to c}|f(x)| = |L|$.

113. Let

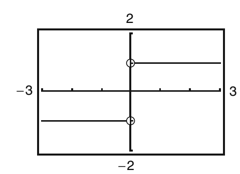
$$f(x) = \begin{cases} 4, & \text{if } x \ge 0\\ -4, & \text{if } x < 0\end{cases}$$

$\lim_{x\to 0}|f(x)| = \lim_{x\to 0} 4 = 4.$

$\lim_{x\to 0} f(x)$ does not exist because for $x < 0$, $f(x) = -4$ and for $x \ge 0$, $f(x) = 4$.

114. The graphing utility was set in degree mode, instead of *radian* mode.

115. The limit does not exist because the function approaches 1 from the right side of 0 and approaches -1 from the left side of 0.



116. False. $\lim_{x\to\pi}\dfrac{\sin x}{x} = \dfrac{0}{\pi} = 0$

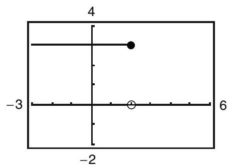
117. True.

118. False. Let

$$f(x) = \begin{cases} x & x \neq 1\\ 3 & x = 1\end{cases}, \quad c = 1.$$

Then $\lim_{x\to 1} f(x) = 1$ but $f(1) \neq 1$.

119. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2.



120. False. Let $f(x) = \frac{1}{2}x^2$ and $g(x) = x^2$.

Then $f(x) < g(x)$ for all $x \neq 0$. But

$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0.$

121.
$$\lim_{x\to 0}\frac{1-\cos x}{x} = \lim_{x\to 0}\frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x}$$
$$= \lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}$$
$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{1+\cos x}$$
$$= \left[\lim_{x\to 0}\frac{\sin x}{x}\right]\left[\lim_{x\to 0}\frac{\sin x}{1+\cos x}\right]$$
$$= (1)(0) = 0$$

122. $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$

$g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$

$\lim_{x\to 0} f(x)$ does not exist.

No matter how "close to" 0 x is, there are still an infinite number of rational and irrational numbers so that $\lim_{x\to 0} f(x)$ does not exist.

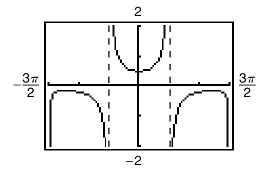
$\lim_{x\to 0} g(x) = 0$

when x is "close to" 0, both parts of the function are "close to" 0.

123. $f(x) = \dfrac{\sec x - 1}{x^2}$

(a) The domain of f is all $x \neq 0, \pi/2 + n\pi$.

(b)



The domain is not obvious. The hole at $x = 0$ is not apparent.

(c) $\lim_{x\to 0} f(x) = \dfrac{1}{2}$

(d)
$$\frac{\sec x - 1}{x^2} = \frac{\sec x - 1}{x^2}\cdot\frac{\sec x + 1}{\sec x + 1} = \frac{\sec^2 x - 1}{x^2(\sec x + 1)}$$
$$= \frac{\tan^2 x}{x^2(\sec x + 1)} = \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{x^2}\right)\frac{1}{\sec x + 1}$$

So, $\lim_{x\to 0}\dfrac{\sec x - 1}{x^2} = \lim_{x\to 0}\dfrac{1}{\cos^2 x}\left(\dfrac{\sin^2 x}{x^2}\right)\dfrac{1}{\sec x + 1}$
$$= 1(1)\left(\frac{1}{2}\right) = \frac{1}{2}.$$

124. (a)
$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x}$$
$$= \lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$$
$$= \lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\frac{1}{1+\cos x}$$
$$= (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

(b) From part (a),
$$\frac{1-\cos x}{x^2} \approx \frac{1}{2} \Rightarrow 1 - \cos x$$
$$\approx \frac{1}{2}x^2 \Rightarrow \cos x$$
$$\approx 1 - \frac{1}{2}x^2 \text{ for } x$$
$$\approx 0.$$

(c) $\cos(0.1) \approx 1 - \frac{1}{2}(0.1)^2 = 0.995$

(d) $\cos(0.1) \approx 0.9950$, which agrees with part (c).

Section 1.4 Continuity and One-Sided Limits

1. (a) $\lim_{x\to 4^+} f(x) = 3$

(b) $\lim_{x\to 4^-} f(x) = 3$

(c) $\lim_{x\to 4} f(x) = 3$

The function is continuous at $x = 4$ and is continuous on $(-\infty, \infty)$.

2. (a) $\lim_{x\to -2^+} f(x) = -2$

(b) $\lim_{x\to -2^-} f(x) = -2$

(c) $\lim_{x\to -2} f(x) = -2$

The function is continuous at $x = -2$.

3. (a) $\lim_{x\to 3^+} f(x) = 0$

(b) $\lim_{x\to 3^-} f(x) = 0$

(c) $\lim_{x\to 3} f(x) = 0$

The function is NOT continuous at $x = 3$.

4. (a) $\lim_{x\to -3^+} f(x) = 3$

(b) $\lim_{x\to -3^-} f(x) = 3$

(c) $\lim_{x\to -3} f(x) = 3$

The function is NOT continuous at $x = -3$ because $f(-3) = 4 \neq \lim_{x\to -3} f(x)$.

5. (a) $\lim_{x\to 2^+} f(x) = -3$

(b) $\lim_{x\to 2^-} f(x) = 3$

(c) $\lim_{x\to 2} f(x)$ does not exist

The function is NOT continuous at $x = 2$.

6. (a) $\lim_{x\to -1^+} f(x) = 0$

(b) $\lim_{x\to -1^-} f(x) = 2$

(c) $\lim_{x\to -1} f(x)$ does not exist.

The function is NOT continuous at $x = -1$.

7. $\lim_{x\to 8^+} \dfrac{1}{x+8} = \dfrac{1}{8+8} = \dfrac{1}{16}$

8. $\lim_{x\to 2^-} \dfrac{2}{x+2} = \dfrac{2}{2+2} = \dfrac{1}{2}$

9. $$\lim_{x\to 5^+} \frac{x-5}{x^2-25} = \lim_{x\to 5^+} \frac{x-5}{(x+5)(x-5)} = \lim_{x\to 5^+} \frac{1}{x+5} = \frac{1}{10}$$

10. $$\lim_{x\to 4^+} \frac{4-x}{x^2-16} = \lim_{x\to 4^+} \frac{-(x-4)}{(x+4)(x-4)} = \lim_{x\to 4^+} \frac{-1}{x+4} = \frac{-1}{4+4} = -\frac{1}{8}$$

11. $\lim_{x\to -3^-} \dfrac{x}{\sqrt{x^2-9}}$ does not exist because $\dfrac{x}{\sqrt{x^2-9}}$ decreases without bound as $x \to -3^-$.

12. $$\lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4^-} \frac{x-4}{(x-4)(\sqrt{x}+2)} = \lim_{x\to 4^-} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

13. $\lim_{x\to 0^-} \dfrac{|x|}{x} = \lim_{x\to 0^-} \dfrac{-x}{x} = -1$

14. $\lim_{x\to 10^+} \dfrac{|x-10|}{x-10} = \lim_{x\to 10^+} \dfrac{x-10}{x-10} = 1$

15. $$\lim_{\Delta x\to 0^-} \frac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x} = \lim_{\Delta x\to 0^-} \frac{x-(x+\Delta x)}{x(x+\Delta x)} \cdot \frac{1}{\Delta x} = \lim_{\Delta x\to 0^-} \frac{-\Delta x}{x(x+\Delta x)} \cdot \frac{1}{\Delta x} = \lim_{\Delta x\to 0^-} \frac{-1}{x(x+\Delta x)} = \frac{-1}{x(x+0)} = -\frac{1}{x^2}$$

16. $\lim_{\Delta x\to 0^+} \frac{(x+\Delta x)^2 + (x+\Delta x) - (x^2+x)}{\Delta x} = \lim_{\Delta x\to 0^+} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - x^2 - x}{\Delta x}$

$= \lim_{\Delta x\to 0^+} \frac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x}$

$= \lim_{\Delta x\to 0^+} (2x + \Delta x + 1)$

$= 2x + 0 + 1 = 2x + 1$

17. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x+2}{2} = \frac{5}{2}$

18. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x^2 - 4x + 6) = 9 - 12 + 6 = 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-x^2 + 4x - 2) = -9 + 12 - 2 = 1$

Since these one-sided limits disagree, $\lim_{x\to 3} f(x)$ does not exist.

19. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x+1) = 2$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^3+1) = 2$

$\lim_{x\to 1} f(x) = 2$

20. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (1-x) = 0$

21. $\lim_{x\to\pi} \cot x$ does not exist because

$\lim_{x\to\pi^+} \cot x$ and $\lim_{x\to\pi^-} \cot x$ do not exist.

22. $\lim_{x\to\pi/2} \sec x$ does not exist because

$\lim_{x\to(\pi/2)^+} \sec x$ and $\lim_{x\to(\pi/2)^-} \sec x$ do not exist.

23. $\lim_{x\to 4^-} (5[\![x]\!] - 7) = 5(3) - 7 = 8$

$([\![x]\!] = 3 \text{ for } 3 \le x < 4)$

24. $\lim_{x\to 2^+} (2x - [\![x]\!]) = 2(2) - 2 = 2$

25. $\lim_{x\to 3} (2 - [\![-x]\!])$ does not exist because

$\lim_{x\to 3^-} (2 - [\![-x]\!]) = 2 - (-3) = 5$

and

$\lim_{x\to 3^+} (2 - [\![-x]\!]) = 2 - (-4) = 6.$

26. $\lim_{x\to 1} \left(1 - \left[\!\!\left[-\frac{x}{2}\right]\!\!\right]\right) = 1 - (-1) = 2$

27. $f(x) = \frac{1}{x^2 - 4}$

has discontinuities at $x = -2$ and $x = 2$ because $f(-2)$ and $f(2)$ are not defined.

28. $f(x) = \frac{x^2 - 1}{x + 1}$

has a discontinuity at $x = -1$ because $f(-1)$ is not defined.

29. $f(x) = \frac{[\![x]\!]}{2} + x$

has discontinuities at each integer k because $\lim_{x\to k^-} f(x) \ne \lim_{x\to k^+} f(x)$.

30. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$ has a discontinuity at

$x = 1$ because $f(1) = 2 \ne \lim_{x\to 1} f(x) = 1$.

31. $g(x) = \sqrt{49 - x^2}$ is continuous on $[-7, 7]$.

32. $f(t) = 3 - \sqrt{9 - t^2}$ is continuous on $[-3, 3]$.

33. $\lim_{x\to 0^-} f(x) = 3 = \lim_{x\to 0^+} f(x)$. f is continuous on $[-1, 4]$.

34. $g(2)$ is not defined. g is continuous on $[-1, 2)$.

35. $f(x) = \frac{6}{x}$ has a nonremovable discontinuity at $x = 0$ because $\lim_{x\to 0} f(x)$ does not exist.

36. $f(x) = \frac{4}{x - 6}$ has a nonremovable discontinuity at $x = 6$ because $\lim_{x\to 6} f(x)$ does not exist.

37. $f(x) = x^2 - 9$ is continuous for all real x.

38. $f(x) = x^2 - 4x + 4$ is continuous for all real x.

39. $f(x) = \dfrac{1}{4 - x^2} = \dfrac{1}{(2 - x)(2 + x)}$ has nonremovable discontinuities at $x = \pm 2$ because $\lim_{x \to 2} f(x)$ and $\lim_{x \to -2} f(x)$ do not exist.

40. $f(x) = \dfrac{1}{x^2 + 1}$ is continuous for all real x.

41. $f(x) = 3x - \cos x$ is continuous for all real x.

42. $f(x) = \cos \dfrac{\pi x}{2}$ is continuous for all real x.

43. $f(x) = \dfrac{x}{x^2 - x}$ is not continuous at $x = 0, 1$.

Because $\dfrac{x}{x^2 - x} = \dfrac{1}{x - 1}$ for $x \neq 0$, $x = 0$ is a removable discontinuity, whereas $x = 1$ is a nonremovable discontinuity.

44. $f(x) = \dfrac{x}{x^2 - 4}$ has nonremovable discontinuities at $x = 2$ and $x = -2$ because $\lim_{x \to 2} f(x)$ and $\lim_{x \to -2} f(x)$ do not exist.

45. $f(x) = \dfrac{x}{x^2 + 1}$ is continuous for all real x.

46. $f(x) = \dfrac{x - 5}{x^2 - 25} = \dfrac{x - 5}{(x + 5)(x - 5)}$

has a nonremovable discontinuity at $x = -5$ because $\lim_{x \to -5} f(x)$ does not exist, and has a removable discontinuity at $x = 5$ because

$$\lim_{x \to 5} f(x) = \lim_{x \to 5} \frac{1}{x + 5} = \frac{1}{10}.$$

47. $f(x) = \dfrac{x + 2}{x^2 - 3x - 10} = \dfrac{x + 2}{(x + 2)(x - 5)}$

has a nonremovable discontinuity at $x = 5$ because $\lim_{x \to 5} f(x)$ does not exist, and has a removable discontinuity at $x = -2$ because

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{1}{x - 5} = -\frac{1}{7}.$$

48. $f(x) = \dfrac{x + 2}{x^2 - x - 6} = \dfrac{x + 2}{(x - 3)(x + 2)}$

has a nonremovable discontinuity at $x = 3$ because $\lim_{x \to 3} f(x)$ does not exist, and has a removable discontinuity at $x = -2$ because

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{1}{x - 3} = -\frac{1}{5}.$$

49. $f(x) = \dfrac{|x + 7|}{x + 7}$

has a nonremovable discontinuity at $x = -7$ because $\lim_{x \to -7} f(x)$ does not exist.

50. $f(x) = \dfrac{|x - 5|}{x - 5}$

has a nonremovable discontinuity at $x = 5$ because $\lim_{x \to 5} f(x)$ does not exist.

51. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

has a **possible** discontinuity at $x = 1$.

1. $f(1) = 1$

2. $\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1 \\ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 = 1 \end{array}\right\} \lim_{x \to 1} f(x) = 1$

3. $f(-1) = \lim_{x \to 1} f(x)$

f is continuous at $x = 1$, therefore, f is continuous for all real x.

52. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \ge 1 \end{cases}$

has a **possible** discontinuity at $x = 1$.

1. $f(1) = 1^2 = 1$

2. $\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-2x + 3) = 1 \\ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 = 1 \end{array}\right\} \lim_{x \to 1} f(x) = 1$

3. $f(1) = \lim_{x \to 1} f(x)$

f is continuous at $x = 1$, therefore, f is continuous for all real x.

53. $f(x) = \begin{cases} \dfrac{x}{2} + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

has a **possible** discontinuity at $x = 2$.

1. $f(2) = \dfrac{2}{2} + 1 = 2$

2. $\left.\begin{array}{l} \lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-}\left(\dfrac{x}{2} + 1\right) = 2 \\ \lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+}(3 - x) = 1 \end{array}\right\}$ $\lim\limits_{x\to 2} f(x)$ does not exist.

Therefore, f has a nonremovable discontinuity at $x = 2$.

54. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

has a **possible** discontinuity at $x = 2$.

1. $f(2) = -2(2) = -4$

2. $\left.\begin{array}{l} \lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-}(-2x) = -4 \\ \lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+}(x^2 - 4x + 1) = -3 \end{array}\right\}$ $\lim\limits_{x\to 2} f(x)$ does not exist.

Therefore, f has a nonremovable discontinuity at $x = 2$.

55. $f(x) = \begin{cases} \tan\dfrac{\pi x}{4}, & |x| < 1 \\ x, & |x| \ge 1 \end{cases}$

$= \begin{cases} \tan\dfrac{\pi x}{4}, & -1 < x < 1 \\ x, & x \le -1 \text{ or } x \ge 1 \end{cases}$

has **possible** discontinuities at $x = -1, x = 1$.

1. $f(-1) = -1$ $\quad f(1) = 1$

2. $\lim\limits_{x\to -1} f(x) = -1$ $\quad \lim\limits_{x\to 1} f(x) = 1$

3. $f(-1) = \lim\limits_{x\to -1} f(x)$ $\quad f(1) = \lim\limits_{x\to 1} f(x)$

f is continuous at $x = \pm 1$, therefore, f is continuous for all real x.

56. $f(x) = \begin{cases} \csc\dfrac{\pi x}{6}, & |x - 3| \le 2 \\ 2, & |x - 3| > 2 \end{cases}$

$= \begin{cases} \csc\dfrac{\pi x}{6}, & 1 \le x \le 5 \\ 2, & x < 1 \text{ or } x > 5 \end{cases}$

has **possible** discontinuities at $x = 1, x = 5$.

1. $f(1) = \csc\dfrac{\pi}{6} = 2$ $\quad f(5) = \csc\dfrac{5\pi}{6} = 2$

2. $\lim\limits_{x\to 1} f(x) = 2$ $\quad \lim\limits_{x\to 5} f(x) = 2$

3. $f(1) = \lim\limits_{x\to 1} f(x)$ $\quad f(5) = \lim\limits_{x\to 5} f(x)$

f is continuous at $x = 1$ and $x = 5$, therefore, f is continuous for all real x.

57. $f(x) = \csc 2x$ has nonremovable discontinuities at integer multiples of $\pi/2$.

58. $f(x) = \tan\dfrac{\pi x}{2}$ has nonremovable discontinuities at each $2k + 1$, k is an integer.

59. $f(x) = [\![x - 8]\!]$ has nonremovable discontinuities at each integer k.

60. $f(x) = 5 - [\![x]\!]$ has nonremovable discontinuities at each integer k.

61. $f(1) = 3$

Find a so that $\lim_{x\to 1^-}(ax - 4) = 3$

$$a(1) - 4 = 3$$
$$a = 7.$$

62. $f(1) = 3$

Find a so that $\lim_{x\to 1^+}(ax + 5) = 3$

$$a(1) + 5 = 3$$
$$a = -2.$$

63. $f(2) = 8$

Find a so that $\lim_{x\to 2^+} ax^2 = 8 \Rightarrow a = \frac{8}{2^2} = 2.$

64. $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \frac{4\sin x}{x} = 4$

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (a - 2x) = a$

Let $a = 4$.

65. Find a and b such that $\lim_{x\to -1^+}(ax + b) = -a + b = 2$ and $\lim_{x\to 3^-}(ax + b) = 3a + b = -2$.

$$\begin{aligned} a - b &= -2 \\ (+)3a + b &= -2 \\ \hline 4a &= -4 \\ a &= -1 \\ b &= 2 + (-1) = 1 \end{aligned}$$

$$f(x) = \begin{cases} 2, & x \le -1 \\ -x + 1, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$$

66. $\lim_{x\to a} g(x) = \lim_{x\to a} \frac{x^2 - a^2}{x - a}$

$$= \lim_{x\to a}(x + a) = 2a$$

Find a such $2a = 8 \Rightarrow a = 4$.

67. $f(g(x)) = (x - 1)^2$

Continuous for all real x

68. $f(g(x)) = 5(x^3) + 1 = 5x^3 + 1$

Continuous for all real x

69. $f(g(x)) = \frac{1}{(x^2 + 5) - 6} = \frac{1}{x^2 - 1}$

Nonremovable discontinuities at $x = \pm 1$

70. $f(g(x)) = \frac{1}{\sqrt{x - 1}}$

Nonremovable discontinuity at $x = 1$; continuous for all $x > 1$

71. $f(g(x)) = \tan\frac{x}{2}$

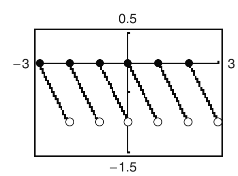
Not continuous at $x = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$ Continuous on the open intervals $\ldots, (-3\pi, -\pi), (-\pi, \pi), (\pi, 3\pi), \ldots$

72. $f(g(x)) = \sin x^2$

Continuous for all real x

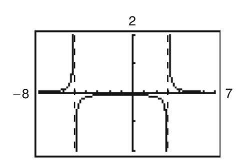
73. $y = [\![x]\!] - x$

Nonremovable discontinuity at each integer



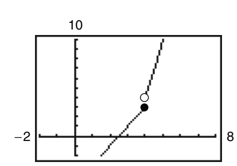
74. $h(x) = \frac{1}{x^2 + 2x - 15} = \frac{1}{(x + 5)(x - 3)}$

Nonremovable discontinuities at $x = -5$ and $x = 3$



75. $g(x) = \begin{cases} x^2 - 3x, & x > 4 \\ 2x - 5, & x \le 4 \end{cases}$

Nonremovable discontinuity at $x = 4$

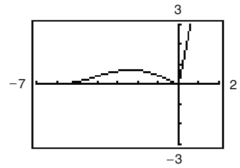


76. $f(x) = \begin{cases} \dfrac{\cos x - 1}{x}, & x < 0 \\ 5x, & x \geq 0 \end{cases}$

$f(0) = 5(0) = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \dfrac{(\cos x - 1)}{x} = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (5x) = 0$



Therefore, $\lim_{x \to 0} f(x) = 0 = f(0)$ and f is continuous on the entire real line.

$(x = 0$ was the only possible discontinuity.$)$

77. $f(x) = \dfrac{x}{x^2 + x + 2}$

Continuous on $(-\infty, \infty)$

78. $f(x) = \dfrac{x + 1}{\sqrt{x}}$

Continuous on $(0, \infty)$

79. $f(x) = 3 - \sqrt{x}$

Continuous on $[0, \infty)$

80. $f(x) = x\sqrt{x + 3}$

Continuous on $[-3, \infty)$

81. $f(x) = \sec\dfrac{\pi x}{4}$

Continuous on:

$\ldots, (-6, -2), (-2, 2), (2, 6), (6, 10), \ldots$

82. $f(x) = \cos\dfrac{1}{x}$

Continuous on $(-\infty, 0)$ and $(0, \infty)$

83. $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$

Since $\lim_{x \to 1} f(x) = \lim_{x \to 1} \dfrac{x^2 - 1}{x - 1} = \lim_{x \to 1} \dfrac{(x - 1)(x + 1)}{x - 1}$

$= \lim_{x \to 1} (x + 1) = 2,$

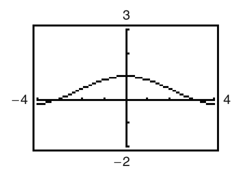
f is continuous on $(-\infty, \infty)$.

84. $f(x) = \begin{cases} 2x - 4, & x \neq 3 \\ 1, & x = 3 \end{cases}$

Since $\lim_{x \to 3} f(x) = \lim_{x \to 3} (2x - 4) = 2 \neq 1,$

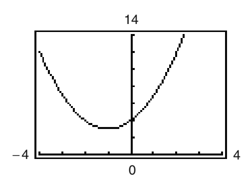
f is continuous on $(-\infty, 3)$ and $(3, \infty)$.

85. $f(x) = \dfrac{\sin x}{x}$



The graph **appears** to be continuous on the interval $[-4, 4]$. Because $f(0)$ is not defined, you know that f has a discontinuity at $x = 0$. This discontinuity is removable so it does not show up on the graph.

86. $f(x) = \dfrac{x^3 - 8}{x - 2}$



The graph **appears** to be continuous on the interval $[-4, 4]$. Because $f(2)$ is not defined, you know that f has a discontinuity at $x = 2$. This discontinuity is removable so it does not show up on the graph.

87. $f(x) = \frac{1}{12}x^4 - x^3 + 4$ is continuous on the interval $[1, 2]$. $f(1) = \frac{37}{12}$ and $f(2) = -\frac{8}{3}$. By the Intermediate Value Theorem, there exists a number c in $[1, 2]$ such that $f(c) = 0$.

88. $f(x) = x^3 + 5x - 3$ is continuous on the interval $[0, 1]$.

$f(0) = -3$ and $f(1) = 3$. By the Intermediate Value Theorem, there exists a number c in $[0, 1]$ such that $f(c) = 0$.

89. $f(x) = x^2 - 2 - \cos x$ is continuous on $[0, \pi]$.

$f(0) = -3$ and $f(\pi) = \pi^2 - 1 \approx 8.87 > 0$. By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and π.

90. $f(x) = -\dfrac{5}{x} + \tan\left(\dfrac{\pi x}{10}\right)$ is continuous on the interval $[1, 4]$.

$f(1) = -5 + \tan\left(\dfrac{\pi}{10}\right) \approx -4.7$ and

$f(4) = -\dfrac{5}{4} + \tan\left(\dfrac{2\pi}{5}\right) \approx 1.8$. By the Intermediate Value Theorem, there exists a number c in $[1, 4]$ such that $f(c) = 0$.

91. $f(x) = x^3 + x - 1$

$f(x)$ is continuous on $[0, 1]$.

$f(0) = -1$ and $f(1) = 1$

By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $f(x)$, you find that $x \approx 0.68$. Using the *root* feature, you find that $x \approx 0.6823$.

92. $f(x) = x^4 - x^2 + 3x - 1$

$f(x)$ is continuous on $[0, 1]$.

$f(0) = -1$ and $f(1) = 2$

By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $f(x)$, you find that $x \approx 0.37$. Using the *root* feature, you find that $x \approx 0.3733$.

93. $g(t) = 2\cos t - 3t$

g is continuous on $[0, 1]$.

$g(0) = 2 > 0$ and $g(1) \approx -1.9 < 0$.

By the Intermediate Value Theorem, $g(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $g(t)$, you find that $t \approx 0.56$. Using the root feature, you find that $t \approx 0.5636$.

94. $h(\theta) = \tan\theta + 3\theta - 4$ is continuous on $[0, 1]$.

$h(0) = -4$ and $h(1) = \tan(1) - 1 \approx 0.557$.

By the Intermediate Value Theorem, $h(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $h(\theta)$, you find that $\theta \approx 0.91$. Using the *root* feature, you obtain $\theta \approx 0.9071$.

95. $f(x) = x^2 + x - 1$

f is continuous on $[0, 5]$.

$f(0) = -1$ and $f(5) = 29$

$-1 < 11 < 29$

The Intermediate Value Theorem applies.

$$x^2 + x - 1 = 11$$
$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$$x = -4 \text{ or } x = 3$$

$c = 3$ ($x = -4$ is not in the interval.)

So, $f(3) = 11$.

96. $f(x) = x^2 - 6x + 8$

f is continuous on $[0, 3]$.

$f(0) = 8$ and $f(3) = -1$

$-1 < 0 < 8$

The Intermediate Value Theorem applies.

$$x^2 - 6x + 8 = 0$$
$$(x - 2)(x - 4) = 0$$
$$x = 2 \text{ or } x = 4$$

$c = 2$ ($x = 4$ is not in the interval.)

So, $f(2) = 0$.

97. $f(x) = x^3 - x^2 + x - 2$

f is continuous on $[0, 3]$.

$f(0) = -2$ and $f(3) = 19$

$-2 < 4 < 19$

The Intermediate Value Theorem applies.

$$x^3 - x^2 + x - 2 = 4$$
$$x^3 - x^2 + x - 6 = 0$$
$$(x - 2)(x^2 + x + 3) = 0$$
$$x = 2$$

$(x^2 + x + 3$ has no real solution.$)$

$$c = 2$$

So, $f(2) = 4$.

98. $f(x) = \dfrac{x^2 + x}{x - 1}$

f is continuous on $\left[\frac{5}{2}, 4\right]$. The nonremovable discontinuity, $x = 1$, lies outside the interval.

$f\left(\frac{5}{2}\right) = \frac{35}{6}$ and $f(4) = \frac{20}{3}$

$\frac{35}{6} < 6 < \frac{20}{3}$

The Intermediate Value Theorem applies.

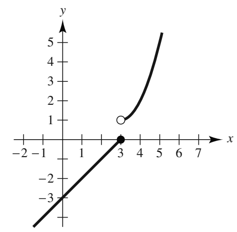
$$\frac{x^2 + x}{x - 1} = 6$$
$$x^2 + x = 6x - 6$$
$$x^2 - 5x + 6 = 0$$
$$(x - 2)(x - 3) = 0$$
$$x = 2 \text{ or } x = 3$$

$c = 3$ $(x = 2$ is not in the interval.$)$

So, $f(3) = 6$.

99. (a) The limit does not exist at $x = c$.
(b) The function is not defined at $x = c$.
(c) The limit exists at $x = c$, but it is not equal to the value of the function at $x = c$.
(d) The limit does not exist at $x = c$.

100. Answers will vary. *Sample answer*:



The function is not continuous at $x = 3$ because $\lim_{x\to 3^+} f(x) = 1 \neq 0 = \lim_{x\to 3^-} f(x)$.

101. If f and g are continuous for all real x, then so is $f + g$ (Theorem 1.11, part 2). However, f/g might not be continuous if $g(x) = 0$. For example, let $f(x) = x$ and $g(x) = x^2 - 1$. Then f and g are continuous for all real x, but f/g is not continuous at $x = \pm 1$.

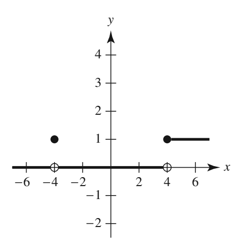
102. A discontinuity at c is removable if the function f can be made continuous at c by appropriately defining (or redefining) $f(c)$. Otherwise, the discontinuity is nonremovable.

(a) $f(x) = \dfrac{|x - 4|}{x - 4}$

(b) $f(x) = \dfrac{\sin(x + 4)}{x + 4}$

(c) $f(x) = \begin{cases} 1, & x \ge 4 \\ 0, & -4 < x < 4 \\ 1, & x = -4 \\ 0, & x < -4 \end{cases}$

$x = 4$ is nonremovable, $x = -4$ is removable



103. True

1. $f(c) = L$ is defined.

2. $\lim_{x\to c} f(x) = L$ exists.

3. $f(c) = \lim_{x\to c} f(x)$

All of the conditions for continuity are met.

104. True. If $f(x) = g(x), x \neq c$, then $\lim_{x\to c} f(x) = \lim_{x\to c} g(x)$ (if they exist) and at least one of these limits then does not equal the corresponding function value at $x = c$.

105. False. A rational function can be written as $P(x)/Q(x)$ where P and Q are polynomials of degree m and n, respectively. It can have, at most, n discontinuities.

106. False. $f(1)$ is not defined and $\lim_{x\to 1} f(x)$ does not exist.

107. The functions agree for integer values of x:

$$\left.\begin{aligned} g(x) &= 3 - [\![-x]\!] = 3 - (-x) = 3 + x \\ f(x) &= 3 + [\![x]\!] = 3 + x \end{aligned}\right\} \text{for } x \text{ an integer}$$

However, for non-integer values of x, the functions differ by 1.

$$f(x) = 3 + [\![x]\!] = g(x) - 1 = 2 - [\![-x]\!].$$

For example,

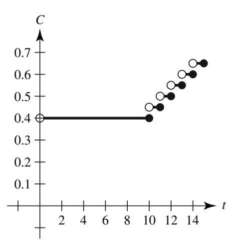
$$f\left(\tfrac{1}{2}\right) = 3 + 0 = 3, \ g\left(\tfrac{1}{2}\right) = 3 - (-1) = 4.$$

108. $\lim_{t\to 4^-} f(t) \approx 28$

$\lim_{t\to 4^+} f(t) \approx 56$

At the end of day 3, the amount of chlorine in the pool has decreased to about 28 oz. At the beginning of day 4, more chlorine was added, and the amount is now about 56 oz.

109. $$C(t) = \begin{cases} 0.40, & 0 < t \leq 10 \\ 0.40 + 0.05[\![t - 9]\!], & t > 10, t \text{ not an integer} \\ 0.40 + 0.05(t - 10), & t > 10, t \text{ an integer} \end{cases}$$



There is a nonremovable discontinuity at each integer greater than or equal to 10.

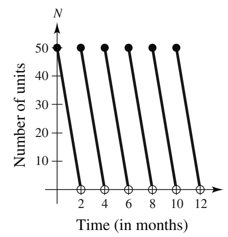
Note: You could also express C as

$$C(t) = \begin{cases} 0.40, & 0 < t \leq 10 \\ 0.40 - 0.05[\![10 - t]\!], & t > 10 \end{cases}$$

110. $N(t) = 25\left(2\left[\!\!\left[\dfrac{t+2}{2}\right]\!\!\right] - t\right)$

t	0	1	1.8	2	3	3.8
$N(t)$	50	25	5	50	25	5

Discontinuous at every positive even integer. The company replenishes its inventory every two months.



111. Let $s(t)$ be the position function for the run up to the campsite. $s(0) = 0$ ($t = 0$ corresponds to 8:00 A.M., $s(20) = k$ (distance to campsite)). Let $r(t)$ be the position function for the run back down the mountain: $r(0) = k, r(10) = 0$. Let $f(t) = s(t) - r(t)$.

When $t = 0$ (8:00 A.M.), $f(0) = s(0) - r(0) = 0 - k < 0$.

When $t = 10$ (8:00 A.M.), $f(10) = s(10) - r(10) > 0$.

Because $f(0) < 0$ and $f(10) > 0$, then there must be a value t in the interval $[0, 10]$ such that $f(t) = 0$. If $f(t) = 0$, then $s(t) - r(t) = 0$, which gives us $s(t) = r(t)$. Therefore, at some time t, where $0 \leq t \leq 10$, the position functions for the run up and the run down are equal.

112. Let $V = \dfrac{4}{3}\pi r^3$ be the volume of a sphere with radius r.

V is continuous on $[5, 8]$. $V(5) = \dfrac{500\pi}{3} \approx 523.6$ and $V(8) = \dfrac{2048\pi}{3} \approx 2144.7$. Because $523.6 < 1500 < 2144.7$, the Intermediate Value Theorem guarantees that there is at least one value r between 5 and 8 such that $V(r) = 1500$. (In fact, $r \approx 7.1012$.)

113. Suppose there exists x_1 in $[a, b]$ such that $f(x_1) > 0$ and there exists x_2 in $[a, b]$ such that $f(x_2) < 0$. Then by the Intermediate Value Theorem, $f(x)$ must equal zero for some value of x in $[x_1, x_2]$ (or $[x_2, x_1]$ if $x_2 < x_1$). So, f would have a zero in $[a, b]$, which is a contradiction. Therefore, $f(x) > 0$ for all x in $[a, b]$ or $f(x) < 0$ for all x in $[a, b]$.

114. Let c be any real number. Then $\lim_{x \to c} f(x)$ does not exist because there are both rational and irrational numbers arbitrarily close to c. Therefore, f is not continuous at c.

115. If $x = 0$, then $f(0) = 0$ and $\lim_{x \to 0} f(x) = 0$. So, f is continuous at $x = 0$.

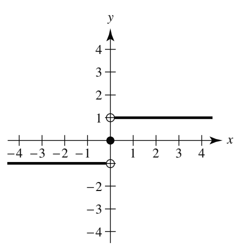
If $x \neq 0$, then $\lim_{t \to x} f(t) = 0$ for x rational, whereas $\lim_{t \to x} f(t) = \lim_{t \to x} kt = kx \neq 0$ for x irrational. So, f is not continuous for all $x \neq 0$.

116. $\text{sgn}(x) = \begin{cases} -1, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 1, & \text{if } x > 0 \end{cases}$

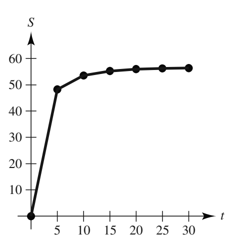
(a) $\lim_{x \to 0^-} \text{sgn}(x) = -1$

(b) $\lim_{x \to 0^+} \text{sgn}(x) = 1$

(c) $\lim_{x \to 0} \text{sgn}(x)$ does not exist.

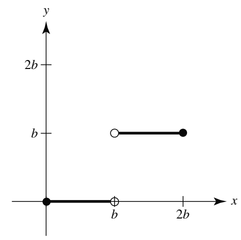


117. (a)



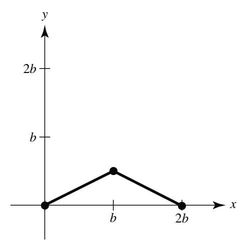
(b) There appears to be a limiting speed and a possible cause is air resistance.

118. (a) $f(x) = \begin{cases} 0, & 0 \le x < b \\ b, & b < x \le 2b \end{cases}$



NOT continuous at $x = b$.

(b) $g(x) = \begin{cases} \dfrac{x}{2}, & 0 \le x \le b \\ b - \dfrac{x}{2}, & b < x \le 2b \end{cases}$



Continuous on $[0, 2b]$.

119. $f(x) = \begin{cases} 1 - x^2, & x \le c \\ x, & x > c \end{cases}$

f is continuous for $x < c$ and for $x > c$. At $x = c$, you need $1 - c^2 = c$. Solving $c^2 + c - 1$, you obtain

$$c = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}.$$

120. Let y be a real number. If $y = 0$, then $x = 0$. If $y > 0$, then let $0 < x_0 < \pi/2$ such that $M = \tan x_0 > y$ (this is possible since the tangent function increases without bound on $[0, \pi/2)$). By the Intermediate Value Theorem, $f(x) = \tan x$ is continuous on $[0, x_0]$ and $0 < y < M$, which implies that there exists x between 0 and x_0 such that $\tan x = y$. The argument is similar if $y < 0$.

121. $f(x) = \dfrac{\sqrt{x + c^2} - c}{x}, c > 0$

Domain: $x + c^2 \ge 0 \Rightarrow x \ge -c^2$ and $x \ne 0, [-c^2, 0) \cup (0, \infty)$

$$\lim_{x\to 0}\frac{\sqrt{x + c^2} - c}{x} = \lim_{x\to 0}\frac{\sqrt{x + c^2} - c}{x}\cdot\frac{\sqrt{x + c^2} + c}{\sqrt{x + c^2} + c} = \lim_{x\to 0}\frac{(x + c^2) - c^2}{x\left[\sqrt{x + c^2} + c\right]} = \lim_{x\to 0}\frac{1}{\sqrt{x + c^2} + c} = \frac{1}{2c}$$

Define $f(0) = 1/(2c)$ to make f continuous at $x = 0$.

122. 1. $f(c)$ is defined.

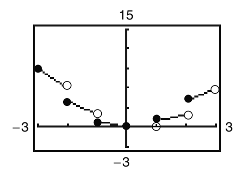
2. $\lim_{x\to c} f(x) = \lim_{\Delta x\to 0} f(c + \Delta x) = f(c)$ exists.

[Let $x = c + \Delta x$. As $x \to c, \Delta x \to 0$]

3. $\lim_{x\to c} f(x) = f(c)$.

Therefore, f is continuous at $x = c$.

123. $h(x) = x[\![x]\!]$



h has nonremovable discontinuities at $x = \pm 1, \pm 2, \pm 3, \ldots$.

124. (a) Define $f(x) = f_2(x) - f_1(x)$. Because f_1 and f_2 are continuous on $[a, b]$, so is f.

$f(a) = f_2(a) - f_1(a) > 0$ and $f(b) = f_2(b) - f_1(b) < 0$

By the Intermediate Value Theorem, there exists c in $[a, b]$ such that $f(c) = 0$.

$f(c) = f_2(c) - f_1(c) = 0 \Rightarrow f_1(c) = f_2(c)$

(b) Let $f_1(x) = x$ and $f_2(x) = \cos x$, continuous on $[0, \pi/2]$, $f_1(0) < f_2(0)$ and $f_1(\pi/2) > f_2(\pi/2)$.

So by part (a), there exists c in $[0, \pi/2]$ such that $c = \cos(c)$.

Using a graphing utility, $c \approx 0.739$.

125. The statement is true.

If $y \ge 0$ and $y \le 1$, then $y(y - 1) \le 0 \le x^2$, as desired. So assume $y > 1$. There are now two cases.

Case 1: If $x \le y - \frac{1}{2}$, then $2x + 1 \le 2y$ and

$$\begin{aligned} y(y - 1) &= y(y + 1) - 2y \\ &\le (x + 1)^2 - 2y \\ &= x^2 + 2x + 1 - 2y \\ &\le x^2 + 2y - 2y \\ &= x^2 \end{aligned}$$

Case 2: If $x \ge y - \frac{1}{2}$

$$\begin{aligned} x^2 &\ge \left(y - \tfrac{1}{2}\right)^2 \\ &= y^2 - y + \tfrac{1}{4} \\ &> y^2 - y \\ &= y(y - 1) \end{aligned}$$

In both cases, $y(y - 1) \le x^2$.

126. $P(1) = P(0^2 + 1) = P(0)^2 + 1 = 1$

$P(2) = P(1^2 + 1) = P(1)^2 + 1 = 2$

$P(5) = P(2^2 + 1) = P(2)^2 + 1 = 5$

Continuing this pattern, you see that $P(x) = x$ for infinitely many values of x. So, the finite degree polynomial must be constant: $P(x) = x$ for all x.

Section 1.5 Infinite Limits

1. $\lim_{x\to -2^+} 2\left|\frac{x}{x^2-4}\right| = \infty$

$\lim_{x\to -2^-} 2\left|\frac{x}{x^2-4}\right| = \infty$

2. $\lim_{x\to -2^+} \frac{1}{x+2} = \infty$

$\lim_{x\to -2^-} \frac{1}{x+2} = -\infty$

3. $\lim_{x\to -2^+} \tan\frac{\pi x}{4} = -\infty$

$\lim_{x\to -2^-} \tan\frac{\pi x}{4} = \infty$

4. $\lim_{x\to -2^+} \sec\frac{\pi x}{4} = \infty$

$\lim_{x\to -2^-} \sec\frac{\pi x}{4} = -\infty$

5. $f(x) = \frac{1}{x-4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$\lim_{x\to 4^-} f(x) = -\infty$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$\lim_{x\to 4^+} f(x) = \infty$

6. $f(x) = \frac{-1}{x-4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$\lim_{x\to 4^-} f(x) = \infty.$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$\lim_{x\to 4^+} f(x) = -\infty.$

7. $f(x) = \frac{1}{(x-4)^2}$

As x approaches 4 from the left or right, $(x-4)^2$ is a small positive number. So,

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x) = \infty.$

8. $f(x) = \frac{-1}{(x-4)^2}$

As x approaches 4 from the left or right, $(x-4)^2$ is a small positive number. So,

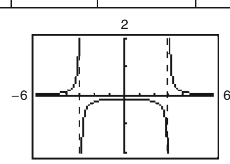
$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = -\infty.$

9. $f(x) = \frac{1}{x^2-9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	0.308	1.639	16.64	166.6	–166.7	–16.69	–1.695	–0.364

$\lim_{x\to -3^-} f(x) = \infty$

$\lim_{x\to -3^+} f(x) = -\infty$

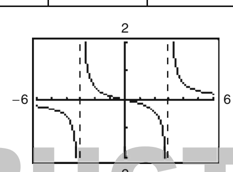


10. $f(x) = \frac{x}{x^2-9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	–1.077	–5.082	–50.08	–500.1	499.9	49.92	4.915	0.9091

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

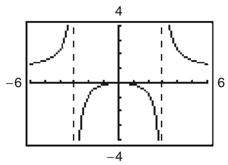


11. $f(x) = \dfrac{x^2}{x^2 - 9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	3.769	15.75	150.8	1501	–1499	–149.3	–14.25	–2.273

$\lim\limits_{x \to -3^-} f(x) = \infty$

$\lim\limits_{x \to -3^+} f(x) = -\infty$

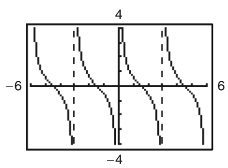


12. $f(x) = \cot \dfrac{\pi x}{3}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	–1.7321	–9.514	–95.49	–954.9	954.9	95.49	9.514	1.7321

$\lim\limits_{x \to -3^-} f(x) = -\infty$

$\lim\limits_{x \to -3^+} f(x) = \infty$



13. $f(x) = \dfrac{1}{x^2}$

$\lim\limits_{x \to 0^+} \dfrac{1}{x^2} = \infty = \lim\limits_{x \to 0^-} \dfrac{1}{x^2}$

Therefore, $x = 0$ is a vertical asymptote.

14. $f(x) = \dfrac{2}{(x - 3)^3}$

$\lim\limits_{x \to 3^-} \dfrac{2}{(x - 3)^3} = -\infty$

$\lim\limits_{x \to 3^+} \dfrac{2}{(x - 3)^3} = \infty$

Therefore, $x = 3$ is a vertical asymptote.

15. $f(x) = \dfrac{x^2}{x^2 - 4} = \dfrac{x^2}{(x + 2)(x - 2)}$

$\lim\limits_{x \to -2^-} \dfrac{x^2}{x^2 - 4} = \infty$ and $\lim\limits_{x \to -2^+} \dfrac{x^2}{x^2 - 4} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim\limits_{x \to 2^-} \dfrac{x^2}{x^2 - 4} = -\infty$ and $\lim\limits_{x \to 2^+} \dfrac{x^2}{x^2 - 4} = \infty$

Therefore, $x = 2$ is a vertical asymptote.

16. $f(x) = \dfrac{3x}{x^2 + 9}$

No vertical asymptotes because the denominator is never zero.

17. $g(t) = \dfrac{t - 1}{t^2 + 1}$

No vertical asymptotes because the denominator is never zero.

18. $h(s) = \dfrac{3s + 4}{s^2 - 16} = \dfrac{3s + 4}{(s - 4)(s + 4)}$

$\lim\limits_{s \to 4^-} \dfrac{3s + 4}{s^2 - 16} = -\infty$ and $\lim\limits_{s \to 4^+} \dfrac{3s + 4}{s^2 - 16} = \infty$

Therefore, $s = 4$ is a vertical asymptote.

$\lim\limits_{s \to -4^-} \dfrac{3s + 4}{s^2 - 16} = -\infty$ and $\lim\limits_{s \to -4^+} \dfrac{3s + 4}{s^2 - 16} = \infty$

Therefore, $s = -4$ is a vertical asymptote.

19. $f(x) = \dfrac{3}{x^2 + x - 2} = \dfrac{3}{(x + 2)(x - 1)}$

$\lim\limits_{x \to -2^-} \dfrac{3}{x^2 + x - 2} = \infty$ and $\lim\limits_{x \to -2^+} \dfrac{3}{x^2 + x - 2} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim\limits_{x \to 1^-} \dfrac{3}{x^2 + x - 2} = -\infty$ and $\lim\limits_{x \to 1^+} \dfrac{3}{x^2 + x - 2} = \infty$

Therefore, $x = 1$ is a vertical asymptote.

20.
$$g(x) = \frac{x^3 - 8}{x - 2} = \frac{(x - 2)(x^2 + 2x + 4)}{x - 2}$$
$$= x^2 + 2x + 4, \; x \neq 2$$

$\lim\limits_{x \to 2} g(x) = 4 + 4 + 4 = 12$

There are no vertical asymptotes. The graph has a hole at $x = 2$.

21. $f(x) = \dfrac{4(x^2 + x - 6)}{x(x^3 - 2x^2 - 9x + 18)}$

$= \dfrac{4(x+3)(x-2)}{x(x-2)(x^2-9)}$

$= \dfrac{4}{x(x-3)}, x \neq -3, 2$

$\lim_{x \to 0^-} f(x) = \infty$ and $\lim_{x \to 0^+} f(x) = -\infty$

Therefore, $x = 0$ is a vertical asymptote.

$\lim_{x \to 3^-} f(x) = -\infty$ and $\lim_{x \to 3^+} f(x) = \infty$

Therefore, $x = 3$ is a vertical asymptote.

$\lim_{x \to 2} f(x) = \dfrac{4}{2(2-3)}$

$= -2$ and $\lim_{x \to -3} f(x)$

$= \dfrac{4}{-3(-3-3)} = \dfrac{2}{9}$

Therefore, the graph has holes at $x = 2$ and $x = -3$.

22. $h(x) = \dfrac{x^2 - 9}{x^3 + 3x^2 - x - 3}$

$= \dfrac{(x-3)(x+3)}{(x-1)(x+1)(x+3)}$

$= \dfrac{x-3}{(x+1)(x-1)}, x \neq -3$

$\lim_{x \to -1^-} h(x) = -\infty$ and $\lim_{x \to -1^+} h(x) = \infty$

Therefore, $x = -1$ is a vertical asymptote.

$\lim_{x \to 1^-} h(x) = \infty$ and $\lim_{x \to 1^+} h(x) = -\infty$

Therefore, $x = 1$ is a vertical asymptote.

$\lim_{x \to -3} h(x) = \dfrac{-3-3}{(-3+1)(-3-1)} = -\dfrac{3}{4}$

Therefore, the graph has a hole at $x = -3$.

23. $f(x) = \dfrac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

$= \dfrac{(x-5)(x+3)}{(x-5)(x^2+1)}$

$= \dfrac{x+3}{x^2+1}, x \neq 5$

$\lim_{x \to 5} f(x) = \dfrac{5+3}{5^2+1} = \dfrac{15}{26}$

There are no vertical asymptotes. The graph has a hole at $x = 5$.

24. $h(t) = \dfrac{t^2 - 2t}{t^4 - 16} = \dfrac{t(t-2)}{(t-2)(t+2)(t^2+4)}$

$= \dfrac{t}{(t+2)(t^2+4)}, t \neq 2$

$\lim_{t \to -2^-} h(t) = \infty$ and $\lim_{t \to -2^+} h(t) = -\infty$

Therefore, $t = -2$ is a vertical asymptote.

$\lim_{t \to 2} h(t) = \dfrac{2}{(2+2)(2^2+4)} = \dfrac{1}{16}$

Therefore, the graph has a hole at $t = 2$.

25. $f(x) = \csc \pi x = \dfrac{1}{\sin \pi x}$

Let n be any integer.

$\lim_{x \to n} f(x) = -\infty$ or ∞

Therefore, the graph has vertical asymptotes at $x = n$.

26. $f(x) = \tan \pi x = \dfrac{\sin \pi x}{\cos \pi x}$

$\cos \pi x = 0$ for $x = \dfrac{2n+1}{2}$, where n is an integer.

$\lim_{x \to \frac{2n+1}{2}} f(x) = \infty$ or $-\infty$

Therefore, the graph has vertical asymptotes at $x = \dfrac{2n+1}{2}$.

27. $s(t) = \dfrac{t}{\sin t}$

$\sin t = 0$ for $t = n\pi$, where n is an integer.

$\lim_{t \to n\pi} s(t) = \infty$ or $-\infty$ (for $n \neq 0$)

Therefore, the graph has vertical asymptotes at $t = n\pi$, for $n \neq 0$.

$\lim_{t \to 0} s(t) = 1$

Therefore, the graph has a hole at $t = 0$.

28. $g(\theta) = \dfrac{\tan \theta}{\theta} = \dfrac{\sin \theta}{\theta \cos \theta}$

$\cos \theta = 0$ for $\theta = \dfrac{\pi}{2} + n\pi$, where n is an integer.

$\lim_{\theta \to \frac{\pi}{2} + n\pi} g(\theta) = \infty$ or $-\infty$

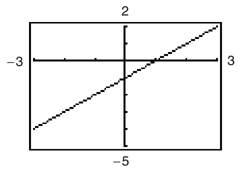
Therefore, the graph has vertical asymptotes at $\theta = \dfrac{\pi}{2} + n\pi$.

$\lim_{\theta \to 0} g(\theta) = 1$

Therefore, the graph has a hole at $\theta = 0$.

29. $\lim_{x\to -1} \dfrac{x^2 - 1}{x + 1} = \lim_{x\to -1}(x - 1) = -2$

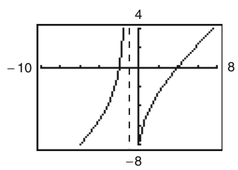
Removable discontinuity at $x = -1$



30. $\lim_{x\to -1^-} \dfrac{x^2 - 2x - 8}{x + 1} = \infty$

$\lim_{x\to -1^+} \dfrac{x^2 - 2x - 8}{x + 1} = -\infty$

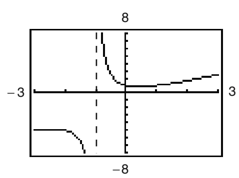
Vertical asymptote at $x = -1$



31. $\lim_{x\to -1^+} \dfrac{x^2 + 1}{x + 1} = \infty$

$\lim_{x\to -1^-} \dfrac{x^2 + 1}{x + 1} = -\infty$

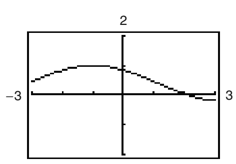
Vertical asymptote at $x = -1$



32. $\lim_{x\to -1} \dfrac{\sin(x + 1)}{x + 1} = 1$

Removable discontinuity at

$x = -1$



33. $\lim_{x\to -1^+} \dfrac{1}{x + 1} = \infty$

34. $\lim_{x\to 1^-} \dfrac{-1}{(x - 1)^2} = -\infty$

35. $\lim_{x\to 2^+} \dfrac{x}{x - 2} = \infty$

36. $\lim_{x\to 2^-} \dfrac{x^2}{x^2 + 4} = \dfrac{4}{4 + 4} = \dfrac{1}{2}$

37. $\lim_{x\to -3^-} \dfrac{x + 3}{(x^2 + x - 6)} = \lim_{x\to -3^-} \dfrac{x + 3}{(x + 3)(x - 2)}$

$= \lim_{x\to -3^-} \dfrac{1}{x - 2} = -\dfrac{1}{5}$

38. $\lim_{x\to -(1/2)^+} \dfrac{6x^2 + x - 1}{4x^2 - 4x - 3} = \lim_{x\to -(1/2)^+} \dfrac{(3x - 1)(2x + 1)}{(2x - 3)(2x + 1)}$

$= \lim_{x\to -(1/2)^+} \dfrac{3x - 1}{2x - 3} = \dfrac{5}{8}$

39. $\lim_{x\to 0^-} \left(1 + \dfrac{1}{x}\right) = -\infty$

40. $\lim_{x\to 0^+} \left(6 - \dfrac{1}{x^3}\right) = -\infty$

41. $\lim_{x\to -4^-} \left(x^2 + \dfrac{2}{x + 4}\right) = -\infty$

42. $\lim_{x\to 3^+} \left(\dfrac{x}{3} + \cot \dfrac{\pi x}{2}\right) = \infty$

43. $\lim_{x\to 0^+} \dfrac{2}{\sin x} = \infty$

44. $\lim_{x\to (\pi/2)^+} \dfrac{-2}{\cos x} = \infty$

45. $\lim_{x\to \pi^+} \dfrac{\sqrt{x}}{\csc x} = \lim_{x\to \pi^+} (\sqrt{x} \sin x) = 0$

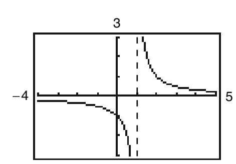
46. $\lim_{x\to 0^-} \dfrac{x + 2}{\cot x} = \lim_{x\to 0^-} [(x + 2) \tan x] = 0$

47. $\lim_{x\to (1/2)^-} x \sec \pi x = \lim_{x\to (1/2)^-} \dfrac{x}{\cos \pi x} = \infty$

48. $\lim_{x\to (1/2)^+} x^2 \tan \pi x = -\infty$

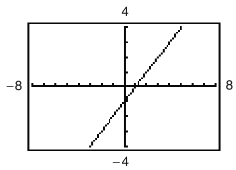
49. $f(x) = \dfrac{x^2 + x + 1}{x^3 - 1} = \dfrac{x^2 + x + 1}{(x - 1)(x^2 + x + 1)}$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \dfrac{1}{x - 1} = \infty$



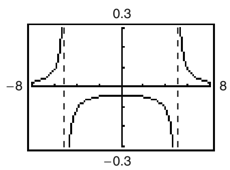
50. $f(x) = \dfrac{x^3 - 1}{x^2 + x + 1} = \dfrac{(x-1)(x^2+x+1)}{x^2+x+1}$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}(x - 1) = 0$



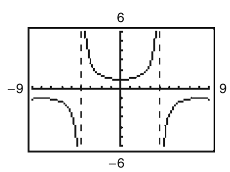
51. $f(x) = \dfrac{1}{x^2 - 25}$

$\lim\limits_{x\to 5^-} f(x) = -\infty$



52. $f(x) = \sec \dfrac{\pi x}{8}$

$\lim\limits_{x\to 4^+} f(x) = -\infty$



53. A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an infinite limit. ∞ is not a number. Rather, the symbol

$\lim\limits_{x\to c} f(x) = \infty$

says how the limit fails to exist.

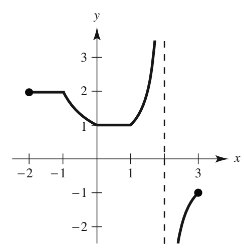
54. The line $x = c$ is a vertical asymptote if the graph of f approaches $\pm\infty$ as x approaches c.

55. One answer is

$f(x) = \dfrac{x - 3}{(x-6)(x+2)} = \dfrac{x-3}{x^2 - 4x - 12}.$

56. No. For example, $f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

57.

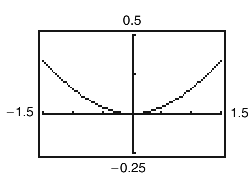


58. $m = \dfrac{m_0}{\sqrt{1 - (v^2/c^2)}}$

$\lim\limits_{v\to c^-} m = \lim\limits_{v\to c^-} \dfrac{m_0}{\sqrt{1-(v^2/c^2)}} = \infty$

59. (a)

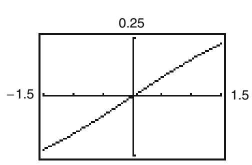
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0411	0.0067	0.0017	≈ 0	≈ 0	≈ 0



$\lim\limits_{x\to 0^+} \dfrac{x - \sin x}{x} = 0$

(b)

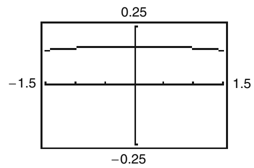
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0823	0.0333	0.0167	0.0017	≈ 0	≈ 0



$\lim\limits_{x\to 0^+} \dfrac{x - \sin x}{x^2} = 0$

(c)

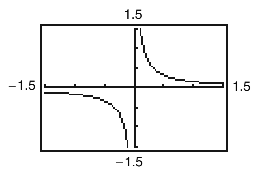
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.1646	0.1663	0.1666	0.1667	0.1667	0.1667



$$\lim_{x\to 0^+} \frac{x - \sin x}{x^3} = 0.1667\ (1/6)$$

(d)

x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.3292	0.8317	1.6658	16.67	166.7	1667.0



$$\lim_{x\to 0^+} \frac{x - \sin x}{x^4} = \infty$$

or $n > 3$, $\displaystyle\lim_{x\to 0^+} \frac{x - \sin x}{x^n} = \infty$.

60. $\displaystyle\lim_{V\to 0^+} P = \infty$

As the volume of the gas decreases, the pressure increases.

61. (a) $r = \dfrac{2(7)}{\sqrt{625 - 49}} = \dfrac{7}{12}$ ft/sec

(b) $r = \dfrac{2(15)}{\sqrt{625 - 225}} = \dfrac{3}{2}$ ft/sec

(c) $\displaystyle\lim_{x\to 25^-} \frac{2x}{\sqrt{625 - x^2}} = \infty$

62. (a) $\text{Average speed} = \dfrac{\text{Total distance}}{\text{Total time}}$

$$50 = \frac{2d}{(d/x) + (d/y)}$$

$$50 = \frac{2xy}{y + x}$$

$$50y + 50x = 2xy$$

$$50x = 2xy - 50y$$

$$50x = 2y(x - 25)$$

$$\frac{25x}{x - 25} = y$$

Domain: $x > 25$

(b)

x	30	40	50	60
y	150	66.667	50	42.857

(c) $\displaystyle\lim_{x\to 25^+} \frac{25x}{\sqrt{x - 25}} = \infty$

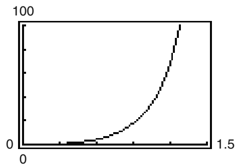
As x gets close to 25 mi/h, y becomes larger and larger.

63. (a) $A = \frac{1}{2}bh - \frac{1}{2}r^2\theta = \frac{1}{2}(10)(10\tan\theta) - \frac{1}{2}(10)^2\theta = 50\tan\theta - 50\theta$

Domain: $\left(0, \frac{\pi}{2}\right)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\lim_{\theta\to\pi/2^-} A = \infty$

64. (a) Because the circumference of the motor is half that of the saw arbor, the saw makes $1700/2 = 850$ revolutions per minute.

(b) The direction of rotation is reversed.

(c) $2(20\cot\phi) + 2(10\cot\phi)$: straight sections. The angle subtended in each circle is $2\pi - \left(2\left(\frac{\pi}{2} - \phi\right)\right) = \pi + 2\phi$.

So, the length of the belt around the pulleys is $20(\pi + 2\phi) + 10(\pi + 2\phi) = 30(\pi + 2\phi)$.

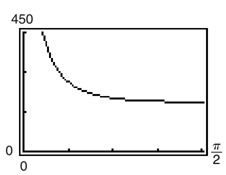
Total length $= 60\cot\phi + 30(\pi + 2\phi)$

Domain: $\left(0, \frac{\pi}{2}\right)$

(d)

ϕ	0.3	0.6	0.9	1.2	1.5
L	306.2	217.9	195.9	189.6	188.5

(e)



(f) $\lim_{\phi\to(\pi/2)^-} L = 60\pi \approx 188.5$

(All the belts are around pulleys.)

(g) $\lim_{\phi\to0^+} L = \infty$

65. False. For instance, let

$f(x) = \frac{x^2 - 1}{x - 1}$ or

$g(x) = \frac{x}{x^2 + 1}$.

66. True

67. False. The graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$ and $y = \csc x$ have vertical asymptotes.

68. False. Let

$$f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 3, & x = 0. \end{cases}$$

The graph of f has a vertical asymptote at $x = 0$, but $f(0) = 3$.

69. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x^4}$, and $c = 0$.

$\lim_{x\to 0}\frac{1}{x^2} = \infty$ and $\lim_{x\to 0}\frac{1}{x^4} = \infty$, but $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{x^4}\right) = \lim_{x\to 0}\left(\frac{x^2 - 1}{x^4}\right) = -\infty \neq 0$.

70. Given $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$:

(1) Difference:

Let $h(x) = -g(x)$. Then $\lim_{x\to c} h(x) = -L$, and $\lim_{x\to c}\left[f(x) - g(x)\right] = \lim_{x\to c}\left[f(x) + h(x)\right] = \infty$, by the Sum Property.

(2) Product:

If $L > 0$, then for $\varepsilon = L/2 > 0$ there exists $\delta_1 > 0$ such that $\left|g(x) - L\right| < L/2$ whenever $0 < \left|x - c\right| < \delta_1$. So, $L/2 < g(x) < 3L/2$. Because $\lim_{x\to c} f(x) = \infty$ then for $M > 0$, there exists $\delta_2 > 0$ such that $f(x) > M(2/L)$ whenever $\left|x - c\right| < \delta_2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < \left|x - c\right| < \delta$, you have $f(x)g(x) > M(2/L)(L/2) = M$. Therefore $\lim_{x\to c} f(x)g(x) = \infty$. The proof is similar for $L < 0$.

(3) Quotient: Let $\varepsilon > 0$ be given.

There exists $\delta_1 > 0$ such that $f(x) > 3L/2\varepsilon$ whenever $0 < \left|x - c\right| < \delta_1$ and there exists $\delta_2 > 0$ such that $\left|g(x) - L\right| < L/2$ whenever $0 < \left|x - c\right| < \delta_2$. This inequality gives us $L/2 < g(x) < 3L/2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < \left|x - c\right| < \delta$, you have

$$\left|\frac{g(x)}{f(x)}\right| < \frac{3L/2}{3L/2\varepsilon} = \varepsilon.$$

Therefore, $\lim_{x\to c}\frac{g(x)}{f(x)} = 0$.

71. Given $\lim_{x\to c} f(x) = \infty$, let $g(x) = 1$. Then

$\lim_{x\to c}\frac{g(x)}{f(x)} = 0$ by Theorem 1.15.

72. Given $\lim_{x\to c}\frac{1}{f(x)} = 0$. Suppose $\lim_{x\to c} f(x)$ exists and equals L.

Then, $\lim_{x\to c}\frac{1}{f(x)} = \frac{\lim_{x\to c} 1}{\lim_{x\to c} f(x)} = \frac{1}{L} = 0$.

This is not possible. So, $\lim_{x\to c} f(x)$ does not exist.

73. $f(x) = \frac{1}{x - 3}$ is defined for all $x > 3$.

Let $M > 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{1}{x - 3} > M$ whenever $3 < x < 3 + \delta$.

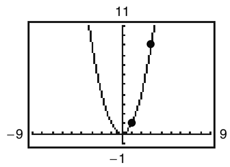
Equivalently, $x - 3 < \frac{1}{M}$ whenever $\left|x - 3\right| < \delta, x > 3$.

So take $\delta = \frac{1}{M}$. Then for $x > 3$ and $\left|x - 3\right| < \delta, \frac{1}{x - 3} > \frac{1}{8} = M$ and so $f(x) > M$.

74. $f(x) = \frac{1}{x - 5}$ is defined for all $x < 5$. Let $N < 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{1}{x - 5} < N$ whenever $5 - \delta < x < 5$. Equivalently, $x - 5 > \frac{1}{N}$ whenever $\left|x - 5\right| < \delta, x < 5$. Equivalently, $\frac{1}{\left|x - 5\right|} < -\frac{1}{N}$ whenever $\left|x - 5\right| < \delta, x < 5$. So take $\delta = -\frac{1}{N}$. Note that $\delta > 0$ because $N < 0$. For $\left|x - 5\right| < \delta$ and $x < 5, \frac{1}{\left|x - 5\right|} > \frac{1}{\delta} = -N$, and $\frac{1}{x - 5} = -\frac{1}{\left|x - 5\right|} < N$.

Review Exercises for Chapter 1

1. Calculus required. Using a graphing utility, you can estimate the length to be 8.3. Or, the length is slightly longer than the distance between the two points, approximately 8.25.

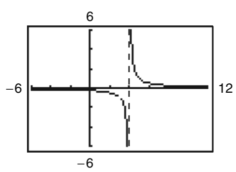


2. Precalculus. $L = \sqrt{(9-1)^2 + (3-1)^2} \approx 8.25$

3. $f(x) = \dfrac{x-3}{x^2 - 7x + 12}$

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	–0.9091	–0.9901	–0.9990	?	–1.0010	–1.0101	–1.1111

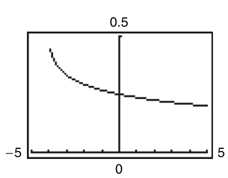
$\lim_{x\to 3} f(x) \approx -1.0000$ (Actual limit is -1.)



4. $f(x) = \dfrac{\sqrt{x+4} - 2}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.2516	0.2502	0.2500	?	0.2500	0.2498	0.2485

$\lim_{x\to 0} f(x) \approx 0.2500$ $\left(\text{Actual limit is } \tfrac{1}{4}.\right)$



5. $h(x) = \dfrac{4x - x^2}{x} = \dfrac{x(4-x)}{x} = 4 - x, x \neq 0$

(a) $\lim_{x\to 0} h(x) = 4 - 0 = 4$

(b) $\lim_{x\to -1} h(x) = 4 - (-1) = 5$

6. $g(x) = \dfrac{-2x}{x-3}$

(a) $\lim_{x\to 3} g(x)$ does not exist

(b) $\lim_{x\to 0} g(x) = \dfrac{-2(0)}{0-3} = 0$

7. $\lim_{x \to 1} (x + 4) = 1 + 4 = 5$

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then for $0 < |x - 1| < \delta = \varepsilon$, you have

$$\begin{aligned} |x - 1| &< \varepsilon \\ |(x + 4) - 5| &< \varepsilon \\ |f(x) - L| &< \varepsilon. \end{aligned}$$

8. $\lim_{x \to 9} \sqrt{x} = \sqrt{9} = 3$

Let $\varepsilon > 0$ be given. You need

$\left|\sqrt{x} - 3\right| < \varepsilon \Rightarrow \left|\sqrt{x} + 3\right|\left|\sqrt{x} - 3\right| < \varepsilon\left|\sqrt{x} + 3\right| \Rightarrow |x - 9| < \varepsilon\left|\sqrt{x} + 3\right|.$

Assuming $4 < x < 16$, you can choose $\delta = 5\varepsilon$.

So, for $0 < |x - 9| < \delta = 5\varepsilon$, you have

$$\begin{aligned} |x - 9| < 5\varepsilon &< \left|\sqrt{x} + 3\right|\varepsilon \\ \left|\sqrt{x} - 3\right| &< \varepsilon \\ |f(x) - L| &< \varepsilon. \end{aligned}$$

9. $\lim_{x \to 2} (1 - x^2) = 1 - 2^2 = -3$

Let $\varepsilon > 0$ be given. You need

$\left|1 - x^2 - (-3)\right| < \varepsilon \Rightarrow \left|x^2 - 4\right| = |x - 2||x + 2| < \varepsilon \Rightarrow |x - 2| < \dfrac{1}{|x + 2|}\varepsilon$

Assuming $1 < x < 3$, you can choose $\delta = \dfrac{\varepsilon}{5}$.

So, for $0 < |x - 2| < \delta = \dfrac{\varepsilon}{5}$, you have

$$\begin{aligned} |x - 2| < \frac{\varepsilon}{5} &< \frac{\varepsilon}{|x + 2|} \\ |x - 2||x + 2| &< \varepsilon \\ \left|x^2 - 4\right| &< \varepsilon \\ \left|4 - x^2\right| &< \varepsilon \\ \left|(1 - x^2) - (-3)\right| &< \varepsilon \\ |f(x) - L| &< \varepsilon. \end{aligned}$$

10. $\lim_{x \to 5} 9 = 9$. Let $\varepsilon > 0$ be given. δ can be any positive number. So, for $0 < |x - 5| < \delta$, you have

$$\begin{aligned} |9 - 9| &< \varepsilon \\ |f(x) - L| &< \varepsilon. \end{aligned}$$

11. $\lim_{x \to -6} x^2 = (-6)^2 = 36$

12. $\lim_{x \to 0} (5x - 3) = 5(0) - 3 = -3$

13. $\lim_{t \to 4} \sqrt{t + 2} = \sqrt{4 + 2} = \sqrt{6} \approx 2.45$

14. $\lim_{x \to -5} \sqrt[3]{x - 3} = \sqrt[3]{(-5) - 3} = \sqrt[3]{-8} = -2$

15. $\lim_{x \to 6} (x - 2)^2 = (6 - 2)^2 = 16$

16. $\lim_{x \to 7} (x - 4)^3 = (7 - 4)^3 = 3^3 = 27$

17. $\lim_{x\to 4}\frac{4}{x-1} = \frac{4}{4-1} = \frac{4}{3}$

18. $\lim_{x\to 2}\frac{x}{x^2+1} = \frac{2}{2^2+1} = \frac{2}{4+1} = \frac{2}{5}$

19. $\lim_{t\to -2}\frac{t+2}{t^2-4} = \lim_{t\to -2}\frac{1}{t-2} = -\frac{1}{4}$

20. $\lim_{t\to 4}\frac{t^2-16}{t-4} = \lim_{t\to 4}\frac{(t-4)(t+4)}{t-4}$
$= \lim_{t\to 4}(t+4) = 4+4 = 8$

21. $\lim_{x\to 4}\frac{\sqrt{x-3}-1}{x-4} = \lim_{x\to 4}\frac{\sqrt{x-3}-1}{x-4}\cdot\frac{\sqrt{x-3}+1}{\sqrt{x-3}+1}$
$= \lim_{x\to 4}\frac{(x-3)-1}{(x-4)\left(\sqrt{x-3}+1\right)}$
$= \lim_{x\to 4}\frac{1}{\sqrt{x-3}+1} = \frac{1}{2}$

22. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}\cdot\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$
$= \lim_{x\to 0}\frac{1}{\sqrt{4+x}+2} = \frac{1}{4}$

23. $\lim_{x\to 0}\frac{\left[1/(x+1)\right]-1}{x} = \lim_{x\to 0}\frac{1-(x+1)}{x(x+1)}$
$= \lim_{x\to 0}\frac{-1}{x+1} = -1$

24. $\lim_{s\to 0}\frac{\left(1/\sqrt{1+s}\right)-1}{s} = \lim_{s\to 0}\left[\frac{\left(1/\sqrt{1+s}\right)-1}{s}\cdot\frac{\left(1/\sqrt{1+s}\right)+1}{\left(1/\sqrt{1+s}\right)+1}\right]$
$= \lim_{s\to 0}\frac{\left[1/(1+s)\right]-1}{s\left[\left(1/\sqrt{1+s}\right)+1\right]} = \lim_{s\to 0}\frac{-1}{(1+s)\left[\left(1/\sqrt{1+s}\right)+1\right]} = -\frac{1}{2}$

25. $\lim_{x\to 0}\frac{1-\cos x}{\sin x} = \lim_{x\to 0}\left(\frac{x}{\sin x}\right)\left(\frac{1-\cos x}{x}\right) = (1)(0) = 0$

26. $\lim_{x\to(\pi/4)}\frac{4x}{\tan x} = \frac{4(\pi/4)}{1} = \pi$

27. $\lim_{\Delta x\to 0}\frac{\sin\left[(\pi/6)+\Delta x\right]-(1/2)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin(\pi/6)\cos\Delta x+\cos(\pi/6)\sin\Delta x-(1/2)}{\Delta x}$
$= \lim_{\Delta x\to 0}\frac{1}{2}\cdot\frac{(\cos\Delta x-1)}{\Delta x} + \lim_{\Delta x\to 0}\frac{\sqrt{3}}{2}\cdot\frac{\sin\Delta x}{\Delta x} = 0 + \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$

28. $\lim_{\Delta x\to 0}\frac{\cos(\pi+\Delta x)+1}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cos\pi\cos\Delta x-\sin\pi\sin\Delta x+1}{\Delta x}$
$= \lim_{\Delta x\to 0}\left[-\frac{(\cos\Delta x-1)}{\Delta x}\right] - \lim_{\Delta x\to 0}\left[\sin\pi\frac{\sin\Delta x}{\Delta x}\right]$
$= -0-(0)(1) = 0$

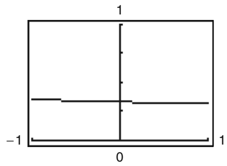
29. $\lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c}f(x)\right]\left[\lim_{x\to c}g(x)\right]$
$= (-6)\left(\frac{1}{2}\right) = -3$

30. $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c}f(x)}{\lim_{x\to c}g(x)} = \frac{-6}{\left(\frac{1}{2}\right)} = -12$

31. $\lim_{x\to c}\left[f(x)+2g(x)\right] = \lim_{x\to c}f(x) + 2\lim_{x\to c}g(x)$
$= -6+2\left(\frac{1}{2}\right) = -5$

32. $\lim_{x\to c}\left[f(x)\right]^2 = \left[\lim_{x\to c}f(x)\right]^2$
$= (-6)^2 = 36$

33. $f(x) = \dfrac{\sqrt{2x+9} - 3}{x}$



The limit appears to be $\dfrac{1}{3}$.

x	–0.01	–0.001	0	0.001	0.01
$f(x)$	0.3335	0.3333	?	0.3333	0.331

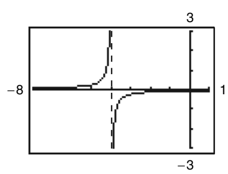
$$\lim_{x\to 0} f(x) \approx 0.3333$$

$$\lim_{x\to 0} \frac{\sqrt{2x+9} - 3}{x} \cdot \frac{\sqrt{2x+9} + 3}{\sqrt{2x+9} + 3} = \lim_{x\to 0} \frac{(2x+9) - 9}{x\left[\sqrt{2x+9} + 3\right]}$$

$$= \lim_{x\to 0} \frac{2}{\sqrt{2x+9} + 3}$$

$$= \frac{2}{\sqrt{9} + 3} = \frac{1}{3}$$

34. $f(x) = \dfrac{\left[1/(x+4)\right] - (1/4)}{x}$



The limit appears to be $-\dfrac{1}{16}$

x	–0.01	–0.001	0	0.001	0.01
$f(x)$	–0.0627	–0.0625	?	–0.0625	–0.0623

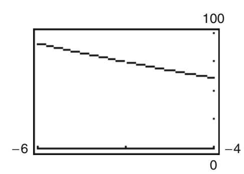
$$\lim_{x\to 0} f(x) \approx -0.0625 = -\frac{1}{16}$$

$$\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x} = \lim_{x\to 0} \frac{4 - (x+4)}{(x+4)4(x)}$$

$$= \lim_{x\to 0} \frac{-1}{(x+4)4}$$

$$= -\frac{1}{16}$$

35. $f(x) = \dfrac{x^3 + 125}{x + 5}$



The limit appears to be 75.

x	–5.01	–5.001	–5	–4.999	–4.99
$f(x)$	75.15	75.015	?	74.985	74.85

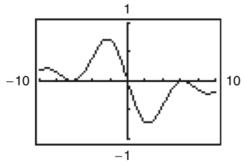
$$\lim_{x\to -5} f(x) \approx 75.000$$

$$\lim_{x\to -5} \frac{x^3 + 125}{x + 5} = \lim_{x\to -5} \frac{(x+5)(x^2 - 5x + 25)}{x + 5}$$

$$= \lim_{x\to -5} (x^2 - 5x + 25)$$

$$= (-5)^2 - 5(-5) + 25 = 75$$

36. $f(x) = \dfrac{\cos x - 1}{x}$



The limit appears to be 0.

x	−0.01	−0.001	0	0.001	0.01
$f(x)$	0.005	0.0005	0	−0.0005	−0.005

$\lim_{x\to 0} f(x) \approx 0.000$

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = \lim_{x\to 0}\frac{\cos x - 1}{x}\cdot\frac{\cos x + 1}{\cos x + 1}$$
$$= \lim_{x\to 0}\frac{\cos^2 x - 1}{x(\cos x + 1)}$$
$$= \lim_{x\to 0}\frac{-\sin^2 x}{x(\cos x + 1)}$$
$$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{-\sin x}{\cos x + 1}\right)$$
$$= (1)\left(\frac{0}{2}\right)$$
$$= 0$$

37. $$v = \lim_{t\to 4}\frac{s(4) - s(t)}{4 - t}$$
$$= \lim_{t\to 4}\frac{\left[-4.9(16) + 250\right] - \left[-4.9t^2 + 250\right]}{4 - t}$$
$$= \lim_{t\to 4}\frac{4.9\left(t^2 - 16\right)}{4 - t}$$
$$= \lim_{t\to 4}\frac{4.9(t - 4)(t + 4)}{4 - t}$$
$$= \lim_{t\to 4}\left[-4.9(t + 4)\right] = -39.2 \text{ m/sec}$$

The object is falling at about 39.2 m/sec.

38. $-4.9t^2 + 250 = 0 \Rightarrow t = \dfrac{50}{7}$ sec

When $a = \dfrac{50}{7}$, the velocity is

$$\lim_{t\to a}\frac{s(a) - s(t)}{a - t} = \lim_{t\to a}\frac{\left[-4.9a^2 + 250\right] - \left[-4.9t^2 + 250\right]}{a - t}$$
$$= \lim_{t\to a}\frac{4.9\left(t^2 - a^2\right)}{a - t}$$
$$= \lim_{t\to a}\frac{4.9(t - a)(t + a)}{a - t}$$
$$= \lim_{t\to a}\left[-4.9(t + a)\right]$$
$$= -4.9(2a) \qquad \left(a = \frac{50}{7}\right)$$
$$= -70 \text{ m/sec.}$$

The velocity of the object when it hits the ground is about 70 m/sec.

39. $$\lim_{x\to 3^+}\frac{1}{x + 3} = \frac{1}{3 + 3} = \frac{1}{6}$$

40. $$\lim_{x\to 6^-}\frac{x - 6}{x^2 - 36} = \lim_{x\to 6^-}\frac{x - 6}{(x - 6)(x + 6)}$$
$$= \lim_{x\to 6^-}\frac{1}{x + 6}$$
$$= \frac{1}{12}$$

41. $$\lim_{x\to 4^-}\frac{\sqrt{x} - 2}{x - 4} = \lim_{x\to 4^-}\frac{\sqrt{x} - 2}{x - 4}\cdot\frac{\sqrt{x} + 2}{\sqrt{x} + 2}$$
$$= \lim_{x\to 4^-}\frac{x - 4}{(x - 4)\left(\sqrt{x} + 2\right)}$$
$$= \lim_{x\to 4^-}\frac{1}{\sqrt{x} + 2}$$
$$= \frac{1}{4}$$

42. $$\lim_{x\to 3^-}\frac{|x - 3|}{x - 3} = \lim_{x\to 3^-}\frac{-(x - 3)}{x - 3} = -1$$

43. $\lim_{x\to 2} f(x) = 0$

44. $\lim_{x\to 1^+} g(x) = 1 + 1 = 2$

45. $\lim_{t\to 1} h(t)$ does not exist because $\lim_{t\to 1^-} h(t) = 1 + 1 = 2$ and $\lim_{t\to 1^+} h(t) = \frac{1}{2}(1 + 1) = 1$.

46. $\lim_{s\to -2} f(s) = 2$

47. $\lim_{x\to 2^-} \left(2[\![x]\!] + 1\right) = 2(1) + 1 = 3$

48. $\lim_{x\to 4} [\![x - 1]\!]$ does not exist. There is a break in the graph at $x = 4$.

49. $f(x) = x^2 - 4$ is continuous for all real x.

50. $f(x) = x^2 - x + 20$ is continuous for all real x.

51. $f(x) = \dfrac{4}{x - 5}$ has a nonremovable discontinuity at $x = 5$ because $\lim_{x\to 5} f(x)$ does not exist.

52. $f(x) = \dfrac{1}{x^2 - 9} = \dfrac{1}{(x - 3)(x + 3)}$

has nonremovable discontinuities at $x = \pm 3$ because $\lim_{x\to 3} f(x)$ and $\lim_{x\to -3} f(x)$ do not exist.

53. $f(x) = \dfrac{x}{x^3 - x} = \dfrac{x}{x(x^2 - 1)} = \dfrac{1}{(x - 1)(x + 1)}, x \neq 0$

has nonremovable discontinuities at $x = \pm 1$ because $\lim_{x\to -1} f(x)$ and $\lim_{x\to 1} f(x)$ do not exist, and has a removable discontinuity at $x = 0$ because

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1}{(x - 1)(x + 1)} = -1.$$

54. $f(x) = \dfrac{x + 3}{x^2 - 3x - 18}$

$$= \frac{x + 3}{(x + 3)(x - 6)}$$

$$= \frac{1}{x - 6}, x \neq -3$$

has a nonremovable discontinuity at $x = 6$ because $\lim_{x\to 6} f(x)$ does not exist, and has a removable discontinuity at $x = -3$ because

$$\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{1}{x - 6} = -\frac{1}{9}.$$

55. $f(2) = 5$

Find c so that $\lim_{x\to 2^+} (cx + 6) = 5$.

$$c(2) + 6 = 5$$
$$2c = -1$$
$$c = -\frac{1}{2}$$

56. $\lim_{x\to 1^+} (x + 1) = 2$

$\lim_{x\to 3^-} (x + 1) = 4$

Find b and c so that $\lim_{x\to 1^-} \left(x^2 + bx + c\right) = 2$ and $\lim_{x\to 3^+} \left(x^2 + bx + c\right) = 4$.

Consequently you get $1 + b + c = 2$ and $9 + 3b + c = 4$.

Solving simultaneously, $b = -3$ and $c = 4$.

57. $f(x) = -3x^2 + 7$

Continuous on $(-\infty, \infty)$

58. $f(x) = \dfrac{4x^2 + 7x - 2}{x + 2} = \dfrac{(4x - 1)(x + 2)}{x + 2}$

Continuous on $(-\infty, -2) \cup (-2, \infty)$. There is a removable discontinuity at $x = -2$.

59. $f(x) = \sqrt{x - 4}$

Continuous on $[4, \infty)$

60. $f(x) = [\![x + 3]\!]$

$\lim_{x\to k^+} [\![x + 3]\!] = k + 3$ where k is an integer.

$\lim_{x\to k^-} [\![x + 3]\!] = k + 2$ where k is an integer.

Nonremovable discontinuity at each integer k

Continuous on $(k, k + 1)$ for all integers k

61. $f(x) = \dfrac{3x^2 - x - 2}{x - 1} = \dfrac{(3x + 2)(x - 1)}{x - 1}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} (3x + 2) = 5$

Removable discontinuity at $x = 1$

Continuous on $(-\infty, 1) \cup (1, \infty)$

62. $f(x) = \begin{cases} 5 - x, & x \le 2 \\ 2x - 3, & x > 2 \end{cases}$

$\lim_{x \to 2^-} (5 - x) = 3$

$\lim_{x \to 2^+} (2x - 3) = 1$

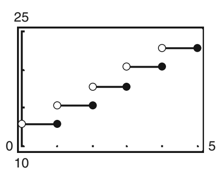
Nonremovable discontinuity at $x = 2$

Continuous on $(-\infty, 2) \cup (2, \infty)$

63. f is continuous on $[1, 2]$. $f(1) = -1 < 0$ and $f(2) = 13 > 0$. Therefore by the Intermediate Value Theorem, there is at least one value c in $(1, 2)$ such that $2c^3 - 3 = 0$.

64. $C(x) = 12.80 + 2.50[-[\![-x]\!] - 1], \quad x > 0$

$= 12.80 - 2.50[[\![-x]\!] + 1], \quad x > 0$



C has a nonremovable discontinuity at each integer 1, 2, 3,….

65. $f(x) = \dfrac{x^2 - 4}{|x - 2|} = (x + 2)\left[\dfrac{x - 2}{|x - 2|}\right]$

(a) $\lim_{x \to 2^-} f(x) = -4$

(b) $\lim_{x \to 2^+} f(x) = 4$

(c) $\lim_{x \to 2} f(x)$ does not exist.

66. $f(x) = \sqrt{(x - 1)x}$

(a) Domain: $(-\infty, 0] \cup [1, \infty)$

(b) $\lim_{x \to 0^-} f(x) = 0$

(c) $\lim_{x \to 1^+} f(x) = 0$

67. $f(x) = \dfrac{3}{x}$

$\lim_{x \to 0^-} \dfrac{3}{x} = -\infty$

$\lim_{x \to 0^+} \dfrac{3}{x} = \infty$

Therefore, $x = 0$ is a vertical asymptote.

68. $f(x) = \dfrac{5}{(x - 2)^4}$

$\lim_{x \to 2^-} \dfrac{5}{(x - 2)^4} = \infty = \lim_{x \to 2^+} \dfrac{5}{(x - 2)^4}$

Therefore, $x = 2$ is a vertical asymptote.

69. $f(x) = \dfrac{x^3}{x^2 - 9} = \dfrac{x^3}{(x + 3)(x - 3)}$

$\lim_{x \to -3^-} \dfrac{x^3}{x^2 - 9} = -\infty$ and $\lim_{x \to -3^+} \dfrac{x^3}{x^2 - 9} = \infty$

Therefore, $x = -3$ is a vertical asymptote.

$\lim_{x \to -3^-} \dfrac{x^3}{x^2 - 9} = -\infty$ and $\lim_{x \to 3^+} \dfrac{x^3}{x^2 - 9} = \infty$

Therefore, $x = 3$ is a vertical asymptote.

70. $f(x) = \dfrac{6x}{36 - x^2} = -\dfrac{6x}{(x + 6)(x - 6)}$

$\lim_{x \to -6^-} \dfrac{6x}{36 - x^2} = \infty$ and $\lim_{x \to -6^+} \dfrac{6x}{36 - x^2} = -\infty$

Therefore, $x = -6$ is a vertical asymptote.

$\lim_{x \to 6^-} \dfrac{6x}{36 - x^2} = \infty$ and $\lim_{x \to 6^+} \dfrac{6x}{36 - x^2} = -\infty$

Therefore, $x = 6$ is a vertical asymptote.

71. $g(x) = \dfrac{2x + 1}{x^2 - 64} = \dfrac{2x + 1}{(x + 8)(x - 8)}$

$\lim_{x \to -8^-} \dfrac{2x + 1}{x^2 - 64} = -\infty$ and $\lim_{x \to -8^+} \dfrac{2x + 1}{x^2 - 64} = \infty$

Therefore, $x = -8$ is a vertical asymptote.

$\lim_{x \to 8^-} \dfrac{2x + 1}{x^2 - 64} = -\infty$ and $\lim_{x \to 8^+} \dfrac{2x + 1}{x^2 - 64} = \infty$

Therefore, $x = 8$ is a vertical asymptote.

72. $f(x) = \csc \pi x = \dfrac{1}{\sin \pi x}$

$\sin \pi x = 0$ for $x = n$, where n is an integer.

$\lim_{x \to n} f(x) = \infty$ or $-\infty$

Therefore, the graph has vertical asymptotes at $x = n$.

73. $\lim_{x \to 1^-} \dfrac{x^2 + 2x + 1}{x - 1} = -\infty$

74. $\lim_{x \to (1/2)^+} \dfrac{x}{2x - 1} = \infty$

75. $\lim_{x \to -1^+} \dfrac{x + 1}{x^3 + 1} = \lim_{x \to -1^+} \dfrac{1}{x^2 - x + 1} = \dfrac{1}{3}$

76. $\lim_{x \to -1^-} \dfrac{x + 1}{x^4 - 1} = \lim_{x \to -1^-} \dfrac{1}{(x^2 + 1)(x - 1)} = -\dfrac{1}{4}$

77. $\lim_{x \to 0^+} \left(x - \dfrac{1}{x^3}\right) = -\infty$

78. $\lim_{x \to 2^-} \dfrac{1}{\sqrt[3]{x^2 - 4}} = -\infty$

79. $\lim_{x\to 0^+} \frac{\sin 4x}{5x} = \lim_{x\to 0^+}\left[\frac{4}{5}\left(\frac{\sin 4x}{4x}\right)\right] = \frac{4}{5}$

80. $\lim_{x\to 0^+} \frac{\sec x}{x} = \infty$

81. $\lim_{x\to 0^+} \frac{\csc 2x}{x} = \lim_{x\to 0^+} \frac{1}{x \sin 2x} = \infty$

82. $\lim_{x\to 0^-} \frac{\cos^2 x}{x} = -\infty$

83. $C = \frac{80{,}000p}{100 - p}, 0 \le p < 100$

(a) $C(15) \approx \$14{,}117.65$

(b) $C(50) = \$80.000$

(c) $C(90) = \$720{,}000$

(d) $\lim_{p\to 100^-} \frac{80{,}000p}{100 - p} = \infty$

84. $f(x) = \frac{\tan 2x}{x}$

(a)

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	2.0271	2.0003	2.0000	2.0000	2.0003	2.0271

$\lim_{x\to 0} \frac{\tan 2x}{x} = 2$

(b) Yes, define $f(x) = \begin{cases} \frac{\tan 2x}{x}, & x \ne 0 \\ 2, & x = 0 \end{cases}.$

Now $f(x)$ is continuous at $x = 0$.

Problem Solving for Chapter 1

1. (a) Perimeter $\Delta PAO = \sqrt{x^2 + (y-1)^2} + \sqrt{x^2 + y^2} + 1$

$= \sqrt{x^2 + (x^2 - 1)^2} + \sqrt{x^2 + x^4} + 1$

Perimeter $\Delta PBO = \sqrt{(x-1)^2 + y^2} + \sqrt{x^2 + y^2} + 1$

$= \sqrt{(x-1)^2 + x^4} + \sqrt{x^2 + x^4} + 1$

(b) $r(x) = \frac{\sqrt{x^2 + (x^2-1)^2} + \sqrt{x^2 + x^4} + 1}{\sqrt{(x-1)^2 + x^4} + \sqrt{x^2 + x^4} + 1}$

x	4	2	1	0.1	0.01
Perimeter ΔPAO	33.02	9.08	3.41	2.10	2.01
Perimeter ΔPBO	33.77	9.60	3.41	2.00	2.00
$r(x)$	0.98	0.95	1	1.05	1.005

(c) $\lim_{x\to 0^+} r(x) = \frac{1 + 0 + 1}{1 + 0 + 1} = \frac{2}{2} = 1$

2. (a) Area $\Delta PAO = \frac{1}{2}bh = \frac{1}{2}(1)(x) = \frac{x}{2}$

Area $\Delta PBO = \frac{1}{2}bh = \frac{1}{2}(1)(y) = \frac{y}{2} = \frac{x^2}{2}$

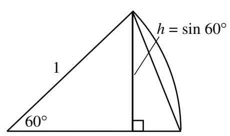
(b) $a(x) = \frac{\text{Area } \Delta PBO}{\text{Area } \Delta PAO} = \frac{x^2/2}{x/2} = x$

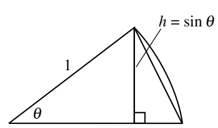
x	4	2	1	0.1	0.01
Area ΔPAO	2	1	1/2	1/20	1/200
Area ΔPBO	8	2	1/2	1/200	1/20,000
$a(x)$	4	2	1	1/10	1/100

(c) $\lim_{x \to 0^+} a(x) = \lim_{x \to 0^+} x = 0$

3. (a) There are 6 triangles, each with a central angle of $60° = \pi/3$. So,

Area hexagon $= 6\left[\frac{1}{2}bh\right] = 6\left[\frac{1}{2}(1)\sin\frac{\pi}{3}\right] = \frac{3\sqrt{3}}{2} \approx 2.598.$





Error $=$ Area (Circle) $-$ Area (Hexagon) $= \pi - \frac{3\sqrt{3}}{2} \approx 0.5435$

(b) There are n triangles, each with central angle of $\theta = 2\pi/n$. So,

$A_n = n\left[\frac{1}{2}bh\right] = n\left[\frac{1}{2}(1)\sin\frac{2\pi}{n}\right] = \frac{n\sin(2\pi/n)}{2}.$

(c)

n	6	12	24	48	96
A_n	2.598	3	3.106	3.133	3.139

(d) As n gets larger and larger, $2\pi/n$ approaches 0. Letting $x = 2\pi/n$, $A_n = \frac{\sin(2\pi/n)}{2/n} = \frac{\sin(2\pi/n)}{(2\pi/n)}\pi = \frac{\sin x}{x}\pi$

which approaches $(1)\pi = \pi$.

4. (a) Slope $= \frac{4-0}{3-0} = \frac{4}{3}$

(b) Slope $= -\frac{3}{4}$ Tangent line: $y - 4 = -\frac{3}{4}(x-3)$

$y = -\frac{3}{4}x + \frac{25}{4}$

(c) Let $Q = (x, y) = \left(x, \sqrt{25-x^2}\right)$

$m_x = \frac{\sqrt{25-x^2} - 4}{x-3}$

(d) $\lim_{x\to 3} m_x = \lim_{x\to 3}\frac{\sqrt{25-x^2}-4}{x-3}\cdot\frac{\sqrt{25-x^2}+4}{\sqrt{25-x^2}+4}$

$= \lim_{x\to 3}\frac{25-x^2-16}{(x-3)\left(\sqrt{25-x^2}+4\right)}$

$= \lim_{x\to 3}\frac{(3-x)(3+x)}{(x-3)\left(\sqrt{25-x^2}+4\right)}$

$= \lim_{x\to 3}\frac{-(3+x)}{\sqrt{25-x^2}+4} = \frac{-6}{4+4} = -\frac{3}{4}$

This is the slope of the tangent line at P.

5. (a) Slope $= -\dfrac{12}{5}$

(b) Slope of tangent line is $\dfrac{5}{12}$.

$$y + 12 = \frac{5}{12}(x - 5)$$

$$y = \frac{5}{12}x - \frac{169}{12} \quad \text{Tangent line}$$

(c) $Q = (x, y) = \left(x, -\sqrt{169 - x^2}\right)$

$$m_x = \frac{-\sqrt{169 - x^2} + 12}{x - 5}$$

(d) $$\lim_{x\to5} m_x = \lim_{x\to5}\frac{12 - \sqrt{169 - x^2}}{x - 5}\cdot\frac{12 + \sqrt{169 - x^2}}{12 + \sqrt{169 - x^2}}$$

$$= \lim_{x\to5}\frac{144 - \left(169 - x^2\right)}{(x - 5)\left(12 + \sqrt{169 - x^2}\right)}$$

$$= \lim_{x\to5}\frac{x^2 - 25}{(x - 5)\left(12 + \sqrt{169 - x^2}\right)}$$

$$= \lim_{x\to5}\frac{(x + 5)}{12 + \sqrt{169 - x^2}} = \frac{10}{12 + 12} = \frac{5}{12}$$

This is the same slope as part (b).

6. $$\frac{\sqrt{a + bx} - \sqrt{3}}{x} = \frac{\sqrt{a + bx} - \sqrt{3}}{x}\cdot\frac{\sqrt{a + bx} + \sqrt{3}}{\sqrt{a + bx} + \sqrt{3}} = \frac{(a + bx) - 3}{x\left(\sqrt{a + bx} + \sqrt{3}\right)}$$

Letting $a = 3$ simplifies the numerator.

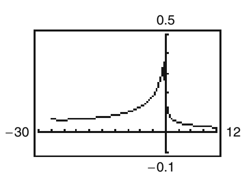
So, $$\lim_{x\to0}\frac{\sqrt{3 + bx} - \sqrt{3}}{x} = \lim_{x\to0}\frac{bx}{x\left(\sqrt{3 + bx} + \sqrt{3}\right)} = \lim_{x\to0}\frac{b}{\sqrt{3 + bx} + \sqrt{3}}$$

Setting $\dfrac{b}{\sqrt{3} + \sqrt{3}} = \sqrt{3}$, you obtain $b = 6$. So, $a = 3$ and $b = 6$.

7. (a) $$3 + x^{1/3} \ge 0$$
$$x^{1/3} \ge -3$$
$$x \ge -27$$

Domain: $x \ge -27, x \ne 1$ or $[-27, 1) \cup (1, \infty)$

(b)



(c) $$\lim_{x\to-27^+} f(x) = \frac{\sqrt{3 + (-27)^{1/3}} - 2}{-27 - 1}$$
$$= \frac{-2}{-28}$$
$$= \frac{1}{14}$$
$$\approx 0.0714$$

(d) $$\lim_{x\to1} f(x) = \lim_{x\to1}\frac{\sqrt{3 + x^{1/3}} - 2}{x - 1}\cdot\frac{\sqrt{3 + x^{1/3}} + 2}{\sqrt{3 + x^{1/3}} + 2}$$

$$= \lim_{x\to1}\frac{3 + x^{1/3} - 4}{(x - 1)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

$$= \lim_{x\to1}\frac{x^{1/3} - 1}{\left(x^{1/3} - 1\right)\left(x^{2/3} + x^{1/3} + 1\right)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

$$= \lim_{x\to1}\frac{1}{\left(x^{2/3} + x^{1/3} + 1\right)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

$$= \frac{1}{(1 + 1 + 1)(2 + 2)} = \frac{1}{12}$$

8. $$\lim_{x\to0^-} f(x) = \lim_{x\to0^-}\left(a^2 - 2\right) = a^2 - 2$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+}\frac{ax}{\tan x} = a\left(\text{because } \lim_{x\to0}\frac{\tan x}{x} = 1\right)$$

Thus,

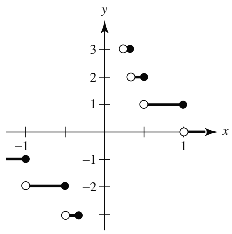
$$a^2 - 2 = a$$
$$a^2 - a - 2 = 0$$
$$(a - 2)(a + 1) = 0$$
$$a = -1, 2$$

9. (a) $\lim_{x\to2} f(x) = 3$: g_1, g_4

(b) f continuous at 2: g_1

(c) $\lim_{x\to2^-} f(x) = 3$: g_1, g_3, g_4

10.



(a) $f\left(\frac{1}{4}\right) = [\![4]\!] = 4$

$f(3) = \left[\!\left[\frac{1}{3}\right]\!\right] = 0$

$f(1) = [\![1]\!] = 1$

(b) $\lim_{x\to 1^-} f(x) = 1$

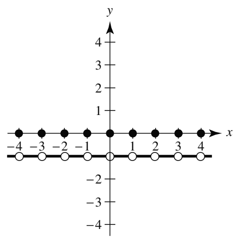
$\lim_{x\to 1^+} f(x) = 0$

$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to 0^+} f(x) = \infty$

(c) f is continuous for all real numbers except $x = 0, \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \ldots$

11.



(a) $f(1) = [\![1]\!] + [\![-1]\!] = 1 + (-1) = 0$

$f(0) = 0$

$f\left(\frac{1}{2}\right) = 0 + (-1) = -1$

$f(-2.7) = -3 + 2 = -1$

(b) $\lim_{x\to 1^-} f(x) = -1$

$\lim_{x\to 1^+} f(x) = -1$

$\lim_{x\to 1/2} f(x) = -1$

(c) f is continuous for all real numbers except $x = 0, \pm 1, \pm 2, \pm 3, \ldots$

12. (a) $v^2 = \dfrac{192{,}000}{r} + v_0^2 - 48$

$\dfrac{192{,}000}{r} = v^2 - v_0^2 + 48$

$r = \dfrac{192{,}000}{v^2 - v_0^2 + 48}$

$\lim_{v\to 0} r = \dfrac{192{,}000}{48 - v_0^2}$

Let $v_0 = \sqrt{48} = 4\sqrt{3}$ mi/sec.

(b) $v^2 = \dfrac{1920}{r} + v_0^2 - 2.17$

$\dfrac{1920}{r} = v^2 - v_0^2 + 2.17$

$r = \dfrac{1920}{v^2 - v_0^2 + 2.17}$

$\lim_{v\to 0} r = \dfrac{1920}{2.17 - v_0^2}$

Let $v_0 = \sqrt{2.17}$ mi/sec $(\approx 1.47$ mi/sec$)$.

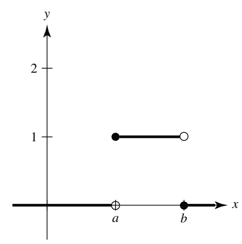
(c) $r = \dfrac{10{,}600}{v^2 - v_0^2 + 6.99}$

$\lim_{v\to 0} r = \dfrac{10{,}600}{6.99 - v_0^2}$

Let $v_0 = \sqrt{6.99} \approx 2.64$ mi/sec.

Because this is smaller than the escape velocity for Earth, the mass is less.

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) $P_{a,b}$ is continuous for all positive real numbers except $x = a, b$.

(d) The area under the graph of U, and above the x-axis, is 1.

14. Let $a \neq 0$ and let $\varepsilon > 0$ be given. There exists $\delta_1 > 0$ such that if $0 < |x - 0| < \delta_1$ then $|f(x) - L| < \varepsilon$. Let $\delta = \delta_1/|a|$. Then for $0 < |x - 0| < \delta = \delta_1/|a|$, you have

$$|x| < \frac{\delta_1}{|a|}$$

$$|ax| < \delta_1$$

$$|f(ax) - L| < \varepsilon.$$

As a counterexample, let

$a = 0$ and $f(x) = \begin{cases} 1, & x \neq 0 \\ 2, & x = 0 \end{cases}$.

Then $\lim_{x\to 0} f(x) = 1 = L$, but

$\lim_{x\to 0} f(ax) = \lim_{x\to 0} f(0) = \lim_{x\to 0} 2 = 2.$

CHAPTER 2
Differentiation

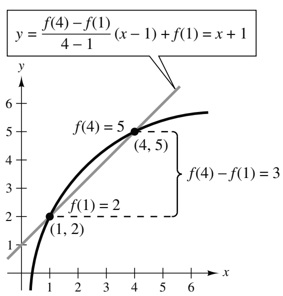
CHAPTER 2
Differentiation

Section 2.1 The Derivative and the Tangent Line Problem

1. At (x_1, y_1), slope $= 0$.
At (x_2, y_2), slope $= \frac{5}{2}$.

2. At (x_1, y_1), slope $= \frac{2}{3}$.
At (x_2, y_2), slope $= -\frac{2}{5}$.

3. (a), (b)



(c)
$$\begin{aligned} y &= \frac{f(4) - f(1)}{4 - 1}(x - 1) + f(1) \\ &= \frac{3}{3}(x - 1) + 2 \\ &= 1(x - 1) + 2 \\ &= x + 1 \end{aligned}$$

4. (a) $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{5 - 2}{3} = 1$

$\dfrac{f(4) - f(3)}{4 - 3} \approx \dfrac{5 - 4.75}{1} = 0.25$

So, $\dfrac{f(4) - f(1)}{4 - 1} > \dfrac{f(4) - f(3)}{4 - 3}$.

(b) The slope of the tangent line at $(1, 2)$ equals $f'(1)$. This slope is steeper than the slope of the line through $(1, 2)$ and $(4, 5)$. So, $\dfrac{f(4) - f(1)}{4 - 1} < f'(1)$.

5. $f(x) = 3 - 5x$ is a line. Slope $= -5$

6. $g(x) = \frac{3}{2}x + 1$ is a line. Slope $= \frac{3}{2}$

7.
$$\begin{aligned} \text{Slope at } (2, -5) &= \lim_{\Delta x \to 0} \frac{g(2 + \Delta x) - g(2)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(2 + \Delta x)^2 - 9 - (-5)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{4 + 4(\Delta x) + (\Delta x)^2 - 4}{\Delta x} \\ &= \lim_{\Delta x \to 0} (4 + \Delta x) = 4 \end{aligned}$$

8.
$$\begin{aligned} \text{Slope at } (3, -4) &= \lim_{\Delta x \to 0} \frac{f(3 + \Delta x) - f(3)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{5 - (3 + \Delta x)^2 - (-4)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{5 - 9 - 6(\Delta x) - (\Delta x)^2 + 4}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-6(\Delta x) - (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} (-6 - \Delta x) = -6 \end{aligned}$$

9.
$$\begin{aligned} \text{Slope at } (0, 0) &= \lim_{\Delta t \to 0} \frac{f(0 + \Delta t) - f(0)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{3(\Delta t) - (\Delta t)^2 - 0}{\Delta t} \\ &= \lim_{\Delta t \to 0} (3 - \Delta t) = 3 \end{aligned}$$

10.
$$\begin{aligned} \text{Slope at } (1, 5) &= \lim_{\Delta t \to 0} \frac{h(1 + \Delta t) - h(1)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{(1 + \Delta t)^2 + 4(1 + \Delta t) - 5}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{1 + 2(\Delta t) + (\Delta t)^2 + 4 + 4(\Delta t) - 5}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{6(\Delta t) + (\Delta t)^2}{\Delta t} \\ &= \lim_{\Delta t \to 0} (6 + \Delta t) = 6 \end{aligned}$$

11. $f(x) = 7$
$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{7 - 7}{\Delta x} \\ &= \lim_{\Delta x \to 0} 0 = 0 \end{aligned}$$

12. $g(x) = -3$
$$\begin{aligned} g'(x) &= \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-3 - (-3)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0 \end{aligned}$$

13. $f(x) = -10x$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-10(x + \Delta x) - (-10x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-10x - 10\Delta x + 10x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-10\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} (-10) = -10 \end{aligned}$$

14. $f(x) = 7x - 3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{7(x + \Delta x) - 3 - (7x - 3)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{7x + 7\Delta x - 3 - 7x + 3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{7(\Delta x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} 7 = 7 \end{aligned}$$

15. $h(s) = 3 + \frac{2}{3}s$

$$\begin{aligned} h'(s) &= \lim_{\Delta s \to 0} \frac{h(s + \Delta s) - h(s)}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}(s + \Delta s) - \left(3 + \frac{2}{3}s\right)}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}s + \frac{2}{3}\Delta s - 3 - \frac{2}{3}s}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{\frac{2}{3}\Delta s}{\Delta s} = \frac{2}{3} \end{aligned}$$

16. $f(x) = 5 - \frac{2}{3}x$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{5 - \frac{2}{3}(x + \Delta x) - \left(5 - \frac{2}{3}x\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{5 - \frac{2}{3}x - \frac{2}{3}\Delta x - 5 + \frac{2}{3}x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{2}{3}(\Delta x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \left(-\frac{2}{3}\right) = -\frac{2}{3} \end{aligned}$$

17. $f(x) = x^2 + x - 3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + (x + \Delta x) - 3 - (x^2 + x - 3)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - 3 - x^2 - x + 3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x + 1) = 2x + 1 \end{aligned}$$

18. $f(x) = x^2 - 5$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 5 - (x^2 - 5)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - 5 - x^2 + 5}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x \end{aligned}$$

19. $f(x) = x^3 - 12x$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 12(x + \Delta x)\right] - \left[x^3 - 12x\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\,\Delta x - x^3 + 12x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\,\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 - 12\right) = 3x^2 - 12
\end{aligned}$$

20. $f(x) = x^3 + x^2$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 + (x + \Delta x)^2\right] - \left[x^3 + x^2\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + x^2 + 2x\,\Delta x + (\Delta x)^2 - x^3 - x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2x\,\Delta x + (\Delta x)^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 + 2x + (\Delta x)\right) = 3x^2 + 2x
\end{aligned}$$

21. $f(x) = \dfrac{1}{x - 1}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x - 1} - \dfrac{1}{x - 1}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x - 1) - (x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)} \\
&= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)} \\
&= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x - 1)(x - 1)} \\
&= -\frac{1}{(x - 1)^2}
\end{aligned}$$

22. $f(x) = \dfrac{1}{x^2}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x + \Delta x)^2} - \dfrac{1}{x^2}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 - (x + \Delta x)^2}{\Delta x(x + \Delta x)^2 x^2} \\
&= \lim_{\Delta x \to 0} \frac{-2x\,\Delta x - (\Delta x)^2}{\Delta x(x + \Delta x)^2 x^2} \\
&= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{(x + \Delta x)^2 x^2} \\
&= \frac{-2x}{x^4} \\
&= -\frac{2}{x^3}
\end{aligned}$$

23. $f(x) = \sqrt{x + 4}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 4} - \sqrt{x + 4}}{\Delta x} \cdot \left(\frac{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 4) - (x + 4)}{\Delta x\left[\sqrt{x + \Delta x + 4} + \sqrt{x + 4}\right]}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}} = \frac{1}{\sqrt{x + 4} + \sqrt{x + 4}} = \frac{1}{2\sqrt{x + 4}}$$

24. $f(x) = \dfrac{4}{\sqrt{x}}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{4}{\sqrt{x + \Delta x}} - \dfrac{4}{\sqrt{x}}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4\sqrt{x} - 4\sqrt{x + \Delta x}}{\Delta x\sqrt{x}\sqrt{x + \Delta x}} \cdot \left(\frac{\sqrt{x} + \sqrt{x + \Delta x}}{\sqrt{x} + \sqrt{x + \Delta x}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{4x - 4(x + \Delta x)}{\Delta x\sqrt{x}\sqrt{x + \Delta x}\left(\sqrt{x} + \sqrt{x + \Delta x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{-4}{\sqrt{x}\sqrt{x + \Delta x}\left(\sqrt{x} + \sqrt{x + \Delta x}\right)}$$

$$= \frac{-4}{\sqrt{x}\sqrt{x}\left(\sqrt{x} + \sqrt{x}\right)} = \frac{-2}{x\sqrt{x}}$$

25. (a) $f(x) = x^2 + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 3\right] - \left(x^2 + 3\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3 - x^2 - 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

At $(-1, 4)$, the slope of the tangent line is

$m = 2(-1) = -2.$

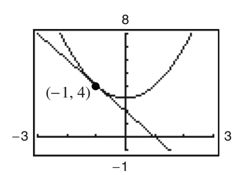
The equation of the tangent line is

$y - 4 = -2(x + 1)$

$y - 4 = -2x - 2$

$y = -2x + 2$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -2$ at $(-1, 4)$.

26. (a) $f(x) = x^2 + 2x - 1$

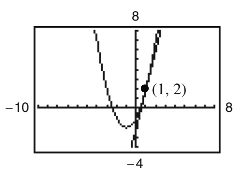
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 2(x + \Delta x) - 1\right] - \left[x^2 + 2x - 1\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x - 1\right] - \left[x^2 + 2x - 1\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x + 2) = 2x + 2$$

At $(1, 2)$, the slope of the tangent line is $m = 2(1) + 2 = 4$.

The equation of the tangent line is

$$y - 2 = 4(x - 1)$$
$$y - 2 = 4x - 4$$
$$y = 4x - 2.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 4$ at $(1, 2)$.

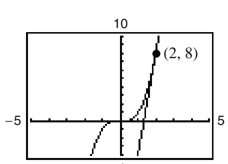
27. (a) $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2\right) = 3x^2$$

At $(2, 8)$, the slope of the tangent is $m = 3(2)^2 = 12$. The equation of the tangent line is

$$y - 8 = 12(x - 2)$$
$$y - 8 = 12x - 24$$
$$y = 12x - 16.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 12$ at $(2, 8)$.

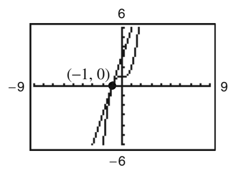
28. (a) $f(x) = x^3 + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 + 1\right] - (x^3 + 1)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 + 1 - x^3 - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left[3x^2 + 3x(\Delta x) + (\Delta x)^2\right] = 3x^2$$

At $(-1, 0)$, the slope of the tangent line is $m = 3(-1)^2 = 3$. The equation of the tangent line is

$$y - 0 = 3(x + 1)$$
$$y = 3x + 3.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 3$ at $(-1, 0)$.

29. (a) $f(x) = \sqrt{x}$

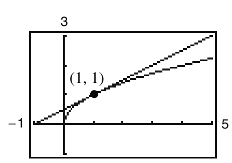
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

At $(1, 1)$, the slope of the tangent line is $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$.

The equation of the tangent line is

$$y - 1 = \frac{1}{2}(x - 1)$$
$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$
$$y = \frac{1}{2}x + \frac{1}{2}.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = \frac{1}{2}$ at $(1, 1)$.

30. (a) $f(x) = \sqrt{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-1} - \sqrt{x-1}}{\Delta x} \cdot \left(\frac{\sqrt{x+\Delta x-1} + \sqrt{x-1}}{\sqrt{x+\Delta x-1} + \sqrt{x-1}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x-1) - (x-1)}{\Delta x\left(\sqrt{x+\Delta x-1} + \sqrt{x-1}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$$

At $(5, 2)$, the slope of the tangent line is $m = \dfrac{1}{2\sqrt{5-1}} = \dfrac{1}{4}$.

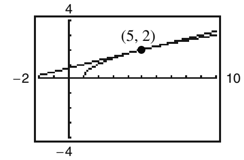
The equation of the tangent line is

$$y - 2 = \frac{1}{4}(x-5)$$

$$y - 2 = \frac{1}{4}x - \frac{5}{4}$$

$$y = \frac{1}{4}x + \frac{3}{4}.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{1}{4}$ at $(5, 2)$.

31. (a) $f(x) = x + \dfrac{4}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x) + \dfrac{4}{x+\Delta x} - \left(x + \dfrac{4}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x(x+\Delta x)(x+\Delta x) + 4x - x^2(x+\Delta x) - 4(x+\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x^3 - x^2(\Delta x) - 4(\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2(\Delta x) + x(\Delta x)^2 - 4(\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + x(\Delta x) - 4}{x(x+\Delta x)}$$

$$= \frac{x^2 - 4}{x^2} = 1 - \frac{4}{x^2}$$

At $(-4, -5)$, the slope of the tangent line is $m = 1 - \dfrac{4}{(-4)^2} = \dfrac{3}{4}$.

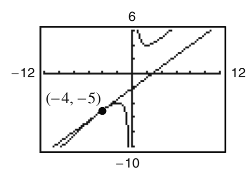
The equation of the tangent line is

$$y + 5 = \frac{3}{4}(x+4)$$

$$y + 5 = \frac{3}{4}x + 3$$

$$y = \frac{3}{4}x - 2.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{3}{4}$ at $(-4, -5)$.

32. (a) $f(x) = x + \dfrac{6}{x+2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{6}{(x+\Delta x)+2} - \dfrac{6}{x+2}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{6x + 12 - 6(x + \Delta x + 2)}{\Delta x(x + \Delta x + 2)(x+2)}$$
$$= \lim_{\Delta x \to 0} \frac{6x + 12 - 6x - 6\Delta x - 12}{\Delta x(x + \Delta x + 2)(x+2)}$$
$$= \lim_{\Delta x \to 0} \frac{-6\Delta x}{\Delta x(x + \Delta x + 2)(x+2)}$$
$$= \frac{-6}{(x+2)^2}$$

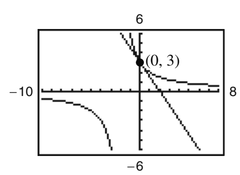
At $(0, 3)$, the slope of the tangent line is

$m = -\frac{6}{4} = -\frac{3}{2}$.

The equation of the tangent line is

$$y - 3 = -\frac{3}{2}(x - 0)$$
$$y - 3 = -\frac{3}{2}x$$
$$y = -\frac{3}{2}x + 3.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -\dfrac{3}{2}$ at $(0, 3)$.

33. Using the limit definition of derivative, $f'(x) = 2x$.

Because the slope of the given line is 2, you have

$2x = 2$

$x = 1$

At the point $(1, 1)$ the tangent line is parallel to $2x - y + 1 = 0$. The equation of this line is

$y - 1 = 2(x - 1)$

$y = 2x - 1.$

34. Using the limit definition of derivative, $f'(x) = 4x$.

Because the slope of the given line is –4, you have

$4x = -4$

$x = -1.$

At the point $(-1, 2)$ the tangent line is parallel to $4x + y + 3 = 0$. The equation of this line is

$y - 2 = -4(x + 1)$

$y = -4x - 2.$

35. From Exercise 27 we know that $f'(x) = 3x^2$.

Because the slope of the given line is 3, you have

$3x^2 = 3$

$x = \pm 1.$

Therefore, at the points $(1, 1)$ and $(-1, -1)$ the tangent lines are parallel to $3x - y + 1 = 0$.

These lines have equations

$y - 1 = 3(x - 1)$ and $y + 1 = 3(x + 1)$

$y = 3x - 2$ $\quad$ $y = 3x + 2.$

36. Using the limit definition of derivative, $f'(x) = 3x^2$.

Because the slope of the given line is 3, you have

$3x^2 = 3$

$x^2 = 1 \Rightarrow x = \pm 1.$

Therefore, at the points $(1, 3)$ and $(-1, 1)$ the tangent lines are parallel to $3x - y - 4 = 0$. These lines have equations

$y - 3 = 3(x - 1)$ and $y - 1 = 3(x + 1)$

$y = 3x$ $\quad$ $y = 3x + 4.$

37. Using the limit definition of derivative,

$f'(x) = \dfrac{-1}{2x\sqrt{x}}.$

Because the slope of the given line is $-\dfrac{1}{2}$, you have

$$-\frac{1}{2x\sqrt{x}} = -\frac{1}{2}$$
$$x = 1.$$

Therefore, at the point $(1, 1)$ the tangent line is parallel to $x + 2y - 6 = 0$. The equation of this line is

$$y - 1 = -\frac{1}{2}(x - 1)$$
$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}.$$

38. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2(x-1)^{3/2}}.$$

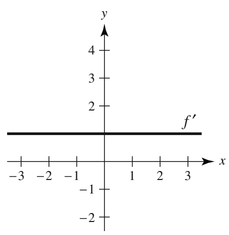
Because the slope of the given line is $-\frac{1}{2}$, you have

$$\frac{-1}{2(x-1)^{3/2}} = -\frac{1}{2}$$
$$1 = (x-1)^{3/2}$$
$$1 = x - 1 \Rightarrow x = 2.$$

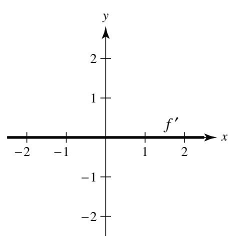
At the point $(2, 1)$, the tangent line is parallel to $x + 2y + 7 = 0$. The equation of the tangent line is

$$y - 1 = -\frac{1}{2}(x - 2)$$
$$y = -\frac{1}{2}x + 2.$$

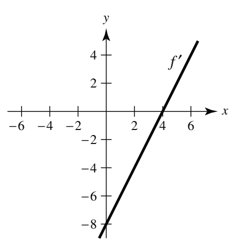
39. The slope of the graph of f is 1 for all x-values.



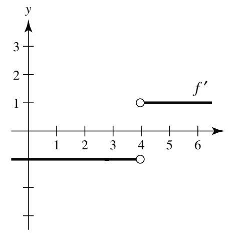
40. The slope of the graph of f is 0 for all x-values.



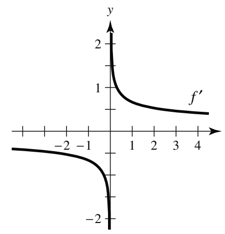
41. The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.



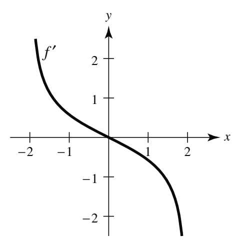
42. The slope of the graph of f is -1 for $x < 4$, 1 for $x > 4$, and undefined at $x = 4$.



43. The slope of the graph of f is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.

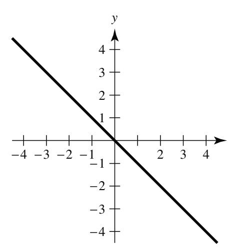


44. The slope is positive for $-2 < x < 0$ and negative for $0 < x < 2$. The slope is undefined at $x = \pm 2$, and 0 at $x = 0$.



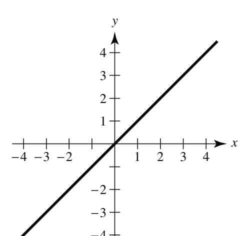
45. Answers will vary.

Sample answer: $y = -x$



46. Answers will vary.

Sample answer: $y = x$



47. $g(4) = 5$ because the tangent line passes through $(4, 5)$.

$$g'(4) = \frac{5 - 0}{4 - 7} = -\frac{5}{3}$$

48. $h(-1) = 4$ because the tangent line passes through $(-1, 4)$.

$$h'(-1) = \frac{6 - 4}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

49. $f(x) = 5 - 3x$ and $c = 1$

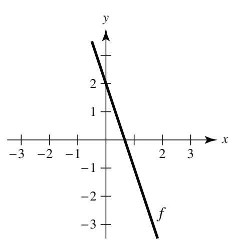
50. $f(x) = x^3$ and $c = -2$

51. $f(x) = -x^2$ and $c = 6$

52. $f(x) = 2\sqrt{x}$ and $c = 9$

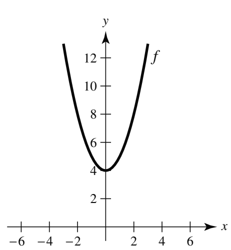
53. $f(0) = 2$ and $f'(x) = -3, -\infty < x < \infty$

$f(x) = -3x + 2$



54. $f(0) = 4, f'(0) = 0; f'(x) < 0$ for $x < 0, f'(x) > 0$ for $x > 0$

Answers will vary: *Sample answer:* $f(x) = x^2 + 4$

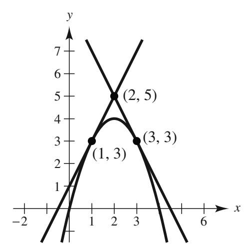


55. Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 4 - 2x$. The slope of the line through $(2, 5)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{5 - y_0}{2 - x_0} = 4 - 2x_0$$
$$5 - y_0 = (2 - x_0)(4 - 2x_0)$$
$$5 - (4x_0 - x_0^2) = 8 - 8x_0 + 2x_0^2$$
$$0 = x_0^2 - 4x_0 + 3$$
$$0 = (x_0 - 1)(x_0 - 3) \Rightarrow x_0 = 1, 3$$

Therefore, the points of tangency are $(1, 3)$ and $(3, 3)$, and the corresponding slopes are 2 and –2. The equations of the tangent lines are:

$$y - 5 = 2(x - 2) \qquad y - 5 = -2(x - 2)$$
$$y = 2x + 1 \qquad y = -2x + 9$$

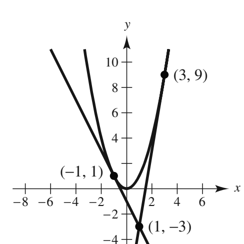


56. Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 2x$. The slope of the line through $(1, -3)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{-3 - y_0}{1 - x_0} = 2x_0$$
$$-3 - y_0 = (1 - x_0)2x_0$$
$$-3 - x_0^2 = 2x_0 - 2x_0^2$$
$$x_0^2 - 2x_0 - 3 = 0$$
$$(x_0 - 3)(x_0 + 1) = 0 \Rightarrow x_0 = 3, -1$$

Therefore, the points of tangency are $(3, 9)$ and $(-1, 1)$, and the corresponding slopes are 6 and –2. The equations of the tangent lines are:

$$y + 3 = 6(x - 1) \qquad y + 3 = -2(x - 1)$$
$$y = 6x - 9 \qquad y = -2x - 1$$



57. (a) $f(x) = x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

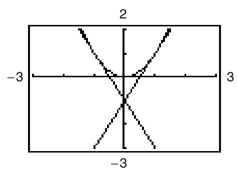
At $x = -1$, $f'(-1) = -2$ and the tangent line is

$y - 1 = -2(x + 1)$ or $y = -2x - 1$.

At $x = 0$, $f'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $f'(1) = 2$ and the tangent line is

$y = 2x - 1$.



For this function, the slopes of the tangent lines are always distinct for different values of x.

(b) $$g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right) = 3x^2$$

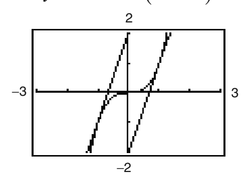
At $x = -1$, $g'(-1) = 3$ and the tangent line is

$y + 1 = 3(x + 1)$ or $y = 3x + 2$.

At $x = 0$, $g'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $g'(1) = 3$ and the tangent line is

$y - 1 = 3(x - 1)$ or $y = 3x - 2$.



For this function, the slopes of the tangent lines are sometimes the same.

58. (a) $g'(0) = -3$

(b) $g'(3) = 0$

(c) Because $g'(1) = -\frac{8}{3}$, g is decreasing (falling) at $x = 1$.

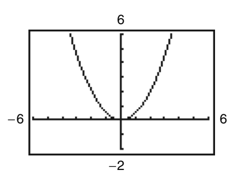
(d) Because $g'(-4) = \frac{7}{3}$, g is increasing (rising) at $x = -4$.

(e) Because $g'(4)$ and $g'(6)$ are both positive, $g(6)$ is greater than $g(4)$, and $g(6) - g(4) > 0$.

(f) No, it is not possible. All you can say is that g is decreasing (falling) at $x = 2$.

59. $f(x) = \dfrac{1}{2}x^2$

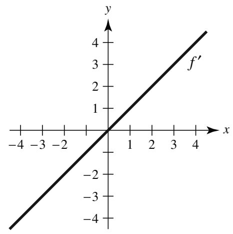
(a)



$f'(0) = 0$, $f'(1/2) = 1/2$, $f'(1) = 1$, $f'(2) = 2$

(b) By symmetry: $f'(-1/2) = -1/2$, $f'(-1) = -1$, $f'(-2) = -2$

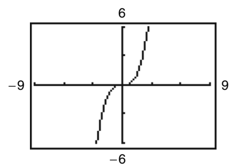
(c)



(d) $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2}(x + \Delta x)^2 - \frac{1}{2}x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2}\left(x^2 + 2x(\Delta x) + (\Delta x)^2\right) - \frac{1}{2}x^2}{\Delta x} = \lim_{\Delta x \to 0}\left(x + \frac{\Delta x}{2}\right) = x$

60. $f(x) = \frac{1}{3}x^3$

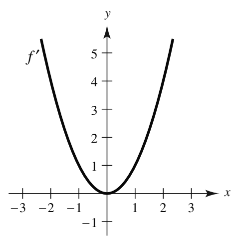
(a)



$f'(0) = 0, f'(1/2) = 1/4, f'(1) = 1, f'(2) = 4, f'(3) = 9$

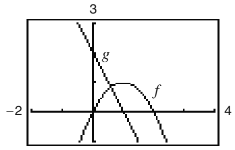
(b) By symmetry: $f'(-1/2) = 1/4, f'(-1) = 1, f'(-2) = 4, f'(-3) = 9$

(c)



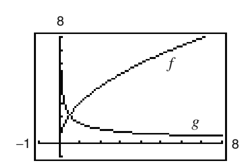
(d) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\frac{1}{3}(x + \Delta x)^3 - \frac{1}{3}x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\frac{1}{3}\left(x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3\right) - \frac{1}{3}x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0}\left[x^2 + x(\Delta x) + \frac{1}{3}(\Delta x)^2\right] = x^2$$

61. $$g(x) = \frac{f(x + 0.01) - f(x)}{0.01}$$
$$= \left[2(x + 0.01) - (x + 0.01)^2 - 2x + x^2\right]100$$
$$= 2 - 2x - 0.01$$



The graph of $g(x)$ is approximately the graph of $f'(x) = 2 - 2x$.

62. $$g(x) = \frac{f(x + 0.01) - f(x)}{0.01}$$
$$= \left(3\sqrt{x + 0.01} - 3\sqrt{x}\right)100$$



The graph of $g(x)$ is approximately the graph of $f'(x) = \frac{3}{2\sqrt{x}}$.

63. $f(2) = 2(4 - 2) = 4, f(2.1) = 2.1(4 - 2.1) = 3.99$

$$f'(2) \approx \frac{3.99 - 4}{2.1 - 2} = -0.1 \quad \left[\text{Exact: } f'(2) = 0\right]$$

64. $f(2) = \frac{1}{4}(2^3) = 2, f(2.1) = 2.31525$

$$f'(2) \approx \frac{2.31525 - 2}{2.1 - 2} = 3.1525\left[\text{Exact: } f'(2) = 3\right]$$

65. $f(x) = x^2 - 5, c = 3$

$$\begin{aligned} f'(3) &= \lim_{x\to3}\frac{f(x) - f(3)}{x - 3} \\ &= \lim_{x\to3}\frac{x^2 - 5 - (9 - 5)}{x - 3} \\ &= \lim_{x\to3}\frac{(x - 3)(x + 3)}{x - 3} \\ &= \lim_{x\to3}(x + 3) = 6 \end{aligned}$$

66. $g(x) = x^2 - x, c = 1$

$$\begin{aligned} g'(1) &= \lim_{x\to1}\frac{g(x) - g(1)}{x - 1} \\ &= \lim_{x\to1}\frac{x^2 - x - 0}{x - 1} \\ &= \lim_{x\to1}\frac{x(x - 1)}{x - 1} \\ &= \lim_{x\to1} x = 1 \end{aligned}$$

67. $f(x) = x^3 + 2x^2 + 1, c = -2$

$$\begin{aligned} f'(-2) &= \lim_{x\to-2}\frac{f(x) - f(-2)}{x + 2} \\ &= \lim_{x\to-2}\frac{(x^3 + 2x^2 + 1) - 1}{x + 2} \\ &= \lim_{x\to-2}\frac{x^2(x + 2)}{x + 2} = \lim_{x\to-2} x^2 = 4 \end{aligned}$$

68. $f(x) = x^3 + 6x, c = 2$

$$\begin{aligned} f'(2) &= \lim_{x\to2}\frac{f(x) - f(2)}{x - 2} \\ &= \lim_{x\to2}\frac{(x^3 + 6x) - 20}{x - 2} \\ &= \lim_{x\to2}\frac{(x - 2)(x^2 + 2x + 10)}{x - 2} \\ &= \lim_{x\to2}(x^2 + 2x + 10) = 18 \end{aligned}$$

69. $g(x) = \sqrt{|x|}, c = 0$

$g'(0) = \lim_{x\to0}\frac{g(x) - g(0)}{x - 0} = \lim_{x\to0}\frac{\sqrt{|x|}}{x}$. Does not exist.

As $x \to 0^-, \frac{\sqrt{|x|}}{x} = \frac{-1}{\sqrt{|x|}} \to -\infty$.

As $x \to 0^+, \frac{\sqrt{|x|}}{x} = \frac{1}{\sqrt{x}} \to \infty$.

Therefore $g(x)$ is not differentiable at $x = 0$.

70. $f(x) = \frac{3}{x}, c = 4$

$$\begin{aligned} f'(4) &= \lim_{x\to4}\frac{f(x) - f(4)}{x - 4} \\ &= \lim_{x\to4}\frac{\frac{3}{x} - \frac{3}{4}}{x - 4} \\ &= \lim_{x\to4}\frac{12 - 3x}{4x(x - 4)} \\ &= \lim_{x\to4}\frac{-3(x - 4)}{4x(x - 4)} \\ &= \lim_{x\to4} -\frac{3}{4x} = -\frac{3}{16} \end{aligned}$$

71. $f(x) = (x - 6)^{2/3}, c = 6$

$$\begin{aligned} f'(6) &= \lim_{x\to6}\frac{f(x) - f(6)}{x - 6} \\ &= \lim_{x\to6}\frac{(x - 6)^{2/3} - 0}{x - 6} = \lim_{x\to6}\frac{1}{(x - 6)^{1/3}}. \end{aligned}$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

72. $g(x) = (x + 3)^{1/3}, c = -3$

$$\begin{aligned} g'(-3) &= \lim_{x\to-3}\frac{g(x) - g(-3)}{x - (-3)} \\ &= \lim_{x\to-3}\frac{(x + 3)^{1/3} - 0}{x + 3} = \lim_{x\to-3}\frac{1}{(x + 3)^{2/3}}. \end{aligned}$$

Does not exist.

Therefore $g(x)$ is not differentiable at $x = -3$.

73. $h(x) = |x + 7|, c = -7$

$$\begin{aligned} h'(-7) &= \lim_{x\to-7}\frac{h(x) - h(-7)}{x - (-7)} \\ &= \lim_{x\to-7}\frac{|x + 7| - 0}{x + 7} = \lim_{x\to-7}\frac{|x + 7|}{x + 7}. \end{aligned}$$

Does not exist.

Therefore $h(x)$ is not differentiable at $x = -7$.

74. $f(x) = |x - 6|, c = 6$

$$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6}$$

$$= \lim_{x \to 6} \frac{|x - 6| - 0}{x - 6} = \lim_{x \to 6} \frac{|x - 6|}{x - 6}.$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

75. $f(x)$ is differentiable everywhere except at $x = 3$. (Discontinuity)

76. $f(x)$ is differentiable everywhere except at $x = \pm 3$. (Sharp turns in the graph)

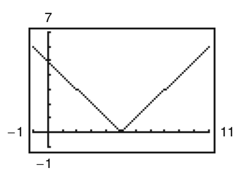
77. $f(x)$ is differentiable everywhere except at $x = -4$. (Sharp turn in the graph)

78. $f(x)$ is differentiable everywhere except at $x = \pm 2$. (Discontinuities)

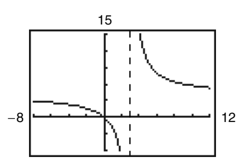
79. $f(x)$ is differentiable on the interval $(1, \infty)$. (At $x = 1$ the tangent line is vertical.)

80. $f(x)$ is differentiable everywhere except at $x = 0$. (Discontinuity)

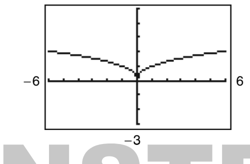
81. $f(x) = |x - 5|$ is differentiable everywhere except at $x = -5$. There is a sharp corner at $x = 5$.



82. $f(x) = \dfrac{4x}{x - 3}$ is differentiable everywhere except at $x = 3$. f is not defined at $x = 3$. (Vertical asymptote)

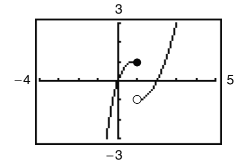


83. $f(x) = x^{2/5}$ is differentiable for all $x \neq 0$. There is a sharp corner at $x = 0$.



84. f is differentiable for all $x \neq 1$.

f is not continuous at $x = 1$.



85. $f(x) = |x - 1|$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{|x - 1| - 0}{x - 1} = -1.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{|x - 1| - 0}{x - 1} = 1.$$

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

86. $f(x) = \sqrt{1 - x^2}$

The derivative from the left does not exist because

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{\sqrt{1 - x^2} - 0}{x - 1}$$

$$= \lim_{x \to 1^-} \frac{\sqrt{1 - x^2}}{x - 1} \cdot \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^2}}$$

$$= \lim_{x \to 1^-} -\frac{1 + x}{\sqrt{1 - x^2}} = -\infty.$$

(Vertical tangent)

The limit from the right does not exist since f is undefined for $x > 1$. Therefore, f is not differentiable at $x = 1$.

87. $f(x) = \begin{cases} (x - 1)^3, & x \leq 1 \\ (x - 1)^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{(x - 1)^3 - 0}{x - 1}$$

$$= \lim_{x \to 1^-} (x - 1)^2 = 0.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{(x - 1)^2 - 0}{x - 1}$$

$$= \lim_{x \to 1^+} (x - 1) = 0.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 1$. $\left(f'(1) = 0\right)$

88. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{x - 1}{x - 1} = \lim_{x \to 1^-} 1 = 1.$$

The derivative from the right is

$$\lim_{x \to 1^+} = \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1^+} (x + 1) = 2.$$

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

89. Note that f is continuous at $x = 2$.

$f(x) = \begin{cases} x^2 + 1, & x \le 2 \\ 4x - 3, & x > 2 \end{cases}$

The derivative from the left is

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{(x^2 + 1) - 5}{x - 2} = \lim_{x \to 2^-} (x + 2) = 4.$$

The derivative from the right is

$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{(4x - 3) - 5}{x - 2} = \lim_{x \to 2^+} 4 = 4.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $\left(f'(2) = 4\right)$

90. Note that f is continuous at $x = 2$.

$f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \ge 2 \end{cases}$

The derivative from the left is

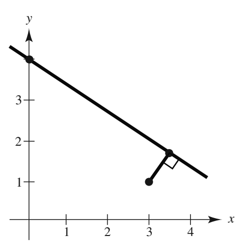
$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{\left(\frac{1}{2}x + 1\right) - 2}{x - 2} = \lim_{x \to 2^-} \frac{\frac{1}{2}(x - 2)}{x - 2} = \frac{1}{2}.$$

The derivative from the right is

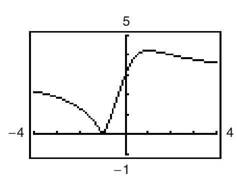
$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{\sqrt{2x} - 2}{x - 2} \cdot \frac{\sqrt{2x} + 2}{\sqrt{2x} + 2} = \lim_{x \to 2^+} \frac{2x - 4}{(x - 2)(\sqrt{2x} + 2)} = \lim_{x \to 2^+} \frac{2(x - 2)}{(x - 2)(\sqrt{2x} + 2)} = \lim_{x \to 2^+} \frac{2}{\sqrt{2x} + 2} = \frac{1}{2}.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $\left(f'(2) = \frac{1}{2}\right)$

91. (a) The distance from $(3, 1)$ to the line $mx - y + 4 = 0$ is $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \dfrac{|m(3) - 1(1) + 4|}{\sqrt{m^2 + 1}} = \dfrac{|3m + 3|}{\sqrt{m^2 + 1}}$



(b)



The function d is not differentiable at $m = -1$. This corresponds to the line $y = -x + 4$, which passes through the point $(3, 1)$.