Chapter 2 Applications of the Derivative

2.1 Describing Graphs of Functions

1. (a), (e), (f)

2. (c), (d)

3. (b), (c), (d)

4. (a), (e)

5. Increasing for $x < .5$, relative maximum point at $x = .5$, maximum value = 1, decreasing for $x > .5$, concave down, y-intercept (0, 0), x-intercepts (0, 0) and (1, 0).

6. Increasing for $x < -.4$, relative maximum point at $x = -.4$, relative maximum value = 5.1, decreasing for $x > -.4$, concave down for $x < 3$, inflection point (3, 3), concave up for $x > 3$, y-intercept (0, 5), x-intercept (–3.5, 0). The graph approaches the x-axis as a horizontal asymptote.

7. Decreasing for $x < 0$, relative minimum point at $x = 0$, relative minimum value = 2, increasing for $0 < x < 2$, relative maximum point at $x = 2$, relative maximum value = 4, decreasing for $x > 2$, concave up for $x < 1$, inflection point at (1, 3), concave down for $x > 1$, y-intercept at (0, 2), x-intercept (3.6, 0).

8. Increasing for $x < -1$, relative maximum at $x = -1$, relative maximum value = 5, decreasing for $-1 < x < 2.9$, relative minimum at $x = 2.9$, relative minimum value = –2, increasing for $x > 2.9$, concave down for $x < 1$, inflection point at (1, .5), concave up for $x > 1$, y-intercept (0, 3.3), x-intercepts (–2.5, 0), (1.3, 0), and (4.4, 0).

9. Decreasing for $x < 2$, relative minimum at $x = 2$, minimum value = 3, increasing for $x > 2$, concave up for all x, no inflection point, defined for $x > 0$, the line $y = x$ is an asymptote, the y-axis is an asymptote.

10. Increasing for all x, concave down for $x < 3$, inflection point at (3, 3), concave up for $x > 3$, y-intercept (0, 1), x-intercept (–.5, 0).

11. Decreasing for $1 \le x < 3$, relative minimum at $x = 3$, relative minimum value = .9, increasing for $x > 3$, maximum value = 6 (at $x = 1$), minimum value = .9 (at $x = 3$), concave up for $1 \le x < 4$, inflection point at (4, 1.5), concave down for $x > 4$; the line $y = 4$ is an asymptote.

12. Increasing for $x < -1.5$, relative maximum at $x = -1.5$, relative maximum value = 3.5, decreasing for $-1.5 < x < 2$, relative minimum at $x = 2$, relative minimum value = –1.6, increasing from $2 < x < 5.5$, relative maximum at $x = 5.5$, relative maximum value = 3.4, decreasing for $x > 5.5$, concave down for $x < 0$, inflection point at (0, 1), concave up for $0 < x < 4$, inflection point at (4, 1), concave down for $x > 4$, y-intercept (0, 1), x-intercepts (–2.8, 0), (.6, 0), (3.5, 0), and (6.7, 0).

13. The slope decreases for all x.

14. Slope decreases for $x < 3$, increases for $x > 3$.

15. Slope decreases for $x < 1$, increases for $x > 1$. Minimum slope occurs at $x = 1$.

16. Slope decreases for $x < 3$, increases for $x > 3$.

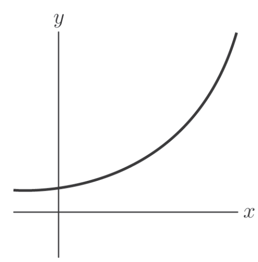
17. **a.** *C, F* **b.** *A, B, F*

c. *C*

18. **a.** *A, E* **b.** *D*

c. *E*

19.



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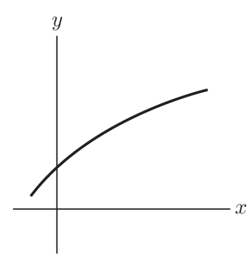

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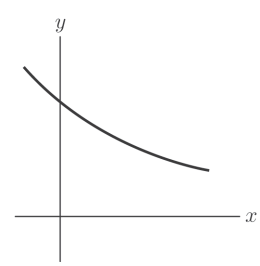

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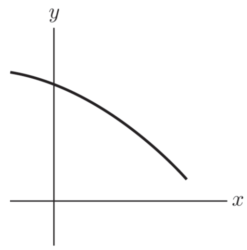

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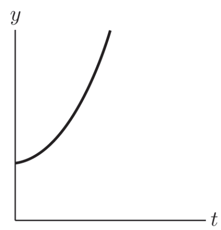

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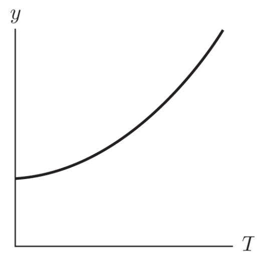

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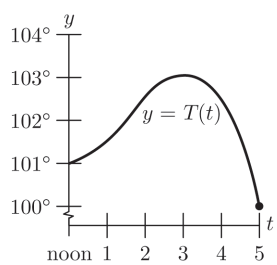

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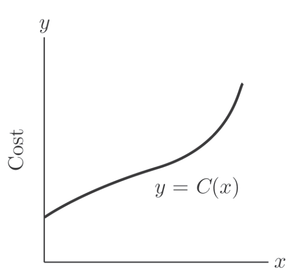

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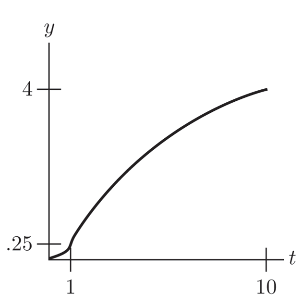


28. Oxygen content decreases until time a, at which time it reaches a minimum. After a, oxygen content steadily increases. The rate at which oxygen content grows increases until b, and then decreases. Time b is the time when oxygen content is increasing fastest.

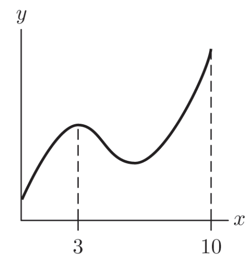
29. 1960

30. 1999; 1985

31. The parachutist's speed levels off to 15 ft/sec.

32. Bacteria population stabilizes at 25,000,000.

33.



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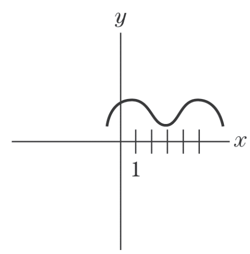

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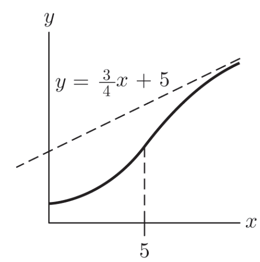

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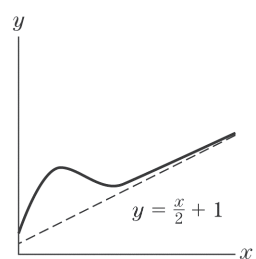
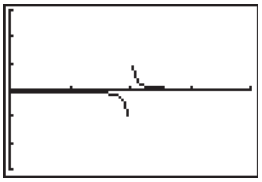


37. **a.** Yes; there is a relative minimum point between the two relative maximum points.

b. Yes; there is an inflection point between the two relative extreme points.

38. No

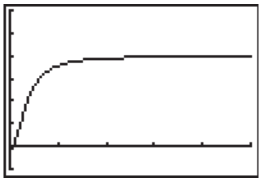
39.



[0, 4] by [−15, 15]

Vertical asymptote: $x = 2$

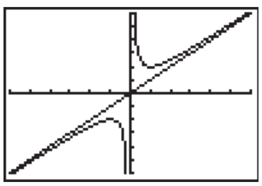
40.



[0, 50] by [−1, 6]

$c = 4$

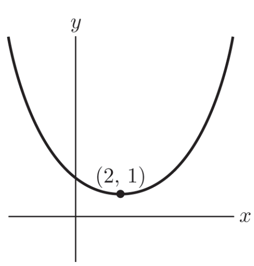
41.

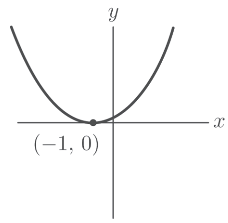


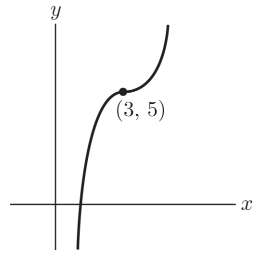
[−6, 6] by [−6, 6]

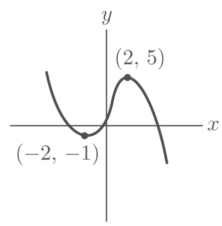
The line $y = x$ is the asymptote of the first function, $y = \frac{1}{x} + x$.

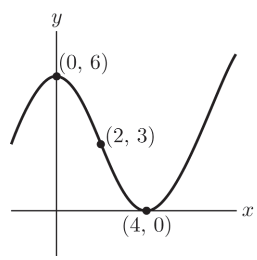
2.2 The First and Second Derivative Rules

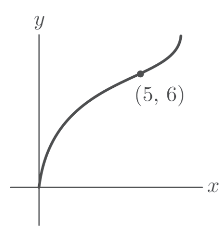
1. (e)
2. (b), (c), (f)
3. (a), (b), (d), (e)
4. (f)
5. (d)
6. (c)
7. 

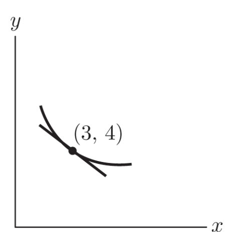
8. 

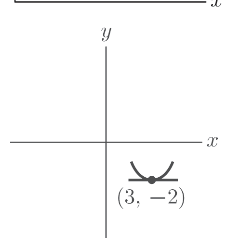
9. 

10. 

11. 

12. 

13. 

14. 

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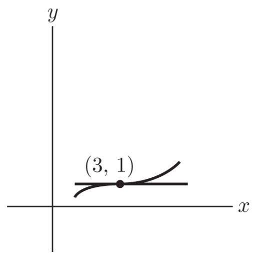

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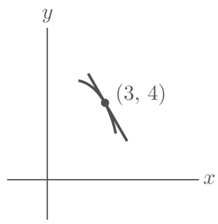

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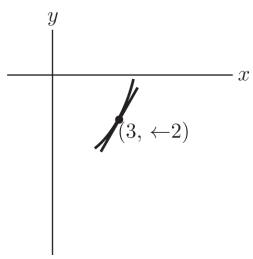

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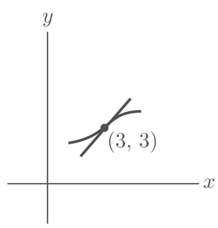


19.

	f	f'	f''
A	POS	POS	NEG
B	0	NEG	0
C	NEG	0	POS

20. a. $f'(x) = 0$ at $x = 2$ or $x = 4$; however $f''(x) = 0$ at $x = 4$, so there is a relative extreme point at $x = 2$.

b. $f''(x) = 0$ at $x = 3$ or $x = 4$, so there are inflection points at $x = 3$ and at $x = 4$.

21. $t = 1$ because the slope is more positive at $t = 1$.

22. $t = 2$ because the $v(2)$ is more positive than $v(1)$.

23. a. $f'(9) < 0$, so $f(x)$ is decreasing at $x = 9$.

b. The function $f(x)$ is increasing for $1 \le x < 2$ because the values of $f'(x)$ are positive. The function $f(x)$ is decreasing for $2 < x \le 3$ because the values of $f'(x)$ are negative. Therefore, $f(x)$ has a relative maximum at $x = 2$. Since $f(2) = 9$, the coordinates of the relative maximum point are (2, 9).

c. The function $f(x)$ is decreasing for $9 \le x < 10$ because the values of $f'(x)$ are negative. The function $f(x)$ is increasing for $10 < x \le 11$ because the values of $f'(x)$ are positive. Therefore, $f(x)$ has a relative minimum at $x = 10$.

d. $f''(2) < 0$, so the graph is concave down.

e. $f''(x) = 0$, so the inflection point is at $x = 6$. Since $f(6) = 5$, the coordinates of the inflection point are (6, 5).

f. The x-coordinate where $f'(x) = 6$ is $x = 15$.

24. a. $f(2) = 3$

b. $t = 4$ or $t = 6$

c. $f(t)$ attains its greatest value after 1 minute, at $t = 1$. To confirm this, observe that $f'(t) > 0$ for $0 \le t < 1$ and $f'(t) < 0$ for $1 < t \le 2$.

d. $f(t)$ attains its least value after 5 minutes, at $t = 5$. To confirm this, observe that $f'(t) < 0$ for $4 \le t < 5$ and $f'(t) > 0$ for $5 < t \le 6$.

e. Since $f'(7.5) = 1$, the rate of change is 1 unit per minute.

f. The solutions to $f'(t) = -1$ are $t = 2.5$ and $t = 3.5$, so $f'(t)$ is decreasing at the rate of 1 unit per minute after 2.5 minutes and after 3.5 minutes.

g. The greatest rate of decrease occurs when $f'(t)$ is most negative, at $t = 3$ (after 3 minutes).

h. The greatest rate of increase occurs when $f'(t)$ is most positive, at $t = 7$ (after 7 minutes).

25. The slope is positive because $f'(6) = 2$.

26. The slope is negative because $f'(4) = -1$.

27. The slope is 0 because $f'(3) = 0$. Also $f'(x)$ is positive for x slightly less than 3, and $f'(x)$ is negative for x slightly greater than 3. Hence $f(x)$ changes from increasing to decreasing at $x = 3$.

28. The slope is 0 because $f'(5) = 0$. Also $f'(x)$ is negative for x slightly less than 5, and $f'(x)$ is positive for x slightly greater than 5. Hence $f(x)$ changes from decreasing to increasing at $x = 5$.

29. $f'(x)$ is increasing at $x = 0$, so the graph of $f(x)$ is concave up.

30. $f'(x)$ is decreasing at $x = 2$, so the graph of $f(x)$ is concave down.

31. At $x = 1$, $f'(x)$ changes from increasing to decreasing, so the slope of the graph of $f(x)$ changes from increasing to decreasing. The concavity of the graph of $f(x)$ changes from concave up to concave down.

32. At $x = 4$, $f'(x)$ changes from decreasing to increasing, so the slope of the graph of $f(x)$ changes from decreasing to increasing. The concavity of the graph of $f(x)$ changes from concave down to concave up.

33. $f'(x) = 2$, so $m = 2 \Rightarrow y - 3 = 2(x-6) \Rightarrow$ $y = 2x - 9$

34. $f(6.5) \approx f(6) + f'(6)(.5) = 8 + \frac{1}{2}(2) = 9$

35. $f(0.25) \approx f(0) + f'(0)(.25)$
$= 3 + (1)(.25) = 3.25$

36. $f(0) = 3$, $f'(0) = 1 \Rightarrow y - 3 = 1(x-0) \Rightarrow$ $y = x + 3$

37. a. $h(100.5) \approx h(100) + h'(100)(.5)$
The change $= h(100.5) - h(100)$
$\approx h'(100)(.5) = \frac{1}{3} \cdot \frac{1}{2} - = \frac{1}{6}$ inch.

b. (ii) because the water level is falling.

38. a. $T(10) - T(10.75) \approx T'(10)(0.75) = 4 \cdot \frac{3}{4}$
$= 3$ degrees

b. (ii) because the temperature is falling (assuming cooler is better).

39. $f'(x) = 4(3x^2+1)^3(6x) = 24x(3x^2+1)^3$
Graph II cannot be the graph of $f(x)$ because $f'(x)$ is always positive for $x > 0$.

40. $f'(x) = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8)$
$= 3(x-2)(x-4)$
Graph I cannot be the graph because it does not have horizontal tangents at $x = 2$ and $x = 4$.

41. $f'(x) = \frac{5}{2}x^{3/2}$; $f''(x) = \frac{15}{4}x^{1/2}$
Graph I could be the graph of $f(x)$ since $f''(x) > 0$
for $x > 0$.

42. a. (C) **b.** (D)
c. (B) **d.** (A)
e. (E)

43. a. Since $f(65) \approx 2$, there were about 2 million farms.

b. Since $f'(65) \approx -.03$, the rate of change was –0.03 million farms per year. The number of farms was declining at the rate of about 30,000 farms per year.

c. The solution of $f(t) = 6$ is $t \approx 15$, so there were 6 million farms in 1940.

d. The solutions of $f'(t) = -.06$ are $t \approx 20$ and $t \approx 53$, so the number of farms was declining at the rate of 60,000 farms per year in 1945 and in 1978.

e. The graph of $f'(t)$ reaches its minimum at $t \approx 35$. Confirm this by observing that the graph of $y = f''(t)$ crosses the t-axis at $t \approx 35$. The number of farms was decreasing fastest in 1960.

44. a. Since $f'(5) < 0$, the amount is decreasing.

b. Since $f''(5) > 0$, the graph of $f(t)$ is concave up.

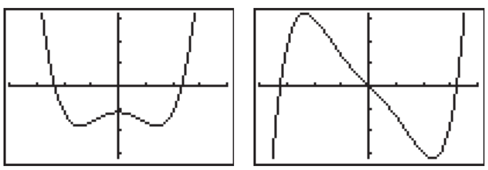
c. The graph of $f'(t)$ reaches its minimum at $t = 4$. Confirm this by observing that the graph of $f''(t)$ crosses the t-axis at $t = 4$. The level is decreasing fastest at $t = 4$ (after 4 hours).

d. Since $f'(t)$ is positive for $0 \le t < 2$ and $f'(t)$ is negative for $t > 2$, the greatest level of drug in the bloodstream is reached at $t = 2$ (after 2 hours).

e. The solutions of $f'(t) = -3$ are $t \approx 2.6$ and $t \approx 7$, so the drug level is decreasing at the rate of 3 units per hour after 2.6 hours and after 7 hours.

45. $f(x) = 3x^5 - 20x^3 - 120x$

$y = f'(x)$ $\quad$ $y = f(x)$

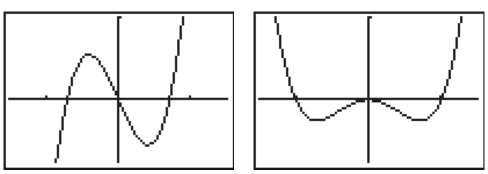


[−4, 4] by [−325, 325]

Note that $f'(x) = 15x^4 - 60x^2 - 120$, or use the calculator's ability to graph numerical derivatives.
Relative maximum: $x \approx -2.34$
Relative minimum: $x \approx 2.34$
Inflection point: $x \approx \pm 1.41$, $x = 0$

46. $f(x) = x^4 - x^2$

$y = f'(x)$ $\quad$ $y = f(x)$



[−1.5, 1.5] by [−.75, 1]

Note that $f'(x) = 4x^3 - 2x$, or use the calculator's ability to graph numerical derivatives.
Relative maximum: $x = 0$
Relative (and absolute) minimum: $x \approx \pm.71$
Inflection points: $x \approx \pm.41$

2.3 The First and Second Derivative Tests and Curve Sketching

1. $f(x) = x^3 - 27x$

$f'(x) = 3x^2 - 27 = 3(x^2 - 9) = 3(x-3)(x+3)$
$f'(x) = 0$ if $x = -3$ or $x = 3$
$f(-3) = 54$, $f(3) = -54$
Critical points: (−3, 54), (3, −54)

Critical Points, Intervals	$x < -3$	$-3 < x < 3$	$3 < x$
$x - 3$	−	−	+
$x + 3$	−	+	+
$f'(x)$	+	−	+
$f(x)$	Increasing on $(-\infty, -3)$	Decreasing on $(-3, 3)$	Increasing on $(3, \infty)$

Relative maximum at (−3, 54), relative minimum at (3, −54).

2. $f(x) = x^3 - 6x^2 + 1$

$f'(x) = 3x^2 - 12x = 3x(x-4)$
$f'(x) = 0$ if $x = 0$ or $x = 4$
$f(0) = 1$; $f(4) = -31$
Critical points: (0, 1), (4, −31)

Critical Points, Intervals	$x < 0$	$0 < x < 4$	$4 < x$
$3x$	−	+	+
$x - 4$	−	−	+
$f'(x)$	+	−	+
$f(x)$	Increasing on $(-\infty, 0)$	Decreasing on $(0, 4)$	Increasing on $(4, \infty)$

Relative maximum at (0, 1), relative minimum at (4, −31).

3. $f(x) = -x^3 + 6x^2 - 9x + 1$

$f'(x) = -3x^2 + 12x - 9$
$= -3(x^2 - 4x + 3) = -3(x-1)(x-3)$
$f'(x) = 0$ if $x = 1$ or $x = 3$
$f(1) = -3$, $f(3) = 1$
Critical points: (1, −3), (3, 1)

Critical Points, Intervals	$x < 1$	$1 < x < 3$	$3 < x$
$-3(x-1)$	+	−	−
$x - 3$	−	−	+
$f'(x)$	−	+	−
$f(x)$	Decreasing on $(-\infty, 1)$	Increasing on $(1, 3)$	Decreasing on $(3, \infty)$

Relative maximum at (3, 1), relative minimum at (1, −3).

4. $f(x) = -6x^3 - \frac{3}{2}x^2 + 3x - 3$

$f'(x) = -18x^2 - 3x + 3 = -3(6x^2 + x - 1)$
$= -3(2x+1)(3x-1)$

$f'(x) = 0$ if $x = -\frac{1}{2}$ or $x = \frac{1}{3}$

$f\left(-\frac{1}{2}\right) = -\frac{33}{8}, f\left(\frac{1}{3}\right) = -\frac{43}{18}$

Critical points: $\left(-\frac{1}{2}, -\frac{33}{8}\right), \left(\frac{1}{3}, -\frac{43}{18}\right)$

Critical Points, Intervals	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < \frac{1}{3}$	$\frac{1}{3} < x$
$2x+1$	−	+	+
$3x-1$	−	−	+
$f'(x)$	+	−	+
$f(x)$	Increasing on $\left(-\infty, -\frac{1}{2}\right)$	Decreasing on $\left(-\frac{1}{2}, \frac{1}{3}\right)$	Increasing on $\left(\frac{1}{3}, \infty\right)$

Relative maximum at $\left(\frac{1}{3}, -\frac{43}{18}\right)$, relative minimum at $\left(-\frac{1}{2}, -\frac{33}{8}\right)$.

5. $f(x) = \frac{1}{3}x^3 - x^2 + 1$

$f'(x) = x^2 - 2x = x(x-2)$
$f'(x) = 0$ if $x = 0$ or $x = 2$

$f(0) = 1;\ f(2) = -\frac{1}{3}$

Critical points: (0, 1), $\left(2, -\frac{1}{3}\right)$

Critical Points, Intervals	$x < 0$	$0 < x < 2$	$2 < x$
x	−	+	+
$x-2$	−	−	+
$f'(x)$	+	−	+
$f(x)$	Increasing on $(-\infty, 0)$	Decreasing on $(0, 2)$	Increasing on $(2, \infty)$

Relative maximum at (0, 1), relative minimum at $\left(2, -\frac{1}{3}\right)$.

6. $f(x) = \frac{4}{3}x^3 - x + 2$

$f'(x) = 4x^2 - 1 = (2x+1)(2x-1)$

$f'(x) = 0$ if $x = -\frac{1}{2}$ or $x = \frac{1}{2}$

$f\left(-\frac{1}{2}\right) = \frac{7}{3};\ f\left(\frac{1}{2}\right) = \frac{5}{3}$

Critical points: $\left(-\frac{1}{2}, \frac{7}{3}\right), \left(\frac{1}{2}, \frac{5}{3}\right)$

Critical Points, Intervals	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < \frac{1}{2}$	$\frac{1}{2} < x$
$2x+1$	−	+	+
$2x-1$	−	−	+
$f'(x)$	+	−	+
$f(x)$	Increasing on $\left(-\infty, -\frac{1}{2}\right)$	Decreasing on $\left(-\frac{1}{2}, \frac{1}{2}\right)$	Increasing on $\left(\frac{1}{2}, \infty\right)$

Relative maximum $\left(-\frac{1}{2}, \frac{7}{3}\right)$, relative minimum $\left(\frac{1}{2}, \frac{5}{3}\right)$.

7. $f(x) = -x^3 - 12x^2 - 2$

$f'(x) = -3x^2 - 24x = -3x(x+8)$
$f'(x) = 0$ if $x = -8$ or $x = 0$
$f(-8) = -258, f(0) = -2$

Critical points: $(-8, -258), (0, -2)$

Critical Points, Intervals	$x < -8$	$-8 < x < 0$	$0 < x$
$x+8$	−	+	+
$-3x$	+	+	−
$f'(x)$	−	+	−
$f(x)$	Decreasing on $(-\infty, -8)$	Increasing on $(-8, 0)$	Decreasing on $(0, \infty)$

Relative maximum at (0, −2), relative minimum at.(−8, −258).

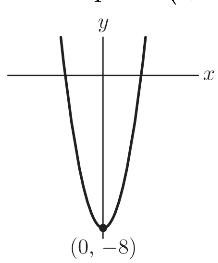
8. $f(x) = 2x^3 + 3x^2 - 3$

$f'(x) = 6x^2 + 6x = 6x(x+1)$
$f'(x) = 0$ if $x = -1$ or $x = 0$
$f(-1) = -2, f(0) = -3$
Critical points: $(-1,-2)$, $(0,-3)$

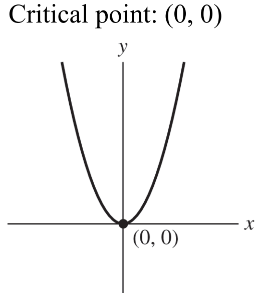
Critical Points, Intervals	$x < -1$	$-1 < x < 0$	$0 < x$
$x+1$	−	−	+
$6x$	−	+	+
$f'(x)$	+	−	+
$f(x)$	Increasing on $(-\infty,-1)$	Decreasing on $(-1,0)$	Increasing on $(0,\infty)$

Relative maximum at $(-1,-2)$, relative minimum at. $(0,-3)$.

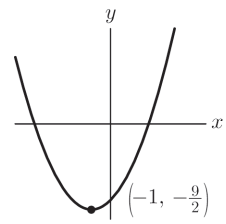
9. $f(x) = 2x^3 - 8$
$f'(x) = 6x^2$
$f'(x) = 0$ if $x = 0$
$f(0) = -8$
Critical point: $(0, -8)$



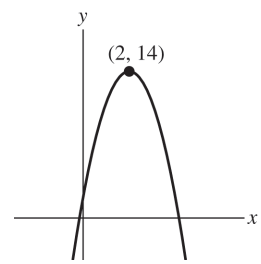
10. $f(x) = x^2$
$f'(x) = 2x$
$f'(x) = 0$ if $x = 0$
$f(0) = 0$
Critical point: (0, 0)



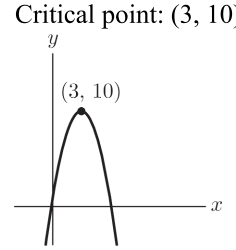
11. $f(x) = \frac{1}{2}x^2 + x - 4$
$f'(x) = x + 1$
$f'(x) = 0$ if $x = -1$
$f(-1) = -\frac{9}{2}$
Critical point: $\left(-1, -\frac{9}{2}\right)$



12. $f(x) = -3x^2 + 12x + 2$
$f'(x) = -6x + 12$
$f'(x) = 0$ if $x = 2$
$f(2) = 14$
Critical point: (2, 14)



13. $f(x) = 1 + 6x - x^2$
$f'(x) = 6 - 2x$
$f'(x) = 0$ if $x = 3$
$f(3) = 10$
Critical point: (3, 10)



14. $f(x) = \frac{1}{2}x^2 + \frac{1}{2}$
$f'(x) = x$
$f'(x) = 0$ if $x = 0$
$f(0) = \frac{1}{2}$
Critical point: $\left(0, \frac{1}{2}\right)$

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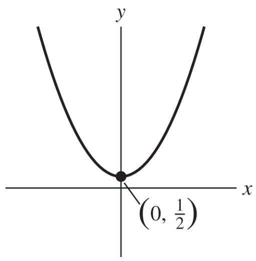
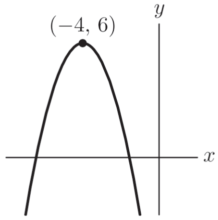
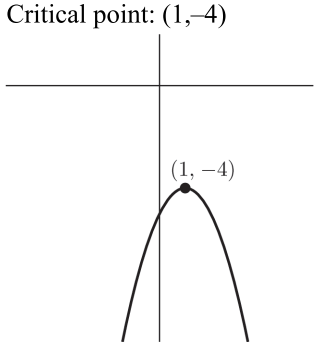


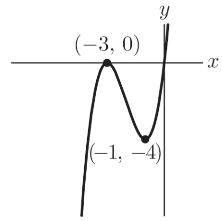
15. $f(x) = -x^2 - 8x - 10$
$f'(x) = -2x - 8$
$f'(x) = 0$ if $x = -4$
$f(-4) = 6$
Critical point: (–4, 6)



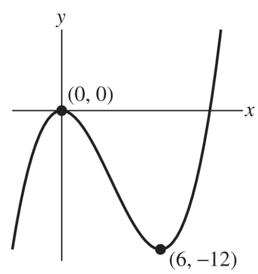
16. $f(x) = -x^2 + 2x - 5$
$f'(x) = -2x + 2$
$f'(x) = 0$ if $x = 1$
$f(1) = -4$
Critical point: (1,–4)



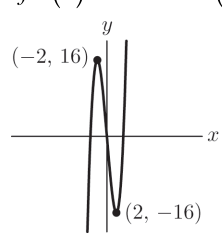
17. $f(x) = x^3 + 6x^2 + 9x$
$f'(x) = 3x^2 + 12x + 9$
$f''(x) = 6x + 12$
$f'(x) = 0$ if $x = -3$ or $x = -1$
$f(-3) = 0 \Rightarrow (-3, 0)$ is a critical pt.
$f(-1) = -4 \Rightarrow (-1, -4)$ is a critical pt.
$f''(-3) = -6 < 0 \Rightarrow (-3, 0)$ is a local max.
$f''(-1) = 18 > 0 \Rightarrow (-1, -4)$ is a local min.



18. $f(x) = \frac{1}{9}x^3 - x^2$
$f'(x) = \frac{1}{3}x^2 - 2x$
$f''(x) = \frac{2}{3}x - 2$
$f'(x) = 0$ if $x = 0$ or $x = 6$
$f(0) = 0 \Rightarrow (0, 0)$ is a critical pt.
$f(6) = -12 \Rightarrow (6, -12)$ is a critical pt.
$f''(0) = 0$
Use first derivative test to determine concavity.
$f''(6) = 2 > 0 \Rightarrow (6, -12)$ is a local min.



19. $f(x) = x^3 - 12x$
$f'(x) = 3x^2 - 12$
$f''(x) = 6x$
$f'(x) = 0$ if $x = -2$ or $x = 2$
$f(-2) = 16 \Rightarrow (-2, 16)$ is a critical pt.
$f(2) = -4 \Rightarrow (2, -16)$ is a critical pt.
$f''(-2) = -12 < 0 \Rightarrow (-2, 16)$ is a local max.
$f''(2) = 12 > 0 \Rightarrow (2, -16)$ is a local min.



20. $f(x) = -\frac{1}{3}x^3 + 9x - 2$

$f'(x) = -x^2 + 9$

$f''(x) = -2x$

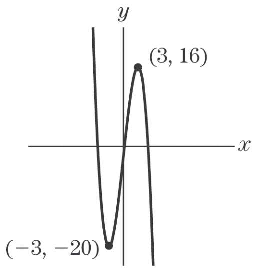
$f'(x) = 0$ if $x = -3$ or $x = 3$

$f(-3) = -20 \Rightarrow (-3, -20)$ is a critical pt.

$f(3) = 16 \Rightarrow (3, 16)$ is a critical pt.

$f''(-3) = 6 > 0 \Rightarrow (-3, -20)$ is a local max.

$f''(3) = -6 < 0 \Rightarrow (3, 16)$ is a local min.



21. $f(x) = -\frac{1}{9}x^3 + x^2 + 9x$

$f'(x) = -\frac{1}{3}x^2 + 2x + 9$

$f''(x) = -\frac{2}{3}x + 2$

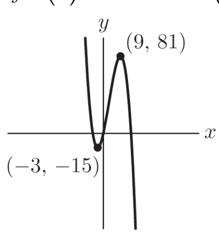
$f'(x) = 0$ if $x = -3$ or $x = 9$

$f(-3) = 0 \Rightarrow (-3, -15)$ is a critical pt.

$f(9) = 81 \Rightarrow (9, 81)$ is a critical pt.

$f''(-3) = 4 > 0 \Rightarrow (-3, -15)$ is a local min.

$f''(9) = -4 < 0 \Rightarrow (9, 81)$ is a local max.



22. $f(x) = 2x^3 - 15x^2 + 36x - 24$

$f'(x) = 6x^2 - 30x + 36$

$f''(x) = 12x - 30$

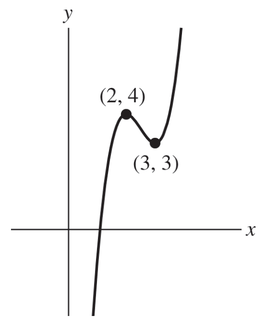
$f'(x) = 0$ if $x = 2$ or $x = 3$

$f(2) = 4 \Rightarrow (2, 4)$ is a critical pt.

$f(3) = 81 \Rightarrow (3, 3)$ is a critical pt.

$f''(2) = -6 < 0 \Rightarrow (2, 4)$ is a local max.

$f''(3) = 6 > 0 \Rightarrow (3, 3)$ is a local min.



23. $f(x) = -\frac{1}{3}x^3 + 2x^2 - 12$

$f'(x) = -x^2 + 4x$

$f''(x) = -2x + 4$

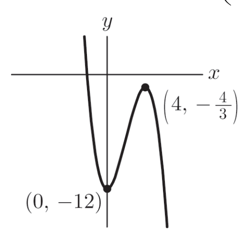
$f'(x) = 0$ if $x = 0$ or $x = 4$

$f(0) = -12 \Rightarrow (0, -12)$ is a critical pt.

$f(4) = -\frac{4}{3} \Rightarrow \left(4, -\frac{4}{3}\right)$ is a critical pt.

$f''(0) = 4 > 0 \Rightarrow (-3, 0)$ is a local min.

$f''(4) = -4 < 0 \Rightarrow \left(4, -\frac{4}{3}\right)$ is a local max.



24. $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x + \frac{8}{3}$

$f'(x) = x^2 + 4x - 5$

$f''(x) = 2x + 4$

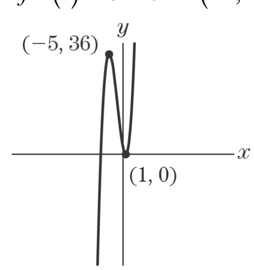
$f'(x) = 0$ if $x = -5$ or $x = 1$

$f(-5) = 36 \Rightarrow (-5, 36)$ is a critical pt.

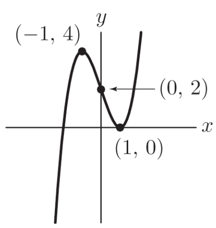
$f(1) = 81 \Rightarrow (1, 0)$ is a critical pt.

$f''(-5) = -6 < 0 \Rightarrow (-3, 0)$ is a local max.

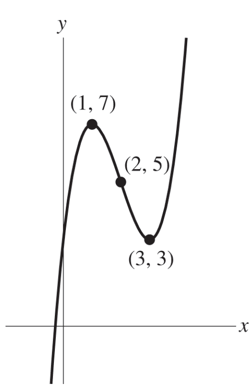
$f''(1) = 6 > 0 \Rightarrow (-1, -4)$ is a local min.



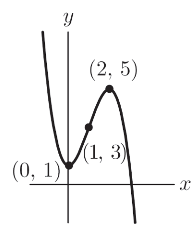
25. $y = x^3 - 3x + 2$
$y' = 3x^2 - 3$
$y'' = 6x$
$y' = 0$ if $x = -1$ or $x = 1$
$y(-1) = 4 \Rightarrow (-1, 4)$ is a critical pt.
$y(1) = 0 \Rightarrow (0, 0)$ is a critical pt.
$y''(-1) = -6 < 0 \Rightarrow (-1, 4)$ is a local max.
$y''(1) = 6 > 0 \Rightarrow (1, 0)$ is a local min.
Concavity reverses between $x = -1$ and $x = 1$, so there must be an inflection point.
$y'' = 0$ when $x = 0$.
$y(0) = 2 \Rightarrow (0, 2)$ is an inflection pt.



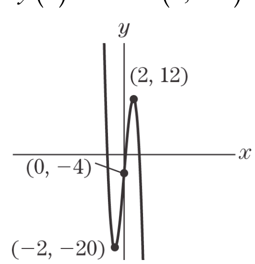
26. $y = x^3 - 6x^2 + 9x + 3$
$y' = 3x^2 - 12x + 9$
$y'' = 6x - 12$
$y' = 0$ if $x = 1$ or $x = 3$
$y(1) = 7 \Rightarrow (1, 7)$ is a critical pt.
$y(3) = 3 \Rightarrow (3, 3)$ is a critical pt.
$y''(1) = -6 < 0 \Rightarrow (1, 7)$ is a local max.
$y''(3) = 6 > 0 \Rightarrow (3, 3)$ is a local min.
Concavity reverses between $x = 1$ and $x = 3$, so there must be an inflection point.
$y'' = 0$ when $x = 2$.
$y(2) = 5 \Rightarrow (2, 5)$ is an inflection pt.



27. $y = 1 + 3x^2 - x^3$
$y' = 6x - 3x^2$
$y'' = 6 - 6x$
$y' = 0$ if $x = 0$ or $x = 2$
$y(0) = 1 \Rightarrow (0, 1)$ is a critical pt.
$y(2) = 5 \Rightarrow (2, 5)$ is a critical pt.
$y''(0) = 6 > 0 \Rightarrow (0, 1)$ is a local min.
$y''(2) = -6 < 0 \Rightarrow (2, 5)$ is a local max.
Concavity reverses between $x = 0$ and $x = 2$, so there must be an inflection point.
$y'' = 0$ when $x = 1$.
$y(1) = 3 \Rightarrow (1, 3)$ is an inflection pt.



28. $y = -x^3 + 12x - 4$
$y' = -3x^2 + 12$
$y'' = -6x$
$y' = 0$ if $x = -2$ or $x = 2$
$y(-2) = -20 \Rightarrow (-2, -20)$ is a critical pt.
$y(2) = 12 \Rightarrow (2, 12)$ is a critical pt.
$y''(-2) = 12 > 0 \Rightarrow (-2, -20)$ is a local min.
$y''(2) = -12 < 0 \Rightarrow (2, 12)$ is a local max.
Concavity reverses between $x = -2$ and $x = 2$, so there must be an inflection point.
$y'' = 0$ when $x = 0$.
$y(0) = -4 \Rightarrow (0, -4)$ is an inflection pt.



29. $y = \frac{1}{3}x^3 - x^2 - 3x + 5$

$y' = x^2 - 2x - 3$

$y'' = 2x - 2$

$y' = 0$ if $x = -1$ or $x = 3$

$y(-1) = \frac{20}{3} \Rightarrow \left(-1, \frac{20}{3}\right)$ is a critical pt.

$y(3) = -4 \Rightarrow (3, -4)$ is a critical pt.

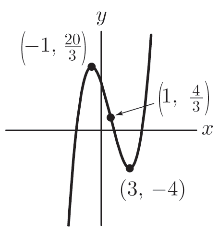
$y''(-1) = -4 < 0 \Rightarrow \left(1, \frac{20}{3}\right)$ is a local max.

$y''(3) = 4 > 0 \Rightarrow (3, -4)$ is a local min.

Concavity reverses between $x = -1$ and $x = 3$, so there must be an inflection point.

$y'' = 0$ when $x = 1$.

$y(1) = \frac{4}{3} \Rightarrow \left(1, \frac{4}{3}\right)$ is an inflection pt.



30. $y = x^4 + \frac{1}{3}x^3 - 2x^2 - x + 1$

$y' = 4x^3 + x^2 - 4x - 1$

$y'' = 12x^2 + 2x - 4$

$y' = 0$ if $x = \pm 1$ or $x = -\frac{1}{4}$

$y(-1) = \frac{2}{3} \Rightarrow \left(-1, \frac{2}{3}\right)$ is a critical pt.

$y\left(-\frac{1}{4}\right) = \frac{863}{768} \Rightarrow \left(-\frac{1}{4}, \frac{863}{768}\right)$ is a critical pt.

$y(1) = -\frac{2}{3} \Rightarrow \left(1, -\frac{2}{3}\right)$ is a critical pt.

$y''(-1) = 6 > 0 \Rightarrow \left(-1, \frac{2}{3}\right)$ is a local min.

$y''\left(-\frac{1}{4}\right) = -\frac{15}{4} < 0 \Rightarrow$

$\left(-\frac{1}{4}, \frac{863}{768}\right)$ is a local max.

$y''(1) = 10 > 0 \Rightarrow \left(1, -\frac{2}{3}\right)$ is a local min.

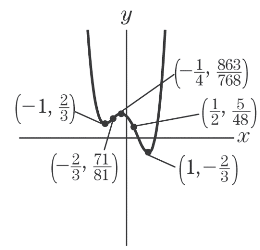
Concavity reverses between $x = -1$ and $x = -\frac{1}{4}$, and $x = -\frac{1}{4}$ and $x = 1$, so there must be inflection points.

$y'' = 0$ when $x = -\frac{2}{3}$ and $x = \frac{1}{2}$.

$y\left(-\frac{2}{3}\right) = \frac{71}{81} \Rightarrow \left(-\frac{2}{3}, \frac{71}{81}\right)$ is an inflection pt.

$y\left(\frac{1}{2}\right) = \frac{5}{48} \Rightarrow \left(\frac{1}{2}, \frac{5}{48}\right)$ is an inflection pt.

Note that $\left(1, -\frac{2}{3}\right)$ is an absolute minimum.



31. $y = 2x^3 - 3x^2 - 36x + 20$

$y' = 6x^2 - 6x - 36$

$y'' = 12x - 6$

$y' = 0$ if $x = -2$ or $x = 3$

$y(-2) = 64 \Rightarrow (-2, 64)$ is a critical pt.

$y(3) = -61 \Rightarrow (3, -61)$ is a critical pt.

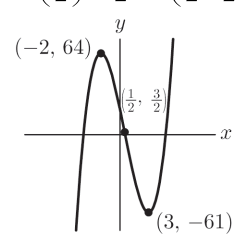
$y''(-2) = -30 < 0 \Rightarrow (-2, 64)$ is a local max.

$y''(3) = 30 > 0 \Rightarrow (3, -61)$ is a local min.

Concavity reverses between $x = -2$ and $x = 3$, so there must be an inflection point.

$y'' = 0$ when $x = \frac{1}{2}$.

$y\left(\frac{1}{2}\right) = \frac{3}{2} \Rightarrow \left(\frac{1}{2}, \frac{3}{2}\right)$ is an inflection pt.



32. $y = x^4 - \frac{4}{3}x^3$

$y' = 4x^3 - 4x^2 = 4x^2(x - 1)$

$y'' = 12x^2 - 8x$

$y' = 0$ if $x = 0$ or $x = 1$

$y(0) = 0 \Rightarrow (0, 0)$ is a critical pt.

$y(1) = -\frac{1}{3} \Rightarrow \left(1, -\frac{1}{3}\right)$ is a critical pt.

$y''(0) = 0,$ so use the first derivative test.

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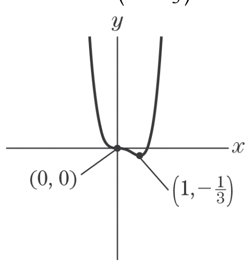
Critical Points, Intervals	$x < 0$	$0 < x < 1$	$1 < x$
$4x^2$	+	+	+
$(x-1)$	−	−	+
$f'(x)$	+	−	+
$f(x)$	Decreasing on $(-\infty, 0)$	Decreasing on $(0, 1)$	Increasing on $(1, \infty)$

We have identified (0, 0) as a critical point. However, it is neither a local maximum, nor a local minimum. Therefore, it must be an inflection point. Verify this by confirming that $y'' = 0$ when $x = 0$.

Note that $\left(1, -\frac{1}{3}\right)$ is an absolute minimum.



33. $f'(x) = 2ax + b;\ f''(x) = 2a$
It is not possible for the graph of $f(x)$ to have an inflection point because $f''(x) = 2a \neq 0$.

34. $f'(x) = 3ax^2 + 2bx + c;\ f''(x) = 6ax + 2b$
No, $f''(x)$ is a linear function of x and hence can be zero for at most one value of x.

35. $f(x) = \frac{1}{4}x^2 - 2x + 7;\ f'(x) = \frac{1}{2}x - 2;$

$f''(x) = \frac{1}{2}$

Set $f'(x) = 0$ and solve for x,

$\frac{1}{2}x - 2 = 0, x = 4$;

$f(4) = \frac{1}{4}(4)^2 - 2(4) + 7 = 3;\ \ f''(4) = \frac{1}{2}$

Since $f''(4)$ is positive, the graph is concave up at $x = 3$ and therefore (4, 3) is a relative minimum point.

36. $f(x) = 5 - 12x - 2x^2; f'(x) = -12 - 4x$;
$f''(x) = -4$
Set $f'(x) = 0$ and solve for x.
$-12 - 4x = 0 \Rightarrow x = -3$
$f(-3) = 5 - 12(-3) - 2(-3)^2 = 23$
$f''(-3) = -4$
Since $f''(3)$ is negative, the graph is concave down at $x = -3$ and therefore (–3, 23) is a relative maximum point.

37. $g(x) = 3 + 4x - 2x^2; g'(x) = 4 - 4x$
$g''(x) = -4$
Set $g'(x) = 0$ and solve for x.
$4 - 4x = 0 \Rightarrow x = 1$
$g(1) = 3 + 4(1) - 2(1)^2 = 5; g''(1) = -4$
Since $g''(1)$ is negative, the graph is concave down at $x = 1$ and therefore (1, 5) is a relative maximum point.

38. $g(x) = x^2 + 10x + 10; g'(x) = 2x + 10$
$g''(x) = 2$
Set $g'(x) = 0$ and solve for x.
$2x + 10 = 0 \Rightarrow x = -5$
$g(-5) = (-5)^2 + 10(-5) + 10 = -15$
$g''(-5) = 2$
Since $g''(-5)$ is positive, the graph is concave up at $x = -5$ and therefore (–5, –15) is a relative minimum point.

39. $f(x) = 5x^2 + x - 3; f'(x) = 10x + 1; f''(x) = 10$
Set $f'(x) = 0$ and solve for x.

$10x + 1 = 0 \Rightarrow x = -\frac{1}{10} = -.1$

$f(-.1) = 5(-.1)^2 + (-.1) - 3 = -3.05$

$f''(-.1) = 10$

Since $f''(-.1)$ is positive, the graph is concave up at $x = -.1$ and therefore (–.1, –3.05) is a relative minimum point.

40. $f(x) = 30x^2 - 1800x + 29,000;$
$f'(x) = 60x - 1800;\ f''(x) = 60$
Set $f'(x) = 0$ and solve for x.
$60x - 1800 = 0 \Rightarrow x = 30$
$f(30) = 30(30)^2 - 1800(30) + 29,000 = 2000$;
$f''(30) = 60$
Since $f''(30)$ is positive, the graph is concave up at $x = 30$ and therefore (30, 2000) is a relative minimum point.

41. $y = g(x)$ is the derivative of $y = f(x)$ because the zero of $g(x)$ corresponds to the extreme point of $f(x)$.

42. $y = g(x)$ is the derivative of $y = f(x)$ because the zero of $g(x)$ corresponds to the extreme point of $f(x)$.

43. a. f has a relative minimum.

b. f has an inflection point.

44. a. Since $f(125) = 125$, the population was 125 million.

b. The solution of $f(t) = 25$ is $t = 50$, so the population was 25 million in 1850.

c. Since $f'(150) = 2.2$, the population was growing at the rate of 2.2 million per year.

d. The solutions of $f'(t) = 1.8$ are $t \approx 110$ and $t \approx 175$, corresponding to the years 1910 and 1975. The desired answer is 1975.

e. The maximum value of $f'(t)$ appears to occur at $t \approx 140$. To confirm, observe that the graph of $f''(t)$ crosses the t-axis at $t = 140$. The population was growing at the greatest rate in 1940.

45. a. $(.47, 41), (.18, 300);\ m = \dfrac{300 - 41}{.18 - .47} - -\dfrac{259}{.29}$;

$$y - 41 = -\frac{259}{.29}(x - .47) \Rightarrow$$
$$y = -\frac{259}{.29}x + \frac{133.62}{.29}$$
$$\approx -893.103x + 460.759$$
$A(x) = -893.103x + 460.759$ billion dollars

b. $R(x) = \dfrac{x}{100} \cdot A(x)$

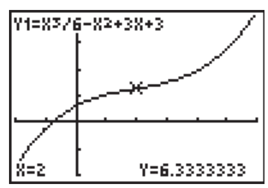
$$= \frac{x}{100}(-893.103x + 460.759)$$
$R(.3) \approx \$.578484$ billion or \$578.484 million
$R(.1) \approx \$.371449$ billion or \$371.449 million

c. $R'(x) = -17.8621x + 4.6076$
$R'(x) = 0$ when $x = .258$, The fee that maximizes revenue is .258% and the maximum revenue is
$R(.258) = \$.594273$ billion or \$594.273 million

46. $C(x) = -2.5x + 1;\ P(x) = R(x) - C(x);$

$$P(x) = \frac{x}{100}(-893.103x + 460.759) - (-2.5x + 1)$$
$$= -8.93103x^2 + 7.10759x - 1$$
$P'(x) = -17.86206x + 7.10759$
$P'(x) = 0$ when $x = .398$.
Profit is maximized when the fee is .398%.
$P(.3) = \$.3285$ billion, $P(.1) = -\$.3786$ billion.
They were better off before lowering their fees.

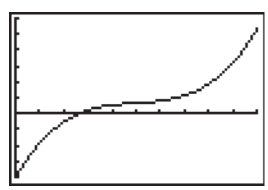
47.



[−2, 6] by [−10, 20]

Since $f(x)$ is always increasing, $f'(x)$ is always nonnegative.

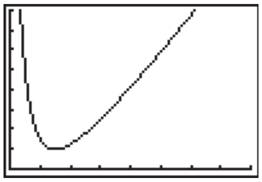
48.



[0, 10] by [−20, 30]

Note that $f'(x) = \frac{1}{2}x^2 - 5x + 13$ and $f''(x) = x - 5$. Solving $f''(x) = 0$, the inflection point occurs at $x = 5$. Since $f(5) = \frac{10}{3}$, the coordinates of the inflection point are $\left(5, \frac{10}{3}\right)$.

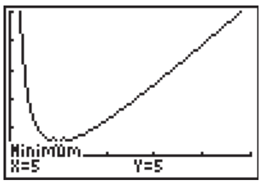
49.



[0, 16] by [0, 16]

This graph is like the graph of a parabola that opens upward because (for $x > 0$) the entire graph is concave up and it has a minimum value. Unlike a parabola, it is not symmetric. Also, this graph has a vertical asymptote ($x = 0$), while a parabola does not have an asymptote.

50.



[0, 25] by [0, 50]

The relative minimum occurs at (5, 5).
To determine this algebraically, observe that $f'(x) = 3 - \frac{75}{x^2}$. Solving $f'(x) = 0$ gives $x^2 = 25$, or $x = \pm 5$. This confirms that the relative extreme value (for $x > 0$) occurs at $x = 5$.To show that there are no inflection points, observe that $f''(x) = \frac{150}{x^3}$. Since $f''(x)$ changes sign only at $x = 0$ (where $f(x)$ is undefined), there are no inflection points.

2.4 Curve Sketching (Conclusion)

1. $y = x^2 - 3x + 1$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{5}}{2}$$

The x-intercepts are $\left(\frac{3+\sqrt{5}}{2}, 0\right)$ and $\left(\frac{3-\sqrt{5}}{2}, 0\right)$

2. $y = x^2 + 5x + 5$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(5)}}{2(1)} = \frac{-5 \pm \sqrt{5}}{2}$$

The x-intercepts are $\left(\frac{-5+\sqrt{5}}{2}, 0\right)$ and $\left(\frac{-5-\sqrt{5}}{2}, 0\right)$.

3. $y = 2x^2 + 5x + 2$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(2)}}{2(2)} = \frac{-5 \pm 3}{4} = -\frac{1}{2}, -2$$

The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and (–2, 0).

4. $y = 4 - 2x - x^2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(4)}}{2(-1)} = \frac{2 \pm 2\sqrt{5}}{-2}$$

$$= -1 \pm \sqrt{5}$$

The x-intercepts are $(-1+\sqrt{5}, 0)$ and $(-1-\sqrt{5}, 0)$.

5. $y = 4x - 4x^2 - 1$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-4)(-1)}}{2(-4)} = \frac{-4 \pm 0}{-8}$$

The x-intercept is $\left(\frac{1}{2}, 0\right)$.

6. $y = 3x^2 + 10x + 3$

$$x = \frac{10 \pm \sqrt{10^2 - 4(3)(3)}}{2(3)} = \frac{-10 \pm 8}{6}$$

The x-intercepts are $\left(-\frac{1}{3}, 0\right)$ and (–3, 0).

7. $f(x) = \frac{1}{3}x^3 - 2x^2 + 5x; f'(x) = x^2 - 4x + 5$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} = \frac{4 \pm \sqrt{-4}}{2}$$

Since $f'(x)$ has no real zeros, $f(x)$ has no relative extreme points.

8. $f(x) = -x^3 + 2x^2 - 6x + 3$
$f'(x) = -3x^2 + 4x - 6$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-3)(-6)}}{2(-3)} = \frac{4 \pm \sqrt{-56}}{-6}$$

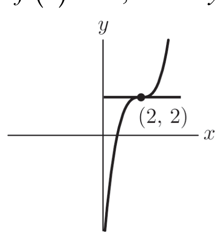
Since $f'(x)$ has noreal zeros, $f(x)$ has no relative extreme points. Since, $f'(x) < 0$ for all x, $f(x)$ is always decreasing.

9. $f(x) = x^3 - 6x^2 + 12x - 6$
$f'(x) = 3x^2 - 12x + 12$
$f''(x) = 6x - 12$
To find possible extrema, set $f'(x) = 0$ and solve for x.
$3x^2 - 12x + 12 = 0$
$3(x^2 - 4x + 4) = 0$
$(x-2)^2 = 0 \Rightarrow x = 2$
$f(2) = 2^3 - 6 \cdot 2^2 + 12 \cdot 2 - 6 = 2$
Thus, (2, 2) is a critical point.

(*continued*)

Critical Points, Intervals	$x < 2$	$2 < x$
$x - 2$	$-$	$+$
$f'(x)$	$+$	$+$
$f(x)$	Increasing on $(-\infty, 2]$	Increasing on $[2, \infty)$

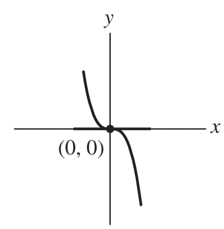
No relative maximum or relative minimum.
Since $f'(x) \geq 0$ for all x, the graph is always increasing.
To find possible inflection points, set $f''(x) = 0$ and solve for x.
$6x - 12 = 0 \Rightarrow x = 2$
Since $f''(x) < 0$ for $x < 2$ (meaning the graph is concave down) and $f''(x) > 0$ for $x > 2$ (meaning the graph is concave up), the point (2, 2) is an inflection point.
$f(0) = -6$, so the y-intercept is (0, −6).



10. $f(x) = -x^3$
$f'(x) = -3x^2$
$f''(x) = -6x$
To find possible extrema, set $f'(x) = 0$ and solve for x.
$-3x^2 = 0 \Rightarrow x = 0$
$f(0) = -0^3 = 0$
Thus, (0, 0) is a critical point.

Critical Points, Intervals	$x < 0$	$x > 0$
x	$+$	$-$
$f'(x)$	$-$	$-$
$f(x)$	Decreasing on $(-\infty, 0]$	Decreasing on $[0, \infty)$

No relative maximum or relative minimum.
Since $f'(x) \leq 0$ for all x, the graph is always decreasing.
To find possible inflection points, set $f''(x) = 0$ and solve for x.
$-6x = 0 \Rightarrow x = 0$
Since $f''(x) > 0$ for $x < 0$ (meaning the graph is concave up) and $f''(x) < 0$ for $x > 0$ (meaning the graph is concave down), the point (0, 0) is an inflection point.
$f(0) = 0$, so the y-intercept is (0, 0).



11. $f(x) = x^3 + 3x + 1$
$f'(x) = 3x^2 + 3$
$f''(x) = 6x$
To find possible extrema, set $f'(x) = 0$ and solve for x.
$3x^2 + 3 = 0 \Rightarrow$ no real solution
Thus, there are no extrema.
Since $f'(x) \geq 0$ for all x, the graph is always increasing.
To find possible inflection points, set $f''(x) = 0$ and solve for x.
$6x = 0 \Rightarrow x = 0$
$f(0) = 1$
Since $f''(x) < 0$ for $x < 0$ (meaning the graph is concave down) and $f''(x) > 0$ for $x > 0$ (meaning the graph is concave up), the point (0, 1) is an inflection point. This is also the y-intercept.

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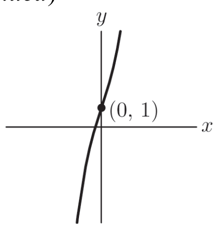
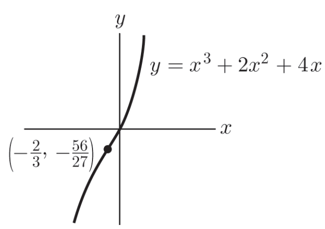
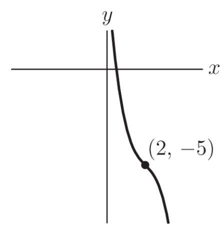


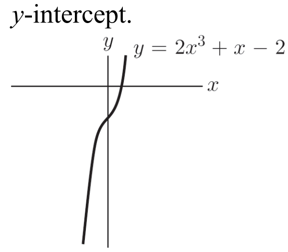
12. $f(x) = x^3 + 2x^2 + 4x$
$f'(x) = 3x^2 + 4x + 4$
$f''(x) = 6x + 4$
To find possible extrema, set $f'(x) = 0$ and solve for *x*.
$3x^2 + 4x + 4 = 0 \Rightarrow$ no real solution
Thus, there are no extrema.
Since $f'(x) \geq 0$ for all *x*, the graph is always increasing.
To find possible inflection points, set $f''(x) = 0$ and solve for *x*.
$6x + 4 = 0 \Rightarrow x = -\dfrac{2}{3}$
$f\left(-\dfrac{2}{3}\right) = -\dfrac{56}{27}$
Since $f''(x) < 0$ for $x < -\frac{2}{3}$ (meaning the graph is concave down) and $f''(x) > 0$ for $x > -\frac{2}{3}$ (meaning the graph is concave up), the point $\left(-\frac{2}{3}, -\frac{56}{27}\right)$ is an inflection point.
$f(0) = 0$, so the *y*-intercept is (0, 0).



13. $f(x) = 5 - 13x + 6x^2 - x^3$
$f'(x) = -13 + 12x - 3x^2$
$f''(x) = 12 - 6x$
To find possible extrema, set $f'(x) = 0$ and solve for *x*.
$-13 + 12x - 3x^2 = 0 \Rightarrow$ no real solution
Thus, there are no extrema.
Since $f'(x) \leq 0$ for all *x*, the graph is always decreasing.
To find possible inflection points, set $f''(x) = 0$ and solve for *x*.
$12 - 6x = 0 \Rightarrow x = 2$
$f(2) = -5$
Since $f''(x) > 0$ for $x < 2$ (meaning the graph is concave up) and $f''(x) < 0$ for $x > 2$ (meaning the graph is concave down), the point (2, –5) is an inflection point.
$f(0) = 5$, so the *y*-intercept is (0, 5).



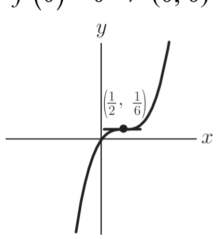
14. $f(x) = 2x^3 + x - 2$
$f'(x) = 6x^2 + 1$
$f''(x) = 12x$
To find possible extrema, set $f'(x) = 0$ and solve for *x*.
$6x^2 + 1 = 0 \Rightarrow$ no real solution
Thus, there are no extrema.
Since $f'(x) \geq 0$ for all *x*, the graph is always increasing.
To find possible inflection points, set $f''(x) = 0$ and solve for *x*.
$12x = 0 \Rightarrow x = 0$
$f(0) = -2$
Since $f''(x) < 0$ for $x < 0$ (meaning the graph is concave down) and $f''(x) > 0$ for $x > 0$ (meaning the graph is concave up), the point (0, 2) is an inflection point. This is also the *y*-intercept.



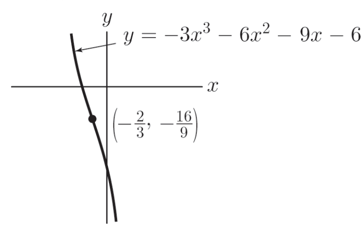
15. $f(x)=\frac{4}{3}x^3-2x^2+x$
$f'(x)=4x^2-4x=4x(x-1)$
$f''(x)=8x-4$
To find possible extrema, set $f'(x)=0$ and solve for x.
$4x^2-4x=0 \Rightarrow x=0, 1$
$f(0)=0 \Rightarrow (0, 0)$ is a critical point
$f(1)=\frac{1}{3} \Rightarrow \left(1, \frac{1}{3}\right)$ is a critical point

Critical Points, Intervals	$x<0$	$0<x<1$	$1<x$
$4x$	–	+	+
$(x-1)$	–	–	+
$f'(x)$	+	–	+
$f(x)$	Decreasing on $(-\infty, 0)$	Decreasing on $(0, 1)$	Increasing on $(1, \infty)$

We have identified (0, 0) and $\left(1, \frac{1}{3}\right)$ as critical points. However, neither is a local maximum, nor a local minimum. Therefore, they may be inflection points. However, $f''(0)\neq 0$ and $f''\neq 0,$ so neither is an inflection point.
Since $f'(x)\geq 0$ for all x, the graph is always increasing.
To find possible inflection points, set $f''(x)=0$ and solve for x.
$8x-4=0 \Rightarrow x=\frac{1}{2}$
$f\left(\frac{1}{2}\right)=\frac{1}{6}$
Since $f''(x)<0$ for $x<\frac{1}{2}$ (meaning the graph is concave down) and $f''(x)>0$ for $x>\frac{1}{2}$ (meaning the graph is concave up), the point $\left(\frac{1}{2}, \frac{1}{6}\right)$ is an inflection point.
$f(0)=0 \Rightarrow$ (0, 0) is the y-intercept.



16. $f(x)=-3x^3-6x^2-9x-6$
$f'(x)=-9x^2-12x-9$
$f''(x)=-18x-12$
To find possible extrema, set $f'(x)=0$ and solve for x.
$-9x^2-12x-9=0 \Rightarrow$ no real solution
Thus, there are no extrema.
Since $f'(x)\leq 0$ for all x, the graph is always decreasing.
To find possible inflection points, set $f''(x)=0$ and solve for x.
$-18x-12=0 \Rightarrow x=-\frac{2}{3}$
$f\left(-\frac{2}{3}\right)=-\frac{16}{9}$
Since $f''(x)>0$ for $x<-\frac{2}{3}$ (meaning the graph is concave up) and $f''(x)<0$ for $x>-\frac{2}{3}$ (meaning the graph is concave down), the point $\left(-\frac{2}{3}, -\frac{16}{9}\right)$ is an inflection point.
$f(0)=-6,$ so the y-intercept is (0, –6).



17. $f(x)=1-3x+3x^2-x^3$
$f'(x)=-3+6x-3x^2=-3\left(x^2-2x+1\right)$
$f''(x)=6-6x$
To find possible extrema, set $f'(x)=0$ and solve for x.
$-3+6x-3x^2=0 \Rightarrow x=1$
$f(1)=0$
Since $f'(x)\leq 0$ for all x, the graph is always decreasing, and thus, there are no extrema. Therefore, (1, 0) may be an inflection point.
Set $f''(x)=0$ and solve for x.
$6-6x=0 \Rightarrow x=1$
Since $f''(x)>0$ for $x<1$ (meaning the graph is concave up) and $f''(x)<0$ for $x>1$ (meaning the graph is concave down), the point (1, 0) is an inflection point.
$f(0)=1,$ so the y-intercept is (0, 1).

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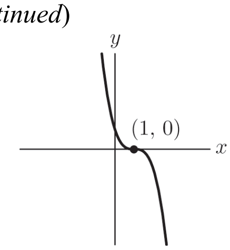


18. $f(x) = \frac{1}{3}x^3 - 2x^2$

$f'(x) = x^2 - 4x$

$f''(x) = 2x - 4$

$f'(x) = 0$ if $x = 0$ or $x = 4$

$f(0) = 0 \Rightarrow (0, 0)$ is a critical pt.

$f(4) = -\frac{32}{3} \Rightarrow \left(4, -\frac{32}{3}\right)$ is a critical pt.

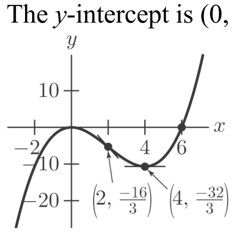
$f''(0) = -4 < 0,$ so the graph is concave down at $x = 0$, and (0, 0) is a relative maximum.

$f''(4) = 4 > 0,$ so the graph is concave up at $x = 4$, and $\left(4, -\frac{32}{3}\right)$ is a relative minimum.

$f''(x) = 0$ when $x = 2$.

$f(2) = -\frac{16}{3} \Rightarrow \left(2, -\frac{16}{3}\right)$ is an inflection pt.

The y-intercept is (0, 0).



19. $f(x) = x^4 - 6x^2$

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$

To find possible extrema, set $f'(x) = 0$ and solve for x.

$4x^3 - 12x = 0$

$4x\left(x^2 - 3\right) = 0 \Rightarrow x = 0,\ x = -\sqrt{3},\ x = \sqrt{3}$

$f(0) = 0^4 - 6 \cdot 0^2 = 0$

$f\left(-\sqrt{3}\right) = \left(-\sqrt{3}\right)^4 - 6 \cdot \left(-\sqrt{3}\right)^2 = 9 - 18 = -9$

$f\left(\sqrt{3}\right) = \left(\sqrt{3}\right)^4 - 6 \cdot \left(\sqrt{3}\right)^2 = 9 - 18 = -9$

Thus, (0, 0), $\left(-\sqrt{3}, -9\right)$, and $\left(\sqrt{3}, -9\right)$ are critical points.

$f''(0) = -12,$ so the graph is concave down at $x = 0$, and (0, 0) is a relative maximum.

$f''\left(-\sqrt{3}\right) = 12\left(-\sqrt{3}\right)^2 - 12 = 24$ so the graph is concave up at $x = -\sqrt{3}$, and $\left(-\sqrt{3}, -9\right)$, is a relative minimum.

$f''\left(\sqrt{3}\right) = 12\left(\sqrt{3}\right)^2 - 12 = 24$ so the graph is concave up at $x = \sqrt{3}$, and $\left(\sqrt{3}, -9\right)$, is a relative minimum.

The concavity of this function reverses twice, so there must be at least two inflection points.

Set $f''(x) = 0$ and solve for x:

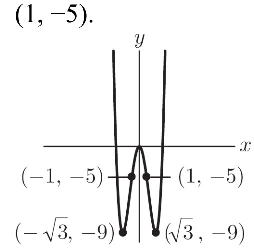
$12x^2 - 12 = 0 \Rightarrow 12\left(x^2 - 1\right) = 0 \Rightarrow$

$(x - 1)(x + 1) = 0 \Rightarrow x = \pm 1$

$f(-1) = (-1)^4 - 6(-1)^2 = -5$

$f(1) = 1^4 - 6(1)^2 = -5$

Thus, the inflection points are (−1, −5) and (1, −5).



20. $f(x) = 3x^4 - 6x^2 + 3$

$f'(x) = 12x^3 - 12x$

$f''(x) = 36x^2 - 12$

To find possible extrema, set $f'(x) = 0$ and solve for x.

$12x^3 - 12x = 0$

$12x\left(x^2 - 1\right) = 0 \Rightarrow x = 0,\ x = \pm 1$

$f(0) = 3 \cdot 0^4 - 6 \cdot 0^2 + 3 = 3$

$f(-1) = 3 \cdot (-1)^4 - 6 \cdot (-1)^2 + 3 = 0$

$f(1) = 3 \cdot (1)^4 - 6 \cdot (1)^2 + 3 = 0$

Thus, (0, 3), (−1, 0) and (1, 0) are critical points.

$f''(0) = -12,$ so the graph is concave down at $x = 0$, and (0, 3) is a relative maximum.

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(*continued*)

$f''(-1) = 36(-1)^2 - 12 = 24$ so the graph is concave up at $x = -1$, and $(-1, 0)$ is a relative minimum.

$f''(1) = 36(1)^2 - 12 = 24$ so the graph is concave up at $x = 1$, and $(1, 0)$ is a relative minimum.

The concavity of this function reverses twice, so there must be at least two inflection points. Set $f''(x) = 0$ and solve for x:

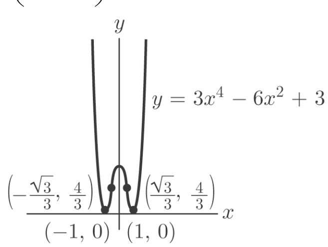
$$36x^2 - 12 = 0 \Rightarrow$$

$$x = \frac{0 \pm \sqrt{0^2 - 4(36)(-12)}}{2 \cdot 36} = \pm\frac{\sqrt{3}}{3}$$

$$f\left(-\frac{\sqrt{3}}{3}\right) = 3\cdot\left(-\frac{\sqrt{3}}{3}\right)^4 - 6\left(-\frac{\sqrt{3}}{3}\right)^2 + 3 = \frac{4}{3}$$

$$f\left(\frac{\sqrt{3}}{3}\right) = 3\cdot\left(\frac{\sqrt{3}}{3}\right)^4 - 6\left(\frac{\sqrt{3}}{3}\right)^2 + 3 = \frac{4}{3}$$

Thus, the inflection points are $\left(-\frac{\sqrt{3}}{3}, \frac{4}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \frac{4}{3}\right)$.



21. $f(x) = (x-3)^4$
$f'(x) = 4(x-3)^3$
$f''(x) = 12(x-3)$

To find possible extrema, set $f'(x) = 0$ and solve for x.

$4(x-3)^3 = 0 \Rightarrow x = 3$

$f(3) = 0$

Thus, (3, 0) is a critical point.

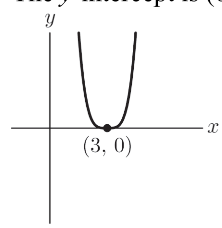
$f''(3) = 0$, so we must use the first derivative rule to determine if (3, 0) is a local maximum or minimum.

Critical Points, Intervals	$x < 3$	$x > 3$
$x - 3$	−	+
$f'(x)$	−	+
$f(x)$	Decreasing on $(-\infty, 3)$	Increasing on $(3, \infty)$

Thus, (3, 0) is local minimum.

Since $f''(x) = 0$, when $x = 3$, (3, 0) is also an inflection point.

The y-intercept is (0, 81).



22. $f(x) = (x+2)^4 - 1$
$f'(x) = 4(x+2)^3$
$f''(x) = 12(x+2)$

To find possible extrema, set $f'(x) = 0$ and solve for x.

$4(x+2)^3 = 0 \Rightarrow x = -2$

$f(-2) = -1$

Thus, (–2, –1) is a critical point.

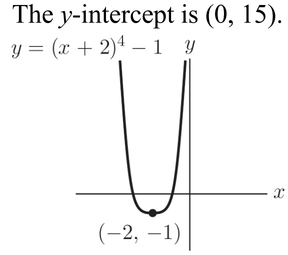
$f''(-2) = 0$, so we must use the first derivative rule to determine if (–2, –1) is a local maximum or minimum.

Critical Points, Intervals	$x < -2$	$x > -2$
$x + 2$	−	+
$f'(x)$	−	+
$f(x)$	Decreasing on $(-\infty, -2)$	Increasing on $(-2, \infty)$

Thus, (–2, –1) is local minimum.

Since $f''(x) = 0$, when $x = -2$, (–2, –1) is also an inflection point.

The y-intercept is (0, 15).



23. $y = \frac{1}{x} + \frac{1}{4}x,\ x > 0$

$y' = -\frac{1}{x^2} + \frac{1}{4}$

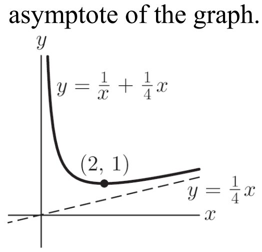
$y'' = \frac{2}{x^3}$

To find possible extrema, set $y' = 0$ and solve for x:

$-\frac{1}{x^2} + \frac{1}{4} = 0 \Rightarrow x = 2$

Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = 2$, $y = 1$, and $y'' = \frac{1}{4} > 0$, so the graph is concave up, and (2, 1) is a relative minimum.

Since y'' can never be zero, there are no inflection points. The term $\frac{1}{x}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = \frac{1}{4}x$, so this is also an asymptote of the graph.



24. $y = \frac{2}{x},\ x > 0$

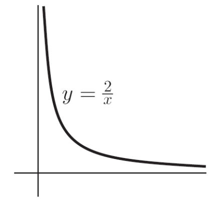
$y' = -\frac{2}{x^2}$

$y'' = \frac{4}{x^3}$

To find possible extrema, set $y' = 0$ and solve for x:

$-\frac{2}{x^2} = 0 \Rightarrow$ no solution, so there are no extrema.

Since $y' \le 0$ for all x, the graph is always decreasing. Since y'' can never be zero, there are no inflection points. The term $\frac{2}{x}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = 0$, so this is also an asymptote of the graph.



25. $y = \frac{9}{x} + x + 1,\ x > 0$

$y' = -\frac{9}{x^2} + 1$

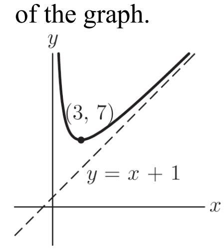
$y'' = \frac{18}{x^3}$

To find possible extrema, set $y' = 0$ and solve for x:

$-\frac{9}{x^2} + 1 = 0 \Rightarrow -\frac{9}{x^2} = -1 \Rightarrow 9 = x^2 \Rightarrow x = 3$

Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = 3$, $y = \frac{9}{3} + 3 + 1 = 7$, and $y'' = \frac{18}{3^3} > 0$, so the graph is concave up, and (3, 7) is a relative minimum.

Since y'' can never be zero, there are no inflection points. The term $\frac{9}{x}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = x + 1$, so this is an asymptote of the graph.



26. $y = \frac{12}{x} + 3x + 1,\ x > 0$

$y' = -\frac{12}{x^2} + 3$

$y'' = \frac{24}{x^3}$

To find possible extrema, set $y' = 0$ and solve for x:

$-\frac{12}{x^2} + 3 = 0 \Rightarrow -\frac{12}{x^2} = -3 \Rightarrow 4 = x^2 \Rightarrow x = 2$

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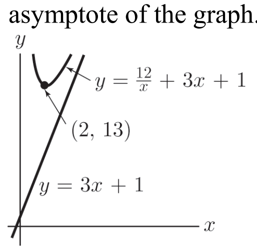
Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = 2$, $y = 13$ and $y'' = \frac{24}{2^3} > 0$, so the graph is concave up, and $(2, 13)$ is a relative minimum.
Since y'' can never be zero, there are no inflection points. The term $\frac{12}{x}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = 3x + 1$, so this is also an asymptote of the graph.



27. $y = \frac{2}{x} + \frac{x}{2} + 2,\ x > 0$

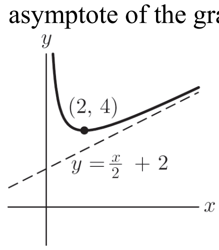
$y' = -\frac{2}{x^2} + \frac{1}{2}$

$y'' = \frac{4}{x^3}$

To find possible extrema, set $y' = 0$ and solve for x:

$-\frac{2}{x^2} + \frac{1}{2} = 0 \Rightarrow -\frac{2}{x^2} = -\frac{1}{2} \Rightarrow x = 2$

Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = 2$, $y = 4$ and $y'' = \frac{4}{2^3} > 0$, so the graph is concave up, and $(2, 4)$ is a relative minimum.
Since y'' can never be zero, there are no inflection points. The term $\frac{2}{x}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = \frac{x}{2} + 2$, so this is also an asymptote of the graph.



28. $y = \frac{1}{x^2} + \frac{x}{4} - \frac{5}{4},\ x > 0$

$y' = -\frac{2}{x^3} + \frac{1}{4}$

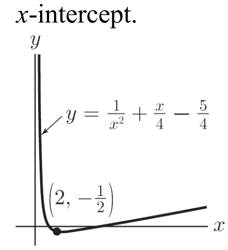
$y'' = \frac{6}{x^4}$

To find possible extrema, set $y' = 0$ and solve for x:

$-\frac{2}{x^3} + \frac{1}{4} = 0 \Rightarrow -\frac{2}{x^3} = -\frac{1}{4} \Rightarrow 8 = x^3 \Rightarrow x = 2$

Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = 2$,

$y = \frac{1}{2^2} + \frac{2}{4} - \frac{5}{4} = -\frac{1}{2}$, and $y'' = \frac{6}{x^4} > 0$, so the graph is concave up, and $\left(2, -\frac{1}{2}\right)$ is a relative minimum. Since y'' can never be zero, there are no inflection points. The term $\frac{1}{x^2}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = \frac{x}{4} - \frac{5}{4}$, so this is an asymptote of the graph. If $x = 1$, then $y =\ y = \frac{1}{1^2} + \frac{1}{4} - \frac{5}{4} = 0$, so $(1, 0)$ is an x-intercept.



29. $y = 6\sqrt{x} - x,\ x > 0$

$y' = 3x^{-1/2} - 1 = \frac{3}{\sqrt{x}} - 1$

$y'' = -\frac{3}{2}x^{-3/2}$

To find possible extrema, set $y' = 0$ and solve for x:

$\frac{3}{\sqrt{x}} - 1 = 0 \Rightarrow x = 9$

Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = 9$, $y = 9$, and $y'' < 0$, so the graph is concave down, and $(9, 9)$ is a relative maximum. Since y'' can never be zero, there are no inflection points.

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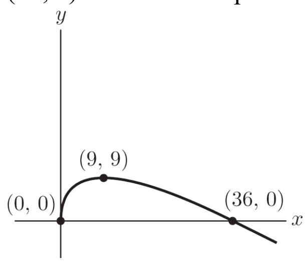
When $x = 0$, $y = 0$, so (0, 0) is the y-intercept.
$y = 6\sqrt{x} - x = 0 \Rightarrow 36x = x^2 \Rightarrow x = 36,$ so (36, 0) is an x-intercept



30. $y = \frac{1}{\sqrt{x}} + \frac{x}{2},\ x > 0$

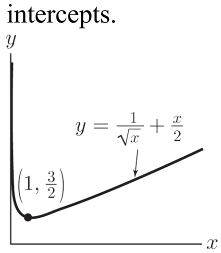
$y' = -\frac{1}{2x^{3/2}} + \frac{1}{2}$

$y'' = \frac{3}{4x^{5/2}}$

To find possible extrema, set $y' = 0$ and solve for x:

$-\frac{1}{2x^{3/2}} + \frac{1}{2} = 0 \Rightarrow x = 1$

When $x = 1$, $y = \frac{3}{2}$, and $y'' = \frac{3}{4} > 0$, so the graph is concave up, and $\left(1, \frac{3}{2}\right)$ is a relative minimum. Since y'' can never be zero, there are no inflection points. The term $\frac{1}{\sqrt{x}}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = \frac{x}{2}$, so this is an asymptote of the graph. The graph has no intercepts.



31. $g(x) = f'(x)$. The 3 zeros of $g(x)$ correspond to the 3 extreme points of $f(x)$. $f(x) \neq g'(x)$, the zeros of $f(x)$ do not correspond with the extreme points of $g(x)$.

32. $g(x) = f'(x)$. The zeros of $g(x)$ correspond to the extreme points of $f(x)$. But the zeros of $f(x)$ also correspond to the extreme points of $g(x)$. Observe that at points where $f(x)$ is decreasing, $g(x) < 0$ and that at points where $f(x)$ is increasing, $g(x) > 0$. But at points where $g(x)$ is increasing, $f(x) < 0$ and at points where $g(x)$ is decreasing, $f(x) > 0$.

33. $f(x) = ax^2 + bx + c;\ f'(x) = 2ax + b$

$f'(0) = b = 0$ (There is a local maximum at $x = 0 \Rightarrow f'(0) = 0$).

Therefore, $f(x) = ax^2 + c; f(0) = c = 1;$
$f(2) = 0 \Rightarrow 4a + 2b + c = 0 \Rightarrow$
$4a + 1 = 0 \Rightarrow a = -\frac{1}{4}:$

Thus, $f(x) = -\frac{1}{4}x^2 + 1.$

34. $f(x) = ax^2 + bx + c;\ f'(x) = 2ax + b$

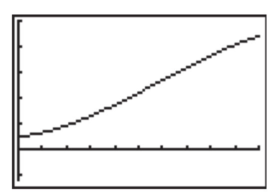
$f'(1) = 2a + b = 0$ (There is a local maximum at $x = 1 \Rightarrow f'(1) = 0$); $b = -2a$

Therefore, $f(x) = ax^2 - 2ax + c; f(0) = c = 1;$
$f(1) = a - 2a + 1 = -1 \Rightarrow a = 2, b = -4:$
$f(x) = 2x^2 - 4x + 1.$

35. Since $f'(a) = 0$ and $f'(x)$ is increasing at $x = a$, $f' < 0$ for $x < a$ and $f' > 0$ for $x > a$. According to the first derivative test, f has a local minimum at $x = a$.

36. Since $f'(a) = 0$ and $f'(x)$ is decreasing at $x = a$, $f' > 0$ for $x < a$ and $f' < 0$ for $x > a$. According to the first derivative test, f has a local maximum at $x = a$.

37. a.

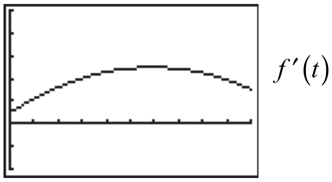


[0, 20] by [−12, 50]

b. Since $f(7) = 15.0036$, the rat weighed about 15.0 grams.

c. Using graphing calculator techniques, solve $f(t) = 27$ to obtain $t \approx 12.0380$. The rat's weight reached 27 grams after about 12.0 days.

d.



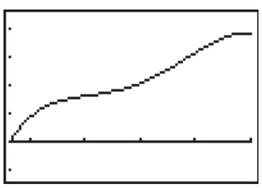
[0, 20] by [−2, 5]

Note that $f'(t) = .48 + .34t - .0144t^2$. Since $f'(4) = 1.6096$, the rat was gaining weight at the rate of about 1.6 grams per day.

e. Using graphing calculator techniques, solve $f'(t) = 2$ to obtain $t \approx 5.990$ or $t \approx 17.6207$. The rat was gaining weight at the rate of 2 grams per day after about 6.0 days and after about 17.6 days.

f. The maximum value of $f'(t)$ appears to occur at $t \approx 11.8$. To confirm, note that $f''(t) = .34 - .0288x$, so the solution of $f''(t) = 0$ is $t \approx 11.8056$. The rat was growing at the fastest rate after about 11.8 days.

38. a.



[32, 250] by [−1.2, 4.5]

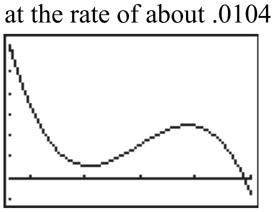
b. Since $f(100) = 1.63$, the canopy was 1.63 meters tall.

c. The solution of $f(t) = 2$ is $t \approx 143.9334$. The canopy was 2 meters high after about 144 days.

d. Note that

$$f'(t) = .142 - .0032t + .0000237t^2 - .0000000532t^3$$

(Alternately, use the calculator's numerical differentiation capability.) The graph of $y = f'(t)$ is shown. Since $f'(80) \approx .0104$, the canopy was growing at the rate of about .0104 meters per day.



[32, 250] by [−.01, .065]

e. The solutions of $f'(t) = .02$ are $t \approx 64.4040$, $t \approx 164.0962$, and $t \approx 216.9885$. The canopy was growing at the rate of .02 meters per day after about 64.4 days, after about 164.1 days, and after 217.0 days.

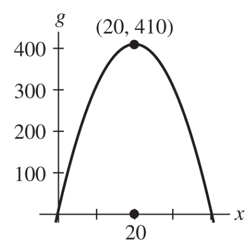
f. Since the solution to $f'(t) = 0$ is $t \approx 243.4488$, the canopy has completely stopped growing at this time and we may say that the canopy was growing slowest after about 243.4 days (see the graph in part (d)). (The growth rate also has a relative minimum after about 103.8 days.)

g. The graph shown in part (d) shows that $f'(t)$ was greatest at $t = 32$, after 32 days. (The growth rate also has a relative maximum after about 193.2 days.)

2.5 Optimization Problems

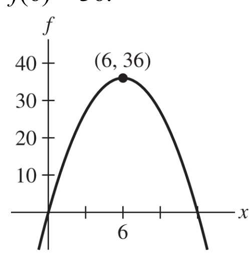
1. $g(x) = 10 + 40x - x^2 \Rightarrow g'(x) = 40 - 2x \Rightarrow g''(x) = -2$

The maximum value of $g(x)$ occurs at $x = 20$; $g(20) = 410$.

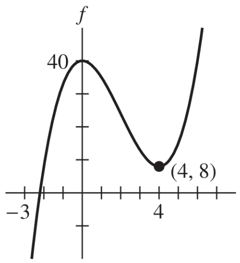


2. $f(x) = 12x - x^2 \Rightarrow f'(x) = 12 - 2x \Rightarrow f''(x) = -2$

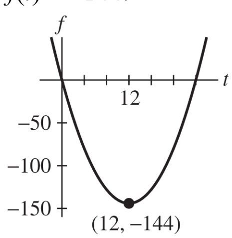
The maximum value of $f(x)$ occurs at $x = 6$; $f(6) = 36$.



3. $f(t)=t^3-6t^2+40 \Rightarrow f'(t)=3t^2-12t \Rightarrow$
$f''(t)=6t-12$
The minimum value for $t \ge 0$ occurs at $t = 4$; $f(4) = 8$.



4. $f(t)=t^2-24t \Rightarrow f'(t)=2t-24 \Rightarrow$
$f''(t)=2$
The minimum value of $f(t)$ occurs at $t = 12$; $f(t) = -144$.



5. Solving $x + y = 2$ for y gives $y = 2 - x$. Substituting into $Q = xy$ gives
$Q(x)=x(2-x)=2x-x^2$.
$\frac{dQ}{dx}=2-2x$
$\frac{dQ}{dx}=0 \Rightarrow 2-2x=0 \Rightarrow x=1$
$\frac{d^2Q}{dx^2}=-2$
The maximum value of $Q(x)$ occurs at $x = 1$, $y = 1$. $Q(1)=2(1)-(1)^2=1$.

6. Solving $x + y = 2$ for y gives $y = 2 - x$. Substituting into $Q=x^2y$ yields
$Q(x)=x^2(2-x)=2x^2-x^3$.
$\frac{dQ}{dx}=4x-3x^2$
$\frac{dQ}{dx}=0 \to 4x-3x^2=0 \to x=0 \text{ or } x=\frac{4}{3}$
$\frac{d^2Q}{dx^2}=4-6x, \left.\frac{d^2Q}{dx^2}\right|_{x=0}=4, \left.\frac{d^2Q}{dx^2}\right|_{x=\frac{4}{3}}=-4$
The maximum value of $Q(x)$ occurs at $x=\frac{4}{3}$.
Then $y=2-\frac{4}{3}=\frac{2}{3}$.

7. $x+y=6 \Rightarrow y=6-x$
$Q(x)=x^2+(6-x)^2=2x^2-12x+36$
$\frac{dQ}{dx}=4x-12; \quad \frac{d^2Q}{dx^2}=4$
$\frac{dQ}{dx}=0 \Rightarrow 4x-12=0 \Rightarrow x=3$
The minimum of $Q(x)$ occurs at $x = 3$. The minimum is $Q(3)=3^2+(6-3)^2=18$

8. No maximum. $\frac{d^2Q}{dx^2}=4$, so the function is concave upward at all points.

9. $xy=36 \to y=\frac{36}{x}$
$S(x)=x+\frac{36}{x}$
$S'(x)=1-\frac{36}{x^2}$
$S'(x)=0 \to 1-\frac{36}{x^2}=0 \to x=6 \text{ or } -6$
$S''(x)=1+\frac{72}{x^3}, S''(6)=\frac{4}{3}$
The positive value $x = 6$ minimizes $S(x)$, and $y=\frac{36}{6}=6$. $S(6,6)=6+6=12$

10. $x+y=1 \to y=1-x$
$y+z=2 \to z=2-y=1+x$
$Q(x)=x(1-x)(1+x)=x-x^3$
$Q'(x)=1-3x^2$
$Q'(x)=0 \to 1-3x^2=0 \to x=\frac{\sqrt{3}}{3}$
$Q''(x)=-6x, Q''\left(\frac{\sqrt{3}}{3}\right)=-2\sqrt{3}$
$Q(x)$ is a maximum when $x=\frac{\sqrt{3}}{3}$,
$y=1-\frac{\sqrt{3}}{3}=\frac{3-\sqrt{3}}{3}$, and $z=1+\frac{\sqrt{3}}{3}=\frac{3+\sqrt{3}}{3}$.
The maximum value of $Q(x)$ is
$Q\left(\frac{\sqrt{3}}{3}\right)=\frac{\sqrt{3}}{3}-\left(\frac{\sqrt{3}}{3}\right)^3=\frac{2\sqrt{3}}{9}$.

11. Let A = area.

a. Objective equation: $A = xy$
Constraint equation: $8x + 4y = 320$

b. Solving constraint equation for y in terms of x gives $y = 80 - 2x$. Substituting into objective equation yields
$A = x(80 - 2x) = -2x^2 + 80x$.

c. $\frac{dA}{dx} = -4x + 80 \Rightarrow \frac{d^2A}{dx^2} = -4$
The maximum value of A occurs at $x = 20$. Substituting this value into the equation for y in part b gives $y = 80 - 40 = 40$.
Answer: $x = 20$ ft, $y = 40$ ft

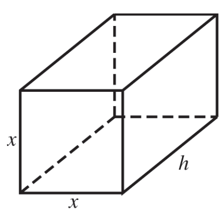
12. Let S = surface area.

a. Objective equation: $S = x^2 + 4xh$
Constraint: $x^2h = 32$

b. From constraint equation, $h = \frac{32}{x^2}$. Thus,
$S = x^2 + 4x\left(\frac{32}{x^2}\right) = x^2 + \frac{128}{x}$.

c. $\frac{dS}{dx} = 2x - \frac{128}{x^2}$; $\frac{d^2S}{dx^2} = 2 + \frac{256}{x^3}$
The minimum value of S for $x > 0$ occurs at $x = 4$. Solving for h gives $h = \frac{32}{4^2} = 2$.
Answer: $x = 4$ ft, $h = 2$ ft

13. a.



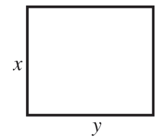
b. length + girth = $h + 4x$

c. Objective equation: $V = x^2h$
Constraint equation: $h + 4x = 84$ or $h = 84 - 4x$

d. Substituting $h = 84 - 4x$ into the objective equation, we have
$V = x^2(84 - 4x) = -4x^3 + 84x^2$.

e. $V' = -12x^2 + 168x$
$V'' = -24x + 168$
The maximum value of V for $x > 0$ occurs at $x = 14$ in. Solving for h gives $h = 84 - 4(14) = 28$ in.

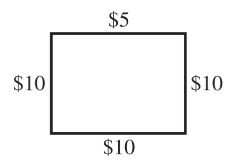
14. a.



b. Let P = perimeter.
Objective: $P = 2x + 2y$
Constraint: $100 = xy$

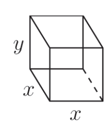
c. From the constraint, $y = \frac{100}{x}$. So
$P = 2x + 2\left(\frac{100}{x}\right) = 2x + \frac{200}{x}$
$\frac{dP}{dx} = 2 - \frac{200}{x^2}$; $\frac{d^2P}{dx^2} = 400x^3$
The minimum value of P for $x > 0$ occurs at $x = 10$. Solving for y gives
$y = \frac{100}{10} = 10$.
Answer: $x = 10$ m, $y = 10$ m

15.



Let C = cost of materials.
Objective: $C = 15x + 20y$
Constraint: $xy = 75$
Solving the constraint for y and substituting gives $C = 15x + 20\left(\frac{75}{x}\right) = 15x + \frac{1500}{x}$;
$\frac{dC}{dx} = 15 - \frac{1500}{x^2}$; $\frac{d^2C}{dx^2} = \frac{3000}{x^3}$
The minimum value for $x > 0$ occurs at $x = 10$.
Answer: $x = 10$ ft, $y = 7.5$ ft

16.



Let C = cost of materials.
Constraint: $x^2y = 12$
Objective: $C = 2x^2 + 4xy + x^2 = 3x^2 + 4xy$
Solving the constraint for y and substituting gives $C = 2x^2 + 4x\left(\frac{12}{x^2}\right) = 3x^2 + \frac{48}{x}$;
$\frac{dC}{dx} = 6x - \frac{48}{x^2}$; $\frac{d^2C}{dx^2} = 6 + \frac{96}{x^3}$
The minimum value of C for $x > 0$ occurs at $x = 2$. Answer: $x = 2$ ft, $y = 3$ ft

17. Let x = length of base, h = height, M = surface area.

Constraint: $x^2h = 8000 \Rightarrow h = \frac{8000}{x^2}$

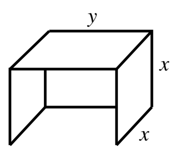
Objective: $M = 2x^2 + 4xh$

Solving the constraint for y and substituting gives $M = 2x^2 + 4x\left(\frac{8000}{x^2}\right) = 2x^2 + \frac{32,000}{x}$

$\frac{dM}{dx} = 4x - \frac{32,000}{x^2}$; $\frac{d^2M}{dx^2} = 4 + \frac{64,000}{x^2}$

The minimum value of M for $x > 0$ occurs at $x = 20$. Answer: 20 cm × 20 cm × 20 cm

18.



Let C = cost of materials.

Constraint: $x^2y = 250$

Objective: $C = 2x^2 + 2xy$

Solving the constraint for y and substituting gives $C = 2x^2 + \frac{500}{x}$

$\frac{dC}{dx} = 4x - \frac{500}{x^2}$; $\frac{d^2C}{dx^2} = 4 + \frac{1000}{x^3}$

The minimum value of C for $x > 0$ occurs at $x = 5$. Answer: $x = 5$ ft, $y = 10$ ft

19. Let x = length of side parallel to river, y = length of side perpendicular to river.

Constraint: $6x + 15y = 1500$

Objective: $A = xy$

Solving the constraint for y and substituting gives $A = x\left[-\frac{2}{5}x + 100\right] = -\frac{2}{5}x^2 + 100x$

$\frac{dA}{dx} = -\frac{4}{5}x + 100$; $\frac{d^2A}{dx^2} = -\frac{4}{5}$

The minimum value of A for $x > 0$ occurs at $x = 125$. Answer: $x = 125$ ft, $y = 50$ ft

20. Let x = length, y = width of garden.

Constraint: $2x + 2y = 300$

Objective: $A = xy$

Solving the constraint for y and substituting gives $A = x(150 - x) = -x^2 + 150x$

$\frac{dA}{dx} = -2x + 150$; $\frac{d^2A}{dx^2} = -2$

The maximum value of A occurs at $x = 75$.

Answer: 75 ft × 75 ft

21. Constraint: $x + y = 100$

Objective: $P = xy$

Solving the constraint for y and substituting gives $P = x(100 - x) = -x^2 + 100x$

$\frac{dP}{dx} = -2x + 100$; $\frac{d^2P}{dx^2} = -2$

The maximum value of P occurs at $x = 50$.

Answer: $x = 50$, $y = 50$

22. Constraint: $xy = 100$

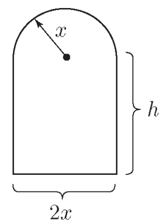
Objective: $S = x + y$

Solving the constraint for y and substituting gives $S = x + \frac{100}{x}$

$\frac{dS}{dx} = 1 - \frac{100}{x^2}$; $\frac{d^2S}{dx^2} = \frac{200}{x^3}$

The minimum value of S for $x > 0$ occurs at $x = 10$. Answer: $x = 10$, $y = 10$

23.



Constraint: $2x + 2h + \pi x = 14$ or $(2 + \pi)x + 2h = 14$

Objective: $A = 2xh + \frac{\pi}{2}x^2$

Solving the constraint for h and substituting gives

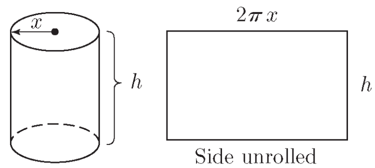
$$A = 2x\left(7 - \frac{2+\pi}{2}x\right) + \frac{\pi}{2}x^2$$
$$= 14x - \left(\frac{\pi}{2} + 2\right)x^2$$

$\frac{dA}{dx} = 14 - (4 + \pi)x$; $\frac{d^2A}{dx^2} = -4 - \pi$

The maximum value of A occurs at $x = \frac{14}{4+\pi}$.

Answer: $x = \frac{14}{4+\pi}$ ft

24.



Let S = surface area.

Constraint: $\pi x^2 h = 16\pi$ or $x^2 h = 16$

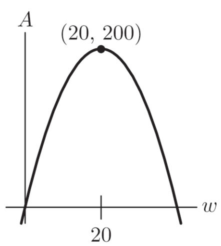
Objective: $S = 2\pi x^2 + 2\pi xh$

Solving the constraint for h and substituting gives $S = 2\pi x^2 + 2\pi x\left(\frac{16}{x^2}\right) = 2\pi\left(x^2 + \frac{16}{x}\right)$

$\frac{dS}{dx} = 2\pi\left(2x - \frac{16}{x^2}\right)$; $\frac{d^2S}{dx^2} = 2\pi\left(2 + \frac{32}{x^3}\right)$

The minimum value of S for $x > 0$ occurs at $x = 2$. Answer: $x = 2$ in., $h = 4$ in.

25. $A = 20w - \frac{1}{2}w^2$; $\frac{dA}{dw} = 20 - w$; $\frac{d^2A}{dw^2} = -1$



The maximum value of A occurs at $w = 20$.

$x = 20 - \frac{1}{2}w = 20 - \frac{1}{2}(20) = 10$

Answer: $w = 20$ ft, $x = 10$ ft

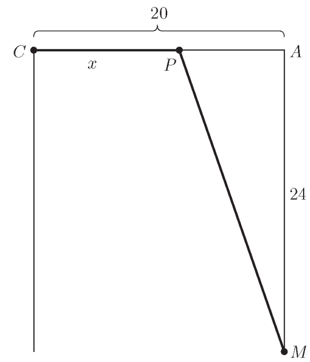
26. Let x miles per hour be the speed. $d = s \cdot t$, so time of the journey is $\frac{500}{x}$ hours. Cost per hour is $5x^2 + 2000$ dollars. Cost of the journey is

$C = (5x^2 + 2000)\cdot\left(\frac{500}{x}\right) = 2500x + \frac{1,000,000}{x}$

$\frac{dC}{dx} = 2500 - \frac{1,000,000}{x^2}$. Set $\frac{dC}{dx} = 0$, and we obtain $x^2 = 400 \Rightarrow x = 20$.

The speed is 20 miles per hour.

27.



If $x =$ distance from C to P, let $y =$ be the distance from P to M. Then cost is the objective: $C = 6x + 10y$ and the constraint

$y^2 = (20 - x)^2 + 24^2 = 976 - 40x + x^2$.

Solving the constraint for y and substituting gives $C = 6x + 10\left(976 - 40x + x^2\right)^{1/2}$.

$\frac{dC}{dx} = 6 + 5(976 - 40x + x^2)^{-1/2}(-40 + 2x)$.

$= 6 + \frac{5(-40 + 2x)}{\sqrt{976 - 40x + x^2}}$

Solve $\frac{dC}{dx} = 0$:

$6 + \frac{5(-40 + 2x)}{\sqrt{976 - 40x + x^2}} = 0$

$\frac{5(-40 + 2x)}{\sqrt{976 - 40x + x^2}} = -6$

$-200 + 10x = -6\sqrt{976 - 40x + x^2}$

$40000 - 4000x + 100x^2 = 36x^2 - 1440x + 35136$

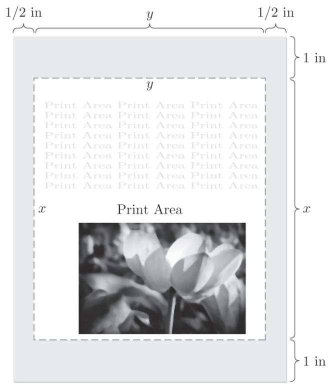
$64x^2 - 2560x + 4864 = 0$

$x^2 - 40x + 76 = 0 \Rightarrow x = 2,\ x = 38.$

But $x < 20 \Rightarrow x \neq 38$ and $\left.\frac{d^2C}{dx^2}\right|_{x=2} > 0.$

Therefore, the value of x that minimizes the cost of installing the cable is $x = 2$ meters and the minimum cost is $C = \$312$.

28.



Let P be the amount of paper used. The objective is $P = (x+2)(y+1)$ and the constraint is $x \cdot y = 50$. Solving the constraint for y and substituting gives

$P = (x+2)\left(\frac{50}{x}+1\right) = 52 + x + \frac{100}{x}$ and

$\frac{dP}{dx} = 1 - \frac{100}{x^2}$. Solve $\frac{dP}{dx} = 0$:

$1 - \frac{100}{x^2} = 0 \Rightarrow x = 10, x = -10$. But $x > 0$ and

$\left.\frac{d^2P}{dx^2}\right|_{x=10} > 0$. Therefore, $x = 10$, $y = 5$ and the dimensions of the page that minimize the amount of paper used: 6 in. × 12 in.

29. Distance = $\sqrt{(x-2)^2 + y^2}$

By the hint we minimize

$D = (x-2)^2 + y^2 = (x-2)^2 + x$, since $y = \sqrt{x}$.

$\frac{dD}{dx} = 2(x-2) + 1$

Set $\frac{dD}{dx} = 0$ to give: $2x = 3$, or

$x = \frac{3}{2}, y = \sqrt{\frac{3}{2}}$. So the point is $\left(\frac{3}{2}, \sqrt{\frac{3}{2}}\right)$.

30. Let D be the total distance.

$D(x) = d_1 + d_2 = \sqrt{x^2+36} + \sqrt{16 + (11-x)^2}$

$D'(x) = \frac{x}{\sqrt{x^2+36}} + \frac{x-11}{\sqrt{x^2-22x+137}}$

Now set $D'(x) = 0$ and solve for x:

$$\frac{x}{\sqrt{x^2+36}} + \frac{x-11}{\sqrt{x^2-22x+137}} = 0$$

$$x\sqrt{x^2-22x+137} + (x-11)\sqrt{x^2+36} = 0$$

$$x\sqrt{x^2-22x+137} = -(x-11)\sqrt{x^2+36}$$

$$\left(x\sqrt{x^2-22x+137}\right)^2 = \left(-(x-11)\sqrt{x^2+36}\right)^2$$

$$x^4 - 22x^3 + 137x^2$$
$$= x^4 - 22x^3 + 157x^2 - 792x + 4356$$

$$20x^2 - 792x + 4356 = 0$$

$$x = \frac{792 \pm \sqrt{(-792)^2 - 4(20)(4356)}}{2(20)}$$
$$= 33 \text{ or } 6.6$$

Since $0 \le x \le 11$, we have $x = 6.6$.
The minimum total distance is

$$D(6.6) = \sqrt{6.6^2 + 36} + \sqrt{16 + (11-6.6)^2}$$
$$= \sqrt{221} \approx 14.87 \text{ miles.}$$

31. Distance $= \sqrt{x^2 + y^2} = \sqrt{x^2 + (-2x+5)^2}$
$= \sqrt{5x^2 - 20x + 25}$

The distance has its smallest value when $5x^2 - 20x + 25$ does, so we minimize

$D(x) = 5x^2 - 20x + 25 \Rightarrow D'(x) = 10x - 20$

Now set $D'(x) = 0$ and solve for x:

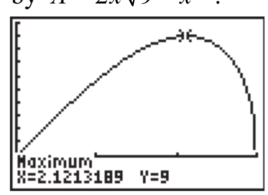
$10x - 20 = 0 \Rightarrow x = 2$
$y = -2(2) + 5 = 1$

The point is (2, 1).

32. Let A = area of rectangle.
Objective: $A = 2xy$

Constraint: $y = \sqrt{9 - x^2}$

Substituting, the area of the rectangle is given by $A = 2x\sqrt{9-x^2}$.



[0, 3] by [−2, 10]

Using graphing calculator techniques, this function has its maximum at $x \approx 2.1213$.
To confirm this, use the calculator's numerical differentiation capability to graph the derivative, and observe that the solution of $\frac{dA}{dx} = 0$ is $x \approx 2.1213$.

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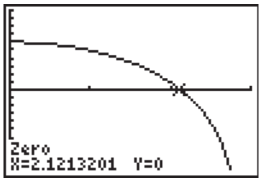
(*continued*)



[0, 3] by [−10, 10]

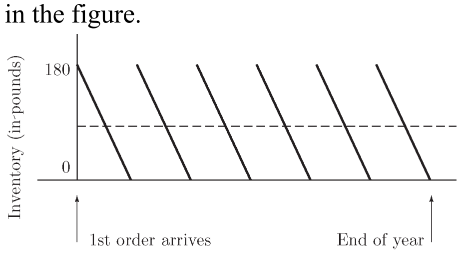
The maximum area occurs when $x \approx 2.12$.

2.6 Further Optimization Problems

1. a. At any given time during the order-reorder period, the inventory is between 180 pounds and 0 pounds. The average is $\frac{180}{2} = 90$ pounds.

b. The maximum is 180 pounds.

c. The number of orders placed during the year can be found by counting the peaks in the figure.



There were 6 orders placed during the year.

d. There were 180 pounds of cherries sold in each order-reorder period, and there were 6-order-reorder periods in the year. So there were $6 \cdot 180 = 1080$ pounds sold in one year.

2. a. There are 6 orders in a year, so the ordering cost is $6 \cdot 50 = \$300$. The average inventory is 90 pounds, so the carrying cost is $90 \cdot 7 = \$630$. The inventory cost is $\$300 + \$630 = \$930$.

b. The maximum inventory is 180 pounds, so the carrying cost is $7 \cdot 180 = \$1260$. The inventory cost is $\$300 + \$1260 = \$1560$.

3. a. The order cost is $16r$, and the carrying cost is $4 \cdot \frac{x}{2} = 2x$. The inventory cost C is $C = 2x + 16r$.

b. The order quantity multiplied by the number of orders per year gives the total number of packages ordered per year. The constraint function is then $rx = 800$.

c. Solving the constraint function for r gives $r = \frac{800}{x}$. Substituting into the cost equation yields $C(x) = 2x + \frac{12,800}{x}$.

$$C'(x) = 2 - \frac{12,800}{x^2} \Rightarrow 2 - \frac{12,800}{x^2} = 0 \Rightarrow$$

$$x^2 = \frac{12,800}{2} = 6400 \Rightarrow x = 80,\ r = 10$$

The minimum inventory cost is $C(80) = \$320$.

4. a. The order cost is $160r$, and the carrying cost is $32 \cdot \frac{x}{2} = 16x$. The inventory cost C is $C = 160r + 16x$.

b. The order quantity times the number of orders per year gives the total number of sofas ordered. The constraint function is $rx = 640$.

c. Solving the constraint function for r gives $r = \frac{640}{x}$. Substituting into the cost equation yields

$$C(x) = \frac{102,400}{x} + 16x.$$

$$C'(x) = 16 - \frac{102,400}{x^2}$$

$$C'(x) = 0 \Rightarrow 16 - \frac{102,400}{x^2} = 0 \Rightarrow x = 80$$

The minimum inventory cost is $C(80) = \$2560$.

5. Let x be the order quantity and r the number of orders placed in the year. Then the inventory cost is $C = 80r + 5x$. The constraint is $rx = 10,000$, so $r = \frac{10,000}{x}$ and we can write

$$C(x) = \frac{800,000}{x} + 5x.$$

a. $C(500) = \frac{800,000}{500} + 5(500) = \$4100$

b. $C'(x) = -\dfrac{800,000}{x^2} + 5 \Rightarrow$

$-\dfrac{800,000}{x^2} + 5 = 0 \Rightarrow$

$x^2 = \dfrac{800,000}{5} = 160,000 \Rightarrow x = 400$

The minimum value of $C(x)$ occurs at $x = 400$.

6. Let x be the number of tires produced in each production run, and let r be the number of runs in the year. Then the production cost is $C = 15{,}000r + 2.5x$. The constraint is $rx = 600{,}000$, so $x = \dfrac{600,000}{r}$ and $r = \dfrac{600,000}{x}$.

Then $C(r) = 15,000r + \dfrac{1,500,000}{r}$ and

$C(x) = \dfrac{15,000(600,000)}{x} + 2.5x$.

a. $C(10) = 15,000(10) + \dfrac{1,500,000}{10} = 300,000$

b. $C'(x) = \dfrac{-9 \cdot 10^9}{x^2} + 2.5 \Rightarrow$

$\dfrac{-9 \cdot 10^9}{x^2} + 2.5 = 0 \Rightarrow x^2 = \dfrac{-9 \cdot 10^9}{-2.5} \Rightarrow$

$x^2 = \dfrac{9 \cdot 10^9}{.25 \cdot 10} \Rightarrow x = \dfrac{3}{.5} \cdot 10^4 \Rightarrow x = 60,000$

Each run should produce 60,000 tires.

7. Let x be the number of microscopes produced in each run and let r be the number of runs. The objective function is

$C = 2500r + 15x + 20\left(\dfrac{x}{2}\right) = 2500r + 25x$.

The constraint is $xr = 1600$, $x = \dfrac{1600}{r}$, so

$C(r) = 2500r + \dfrac{40,000}{r}$.

$C'(r) = 2500 - \dfrac{40,000}{r^2} \Rightarrow$

$2500 - \dfrac{40,000}{r^2} = 0 \Rightarrow r^2 = \dfrac{40,000}{2500} \Rightarrow r = 4$

C has a minimum at $r = 4$. There should be 4 production runs.

8. Let x be the size of each order and let r be the number of orders placed in the year. Then the inventory cost is $C = 40r + 2x$ and $rx = 8000$, so $x = \dfrac{8000}{r}$, $C(r) = 40r + \dfrac{1600}{r}$

$C'(r) = 40 - \dfrac{16,000}{r^2} \Rightarrow 40 - \dfrac{16,000}{r^2} = 0 \Rightarrow$

$r^2 = \dfrac{16,000}{40} \Rightarrow r = 20$

The minimum value for C occurs at $r = 20$ (for $r > 0$).

9. The inventory cost is $C = hr + s\left(\dfrac{x}{2}\right)$ where r is the number of orders placed and x is the order size. The constraint is $rx = Q$, so $r = \dfrac{Q}{x}$ and we can write $C(x) = \dfrac{hQ}{x} + \dfrac{sx}{2}$.

$C'(x) = \dfrac{-hQ}{x^2} + \dfrac{s}{2}$. Setting $C'(x) = 0$ gives

$\dfrac{-hQ}{x^2} + \dfrac{s}{2} = 0$, $x^2 = \dfrac{2hQ}{s}$, $x = \pm\sqrt{\dfrac{2hQ}{s}}$. The positive value $\sqrt{\dfrac{2hQ}{s}}$ gives the minimum value for $C(x)$ for $x > 0$.

10. In this case, the inventory cost becomes

$$C = \begin{cases} 75r + 4x & \text{for } x < 600 \\ (75 - (x - 600))r + 4x & \text{for } x \ge 600 \end{cases}$$

Since $r = \dfrac{1200}{x}$,

$$C(x) = \begin{cases} \frac{90,000}{x} + 4x & \text{for } x < 600 \\ \frac{810,000}{x} + 4x - 1200 & \text{for } x \ge 600 \end{cases}$$

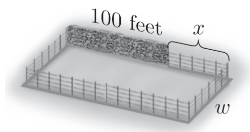
Now the function

$f(x) = \dfrac{810,000}{x} + 4x - 1200$ has

$f'(x) = -\dfrac{810,000}{x^2} + 4$, $f'(450) = 0$ and

$f'(x) > 0$ for $x > 450$.

Thus, $C(x)$ is increasing for $x > 600$ so the optimal order quantity does not change.

11. 

The objective is $A = (x + 100)w$ and the constraint is
$x + (x + 100) + 2w = 2x + 2w + 100 = 400$; or
$x + w = 150$, $w = 150 - x$.

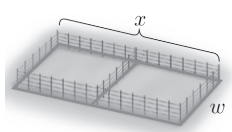
$$A(x) = (x+100)(150-x)$$
$$= -x^2 + 50x + 15,000$$
$$A'(x) = -2x + 50,\ A'(25) = 0$$

The maximum value of A occurs at $x = 25$. Thus the optimal values are $x = 25$ ft, $w = 150 - 25 = 125$ ft.

12. Refer to the figure for exercise 11. The objective remains $A = (x + 100)w$, but the constraint becomes $2x + 2w + 100 = 200$; or $x + w = 50$, so $A(x) = (x + 100)(50 - x)$

$$A(x) = (x+100)(50-x) = -x^2 - 50x + 5000,$$
$$A'(x) = -2x - 50$$
$$A'(x) = 0 \Rightarrow -2x - 50 = 0 \Rightarrow x = -25.$$

In this case, the maximum value of A occurs at $x = -25$, and $A(x)$ is decreasing for $x > -25$. Thus, the best non-negative value for x is $x = 0$. The optimal dimensions are $x = 0$ ft, $w = 50$ ft.

13. 

The objective is $F = 2x + 3w$, and the constraint is $xw = 54$, or $w = \frac{54}{x}$, so

$$F(x) = 2x + \frac{162}{x},$$
$$F'(x) = 2 - \frac{162}{x^2} \Rightarrow 2 - \frac{162}{x^2} = 0 \Rightarrow$$
$$x^2 = \frac{162}{2} \Rightarrow x = 9.$$

The minimum value of F for $x > 0$ is $x = 9$. The optimal dimensions are thus $x = 9$ m, $w = 6$ m.

14. Refer to the figure for exercise 13. The objective is
$C = 2(5x) + 2(5w) + 2w = 10x + 12w.$

The constraint is $xw = 54$, so $w = \frac{54}{x}$ and

$$C(x) = 10x + \frac{648}{x}.$$
$$C'(x) = 10 - \frac{648}{x^2}$$
$$C'(x) = 0 \Rightarrow 10 - \frac{648}{x^2} = 0 \Rightarrow x = \frac{18}{\sqrt{5}}$$

The optimal dimensions are $x = \frac{18}{\sqrt{5}}$ m, $w = 3\sqrt{5}$ m.

15. **a.** $(0,1000)$, $(5,1500) \Rightarrow$

$$m = \frac{1500-1000}{5-0} = 100.$$
$$y - 1500 = 100(x-5);\ y = 100x + 1000 \Rightarrow$$
$$A(x) = 100x + 1000.$$

b. Let x be the discount per pizza. Then, for $0 \le x \le 18$,

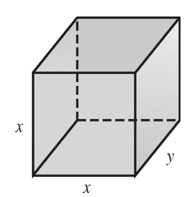
$$\text{revenue } = R(x) = (100x+1000)(18-x)$$
$$= 18000 + 800x - 100x^2$$
$$R'(x) = 800 - 200x \Rightarrow$$
$$800 - 200x = 0 \Rightarrow x = 4$$

Therefore, revenue is maximized when the discount is $x = \$4$.

c. Let each pizza cost \$9 and let x be the discount per pizza. Then $A(x) = 100x + 1000$ and, for $0 \le x \le 9$,

$$\text{revenue } = R(x) = (100x+1000)(9-x).$$
$$R(x) = 9000 - 100x - 100x^2$$
$$R'(x) = -100 - 200x \Rightarrow$$
$$-100 - 200x = 0 \Rightarrow x = -.5$$

In this case, revenue is maximized when the discount is $x = -\$.50$. Since $0 \le x \le 9$, the revenue is maximized when $x = 0$.

16. 

The objective is $S = 2x^2 + 3xy$ where x and y are the dimensions of the box. The constraint is $x^2y = 36$, so $y = \frac{36}{x^2}$ and

$$S(x) = 2x^2 + 3x\left(\frac{36}{x^2}\right) = 2x^2 + \frac{108}{x}.$$

(*continued on next page*)

(*continued*)

$S'(x) = 4x - \frac{108}{x^2}$

$S'(x) = 0 \Rightarrow 4x - \frac{108}{x^2} = 0 \Rightarrow x = 3$.

$x^2 y = 36 \Rightarrow 9y = 36 \Rightarrow y = 4$

The optimal dimensions are
3 in. $\times$ 3 in. $\times$ 4 in.

17. Let x be the length and width of the base and let y be the height of the shed. The objective is $C = 4x^2 + 2x^2 + 4 \cdot 2.5xy = 6x^2 + 10xy$. The constraint is $x^2 y = 150 \Rightarrow y = \frac{150}{x^2}$.

$C(x) = 6x^2 + \frac{1500}{x}, C'(x) = 12x - \frac{1500}{x^2}$

$C'(x) = 0 \Rightarrow 12x - \frac{1500}{x^2} = 0 \Rightarrow x = 5$

The optimal dimensions are 5 ft $\times$ 5 ft $\times$ 6 ft.

18. Let x be the length of the front of the building and let y be the other dimension. The objective is $C = 70x + 2 \cdot 50y + 50x = 120x + 100y$ and the constraint is $xy = 12{,}000 \Rightarrow y = \frac{12{,}000}{x}$.

So $C(x) = 120x + \frac{1{,}200{,}000}{x}$,

$C'(x) = 120 - \frac{1{,}200{,}000}{x^2}, C'(100) = 0$.

The optimal dimensions are $x = 100$ ft, $y = 120$ ft.

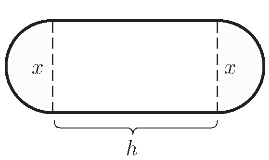
19. Let x be the length of the square end and let h be the other dimension. The objective is $V = x^2 h$ and the constraint is $2x + h = 120 \Rightarrow h = 120 - 2x$.

$V(x) = 120x^2 - 2x^3$, $V'(x) = 240x - 6x^2 \Rightarrow$

$V'(x) = 0 \Rightarrow 240x - 6x^2 = 0 \Rightarrow$
$6x(40 - x) = 0 \Rightarrow x = 0$ or $x = 40$.

The maximum value of V for $x > 0$ occurs at $x = 40$ cm, $h = 40$ cm.
The optimal dimensions are 40 cm $\times$ 40 cm $\times$ 40 cm.

20.



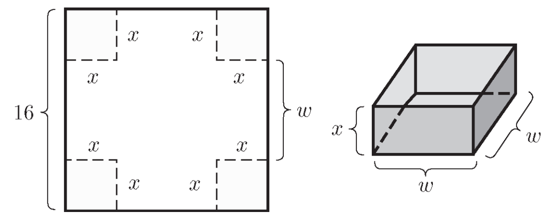
The objective is $A = xh$ and the constraint is $\pi x + 2h = 440 \Rightarrow h = 220 - \frac{\pi}{2}x$.

$A(x) = x\left(220 - \frac{\pi}{2}x\right) = 220x - \frac{\pi}{2}x^2$,

$A'(x) = 220 - \pi x \Rightarrow 220 - \pi x = 0 \Rightarrow x = \frac{220}{\pi}$

The optimal dimensions are $x = \frac{220}{\pi}$ yd, $h = 110$ yd.

21.



The objective equation is $V = w^2 x$ and the constraint is $w + 2x = 16 \Rightarrow w = 16 - 2x.$

$V(x) = (16 - 2x)^2 x = 4x^3 - 64x^2 + 256x$

$V'(x) = 12x^2 - 128x + 256$

$V'(x) = 0 \Rightarrow 12x^2 - 128x + 256 = 0 \Rightarrow$

$4(x - 8)(3x - 8) = 0 \Rightarrow x = 8$ or $x = \frac{3}{8}$

$V''\left(\frac{8}{3}\right) < 0, V''(8) > 0$

The maximum value of V for x between 0 and 8 occurs at $x = \frac{8}{3}$ in.

22. Let x be the width of the base and let h be the other dimension. The objective is $V = 2x^2 h$ and the constraint is

$2(2x^2) + 2xh + 2(2xh) = 27$, or

$4x^2 + 6xh = 27 \Rightarrow h = \frac{27 - 4x^2}{6x}$. Thus,

$V = \frac{2x^2(27 - 4x^2)}{6x} = 9x - \frac{4}{3}x^3$.

$V'(x) = 9 - 4x^2 \Rightarrow 9 - 4x^2 = 0 \Rightarrow x^2 = \frac{9}{4} \Rightarrow$

$x = \frac{3}{2}$

The optimal values are $x = \frac{3}{2}$, $h = 2$. The dimensions should be $\frac{3}{2}$ ft $\times$ 3 ft $\times$ 2 ft .

23. We want to find the maximum value of $f'(t)$.

$f'(t) = \frac{10}{(t+10)^2} - \frac{200}{(t+10)^3}$;

$f''(t) = \frac{-20}{(t+10)^3} + \frac{600}{(t+10)^4}$. Setting $f''(t) = 0$ gives

$\frac{20}{(t+10)^3} = \frac{600}{(t+10)^4} \Rightarrow 20 = \frac{600}{(t+10)} \Rightarrow t = 20.$

$f'''(t) = \frac{60}{(t+10)^4} - \frac{2400}{(t+10)^5}$; $f'''(20) < 0$, so $t = 20$ is the maximum value of $f'(t)$. Oxygen content is increasing fastest after 20 days.

24. We want to find the maximum value of $f'(t) = 40 + 2t - \frac{1}{5}t^2$.

$f''(t) = 2 - \frac{2}{5}t \Rightarrow 2 - \frac{2}{5}t = 0 \Rightarrow t = 5$.

The maximum rate of output occurs at $t = 5$. The maximum output rate is $f'(5) = 45$ tons/hour.

25. Let (x, y) be the top right-hand corner of the window. The objective is $A = 2xy$ and the constraint is $y = 9 - x^2$. Thus,

$A(x) = 2x(9 - x^2) = 18x - 2x^3$,

$A'(x) = 18 - 6x^2$

$A'(x) = 0 \Rightarrow 18 - 6x^2 = 0 \Rightarrow x = \sqrt{3}$.

The maximum value of A for $x > 0$ occurs at $x = \sqrt{3}$. Thus, the window should be 6 units high and $2\sqrt{3}$ units wide.

26. We want to find the minimum value of

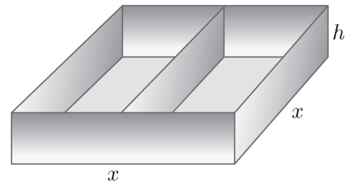
$f'(t) = \frac{-1000}{(t+8)^2} + \frac{8000}{(t+8)^3}$;

$f''(t) = \frac{2000}{(t+8)^3} - \frac{24,000}{(t+8)^4}$. Setting $f''(t) = 0$ gives

$\frac{2000}{(t+8)^3} = \frac{24,000}{(t+8)^4} \Rightarrow 2000 = \frac{24,000}{t+8} \Rightarrow t = 4.$

$f'''(t) = \frac{-6000}{(t+8)^4} + \frac{96,000}{(t+8)^5}$, $f'''(4) > 0$, so $t =$ 4 gives the minimum value of $f'(t)$. Sales fall the fastest after 4 weeks.

27.



$A = x^2 + 5xh$ (Area: where x is the length of the square base and h is the height.)

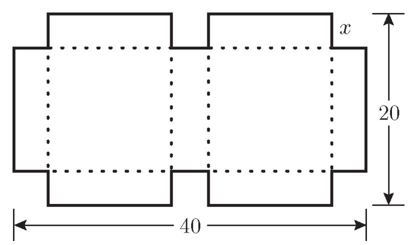
$V = x^2h = 400 \Rightarrow h = \frac{400}{x^2}$, so

$A = x^2 + \frac{2000}{x}$, and $\frac{dA}{dx} = 2x - \frac{2000}{x^2}$.

Setting $\frac{dA}{dx} = 0$ gives $2x^3 = 2000$ or $x = 10$ which in turn yields $h = 4$ in. The dimensions should be 10 in. × 10 in. × 4 in.

28. Since $f'(x)$ is negative on the interval $0 \le x \le 5$, $f(x)$ is decreasing on the interval 0. Therefore $f(x)$ has its greatest value at zero.

29.



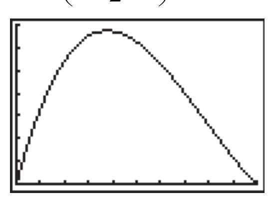
Let V = volume of box, and let l and w represent the dimensions of the base of the box.

Objective: $V = lwx$

Constraints: $l = \frac{40 - 3x}{2}$, $w = 20 - 2x$

Substituting, the volume of the box is given by

$V = \left(\frac{40 - 3x}{2}\right)(20 - 2x)x = 3x^3 - 70x^2 + 400x$.

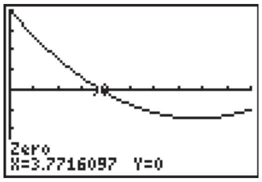


[0, 10] by [0, 700]

Since we require the dimensions of the box to be positive, the appropriate domain is $0 < x < 10$. Using graphing calculator techniques, the maximum function value on this domain occurs at $x \approx 3.7716$.

(*continued on next page*)

(*continued*)

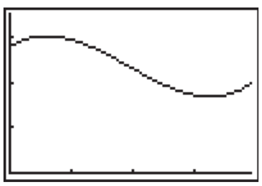


[0, 10] by [−400, 400]

To confirm this, use the calculator's numerical differentiation capability or the function $\frac{dV}{dx} = 9x^2 - 140x + 400$ to graph the derivative, and observe that the solution of $\frac{dV}{dx} = 0$ is $x \approx 3.7716$.

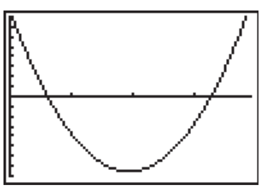
The maximum volume occurs when $x \approx 3.77$ cm.

30. a.



[0, 39] by [0, 3.5]

b. Note that $f'(x) = .0848 - .01664x + .000432x^2$.

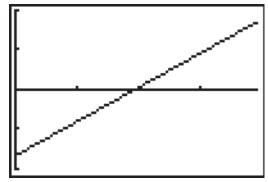


[0, 39] by [−.08, .08]

The solutions of $f'(x) = 0$ are $x \approx 6.0448$ and $x \approx 32.4738$. The solution corresponding to the least coffee consumption is $x \approx 32.4738$, which corresponds to the year 1988. The coffee consumption at that time was $f(32.4738) \approx 1.7$ cups per day per adult.

c. The solution of $f'(x) = 0$ corresponding to the greatest coffee consumption is $x \approx 6.0448$, which corresponds to the year 1961. The coffee consumption at that time was $f(6.0448) \approx 3.0$ cups per day per adult.

d. Note that $f''(x) = -.01664 + .000864x$.



[0, 39] by [−.02, .02]

The solution of $f''(x) = 0$ is $x \approx 19.2593$, which corresponds to the year 1975. Coffee consumption was decreasing at the greatest rate in 1975.

2.7 Applications of Derivatives to Business and Economics

1. The marginal cost function is

$M(x) = C'(x) = 3x^2 - 12x + 13$.

$M'(x) = 6x - 12$

$M'(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow x = 2$

The minimum value of $M(x)$ occurs at $x = 2$. The minimum marginal cost is $M(2) = \$1$.

2. $M(x) = C'(x) = .0003x^2 - .12x + 12$

$M'(x) = .0006x - .12, M'(100) = -.06 < 0,$ so the marginal cost is decreasing at $x = 100$.

$M'(x) = 0 \Rightarrow .0006x - .12 = 0 \Rightarrow x = 200$

The minimal marginal cost is $M(200) = \$0$.

3. $R(x) = 200 - \frac{1600}{x+8} - x, R'(x) = \frac{1600}{(x+8)^2} - 1,$

$R'(x) = 0 \Rightarrow \frac{1600}{(x+8)^2} - 1 = 0 \Rightarrow$

$1600 = (x+8)^2 \Rightarrow 40 = x + 8 \Rightarrow x = 32$

The maximum value of $R(x)$ occurs at $x = 32$.

4. $R(x) = 4x - .0001x^2, R'(x) = 4 - .0002x$,

$R'(x) = 0 \Rightarrow 4 - .0002x = 0 \Rightarrow x = 20{,}000$

The maximum value of $R(x)$ occurs at $x = 20{,}000$. The maximum possible revenue is $R(20{,}000) = 40{,}000$.

5. The profit function is

$$P(x) = R(x) - C(x) = 28x - (x^3 - 6x^2 + 13x + 15) = -x^3 + 6x^2 + 15x - 15$$

$P'(x) = -3x^2 + 12x + 15$

$P'(x) = 0 \Rightarrow -3x^2 + 12x + 15 = 0 \Rightarrow$,

$-3(x-5)(x+1) = 0 \Rightarrow x = 5$ or $x = -1$

The maximum value of $P(x)$ for $x > 0$ occurs at $x = 5$.

6. The revenue function is $R(x) = 3.5x$. Thus, the profit function is $P(x) = R(x) - C(x)$

$$P(x) = R(x) - C(x)$$
$$= 3.5x - (.0006x^3 - .03x^2 + 2x + 20)$$
$$= -.0006x^3 + .03x^2 + 1.5x - 20$$
$$P'(x) = -.0018x^2 + .06x + 1.5$$
$$P'(x) = 0 \Rightarrow -.0018x^2 + .06x + 1.5 = 0 \Rightarrow$$
$$x = 50 \text{ or } x = -\frac{50}{3}$$

Thus, the maximum value of $P(x)$ for $x > 0$ occurs at $x = 50$.

7. The revenue function is

$$R(x) = x\left(\frac{1}{12}x^2 - 10x + 300\right)$$
$$= \frac{1}{12}x^3 - 10x^2 + 300x$$
$$R'(x) = \frac{1}{4}x^2 - 20x + 300 \Rightarrow R'(x) = 0 \Rightarrow$$
$$\frac{1}{4}x^2 - 20x + 300 = 0 \Rightarrow x^2 - 80x + 1200 = 0 \Rightarrow$$
$$(x-60)(x-20) = 0 \Rightarrow x = 60 \text{ or } x = 20$$
$$R''(x) = \frac{1}{2}x - 20$$
$$R''(20) < 0, R''(60) > 0$$

The maximum value of $R(x)$ occurs at $x = 20$.

The corresponding price is $\$133\frac{1}{3}$ or \$133.33 .

8. The revenue function is

$$R(x) = x(2 - .001x) = 2x - .001x^2.$$
$$R'(x) = 2 - .002x$$
$$R'(x) = 0 \Rightarrow 2 - .002x = 0 \Rightarrow x = 1000$$

The maximum value of $R(x)$ occurs at $x = 1000$. The corresponding price is $p = 2 - .001(1000) = \$1$.

9. The revenue function is

$R(x) = x(256 - 50x) = 256x - 50x^2$. Thus, the profit function is

$$P(x) = R(x) - C(x) = 256x - 50x^2 - 182 - 56x$$
$$= -50x^2 + 200x - 182$$
$$P'(x) = -100x + 200$$
$$P'(x) = 0 \Rightarrow -100x + 200 = 0 \Rightarrow x = 2$$

The maximum profit occurs at $x = 2$ (million tons). The corresponding price is $256 - 50(2) = 156$ dollars per ton.

10. The objective is $A = xy$ and the constraint is $y = 30 - x$. $A(x) = x(30 - x) = 30x - x^2$,

$$A'(x) = 30 - 2x$$
$$A'(x) = 0 \Rightarrow 30 - 2x = 0 \Rightarrow x = 15$$

The maximum value of $A(x)$ occurs at $x = 15$. Thus, the optimal values are $a = 15$, $b = 15$. If $y = 30 - x$ is a demand curve, then $A(x)$ above corresponds to the revenue function $R(x)$ and the optimal values a, b correspond to the revenue-maximizing quantity and price, respectively.

11. a. Let p stand for the price of hamburgers and let x be the quantity. Using the point-slope equation,

$$p - 4 = \frac{4.4 - 4}{8000 - 10,000}(x - 10,000) \text{ or}$$

$p = -.0002x + 6$. Thus, the revenue function is

$$R(x) = x(-.0002x + 6) = -.0002x^2 + 6x.$$
$$R'(x) = -.0004x + 6$$
$$R'(x) = 0 \Rightarrow -.0004x + 6 = 0 \Rightarrow$$
$$x = 15,000$$

The maximum value of $R(x)$ occurs at $x = 15,000$. The optimal price is thus $-.0002(15,000) + 6 = \$3.00$

b. The cost function is $C(x) = 1000 + .6x$, so the profit function is $P(x) = R(x) - C(x)$

$$P(x) = R(x) - C(x)$$
$$= -.0002x^2 + 6x - (1000 + .6x)$$
$$= -.0002x^2 + 5.4x - 1000$$
$$P'(x) = -.0004x + 5.4$$
$$P'(x) = 0 \Rightarrow -.0004x + 5.4 = 0 \Rightarrow$$
$$x = 13,500$$

The maximum value of $P(x)$ occurs at $x = 13,500$. The optimal price is $-.0002(13,500) + 6 = \$3.30$.

12. Let $50 + x$ denote the ticket price and y the attendance. Since a \$2 increase in price lowers the attendance by 200, we have $y = 4000 - 100x$.

We now have

$$\text{Revenue} = R = \text{price} \times \text{attendance}$$
$$= (50 + x)(4000 - 100x)$$
$$= -100x^2 - 1000x + 200,000$$
$$R'(x) = -200x - 1000$$
$$R'(x) = 0 \Rightarrow -200x - 1000 = 0 \Rightarrow x = -5$$

$R = (50 - 5)(4000 - 100(-5)) = 202,500$

Answer: Charge \$45 per ticket.

Revenue = \$202,500

13. Let x be the number of prints the artist sells. Then his revenue = [price] · [quantity].

$$\begin{cases}(400-5(x-50))x & \text{if } x>50\\ 400x & \text{if } x\le 50\end{cases}$$

For $x > 50$, $r(x) = -5x^2 + 650x$,

$r'(x) = -10x + 650$

$r'(x) = 0 \Rightarrow -10x + 650 = 0 \Rightarrow x = 65$

The maximum value of $r(x)$ occurs at $x = 65$. The artist should sell 65 prints.

14. Let x be the number of memberships the club sells. Then their revenue is

$$r(x) = \begin{cases}200x & \text{if } x\le 100\\ (200-(x-100))x & \text{if } 100<x\le 160\end{cases}$$
$$= \begin{cases}200x & \text{if } x\le 100\\ -x^2+300x & \text{if } 100<x\le 160\end{cases}$$

For $100 < x \le 160, r'(x) = -2x + 300$

$r'(x) = 0 \Rightarrow -2x + 300 = 0 \Rightarrow x = 150$

The maximum value of $r(x)$ occurs at $x = 150$. The club should try to sell 150 memberships.

15. Let $P(x)$ be the profit from x tables.

Then $P(x) = (10-(x-12)(.5)x = -.5x^2 + 16x$

For $x \ge 12$, $P'(x) = 16 - x$

$P'(x) = 0 \Rightarrow 16 - x = 0 \Rightarrow x = 16$

The maximum value of $P(x)$ occurs at $x = 16$. The cafe should provide 16 tables.

16. The revenue function is

$$R(x) = x\left(36{,}000 - 300\left(x-100\right)\right)$$
$$= -300x^2 + 66{,}000x$$

where x is the price in cents and $x \ge 100$.

$R'(x) = 66{,}000 - 600x$

$R'(x) = 0 \Rightarrow 66{,}000 - 600x = 0 \Rightarrow x = 110$

The maximum value occurs at $x = 110$. The toll should be $1.10.

17. a. $R(x) = x\left(60 - 10^{-5}x\right) = 60x - 10^{-5}x^2$; so the profit function is $P(x) = R(x) - C(x)$

$$P(x) = R(x) - C(x)$$
$$= \left(60x - 10^{-5}x^2\right) - \left(7\cdot 10^6 + 30x\right)$$
$$= -10^{-5}x^2 + 30x - 7\cdot 10^6$$

$P'(x) = -2\cdot 10^{-5}x + 30$

$P'(x) = 0 \Rightarrow -2\cdot 10^{-5}x + 30 = 0 \Rightarrow$

$x = 15\cdot 10^5$

The maximum value of $P(x)$ occurs at $x = 1.5\cdot 10^5$ (thousand kilowatt-hours). The corresponding price is

$p = 60 - 10^{-5}\left(15\cdot 10^5\right) = 45.$

This represents $45/thousand kilowatt-hours.

b. The new profit function is

$$P_1(x) = R(x) - C_1(x)$$
$$= 60x - 10^{-5}x^2 - 7\cdot 10^6 - 40x$$
$$= -10^{-5}x^2 + 20x - 7\cdot 10^6$$

$P_1'(x) = -2\cdot 10^{-5}x + 20$

$P_1'(x) = 0 \Rightarrow -2\cdot 10^{-5}x + 20 = 0 \Rightarrow x = 10^6.$

The maximum value of $P_1(x)$ occurs at $x = 10^6$ (thousand kilowatt-hours). The corresponding price is

$p = 60 - 10^{-5}(10^6) = 50$, representing $50/thousand kilowatt-hours.

The maximum profit will be obtained by charging $50/thousand kilowatt-hours. Since this represents an increase of only $5/thousand kilowatt-hours over the answer to part (a), the utility company should not pass all of the increase on to consumers.

18. a. $R(x) = x(200 - 3x) = 200x - 3x^2$, so the profit function is

$$P(x) = C(x) - R(x)$$
$$= 200x - 3x^2 - (75 + 80x - x^2)$$
$$= -2x^2 + 120x - 75$$

$P'(x) = -4x + 120$

$P'(x) = 0 \Rightarrow -4x + 120 = 0 \Rightarrow x = 30$

The corresponding price is $p = 200 - 3(30) = 110$. Thus, $x = 30$ and the price is $110.

b. The tax increases the cost function by $4x$, so the new cost function is

$C(x) = 75 + 84x - x^2$ and the profit function is now

$P(x) = R(x) - C(x)$

$$P(x) = R(x) - C(x)$$
$$= 200x - 3x^2 - (75 + 84x - x^2)$$
$$= -2x^2 + 116x - 75$$

$P'(x) = -4x + 116$

$P'(x) = 0 \Rightarrow -4x + 116 = 0 \Rightarrow x = 29$

The corresponding price is $p = 200 - 3(29) = 113$, or $113.

c. The profit function is now

$$P(x) = R(x) - C(x)$$
$$= 200x - 3x^2 - \left[75 + (80+T)x - x^2\right]$$
$$= -2x^2 + (120-T)x - 75$$
$$P'(x) = -4x + (120-T)$$
$$P'(x) = 0 \Rightarrow -4x + (120-T) \Rightarrow x = 30 - \frac{T}{4}$$

The new value of x is $30 - \frac{T}{4}$.

The government's tax revenue is given by

$$G(T) = Tx = T\left(30 - \frac{T}{4}\right) = 30T - \frac{1}{4}T^2,$$
$$G'(T) = 30 - \frac{1}{2}T$$
$$G'(T) = 0 \Rightarrow 30 - \frac{1}{2}T = 0 \Rightarrow T = 60.$$

The maximum value of $G(T)$ occurs at $T = 60$. Thus a tax of \$60/unit will maximize the government's tax revenue.

19. Let r be the percentage rate of interest ($r = 4$ represents a 4% interest rate).
Total deposit is \$1,000,000r. Total interest paid out in one year is $10,000r^2$. Total interest received on the loans of 1,000,000r is 100,000r.

$$P = 100,000r - 10,000r^2$$
$$\frac{dP}{dr} = 100,000 - 20,000r$$

Set $\frac{dP}{dr} = 0$ and solve for r:

$$100,000 - 20,000r = 0 \Rightarrow r = 5$$

An interest rate of 5% generates the greatest profit.

20. a. $P(0)$ is the profit with no advertising budget.

b. As money is spent on advertising, the marginal profit initially increases. However, at some point the marginal profit begins to decrease.

c. Additional money spent on advertising is most advantageous at the inflection point.

21. a. Since $R(40) = 75$, the revenue is \$75,000.

b. Since $R'(17.5) \approx 3.2$, the marginal revenue is about \$3200 per unit.

c. Since the solution of $R(x) = 45$ is $x = 15$, the production level in 15 units.

d. Since the solution of $R'(x) = .8$ is $x = 32.5$, the production level is 32.5 units.

e. Looking at the graph of $y = R(x)$, the revenue appears to be greatest at $x \approx 35$. To confirm, observe that the graph of $y = R'(x)$ crosses the x-axis at $x = 35$. The revenue is greatest at a production level of 35 units.

22. a. Since $C(60) = 1100$, the cost is \$1100.

b. Since $C'(40) = 12.5$, the marginal cost is \$12.50.

c. Since the solution of $C(x) = 1200$ is $x = 100$, the production level is 100 units.

d. Since the solutions of $C'(x) = 22.5$ are $x = 20$ and $x = 140$, the production levels are 20 units and 140 units.

e. Looking at the graph of $y = C'(x)$, the marginal cost appears to be least at $x \approx 80$. The production level is 80 units, and the marginal cost is \$5.

Chapter 2 Fundamental Concept Check Exercises

1. Increasing and decreasing functions
relative maximum and minimum points
absolute maximum and minimum points
concave up and concave down
inflection point, intercepts, asymptotes

2. A point is a relative maximum at $x = 2$ if the function attains a maximum at $x = 2$ relative to nearby points on the graph. The function has an absolute maximum at $x = 2$ if it attains its largest value at $x = 2$.

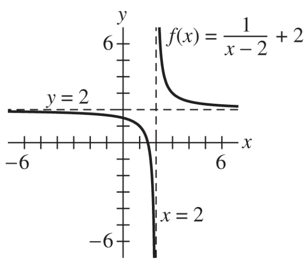
3. Concave up at $x = 2$: The graph "opens" up as it passes through the point at $x = 2$; there is an open interval containing $x = 2$ throughout which the graph lies above its tangent line; the slope of the tangent line increases as we move from left to right through the point at $x = 2$.
Concave down at $x = 2$: The graph "opens" down as it passes through the point at $x = 2$; there is an open interval containing $x = 2$ throughout which the graph lies below its tangent line; the slope of the tangent line decreases as we move from left to right through the point at $x = 2$.

4. $f(x)$ has an inflection point at $x = 2$ if the concavity of the graph changes at the point $(2, f(2))$.

5. The x-coordinate of the x-intercept is a zero of the function.

6. To determine the y-intercept, set $x = 0$ and compute $f(0)$.

7. An asymptote is a line that a curve approaches as the curve approaches infinity. There are three types of asymptotes: horizontal, vertical, and oblique (or slant) asymptotes. Note that the distance between the curve and the asymptote approaches zero. For example, in the figure $y = 2$ is a vertical asymptote and $x = 2$ is a horizontal asymptote.



8. First derivative rule: If $f'(a) > 0,$ then f is increasing at $x = a$. If $f'(a) < 0,$ then f is decreasing at $x = a$.
Second derivative rule: If $f''(a) > 0,$ then f is concave up at $x = a$. If $f''(a) < 0,$ then f is concave down at $x = a$.

9. We can think of the derivative of $f(x)$ as a "slope function" for $f(x)$. The y-values on the graph of $y = f'(x)$ are the slopes of the corresponding points on the graph of $y = f(x)$. Thus, on an interval where $f'(x) > 0$, f is increasing. On an interval where $f'(x)$ is increasing, f is concave up.

10. Solve $f'(x) = 0.$ Let a solution be represented by a. If f' changes from positive to negative at $x = a$, then f has a local maximum at a. If f' changes from negative to positive at $x = a$, then f has a local minimum at a. If f' does not change sign at a (that is, f' is either positive on both sides of a or negative on both sides of a, then f has no local extremum at a.

11. Solve $f''(x) = 0.$ Let a solution be represented by a. If $f''(a) = 0$ and $f''(x)$ changes sign as we move from left to right through $x = a$, then there is an inflection point at $x = a$.

12. See pages 161−162 in section 2.4 for more detail.

1. Compute $f'(x)$ and $f''(x)$.
2. Find all relative extreme points.
   a. Apply the first and second derivative tests to find the relative extreme points. Set $f'(x) = 0,$ and solve for x to find the critical value $x = a$.
      (i) If $f''(a) > 0,$ the curve has a relative minimum at $x = a$.
      (ii) If $f''(a) < 0,$ the curve has a relative maximum at $x = a$.
      (iii) If $f''(a) = 0,$ there is an inflection point at $x = a$.
   b. Repeat the preceding steps for each solution to $f'(x) = 0.$
3. Find all the inflection points of $f(x)$ using the second derivative test.
4. Consider other properties of the function and complete the sketch.

13. In an optimization problem, the quantity to be optimized (maximized or minimized) is given by the objective equation.

14. A constraint equation is an equation that places a limit, or a constraint, on the variables in an optimization problem.

15.
1. Draw a picture, if possible.
2. Decide what quantity Q is to be maximized or minimized.
3. Assign variables to other quantities in the problem.
4. Determine the objective equation that expresses Q as a function of the variables assigned in step 3.
5. Find the constraint equation that relates the variable to each other and to any constants that are given in the problem.
6. Use the constraint equation to simplify the objective equation in such a way that Q becomes a function of only one variable. Determine the domain of this function.

(*continued on next page*)

(*continued*)

7. Sketch the graph of the function obtained in step 6 and use this graph to solve the optimization problem. Alternatively, use the second derivative test.

16. $P(x) = R(x) - C(x)$

Chapter 2 Review Exercises

1. a. The graph of $f(x)$ is increasing when $f'(x) > 0$: $-3 < x < 1$, $x > 5$.
The graph of $f(x)$ is decreasing when $f'(x) < 0$: $x < -3$, $1 < x < 5$.

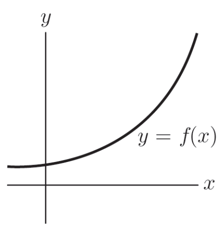
b. The graph of $f(x)$ is concave up when $f'(x)$ is increasing: $x < -1$, $x > 3$.
The graph of $f(x)$ is concave down when $f'(x)$ is decreasing: $-1 < x < 3$.

2. a. $f(3) = 2$

b. The tangent line has slope $\frac{1}{2}$, so $f'(3) = \frac{1}{2}$.

c. Since the point (3, 2) appears to be an inflection point, $f''(3) = 0$.

3.



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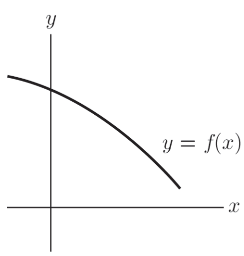
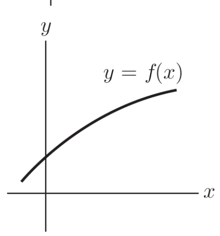
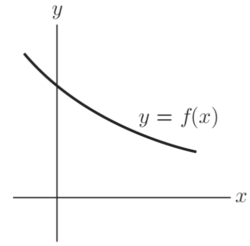


5.



6.



7. (d), (e) **8.** (b)

9. (c), (d) **10.** (a)

11. (e) **12.** (b)

13. Graph goes through (1, 2), increasing at $x = 1$.

14. Graph goes through (1, 5), decreasing at $x = 1$.

15. Increasing and concave up at $x = 3$.

16. Decreasing and concave down at $x = 2$.

17. (10, 2) is a relative minimum point.

18. Graph goes through (4, –2), increasing and concave down at $x = 4$.

19. Graph goes through (5, –1), decreasing at $x = 5$.

20. (0, 0) is a relative minimum point.

21. a. $f(t) = 1$ at $t = 2$, after 2 hours.

b. $f(5) = .8$

c. $f'(t) = -.08$ at $t = 3$, after 3 hours.

d. Since $f'(8) = -.02$, the rate of change is –.02 unit per hour.

22. a. Since $f(50) = 400$, the amount of energy produced was 400 trillion kilowatt-hours.

b. Since $f'(50) = 35$, the rate of change was 35 trillion kilowatt-hours per year.

c. Since $f(t) = 3000$ at $t = 95$, the production level reached 300 trillion kilowatt-hours in 1995.

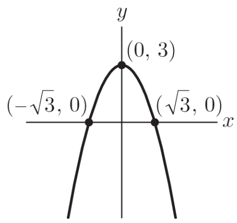
d. Since $f'(t) = 10$ at $t = 35$, the production level was rising at the rate of 10 trillion kilowatt-hours per year in 1935.

e. Looking at the graph of $y = f'(t)$, the value of $f'(t)$ appears to be greatest at $t = 70$. To confirm, observe that the graph of $y = f''(t)$ crosses the t-axis at $t = 70$. Energy production was growing at the greatest rate in 1970. Since $f(70) = 1600$, the production level at that time was 1600 trillion kilowatt-hours.

23. $y = 3 - x^2$
$y' = -2x$
$y'' = -2$
$y' = 0$ if $x = 0$

If $x = 0$, $y = 3$, so (0, 3) is a critical point and the y-intercept. $y'' < 0$, so (0, 3) is a relative maximum.

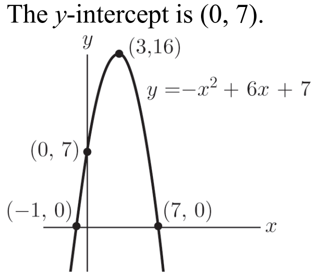
$0 = 3 - x^2 \Rightarrow x = \pm\sqrt{3}$, so the x-intercepts are $\left(\pm\sqrt{3},\ 0\right)$.



24. $y = 7 + 6x - x^2$
$y' = 6 - 2x$
$y'' = -2$
$y' = 0$ if $x = 3$

If $x = 3$, $y = 16$, so (3, 16) is a critical point. $y'' < 0$, so (3, 16) is a relative maximum.

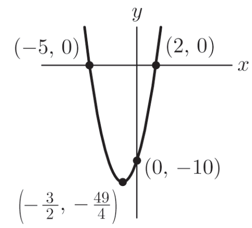
$0 = 7 + 6x - x^2 \Rightarrow x = -1$ or $x = 7$, so the x-intercepts are (–1, 0) and (7, 0).
The y-intercept is (0, 7).



25. $y = x^2 + 3x - 10$
$y' = 2x + 3$
$y'' = 2$
$y' = 0$ if $x = -\dfrac{3}{2}$

If $x = -\frac{3}{2}$, $y = -\frac{49}{4}$ so $\left(-\frac{3}{2}, -\frac{49}{4}\right)$ is a critical point. $y'' > 0$, so $\left(-\frac{3}{2}, -\frac{49}{4}\right)$ is a relative minimum.

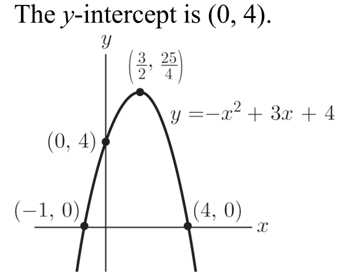
$x^2 + 3x - 10 = 0 \Rightarrow x = -5$ or $x = 2$, so the x-intercepts are (–5, 0) and (2, 0).
The y-intercept is (0, –10).



26. $y = 4 + 3x - x^2$
$y' = 3 - 2x$
$y'' = -2$
$y' = 0$ if $x = \dfrac{3}{2}$

If $x = \frac{3}{2}$, $y = -\frac{49}{4}$ so $\left(\frac{3}{2}, \frac{25}{4}\right)$ is a critical point. $y'' < 0$, so $\left(\frac{3}{2}, \frac{25}{4}\right)$ is a relative maximum.

$0 = 4 + 3x - x^2 \Rightarrow x = -1$ or $x = 4$, so the x-intercepts are (–1, 0) and (4, 0).
The y-intercept is (0, 4).



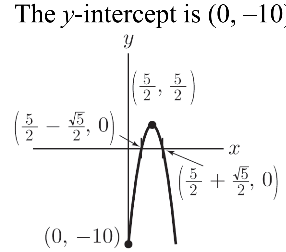
27. $y = -2x^2 + 10x - 10$
$y' = -4x + 10$
$y'' = -4$
$y' = 0$ if $x = \dfrac{5}{2}$

If $x = \frac{5}{2}$, $y = \frac{5}{2}$ so $\left(\frac{5}{2}, \frac{5}{2}\right)$ is a critical point.

$y'' < 0$, so $\left(\frac{5}{2}, \frac{5}{2}\right)$ is a relative maximum.

$0 = -2x^2 + 10x - 10 \Rightarrow x = \frac{5\pm\sqrt{5}}{2}$, so the x-intercepts are $\left(\frac{5-\sqrt{5}}{2}, 0\right)$ and $\left(\frac{5+\sqrt{5}}{2}, 0\right)$.

The y-intercept is (0, –10).

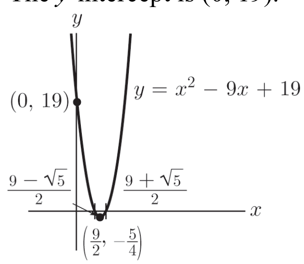


28. $y = x^2 - 9x + 19$
$y' = 2x - 9$
$y'' = 2$
$y' = 0 \text{ if } x = \dfrac{9}{2}$

If $x = \frac{9}{2}$, $y = -\frac{5}{4}$ so $\left(\frac{9}{2}, -\frac{5}{4}\right)$ is a critical point. $y'' > 0$, so $\left(\frac{9}{2}, -\frac{5}{4}\right)$ is a relative minimum.

$0 = x^2 - 9x + 19 \Rightarrow x = \frac{9 \pm \sqrt{5}}{2}$, so the x-intercepts are $\left(\frac{9-\sqrt{5}}{2}, 0\right)$ and $\left(\frac{9+\sqrt{5}}{2}, 0\right)$.

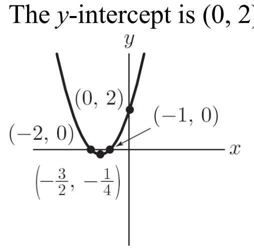
The y-intercept is (0, 19).



29. $y = x^2 + 3x + 2$
$y' = 2x + 3$
$y'' = 2$
$y' = 0 \text{ if } x = -\dfrac{3}{2}$

If $x = -\frac{3}{2}$, $y = -\frac{1}{4}$ so $\left(-\frac{3}{2}, -\frac{1}{4}\right)$ is a critical point. $y'' > 0$, so $\left(-\frac{3}{2}, -\frac{1}{4}\right)$ is a relative minimum.

$0 = x^2 + 3x + 2 \Rightarrow x = -2$ or $x = -1$, so the x-intercepts are (–2, 0) and (–1, 0).
The y-intercept is (0, 2).

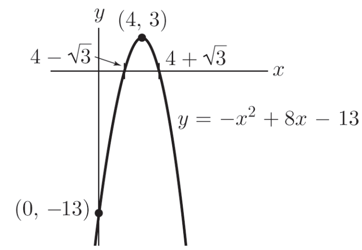


30. $y = -x^2 + 8x - 13$
$y' = -2x + 8$
$y'' = -2$
$y' = 0 \text{ if } x = 4$

If $x = 4$, $y = 3$, so (4, 3) is a critical point.
$y'' < 0$, so (4, 3) is a relative maximum.

$0 = -x^2 + 8x - 13 \Rightarrow x = 4 \pm \sqrt{3}$, so the x-intercepts are $\left(4 - \sqrt{3}, 0\right)$ and $\left(4 + \sqrt{3}, 0\right)$.

The y-intercept is (0, –13).

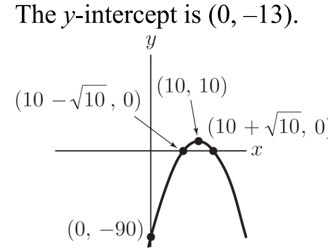


31. $y = -x^2 + 20x - 90$
$y' = -2x + 20$
$y'' = -2$
$y' = 0 \text{ if } x = 10$

If $x = 10$, $y = 10$, so (10, 10) is a critical point.
$y'' < 0$, so (10, 10) is a relative maximum.

$0 = -x^2 + 20x - 90 \Rightarrow x = 10 \pm \sqrt{10}$, so the x-intercepts are $\left(10 - \sqrt{10}, 0\right)$ and $\left(10 + \sqrt{10}, 0\right)$.

The y-intercept is (0, –13).

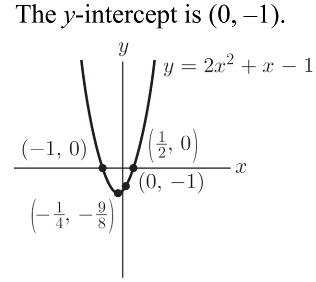


32. $y = 2x^2 + x - 1$
$y' = 4x + 1$
$y'' = 4$
$y' = 0 \text{ if } x = -\dfrac{1}{4}$

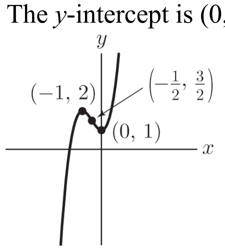
If $x = -\frac{1}{4}$, $y = -\frac{9}{8}$ so $\left(-\frac{1}{4}, -\frac{9}{8}\right)$ is a critical point. $y'' > 0$, so $\left(-\frac{1}{4}, -\frac{9}{8}\right)$ is a relative minimum.

$0 = 2x^2 + x - 1 \Rightarrow x = -1$ or $x = \frac{1}{2}$, so the x-intercepts are (–1, 0) and $\left(\frac{1}{2}, 0\right)$.

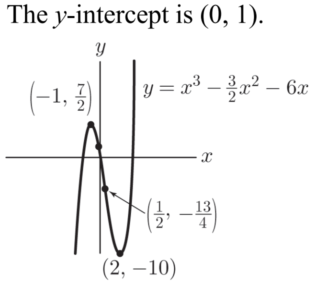
The y-intercept is (0, –1).



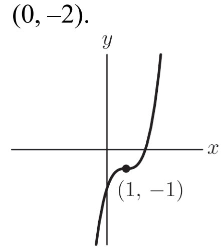
33. $f(x)=2x^3+3x^2+1$
$f'(x)=6x^2+6x$
$f''(x)=12x+6$
$f'(x)=0$ if $x=0$ or $x=-1$
$f(0)=1\Rightarrow(0,1)$ is a critical pt.
$f(-1)=2\Rightarrow(-1,2)$ is a critical pt.
$f''(0)=6>0,$ so the graph is concave up at $x=0$, and (0, 1) is a relative minimum.
$f''(-1)=-6<0,$ so the graph is concave down at $x=-1$, and (–1, 2) is a relative maximum.
$f''(x)=0$ when $x=-\frac{1}{2}.$
$f\left(-\frac{1}{2}\right)=\frac{3}{2}\Rightarrow\left(-\frac{1}{2},\frac{3}{2}\right)$ is an inflection pt.
The y-intercept is (0, 1).



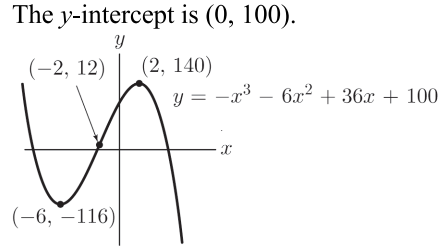
34. $f(x)=x^3-\frac{3}{2}x^2-6x$
$f'(x)=3x^2-3x-6$
$f''(x)=6x-3$
$f'(x)=0$ if $x=-1$ or $x=2$
$f(-1)=\frac{7}{2}\Rightarrow\left(-1,\frac{7}{2}\right)$ is a critical pt.
$f(2)=-10\Rightarrow(2,-10)$ is a critical pt.
$f''(-1)=-9<0,$ so the graph is concave down at $x=-1$, and $\left(-1,\frac{7}{2}\right)$ is a relative maximum.
$f''(2)=9>0,$ so the graph is concave up at $x=2$, and (2, –10) is a relative minimum.
$f''(x)=0$ when $x=\frac{1}{2}.$
$f\left(\frac{1}{2}\right)=-\frac{13}{4}\Rightarrow\left(\frac{1}{2},-\frac{13}{4}\right)$ is an inflection pt.
The y-intercept is (0, 1).



35. $f(x)=x^3-3x^2+3x-2$
$f'(x)=3x^2-6x+3$
$f''(x)=6x-6$
To find possible extrema, set $f'(x)=0$ and solve for x.
$3x^2-6x+3=0\Rightarrow x=1$
$f(1)=-1,$ so (1, –1) is a critical point.
Since $f'(x)\ge 0$ for all x, the graph is always increasing, and (1, –1) is neither a relative maximum nor a relative minimum.
To find possible inflection points, set $f''(x)=0$ and solve for x.
$6x-6=0\Rightarrow x=1$
Since $f''(x)<0$ for $x<1$ (meaning the graph is concave down) and $f''(x)>0$ for $x>1$ (meaning the graph is concave up), the point (1, –1) is an inflection point. The y-intercept is (0, –2).



36. $f(x)=100+36x-6x^2-x^3$
$f'(x)=36-12x-3x^2$
$f''(x)=-12-6x$
$f'(x)=0$ if $x=-6$ or $x=2$
$f(-6)=-116\Rightarrow(-6,-116)$ is a critical pt.
$f(2)=140\Rightarrow(2,140)$ is a critical pt.
$f''(-6)=24>0,$ so the graph is concave up at $x=-6$, and (–6, –116) is a relative minimum.
$f''(2)=-24<0,$ so the graph is concave down at $x=-1$, and (2, 140) is a relative maximum.
$f''(x)=0$ when $x=-2.$
$f(-2)=12\Rightarrow(-2,12)$ is an inflection pt.
The y-intercept is (0, 100).



37. $f(x)=\frac{11}{3}+3x-x^2-\frac{1}{3}x^3$

$f'(x)=3-2x-x^2$

$f''(x)=-2-2x$

$f'(x)=0$ if $x=-3$ or $x=1$

$f(-3)=-\frac{16}{3}\Rightarrow\left(-3,-\frac{16}{3}\right)$ is a critical pt.

$f(1)=\frac{16}{3}\Rightarrow\left(1,\frac{16}{3}\right)$ is a critical pt.

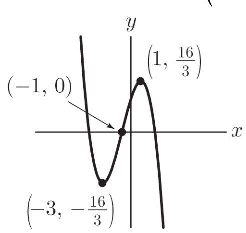
$f''(-3)=4>0,$ so the graph is concave up at $x=-3$, and $\left(-3,-\frac{16}{3}\right)$ is a relative minimum.

$f''(1)=-4<0,$ so the graph is concave down at $x=1$, and $\left(1,\frac{16}{3}\right)$ is a relative maximum.

$f''(x)=0$ when $x=-1$.

$f(-1)=0\Rightarrow(-1,0)$ is an inflection pt.

The y-intercept is $\left(0,\frac{11}{3}.\right)$



38. $f(x)=x^3-3x^2-9x+7$

$f'(x)=3x^2-6x-9$

$f''(x)=6x-6$

$f'(x)=0$ if $x=-1$ or $x=3$

$f(-1)=12\Rightarrow(-1,12)$ is a critical pt.

$f(3)=-10\Rightarrow(3,-20)$ is a critical pt.

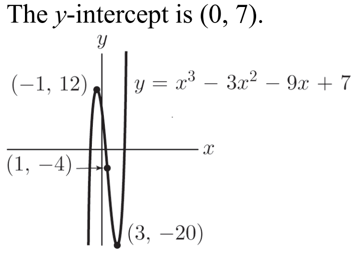
$f''(-1)=-12<0,$ so the graph is concave down at $x=-1$, and (–1, 12) is a relative maximum.

$f''(3)=12>0,$ so the graph is concave up at $x=3$, and (3, –20) is a relative minimum.

$f''(x)=0$ when $x=1$.

$f(1)=-4\Rightarrow(1,-4)$ is an inflection pt.

The y-intercept is (0, 7).



39. $f(x)=-\frac{1}{3}x^3-2x^2-5x$

$f'(x)=-x^2-4x-5$

$f''(x)=-2x-4$

To find possible extrema, set $f'(x)=0$ and solve for x.

$-x^2-4x-5=0\Rightarrow$ no real solution

Thus, there are no extrema.

Since $f'(x)\le 0$ for all x, the graph is always decreasing.

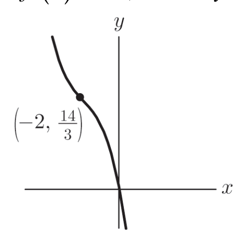
To find possible inflection points, set $f''(x)=0$ and solve for x.

$-2x-4=0\Rightarrow x=-2$

$f(-2)=\frac{14}{3}$

Since $f''(x)>0$ for $x<\frac{14}{3}$ (meaning the graph is concave up) and $f''(x)<0$ for $x>\frac{14}{3}$ (meaning the graph is concave down), the point $\left(-2,\frac{14}{3}\right)$ is an inflection point.

$f(0)=-6,$ so the y-intercept is (0, 0).



40. $y=x^3-6x^2-15x+50$

$y'=3x^2-12x-15$

$y''=6x-12$

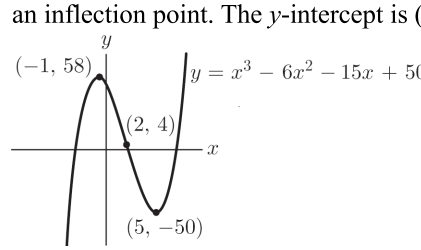
$y'=0$ if $x=-1$ or $x=5$

If $x=-1$, $y=58$. If $x=5$, $y=-50$. So, (–1, 58) and (5, –50) are critical points.

If $x=-1$, $y''=-18<0,$ so the graph is concave down and (–1, 58) is a relative maximum.

If $x=5$, $y''=18>0,$ so the graph is concave up and (5, –50) is a relative minimum.

$y''=0$ when $x=2$. If $x=2$, $y=4$, so (2, 4) is an inflection point. The y-intercept is (0, 50).



41. $y = x^4 - 2x^2$

$y' = 4x^3 - 4x$

$y'' = 12x^2 - 4$

$y' = 0$ if $x = 0$, $x = -1$, or $x = 1$

If $x = -1$, $y = -1$. If $x = 0$, $y = 0$. If $x = 1$, $y = -1$. So, $(-1, -1)$, $(0, 0)$, and $(1, -1)$ are critical points.

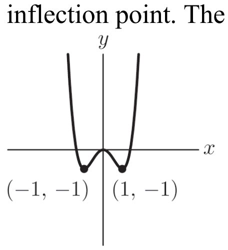
If $x = -1$, $y'' = 8 > 0$, so the graph is concave up and $(-1, -1)$ is a relative minimum.

If $x = 1$, $y = -1$, $y'' = 8 > 0$, so the graph is concave up and $(1, -1)$ is a relative minimum.

If $x = 0$, $y'' = 0$, so we must use the first derivative test. Since $y' < 0$ when $x < 0$ and also when $x > 0$, $(0, 0)$ is a relative maximum.

$y'' = 0$ when $x = \pm\frac{1}{\sqrt{3}}$. If $x = -\frac{1}{\sqrt{3}}$, $y = -\frac{5}{9}$, so $\left(-\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$ is an inflection point.

If $x = \frac{1}{\sqrt{3}}$, $y = -\frac{5}{9}$, so $\left(\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$ is an inflection point. The y-intercept is $(0, 0)$.



42. $y = x^4 - 4x^3$

$y' = 4x^3 - 12x^2 = 4x^2(x - 3)$

$y'' = 12x^2 - 24x$

$y' = 0$ if $x = 0$ or $x = 3$

If $x = 0$, $y = 0$. If $x = 0$, $y = 0$. If $x = 3$, $y = -27$. So, $(0, 0)$, and $(3, -27)$ are critical points.

If $x = 3$, $y'' = 36 > 0$, so the graph is concave up and $(3, -27)$ is a relative minimum.

If $x = 0$, $y'' = 0$, so we must use the first derivative test.

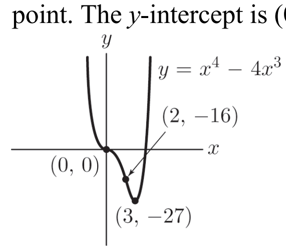
Critical Points, Intervals	$x < 0$	$0 < x < 3$	$3 < x$
$4x^2$	+	+	+
$x - 3$	–	–	+
y'	–	–	+
y	Decreasing on $(-\infty, 0)$	Decreasing on $(0, 3)$	Increasing on $(3, \infty)$

Thus, $(0, 0)$ is neither a relative maximum nor a relative minimum. It may be an inflection point. Verify by using the second derivative test.

$y'' = 0$ when $x = 0$ or $x = 2$.

Critical Points, Intervals	$x < 0$	$0 < x < 2$	$2 < x < 3$	$3 < x$
$12x$	–	+	+	+
$x - 2$	–	–	+	+
y''	+	–	+	+
Concavity	up	down	up	up

If $x = 0$, $y = 0$ so $(0, 0)$ is an inflection point. If $x = 2$, $y = -16$ so $(2, -16)$ is an inflection point. The y-intercept is $(0, 0)$.



43. $y = \frac{x}{5} + \frac{20}{x} + 3$. $x > 0$

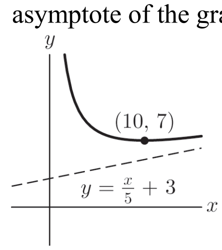
$y' = \frac{1}{5} - \frac{20}{x^2}$

$y'' = \frac{40}{x^3}$

$y' = 0$ if $x = \pm 10$

Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = 10$, $y = 7$, and $y'' = \frac{1}{25} > 0$, so the graph is concave up and $(10, 7)$ is a relative minimum.

Since y'' can never be zero, there are no inflection points. The term $\frac{20}{x}$ tells us that the y-axis is an asymptote. As $x \to \infty$, the graph approaches $y = \frac{x}{4} + 3$, so this is also an asymptote of the graph.



44. $y = \frac{1}{2x} + 2x + 1,\ x > 0$

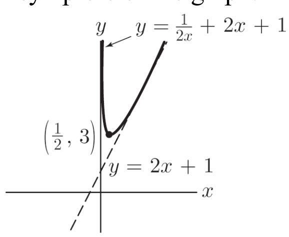
$y' = -\frac{1}{2x^2} + 2$

$y'' = \frac{1}{x^3}$

$y' = 0$ if $x = \pm\frac{1}{2}$

Note that we need to consider the positive solution only because the function is defined only for $x > 0$. When $x = \frac{1}{2},\ y = 3$, and $y'' = 8 > 0,$ so the graph is concave up and $\left(\frac{1}{2}, 3\right)$ is a relative minimum.

Since y'' can never be zero, there are no inflection points. The term $\frac{1}{2x}$ tells us that the y-axis is an asymptote. As $x \to \infty,$ the graph approaches $y = 2x + 1,$ so this is also an asymptote of the graph.



45. $f'(x) = \frac{3}{2}\left(x^2 + 2\right)^{1/2}(2x) = 3x\left(x^2 + 2\right)^{1/2}$

Since $f'(0) = 0,$ f has a possible extreme value at $x = 0$.

46. $f'(x) = \frac{3}{2}\left(2x^2 + 3\right)^{1/2}(4x) = 6x\left(2x^2 + 3\right)^{1/2}$

$2x^2 + 3 > 0$ for all x, so the sign of $f'(x)$ is determined by the sign of $4x$. Therefore, $f'(x) > 0$ if $x > 0,$ $f'(x) < 0$ if $x < 0$. This means that $f(x)$ is decreasing for $x < 0$ and increasing for $x > 0$.

47. $f''(x) = \frac{-2x}{(1 + x^2)^2},$ so $f''(0) = 0.$ Since $f'(x) > 0$ for all x, it follows that 0 must be an inflection point.

48. $f''(x) = \frac{1}{2}\left(5x^2 + 1\right)^{-1/2}(10x) = \frac{5x}{\sqrt{5x^2 + 1}},$ so $f''(0) = 0.$ Since $f'(x) > 0$ for all x, $f''(x)$ is positive for $x > 0$ and negative for $x < 0$, and it follows that 0 must be an inflection point.

49. A – c, B – e, C – f, D – b, E – a, F – d

50. A – c, B – e, C – f, D – b, E – a, F – d

51. **a.** The number of people living between $10 + h$ and 10 miles from the center of the city.

b. If so, $f(x)$ would be decreasing at $x = 10$, which is not possible.

52. $f(x) = \frac{1}{4}x^2 - x + 2\ \ (0 \le x \le 8)$

$f'(x) = \frac{1}{2}x - 1$

$f'(x) = 0 \Rightarrow \frac{1}{2}x - 1 = 0 \Rightarrow x = 2$

$f''(x) = \frac{1}{2}$

Since $f'(2)$ is a relative minimum, the maximum value of $f(x)$ must occur at one of the endpoints. $f(0) = 2, f(8) = 10$. 10 is the maximum value, attained $x = 8$.

53. $f(x) = 2 - 6x - x^2\ \ (0 \le x \le 5)$

$f'(x) = -6 - 2x$

Since $f'(x) < 0$ for all $x > 0$, $f(x)$ is decreasing on the interval [0, 5]. Thus, the maximum value occurs at $x = 0$. The maximum value is $f(0) = 2$.

54. $g(t) = t^2 - 6t + 9\ \ (1 \le t \le 6)$

$g'(t) = 2t - 6$

$g'(t) = 0 \Rightarrow 2t - 6 = 0 \Rightarrow t = 3$

$g''(t) = 2$

The minimum value of $g(t)$ is $g(3) = 0$.

55. Let x be the width and h be the height. The objective is $S = 2xh + 4x + 8h$ and the constraint is $4xh = 200 \Rightarrow h = \frac{50}{x}$.

Thus,

$$S(x) = 2x\left(\frac{50}{x}\right) + 4x + \frac{400}{x} = 100 + 4x + \frac{400}{x}.$$

$$S'(x) = 4 - \frac{400}{x^2}$$

$$S'(x) = 0 \Rightarrow 4 - \frac{400}{x^2} = 0 \Rightarrow x = 10$$

$$h = \frac{50}{x} = \frac{50}{10} = 5$$

The minimum value of $S(x)$ for $x > 0$ occurs at $x = 10$. Thus, the dimensions of the box should be 10 ft × 4 ft × 5 ft.

56. Let x be the length of the base of the box and let y be the other dimension. The objective is $V = x^2y$ and the constraint is

$$3x^2 + x^2 + 4xy = 48$$

$$y = \frac{48 - 4x^2}{4x} = \frac{12 - x^2}{x}$$

$$V(x) = x^2 \cdot \frac{12 - x^2}{x} = 12x - x^3$$

$$V'(x) = 12 - 3x^2$$

$$V'(x) = 0 \Rightarrow 12 - 3x^2 = 0 \Rightarrow x = 2$$

$$V''(x) = -6x;\ V''(2) < 0.$$

The maximum value for x for $x > 0$ occurs at $x = 2$. The optimal dimensions are thus 2 ft × 2 ft × 4 ft.

57. Let x be the number of inches turned up on each side of the gutter. The objective is $A(x) = (30 - 2x)x$

(A is the cross-sectional area of the gutter—maximizing this will maximize the volume).

$$A'(x) = 30 - 4x$$

$$A'(x) = 0 \Rightarrow 30 - 4x = 0 \Rightarrow x = \frac{15}{2}$$

$$A''(x) = -4,\ A''\left(\frac{15}{2}\right) < 0$$

$x = \frac{15}{2}$ inches gives the maximum value for A.

58. Let x be the number of trees planted. The objective is $f(x) = \left(25 - \frac{1}{2}(x - 40)\right)x$ $(x \geq 40)$. $f(x) = 45x - \frac{1}{2}x^2$

$$f'(x) = 45 - x$$

$$f'(x) = 0 \Rightarrow 45 - x = 0 \Rightarrow x = 45$$

$$f''(x) = -1;\ f''(45) < 0.$$

The maximum value of $f(x)$ occurs at $x = 45$. Thus, 45 trees should be planted.

59. Let r be the number of production runs and let x be the lot size. Then the objective is $C = 1000r + .5\left(\frac{x}{2}\right)$ and the constraint is

$$rx = 400{,}000 \Rightarrow r = \frac{400{,}000}{x}$$

so $C(x) = \frac{4 \cdot 10^8}{x} + \frac{x}{4}$

$$C'(x) = \frac{-4 \cdot 10^8}{x^2} + \frac{1}{4}$$

$$C'(x) = 0 \Rightarrow \frac{-4 \cdot 10^8}{x^2} + \frac{1}{4} = 0 \Rightarrow x = 4 \cdot 10^4$$

$$C''(x) = \frac{8 \cdot 10^8}{x^3};\ C''(4 \cdot 10^4) > 0.$$

The minimum value of $C(x)$ for $x > 0$ occurs at $x = 4 \cdot 10^4 = 40{,}000$. Thus the economic lot size is 40,000 books/run.

60. The revenue function is

$$R(x) = (150 - .02x)x = 150x - .02x^2.$$

Thus, the profit function is

$$P(x) = (150x - .02x^2) - (10x + 300)$$
$$= -.02x^2 + 140x - 300$$

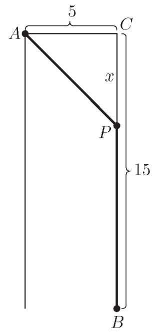
$$P'(x) = -.04x + 140$$

$$P'(x) = 0 \Rightarrow -.04x + 140 = 0 \Rightarrow x = 3500$$

$$P''(x) = -.04;\ P''(3500) < 0.$$

The maximum value of $P(x)$ occurs at $x = 3500$.

61.



The distance from point *A* to point *P* is $\sqrt{25+x^2}$ and the distance from point *P* to point *B* is $15-x$. The time it takes to travel from point *A* to point *P* is $\frac{\sqrt{25+x^2}}{8}$ and the time it takes to travel from point *P* to point *B* is $\frac{15-x}{17}$. Therefore, the total trip takes $T(x)=\frac{1}{8}\left(25+x^2\right)^{1/2}+\frac{1}{17}(15-x)$ hours.

$$T'(x)=\frac{1}{16}(25+x^2)^{-1/2}(2x)-\frac{1}{17}$$

$$T'(x)=0 \Rightarrow \frac{1}{16}(25+x^2)^{-1/2}(2x)-\frac{1}{17}=0 \Rightarrow x=\frac{8}{3}$$

$$T''(x)=\frac{1}{8}(25+x^2)^{-1/2}-\frac{1}{8}x^2(25+x^2)^{-3/2}$$

$$T''\left(\frac{8}{3}\right)=\frac{1}{8}\left(25+\left(\frac{8}{3}\right)^2\right)^{-1/2}-\frac{1}{8}\left(\frac{8}{3}\right)^2\left(25+\left(\frac{8}{3}\right)^2\right)^{-3/2}=\frac{675}{39304}>0$$

The minimum value for $T(x)$ occurs at $x=\frac{8}{3}$. Thus, Jane should drive from point *A* to point *P*, $\frac{8}{3}$ miles from point *C*, then down to point *B*.

62. Let $12 \le x \le 25$ be the size of the tour group. Then, the revenue generated from a group of *x* people, *R*(*x*), is $R(x)=\left[800-20(x-12)\right]x.$ To maximize revenue:

$$R'(x)=1040-40x$$

$$R'(x)=0 \Rightarrow 1040-40x=0 \Rightarrow R=26$$

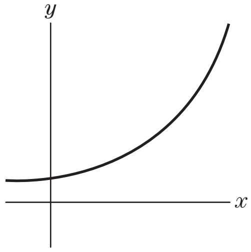
$$R''(x)=-40;\ R''(26)<0$$

Revenue is maximized for a group of 26 people, which exceeds the maximum allowed. Although, *R*(*x*) is an increasing function on [12, 25], therefore *R*(*x*) reaches its maximum at *x* = 25 on the interval $12 \le x \le 25$.
The tour group that produces the greatest revenue is size 25.

CHAPTER 2

Exercises 2.1, page 138

1. (a), (e), (f) **2.** (c), (d) **3.** (b), (c), (d) **4.** (a), (e) **5.** Increasing for $x < \frac{1}{2}$, relative maximum point at $x = \frac{1}{2}$, maximum value $= 1$, decreasing for $x > \frac{1}{2}$, concave down, y-intercept (0, 0), x-intercepts (0, 0) and (1, 0). **6.** Increasing for $x < -.4$, relative maximum point at $x = -.4$, relative maximum value $= 5.1$, decreasing for $x > -.4$, concave down for $x < 3$, inflection point (3, 3), concave up for $x > 3$, y-intercept (0, 5), x-intercept $(-3.5, 0)$. The graph approaches the x-axis as a horizontal asymptote. **7.** Decreasing for $x < 0$, relative minimum point at $x = 0$, relative minimum value $= 2$, increasing for $0 < x < 2$, relative maximum point at $x = 2$, relative maximum value $= 4$, decreasing for $x > 2$, concave up for $x < 1$, concave down for $x > 1$, inflection point at (1, 3), y-intercept (0, 2), x-intercept (3.6, 0). **8.** Increasing for $x < -1$, relative maximum at $x = -1$, relative maximum value $= 5$, decreasing for $-1 < x < 2.9$, relative minimum at $x = 2.9$, relative minimum value $= -2$, increasing for $x > 2.9$, concave down for $x < 1$, inflection point at (1, .5), concave up for $x > 1$, y-intercept (0, 3.3), x-intercepts $(-2.5, 0)$, (1.3, 0), and (4.4, 0). **9.** Decreasing for $x < 2$, relative minimum at $x = 2$, minimum value $= 3$, increasing for $x > 2$, concave up for all x, no inflection point, defined for $x > 0$, the line $y = x$ is an asymptote, the y-axis is an asymptote. **10.** Increasing for all x, concave down for $x < 3$, inflection point at (3, 3), concave up for $x > 3$, y-intercept (0, 1), x-intercept $(-.5, 0)$. **11.** Decreasing for $1 \le x < 3$, relative minimum point at $x = 3$, increasing for $x > 3$, maximum value $= 6$ (at $x = 1$), minimum value $= .9$ (at $x = 3$), inflection point at $x = 4$, concave up for $1 \le x < 4$, concave down for $x > 4$, the line $y = 4$ is an asymptote. **12.** Increasing for $x < -1.5$, relative maximum at $x = -1.5$, relative maximum value $= 3.5$, decreasing for $-1.5 < x < 2$, relative minimum at $x = 2$, relative minimum value $= -1.6$, increasing from $2 < x < 5.5$, relative maximum at $x = 5.5$, relative maximum value $= 3.4$, decreasing for $x > 5.5$, concave down for $x < 0$, inflection point at (0, 1), concave up for $0 < x < 4$, inflection point at (4, 1), concave down for $x > 4$, y-intercept (0, 1), x-intercepts $(-2.8, 0)$, (.6, 0), (3.5, 0), and (6.7, 0). **13.** Slope decreases for all x. **14.** Slope decreases for $x < 3$, increases for $x > 3$. **15.** Slope decreases for $x < 1$, increases for $x > 1$. Minimum slope occurs at $x = 1$. **16.** Slope decreases for $x < 3$, increases for $x > 3$. **17.** **(a)** C, F **(b)** A, B, F **(c)** C
18. **(a)** A, E **(b)** D **(c)** E

19.


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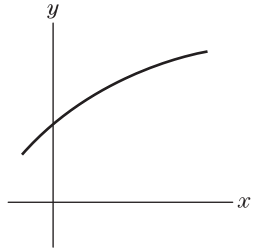

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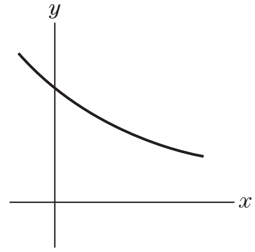

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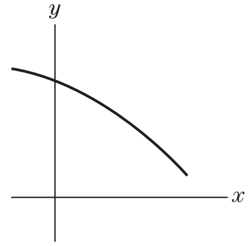

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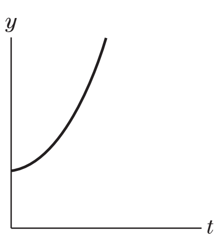

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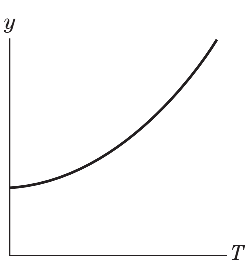

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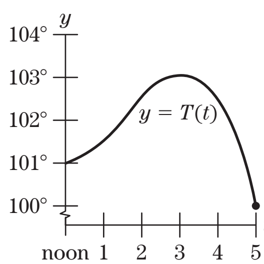

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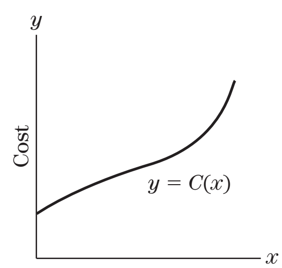

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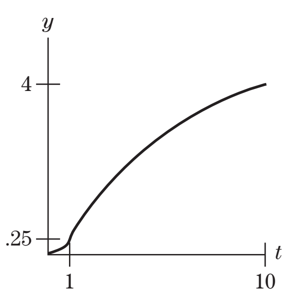

28. Oxygen content decreases until time a, at which time it reaches a minimum. After a, oxygen content steadily increases. The rate of increases until b, and then decreases. Time b is the time when oxygen content is increasing fastest. **29.** 1960 **30.** 1999; 1985 **31.** The parachutist's velocity levels off to 15 ft/sec. **32.** Bacteria population stabilizes at 25,000.

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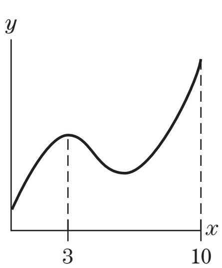

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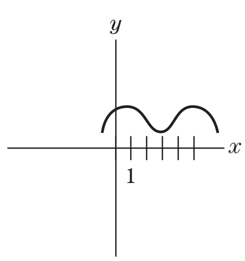

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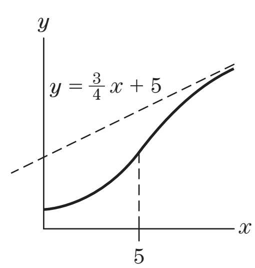

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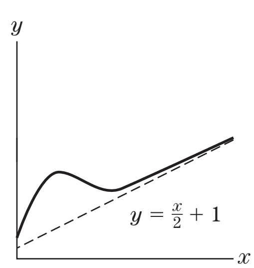
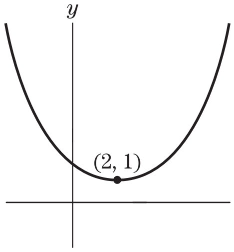


37. **(a)** Yes **(b)** Yes **38.** No **39.** $x = 2$ **40.** $C = 4$ **41.** $y = x$

Exercises 2.2, page 145

1. (e) **2.** (b), (c), (f) **3.** (a), (b), (d), (e) **4.** (f) **5.** (d) **6.** (c)

7.



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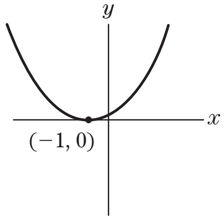

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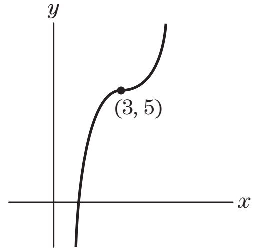

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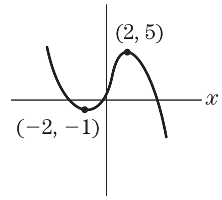

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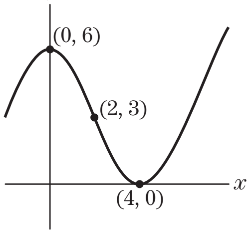

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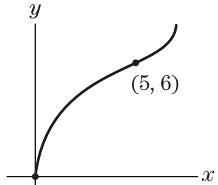

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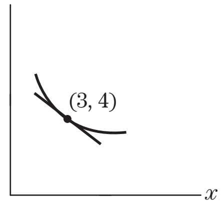

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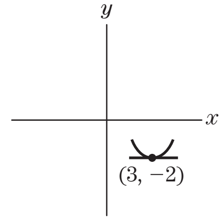

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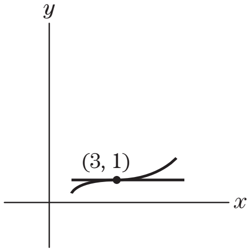

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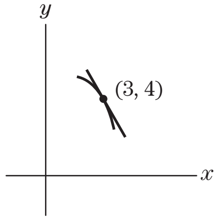

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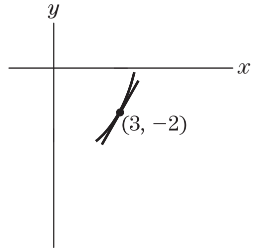

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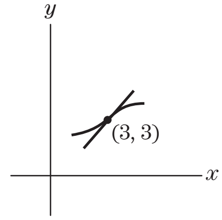
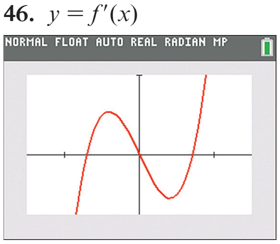


19.

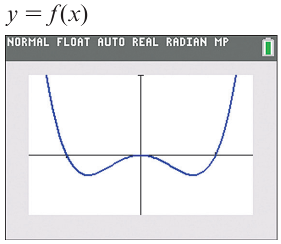
	f	f'	f''
A	POS	POS	NEG
B	0	NEG	0
C	NEG	0	POS

20. (a) $x = 2$ **(b)** $x = 3$ and $x = 4$ **21.** $t = 1$ **22.** $t = 2$ **23. (a)** Decreasing **(b)** The function $f(x)$ is increasing for $1 \le x < 2$ because the values of $f'(x)$ are positive. The function $f(x)$ is decreasing for $2 < x \le 3$ because the values of $f'(x)$ are negative. Therefore, $f(x)$ has a relative maximum at $x = 2$. Coordinates: (2, 9) **(c)** The function $f(x)$ is decreasing for $9 \le x < 10$ because the values of $f'(x)$ are negative. The function $f(x)$ is increasing for $10 < x \le 11$ because the values of $f'(x)$ are positive. Therefore, $f(x)$ has a relative minimum at $x = 10$. **(d)** Concave down **(e)** At $x = 6$; coordinates: (6, 5) **(f)** $x = 15$ **24. (a)** $f(2) = 3$ **(b)** $t = 4$ or $t = 6$ **(c)** $t = 1$ **(d)** $t = 5$ **(e)** 1 unit per minute **(f)** The solutions to $f'(t) = -1$ are $t = 2.5$ and $t = 3.5$, so $f(t)$ is decreasing at the rate of 1 unit per minute after 2.5 minutes and after 3.5 minutes. **(g)** $t = 3$ **(h)** $t = 7$ **25.** The slope is positive because $f'(6) = 2$, a positive number. **26.** The slope is negative because $f'(4) = -1$. **27.** The slope is 0 because $f'(3) = 0$. Also, $f'(x)$ is positive for x slightly less than 3, and $f'(x)$ is negative for x slightly greater than 3. Hence, $f(x)$ changes from increasing to decreasing at $x = 3$. **28.** The slope is 0 because $f'(5) = 0$. Also, $f'(x)$ is negative for x slightly less than 5, and $f'(x)$ is positive for x slightly greater than 5. Hence, $f(x)$ changes from decreasing to increasing at $x = 5$. **29.** $f'(x)$ is increasing at $x = 0$, so the graph of $f(x)$ is concave up. **30.** $f'(x)$ is decreasing at $x = 2$, so the graph of $f(x)$ is concave down. **31.** At $x = 1$, $f'(x)$ changes from increasing to decreasing, so the concavity of the graph of $f(x)$ changes from concave up to concave down. **32.** At $x = 4$, $f'(x)$ changes from decreasing to increasing, so the slope of the graph of $f(x)$ changes from decreasing to increasing. **33.** $y - 3 = 2(x - 6)$ **34.** 9 **35.** 3.25 **36.** $y - 3 = 1(x - 0)$; $y = x + 3$ **37. (a)** $\frac{1}{6}$ in. **(b)** (ii), Because the water level is falling. **38. (a)** 3 degrees **(b)** (ii) Because the temperature is falling **39.** II. The derivative is positive for $x > 0$, so the function should be increasing. **40.** $f'(x) = 3(x - 2)(x - 4)$ I cannot be the graph since it does not have horizontal tangents at $x = 2, 4$. **41.** I **42. (a)** (C) **(b)** (D) **(c)** (B) **(d)** (A) **(e)** (E) **43. (a)** 2 million **(b)** 30,000 farms per year **(c)** 1940 **(d)** 1945 and 1978 **(e)** 1960 **44. (a)** Decreasing. **(b)** Concave up. **(c)** $t = 4$ (after 4 hours) **(d)** $t = 2$ (after 2 hours) **(e)** After 2.6 hours and after 7 hours **45.** Rel. max: $x \approx -2.34$; rel. min: $x \approx 2.34$; inflection point: $x = 0$, $x \approx \pm 1.41$

46. $y = f'(x)$



$y = f(x)$



Relative max at $x = 0$.
Relative min at $x \approx .71$ and $-.71$.
Inflection point at $x \approx .41$ and $-.41$

Exercises 2.3, page 156

1. $f'(x) = 3(x+3)(x-3)$; relative maximum point $(-3, 54)$; relative minimum point $(3, -54)$

Critical Values		-3		3	
	$x < -3$		$-3 < x < 3$		$3 < x$
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	Increasing on $(-\infty, -3)$ ↗	54 Local maximum $(-3, 54)$	Decreasing on $(-3, 3)$ ↘	-54 Local minimum $(3, -54)$	Increasing on $(3, \infty)$ ↗

2. Relative maximum $(0, 1)$; relative minimum $(4, -31)$

3. $f'(x) = -3(x-1)(x-3)$; relative maximum point $(3, 1)$; relative minimum point $(1, -3)$

Critical Values		1		3	
	$x < 1$		$1 < x < 3$		$3 < x$
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	Decreasing on $(-\infty, 1)$ ↘	-3 Local minimum $(1, -3)$	Increasing on $(1, 3)$ ↗	1 Local maximum $(3, 1)$	Decreasing on $(3, \infty)$ ↘

4. Relative minimum $\left(-\frac{1}{2}, -\frac{33}{8}\right)$; relative maximum $\left(\frac{1}{3}, -\frac{43}{18}\right)$

5. $f'(x) = x(x-2)$; relative maximum point $(0, 1)$; relative minimum point $(2, -1/3)$

Critical Values		0		2	
	$x < 0$		$0 < x < 2$		$2 < x$
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	Increasing on $(-\infty, 0)$ ↗	1 Local maximum $(0, 1)$	Decreasing on $(0, 2)$ ↘	$-1/3$ Local minimum $(2, -1/3)$	Increasing on $(2, \infty)$ ↗

6. Relative maximum $\left(-\frac{1}{2}, \frac{7}{3}\right)$; relative minimum $\left(\frac{1}{2}, \frac{5}{3}\right)$

7. $f'(x) = -3x(x+8)$; relative maximum point $(0, -2)$; relative minimum point $(-8, -258)$

Critical Values		-8		0	
	$x < -8$		$-8 < x < 0$		$0 < x$
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	Decreasing on $(-\infty, -8)$ ↘	-258 Local minimum $(-8, -258)$	Increasing on $(-8, 0)$ ↗	-2 Local maximum $(0, -2)$	Decreasing on $(0, \infty)$ ↘

8. Relative maximum $(-1, -2)$; relative minimum $(0, -3)$

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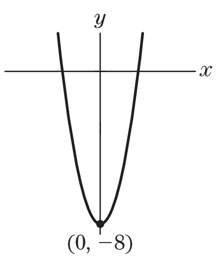

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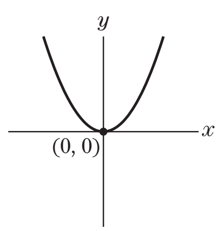

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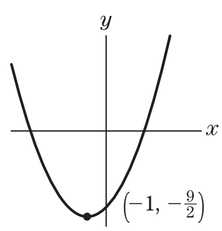

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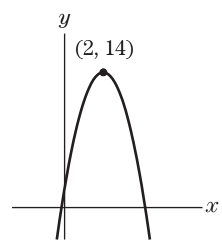

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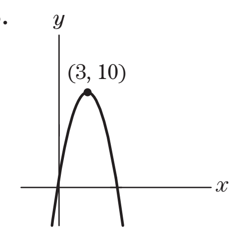

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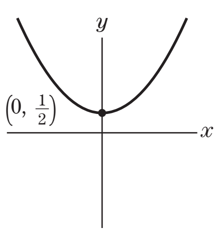

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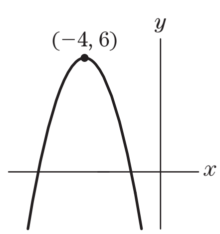

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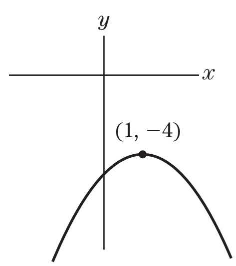

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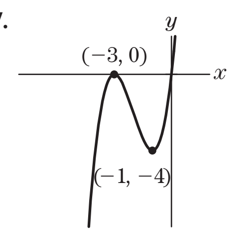

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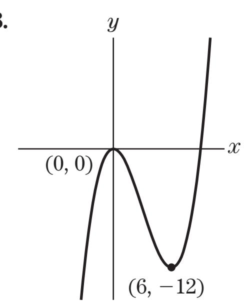

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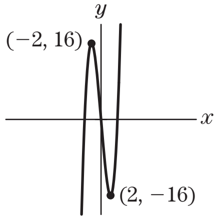

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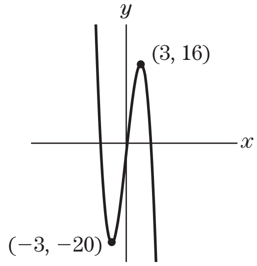

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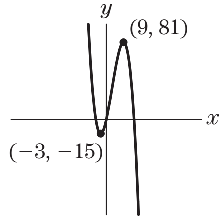

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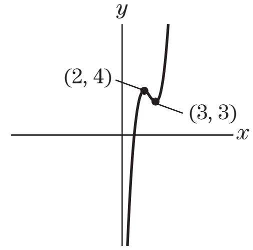

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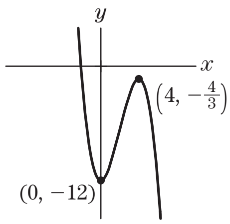

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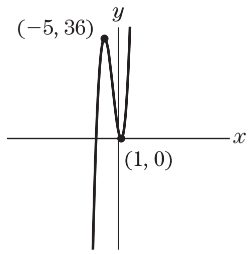

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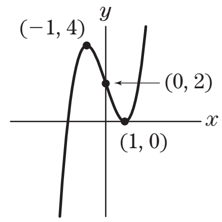

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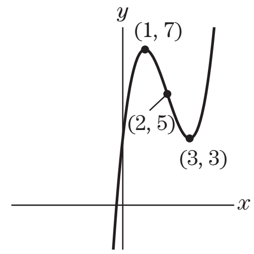

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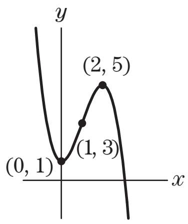

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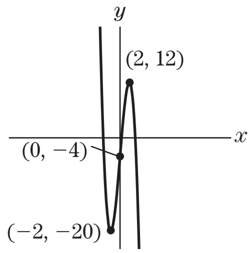

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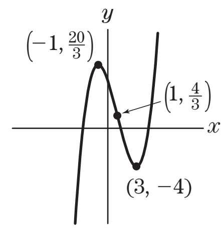

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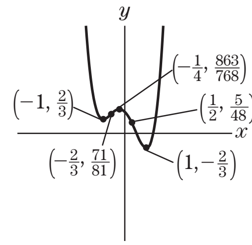

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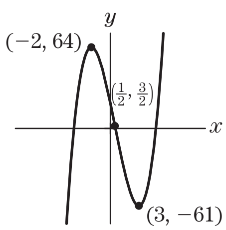

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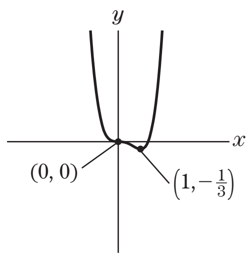

33. No, $f''(x) = 2a \neq 0$. **34.** No, $f''(x)$ is a linear function. It has at most one zero. **35.** (4, 3) min. **36.** (−3, 23) max. **37.** (1, 5) max. **38.** (−5, −15) min. **39.** (−.1, −3.05) min. **40.** (30, 2000) min. **41.** $f'(x) = g(x)$ **42.** $f'(x) = g(x)$ **43. (a)** f has a relative min. **(b)** f has an inflection point. **44. (a)** 125 million **(b)** 1850 **(c)** 2.2 million/year **(d)** 1925 **(e)** 1940. **45. (a)** $A(x) = -893.103x + 460.759$ (billion dollars). **(b)** Revenue is x% of assets or $R(x) = \frac{xA(x)}{100} = \frac{x}{100}(-893.103x + 460.759)$. $R(.3) \approx .578484$ billion dollars or \$578.484 million $R(.1) \approx .371449$ billion dollars or \$371.449 million. **(c)** Maximum revenue when $R'(x) = 0$ or $x \approx .258$. Maximum revenue $R(.258) \approx .594273$ or \$594.273 million. **46.** $x = .398 P(.3) = \$.3285$ billion $P(.1) = \$-.3786$ billion. They were better off not lowering their fees. **47.** $f'(x)$ is always nonnegative. **48.** Inflection point $(5, \frac{10}{3})$ **49.** They both have a minimum point. The parabola does not have a vertical asymptote. **50.** Minimum at (5, 5)

Exercises 2.4, page 162

1. $(\frac{3 \pm \sqrt{5}}{2}, 0)$ **2.** $(\frac{-5 + \sqrt{5}}{2}, 0)$ and $(\frac{-5 - \sqrt{5}}{2}, 0)$ **3.** $(-2, 0), (-\frac{1}{2}, 0)$ **4.** $(-1 + \sqrt{5}, 0), (-1 - \sqrt{5}, 0)$ **5.** $(\frac{1}{2}, 0)$ **6.** $(-\frac{1}{3}, 0), (-3, 0)$ **7.** The derivative $x^2 - 4x + 5$ has no zeros; no relative extreme points. **8.** No relative extreme points; $f(x)$ always decreasing.

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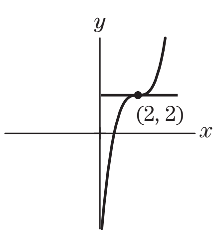

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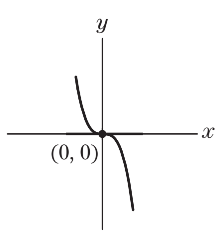

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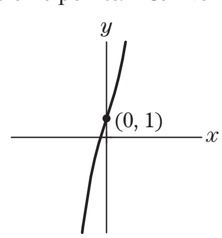

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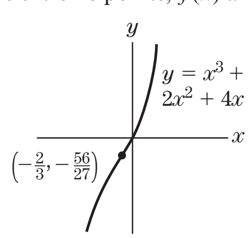

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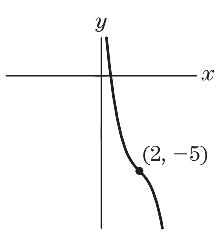

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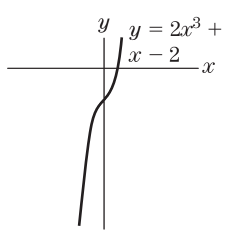

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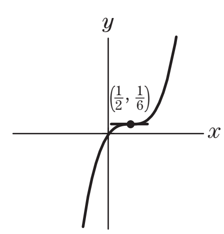

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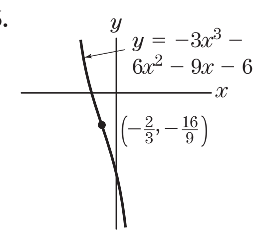

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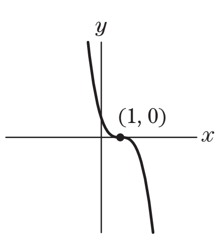

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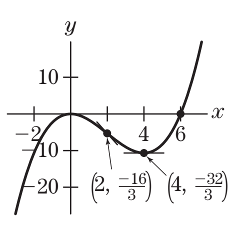

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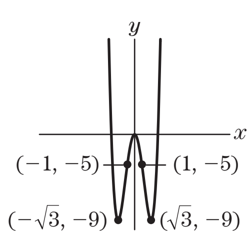

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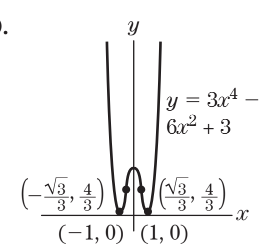
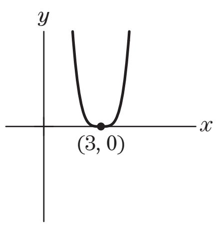
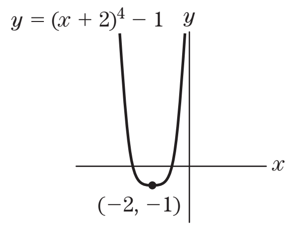
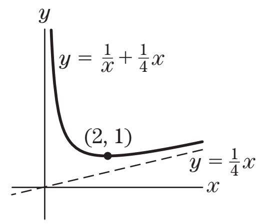
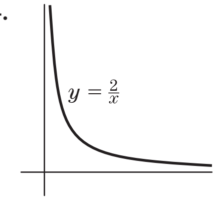
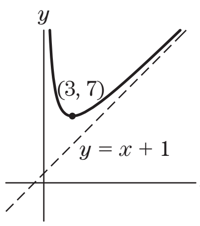


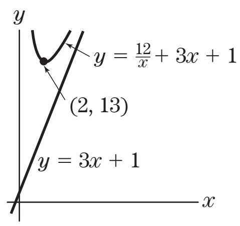
21. 

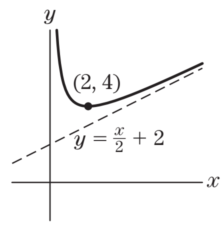
22. 

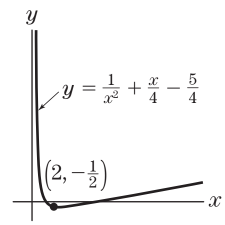
23. 

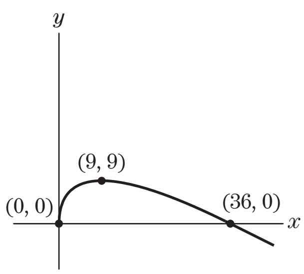
24. 

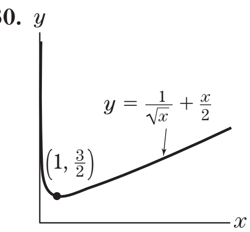
25. 

26. 

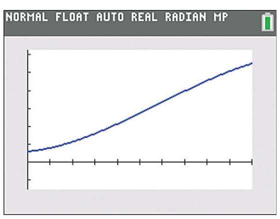
27. 

28. 

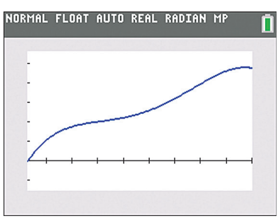
29. 

30. 

31. $g(x) = f'(x)$ **32.** $g(x) = f'(x)$ **33.** $f(2) = 0$ implies $4a + 2b + c = 0$. Local maximum at (0, 1) implies $f'(0) = 0$ and $f(0) = 1$. $a = -1/4$, $b = 0$, $c = 1$, $f(x) = -1/4x^2 + 1$.
34. $f(x) = 2x^2 - 4x + 1$ **35.** If $f'(a) = 0$ and f' is increasing at $x = a$, then $f'(x) < 0$ for $x < a$ and $f'(x) > 0$ for $x > a$. By the first-derivative test (case (b)), f has a local minimum at $x = a$.
36. Minimum at $x = a$

37. (a) 

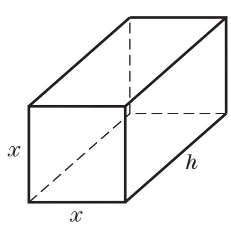
(b) 15.0 g **(c)** after 12.0 days **(d)** 1.6 g/day **(e)** after 6.0 days and after 17.6 days **(f)** after 11.8 days

38. (a) 

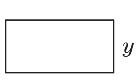
(b) 1.63 m **(c)** About 144 days **(d)** About .0104 meter/day **(e)** After 64 days **(f)** After 243 days **(g)** After 32 days.

Exercises 2.5, page 168

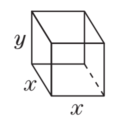
1. 20 **2.** $x = 6$, $f(6) = 36$ **3.** $t = 4$, $f(4) = 8$ **4.** $t = 12$, $f(t) = -144$ **5.** $x = 1$, $y = 1$, maximum = 1 **6.** $x = \frac{4}{3}$, $y = \frac{2}{3}$
7. $x = 3$, $y = 3$, minimum = 18 **8.** No maximum **9.** $x = 6$, $y = 6$, minimum = 12 **10.** $x = \frac{\sqrt{3}}{3}$, $y = 1 - \frac{\sqrt{3}}{3}$, $z = 1 + \frac{\sqrt{3}}{3}$, maximum $\frac{2\sqrt{3}}{9}$ **11. (a)** Objective: $A = xy$; constraint: $8x + 4y = 320$ **(b)** $A = -2x^2 + 80x$ **(c)** $x = 20$ ft, $y = 40$ ft
12. (a) Objective: $S = x^2 + 4xh$; constraint: $x^2h = 32$ **(b)** $S = x^2 + \frac{128}{x}$ **(c)** $x = 4$ ft, $h = 2$ ft

13. (a) 

(b) $h + 4x$ **(c)** Objective: $V = x^2h$; constraint: $h + 4x = 84$ **(d)** $V = -4x^3 + 84x^2$ **(e)** $x = 14$ in., $h = 28$ in.

14. (a) 

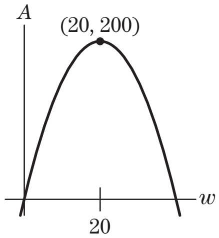
(b) Objective: $P = 2x + 2y$; constraint: $100 = xy$ **(c)** $x = 10$ m, $y = 10$ m **15.** Let x be the length of the fence and y the other dimension. Objective: $C = 15x + 20y$; constraint: $xy = 75$; $x = 10$ ft, $y = 7.5$ ft.

16. 

Optimal values $x = 2$ ft, $y = 3$ ft

17. Let x be the length of each edge of the base and h the height. Objective: $A = 2x^2 + 4xh$; constraint: $x^2h = 8000$; 20 cm by 20 cm by 20 cm **18.** $x = 5$ ft, $y = 10$ ft **19.** Let x be the length of the fence parallel to the river and y the length of each section perpendicular to the river. Objective: $A = xy$; constraint: $6x + 15y = 1500$; $x = 125$ ft, $y = 50$ ft **20.** Maximum area 75 ft $\times$ 75 ft **21.** Objective: $P = xy$; constraint: $x + y = 100$; $x = 50$, $y = 50$ **22.** $x = 10$, $y = 10$ **23.** Objective: $A = \frac{\pi x^2}{2} + 2xh$; constraint: $(2 + \pi)x + 2h = 14$; $x = \frac{14}{4 + \pi}$ ft **24.** $x = 2$ in, $h = 4$ in

25. $w = 20$ ft, $x = 10$ ft



26. 20 miles per hour **27.** $C(x) = 6x + 10\sqrt{(20 - x)^2 + 24^2}$; $C'(x) = 6 - \frac{10(20 - x)}{\sqrt{(20 - x)^2 + 24^2}}$; $C'(x) = 0$ $(0 \le x \le 20)$ implies $x = 2$. Use the first-derivative test to conclude that the minimum cost is $C(2) = \$312$. **28.** $x = 6$ in; $y = 12$ in **29.** $\left(\frac{3}{2}, \sqrt{\frac{3}{2}}\right)$ **30.** $D(6, 6) \approx 14.87$ miles **31.** $x = 2$, $y = 1$ **32.** $x \approx 2.12$

Exercises 2.6, page 175

1. **(a)** 90 **(b)** 180 **(c)** 6 **(d)** 1080 pounds **2.** **(a)** \$930 **(b)** \$1560 **3.** **(a)** $C = 16r + 2x$ **(b)** Constraint $rx = 800$ **(c)** $x = 80$, $r = 10$, minimum inventory cost = \$320 **4.** **(a)** $C = 160r + 16x$ **(b)** $rx = 640$ **(c)** $C(80) = \$2560$ **5.** Let x be the number of cases per order and r the number of orders per year. Objective: $C = 80r + 5x$; constraint: $rx = 10{,}000$ (a) \$4100 (b) 400 cases **6.** **(a)** \$300,000 **(b)** 60,000 tires **7.** Let r be the number of production runs and x the number of microscopes manufactured per run. Objective: $C = 2500r + 25x$; constraint: $rx = 1600$; 4 runs **8.** $r = 20$ **10.** The optimal order quantity does not change **11.** Objective: $A = (100 + x)w$; constraint: $2x + 2w = 300$; $x = 25$ ft, $w = 125$ ft **12.** $x = 0$ ft, $w = 50$ ft **13.** Objective: $F = 2x + 3w$; constraint: $xw = 54$; $x = 9$ m, $w = 6$ m **14.** $x = \frac{18}{\sqrt{5}}$ m, $w = 3\sqrt{5}$ m **15.** **(a)** $A(x) = 100x + 1000$ **(b)** $R(x) = A(x)\cdot(\text{Price}) = (100x + 1000)(18 - x)$ $(0 \le x \le 18)$. The graph of $R(x)$ is a parabola looking downward, with a maximum at $x = 4$. **(c)** $A(x)$ does not change, $R(x) = (100x + 1000)(9 - x)$ $(0 \le x \le 9)$. Maximum value when $x = 0$. **16.** 3 in. $\times$ 3 in. $\times$ 4 in. **17.** Let x be the length of each edge of the base and h the height. Objective: $C = 6x^2 + 10xh$; constraint: $x^2h = 150$; 5 ft by 5 ft by 6 ft **18.** $x = 100$ ft, $y = 120$ ft **19.** Let x be the length of each edge of the end and h the length. Objective: $V = x^2h$; constraint: $2x + h = 120$; 40 cm by 40 cm by 40 cm **20.** $x = \frac{220}{\pi}$ yd, $y = 110$ yd **21.** objective: $V = w^2x$; constraint: $2x + w = 16$; $\frac{8}{3}$ in. **22.** $\frac{3}{2}$ ft $\times$ 3 ft $\times$ 2 ft **23.** After 20 days **24.** $t = 5$, $f'(5) = 45$ or 45 tons per day. **25.** $2\sqrt{3}$ by 6 **26.** After 4 weeks **27.** 10 in. by 10 in. by 4 in. **28.** Greatest value at $x = 0$ **29.** ≈ 3.77 cm **30.** **(b)** $x = 32.47$ or 1988, $f(32.47) \approx 1.7$ cups per day **(c)** $x = 6$ or 1961, $f(6) \approx 3$ or 3 cups per day. **(d)** $x \approx 19.26$ or 1975.

Exercises 2.7, page 183

1. \$1 **2.** Marginal cost is decreasing at $x = 100$. $M(200) = 0$ is the minimal marginal cost. **3.** 32 **4.** $R(20{,}000) = 40{,}000$ is maximum possible. **5.** 5 **6.** Maximum occurs at $x = 50$. **7.** $x = 20$ units, $p = \$133.33$. **8.** $x = 1000$, $p = \$1$ **9.** 2 million tons, \$156 per ton **10.** $x = 15$, $y = 15$. **11.** **(a)** \$3.00 **(b)** \$3.30 **12.** \$45 per ticket **13.** Let x be the number of prints and p the price per print. Demand equation: $p = 650 - 5x$; revenue: $R(x) = (650 - 5x)x$; 65 prints **14.** $x = 150$ memberships **15.** Let x be the number of tables and p the profit per table. $p = 16 - .5x$; profit from the café: $R = (16 - .5x)x$; 16 tables. **16.** Toll should be \$1.10. **17.** **(a)** $x = 15 \cdot 10^5$, $p = \$45$. **(b)** No. Profit is maximized when price is increased to \$50. **18.** **(a)** $x = 30$ **(b)** \$113 **(c)** $x = 30 - \frac{T}{4}$, $T = \$60/\text{unit}$ **19.** 5% **20.** **(a)** $P(0)$ is the profit with no advertising budget **(b)** As money is spent on advertising, the marginal profit initially increases. However, at some point the marginal profit begins to decrease. **(c)** Additional money spent on advertising is most advantageous at the inflection point. **21.** **(a)** \$75,000 **(b)** \$3200 per unit **(c)** 15 units **(d)** 32.5 units **(e)** 35 units **22.** **(a)** \$1,100 **(b)** \$12.5 per unit **(c)** 100 units **(d)** 20 units and 140 units **(e)** 80 units, \$5 per unit.

Chapter 2: Answers to Fundamental Concept Check Exercises, page 189

1. Increasing and decreasing functions, relative maximum and minimum points, absolute maximum and minimum points, concave up and concave down, inflection point, intercepts, asymptotes. **2.** A point is a relative maximum at $x = 2$ if the function attains a maximum at $x = 2$ relative to nearby points on the graph. The function has an absolute maximum at $x = 2$ if it attains its largest value at $x = 2$. **3.** The graph of $f(x)$ is concave up at $x = 2$ if the graph looks up as it goes through the point at $x = 2$. Equivalently, there is an open interval containing $x = 2$ throughout which the graph lies above its tangent line. Equivalently, the graph is concave up at $x = 2$ if the slope of the tangent line increases as we move from left to right through the point at $x = 2$. The graph of $f(x)$ is concave down at $x = 2$ if the graph looks down as it goes through the point at $x = 2$. Equivalently, there is an open interval containing $x = 2$ throughout which the graph lies below its tangent line. Equivalently, the graph is concave down at $x = 2$ if the slope of the tangent line decreases as we move from left to right through the point at $x = 2$. **4.** $f(x)$ has an inflection point at $x = 2$ if the concavity of the graph changes at the point $(2, f(2))$. **5.** The x-coordinate of the x-intercept is a zero of the function. **6.** To determine the y-intercept, set $x = 0$ and compute $f(0)$. **7.** If the graph of a function becomes closer and closer to a straight line, the straight line is an asymptote. For example, $y = 0$ is a horizontal asymptote of $y = \frac{1}{x}$. **8.** First-derivative rule: If $f'(a) > 0$, then f is increasing at $x = a$. If $f'(a) < 0$, then f is decreasing at $x = a$. Second-derivative rule: If $f''(a) > 0$, then f is concave up at $x = a$. If $f''(a) < 0$, then f is concave down at $x = a$ **9.** On an interval where $f'(x) > 0$, f is increasing. On an interval where $f'(x)$ is increasing, f is concave up. **10.** Solve $f'(x) = 0$. If $f'(a) = 0$ and $f'(x)$ changes sign from positive to negative as we move from left to right through $x = a$, then there is a local maximum at

$x = a$. If $f'(a) = 0$ and $f'(x)$ changes sign from negative to positive as we move from left to right through $x = a$, then there is a local minimum at $x = a$. **11.** Solve $f''(x) = 0$. If $f''(a) = 0$ and $f''(x)$ changes sign as we move from left to right through $x = a$, then there is an inflection point at $x = a$. **12.** See the summary of curve sketching at the end of Section 2.4. **13.** In an optimization problem, the quantity to be optimized is given by an objective equation. **14.** The equation that places a limit or a constraint on the variables in an optimization problem is a constraint equation. **15.** See the Suggestions for Solving an Optimization Problem at the end of Section 2.5. **16.** $P(x) = R(x) - C(x)$.

Chapter 2: Review Exercises, page 190

1. (a) increasing: $-3 < x < 1$, $x > 5$; decreasing: $x < -3$, $1 < x < 5$ **(b)** concave up: $x < -1$, $x > 3$; concave down: $-1 < x < 3$
2. (a) $f(3) = 2$ **(b)** $f'(3) = \frac{1}{2}$ **(c)** $f''(3) = 0$

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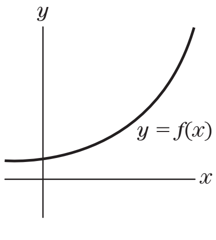

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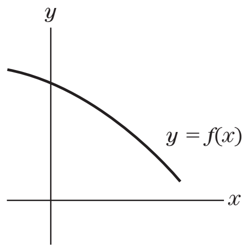

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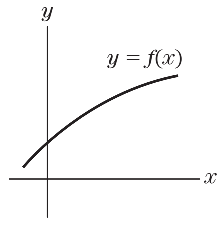

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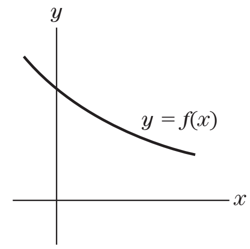

7. d, e **8.** b **9.** c, d **10.** a **11.** e **12.** b **13.** Graph goes through (1, 2), increasing at $x = 1$. **14.** Graph goes through (1, 5), decreasing at $x = 1$. **15.** Increasing and concave up at $x = 3$. **16.** Decreasing and concave down at $x = 2$. **17.** (10, 2) is a relative minimum point. **18.** Graph goes through (4, −2), increasing and concave down at $x = 4$. **19.** Graph goes through (5, −1), decreasing at $x = 5$. **20.** (0, 0) is a relative minimum. **21. (a)** after 2 hours **(b)** .8 **(c)** after 3 hours **(d)** −.02 unit per hour
22. (a) 400 trillion kilowatt-hours **(b)** 35 trillion kilowatt-hours per year **(c)** 1995 **(d)** 10 trillion kilowatt-hours per year in 1935 **(e)** 1600 trillion kilowatt-hours in 1970

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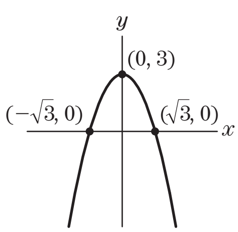

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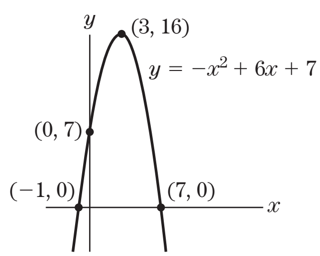

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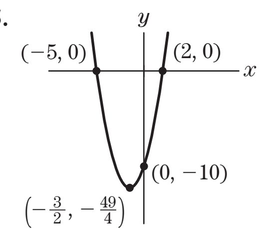

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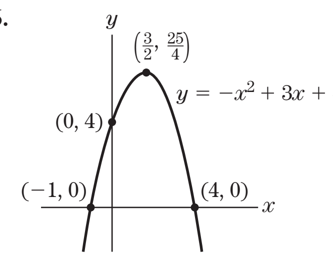

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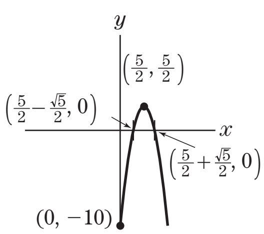

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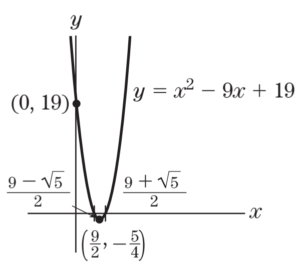

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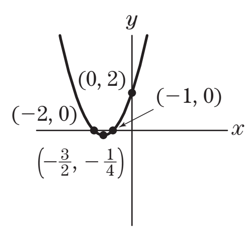

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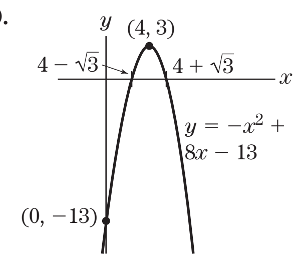

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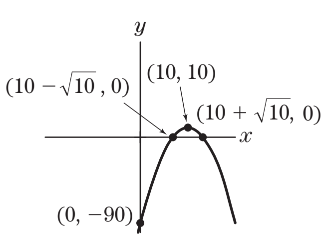

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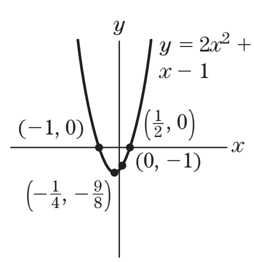

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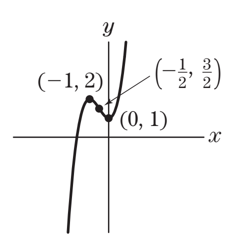

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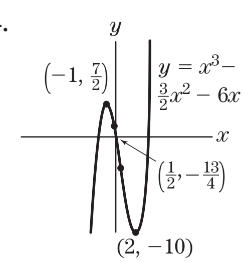

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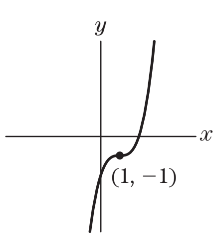

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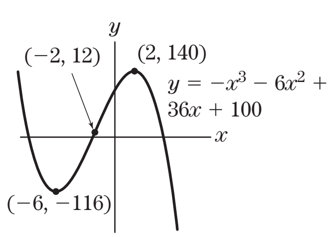

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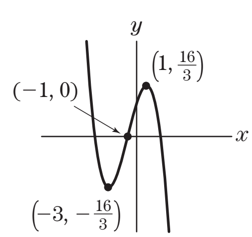

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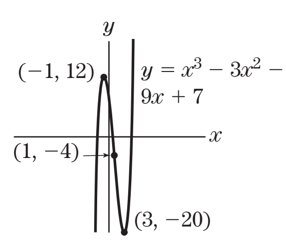

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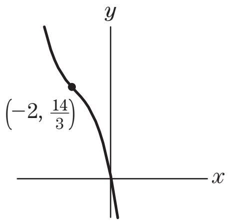

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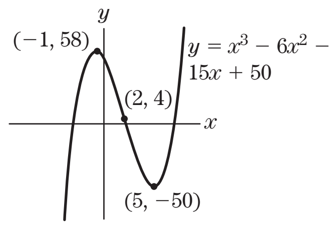

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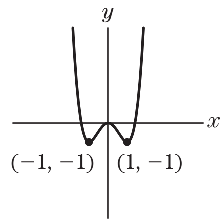

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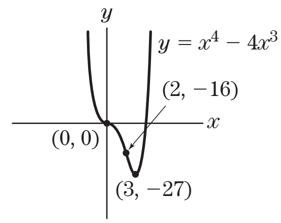

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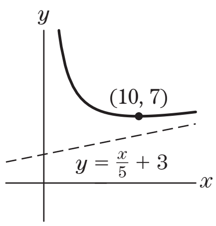

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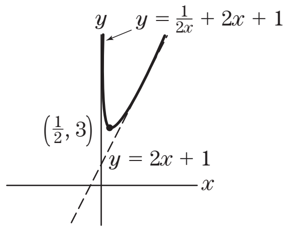

45. $f'(x) = 3x(x^2 + 2)^{1/2}$, $f'(0) = 0$ **46.** $f'(x) = 6x(2x^2 + 3)^{1/2}$, $f'(x) > 0$ if $x > 0$ and $f'(x) < 0$ if $x < 0$. **47.** $f''(x) = -2x(1 + x^2)^{-2}$, $f''(x)$ is positive for $x < 0$ and negative for $x > 0$. **48.** $f''(x) = \frac{1}{2}(5x^2 + 1)^{-1/2}(10x)$, so $f''(0) = 0$. Since $f'(x) > 0$ for all x, $f''(x)$ is positive for $x > 0$ and negative for $x < 0$, and it follows that 0 must be an inflection point. **49.** A–c, B–e, C–f, D–b, E–a, F–d **50.** A–c, B–e, C–f, D–b, E–a, F–d **51.** **(a)** the number of people living between $10 + h$ and 10 mi from the center of the city **(b)** If so, $f(x)$ would be decreasing at $x = 10$. **52.** $x = 8$ **53.** The endpoint maximum value of 2 occurs at $x = 0$. **54.** $g(3) = 0$ **55.** Let x be the width and h the height. Objective: $A = 4x + 2xh + 8h$; constraint: $4xh = 200$; 4 ft by 10 ft by 5 ft **56.** 2 ft × 2 ft × 4 ft **57.** $\frac{15}{2}$ in. **58.** 45 trees **59.** Let r be the number of production runs and x the number of books manufactured per run. Objective: $C = 1000r + (.25)x$; constraint: $rx = 400{,}000$; $x = 40{,}000$ **60.** $x = 3500$ **61.** A to P, $\frac{8}{3}$ miles from C **62.** Let x be the number of people and c the cost. Objective: $R = xc$; constraint: $c = 1040 - 20x$; 25 people.