

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Find the average velocity of the function over the given interval.

1)  $y = x^2 + 6x$ ,  $[6, 9]$  1) \_\_\_\_\_  
 A) 21 B) 15 C) 45 D) 7

2)  $y = 3x^3 - 8x^2 + 6$ ,  $[-8, 5]$  2) \_\_\_\_\_  
 A)  $\frac{181}{13}$  B) 171 C)  $\frac{2223}{5}$  D)  $\frac{181}{5}$

3)  $y = \sqrt{2x}$ ,  $[2, 8]$  3) \_\_\_\_\_  
 A)  $\frac{1}{3}$  B) 7 C) 2 D)  $-\frac{3}{10}$

4)  $y = \frac{3}{x-2}$ ,  $[4, 7]$  4) \_\_\_\_\_  
 A)  $\frac{1}{3}$  B)  $-\frac{3}{10}$  C) 2 D) 7

5)  $y = 4x^2$ ,  $\left[0, \frac{7}{4}\right]$  5) \_\_\_\_\_  
 A) 2 B)  $\frac{1}{3}$  C)  $-\frac{3}{10}$  D) 7

6)  $y = -3x^2 - x$ ,  $[5, 6]$  6) \_\_\_\_\_  
 A) -34 B)  $-\frac{1}{6}$  C) -2 D)  $\frac{1}{2}$

7)  $h(t) = \sin(4t)$ ,  $\left[0, \frac{\pi}{8}\right]$  7) \_\_\_\_\_  
 A)  $\frac{8}{\pi}$  B)  $-\frac{8}{\pi}$  C)  $\frac{\pi}{8}$  D)  $\frac{4}{\pi}$

8)  $g(t) = 3 + \tan t$ ,  $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$  8) \_\_\_\_\_  
 A)  $\frac{4}{\pi}$  B)  $-\frac{8}{5}$  C) 0 D)  $-\frac{4}{\pi}$

Use the table to find the instantaneous velocity of  $y$  at the specified value of  $x$ .

9)  $x = 1$ .

9) \_\_\_\_\_

$x$	$y$
0	0
0.2	0.02
0.4	0.08
0.6	0.18
0.8	0.32
1.0	0.5
1.2	0.72
1.4	0.98

A) 2

B) 0.5

C) 1

D) 1.5

10)  $x = 1$ .

10) \_\_\_\_\_

$x$	$y$
0	0
0.2	0.01
0.4	0.04
0.6	0.09
0.8	0.16
1.0	0.25
1.2	0.36
1.4	0.49

A) 1

B) 0.5

C) 1.5

D) 2

11)  $x = 1$ .

11) \_\_\_\_\_

$x$	$y$
0	0
0.2	0.12
0.4	0.48
0.6	1.08
0.8	1.92
1.0	3
1.2	4.32
1.4	5.88

A) 4

B) 2

C) 6

D) 8

12)  $x = 2$ .

12) \_\_\_\_\_

x	y
0	10
0.5	38
1.0	58
1.5	70
2.0	74
2.5	70
3.0	58
3.5	38
4.0	10

A) 4

B) 8

C) 0

D) -8

13)  $x = 1$ .

13) \_\_\_\_\_

x	y
0.900	-0.05263
0.990	-0.00503
0.999	-0.0005
1.000	0.0000
1.001	0.0005
1.010	0.00498
1.100	0.04762

A) 0

B) -0.5

C) 1

D) 0.5

**Find the slope of the curve for the given value of x.**

14)  $y = x^2 + 5x, x = 4$

14) \_\_\_\_\_

A) slope is  $-\frac{4}{25}$

B) slope is 13

C) slope is -39

D) slope is  $\frac{1}{20}$

15)  $y = x^2 + 11x - 15, x = 1$

15) \_\_\_\_\_

A) slope is  $-\frac{4}{25}$

B) slope is  $\frac{1}{20}$

C) slope is 13

D) slope is -39

16)  $y = x^3 - 5x, x = 1$

16) \_\_\_\_\_

A) slope is -3

B) slope is 1

C) slope is 3

D) slope is -2

17)  $y = x^3 - 3x^2 + 4, x = 1$

17) \_\_\_\_\_

A) slope is 0

B) slope is -3

C) slope is -3

D) slope is 1

18)  $y = 2 - x^3, x = 1$

18) \_\_\_\_\_

A) slope is 0

B) slope is -3

C) slope is -1

D) slope is 3

**Solve the problem.**

19) Given  $\lim_{x \rightarrow 0^-} f(x) = L_L$ ,  $\lim_{x \rightarrow 0^+} f(x) = L_R$ , and  $L_L \neq L_R$ , which of the following statements is true? 19) \_\_\_\_\_

- I.  $\lim_{x \rightarrow 0} f(x) = L_L$
- II.  $\lim_{x \rightarrow 0} f(x) = L_R$
- III.  $\lim_{x \rightarrow 0} f(x)$  does not exist.

A) I                                      B) none                                      C) II                                      D) III

20) Given  $\lim_{x \rightarrow 0^-} f(x) = L_L$ ,  $\lim_{x \rightarrow 0^+} f(x) = L_R$ , and  $L_L = L_R$ , which of the following statements is false? 20) \_\_\_\_\_

- I.  $\lim_{x \rightarrow 0} f(x) = L_L$
- II.  $\lim_{x \rightarrow 0} f(x) = L_R$
- III.  $\lim_{x \rightarrow 0} f(x)$  does not exist.

A) I                                      B) II                                      C) III                                      D) none

21) If  $\lim_{x \rightarrow 0} f(x) = L$ , which of the following expressions are true? 21) \_\_\_\_\_

- I.  $\lim_{x \rightarrow 0^-} f(x)$  does not exist.
- II.  $\lim_{x \rightarrow 0^+} f(x)$  does not exist.
- III.  $\lim_{x \rightarrow 0^-} f(x) = L$
- IV.  $\lim_{x \rightarrow 0^+} f(x) = L$

A) II and III only                      B) III and IV only                      C) I and II only                      D) I and IV only

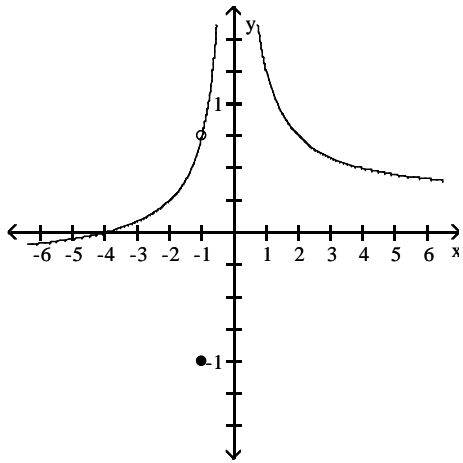
22) What conditions, when present, are sufficient to conclude that a function  $f(x)$  has a limit as  $x$  approaches some value of  $a$ ? 22) \_\_\_\_\_

- A) Either the limit of  $f(x)$  as  $x \rightarrow a$  from the left exists or the limit of  $f(x)$  as  $x \rightarrow a$  from the right exists
- B) The limit of  $f(x)$  as  $x \rightarrow a$  from the left exists, the limit of  $f(x)$  as  $x \rightarrow a$  from the right exists, and at least one of these limits is the same as  $f(a)$ .
- C)  $f(a)$  exists, the limit of  $f(x)$  as  $x \rightarrow a$  from the left exists, and the limit of  $f(x)$  as  $x \rightarrow a$  from the right exists.
- D) The limit of  $f(x)$  as  $x \rightarrow a$  from the left exists, the limit of  $f(x)$  as  $x \rightarrow a$  from the right exists, and these two limits are the same.

Use the graph to evaluate the limit.

23)  $\lim_{x \rightarrow -1} f(x)$

23) \_\_\_\_\_



A) -1

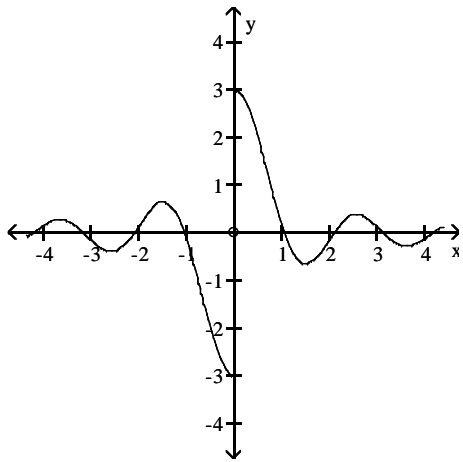
B)  $-\frac{3}{4}$

C)  $\infty$

D)  $\frac{3}{4}$

24)  $\lim_{x \rightarrow 0} f(x)$

24) \_\_\_\_\_



A) does not exist

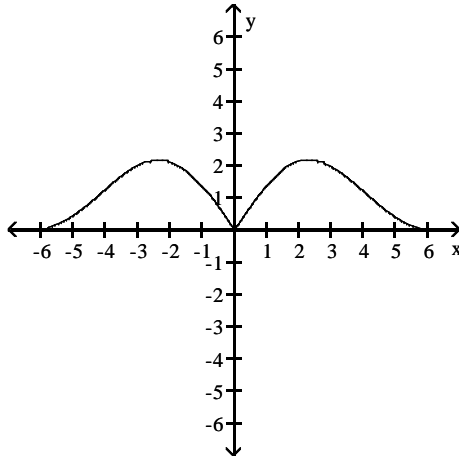
B) 3

C) -3

D) 0

25)  $\lim_{x \rightarrow 0} f(x)$

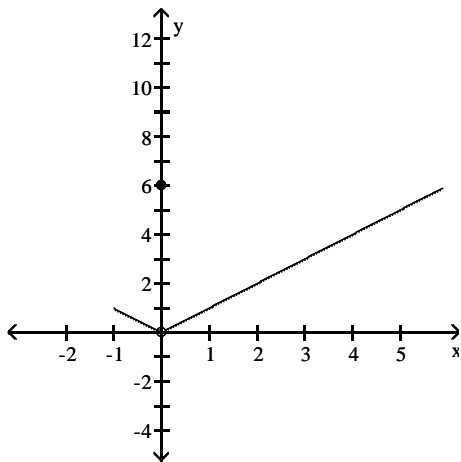
25) \_\_\_\_\_



- A) does not exist      B) 3      C) 0      D) -3

26)  $\lim_{x \rightarrow 0} f(x)$

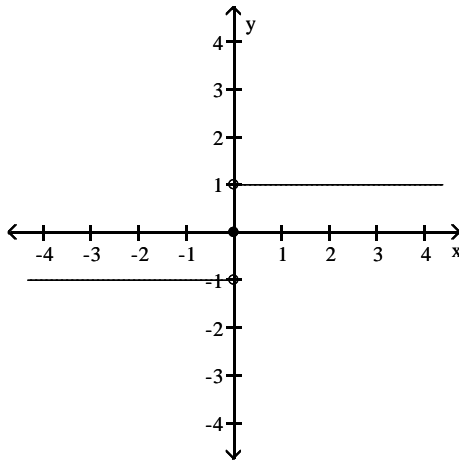
26) \_\_\_\_\_



- A) does not exist      B) 0      C) -1      D) 6

27)  $\lim_{x \rightarrow 0} f(x)$

27) \_\_\_\_\_



A) 1

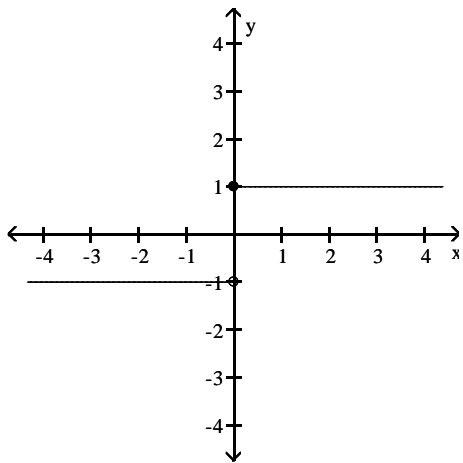
B) does not exist

C) -1

D)  $\infty$

28)  $\lim_{x \rightarrow 0} f(x)$

28) \_\_\_\_\_



A) does not exist

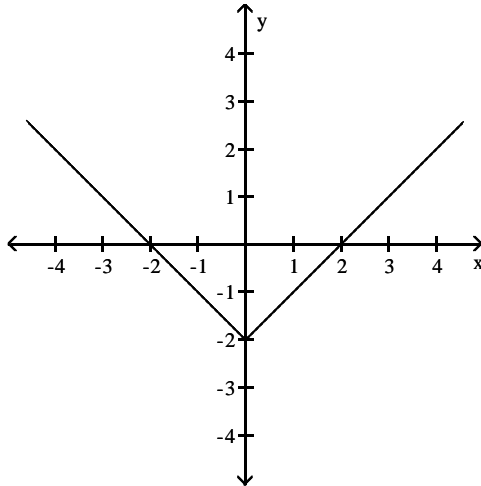
B) -1

C)  $\infty$

D) 1

29)  $\lim_{x \rightarrow 0} f(x)$

29) \_\_\_\_\_



A) 2

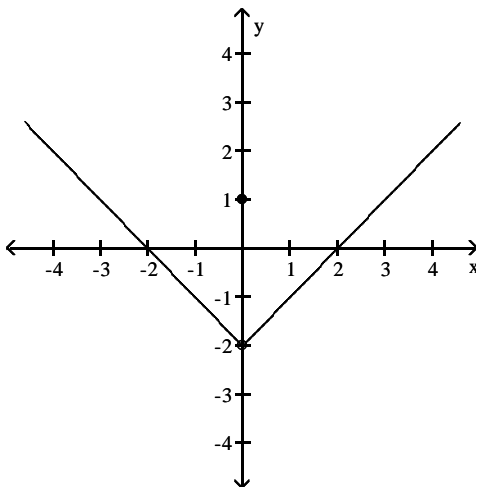
B) 0

C) does not exist

D) -2

30)  $\lim_{x \rightarrow 0} f(x)$

30) \_\_\_\_\_



A) 0

B) does not exist

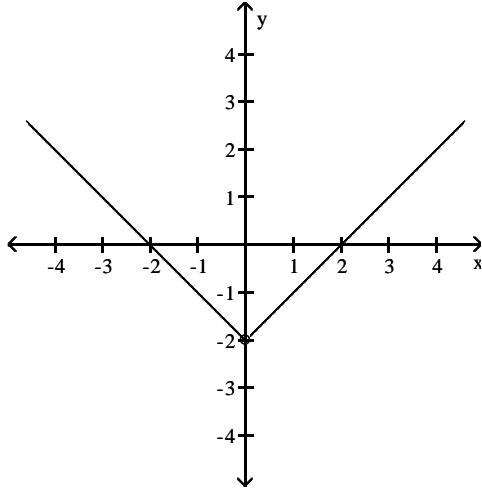
C) 1

D) -2



31)  $\lim_{x \rightarrow 0} f(x)$

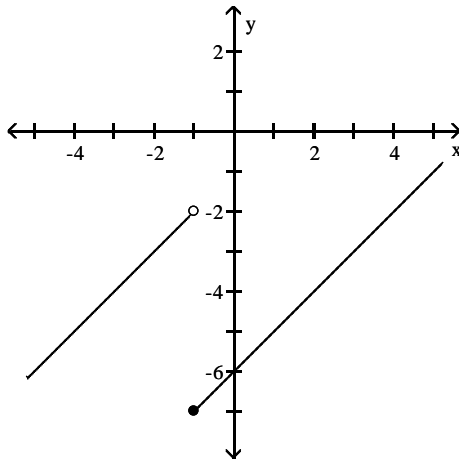
31) \_\_\_\_\_



- A) -2                      B) does not exist                      C) 2                      D) -1

32) Find  $\lim_{x \rightarrow (-1)^-} f(x)$  and  $\lim_{x \rightarrow (-1)^+} f(x)$

32) \_\_\_\_\_



- A) -7; -2                      B) -2; -7                      C) -7; -5                      D) -5; -2

Use the table of values of  $f$  to estimate the limit.

33) Let  $f(x) = x^2 + 8x - 2$ , find  $\lim_{x \rightarrow 2} f(x)$ .

33) \_\_\_\_\_

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

A)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	5.043	5.364	5.396	5.404	5.436	5.763

; limit =  $\infty$

B)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	5.043	5.364	5.396	5.404	5.436	5.763

; limit = 5.40

C)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	16.810	17.880	17.988	18.012	18.120	19.210

; limit = 18.0

D)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	16.692	17.592	17.689	17.710	17.808	18.789

; limit = 17.70

34) Let  $f(x) = \frac{x-4}{\sqrt{x}-2}$ , find  $\lim_{x \rightarrow 4} f(x)$ .

34) \_\_\_\_\_

$x$	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

A)

$x$	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	5.07736	5.09775	5.09978	5.10022	5.10225	5.12236

; limit = 5.10

B)

$x$	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit = 1.20

C)

$x$	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	3.97484	3.99750	3.99975	4.00025	4.00250	4.02485

; limit = 4.0

D)

$x$	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit =  $\infty$

35) Let  $f(x) = x^2 - 5$ , find  $\lim_{x \rightarrow 0} f(x)$ .

35) \_\_\_\_\_

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

A)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = -15.0

B)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit =  $\infty$

C)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-2.9910	-2.9999	-3.0000	-3.0000	-2.9999	-2.9910

; limit = -3.0

D)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-4.9900	-4.9999	-5.0000	-5.0000	-4.9999	-4.9900

; limit = -5.0

36) Let  $f(x) = \frac{x - 3}{x^2 + 2x - 15}$ , find  $\lim_{x \rightarrow 3} f(x)$ .

36) \_\_\_\_\_

x	2.9	2.99	2.999	3.001	3.01	3.1
f(x)						

A)

x	2.9	2.99	2.999	3.001	3.01	3.1
f(x)	0.1266	0.1252	0.1250	0.1250	0.1248	0.1235

; limit = 0.125

B)

x	2.9	2.99	2.999	3.001	3.01	3.1
f(x)	-0.1266	-0.1252	-0.1250	-0.1250	-0.1248	-0.1235

; limit = -0.125

C)

x	2.9	2.99	2.999	3.001	3.01	3.1
f(x)	0.0266	0.0252	0.0250	0.0250	0.0248	0.0235

; limit = 0.025

D)

x	2.9	2.99	2.999	3.001	3.01	3.1
f(x)	0.2266	0.2252	0.2250	0.2250	0.2248	0.2235

; limit = 0.225

37) Let  $f(x) = \frac{x^2 - 5x + 4}{x^2 - 6x + 5}$ , find  $\lim_{x \rightarrow 1} f(x)$ .

37) \_\_\_\_\_

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)						

A)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.7561	0.7506	0.7501	0.7499	0.7494	0.7436

; limit = 0.75

B)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.6561	0.6506	0.6501	0.6499	0.6494	0.6436

; limit = 0.65

C)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.8361	0.8336	0.8334	0.8333	0.8331	0.8305

; limit = 0.8333

D)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.8561	0.8506	0.8501	0.8499	0.8494	0.8436

; limit = 0.85

38) Let  $f(x) = \frac{\sin(6x)}{x}$ , find  $\lim_{x \rightarrow 0} f(x)$ .

38) \_\_\_\_\_

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)		5.99640065			5.99640065	

A) limit = 6

B) limit does not exist

C) limit = 5.5

D) limit = 0

39) Let  $f(\theta) = \frac{\cos(6\theta)}{\theta}$ , find  $\lim_{\theta \rightarrow 0} f(\theta)$ .

39) \_\_\_\_\_

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(θ)	-8.2533561					8.2533561

A) limit does not exist

B) limit = 6

C) limit = 0

D) limit = 8.2533561

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

**Provide an appropriate response.**

40) It can be shown that the inequalities  $1 - \frac{x^2}{6} < \frac{x \sin(x)}{2 - 2 \cos(x)} < 1$  hold for all values of x close to zero. What, if anything, does this tell you about  $\frac{x \sin(x)}{2 - 2 \cos(x)}$ ? Explain. 40) \_\_\_\_\_

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

41) Write the formal notation for the principle "the limit of a quotient is the quotient of the limits" and include a statement of any restrictions on the principle. 41) \_\_\_\_\_

A) If  $\lim_{x \rightarrow a} g(x) = M$  and  $\lim_{x \rightarrow a} f(x) = L$ , then  $\lim_{x \rightarrow a} \frac{g(x)}{f(x)} = \frac{\lim_{x \rightarrow a} g(x)}{\lim_{x \rightarrow a} f(x)} = \frac{M}{L}$ , provided that

$f(a) \neq 0$ .

B) If  $\lim_{x \rightarrow a} g(x) = M$  and  $\lim_{x \rightarrow a} f(x) = L$ , then  $\lim_{x \rightarrow a} \frac{g(x)}{f(x)} = \frac{\lim_{x \rightarrow a} g(x)}{\lim_{x \rightarrow a} f(x)} = \frac{M}{L}$ , provided that

$L \neq 0$ .

C)  $\lim_{x \rightarrow a} \frac{g(x)}{f(x)} = \frac{g(a)}{f(a)}$ .

D)  $\lim_{x \rightarrow a} \frac{g(x)}{f(x)} = \frac{g(a)}{f(a)}$ , provided that  $f(a) \neq 0$ .

42) Provide a short sentence that summarizes the general limit principle given by the formal notation  $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L \pm M$ , given that  $\lim_{x \rightarrow a} f(x) = L$  and  $\lim_{x \rightarrow a} g(x) = M$ . 42) \_\_\_\_\_

A) The sum or the difference of two functions is the sum of two limits.

B) The limit of a sum or a difference is the sum or the difference of the functions.

C) The limit of a sum or a difference is the sum or the difference of the limits.

D) The sum or the difference of two functions is continuous.

43) The statement "the limit of a constant times a function is the constant times the limit" follows from a combination of two fundamental limit principles. What are they? 43) \_\_\_\_\_

A) The limit of a product is the product of the limits, and a constant is continuous.

B) The limit of a product is the product of the limits, and the limit of a quotient is the quotient of the limits.

C) The limit of a constant is the constant, and the limit of a product is the product of the limits.

D) The limit of a function is a constant times a limit, and the limit of a constant is the constant.

**Find the limit.**

44)  $\lim_{x \rightarrow 7} \sqrt{10}$  44) \_\_\_\_\_

A) 7

B)  $\sqrt{7}$

C)  $\sqrt{10}$

D) 10

45)  $\lim_{x \rightarrow 1} (9x - 4)$  45) \_\_\_\_\_

A) -5

B) -13

C) 5

D) 13

46)  $\lim_{x \rightarrow -18} (15 - 2x)$  46) \_\_\_\_\_

A) 21

B) -21

C) 51

D) -51

**Give an appropriate answer.**

47) Let  $\lim_{x \rightarrow 9} f(x) = 5$  and  $\lim_{x \rightarrow 9} g(x) = -8$ . Find  $\lim_{x \rightarrow 9} [f(x) - g(x)]$ . 47) \_\_\_\_\_

- A) 9                                      B) 13                                      C) 5                                      D) -3

48) Let  $\lim_{x \rightarrow -4} f(x) = -9$  and  $\lim_{x \rightarrow -4} g(x) = 4$ . Find  $\lim_{x \rightarrow -4} [f(x) \cdot g(x)]$ . 48) \_\_\_\_\_

- A) -5                                      B) -4                                      C) -36                                      D) 4

49) Let  $\lim_{x \rightarrow -9} f(x) = 9$  and  $\lim_{x \rightarrow -9} g(x) = -4$ . Find  $\lim_{x \rightarrow -9} \frac{f(x)}{g(x)}$ . 49) \_\_\_\_\_

- A)  $-\frac{4}{9}$                                       B) -9                                      C) 13                                      D)  $-\frac{9}{4}$

50) Let  $\lim_{x \rightarrow -2} f(x) = 16$ . Find  $\lim_{x \rightarrow -2} \sqrt{f(x)}$ . 50) \_\_\_\_\_

- A) 2.0000                                      B) -2                                      C) 4                                      D) 16

51) Let  $\lim_{x \rightarrow 1} f(x) = 1$  and  $\lim_{x \rightarrow 1} g(x) = -7$ . Find  $\lim_{x \rightarrow 1} [f(x) + g(x)]^2$ . 51) \_\_\_\_\_

- A) 50                                      B) -6                                      C) 8                                      D) 36

52) Let  $\lim_{x \rightarrow 6} f(x) = 243$ . Find  $\lim_{x \rightarrow 6} \sqrt[5]{f(x)}$ . 52) \_\_\_\_\_

- A) 3                                      B) 6                                      C) 5                                      D) 243

53) Let  $\lim_{x \rightarrow 1} f(x) = -8$  and  $\lim_{x \rightarrow 1} g(x) = 6$ . Find  $\lim_{x \rightarrow 1} \left[ \frac{6f(x) - 10g(x)}{-9 + g(x)} \right]$ . 53) \_\_\_\_\_

- A)  $-\frac{14}{3}$                                       B) 1                                      C) 36                                      D) -4

**Find the limit.**

54)  $\lim_{x \rightarrow 2} (x^3 + 5x^2 - 7x + 1)$  54) \_\_\_\_\_

- A) 0                                      B) does not exist                                      C) 29                                      D) 15

55)  $\lim_{x \rightarrow -2} (3x^5 - 3x^4 - 4x^3 + x^2 + 5)$  55) \_\_\_\_\_

- A) -7                                      B) -167                                      C) 41                                      D) -103

56)  $\lim_{x \rightarrow -1} \frac{x}{3x + 2}$  56) \_\_\_\_\_

- A) 1                                      B)  $-\frac{1}{5}$                                       C) 0                                      D) does not exist

57)  $\lim_{x \rightarrow 0} \frac{x^3 - 6x + 8}{x - 2}$  57) \_\_\_\_\_  
 A) -4                      B) 0                      C) Does not exist                      D) 4

58)  $\lim_{x \rightarrow 1} \frac{3x^2 + 7x - 2}{3x^2 - 4x - 2}$  58) \_\_\_\_\_  
 A) Does not exist                      B)  $-\frac{7}{4}$                       C) 0                      D)  $-\frac{8}{3}$

59)  $\lim_{x \rightarrow -2} (x + 3)^2(x - 1)^3$  59) \_\_\_\_\_  
 A) -675                      B) -1                      C) -27                      D) -25

60)  $\lim_{x \rightarrow 7} \sqrt{x^2 + 4x + 4}$  60) \_\_\_\_\_  
 A) 81                      B)  $\pm 9$                       C) 9                      D) does not exist

61)  $\lim_{x \rightarrow 5} \sqrt{7x + 51}$  61) \_\_\_\_\_  
 A) -86                      B)  $\sqrt{86}$                       C)  $-\sqrt{86}$                       D) 86

62)  $\lim_{h \rightarrow 0} \frac{2}{\sqrt{3h + 4} + 2}$  62) \_\_\_\_\_  
 A) 1                      B) 2                      C) 1/2                      D) Does not exist

63)  $\lim_{x \rightarrow 0} \frac{\sqrt{1 + x} - 1}{x}$  63) \_\_\_\_\_  
 A) 1/2                      B) Does not exist                      C) 1/4                      D) 0

**Determine the limit by sketching an appropriate graph.**

64)  $\lim_{x \rightarrow 1^-} f(x)$ , where  $f(x) = \begin{cases} -3x + 4 & \text{for } x < 1 \\ 5x + 5 & \text{for } x \geq 1 \end{cases}$  64) \_\_\_\_\_  
 A) 1                      B) 6                      C) 5                      D) 10

65)  $\lim_{x \rightarrow 6^+} f(x)$ , where  $f(x) = \begin{cases} -2x + 2 & \text{for } x < 6 \\ 4x + 3 & \text{for } x \geq 6 \end{cases}$  65) \_\_\_\_\_  
 A) -10                      B) 4                      C) 27                      D) 3

66)  $\lim_{x \rightarrow 4^+} f(x)$ , where  $f(x) = \begin{cases} x^2 + 4 & \text{for } x \neq 4 \\ 0 & \text{for } x = 4 \end{cases}$  66) \_\_\_\_\_  
 A) 0                      B) 12                      C) 16                      D) 20





76)  $\lim_{x \rightarrow 6} \frac{x^2 - 9x + 18}{x^2 - 3x - 18}$  76) \_\_\_\_\_  
 A) 1                      B)  $-\frac{1}{3}$                       C)  $\frac{1}{3}$                       D) Does not exist

77)  $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$  77) \_\_\_\_\_  
 A) 0                      B) Does not exist                      C)  $3x^2 + 3xh + h^2$                       D)  $3x^2$

78)  $\lim_{x \rightarrow 10} \frac{|10-x|}{10-x}$  78) \_\_\_\_\_  
 A) Does not exist                      B) 0                      C) -1                      D) 1

**Provide an appropriate response.**

79) It can be shown that the inequalities  $-x \leq x \cos\left(\frac{1}{x}\right) \leq x$  hold for all values of  $x \geq 0$ . 79) \_\_\_\_\_

Find  $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right)$  if it exists.

- A) 0.0007                      B) 0                      C) does not exist                      D) 1

80) The inequality  $1 - \frac{x^2}{2} < \frac{\sin x}{x} < 1$  holds when  $x$  is measured in radians and  $|x| < 1$ . 80) \_\_\_\_\_

Find  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$  if it exists.

- A) 1                      B) 0.0007                      C) does not exist                      D) 0

81) If  $x^3 \leq f(x) \leq x$  for  $x$  in  $[-1,1]$ , find  $\lim_{x \rightarrow 0} f(x)$  if it exists. 81) \_\_\_\_\_

- A) -1                      B) 1                      C) does not exist                      D) 0

Compute the values of  $f(x)$  and use them to determine the indicated limit.

82) If  $f(x) = x^2 + 8x - 2$ , find  $\lim_{x \rightarrow 2} f(x)$ .

82) \_\_\_\_\_

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

A)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	5.043	5.364	5.396	5.404	5.436	5.763

; limit =  $\infty$

B)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	16.810	17.880	17.988	18.012	18.120	19.210

; limit = 18.0

C)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	16.692	17.592	17.689	17.710	17.808	18.789

; limit = 17.70

D)

$x$	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	5.043	5.364	5.396	5.404	5.436	5.763

; limit = 5.40

83) If  $f(x) = \frac{x^4 - 1}{x - 1}$ , find  $\lim_{x \rightarrow 1} f(x)$ .

83) \_\_\_\_\_

$x$	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$						

A)

$x$	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	3.439	3.940	3.994	4.006	4.060	4.641

; limit = 4.0

B)

$x$	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	4.595	5.046	5.095	5.105	5.154	5.677

; limit = 5.10

C)

$x$	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	1.032	1.182	1.198	1.201	1.218	1.392

; limit = 1.210

D)

$x$	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	1.032	1.182	1.198	1.201	1.218	1.392

; limit =  $\infty$

84) If  $f(x) = \frac{x^3 - 6x + 8}{x - 2}$ , find  $\lim_{x \rightarrow 0} f(x)$ .

84) \_\_\_\_\_

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

A)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.22843	-1.20298	-1.20030	-1.19970	-1.19699	-1.16858

; limit = -1.20

B)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.22843	-1.20298	-1.20030	-1.19970	-1.19699	-1.16858

; limit =  $\infty$

C)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-2.18529	-2.10895	-2.10090	-2.99910	-2.09096	-2.00574

; limit = -2.10

D)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-4.09476	-4.00995	-4.00100	-3.99900	-3.98995	-3.89526

; limit = -4.0

85) If  $f(x) = \frac{x - 4}{\sqrt{x} - 2}$ , find  $\lim_{x \rightarrow 4} f(x)$ .

85) \_\_\_\_\_

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

A)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	5.07736	5.09775	5.09978	5.10022	5.10225	5.12236

; limit = 5.10

B)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit =  $\infty$

C)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	3.97484	3.99750	3.99975	4.00025	4.00250	4.02485

; limit = 4.0

D)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit = 1.20

86) If  $f(x) = x^2 - 5$ , find  $\lim_{x \rightarrow 0} f(x)$ .

86) \_\_\_\_\_

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

A)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = -15.0

B)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-4.9900	-4.9999	-5.0000	-5.0000	-4.9999	-4.9900

; limit = -5.0

C)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit =  $\infty$

D)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-2.9910	-2.9999	-3.0000	-3.0000	-2.9999	-2.9910

; limit = -3.0

87) If  $f(x) = \frac{\sqrt{x+1}}{x+1}$ , find  $\lim_{x \rightarrow 1} f(x)$ .

87) \_\_\_\_\_

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)						

A)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.21764	0.21266	0.21219	0.21208	0.21160	0.20702

; limit = 0.21213

B)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.72548	0.70888	0.70728	0.70693	0.70535	0.69007

; limit = 0.7071

C)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	2.15293	2.13799	2.13656	2.13624	2.13481	2.12106

; limit = 2.13640

D)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.21764	0.21266	0.21219	0.21208	0.21160	0.20702

; limit =  $\infty$

88) If  $f(x) = \sqrt{x} - 2$ , find  $\lim_{x \rightarrow 4} f(x)$ .

88) \_\_\_\_\_

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

A)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	3.9000	2.9000	1.9000	2.0000	3.0000	4.0000

; limit =  $\infty$

B)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	-0.02516	-0.00250	-0.00025	0.00025	0.00250	0.02485

; limit = 0

C)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.47736	1.49775	1.49977	1.50022	1.50225	1.52236

; limit = 1.50

D)

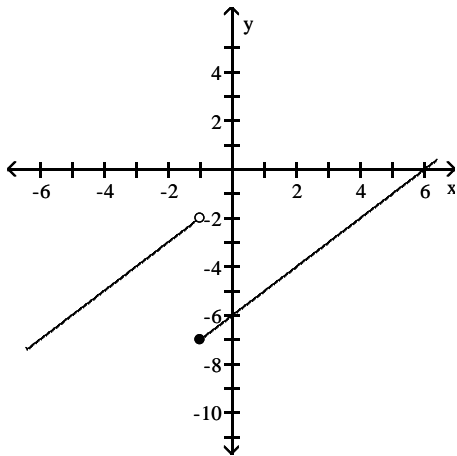
x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	3.9000	2.9000	1.9000	2.0000	3.0000	4.0000

; limit = 1.95

**For the function f whose graph is given, determine the limit.**

89) Find  $\lim_{x \rightarrow -1^-} f(x)$  and  $\lim_{x \rightarrow -1^+} f(x)$ .

89) \_\_\_\_\_



A) -5; -2

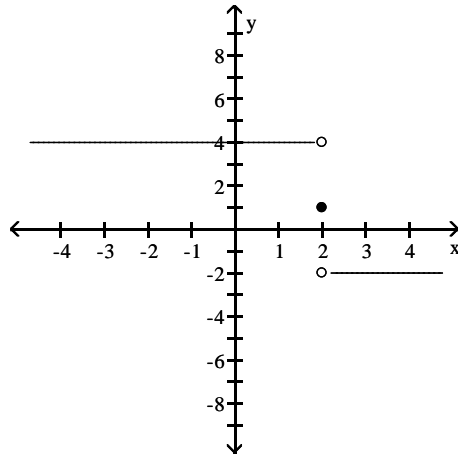
B) -7; -2

C) -7; -5

D) -2; -7

90) Find  $\lim_{x \rightarrow 2^-} f(x)$  and  $\lim_{x \rightarrow 2^+} f(x)$ .

90) \_\_\_\_\_

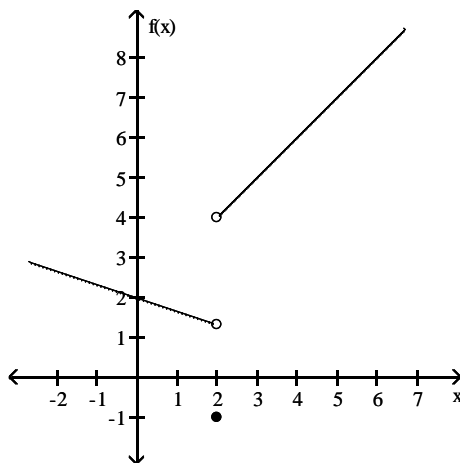


- A) 1; 1
- C) -2; 4

- B) does not exist; does not exist
- D) 4; -2

91) Find  $\lim_{x \rightarrow 2^-} f(x)$ .

91) \_\_\_\_\_



A) -1

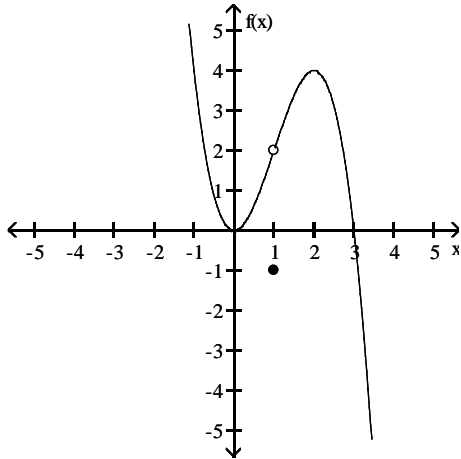
B) 2.3

C) 1.3

D) 4

92) Find  $\lim_{x \rightarrow 1^-} f(x)$ .

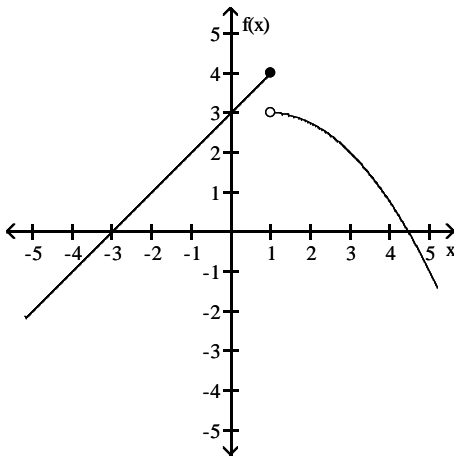
92) \_\_\_\_\_



- A)  $\frac{1}{2}$       B) does not exist      C) 2      D) -1

93) Find  $\lim_{x \rightarrow 1^+} f(x)$ .

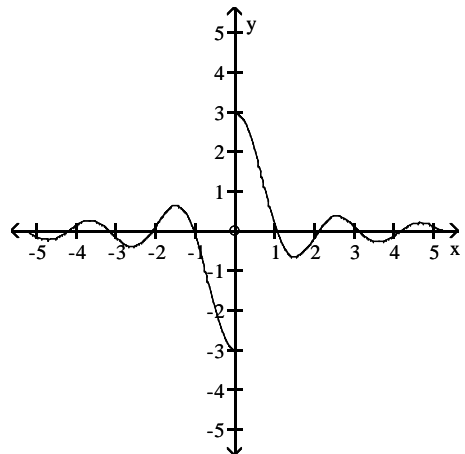
93) \_\_\_\_\_



- A) 3      B)  $3\frac{1}{2}$       C) 4      D) does not exist

94) Find  $\lim_{x \rightarrow 0} f(x)$ .

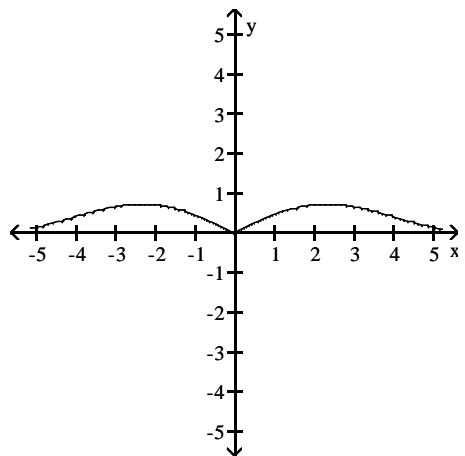
94) \_\_\_\_\_



- A) -3                      B) does not exist                      C) 0                      D) 3

95) Find  $\lim_{x \rightarrow 0} f(x)$ .

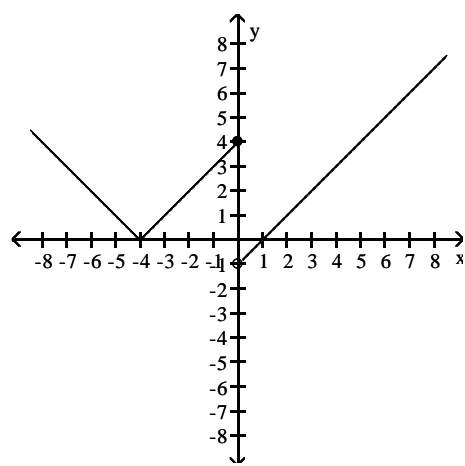
95) \_\_\_\_\_



- A) -1                      B) 0                      C) does not exist                      D) 1

96) Find  $\lim_{x \rightarrow 0} f(x)$ .

96) \_\_\_\_\_

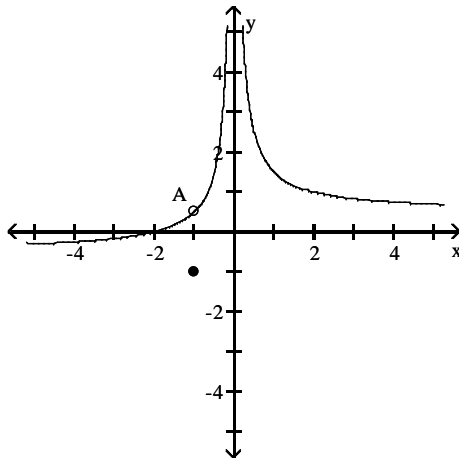


- A) -4                      B) 0                      C) 4                      D) does not exist



97) Find  $\lim_{x \rightarrow -1} f(x)$ .

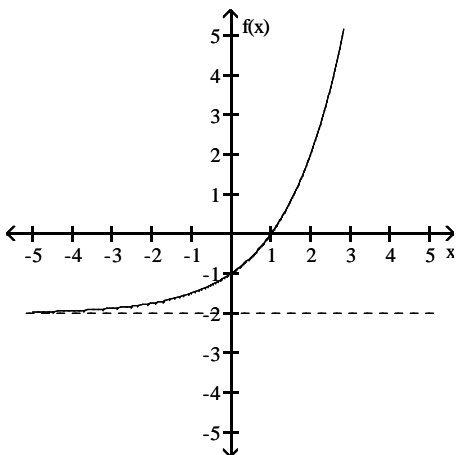
97) \_\_\_\_\_



- A)  $\frac{1}{2}$                       B)  $-\frac{1}{2}$                       C) -1                      D) does not exist

98) Find  $\lim_{x \rightarrow -\infty} f(x)$ .

98) \_\_\_\_\_



- A) 0                      B) does not exist                      C) -2                      D)  $-\infty$

**Find the limit.**

99)  $\lim_{x \rightarrow -2} \frac{1}{x+2}$

99) \_\_\_\_\_

- A) Does not exist                      B)  $-\infty$                       C)  $\infty$                       D)  $1/2$

100)  $\lim_{x \rightarrow -4^-} \frac{1}{x+4}$

100) \_\_\_\_\_

- A)  $-\infty$                       B) -1                      C) 0                      D)  $\infty$

101)  $\lim_{x \rightarrow 10^+} \frac{1}{(x-10)^2}$

101) \_\_\_\_\_

- A) 0                      B)  $\infty$                       C)  $-\infty$                       D) -1

102)  $\lim_{x \rightarrow -2^-} \frac{6}{x^2 - 4}$  102) \_\_\_\_\_  
 A)  $-\infty$  B)  $\infty$  C) 0 D) -1

103)  $\lim_{x \rightarrow 3^+} \frac{1}{x^2 - 9}$  103) \_\_\_\_\_  
 A) 0 B)  $-\infty$  C) 1 D)  $\infty$

104)  $\lim_{x \rightarrow (\pi/2)^+} \tan x$  104) \_\_\_\_\_  
 A)  $\infty$  B) 0 C)  $-\infty$  D) 1

105)  $\lim_{x \rightarrow (-\pi/2)^-} \sec x$  105) \_\_\_\_\_  
 A)  $-\infty$  B) 0 C) 1 D)  $\infty$

106)  $\lim_{x \rightarrow 0^+} (1 + \csc x)$  106) \_\_\_\_\_  
 A) 1 B)  $\infty$  C) 0 D) Does not exist

107)  $\lim_{x \rightarrow 0} (1 - \cot x)$  107) \_\_\_\_\_  
 A)  $-\infty$  B) 0 C)  $\infty$  D) Does not exist

108)  $\lim_{x \rightarrow -3^-} \frac{x^2 - 7x + 12}{x^3 - 9x}$  108) \_\_\_\_\_  
 A) 0 B)  $\infty$  C)  $-\infty$  D) Does not exist

109)  $\lim_{x \rightarrow 3^+} \frac{x^2 - 4x + 3}{x^3 - x}$  109) \_\_\_\_\_  
 A)  $-\infty$  B) 0 C) Does not exist D)  $\infty$

**Find all vertical asymptotes of the given function.**

110)  $g(x) = \frac{4x}{x - 6}$  110) \_\_\_\_\_  
 A) none B)  $x = 4$  C)  $x = -6$  D)  $x = 6$

111)  $f(x) = \frac{x + 9}{x^2 - 49}$  111) \_\_\_\_\_  
 A)  $x = 49, x = -9$  B)  $x = -7, x = 7$   
 C)  $x = 0, x = 49$  D)  $x = -7, x = 7, x = -9$

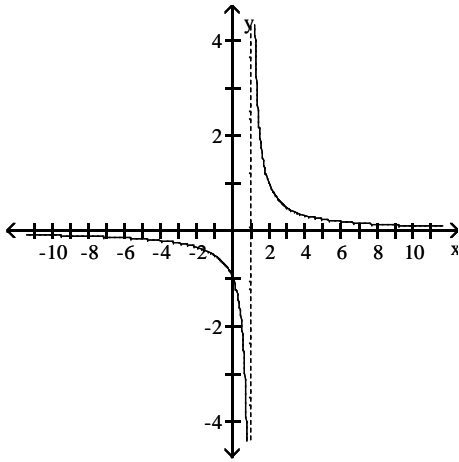
- 112)  $g(x) = \frac{x+3}{x^2+49}$  112) \_\_\_\_\_  
 A)  $x = -7, x = 7$  B) none  
 C)  $x = -7, x = 7, x = -3$  D)  $x = -7, x = -3$
- 113)  $h(x) = \frac{x+11}{x^2+64x}$  113) \_\_\_\_\_  
 A)  $x = 0, x = -8, x = 8$  B)  $x = -8, x = 8$   
 C)  $x = 0, x = -64$  D)  $x = -64, x = -11$
- 114)  $f(x) = \frac{x(x-1)}{x^3+16x}$  114) \_\_\_\_\_  
 A)  $x = 0$  B)  $x = 0, x = -4, x = 4$   
 C)  $x = -4, x = 4$  D)  $x = 0, x = -16$
- 115)  $R(x) = \frac{-3x^2}{x^2+4x-77}$  115) \_\_\_\_\_  
 A)  $x = -77$  B)  $x = -11, x = 7, x = -3$   
 C)  $x = 11, x = -7$  D)  $x = -11, x = 7$
- 116)  $R(x) = \frac{x-1}{x^3+8x^2-33x}$  116) \_\_\_\_\_  
 A)  $x = -3, x = 0, x = 11$  B)  $x = -11, x = 3$   
 C)  $x = -11, x = 0, x = 3$  D)  $x = -3, x = -30, x = 11$
- 117)  $f(x) = \frac{-2x(x+2)}{4x^2-3x-7}$  117) \_\_\_\_\_  
 A)  $x = -\frac{7}{4}, x = 1$  B)  $x = -\frac{4}{7}, x = 1$  C)  $x = \frac{4}{7}, x = -1$  D)  $x = \frac{7}{4}, x = -1$
- 118)  $f(x) = \frac{x-3}{9x-x^3}$  118) \_\_\_\_\_  
 A)  $x = 0, x = 3$  B)  $x = 0, x = -3$   
 C)  $x = -3, x = 3$  D)  $x = 0, x = -3, x = 3$
- 119)  $f(x) = \frac{-x^2+16}{x^2+5x+4}$  119) \_\_\_\_\_  
 A)  $x = -1$  B)  $x = 1, x = -4$  C)  $x = -1, x = 4$  D)  $x = -1, x = -4$

Choose the graph that represents the given function without using a graphing utility.

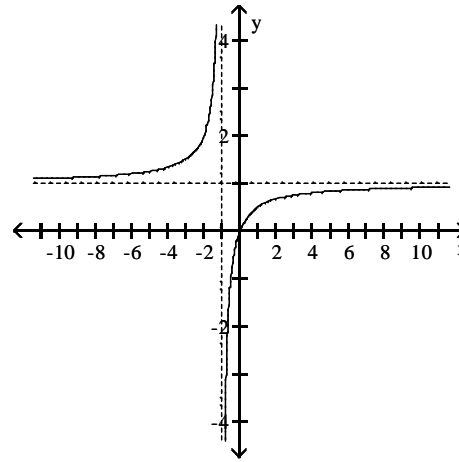
120)  $f(x) = \frac{x}{x-1}$

120) \_\_\_\_\_

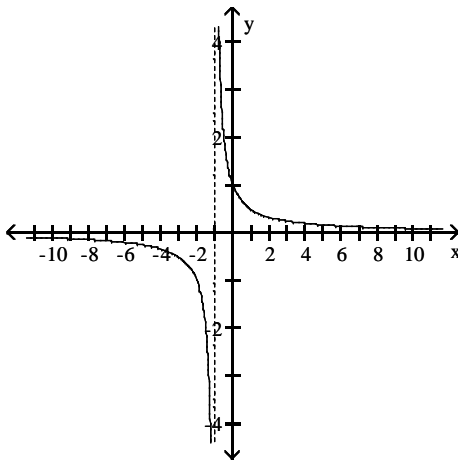
A)



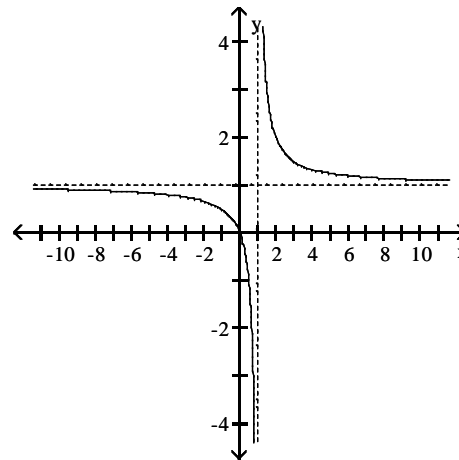
B)



C)



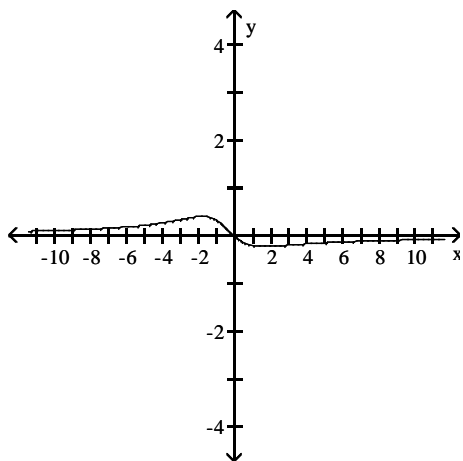
D)



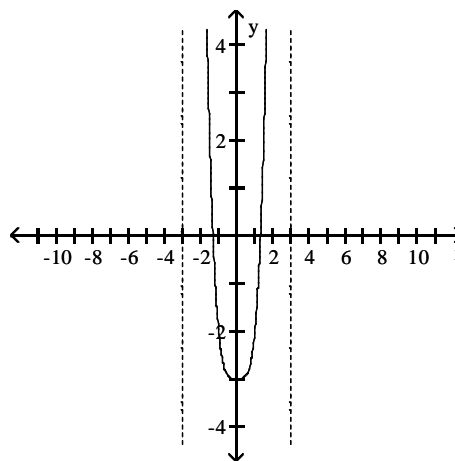
121)  $f(x) = \frac{x}{x^2 + x + 3}$

121) \_\_\_\_\_

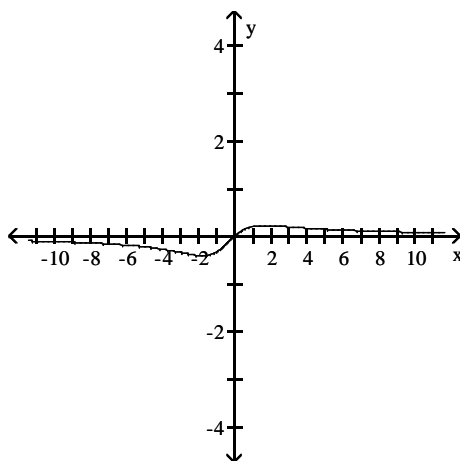
A)



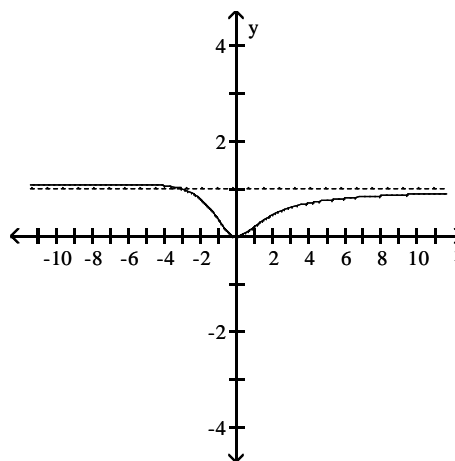
B)



C)



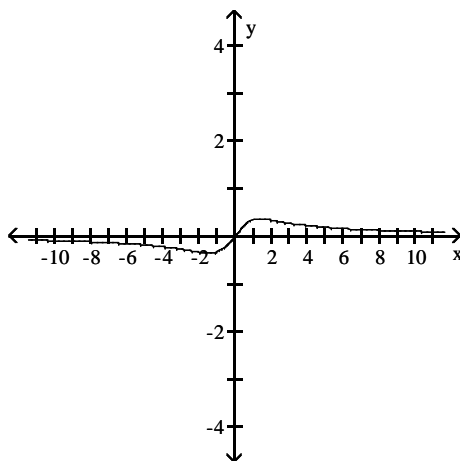
D)



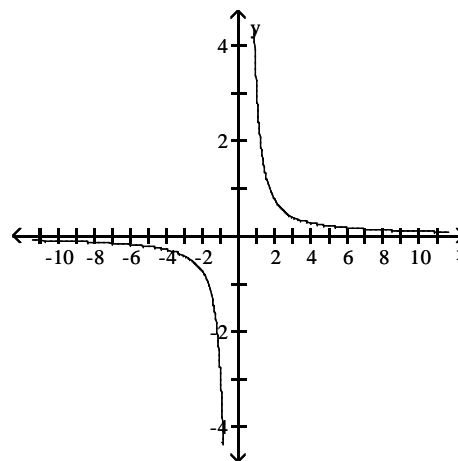
122)  $f(x) = \frac{x^2 - 2}{x^3}$

122) \_\_\_\_\_

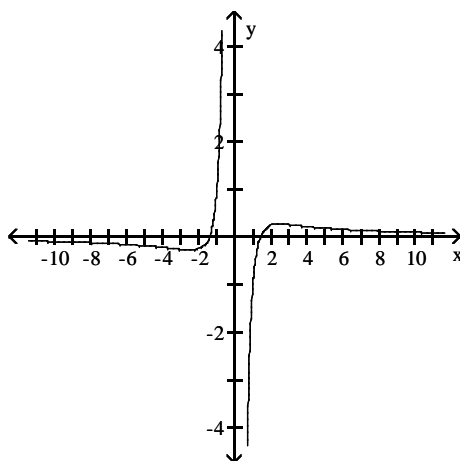
A)



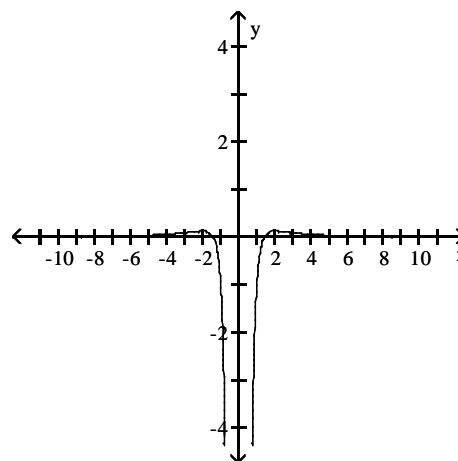
B)



C)



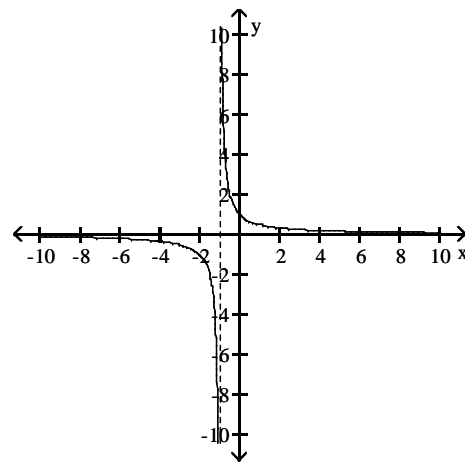
D)



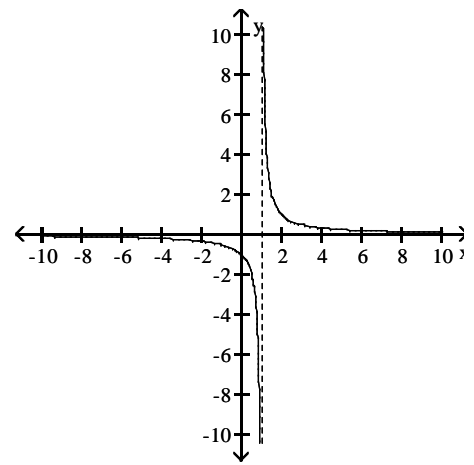
123)  $f(x) = \frac{1}{x+1}$

123) \_\_\_\_\_

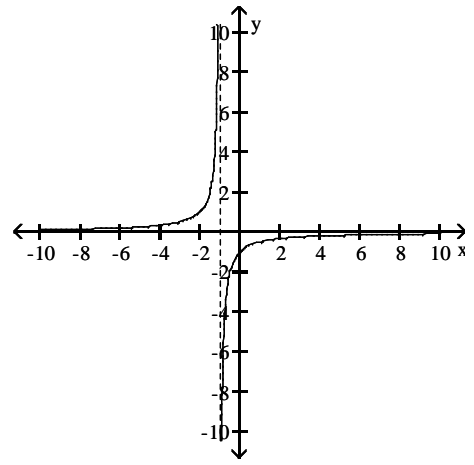
A)



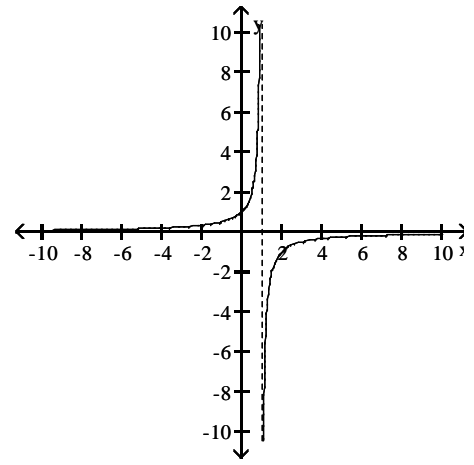
B)



C)



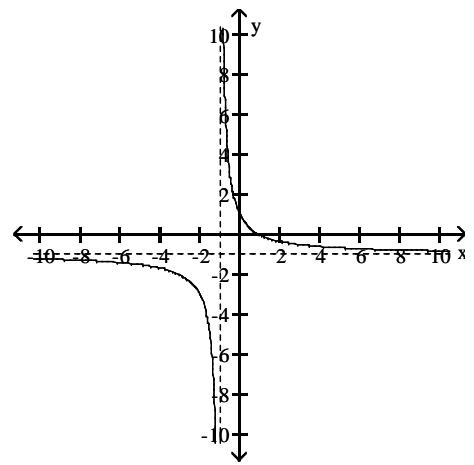
D)



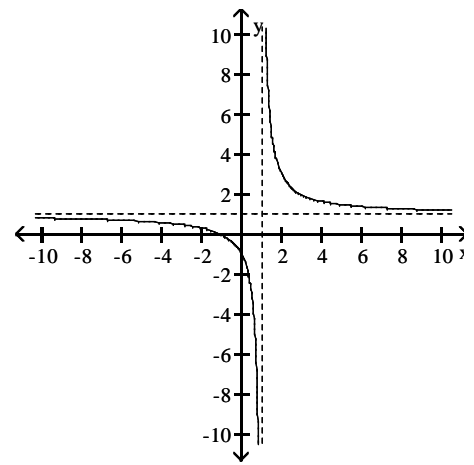
124)  $f(x) = \frac{x-1}{x+1}$

124) \_\_\_\_\_

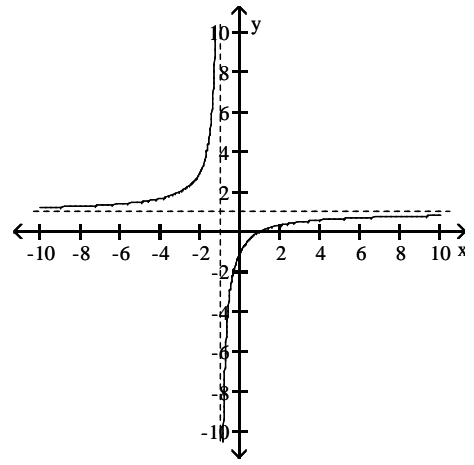
A)



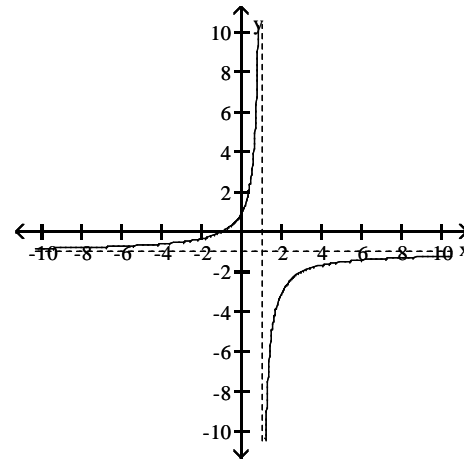
B)



C)



D)

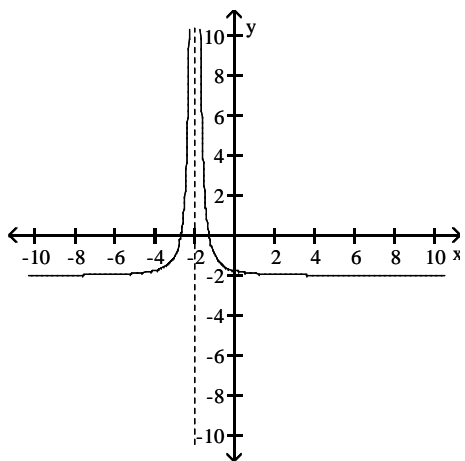




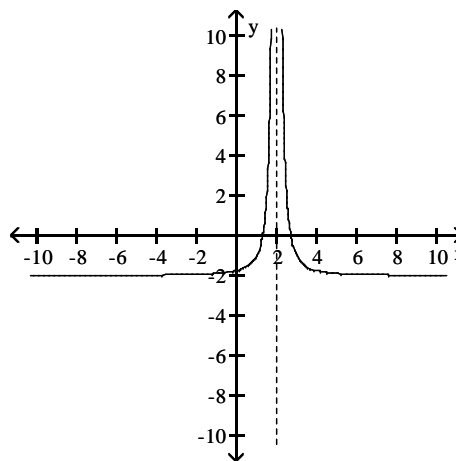
125)  $f(x) = \frac{1}{(x+2)^2}$

125) \_\_\_\_\_

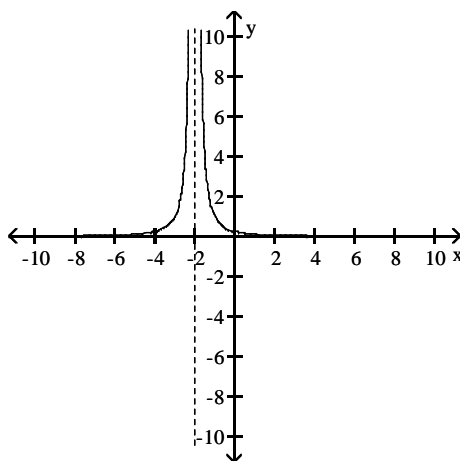
A)



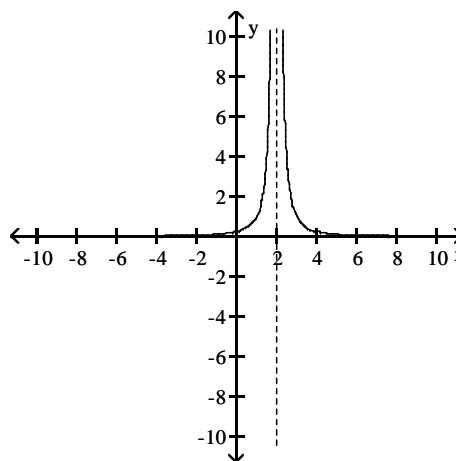
B)



C)



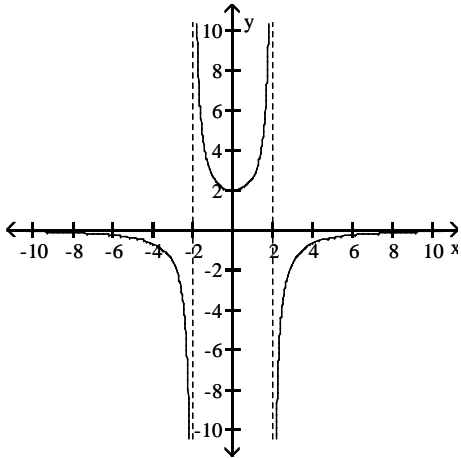
D)



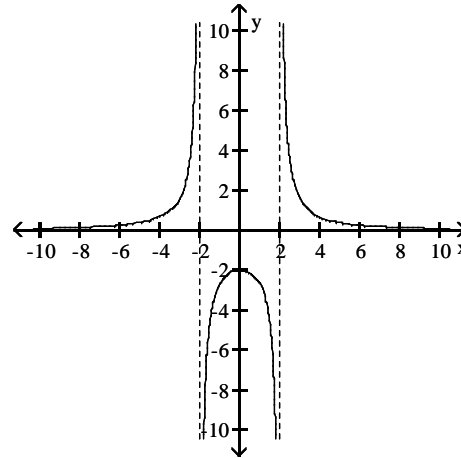
126)  $f(x) = \frac{2x^2}{4 - x^2}$

126) \_\_\_\_\_

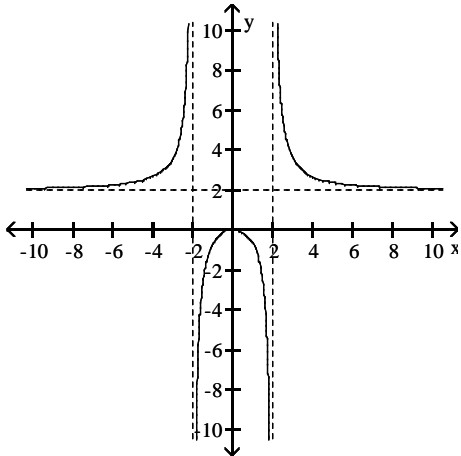
A)



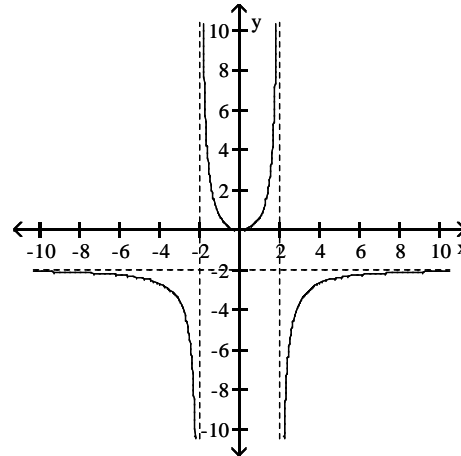
B)



C)



D)



Find the limit.

127)  $\lim_{x \rightarrow \infty} \frac{9}{x} - 1$

127) \_\_\_\_\_

A) -1

B) -10

C) 1

D) 8

128)  $\lim_{x \rightarrow -\infty} \frac{3}{6 - (1/x^2)}$

128) \_\_\_\_\_

A) 3

B)  $\frac{1}{2}$

C)  $-\infty$

D)  $\frac{3}{5}$

129)  $\lim_{x \rightarrow -\infty} \frac{-6 + (2/x)}{6 - (1/x^2)}$

129) \_\_\_\_\_

A)  $-\infty$

B)  $\infty$

C) 1

D) -1

130)  $\lim_{x \rightarrow \infty} \frac{x^2 + 5x + 3}{x^3 - 8x^2 + 13}$  130) \_\_\_\_\_  
 A)  $\infty$  B) 0 C)  $\frac{3}{13}$  D) 1

131)  $\lim_{x \rightarrow -\infty} \frac{-17x^2 + 5x + 7}{-12x^2 - 4x + 11}$  131) \_\_\_\_\_  
 A) 1 B)  $\frac{7}{11}$  C)  $\frac{17}{12}$  D)  $\infty$

132)  $\lim_{x \rightarrow \infty} \frac{6x + 1}{8x - 7}$  132) \_\_\_\_\_  
 A)  $-\frac{1}{7}$  B) 0 C)  $\infty$  D)  $\frac{3}{4}$

133)  $\lim_{x \rightarrow \infty} \frac{9x^3 - 5x^2 + 3x}{-x^3 - 2x + 5}$  133) \_\_\_\_\_  
 A) -9 B)  $\infty$  C)  $\frac{3}{2}$  D) 9

134)  $\lim_{x \rightarrow -\infty} \frac{5x^3 + 2x^2}{x - 7x^2}$  134) \_\_\_\_\_  
 A)  $\infty$  B)  $-\infty$  C) 5 D)  $-\frac{2}{7}$

135)  $\lim_{x \rightarrow -\infty} \frac{\cos 5x}{x}$  135) \_\_\_\_\_  
 A) 5 B) 0 C)  $-\infty$  D) 1

**Divide numerator and denominator by the highest power of x in the denominator to find the limit.**

136)  $\lim_{x \rightarrow \infty} \sqrt{\frac{4x^2}{5 + 49x^2}}$  136) \_\_\_\_\_  
 A)  $\frac{2}{7}$  B)  $\frac{4}{49}$  C) does not exist D)  $\frac{4}{5}$

137)  $\lim_{x \rightarrow \infty} \sqrt{\frac{49x^2 + x - 3}{(x - 15)(x + 1)}}$  137) \_\_\_\_\_  
 A)  $\infty$  B) 7 C) 49 D) 0

138)  $\lim_{x \rightarrow \infty} \frac{-4\sqrt{x} + x - 1}{2x - 3}$  138) \_\_\_\_\_  
 A) 0 B)  $\infty$  C)  $\frac{1}{2}$  D) -2

139)  $\lim_{x \rightarrow \infty} \frac{-4x^{-1} + 2x^{-3}}{-4x^{-2} + x^{-5}}$  139) \_\_\_\_\_  
 A) 0                                      B)  $\infty$                                       C) 1                                      D)  $-\infty$

140)  $\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x} - 7x - 4}{-6x + x^{2/3} - 2}$  140) \_\_\_\_\_  
 A)  $\frac{6}{7}$                                       B) 0                                      C)  $\frac{7}{6}$                                       D)  $-\infty$

141)  $\lim_{t \rightarrow \infty} \frac{\sqrt{81t^2 - 729}}{t - 9}$  141) \_\_\_\_\_  
 A) 81                                      B) does not exist                                      C) 729                                      D) 9

142)  $\lim_{t \rightarrow \infty} \frac{\sqrt{25t^2 - 125}}{t - 5}$  142) \_\_\_\_\_  
 A) does not exist                                      B) 25                                      C) 125                                      D) 5

143)  $\lim_{x \rightarrow \infty} \frac{3x + 7}{\sqrt{7x^2 + 1}}$  143) \_\_\_\_\_  
 A)  $\infty$                                       B)  $\frac{3}{\sqrt{7}}$                                       C) 0                                      D)  $\frac{3}{7}$

**Find all horizontal asymptotes of the given function, if any.**

144)  $h(x) = \frac{5x - 8}{x - 4}$  144) \_\_\_\_\_  
 A)  $y = 0$                                       B)  $y = 4$   
 C)  $y = 5$                                       D) no horizontal asymptotes

145)  $h(x) = 8 - \frac{3}{x}$  145) \_\_\_\_\_  
 A)  $x = 0$                                       B)  $y = 3$   
 C)  $y = 8$                                       D) no horizontal asymptotes

146)  $g(x) = \frac{x^2 + 5x - 8}{x - 8}$  146) \_\_\_\_\_  
 A)  $y = 0$                                       B)  $y = 1$   
 C)  $y = 8$                                       D) no horizontal asymptotes

147)  $h(x) = \frac{8x^2 - 5x - 8}{2x^2 - 4x + 2}$  147) \_\_\_\_\_  
 A)  $y = 0$                                       B)  $y = 4$   
 C)  $y = \frac{5}{4}$                                       D) no horizontal asymptotes

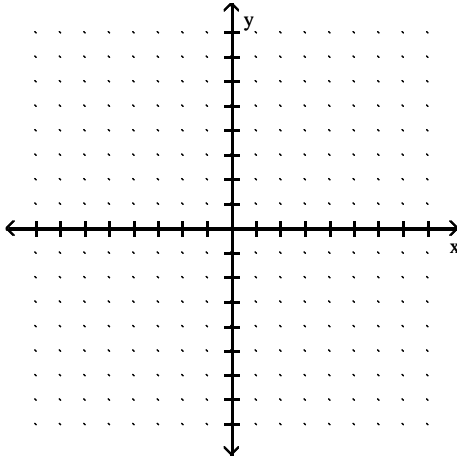
- 148)  $h(x) = \frac{8x^4 - 5x^2 - 9}{5x^5 - 4x + 9}$  148) \_\_\_\_\_
- A)  $y = \frac{8}{5}$  B)  $y = \frac{5}{4}$
- C)  $y = 0$  D) no horizontal asymptotes
- 149)  $h(x) = \frac{2x^3 - 7x}{8x^3 - 3x + 8}$  149) \_\_\_\_\_
- A)  $y = \frac{7}{3}$  B)  $y = \frac{1}{4}$
- C)  $y = 0$  D) no horizontal asymptotes
- 150)  $h(x) = \frac{8x^3 - 7x - 6}{9x^2 + 7}$  150) \_\_\_\_\_
- A)  $y = 8$  B)  $y = 0$
- C)  $y = \frac{8}{9}$  D) no horizontal asymptotes
- 151)  $h(x) = \frac{4x + 1}{x^2 - 36}$  151) \_\_\_\_\_
- A) no horizontal asymptotes B)  $y = 4$
- C)  $y = -6, y = 6$  D)  $y = 0$
- 152)  $R(x) = \frac{-3x^2 + 1}{x^2 + 3x - 40}$  152) \_\_\_\_\_
- A)  $y = -8, y = 5$  B)  $y = -3$
- C)  $y = 0$  D) no horizontal asymptotes
- 153)  $f(x) = \frac{x^2 - 5}{25x - x^4}$  153) \_\_\_\_\_
- A)  $y = -5, y = 5$  B)  $y = 0$
- C) no horizontal asymptotes D)  $y = -1$
- 154)  $f(x) = \frac{49x^4 + x^2 - 7}{x - x^3}$  154) \_\_\_\_\_
- A)  $y = -49$  B)  $y = 0$
- C) no horizontal asymptotes D)  $y = -1, y = 1$

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

Sketch the graph of a function  $y = f(x)$  that satisfies the given conditions.

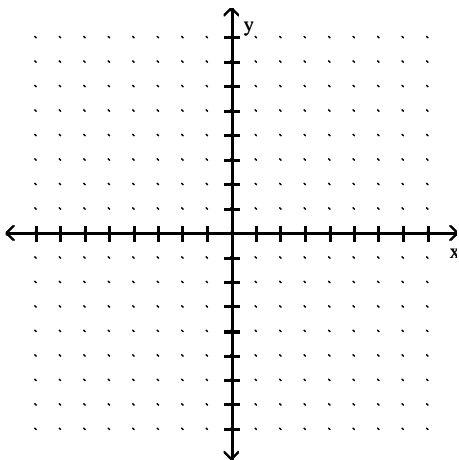
155)  $f(0) = 0, f(1) = 3, f(-1) = -3, \lim_{x \rightarrow -\infty} f(x) = -2, \lim_{x \rightarrow \infty} f(x) = 2.$

155) \_\_\_\_\_



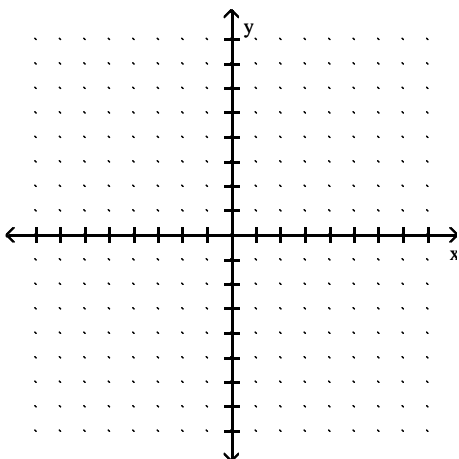
156)  $f(0) = 0, f(1) = 4, f(-1) = 4, \lim_{x \rightarrow \pm\infty} f(x) = -4.$

156) \_\_\_\_\_



157)  $f(0) = 4, f(1) = -4, f(-1) = -4, \lim_{x \rightarrow \pm\infty} f(x) = 0.$

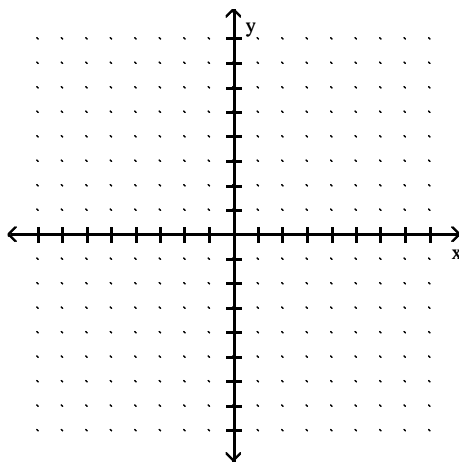
157) \_\_\_\_\_



158)  $f(0) = 0$ ,  $\lim_{x \rightarrow \pm\infty} f(x) = 0$ ,  $\lim_{x \rightarrow 3^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow -3^+} f(x) = -\infty$ ,  $\lim_{x \rightarrow 3^+} f(x) = \infty$ ,

158) \_\_\_\_\_

$\lim_{x \rightarrow -3^-} f(x) = \infty$ .



**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**Find all points where the function is discontinuous.**

159)

159) \_\_\_\_\_

A) None

B)  $x = 4$

C)  $x = 4, x = 2$

D)  $x = 2$

160)

160) \_\_\_\_\_

A) None

B)  $x = -2, x = 1$

C)  $x = -2$

D)  $x = 1$

161)

161) \_\_\_\_\_

A)  $x = 0, x = 2$

C)  $x = -2, x = 0$

B)  $x = -2, x = 0, x = 2$

D)  $x = 2$

162)

162) \_\_\_\_\_

A)  $x = 6$

B) None

C)  $x = -2, x = 6$

D)  $x = -2$

163)

163) \_\_\_\_\_

A)  $x = 1, x = 5$

C) None

B)  $x = 4$

D)  $x = 1, x = 4, x = 5$

164)

164) \_\_\_\_\_

A) None

B)  $x = 1$

C)  $x = 0$

D)  $x = 0, x = 1$

165)

165) \_\_\_\_\_

A)  $x = 0$

B)  $x = 0, x = 3$

C) None

D)  $x = 3$

166)

166) \_\_\_\_\_

A)  $x = -2$

B)  $x = -2, x = 2$

C) None

D)  $x = 2$



167)

167) \_\_\_\_\_

- A)  $x = -2, x = 0, x = 2$
- C)  $x = 0$

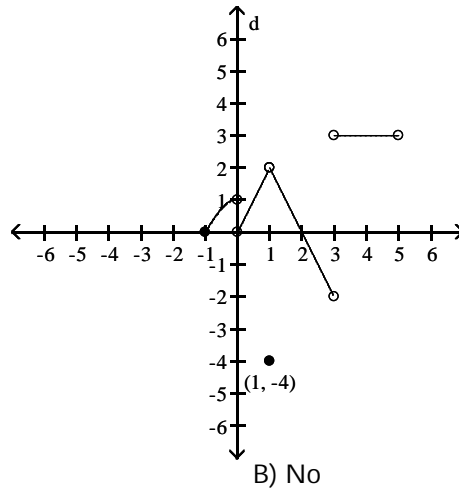
- B) None
- D)  $x = -2, x = 2$

**Provide an appropriate response.**

168) Is  $f$  continuous at  $f(1)$ ?

168) \_\_\_\_\_

$$f(x) = \begin{cases} -x^2 + 1, & -1 \leq x < 0 \\ 2x, & 0 < x < 1 \\ -4, & x = 1 \\ -2x + 4, & 1 < x < 3 \\ 3, & 3 < x < 5 \end{cases}$$



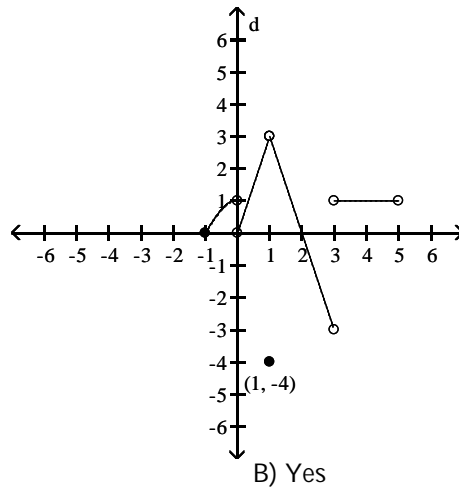
A) Yes

B) No

169) Is  $f$  continuous at  $f(0)$ ?

169) \_\_\_\_\_

$$f(x) = \begin{cases} -x^2 + 1, & -1 \leq x < 0 \\ 3x, & 0 < x < 1 \\ -4, & x = 1 \\ -3x + 6, & 1 < x < 3 \\ 1, & 3 < x < 5 \end{cases}$$



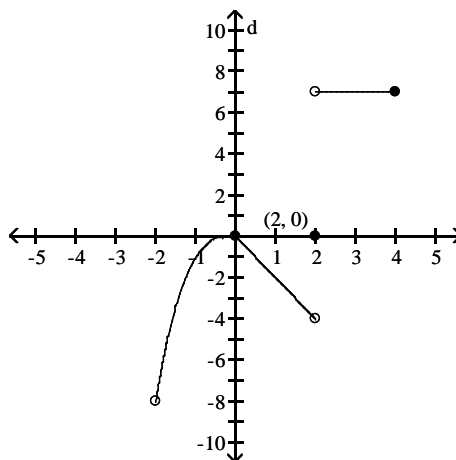
A) No

B) Yes

170) Is  $f$  continuous at  $x = 0$ ?

170) \_\_\_\_\_

$$f(x) = \begin{cases} x^3, & -2 < x \leq 0 \\ -2x, & 0 \leq x < 2 \\ 7, & 2 < x \leq 4 \\ 0, & x = 2 \end{cases}$$



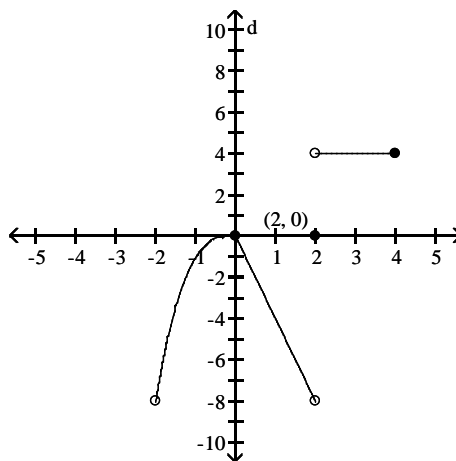
A) Yes

B) No

171) Is  $f$  continuous at  $x = 4$ ?

171) \_\_\_\_\_

$$f(x) = \begin{cases} x^3, & -2 < x \leq 0 \\ -4x, & 0 \leq x < 2 \\ 4, & 2 < x \leq 4 \\ 0, & x = 2 \end{cases}$$



A) Yes

B) No

**Find the intervals on which the function is continuous.**

172)  $y = \frac{3}{x+1} - 5x$

172) \_\_\_\_\_

- A) discontinuous only when  $x = -6$
- C) discontinuous only when  $x = -1$

- B) discontinuous only when  $x = 1$
- D) continuous everywhere

173)  $y = \frac{4}{(x+2)^2 + 4}$

173) \_\_\_\_\_

- A) continuous everywhere
- C) discontinuous only when  $x = -2$

- B) discontinuous only when  $x = -16$
- D) discontinuous only when  $x = 8$

174)  $y = \frac{x+2}{x^2 - 15x + 56}$

174) \_\_\_\_\_

- A) discontinuous only when  $x = 7$  or  $x = 8$
- C) discontinuous only when  $x = 7$

- B) discontinuous only when  $x = -8$  or  $x = 7$
- D) discontinuous only when  $x = -7$  or  $x = 8$

175)  $y = \frac{4}{x^2 - 9}$  175) \_\_\_\_\_

- A) discontinuous only when  $x = 9$   
 C) discontinuous only when  $x = -3$  or  $x = 3$

- B) discontinuous only when  $x = -3$   
 D) discontinuous only when  $x = -9$  or  $x = 9$

176)  $y = \frac{3}{|x| + 4} - \frac{x^2}{8}$  176) \_\_\_\_\_

- A) discontinuous only when  $x = -4$   
 C) discontinuous only when  $x = -12$

- B) continuous everywhere  
 D) discontinuous only when  $x = -8$  or  $x = -4$

177)  $y = \frac{\sin(4\theta)}{3\theta}$  177) \_\_\_\_\_

- A) discontinuous only when  $\theta = \frac{\pi}{2}$   
 C) discontinuous only when  $\theta = \pi$

- B) discontinuous only when  $\theta = 0$   
 D) continuous everywhere

178)  $y = \frac{4 \cos \theta}{\theta + 1}$  178) \_\_\_\_\_

- A) discontinuous only when  $\theta = -1$   
 C) discontinuous only when  $\theta = \frac{\pi}{2}$

- B) continuous everywhere  
 D) discontinuous only when  $\theta = 1$

179)  $y = \sqrt{8x + 8}$  179) \_\_\_\_\_

- A) continuous on the interval  $[-1, \infty)$   
 C) continuous on the interval  $[1, \infty)$

- B) continuous on the interval  $(-1, \infty)$   
 D) continuous on the interval  $(-\infty, -1]$

180)  $y = \sqrt[4]{7x - 8}$  180) \_\_\_\_\_

- A) continuous on the interval  $\left(-\infty, \frac{8}{7}\right]$   
 C) continuous on the interval  $\left[\frac{8}{7}, \infty\right)$

- B) continuous on the interval  $\left[\frac{8}{7}, \infty\right)$   
 D) continuous on the interval  $\left[-\frac{8}{7}, \infty\right)$

181)  $y = \sqrt{x^2 - 3}$  181) \_\_\_\_\_

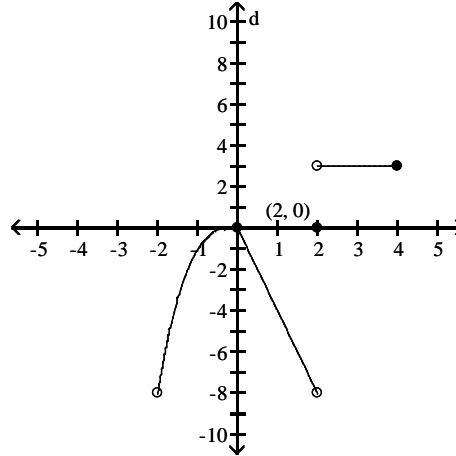
- A) continuous on the interval  $[-\sqrt{3}, \sqrt{3}]$   
 B) continuous on the intervals  $(-\infty, -\sqrt{3}]$  and  $[\sqrt{3}, \infty)$   
 C) continuous everywhere  
 D) continuous on the interval  $[\sqrt{3}, \infty)$

**Provide an appropriate response.**

182) Is  $f$  continuous on  $(-2, 4]$ ?

182) \_\_\_\_\_

$$f(x) = \begin{cases} x^3, & -2 < x \leq 0 \\ -4x, & 0 \leq x < 2 \\ 3, & 2 < x \leq 4 \\ 0, & x = 2 \end{cases}$$



A) No

B) Yes

**Find the limit, if it exists.**

183)  $\lim_{x \rightarrow -3} (x^2 - 16 + \sqrt[3]{x^2 - 36})$

183) \_\_\_\_\_

A) -10

B) -4

C) Does not exist

D) 4

184)  $\lim_{x \rightarrow 4} \sqrt{x^2 + 14x + 49}$

184) \_\_\_\_\_

A) 11

B) 121

C) Does not exist

D)  $\pm 11$

185)  $\lim_{x \rightarrow 3} \sqrt{x - 10}$

185) \_\_\_\_\_

A) 2.64575131

B) Does not exist

C) -2.6457513

D) 0

186)  $\lim_{x \rightarrow 14} \sqrt{x^2 - 9}$

186) \_\_\_\_\_

A) 93.5

B)  $\pm\sqrt{187}$

C)  $\sqrt{187}$

D) Does not exist

187)  $\lim_{x \rightarrow -8^-} \sqrt{x^2 - 64}$

187) \_\_\_\_\_

A)  $8\sqrt{5}$

B) 4

C) 0

D) Does not exist

188)  $\lim_{x \rightarrow 2^+} \frac{4\sqrt{(x-2)^3}}{x-2}$

188) \_\_\_\_\_

A) 0

B) 4

C)  $4\sqrt{2}$

D) Does not exist

189)  $\lim_{t \rightarrow 1^+} \frac{\sqrt{(t+36)(t-1)^2}}{13t-13}$

189) \_\_\_\_\_

A)  $\frac{1}{13}$

B)  $\frac{\sqrt{37}}{13}$

C) 0

D) Does not exist

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

**Provide an appropriate response.**

190) Use the Intermediate Value Theorem to prove that  $7x^3 + 9x^2 - 6x - 5 = 0$  has a solution between -2 and -1. 190) \_\_\_\_\_

191) Use the Intermediate Value Theorem to prove that  $-2x^4 - 5x^3 - 3x - 9 = 0$  has a solution between -2 and -1. 191) \_\_\_\_\_

192) Use the Intermediate Value Theorem to prove that  $x(x - 2)^2 = 2$  has a solution between 1 and 3. 192) \_\_\_\_\_

193) Use the Intermediate Value Theorem to prove that  $4 \sin x = x$  has a solution between  $\frac{\pi}{2}$  and  $\pi$ . 193) \_\_\_\_\_

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**Find numbers a and b, or k, so that f is continuous at every point.**

194) \_\_\_\_\_ 194) \_\_\_\_\_

$$f(x) = \begin{cases} -10, & x < 2 \\ ax + b, & 2 \leq x \leq 4 \\ 2, & x > 4 \end{cases}$$

- A)  $a = 6, b = -22$       B)  $a = -10, b = 2$       C)  $a = 6, b = 26$       D) Impossible

195) \_\_\_\_\_ 195) \_\_\_\_\_

$$f(x) = \begin{cases} x^2, & x < -1 \\ ax + b, & -1 \leq x \leq 3 \\ x + 6, & x > 3 \end{cases}$$

- A)  $a = -2, b = 3$       B)  $a = 2, b = -3$       C)  $a = 2, b = 3$       D) Impossible

196) \_\_\_\_\_ 196) \_\_\_\_\_

$$f(x) = \begin{cases} 3x + 4, & \text{if } x < -8 \\ kx + 2, & \text{if } x \geq -8 \end{cases}$$

- A)  $k = \frac{1}{4}$       B)  $k = -\frac{1}{4}$       C)  $k = \frac{11}{4}$       D)  $k = 5$

197) \_\_\_\_\_ 197) \_\_\_\_\_

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 4 \\ x + k, & \text{if } x > 4 \end{cases}$$

- A)  $k = -4$       B)  $k = 12$       C)  $k = 20$       D) Impossible

198)

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 9 \\ kx, & \text{if } x > 9 \end{cases}$$

A)  $k = \frac{1}{9}$

B)  $k = 9$

C)  $k = 81$

D) Impossible

198) \_\_\_\_\_

**Solve the problem.**199) Select the correct statement for the definition of the limit:  $\lim_{x \rightarrow x_0} f(x) = L$ 

199) \_\_\_\_\_

means that \_\_\_\_\_

A) if given a number  $\varepsilon > 0$ , there exists a number  $\delta > 0$ , such that for all  $x$ ,  $0 < |x - x_0| < \delta$  implies  $|f(x) - L| > \varepsilon$ .B) if given any number  $\varepsilon > 0$ , there exists a number  $\delta > 0$ , such that for all  $x$ ,  $0 < |x - x_0| < \delta$  implies  $|f(x) - L| < \varepsilon$ .C) if given any number  $\varepsilon > 0$ , there exists a number  $\delta > 0$ , such that for all  $x$ ,  $0 < |x - x_0| < \varepsilon$  implies  $|f(x) - L| > \delta$ .D) if given any number  $\varepsilon > 0$ , there exists a number  $\delta > 0$ , such that for all  $x$ ,  $0 < |x - x_0| < \varepsilon$  implies  $|f(x) - L| < \delta$ .

200) Identify the incorrect statements about limits.

200) \_\_\_\_\_

I. The number  $L$  is the limit of  $f(x)$  as  $x$  approaches  $x_0$  if  $f(x)$  gets closer to  $L$  as  $x$  approaches  $x_0$ .II. The number  $L$  is the limit of  $f(x)$  as  $x$  approaches  $x_0$  if, for any  $\varepsilon > 0$ , there corresponds a  $\delta > 0$  such that  $|f(x) - L| < \varepsilon$  whenever  $0 < |x - x_0| < \delta$ .III. The number  $L$  is the limit of  $f(x)$  as  $x$  approaches  $x_0$  if, given any  $\varepsilon > 0$ , there exists a value of  $x$  for which  $|f(x) - L| < \varepsilon$ .

A) I and III

B) II and III

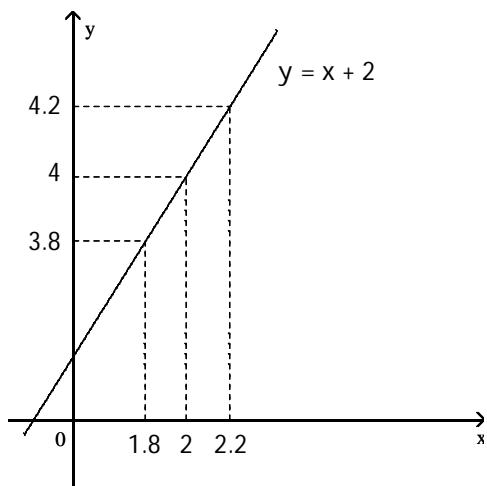
C) I and II

D) I, II, and III

**Use the graph to find a  $\delta > 0$  such that for all  $x$ ,  $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$ .**

201)

201) \_\_\_\_\_



$$\begin{aligned} f(x) &= x + 2 \\ x_0 &= 2 \\ L &= 4 \\ \varepsilon &= 0.2 \end{aligned}$$

NOT TO SCALE

A) 0.1

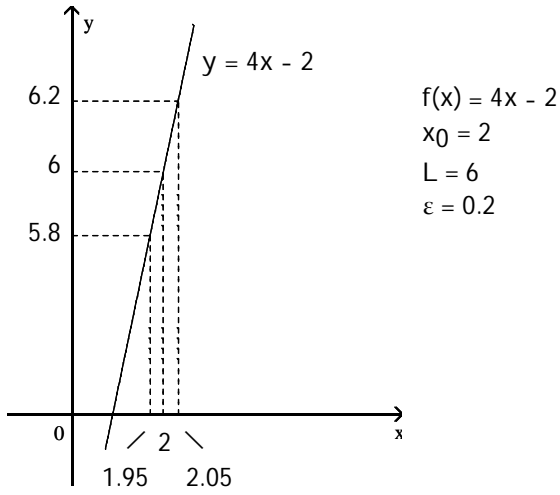
B) 2

C) 0.4

D) 0.2

202)

202) \_\_\_\_\_



NOT TO SCALE

A) 0.1

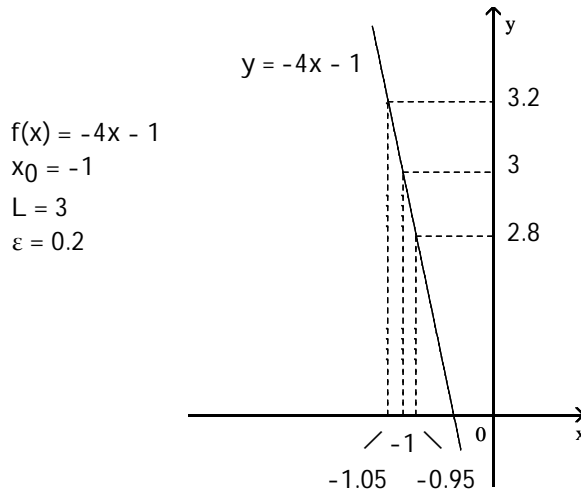
B) 0.05

C) 0.5

D) 4

203)

203) \_\_\_\_\_



NOT TO SCALE

A) -0.05

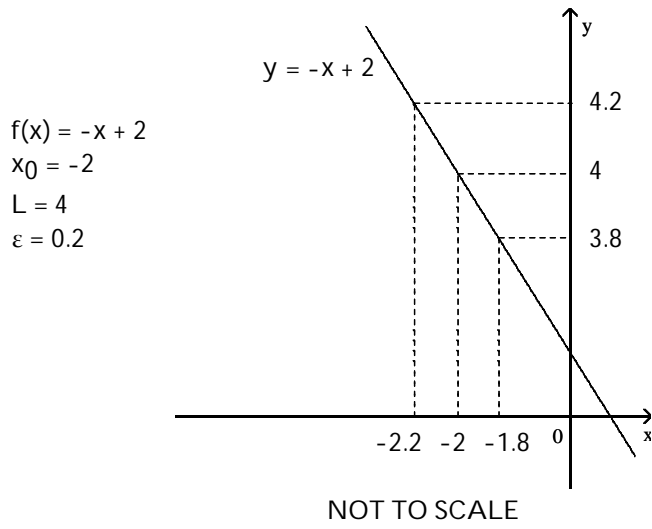
B) 6

C) 0.5

D) 0.05

204)

204) \_\_\_\_\_



A) 0.4

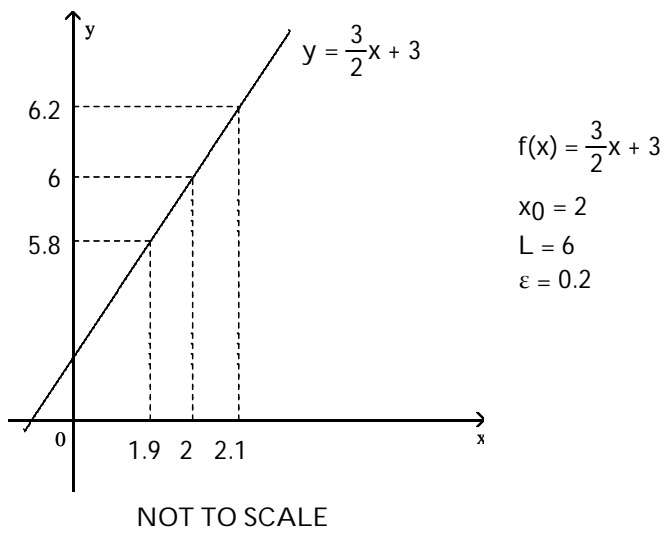
B) -0.2

C) 0.2

D) 6

205)

205) \_\_\_\_\_



A) -0.2

B) 4

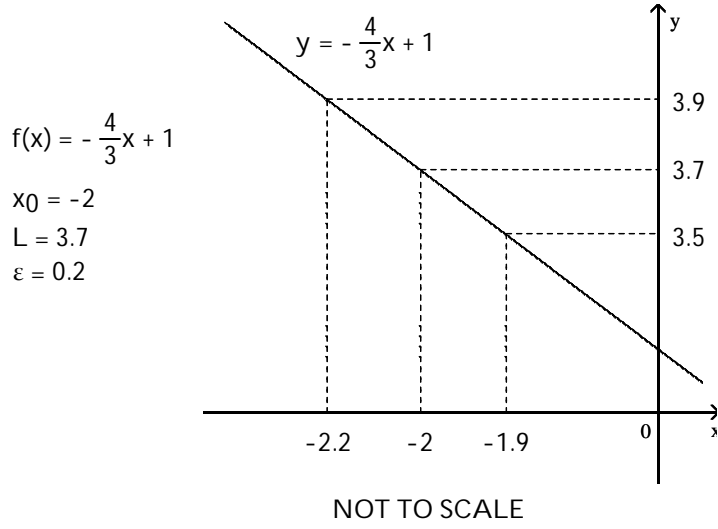
C) 0.1

D) 0.2



206)

206) \_\_\_\_\_



A) -0.3

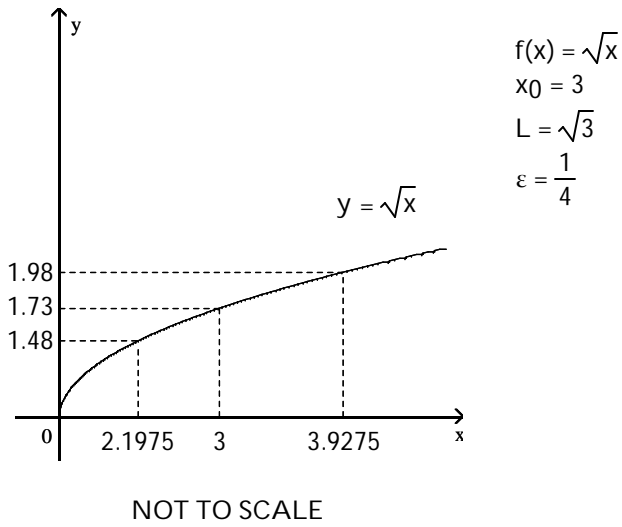
B) 5.7

C) 0.1

D) 0.3

207)

207) \_\_\_\_\_



A) 0.9275

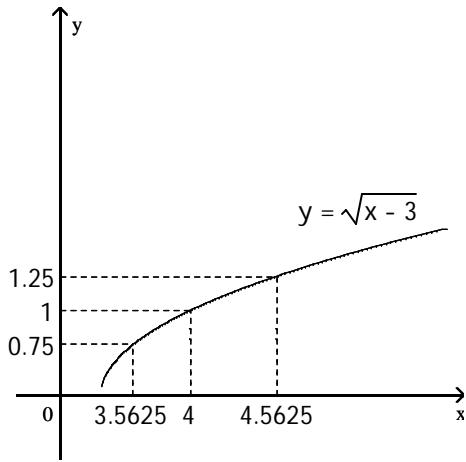
B) 0.8025

C) 1.73

D) -1.27

208)

208) \_\_\_\_\_



$$f(x) = \sqrt{x-3}$$

$$x_0 = 4$$

$$L = 1$$

$$\varepsilon = \frac{1}{4}$$

NOT TO SCALE

A) 0.5625

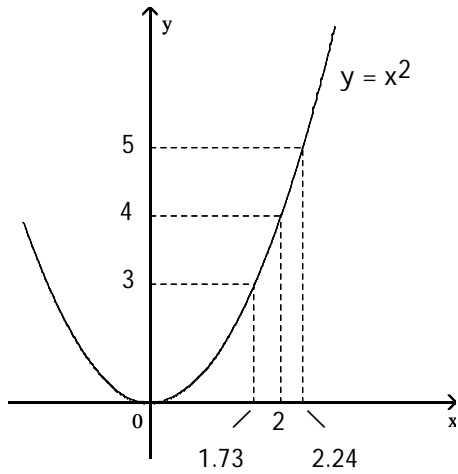
B) 1

C) 0.4375

D) 3

209)

209) \_\_\_\_\_



$$f(x) = x^2$$

$$x_0 = 2$$

$$L = 4$$

$$\varepsilon = 1$$

NOT TO SCALE

A) 2

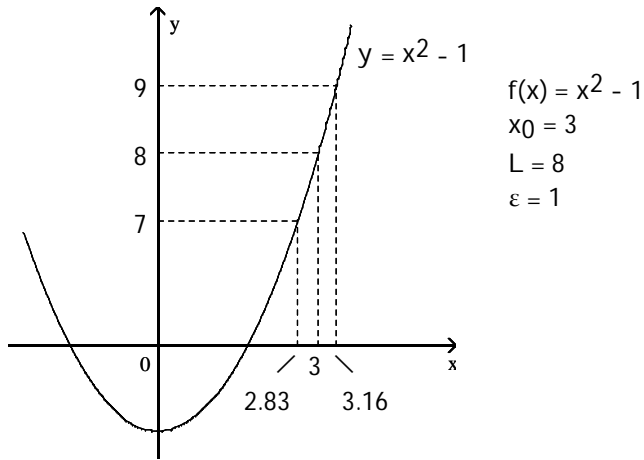
B) 0.27

C) 0.24

D) 0.51

210)

210) \_\_\_\_\_



NOT TO SCALE

A) 0.16

B) 5

C) 0.33

D) 0.17

**A function  $f(x)$ , a point  $x_0$ , the limit of  $f(x)$  as  $x$  approaches  $x_0$ , and a positive number  $\varepsilon$  is given. Find a number  $\delta > 0$  such that for all  $x$ ,  $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$ .**

211)  $f(x) = 9x + 3$ ,  $L = 21$ ,  $x_0 = 2$ , and  $\varepsilon = 0.01$

211) \_\_\_\_\_

A) 0.005556

B) 0.002222

C) 0.001111

D) 0.005

212)  $f(x) = 3x - 10$ ,  $L = -4$ ,  $x_0 = 2$ , and  $\varepsilon = 0.01$

212) \_\_\_\_\_

A) 0.005

B) 0.006667

C) 0.001667

D) 0.003333

213)  $f(x) = -10x + 9$ ,  $L = -1$ ,  $x_0 = 1$ , and  $\varepsilon = 0.01$

213) \_\_\_\_\_

A) 0.002

B) 0.001

C) 0.004

D) -0.01

214)  $f(x) = -9x - 6$ ,  $L = -33$ ,  $x_0 = 3$ , and  $\varepsilon = 0.01$

214) \_\_\_\_\_

A) -0.003333

B) 0.001111

C) 0.002222

D) 0.000556

215)  $f(x) = 3x^2$ ,  $L = 12$ ,  $x_0 = 2$ , and  $\varepsilon = 0.2$

215) \_\_\_\_\_

A) 1.98326

B) 2.0166

C) 0.01674

D) 0.0166

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

**Prove the limit statement**

216)  $\lim_{x \rightarrow 3} (3x - 2) = 7$

216) \_\_\_\_\_

217)  $\lim_{x \rightarrow 8} \frac{x^2 - 64}{x - 8} = 16$

217) \_\_\_\_\_

218)  $\lim_{x \rightarrow 9} \frac{2x^2 - 15x - 27}{x - 9} = 21$

218) \_\_\_\_\_

$$219) \lim_{x \rightarrow 9} \frac{1}{x} = \frac{1}{9}$$

219) \_\_\_\_\_

# Answer Key

Testname: UNTITLED1

- 1) A
- 2) B
- 3) A
- 4) B
- 5) D
- 6) A
- 7) A
- 8) A
- 9) C
- 10) B
- 11) C
- 12) C
- 13) D
- 14) B
- 15) C
- 16) D
- 17) C
- 18) B
- 19) D
- 20) C
- 21) B
- 22) D
- 23) D
- 24) A
- 25) C
- 26) B
- 27) B
- 28) A
- 29) D
- 30) D
- 31) A
- 32) B
- 33) C
- 34) C
- 35) D
- 36) A
- 37) A
- 38) A
- 39) A

40) Answers may vary. One possibility:  $\lim_{x \rightarrow 0} 1 - \frac{x^2}{6} = \lim_{x \rightarrow 0} 1 = 1$ . According to the squeeze theorem, the function

$\frac{x \sin(x)}{2 - 2 \cos(x)}$ , which is squeezed between  $1 - \frac{x^2}{6}$  and 1, must also approach 1 as  $x$  approaches 0. Thus,

$$\lim_{x \rightarrow 0} \frac{x \sin(x)}{2 - 2 \cos(x)} = 1.$$

- 41) B
- 42) C
- 43) C

## Answer Key

Testname: UNTITLED1

- 44) C
- 45) C
- 46) C
- 47) B
- 48) C
- 49) D
- 50) C
- 51) D
- 52) A
- 53) C
- 54) D
- 55) D
- 56) A
- 57) A
- 58) D
- 59) C
- 60) C
- 61) B
- 62) C
- 63) A
- 64) A
- 65) C
- 66) D
- 67) C
- 68) B
- 69) A
- 70) D
- 71) B
- 72) B
- 73) C
- 74) C
- 75) A
- 76) C
- 77) D
- 78) A
- 79) B
- 80) A
- 81) D
- 82) B
- 83) A
- 84) D
- 85) C
- 86) B
- 87) B
- 88) B
- 89) D
- 90) D
- 91) C
- 92) C
- 93) A

## Answer Key

Testname: UNTITLED1

- 94) B
- 95) B
- 96) D
- 97) A
- 98) C
- 99) A
- 100) A
- 101) B
- 102) B
- 103) D
- 104) C
- 105) D
- 106) B
- 107) D
- 108) C
- 109) B
- 110) D
- 111) B
- 112) B
- 113) C
- 114) A
- 115) D
- 116) C
- 117) D
- 118) B
- 119) A
- 120) D
- 121) C
- 122) C
- 123) A
- 124) C
- 125) C
- 126) D
- 127) A
- 128) B
- 129) D
- 130) B
- 131) C
- 132) D
- 133) A
- 134) A
- 135) B
- 136) A
- 137) B
- 138) A
- 139) B
- 140) C
- 141) D
- 142) D
- 143) B

Answer Key

Testname: UNTITLED1

144) C

145) C

146) D

147) B

148) C

149) B

150) D

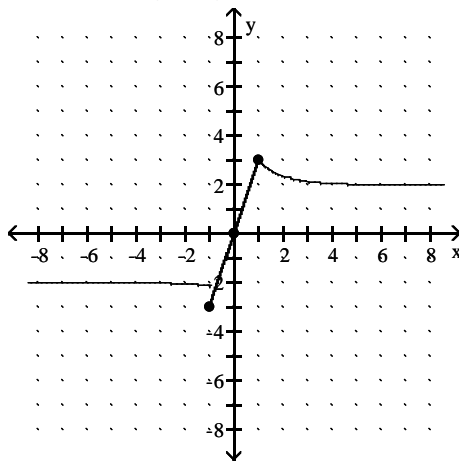
151) D

152) B

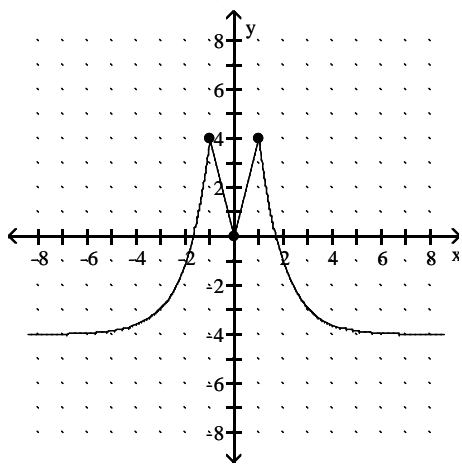
153) B

154) C

155) Answers may vary. One possible answer:



156) Answers may vary. One possible answer:

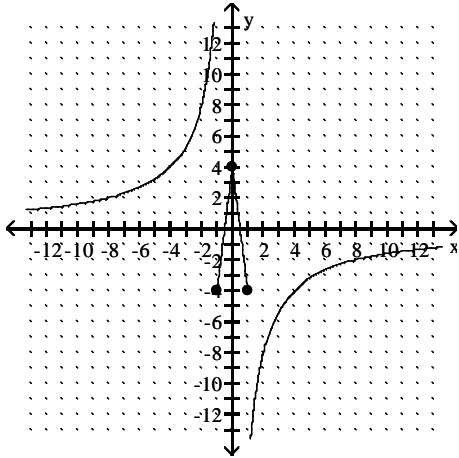




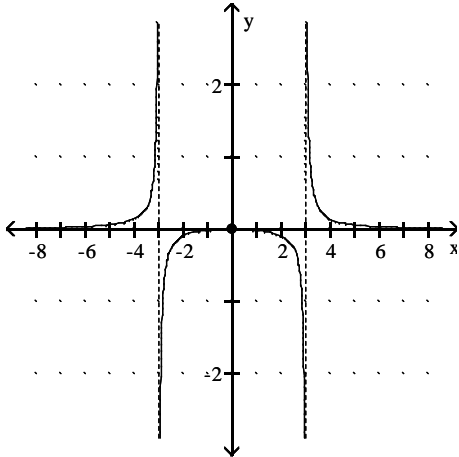
Answer Key

Testname: UNTITLED1

157) Answers may vary. One possible answer:



158) Answers may vary. One possible answer:



- 159) B
- 160) D
- 161) B
- 162) A
- 163) C
- 164) A
- 165) D
- 166) B
- 167) C
- 168) B
- 169) A
- 170) A
- 171) A
- 172) C
- 173) A
- 174) A
- 175) C
- 176) B
- 177) B
- 178) A
- 179) A

## Answer Key

Testname: UNTITLED1

180) C

181) B

182) A

183) A

184) A

185) B

186) C

187) C

188) A

189) B

190) Let  $f(x) = 7x^3 + 9x^2 - 6x - 5$  and let  $y_0 = 0$ .  $f(-2) = -13$  and  $f(-1) = 3$ . Since  $f$  is continuous on  $[-2, -1]$  and since  $y_0 = 0$  is between  $f(-2)$  and  $f(-1)$ , by the Intermediate Value Theorem, there exists a  $c$  in the interval  $(-2, -1)$  with the property that  $f(c) = 0$ . Such a  $c$  is a solution to the equation  $7x^3 + 9x^2 - 6x - 5 = 0$ .

191) Let  $f(x) = -2x^4 - 5x^3 - 3x - 9$  and let  $y_0 = 0$ .  $f(-2) = 5$  and  $f(-1) = -3$ . Since  $f$  is continuous on  $[-2, -1]$  and since  $y_0 = 0$  is between  $f(-2)$  and  $f(-1)$ , by the Intermediate Value Theorem, there exists a  $c$  in the interval  $(-2, -1)$  with the property that  $f(c) = 0$ . Such a  $c$  is a solution to the equation  $-2x^4 - 5x^3 - 3x - 9 = 0$ .

192) Let  $f(x) = x(x - 2)^2$  and let  $y_0 = 2$ .  $f(1) = 1$  and  $f(3) = 3$ . Since  $f$  is continuous on  $[1, 3]$  and since  $y_0 = 2$  is between  $f(1)$  and  $f(3)$ , by the Intermediate Value Theorem, there exists a  $c$  in the interval  $(1, 3)$  with the property that  $f(c) = 2$ . Such a  $c$  is a solution to the equation  $x(x - 2)^2 = 2$ .

193) Let  $f(x) = \frac{\sin x}{x}$  and let  $y_0 = \frac{1}{4}$ .  $f\left(\frac{\pi}{2}\right) \approx 0.6366$  and  $f(\pi) = 0$ . Since  $f$  is continuous on  $\left[\frac{\pi}{2}, \pi\right]$  and since  $y_0 = \frac{1}{4}$  is between  $f\left(\frac{\pi}{2}\right)$  and  $f(\pi)$ , by the Intermediate Value Theorem, there exists a  $c$  in the interval  $\left(\frac{\pi}{2}, \pi\right)$ , with the property that  $f(c) = \frac{1}{4}$ . Such a  $c$  is a solution to the equation  $4 \sin x = x$ .

194) A

195) C

196) C

197) B

198) B

199) B

200) A

201) D

202) B

203) D

204) C

205) C

206) C

207) B

208) C

209) C

210) A

211) C

212) D

213) B

214) B

215) D

## Answer Key

Testname: UNTITLED1

216)

Let  $\varepsilon > 0$  be given. Choose  $\delta = \varepsilon/3$ . Then  $0 < |x - 3| < \delta$  implies that

$$\begin{aligned} |(3x - 2) - 7| &= |3x - 9| \\ &= |3(x - 3)| \\ &= 3|x - 3| < 3\delta = \varepsilon \end{aligned}$$

Thus,  $0 < |x - 3| < \delta$  implies that  $|(3x - 2) - 7| < \varepsilon$ 217) Let  $\varepsilon > 0$  be given. Choose  $\delta = \varepsilon$ . Then  $0 < |x - 8| < \delta$  implies that

$$\begin{aligned} \left| \frac{x^2 - 64}{x - 8} - 16 \right| &= \left| \frac{(x - 8)(x + 8)}{x - 8} - 16 \right| \\ &= |(x + 8) - 16| \quad \text{for } x \neq 8 \\ &= |x - 8| < \delta = \varepsilon \end{aligned}$$

Thus,  $0 < |x - 8| < \delta$  implies that  $\left| \frac{x^2 - 64}{x - 8} - 16 \right| < \varepsilon$ 218) Let  $\varepsilon > 0$  be given. Choose  $\delta = \varepsilon/2$ . Then  $0 < |x - 9| < \delta$  implies that

$$\begin{aligned} \left| \frac{2x^2 - 15x - 27}{x - 9} - 21 \right| &= \left| \frac{(x - 9)(2x + 3)}{x - 9} - 21 \right| \\ &= |(2x + 3) - 21| \quad \text{for } x \neq 9 \\ &= |2x - 18| \\ &= |2(x - 9)| \\ &= 2|x - 9| < 2\delta = \varepsilon \end{aligned}$$

Thus,  $0 < |x - 9| < \delta$  implies that  $\left| \frac{2x^2 - 15x - 27}{x - 9} - 21 \right| < \varepsilon$ 219) Let  $\varepsilon > 0$  be given. Choose  $\delta = \min\{9/2, 81\varepsilon/2\}$ . Then  $0 < |x - 9| < \delta$  implies that

$$\begin{aligned} \left| \frac{1}{x} - \frac{1}{9} \right| &= \left| \frac{9 - x}{9x} \right| \\ &= \frac{1}{|x|} \cdot \frac{1}{9} \cdot |x - 9| \\ &< \frac{1}{9/2} \cdot \frac{1}{9} \cdot \frac{81\varepsilon}{2} = \varepsilon \end{aligned}$$

Thus,  $0 < |x - 9| < \delta$  implies that  $\left| \frac{1}{x} - \frac{1}{9} \right| < \varepsilon$