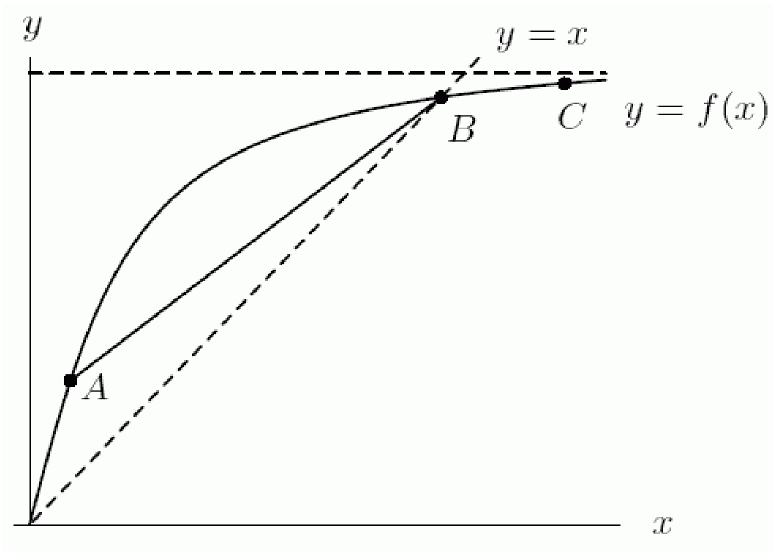
1. For any number r, let $m(r)$ be the slope of the graph of the function $y=(2.3)^x$ at the point $x = r$. Estimate $m(4)$ to 2 decimal places.
   Ans: 23.31
   difficulty: medium section: 2.1

2. If $x(V)=V^{1/3}$ is the length of the side of a cube in terms of its volume, V, calculate the average rate of change of x with respect to V over the interval $3<V<4$ to 2 decimal places.
   Ans: 0.15
   difficulty: easy section: 2.1

3. The length, x, of the side of a cube with volume V is given by $x(V)=V^{1/3}$. Is the average rate of change of x with respect to V increasing or decreasing as the volume V decreases?
   Ans: increasing
   difficulty: medium section: 2.1

4. If the graph of $y = f(x)$ is shown below, arrange the following in ascending order with 1 representing the smallest value and 6 the largest.

   A. $f'(A)$ B. $f'(B)$ C. $f'(C)$ D. slope of $\overline{AB}$
   E. 1 F. 0



Part A: 6
Part B: 3
Part C: 2
Part D: 4
Part E: 5
Part F: 1
difficulty: medium section: 2.1

5. The height of an object in feet above the ground is given in the following table. Compute the average velocity over the interval $1 \le t \le 3$.

t (sec)	0	1	2	3	4	5	6
y (feet)	10	45	70	85	90	85	70

Ans: 20
difficulty: easy section: 2.1

6. The height of an object in feet above the ground is given in the following table. If heights of the object are cut in half, how does the average velocity change, over a given interval?

t (sec)	0	1	2	3	4	5	6
y (feet)	10	45	70	85	90	85	70

A) It is cut in half.
B) It is doubled.
C) It remains the same.
D) It depends on the interval.

Ans: A difficulty: medium section: 2.1

7. The height of an object in feet above the ground is given in the following table, $y = f(t)$. Make a graph of $f(t)$. On your graph , what does the average velocity over a the interval $0 \le t \le 3$ represent?

t (sec)	0	1	2	3	4	5	6
y (feet)	10	45	70	85	90	85	70

A) The average height between $f(0)$ and $f(3)$.
B) The slope of the line between the points $(0, f(0))$, and $(3, f(3))$.
C) The average of the slopes of the tangent lines to the points $(0, f(0))$, and $(3, f(3))$.
D) The distance between the points $(0, f(0))$, and $(3, f(3))$.

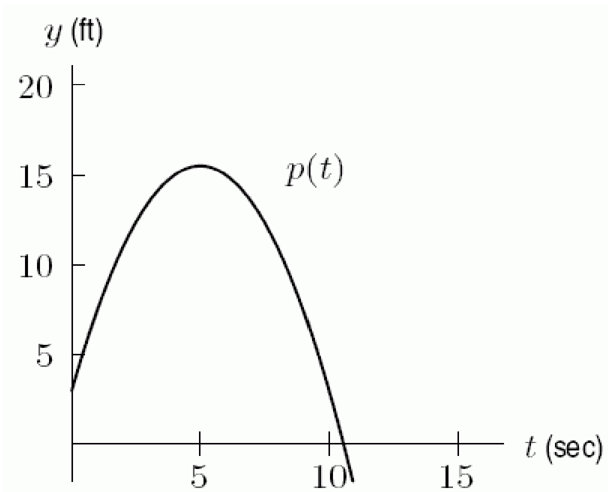
Ans: B difficulty: medium section: 2.1

8. The graph of $p(t)$, in the following figure, gives the position of a particle p at time t. List the following quantities in order, smallest to largest with 1 representing the smallest value.

   A. Average velocity on $1 \le t \le 3$.

   B. Instantaneous velocity at $t = 1$.

   C. Instantaneous velocity at $t = 3$.



Part A: 2
Part B: 3
Part C: 1
difficulty: medium section: 2.1

9. Estimate $\lim\limits_{h \to 0} \dfrac{(6+h)^2 - 36}{h}$ to 2 decimal places by substituting smaller and smaller values of h.
Ans: 12
difficulty: easy section: 2.1

10. Estimate $\lim\limits_{h \to 0} \dfrac{\sin(h^2)}{h}$ to 2 decimal places by substituting smaller and smaller values of h (use radians).
Ans: 0
difficulty: easy section: 2.1

11. A runner planned her strategy for running a half marathon, a distance of 13.1 miles. She planned to run negative splits, faster speeds as time passed during the race. In the actual race, she ran the first 6 miles in 48 minutes, the second 4 miles in 28 minutes and the last 3.1 miles in 18 minutes. What was her average velocity over the first 6 miles? What was her average velocity over the entire race? Did she run negative splits?
A) 7.50 mph for the first 5 miles, 8.36 mph for the race, No
B) 8.36 mph for the first 5 miles, 7.50 mph for the race, No
C) 8.12 mph for the first 5 miles, 7.35 mph for the race, Yes
D) 7.35 mph for the first 5 miles, 8.12 mph for the race, No
Ans: A difficulty: medium section: 2.1

12. Let $f(x) = x^{2/3}$. Use a graph to decide which one of the following statements is true.
A) When x = -5, the derivative is negative; when x = 5, the derivative is positive; and as x approaches infinity, the derivative approaches 0.
B) When x = -6, the derivative is positive; when x = 6, the derivative is also positive, and as x approaches infinity, the derivative approaches 0.
C) When x = -7, the derivative is negative; when x = 7, the derivative is positive, and as x approaches infinity, the derivative approaches infinity.
D) The derivative is positive at at all values of x.
Ans: A difficulty: easy section: 2.1

13. Given the following data about a function f, estimate $f'(4.75)$.

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

Ans: –4
difficulty: medium section: 2.2

14. Given the following data about a function $f(x)$, the equation of the tangent line at x = 5 is approximated by

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

A) $y-5=-4(x-2)$
B) $y-5=-8(x-2)$
C) $y-2=-4(x-5)$
D) $y-2=-8(x-5)$
Ans: C difficulty: medium section: 2.2

15. For $f(x)=2^{-x}$, estimate $f'(0)$ to 3 decimal places.
Ans: –0.693
difficulty: medium section: 2.2

16. Let $f(x) = \log(\log(x))$. Estimate $f'(7)$ to 3 decimal places using any method.
Ans: 0.032
difficulty: hard section: 2.2

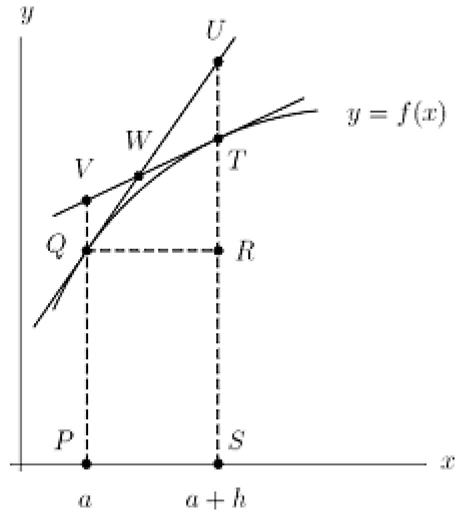
17. For $f(x) = \log x$, estimate $f'(3)$ to 3 decimal places by finding the average slope over intervals containing the value $x = 3$.
Ans: 0.145
difficulty: medium section: 2.2

18. There is a function used by statisticians, called the error function, which is written $y = \text{erf}(x)$. Suppose you have a statistical calculator, which has a button for this function. Playing with your calculator, you discover the following:

x	**erf(*x*)**
1	0.29793972
0.1	0.03976165
0.01	0.00398929
0.001	0.000398942
0	0

Using this information alone, give an estimate for $\text{erf}'(0)$, the derivative of erf at $x = 0$ to 4 decimal places.
Ans: 0.3989
difficulty: medium section: 2.2

19. In the picture



the quantity $f'(a+h)$ is represented by

A) the slope of the line TV
B) the area of the rectangle PQRS
C) the slope of the line *RU*.
D) the length of the line *TV*.
E) the slope of the line *QU*.
F) the length of the line Q*U*.

Ans: A difficulty: medium section: 2.2

20. Given the following table of values for a Bessel function, $J_0(x)$, estimate the derivative at x = 0.5.

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$J_0(x)$	1.0	.9975	.9900	.9776	.9604	.9385	.9120	.8812	.8463	.8075

Ans: –0.242
difficulty: medium section: 2.2

21. The data in the table report the average improvement in scores of six college freshmen who took a writing assessment before and again after they had *x* hours of tutoring by a tutor trained in a new method of instruction. When *f*(*x*)>0 the group showed improvement on average.

x	2	3.5	5	6.5	8	9.5	11
f(x)	-2	-1	0	3	7	9	10

a) Find the average change in score from 6.5 to 9.5 hours of tutoring.
b) Estimate the instantaneous rate of change at 8 hours.
c) Approximate the equation of the tangent line at x = 8 hours.
d) Use the tangent line to estimate *f*(8.5).

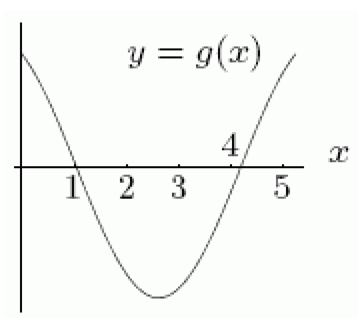
Ans: a) 2.00 points, b) 2.00 points, but answers may vary; c) $y-7=2.00(x-8)$; d) 8 points
difficulty: medium section: 2.2

22. Use the graph of $5.5e^{3x}$ at the point (0, 5.5) to estimate $f'(0)$ to three decimal places.
    A) 16.500 B) 36.823 C) 3.500 D) 146.507
    Ans: A difficulty: easy section: 2.2

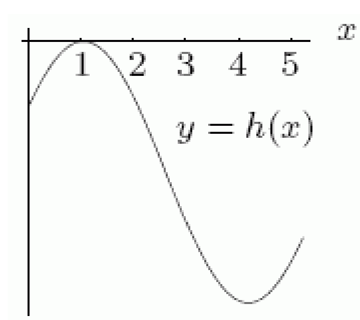
23. A horticulturist conducted an experiment to determine the effects of different amounts of fertilizer on the yield of a plot of green onions. He modeled his results with the function $Y(x) = -0.5(x-2)^2 + 5$ where Y is the yield in bushels and x is the amount of fertilizer in pounds. What are $Y(0.75)$ and $Y'(0.75)$? Give your answers to two decimal places, specify units.
    A) 4.22 bushels, 1.25 bushels/pound, respectively
    B) 4.22 bushels, 6.25 bushels/pound, respectively
    C) 1.25 bushels, 1.56 bushels/pound, respectively
    D) 1.25 bushels, 4.22 bushels/pound, respectively
    Ans: A difficulty: medium section: 2.2

24. Use the limit of the difference quotient to find the derivative of $g(x) = \dfrac{11}{x+1}$ at the point (1, 11/2).
    A) $\dfrac{-11}{4}$ B) $\dfrac{11}{2}$ C) -11 D) $\dfrac{-11}{2}$
    Ans: A difficulty: medium section: 2.2

25. Could the first graph, A be the derivative of the second graph, B?



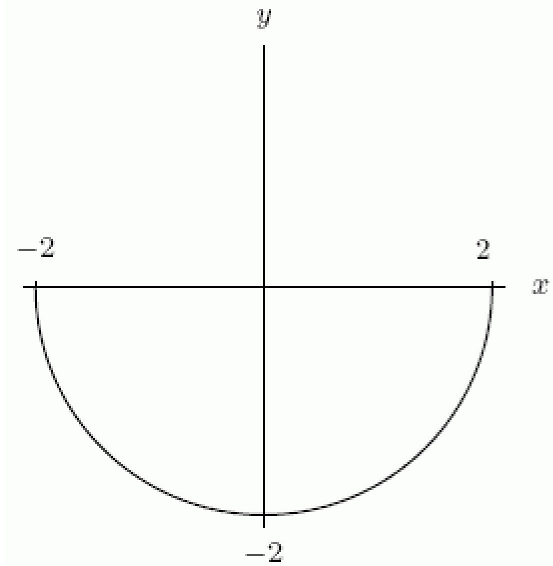
A

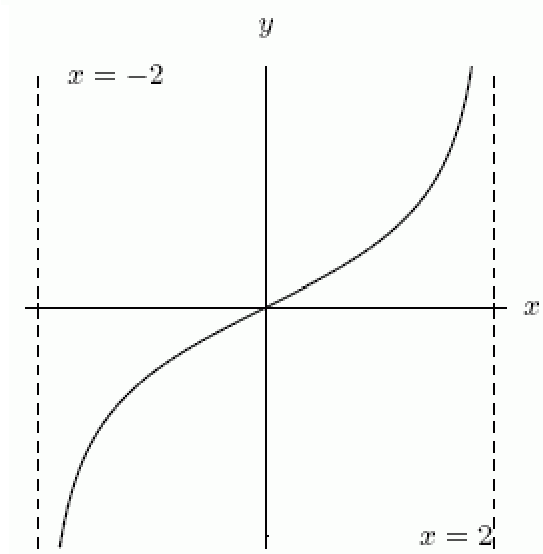


B

Ans: yes
difficulty: medium section: 2.3

26. Could the first graph, A be the derivative of the second graph, B?

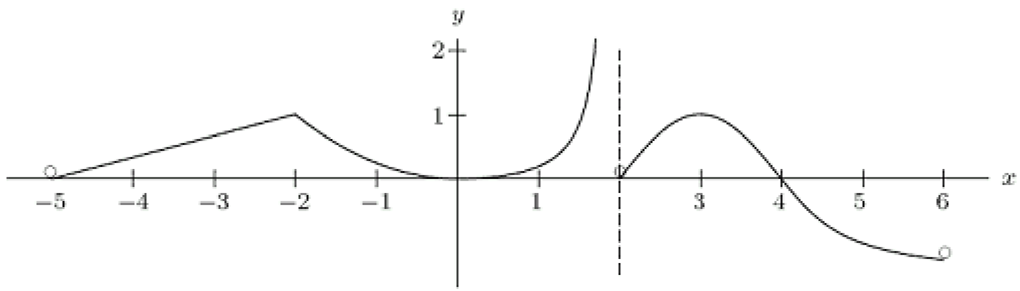




A B

Ans: no
difficulty: medium section: 2.3

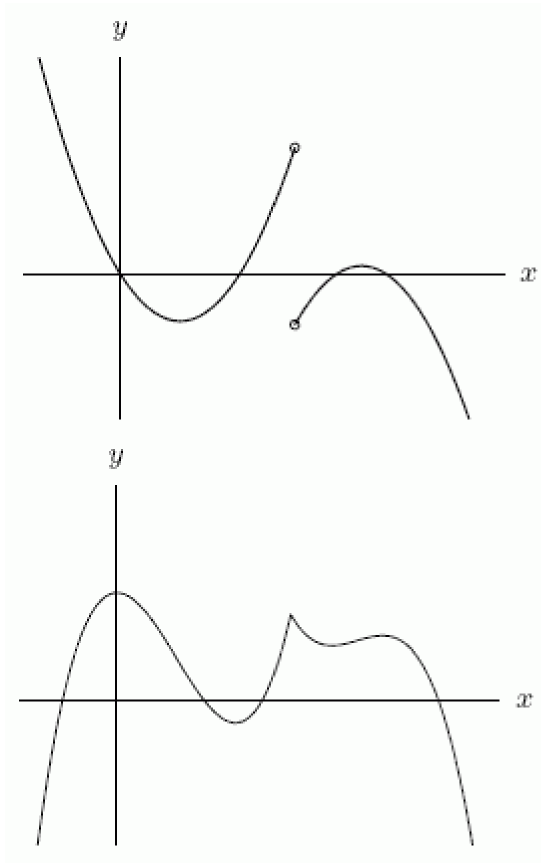
27. Consider the function $y = f(x)$ graphed below. At the point $x = -3$, is $f'(x)$ positive, negative, 0, or undefined?Note: $f(x)$ is defined for $-5 < x < 6$, *except* $x = 2$.



Ans: positive
difficulty: medium section: 2.3

28. Estimate a formula for $f'(x)$ for the function $f(x) = 8^x$. Round constants to 3 decimal places.
Ans: $(2.079)8^x$
difficulty: hard section: 2.3

29. Could the first graph, A be the derivative of the second graph, B?



A B

Ans: yes
difficulty: medium section: 2.3

30. Find the derivative of $g(x) = 2x^2 + 8x - 6$ at $x = 4$ algebraically.
Ans: 24
difficulty: medium section: 2.3

31. To find the derivative of $g(x) = 2x^2 + 5x - 9$ at $x = 8$ algebraically, you evaluate the following expression.

A) $\lim\limits_{h\to 0} \dfrac{2(8+h)^2 - 5(8+h) - 9 - (2\cdot 8^2 + 5\cdot 8 - 9)}{h}$

B) $\dfrac{g(8+1) + g(8)}{h}$

C) $\lim\limits_{h\to\infty} \dfrac{g(h) - g(8)}{h}$

D) All of the above are correct.

E) None of the above is correct.

Ans: A difficulty: medium section: 2.3

32. Find the derivative of $m(x) = 3x^3$ at $x = 1$ algebraically.

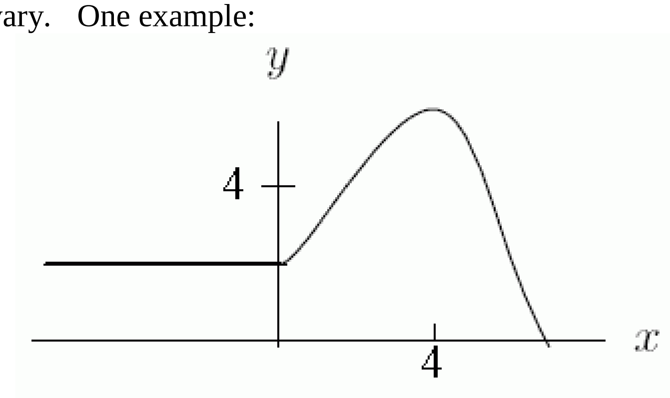
Ans: 9

difficulty: medium section: 2.3

33. Draw the graph of a continuous function $y = g(x)$ that satisfies the following three conditions:

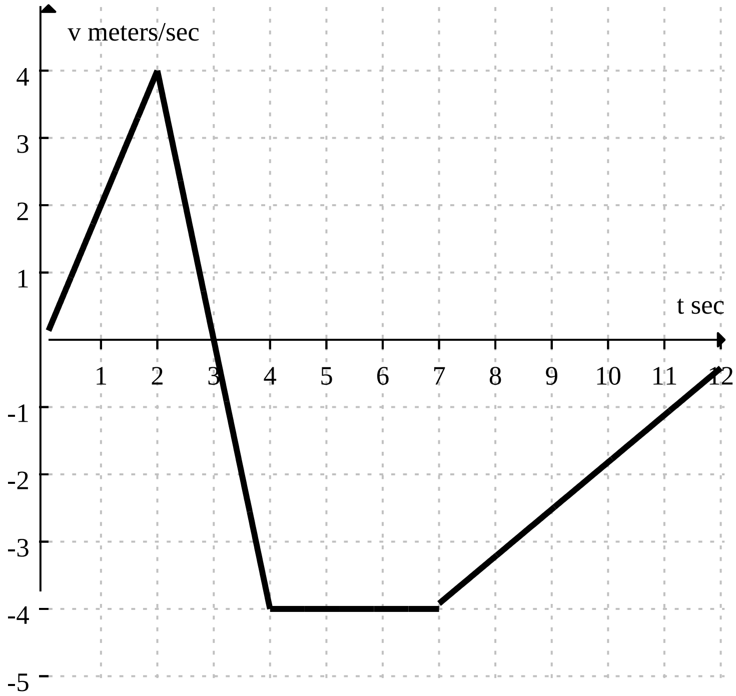
- $g'(x) = 0$ for $x < 0$
- $g'(x) > 0$ for $0 < x < 4$
- $g'(x) < 0$ for $x > 4$

Ans: Answers will vary. One example:



difficulty: medium section: 2.3

34. The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.

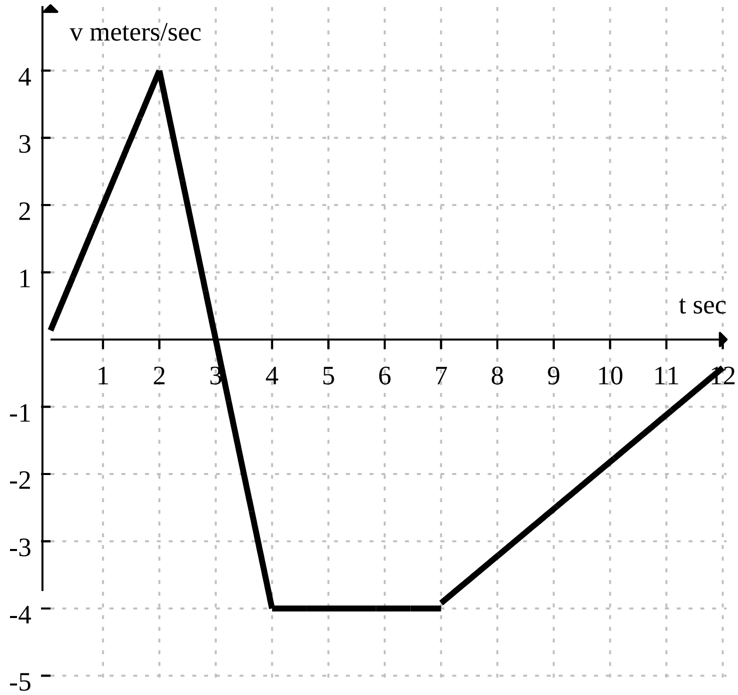


At what time(s) does the bug turn around?

A) At 3 seconds.
B) At 2 seconds and again at 7 seconds.
C) At 4 seconds and again at 7 seconds.
D) Never.

Ans: A difficulty: easy section: 2.3

35. The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.

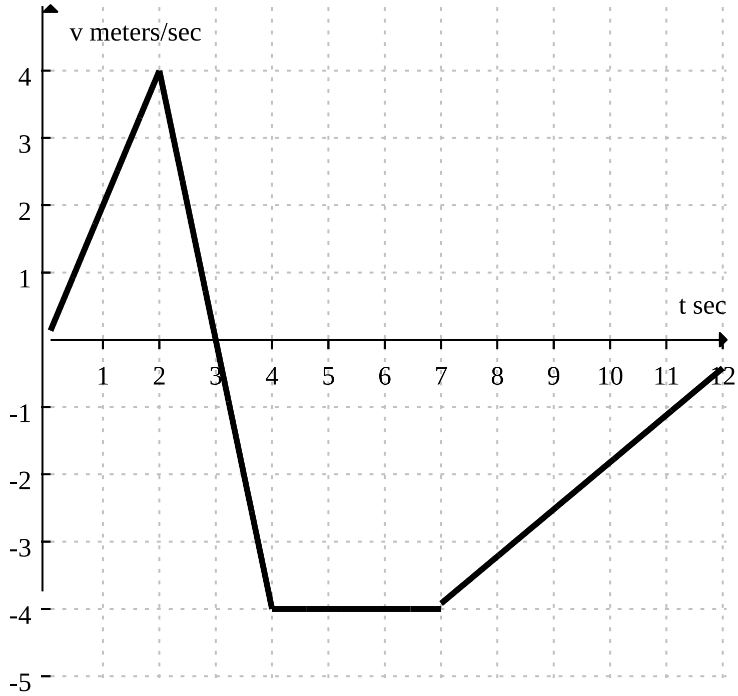


When is the bug moving at a constant speed?

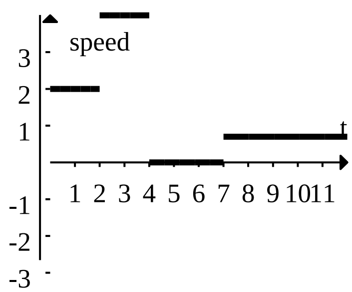
A) Between 4 and 7 seconds.
B) Whenever the velocity is linear with a positive slope.
C) Whenever the velocity is linear with a negative slope.
D) When the velocity is equal to zero.

Ans: A difficulty: easy section: 2.3

36. he graph below shows the velocity of a bug traveling along a straight line on the classroom floor.



Graph the bug's speed at time, t. How does it differ from the bug's velocity?



Ans:

Speed is always non-negative, but has the same magnitude as the velocity.

difficulty: medium section: 2.3

37. Use the definition of the derivative function to find a formula for the slope of the graph of $f(t) = \dfrac{1}{9t+1}$.

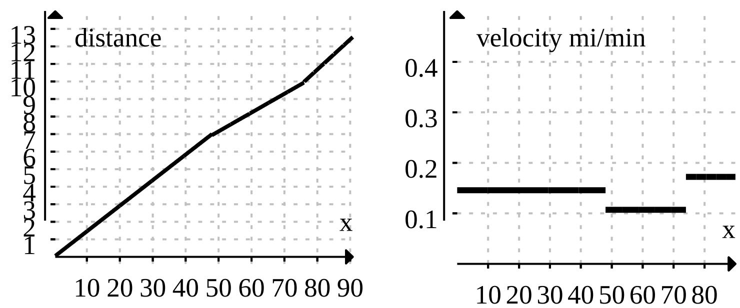
Ans: $\dfrac{-9}{(9t+1)^2}$

difficulty: hard section: 2.3

38. What is the equation of the tangent line to the graph of $f(x) = x^3$ at the point (2, 8)?
A) $y = 12x - 16$ B) $y = 2x + 8$ C) $y = 8x + 2$ D) $y = 4x + 64$
Ans: A difficulty: easy section: 2.3

39. The definition of the derivative function is $f'(x) = \dfrac{f(x+h) - f(x)}{h}$
Ans: False difficulty: easy section: 2.3

40. A runner competed in a half marathon in Anaheim, a distance of 13.1 miles. She ran the first 7 miles at a steady pace in 48 minutes, the second 3 miles at a steady pace in 28 minutes and the last 3.1 miles at a steady pace in 18 minutes.
a) Sketch a well-labeled graph of her distance completed with respect to time.
b) Sketch a well-labeled graph of her velocity with respect to time.

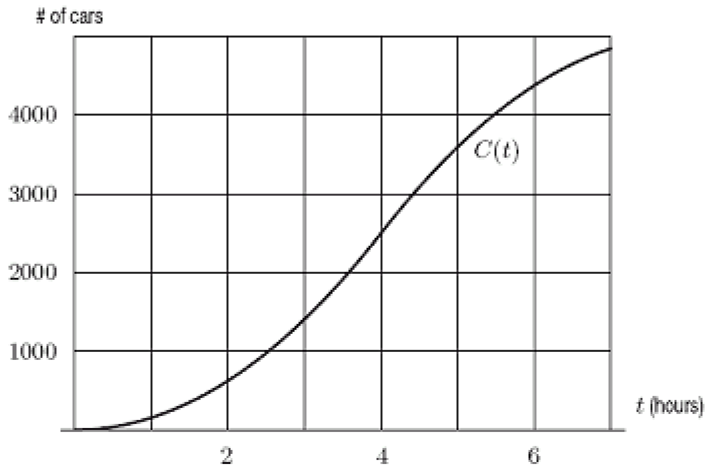


Ans:
Answers will vary. The graphs above give one possibility.
difficulty: medium section: 2.3

41. Which of the following is NOT a way to describe the derivative of a function at a point?
A) slope of the tangent line
B) slope of the curve
C) y-intercept of the tangent line
D) limit of the difference quotient
E) limit of the slopes of secant lines
F) limit of the average rates of change
Ans: C difficulty: easy section: 2.3

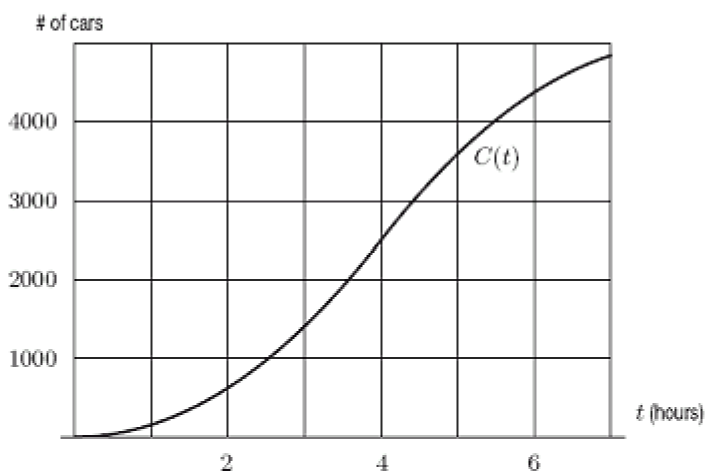
42. Suppose that $f(T)$ is the cost to heat my house, in dollars per day, when the outside temperature is $T\ {}^\circ F$. If $f(28) = 11.10$ and $f'(28) = -0.12$, approximately what is the cost to heat my house when the outside temperature is $25^\circ F$?
Ans: \$11.46
difficulty: easy section: 2.4

43. To study traffic flow along a major road, the city installs a device at the edge of the road at 1:00a.m. The device counts the cars driving past, and records the total periodically. The resulting data is plotted on a graph, with time (in hours since installation) on the horizontal axis and the number of cars on the vertical axis. The graph is shown below; it is the graph of the function $C(t)$ = Total number of cars that have passed by after t hours. When is the traffic flow greatest?



A) 2:00 am B) 3:00 am C) 4:00 am D) 5:00 am
Ans: D difficulty: medium section: 2.4

44. To study traffic flow along a major road, the city installs a device at the edge of the road at 3:00a.m. The device counts the cars driving past, and records the total periodically. The resulting data is plotted on a graph, with time (in hours since installation) on the horizontal axis and the number of cars on the vertical axis. The graph is shown below; it is the graph of the function $C(t)$ = Total number of cars that have passed by after t hours. From the graph, estimate $C'(6)$.



A) 600 B) 900 C) 1200 D) 1500
Ans: A difficulty: medium section: 2.4

45. Every day the Office of Undergraduate Admissions receives inquiries from eager high school students. They keep a running account of the number of inquiries received each day, along with the total number received until that point. Below is a table of *weekly* figures from about the end of August to about the end of October of a recent year.

Week of	**Inquiries That Week**	**Total for Year**
8/28-9/01	1085	11,928
9/04-9/08	1193	13,121
9/11-9/15	1312	14,433
9/18-9/22	1443	15,876
9/25-9/29	1588	17,464
10/02-10/06	1746	19,210
10/09-10/13	1921	21,131
10/16-10/20	2113	23,244
10/23-10/27	2325	25,569

One of these columns can be interpreted as a rate of change. Which one is it?
A) the first B) the second C) the third
Ans: B difficulty: easy section: 2.4

46. Every day the Office of Undergraduate Admissions receives inquiries from eager high school students. They keep a running account of the number of inquiries received each day, along with the total number received until that point. Below is a table of *weekly* figures from about the end of August to about the end of October of a recent year.

Week of	**Inquiries That Week**	**Total for Year**
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9/18-9/22	1443	15,876
9/25-9/29	1588	17,464
10/02-10/06	1746	19,210
10/09-10/13	1921	21,131
10/16-10/20	2113	23,244
10/23-10/27	2325	25,569

Based on the table determine a formula that approximates the total number of inquiries received by a given week. Use your formula to estimate how many inquiries the admissions office will have received by November 24.
Ans: 37,435
difficulty: medium section: 2.4

47. Let $L(r)$ be the amount of board-feet of lumber produced from a tree of radius r (measured in inches). What does $L(16)$ mean in practical terms?
    A) The amount of board-feet of lumber produced from a tree with a radius of 16 inches.
    B) The radius of a tree that will produce 16 board-feet of lumber.
    C) The rate of change of the amount of lumber with respect to radius when the radius is 16 inches (in board-feet per inch).
    D) The rate of change of the radius with respect to the amount of lumber produced when the amount is 16 board-feet (in inches per board-foot).

    Ans: A difficulty: easy section: 2.4

48. Let $t(h)$ be the temperature in degrees Celsius at a height h (in meters) above the surface of the earth. What does $t'(1200)$ mean in practical terms?
    A) The temperature in degrees Celsius at a height 1200 meters above the surface of the earth.
    B) The height above the surface of the earth at which the temperature is 1200 degrees Celsius.
    C) The rate of change of temperature with respect to height at 1200 meters above the surface of the earth (in degrees per meter).
    D) The rate of change of height with respect to temperature when the temperature is 1200 degrees Celsius (in meters per degree).

    Ans: C difficulty: easy section: 2.4

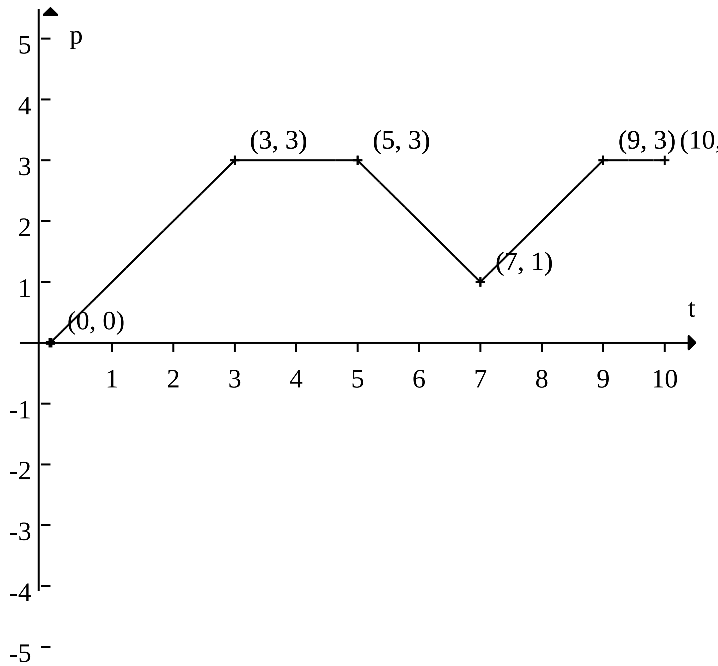
49. Let $t(h)$ be the temperature in degrees Celsius at a height of h meters above the surface of the earth. What does h such that $t(h) = 8$ mean in practical terms?
    A) The temperature in degrees Celsius at a height 8 meters above the surface of the earth.
    B) The height above the surface of the earth at which the temperature is 8 degrees Celsius.
    C) The rate of change of temperature with respect to height at 8 meters above the surface of the earth (in degrees per meter).
    D) The rate of change of height with respect to temperature when the temperature is 8 degrees Celsius (in meters per degree).

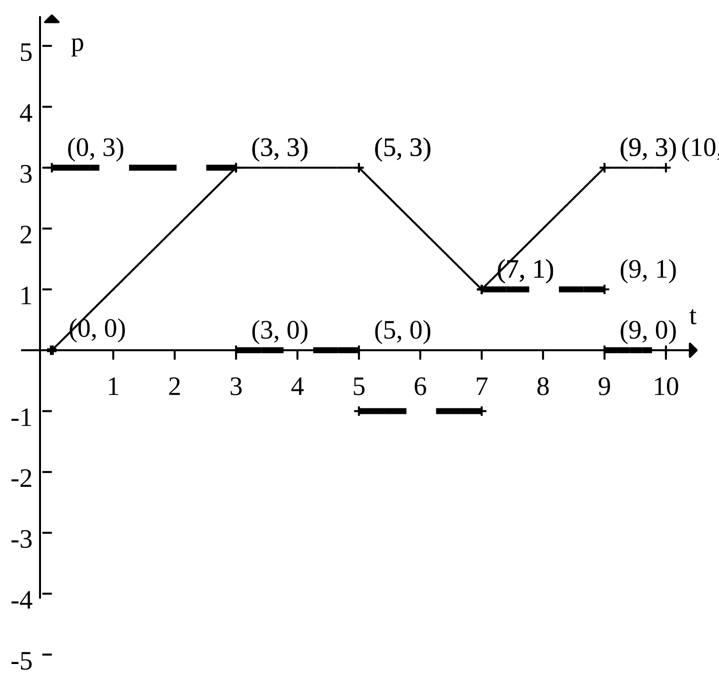
    Ans: B difficulty: easy section: 2.4

50. Let $t(h)$ be the temperature in degrees Celsius at a height of h meters above the surface of the earth. What does $t(h) + 15$ mean in practical terms?
   A) The temperature in degrees Celsius at a height h meters above the surface of the earth plus an additional 15 degrees
   B) The height above the surface of the earth at which the temperature is h degrees Celsius plus an additional 15 meters.
   C) The rate of change of temperature with respect to height at 15 additional meters above the surface of the earth (in degrees per meter).
   D) The rate of change of height with respect to temperature when the temperature is 15 additional degrees Celsius (in meters per degree).
   Ans: A difficulty: easy section: 2.4

51. A concert promoter estimates that the cost of printing p full color posters for a major concert is given by a function Cost = $c(p)$ where p is the number of posters produced.
   a) Interpret the meaning of the statement $c(450) = 5400$.
   b) Interpret the meaning of the statement $c'(450) = 11$.
   Ans: a) It costs $5400.00 to produce 450 posters.
   b) When 450 posters have been produced, it costs $11.00 to produce an additional poster.
   difficulty: easy section: 2.4

52. The graph below gives the position of a spider moving along a straight line on the forest floor for 10 seconds. On the same axes, sketch a graph of the spider's velocity over the 10 seconds. Then write a description of the spider's movement for the 10 second period.





Ans:

The dashed line represents the spider's velocity.

For the first three seconds the spider moves forward at 3 feet/sec. It stops for the next 2 seconds, turns around and goes back in the opposite direction for at a speed of 1 ft/sec for 2 seconds. It turns around again and goes forward at 1 ft/sec for the next two seconds, then stops for the final second.

difficulty: medium section: 2.4

53. A typhoon is a tropical cyclone, like a hurricane, that forms in the northwestern Pacific Ocean. The wind speed of a typhoon is given by a function $W = w(r)$ where W is measured in meters/sec., and r is measured in kilometers from the center of the typhoon. What does the statement that $w'(15) > 0$ tell you about the typhoon?

A) At a distance of 15 kilometers from the center of the typhoon, the wind speed is increasing.

B) At a distance of 15 kilometers from the center of the typhoon, the wind speed is positive.

C) The wind speed of the typhoon is 15 meters per second at any distance from the center of the typhoon.

Ans: A difficulty: medium section: 2.4

54. The cost in dollars to produce q bottles of a prescription skin treatment is given by the function $C(q) = 0.08q^2 + 75q + 900$. The manufacturing process is difficult and costly when large quantities are produced. The marginal cost of producing one additional bottle when q bottles have been produced is the derivative $\dfrac{dC}{dq}$.

a) Find the marginal cost function.

b) Compute $C(50)$ and explain what the number means in terms of cost and production.

c) Compute $C'(50)$ and explain what the number means in terms of cost and production.

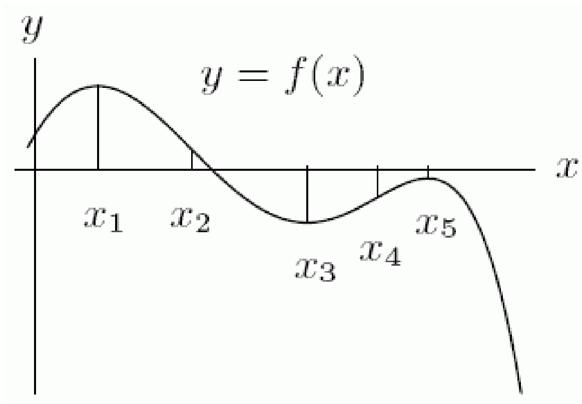
Ans: a) $\dfrac{dC}{dq} = 0.16q + 75$

b) C(50) = \$4850.00 is the cost of producing 50 bottles of the skin treatment.

c) C'(50) = \$83.00 per bottle of the cost of producing an additional bottle when 50 have already been produced.

difficulty: medium section: 2.4

55. The graph of $f(x)$ is given in the following figure. What happens to $f'(x)$ at the point x_1?



A) $f'(x)$ has an inflection point.
B) $f'(x)$ has a local minimum or maximum.
C) $f'(x)$ changes sign.
D) none of the above
Ans: C difficulty: hard section: 2.5

56. Esther is a swimmer who prides herself in having a smooth backstroke. Let $s(t)$ be her position in an Olympic size (50-meter) pool, as a function of time ($s(t)$ is measured in meters, t is seconds). Below we list some values of $s(t)$ for a recent swim. Find Esther's average speed over the entire swim in meters per second. Round to 2 decimal places.

t	0	3.0	8.6	14.64	21.35	28.06	32.33	39.04	46.36.	54.9	61
$s(t)$	0	10	20	30	40	50	40	30	20	10	0

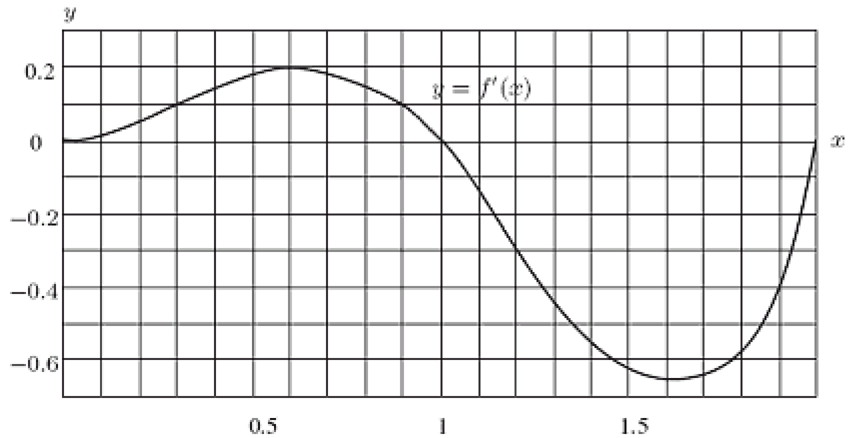
Ans: 1.64
difficulty: medium section: 2.5

57. Esther is a swimmer who prides herself in having a smooth backstroke. Let $s(t)$ be her position in an Olympic size (50-meter) pool, as a function of time ($s(t)$ is measured in meters, t is seconds). Below we list some values of $s(t)$, for a recent swim. Based on the data, was Esther's instantaneous speed ever greater than 3 meters/second?

t	0	3.0	8.6	14.6	20.8	27.6	31.9	38.1	45.8	53.9	60
$s(t)$	0	10	20	30	40	50	40	30	20	10	0

Ans: yes
difficulty: medium section: 2.5

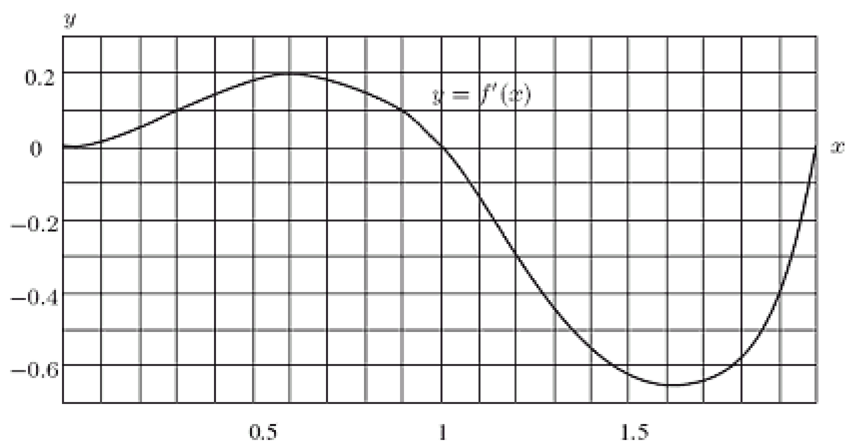
58. The graph below represents the *rate of change* of a function f with respect to x; i.e., it is a graph of f'. You are told that $f(0) = 0$. What can you say about $f(x)$ at the point $x = 1.3$? Mark all that apply.



A) $f(x)$ is decreasing.
B) $f(x)$ is increasing.
C) $f(x)$ is concave up.
D) $f(x)$ is concave down.

Ans: A, D difficulty: easy section: 2.5

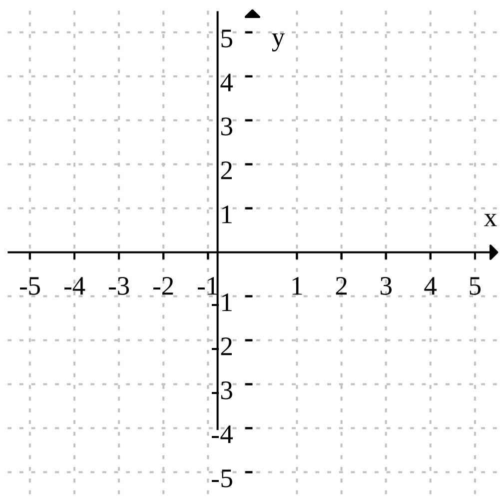
59. The graph below represents the *rate of change* of a function f with respect to x; i.e., it is a graph of f'. You are told that $f(0) = -2$. For approximately what value of x other than $x = 0$ in the interval $0 \le x \le 2$ does $f(x) = -2$?



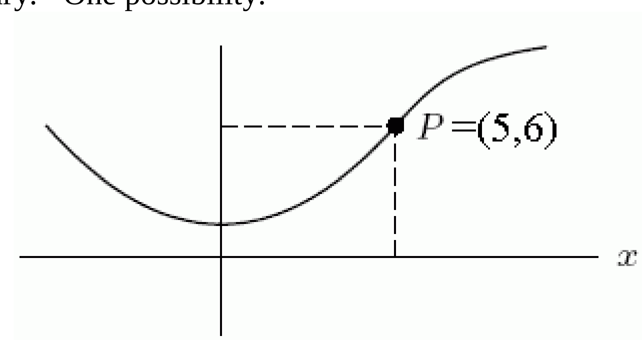
A) 0.6 B) 1 C) 1.4 D) 2 E) None of the above

Ans: C difficulty: medium section: 2.5

60. On the axes below, sketch a smooth, continuous curve (i.e., no sharp corners, no breaks) which passes through the point *P*(5, 6), and which clearly satisfies the following conditions:
- Concave up to the left of *P*
- Concave down to the right of *P*
- Increasing for $x > 0$
- Decreasing for $x < 0$
- Does *not* pass through the origin.

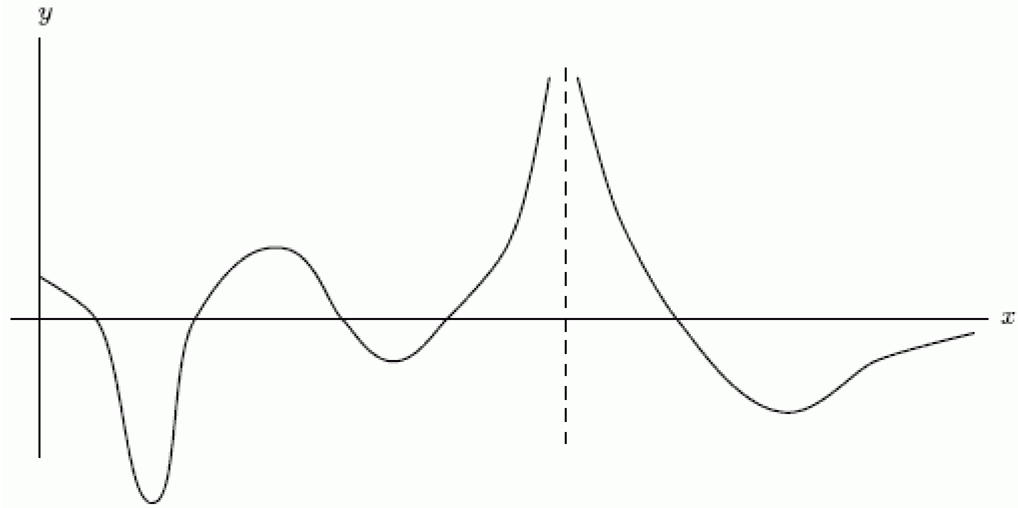


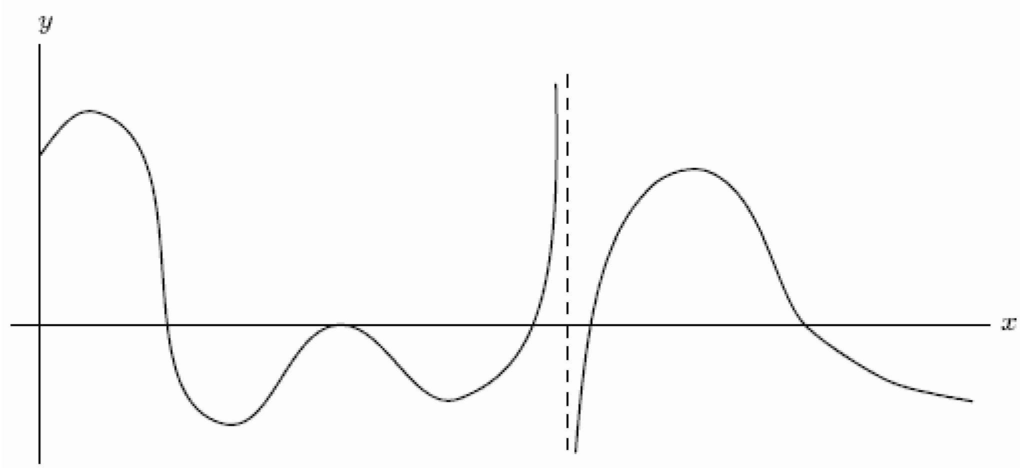
Ans: Answers will vary. One possibility:



difficulty: easy section: 2.5

61. One of the following graphs is of $f(x)$, and the other is of $f'(x)$. Is $f(x)$ the first graph or the second graph?





Ans: second
difficulty: medium section: 2.5

62. Given the following data about a function f, estimate the rate of change of the derivative f' at $x = 4.5$.

.

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

Ans: 4
difficulty: medium section: 2.5

63. A function defined for all x has the following properties:
- f is increasing
- f is concave down
- $f(3) = 2$
- $f'(3) = 1/2$

How many zeros does $f(x)$ have in the interval $1 \le x \le 3$?
Ans: 1
difficulty: medium section: 2.5

64. A function defined for all x has the following properties:
- f is increasing
- f is concave down
- $f(4) = 2$
- $f'(4) = 1/2$

Is it possible that $f'(1) = \frac{1}{4}$?
Ans: no
difficulty: medium section: 2.5

65. Assume that f is a differentiable function defined on all of the real line. Is it possible that $f > 0$ everywhere, $f' > 0$ everywhere, and $f'' < 0$ everywhere?
Ans: no
difficulty: medium section: 2.5

66. Assume that f and g are differentiable functions defined on all of the real line. Is it possible that $f'(x) > g'(x)$ for all x and $f(x) < g(x)$ for all x?
Ans: yes
difficulty: medium section: 2.5

67. Assume that f and g are differentiable functions defined on all of the real line. If $f'(x) = g'(x)$ for all x and if $f(x_0) = g(x_0)$ for some x_0, then must $f(x) = g(x)$ for all x?
Ans: yes
difficulty: medium section: 2.5

68. Assume that f and g are differentiable functions defined on all of the real line. If $f' > 0$ everywhere and $f > 0$ everywhere then must $\lim_{x \to +\infty} f(x) = \infty$?
Ans: no
difficulty: medium section: 2.5

69. Suppose a function is given by a table of values as follows:

x	1.1	1.3	1.5	1.7	1.9	2.1
$f(x)$	14	17	23	25	26	27

Give your best estimate of $f''(1.9)$.
Ans: 0
difficulty: medium section: 2.5

70. If the Figure 1 is $f(x)$, could Figure 2 be $f''(x)$?

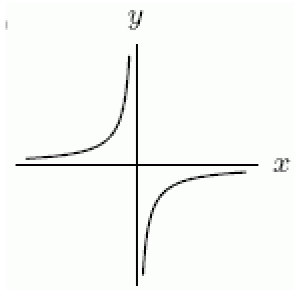


Figure 1

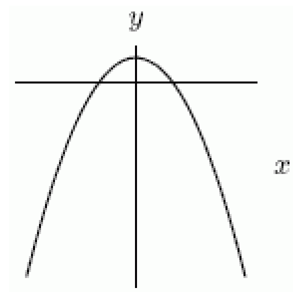


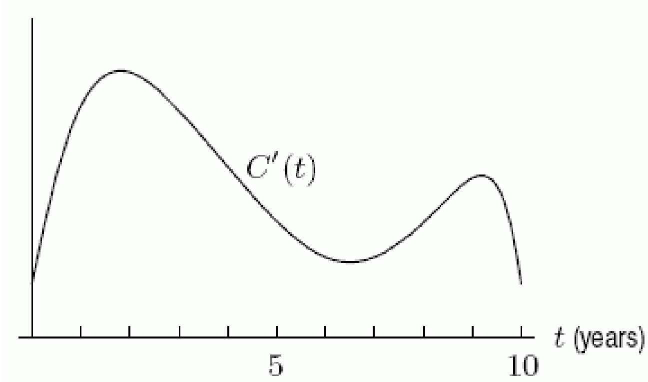
Figure 2

Ans: no
difficulty: medium section: 2.5

71. The cost of mining a ton of coal is rising faster every year. Suppose $C(t)$ is the cost of mining a ton of coal at time t. Must $C''(t)$ be concave up?
Ans: no
difficulty: medium section: 2.5

72. Let $S(t)$ represent the number of students enrolled in school in the year t. If the number of students enrolling is increasing faster and faster, then is $S'(t)$ positive, negative, or 0?
Ans: positive
difficulty: medium section: 2.5

73. A company graphs $C'(t)$, the derivative of the number of pints of ice cream sold over the past ten years. At approximately what year was $C''(t)$ greatest?



Ans: 0
difficulty: medium section: 2.5

74. A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 17.00 meters per second and will attain a height of $s(t) = -0.8t^2 + 17.00t$ meters in t seconds. Find a formula for the velocity of the golf ball at time t.
A) $v(t) = -1.6t + 17.00$ meters/sec
B) $v(t) = -0.8t + 8.50$ meters/sec
C) $v(t) = -1.36$ meters/sec
D) $v(t) = -16t^2 + 17.00$ meters/sec
Ans: A difficulty: medium section: 2.5

75. A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 14.00 meters per second and will attain a height of $s(t) = -0.8t^2 + 14.00t$ meters in t seconds. What is the acceleration of the golf ball at time t?
A) -1.6 meters/sec/sec
B) $-1.6t$ meters/sec^2
C) $-0.8t$ meters/sec/sec
D) 14.00 meters/sec^2
Ans: A difficulty: medium section: 2.5

76. A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 20 meters per second and will attain a height of $s(t) = -0.8t^2 + 20t$ meters in t seconds. How fast is the golf ball going at its high point?
A) 0 meters/sec B) -0.8 meters/sec C) 20 meters/sec D) -20 meters/sec
Ans: A difficulty: easy section: 2.5

77. A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 20 meters per second and will attain a height of $s(t) = -0.8t^2 + 20t$ meters in t seconds. On Earth, its height would be given by $-4.9t^2 + 20t$ .Compare the velocity and acceleration of the golf ball on the moon after two seconds with its velocity and acceleration on Earth after two seconds.
Ans: Comparisons will vary. The numerical results are:
On the moon: velocity -3.2 m/s and acceleration -1.6 m/s^2
On the Earth: velocity -19.6 m/s and acceleration - 9.8 m/s^2

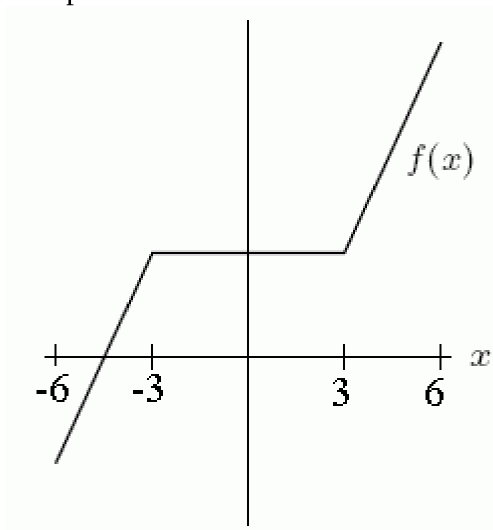
difficulty: medium section: 2.5

78. The Chief Financial Officer of an insurance firm reports to the board of directors that the cost of claims is rising more slowly than last quarter. Let C(t) be the cost of claims. Select all statements that apply.
A) C is positive.
B) C is negative.
C) The first derivative of C is positive.
D) The first derivative of C is negative.
E) The second derivative of C is positive.
F) The second derivative of C is negative.
Ans: A, C, F difficulty: medium section: 2.5

79. A husband and wife purchase life insurance policies. Over the next 40 years, one policy pays out when the husband dies, and the other pays out when both husband and wife die. Their life expectancy is 20 years, and the probability that both die before year t is given by the function $f_T(t) = \frac{1}{1600}t^2$. How fast is the probability that both are dead increasing in 25 years?
A) 0.0313 B) 0.3906 C) 0.0500 D) 50.0006
Ans: A difficulty: hard section: 2.5

80. Sketch a graph $y = f(x)$ that is continuous everywhere on $-6 < x < 6$ but not differentiable at $x = -3$ or $x = 3$.

Ans: Many possible. One example:

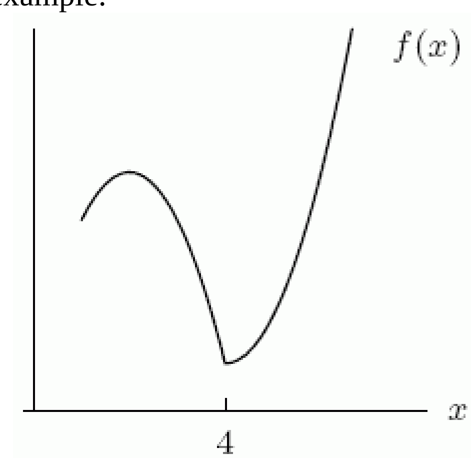


difficulty: easy section: 2.6

81. Sketch a graph of a continuous function $f(x)$ with the following properties:

- $f'(x) < 0$ for $x < 4$
- $f'(x) > 0$ for $x > 4$
- $f'(4)$ is undefined

Ans: Many possible. One example:



difficulty: easy section: 2.6

82. Is the graph of $f(x) = |x + 3|$ continuous at $x = -3$?

Ans: yes

difficulty: easy section: 2.6

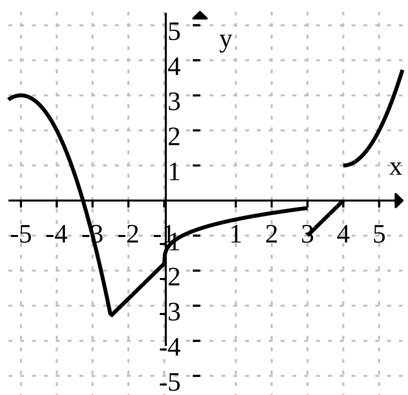
83. Is the graph of $f(x)=\dfrac{1}{x+9}$ continuous at x = –9?
Ans: no
difficulty: easy section: 2.6

84. Given the function $h(r)=\begin{cases}1-\sin(\pi r/2) & -1\le r\le 1\\ 0 & r<-1,\ r>1\end{cases}$. Is $h(r)$ differentiable at r = –1?
Ans: no
difficulty: medium section: 2.6

85. Describe two ways that a continuous function can fail to have a derivative at a point, x = a. Illustrate your description with graphs.
Ans: Answers will vary but will describe two of: cusps, corners, vertical tangents.
difficulty: medium section: 2.6

86. A function that has an instantaneous rate of change of 3 at a point (x, y) can fail to be continuous at that point.
Ans: False difficulty: easy section: 3.6

87. Based on the graph of $f(x)$ below:
a) List all values of x for which f is NOT differentiable.
b) List all values of x for which f is NOT continuous.
c) List all values of x for which $f'(x) = 0$.



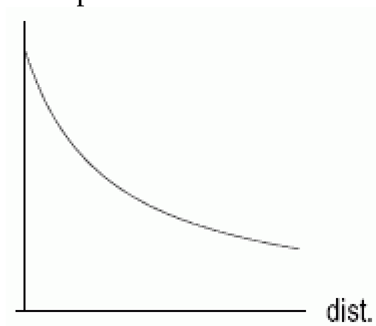
Ans: a) Not differentiable at x = -2.5, -1, 3, 4
b) Not continuous at x = 3, 4
c) Derivative of zero at x = -5.
difficulty: easy section: 3.8

88. Let $f(x) = x^{\sin x}$. Using your calculator, estimate $f'(7)$ to 3 decimal places.
Ans: 5.605
difficulty: medium section: 2 review

89. Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^\circ F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	**Temp. (° F)**	**illumination (%)**
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

Does the following graph show temperature or illumination as a function of distance?



Ans: illumination
difficulty: easy section: 2 review

90. Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^{\circ}F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	**Temp. (° F)**	**illumination (%)**
0	56	100
1	55.5	85
2	54.5	75
3	53	67
4	51	60
5	48	56
6	44.5	53

What is the average rate at which the temperature is changing (in degrees per foot) when the illumination drops from 75% to 56%? Round to 2 decimal places.
Ans: 2.17
difficulty: medium section: 2 review

91. Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^{\circ}F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	**Temp. (° F)**	**illumination (%)**
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

You can still read your watch when the illumination is about 55%, so somewhere between 5 and 6 feet. Can you read your watch at 5.5 feet?
A) yes B) no C) cannot tell
Ans: B difficulty: medium section: 2 review

92. Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^{\circ}F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	**Temp. (° F)**	**illumination (%)**
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

Suppose you know that at 6 feet the instantaneous rate of change of the illumination is –3.5 % candle power/ft. At 7 feet, the illumination is approximately **_____** % candle power.
Ans: 49.5
difficulty: medium section: 2 review

93. Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^{\circ}F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

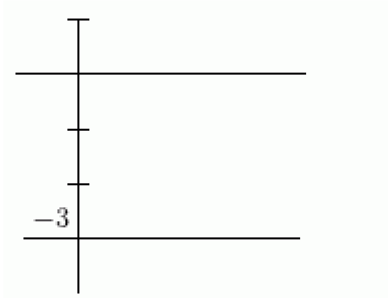
distance(feet)	**Temp. (° F)**	**illumination (%)**
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

You are cold when the temperature is below 40°F. You are in the dark when the illumination is at most 50% of one candle power. Suppose you know that at 6 feet the instantaneous rate of change of the temperature is -4.5° F/ft and the instantaneous rate of change of illumination is -3% candle power/ft. Are you in the dark before you are cold, or cold before you are dark?

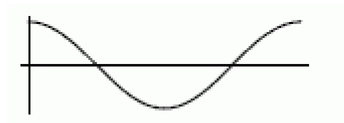
A) You are cold before you are in the dark.
B) You are in the dark before you are cold.

Ans: A difficulty: medium section: 2 review

94. Could the Function 1 be the derivative of the Function 2?



Function 1



Function 2

Ans: no
difficulty: medium section: 2 review

95. Is the function $f(x) = \dfrac{x^2|2x-4|}{x-2}$ continuous at $x = 2$?
Ans: no
difficulty: easy section: 2 review

96. Mark all TRUE statements.
A) $f(x) = |x-3|$ is continuous at $x = 0$.
B) $g(x) = \sqrt[3]{x}$ fails to be differentiable at $x = 0$.
C) $h(x) = |x+5|$ is not continuous at $x = -5$.
D) $r(x) = \dfrac{(x-3)^2}{(x-3)}$ is continuous for all values of x.
E) Any polynomial function is differentiable for all values of x.
Ans: A, B, E difficulty: easy section: 2 review

97. In the lobby of a university mathematics building, there is a large bronze sculpture in the shape of a parabola. When the sun shines on the parabola at a certain time, its shadow falls on a mural with a coordinate plane that reveals the sculpture's height as the function $f(x) = -x^2 + 18$. A spider drops from its web onto the sculpture at the point (1, 17). What is the slope of the parabola at the point where the spider lands?
A) –2 B) –20 C) 17 D) –17 E) None of the above
Ans: A difficulty: easy section: 2 review

98. If $f(x) = x^4$, what is $f'(3)$?
A) 81 B) 3 C) 108 D) 12 E) 4
Ans: C difficulty: easy section: 2 review

99. The derivative of $f(t) = e^{\pi}$ is $\pi e^{\pi-1}$.
Ans: False difficulty: easy section: 2 review