3

MOTION IN A PLANE

Answers to Multiple-Choice Problems

1. B **2.** A **3.** C **4.** C **5.** A **6.** B **7.** A **8.** C **9.** A **10.** C **11.** D **12.** B **13.** B

Solutions to Problems

3.1. Set Up: The magnitude A of a vector is $A = \sqrt{A_x^2 + A_y^2}$. We are given $A = 25.0$ m/s and $A_y = -13.0$ m/s.

Solve: Solving for the x component of the vector, we find

$$A^2 = A_x^2 + A_y^2 \Rightarrow A_x^2 = A^2 - A_y^2 \Rightarrow A_x = \pm\sqrt{A^2 - A_y^2} = \pm\sqrt{(25.0\text{ m/s})^2 - (13.0\text{ m/s})^2} = \pm 21.4\text{ m/s}$$

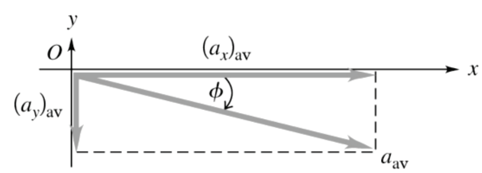
Solve: The x component of the vector A can point in the positive or negative x direction. Both will give the same magnitude because the component is squared in the expression for vector magnitude.

3.2. Set Up: Since the velocity is expressed in units of km/h and the time interval is in seconds, it is convenient to express the acceleration in mixed units of km/h·s.

Solve: (a)

$$(a_x)_{\text{av}} = \frac{\Delta \upsilon_x}{\Delta t} = \frac{848\text{ km/h} - 665\text{ km/h}}{7.47\text{ s} - 4.45\text{ s}} = 60.6\text{ km/h}\cdot\text{s};\ (a_y)_{\text{av}} = \frac{\Delta \upsilon_y}{\Delta t} = \frac{350\text{ km/h} - 420\text{ km/h}}{7.47\text{ s} - 4.45\text{ s}} = -23.2\text{ km/h}\cdot\text{s}$$

(b) $\vec{a}_{\text{av}}$ and its components are shown in the figure below.

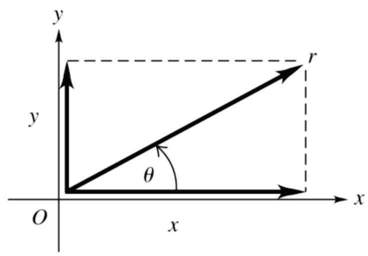


$$a_{\text{av}} = \sqrt{(a_x)_{\text{av}}^2 + (a_y)_{\text{av}}^2} = 64.9\text{ km/h}\cdot\text{s};\ \tan\phi = \frac{(\upsilon_y)_{\text{av}}}{(\upsilon_x)_{\text{av}}}\ \text{ and }\ \phi = 20.9°,\ \text{below the } +x \text{ axis.}$$

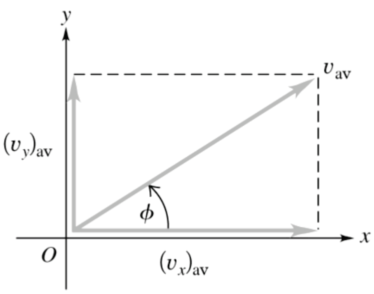
***3.3. Set Up:** Coordinates of point A are (2.0 m, 1.0 m) and for point B they are (10.0 m, 6.0 m).

Solve: (a) At $A, x = 2.0$ m, $y = 1.0$ m.

(b) $\vec{r}$ and its components are shown in Figure (a) below. $r = \sqrt{x^2 + y^2} = 2.2$ m; $\tan\theta = \frac{y}{x}$ and $\theta = 26.6°$, counterclockwise from the $+x$ axis.



(a)



(b)

(c) $(\upsilon_x)_{av} = \frac{\Delta x}{\Delta t} = \frac{10.0 \text{ m} - 2.0 \text{ m}}{1.50 \text{ s}} = 5.3 \text{ m/s}$; $(\upsilon_y)_{av} = \frac{\Delta y}{\Delta t} = \frac{6.0 \text{ m} - 1.0 \text{ m}}{1.50 \text{ s}} = 3.3 \text{ m/s}$

(d) $\vec{\upsilon}_{av}$ and its components are shown in Figure (b) above. $\upsilon_{av} = \sqrt{(\upsilon_x)^2_{av} + (\upsilon_y)^2_{av}} = 6.2$ m/s; $\tan\phi = \frac{(\upsilon_y)_{av}}{(\upsilon_x)_{av}}$ and $\phi = 32°$, counterclockwise from the $+x$ axis.

Reflect: The displacement of the dragonfly is in the direction of $\vec{\upsilon}_{av}$.

3.4. Set Up: Let the x axis point due east and the y axis point due north. The x velocity changes from $+8.00$ m/s to 0 m/s and the y velocity changes from 0 m/s to -8.80 m/s in 4.25 s.

Solve: (a) $a_{av,\,x} = \frac{\Delta \upsilon_x}{\Delta t} = \frac{0 \text{ m/s} - 8.00 \text{ m/s}}{4.25 \text{ s}} = -1.88 \text{ m/s}^2$; $a_{av,\,y} = \frac{\Delta \upsilon_y}{\Delta t} = \frac{-8.80 \text{ m/s} - 0 \text{ m/s}}{4.00 \text{ s}} = -2.07 \text{ m/s}^2$

(b) $a_{av} = \sqrt{a^2_{av,x} + a^2_{av,y}} = \sqrt{(-1.88 \text{ m/s}^2)^2 + (-2.07 \text{ m/s}^2)^2} = 2.80 \text{ m/s}^2$; $\tan\phi = \frac{a_{av,y}}{a_{av,x}} = \frac{-2.07 \text{ m/s}^2}{-1.88 \text{ m/s}^2} = +1.10$ Thus, noting that the acceleration vector is in the third quadrant (since both its components are negative), we have $\phi = 227.7°$, counterclockwise from the $+x$ axis. The direction can also be written as $\theta = 270° - 227.7° = 42.3°$ west of south.

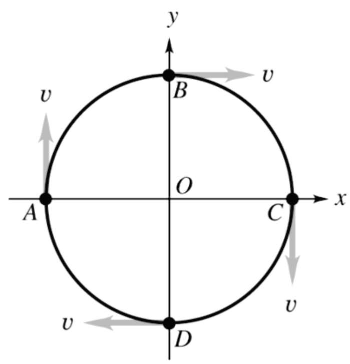
Reflect: Since the westward component of the acceleration vector is slightly less than its southward component, the angle between the acceleration vector and the southward direction is slightly less than 45°.

***3.5. Set Up:** To find the distance traveled by the ball, we need to know the magnitude of its velocity, which we can calculate from the components of the velocity.

Solve: (a) The magnitude of the ball's velocity is $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{(0.5\text{ m/s})^2 + (0.8\text{ m/s})^2} = 0.943\text{ m/s}$. The distance d traveled in the time $t = 0.4$ s is $d = \upsilon t = (0.943\text{ m/s})(0.4\text{ m/s}) = 0.4\text{ m}$.

Reflect: This is a reasonable distance for a pool ball to travel on a pool table.

***3.6. Set Up:** The coordinates of each point are: A, $(-50\text{ m}, 0)$; B, $(0, +50\text{ m})$; C, $(+50\text{ m}, 0)$; D, $(0, -50\text{ m})$. At each point the velocity is tangent to the circular path, as shown in the figure below. The components (υ_x, υ_y) of the velocity at each point are: A, $(0, +6.0\text{ m/s})$; B, $(+6.0\text{ m/s}, 0)$; C, $(0, -6.0\text{ m/s})$; D, $(-6.0\text{ m/s}, 0)$.



Solve: (a) A to B The time for one full lap is

$$t = \frac{2\pi r}{\upsilon} = \frac{2\pi(50\text{ m})}{6.0\text{ m/s}} = 52.4\text{ s}$$

A to B is one-quarter lap and takes $\frac{1}{4}(52.4\text{ s}) = 13.1\text{ s}$.

$$(\upsilon_x)_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{0-(-50\text{ m})}{13.1\text{ s}} = 3.8\text{ m/s};\ (\upsilon_y)_{\text{av}} = \frac{\Delta y}{\Delta t} = \frac{+50\text{ m} - 0}{13.1\text{ s}} = 3.8\text{ m/s}$$

$$(a_x)_{\text{av}} = \frac{\Delta \upsilon_x}{\Delta t} = \frac{6.0\text{ m/s} - 0}{13.1\text{ s}} = 0.46\text{ m/s}^2;\ (a_y)_{\text{av}} = \frac{\Delta \upsilon_y}{\Delta t} = \frac{0 - 6.0\text{ m/s}}{13.1\text{ s}} = -0.46\text{ m/s}^2$$

(b) A to C $t = \frac{1}{2}(52.4\text{ s}) = 26.2\text{ s}$

$$(\upsilon_x)_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{+50\text{ m} - (-50\text{ m})}{26.2\text{ s}} = 3.8\text{ m/s};\ (\upsilon_y)_{\text{av}} = \frac{\Delta y}{\Delta t} = 0$$

$$(a_x)_{\text{av}} = \frac{\Delta \upsilon_x}{\Delta t} = 0;\ (a_y)_{\text{av}} = \frac{\Delta \upsilon_y}{\Delta t} = \frac{-6.0\text{ m/s} - 6.0\text{ m/s}}{26.2\text{ s}} = -0.46\text{ m/s}^2$$

(c) C to D $t = \frac{1}{4}(52.4\text{ s}) = 13.1\text{ s}$

$$(\upsilon_x)_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{0 - 50\text{ m}}{13.1\text{ s}} = -3.8\text{ m/s};\ (\upsilon_y)_{\text{av}} = \frac{\Delta y}{\Delta t} = \frac{-50\text{ m} - 0}{13.1\text{ s}} = -3.8\text{ m/s}$$

$$(a_x)_{\text{av}} = \frac{\Delta \upsilon_x}{\Delta t} = \frac{-6.0\text{ m/s} - 0}{13.1\text{ s}} = -0.46\text{ m/s}^2;\ (a_y)_{\text{av}} = \frac{\Delta \upsilon_y}{\Delta t} = \frac{0 - (-6.0\text{ m/s})}{13.1\text{ s}} = 0.46\text{ m/s}^2$$

(d) A to A $\Delta x = \Delta y = 0$ so $(\upsilon_x)_{\text{av}} = (\upsilon_y)_{\text{av}} = 0,$ and $\Delta\upsilon_x = \Delta\upsilon_y = 0$ so $(a_x)_{\text{av}} = (a_y)_{\text{av}} = 0$

(e) For A to B, $\upsilon_{\text{av}} = \sqrt{(\upsilon_x)_{\text{av}}^2 + (\upsilon_y)_{\text{av}}^2} = \sqrt{(3.8 \text{ m/s})^2 + (3.8 \text{ m/s})^2} = 5.4 \text{ m/s}.$ The speed is constant so the average speed is 6.0 m/s. The average speed is larger than the magnitude of the average velocity because the distance traveled is larger than the displacement.

(f) Velocity is a vector, with both magnitude and direction. The magnitude of the velocity is constant but its direction is changing.

Reflect: For this motion the acceleration describes the rate of change of the direction of the velocity.

3.7. Set Up: The particle starts at rest, so its velocity will be in the same direction as its constant acceleration. We can find the velocity and distance traveled in this given direction and at the given time, and then break these quantities into their x and y components.

Solve: **(a)** At $t = 20$ s, the magnitude of the velocity is $\upsilon = \upsilon_0 + at = at,$ where we have used the fact that the initial velocity υ_0 is zero. Inserting the given time and acceleration gives $\upsilon = at = \left(4 \text{ m/s}^2\right)\left(20 \text{ s}\right) = 80 \text{ m/s}.$ This velocity is oriented at 30° above the positive x axis, so its components are

$$\upsilon_x = \upsilon\cos\theta = \left(80 \text{ m/s}\right)\cos\left(30°\right) = 7\times10^1 \text{ m/s}$$
$$\upsilon_y = \upsilon\sin\theta = \left(80 \text{ m/s}\right)\sin\left(30°\right) = 4\times10^1 \text{ m/s}$$

(b) The displacement d of the particle is $d = \upsilon_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2.$ Inserting the given acceleration and time gives

$$d = \frac{1}{2}at^2 = \frac{1}{2}\left(4 \text{ m/s}^2\right)\left(20 \text{ s}\right)^2 = 800 \text{ m}$$

Again, this displacement is at 30° above the $+x$ axis, so the coordinates of the particle's position at this point in time are

$$d_x = d\cos\theta = \left(800 \text{ m}\right)\cos\left(30°\right) = 7\times10^2 \text{ m}$$
$$d_y = d\sin\theta = \left(800 \text{ m}\right)\sin\left(30°\right) = 4\times10^2 \text{ m}$$

Thus, the projectile's position is $\left(7\times10^2 \text{ m}, 4\times10^2 \text{ m}\right).$

Solve: We retained only a single significant digit in the answers because the acceleration was given to a single significant digit.

3.8. Set Up: We are given the initial speed and angle of the trajectory. Recall that gravity acts only in the y direction, so the horizontal component of the velocity remains constant.

Solve **(a)** At the instant the projectile is fires, the magnitude and direction of the instantaneous velocity are given in the problem: 30 m/s and 60° above the horizontal.

(b) At its maximum height, the y component of the velocity is zero, so the magnitude and direction of the instantaneous velocity is simply due to the horizontal component of velocity, which is

$$\upsilon_{0x} = \upsilon_0\cos\theta = \left(30 \text{ m/s}\right)\cos\left(60°\right) = 15 \text{ m/s}$$

Thus, the magnitude and direction of the instantaneous velocity is $\upsilon = 15 \text{ m/s}$ oriented at 0° with respect to the horizontal.

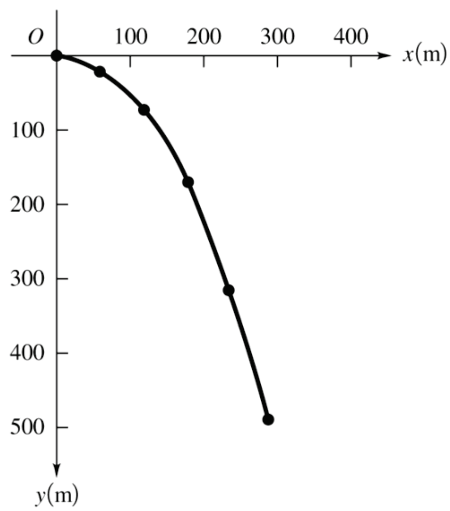
(c) At the moment it hits the ground, the y component of the velocity will have the same magnitude as the initial y component, but it will point in the opposite direction (i.e., down). This will change the magnitude of the final velocity, but it does negate the angle with respect to the horizontal. Thus, the magnitude and direction of the velocity at the moment the projectile hits the ground is 30 m/s and 60° *below* the horizontal.

Reflect This problem illustrates some of the special points in a projectile's trajectory where we know certain things about the projectile's velocity: the vertical component of the velocity is zero at the summit of the trajectory, the horizontal component of the velocity is constant, and the vertical component of the velocity simply changes sign (but not magnitude) as the projectiles travels up past a given height and down past the same height.

3.9. Set Up: Use coordinates with the origin at the initial position of the stone and take $+y$ downward. $\upsilon_{0x} = 70$ m/s, $\upsilon_{0y} = 0$, $a_x = 0$, $a_y = g = 9.8$ m/s^2.

Solve: (a) $x_0 = y_0 = 0$. $x = \upsilon_{0x}t = (70 \text{ m/s})t$ and $y = \frac{1}{2}gt^2 = (4.9 \text{ m/s}^2)t^2$. $t = 2$ s: $x = 14$ m, $y = 19.6$ m; $t = 4$ s: $x = 280$ m, $y = 78.4$ m; $t = 6$ s: $x = 420$ m, $y = 176$ m; $t = 8$ s: $x = 560$ m, $y = 314$ m; $t = 10$ s: $x = 700$ m, $y = 490$ m

(b) The graph of y versus x is shown in the figure below.



3.10. Set Up: We can use the equations for projectile motion with $\upsilon_{0x} = 42.0$ m/s, $\upsilon_{0y} = 0$, and $\Delta x = 60.5$ ft. Note that 1 ft = 0.3048 m.

Solve: $x - x_0 = \upsilon_{0x}t$ gives

$$t = \frac{x - x_0}{\upsilon_{0x}} = \frac{(60.5 \text{ ft})(0.3048 \text{ m/ft})}{42.0 \text{ m/s}} = 0.\underline{439}1 \text{ s}$$

The vertical displacement during this time is $y - y_0 = \upsilon_{0y}t - \frac{1}{2}\text{g}t^2 = -\frac{1}{2}(9.80 \text{ m/s}^2)(0.\underline{439}1 \text{ s})^2 = -0.945$ m. Thus, the ball drops 0.945 m.

Reflect: In the absence of air resistance, the motion of a baseball is simply two-dimensional parabolic motion. In reality, air resistance is very important in baseball—curve balls are deflected by slight differences in air pressure on opposite sides of the spinning baseball.

3.11. Set Up: Take $+y$ downward, so $a_x = 0$, $a_y = +9.80$ m/s^2, $\upsilon_{0x} = 2.00$ m/s and $\upsilon_{0y} = 0$.

Solve: (a) $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_yt^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(0.350 \text{ s})^2 = 0.600$ m. This is the height of the tabletop.

(b) $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_xt^2 = (2.00 \text{ m/s})(0.350 \text{ s}) = 0.700$ m

(c) $\upsilon_x = \upsilon_{0x} = 2.00$ m/s, $\upsilon_y = \upsilon_{0y} + a_yt = (9.80 \text{ m/s}^2)(0.350 \text{ s}) = 3.43$ m/s. $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = 3.97$ m/s.

$$\tan\theta = \frac{|\upsilon_y|}{|\upsilon_x|} = \frac{3.43 \text{ m/s}}{2.00 \text{ m/s}}$$

and $\theta = 59.8°$. The velocity of the book just before it hits the floor has magnitude 3.97 m/s and is directed at 59.8° below the horizontal.

***3.12. Set Up:** Take $+y$ downward, so $a_x = 0, a_y = +9.80\ \text{m/s}^2$ and $\upsilon_{0y} = 0$. When the ball reaches the floor, $y - y_0 = 0.850\ \text{m}$.

Solve: (a) $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_yt^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(0.850\ \text{m})}{9.80\ \text{m/s}^2}} = 0.416\ \text{s}$.

(b) $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_xt^2$ gives $\upsilon_{0x} = \frac{x - x_0}{t} = \frac{1.50\ \text{m}}{0.416\ \text{s}} = 3.60\ \text{m/s}$. Since $\upsilon_{0y} = 0$, $\upsilon_0 = \upsilon_{0x} = 3.60\ \text{m/s}$.

(c) $\upsilon_x = \upsilon_{0x} = 3.60\ \text{m/s}$. $\upsilon_y = \upsilon_{0y} + a_yt = (9.80\ \text{m/s}^2)(0.416\ \text{s}) = 4.08\ \text{m/s}$. $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = 5.44\ \text{m/s}$.

$$\tan\theta = \frac{|\upsilon_y|}{|\upsilon_x|} = \frac{4.08\ \text{m/s}}{3.60\ \text{m/s}}$$

and $\theta = 48.6°$. The final velocity of the ball has magnitude 5.44 m/s and is directed at 48.6° below the horizontal.

Reflect: The time for the ball to reach the floor is the same as if it had been dropped from a height of 0.750 m; the horizontal component of velocity has no effect on the vertical motion.

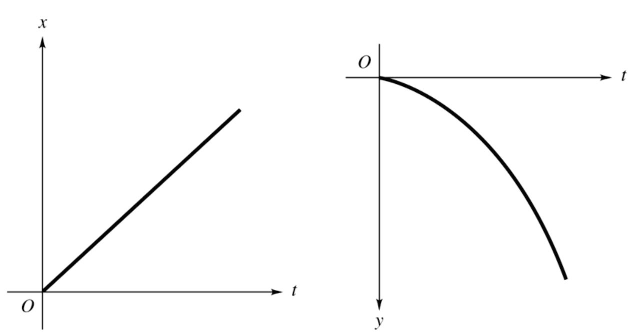
3.13. Set Up: The initial velocity of the bomb is the same as that of the helicopter. Take $+y$ downward, so $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$, $\upsilon_{0x} = 70.0\ \text{m/s}$ and $\upsilon_{0y} = 0$.

Solve: (a) $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_yt^2$ with $y - y_0 = 400\ \text{m}$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(400\ \text{m})}{9.80\ \text{m/s}^2}} = 9.04\ \text{s}$.

(b) The bomb travels a horizontal distance $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_xt^2 = (70.0\ \text{m/s})(9.04\ \text{s}) = 632\ \text{m}$.

(c) $\upsilon_x = \upsilon_{0x} = 70.0\ \text{m/s}$. $\upsilon_y = \upsilon_{0y} + a_yt = (9.80\ \text{m/s}^2)(9.04\ \text{s}) = 88.6\ \text{m/s}$.

(d) Graphs of x versus t and of y versus t are given in the figure below.



(e) Because the airplane and the bomb always have the same x-component of velocity *and* position, the plane will be 400 m directly above the bomb at impact.

3.14. Set Up: Take $+y$ downward. $a_x = 0, a_y = +9.80\ \text{m/s}^2$. $\upsilon_{0x} = \upsilon_0, \upsilon_{0y} = 0$. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

Solve: Time to fall 9.00 m: $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_yt^2$ gives

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(9.00\ \text{m})}{9.80\ \text{m/s}^2}} = 1.36\ \text{s}$$

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_xt^2$ gives

$$\upsilon_0 = \upsilon_{0x} = \frac{x - x_0}{t} = \frac{1.75\ \text{m}}{1.36\ \text{s}} = 1.29\ \text{m/s}$$

3.15. Set Up: Take +*y* to be upward, so $\upsilon_{0x} = 16.5$ m/s and $\upsilon_{0y} = 14.5$ m/s. At the highest point in the trajectory, $\upsilon_{0y} = 0$. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. When the object reaches the ground, $y - y_0 = 0$.

Solve: (a) $\upsilon_y = \upsilon_{0y} + a_y t$ and $t = \dfrac{\upsilon_y - \upsilon_{0y}}{a_y} = \dfrac{0 - 4.5\ \text{m/s}}{-9.80\ \text{m/s}^2} = 1.48\ \text{s}$

(b) The vertical displacement at $t = 1.53$ s is

$$y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2 = (14.5\ \text{m/s})(1.48\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.48\ \text{s})^2 = 10.6\ \text{m}$$

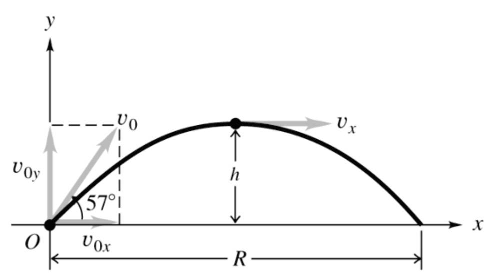
(c) Use $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$. $y - y_0 = 0$ and

$$t = -\frac{2\upsilon_{0y}}{a_y} = -\frac{2(14.5\ \text{m/s})}{-9.80\ \text{m/s}^2} = 2.96\ \text{s}$$

The time from when it is thrown to when it returns to its original height is twice the time it takes it to reach its maximum height.

(d) $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (16.5\ \text{m/s})(2.96\ \text{s}) = 48.8\ \text{m}$

3.16. Set Up: Use coordinates with the origin at the initial position of the ball and +*y* upward. $a_x = 0$ and $a_y = -9.80\ \text{m/s}^2$. The trajectory of the ball is sketched in the figure below.



Solve: (a) From the figure above, $\upsilon_{0x} = \upsilon_0 \cos 60° = 11.5$ m/s and $\upsilon_{0y} = \upsilon_0 \sin 60° = 20$ m/s.

(b) At the maximum height $\upsilon_y = 0$, so $\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0)$ with $y = h$ and $y_0 = 0$ gives

$$h = \frac{\upsilon_y^2 - \upsilon_{0y}^2}{2a_y} = \frac{0 - (20\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 20\ \text{m}$$

(c) $\upsilon_y = \upsilon_{0y} + a_y t$ gives $t = \dfrac{\upsilon_y - \upsilon_{0y}}{a_y} = \dfrac{0 - 20\ \text{m/s}}{-9.80\ \text{m/s}^2} = 2.0\ \text{s}$

(d) $\vec{a} = g$, downward at all points of the trajectory. At the maximum height $\upsilon_y = 0$ and $\upsilon_x = \upsilon_{0x}$, so $\vec{\upsilon} = 11.5$ m/s, horizontal.

(e) The total time in the air is twice the time to the maximum height, so is 4.0 s.

(f) $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2$ with $x = R$ and $x_0 = 0$ gives $R = \upsilon_{0x}t = (11.5\ \text{m/s})(4.0\ \text{s}) = 46$ m.

Reflect: Our results for *h* and *R* agree with the general expressions derived in Example 3.5.

3.17. Set Up: The flare moves with projectile motion. The equations derived in Example 3.5 can be used to find the maximum height *h* and range *R*. From Example 3.5, $h = \dfrac{\upsilon_0^2 \sin^2\theta_0}{2g}$ and $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$.

Solve: (a) $h = \dfrac{(145\ \text{m/s})^2(\sin 46.6°)^2}{2(9.80\ \text{m/s}^2)} = 566\ \text{m}$. $R = \dfrac{(145\ \text{m/s})^2(\sin 93.2°)}{9.80\ \text{m/s}^2} = 2142\ \text{m}$

(b) h and R are proportional to $1/g$, so on the Moon, $h=\left(\dfrac{9.80\text{ m/s}^2}{1.67\text{ m/s}^2}\right)(566\text{ m})=3321\text{ m}$ and

$R=\left(\dfrac{9.80\text{ m/s}^2}{1.67\text{ m/s}^2}\right)(2142\text{ m})=12570\text{ m}.$

Reflect: The projectile travels on a parabolic trajectory. It is incorrect to say that $h=(R/2)\tan\theta_0$.

3.18. Set Up Let the upward direction by the positive y direction and let the horizontal direction be the positive x direction. The x component of the velocity remains constant throughout the trajectory.

Solve: (a) The x component of the velocity is $\upsilon_x=\upsilon_0\cos\theta=(100\text{ m/s})\cos 50°=64.28\text{ m/s}$. The y component is $\upsilon_y=\upsilon_{0y}-gt=\upsilon_0\sin\theta-gt$. At t = 5 s, this gives $\upsilon_y=(100\text{ m/s})\sin 50°-(9.8\text{ m/s}^2)(5\text{ s})=27.6\text{ m/s}$. Thus, the magnitude of the velocity is $\upsilon=\sqrt{\upsilon_x^2+\upsilon_y^2}=\sqrt{(64.28\text{ m/s})^2+(27.60\text{ m/s})^2}=70\text{ m/s}$ and the angle above the horizontal is $\theta=\tan^{-1}\dfrac{\upsilon_y}{\upsilon_x}=\tan^{-1}\dfrac{27.60\text{ m/s}}{64.28\text{ m/s}}=23°$.

(b) At the t = 10 s, the y component of the velocity is $\upsilon_y=(100\text{ m/s})\sin 50°-(9.8\text{ m/s}^2)(10\text{ s})=-21.40\text{ m/s}$. Thus, the magnitude of the velocity is $\upsilon=\sqrt{\upsilon_x^2+\upsilon_y^2}=\sqrt{(64.28\text{ m/s})^2+(-22.40\text{ m/s})^2}=68\text{ m/s}$ and the angle above the horizontal is $\theta=\tan^{-1}\dfrac{\upsilon_y}{\upsilon_x}=\tan^{-1}\dfrac{-21.40\text{ m/s}}{64.28\text{ m/s}}=-18°$, or 18° below the horizontal.

(c) At the t = 15 s, the y component of the velocity is $\upsilon_y=(100\text{ m/s})\sin 50°-(9.8\text{ m/s}^2)(15\text{ s})=-70.40\text{ m/s}$. Thus, the magnitude of the velocity is $\upsilon=\sqrt{\upsilon_x^2+\upsilon_y^2}=\sqrt{(64.28\text{ m/s})^2+(-70.40\text{m/s})^2}=95\text{ m/s}$ and the angle above the horizontal is $\theta=\tan^{-1}\dfrac{\upsilon_y}{\upsilon_x}=\tan^{-1}\dfrac{-70.40\text{ m/s}}{64.28\text{ m/s}}=-48°$, or 48° below the horizontal.

***3.19. Set Up:** The horizontal displacement when the ball returns to its original height is

$$R=\frac{\upsilon_0^2\sin(2\theta_0)}{g}$$

At its maximum height $\upsilon_y=0$. $g=32\text{ ft/s}^2$. $a_x=0$, $a_y=-g$.

Solve: (a) $\upsilon_0=\sqrt{\dfrac{Rg}{\sin(2\theta_0)}}=\sqrt{\dfrac{(355\text{ ft})(32\text{ ft/s}^2)}{\sin(68.0°)}}=111\text{ ft/s}$

(b) $\upsilon_{0y}=\upsilon_0\sin\theta_0=(111\text{ ft/s})\sin 34.0°=62\text{ ft/s}$. $\upsilon_y^2=\upsilon_{0y}^2+2a_y(y-y_0)$.

$$y-y_0=\frac{\upsilon_y^2-\upsilon_{0y}^2}{2a_y}=\frac{0-(62\text{ ft/s})^2}{2(-32\text{ ft/s}^2)}=59.9\text{ ft}$$

Reflect: At the maximum height $\upsilon_y=0$. But $\upsilon\neq 0$ there because the ball still has its constant horizontal component of velocity. The horizontal range equation,

$$R=\frac{\upsilon_0^2\sin(2\theta_0)}{g}$$

can be used only when the initial and final points of the motion are at the same elevation.

3.20. Set Up: Take +y upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. $\upsilon_{0x} = \upsilon_0 \cos\theta_0 = 28.2$ m/s. $\upsilon_{0y} = \upsilon_0 \sin\theta_0 = 19.0$ m/s.

Solve: (a) With $\upsilon_y = 0$ in $\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0)$,

$$y - y_0 = -\frac{\upsilon_{0y}^2}{2a_y} = -\frac{(19.0\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 18.4\ \text{m}$$

(b) $\upsilon_x = \upsilon_{0y} = 28.2$ m/s. With $y - y_0 = -19.0$ m, $\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0)$ gives

$$\upsilon_y = -\sqrt{\upsilon_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(19\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-19.0\ \text{m})} = -27.1\ \text{m/s}$$

Then $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = 39.1$ m/s.

(c) Use the vertical motion to find the time in the air:

$$\upsilon_y = \upsilon_{0y} + a_y t \text{ gives } t = \frac{\upsilon_y - \upsilon_{0y}}{a_y} = \frac{-27.1\ \text{m/s} - 19\ \text{m/s}}{-9.80\ \text{m/s}^2} = 4.70\ \text{s}$$

Then $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (28.2\ \text{m/s})(4.70\ \text{s}) = 132$ m.

***3.21. Set Up:** Use coordinates with the origin at the ground and +y upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. At the maximum height $\upsilon_y = 0$.

Solve: (a) $\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0)$ gives

$$\upsilon_y = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(0.547\ \text{m})} = 3.27\ \text{m/s}$$

$$\upsilon_{0y} = \upsilon_0 \sin\theta_0 \text{ so } \upsilon_0 = \frac{\upsilon_{0y}}{\sin\theta_0} = \frac{3.27\ \text{m/s}}{\sin 58.0°} = 3.86\ \text{m/s}$$

(b) Use the vertical motion to find the time in the air. When the froghopper has returned to the ground, $y - y_0 = 0$.

$$y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2 \text{ gives } t = -\frac{2\upsilon_{0y}}{a_y} = -\frac{2(3.27\ \text{m/s})}{-9.80\ \text{m/s}^2} = 0.667\ \text{s}$$

Then $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (\upsilon_0 \cos\theta_0)t = (3.86\ \text{m/s})(\cos 58.0°)(0.667\ \text{s}) = 1.36$ m

Reflect: $\upsilon_y = 0$ when $t = -\frac{\upsilon_{0y}}{a_y} = -\frac{3.27\ \text{m/s}}{-9.80\ \text{m/s}^2} = 0.334$ s. The total time in the air is twice this.

3.22. Set Up: Use coordinates with the origin at the ground and +y upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$.

Solve: (a) $\upsilon_y = 0$ when $y - y_0 = 0.0674$ m

$$\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0) \text{ gives } \upsilon_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(0.0674\ \text{m})} = 1.15\ \text{m/s}$$

$$\upsilon_{0y} = \upsilon_0 \sin\theta_0 \text{ so } \upsilon_0 = \frac{\upsilon_{0y}}{\sin\theta_0} = \frac{1.15\ \text{m/s}}{\sin 50.0°} = 1.50\ \text{m/s}$$

(b) Use the horizontal motion to find the time in the air. The grasshopper travels horizontally $x - x_0 = 1.06$ m. $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2$ gives

$$t = \frac{x - x_0}{\upsilon_{0x}} = \frac{x - x_0}{\upsilon_0 \cos 50.0°} = 1.10\ \text{s}$$

Find the vertical displacement of the grasshopper at $t = 1.10$ s:

$$y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2 = (1.15\text{ m/s})(1.10\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(1.10\text{ s})^2 = -4.66\text{ m}$$

The height of the cliff is 4.66 m.

3.23. Set Up: Example 3.5 derives $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$.

Solve: The maximum range occurs when $\sin 2\theta_0 = 1$, $2\theta_0 = 90°$ and $\theta_0 = 45°$.

***3.24. Set Up:** Example 3.5 gives $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$ and $t = \dfrac{2\upsilon_0 \sin\theta_0}{g}$.

Solve: (a) The maximum range occurs when $\sin 2\theta_0 = 1$, $2\theta_0 = 90°$ and $\theta_0 = 45°$.

(b) $R_{\text{max}} = \dfrac{\upsilon_0^2}{g} = \dfrac{(26.0\text{ m/s})^2}{9.80\text{ m/s}^2} = 69.0\text{ m}$

(c) $t = \dfrac{2(26.0\text{ m/s})\sin 45°}{g} = 3.75\text{ s}$

Reflect: The time that the balloon is in the air can also be calculated from $t = \dfrac{\Delta x}{\upsilon_{0x}} = \dfrac{69\text{ m}}{(26.0\text{ m/s})\cos 45°} = 3.75$ s.

3.25. Set Up: Example 3.5 derives $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$.

Solve: Use the 43° data:

$$\frac{\upsilon_0^2}{g} = \frac{R}{\sin 2\theta_0} = \frac{215\text{ m}}{\sin 86°} = 215\text{ m}$$

Then for $\theta_0 = 60.0°$, $R = (215\text{ m})\sin 120.0° = 186\text{ m}$. The distance between the two arrows when they land is $215\text{ m} - 186\text{ m} = 29\text{ m}$.

***3.26. Set Up:** We can determine the initial velocity of the bottle rocket from its maximum vertical height by using the equation $h = \dfrac{\upsilon_0^2 \sin^2\theta_0}{2g}$, which is derived in Example 3.5. The maximum range of the projectile can be determined from $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$, which is also derived in Example 3.5.

Solve: When fired vertically we have $\theta_0 = 90°$ and so $h = \dfrac{\upsilon_0^2 \sin^2 90°}{2g} = \dfrac{\upsilon_0^2}{2g}$. Solving for υ_0 we obtain $\upsilon_0 = \sqrt{2gh}$.

The maximum range occurs when $\sin 2\theta_0 = 1$ so $\theta_0 = 45°$.

Substituting this value into $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$ we obtain $R = \dfrac{(2gh)\sin 90°}{g} = 2h = 54.0$ m.

Reflect: When a projectile is fired at a fixed initial speed without air resistance, the maximum range that it can achieve is twice the maximum height that it achieves when fired vertically.

3.27. Set Up: The suitcase moves in projectile motion. The initial velocity of the suitcase equals the velocity of the airplane. Take $+y$ to be upward. $a_x = 0$, $a_y = -g$.

Solve: Use the vertical motion to find the time it takes the suitcase to reach the ground:

$$\upsilon_{0y} = \upsilon_0 \sin 20°,\ a_y = -9.80\text{ m/s}^2,\ y - y_0 = -106\text{ m},\ t = ?\ \ y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2 \text{ gives } t = 8.22\text{ s}$$

The distance the suitcase travels horizontally is $x - x_0 = \upsilon_{0x} = (\upsilon_0 \cos 20.0°)t = 618$ m.

Reflect: An object released from rest at a height of 106 m strikes the ground at $t = \sqrt{\frac{2(y - y_0)}{-g}} = 4.65$ s. The suitcase is in the air much longer than this since it initially has an upward component of velocity.

***3.28.** **Set Up:** The acceleration is a_{rad}. $R = 0.80$ m.

Solve: $a_{\text{rad}} = \frac{\upsilon^2}{R}$ so $\upsilon = \sqrt{Ra_{\text{rad}}} = \sqrt{(0.80 \text{ m})(9.8 \text{ m/s}^2)} = 2.8$ m/s

3.29. **Set Up:** The acceleration is a_{rad}. Part (a): $T = 24 \text{ h} = 8.64 \times 10^4$ s and $R = 6.38 \times 10^6$ m. Part (b): $T = 0.41 \text{ day} = 9.84 \text{ h} = 3.54 \times 10^4$ s and $R = 7.18 \times 10^7$ m.

Solve: **(a)** $a_{\text{rad}} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 (6.38 \times 10^6 \text{ m})}{(8.64 \times 10^4 \text{ s})^2} = 0.0337 \text{ m/s}^2 = 3.44 \times 10^{-3} g$

(b) $a_{\text{rad}} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 (7.18 \times 10^7 \text{ m})}{(3.54 \times 10^4 \text{ s})^2} = 2.3 \text{ m/s}^2 = 0.23g$

3.30. **Set Up:** $560 \text{ rev/min} = 9.34 \text{ rev/s}$, corresponding to a period of

$$T = \frac{1}{9.34 \text{ rev/s}} = 0.107 \text{ s}$$

Each blade tip moves in a circle of radius $R = 3.40$ m.

Solve: **(a)** $\upsilon = \frac{2\pi R}{T} = 200$ m/s

(b) $a_{\text{rad}} = \frac{\upsilon^2}{R} = 1.17 \times 10^4 \text{ m/s}^2 = 1.20 \times 10^3 g$

3.31. **Set Up:** $R = 0.250$ m. $T = 1 \text{ min} = 60.0$ s

Solve: $a_{\text{rad}} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 (0.250 \text{ m})}{(60.0 \text{ s})^2} = 2.74 \times 10^{-3} \text{ m/s}^2$

Reflect: Different points in the hand travel in circles of different radii but have the same period so they have different a_{rad}. a_{rad} is largest for the tip, since it has the largest *R*.

3.32. **Set Up:** The centripetal acceleration for circular motion at a constant speed is given by $a_{\text{rad}} = \frac{\upsilon^2}{r}$. From the appendix we have $1 \text{ mph} = 0.4470$ m/s.

Solve: First convert speeds from mph into m/s:

$(35 \text{ mph})\left(\frac{0.4470 \text{ m/s}}{1 \text{ mph}}\right) = \underline{15}.6$ m/s and $(50 \text{ mph})\left(\frac{0.4470 \text{ m/s}}{1 \text{ mph}}\right) = \underline{22}.4$ m/s. Thus, we have

$$a_{\text{rad}} = \frac{\upsilon^2}{r} = \frac{(\underline{15}.6 \text{ m/s})^2}{52.0 \text{ m}} = 4.7 \text{ m/s}^2 \text{ and } a_{\text{rad}} = \frac{(\underline{22}.4 \text{ m/s})^2}{52.0 \text{ m}} = 9.6 \text{ m/s}^2$$

Reflect: Since $a_{\text{rad}} \propto \upsilon^2$, we can also calculate the second acceleration from the first acceleration:

$$\left(\frac{50 \text{ mph}}{35 \text{ mph}}\right)^2 (4.7 \text{ m/s}^2) = 9.8 \text{ m/s}^2$$

3.33. Set Up: $a_{\text{rad}} = 5.5g = 53.9 \text{ m/s}^2$. 1 mph = 0.4470 m/s.

Solve: $a_{\text{rad}} = \dfrac{\upsilon^2}{R}$ so $\upsilon = \sqrt{Ra_{\text{rad}}} = \sqrt{(350 \text{ m})(53.9 \text{ m/s}^2)} = 140 \text{ m/s} = 310 \text{ mph}$

3.34. Set Up: The speed of an object at the surface of earth is related to the period of rotation of the earth (i.e., one day) as $2\pi R/\upsilon = T$, where $R = 6.371 \times 10^6$ m is the radius of earth.

Solve: The radial acceleration at the surface of earth is $a = \upsilon^2/R$. The period of rotation that makes this acceleration equal that of gravity is

$$a = g = \frac{\upsilon^2}{R}$$

$$g = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.371\times10^6 \text{ m}}{9.8 \text{ m/s}^2}} = (5066 \text{ s})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$$

$$= 1.4 \text{ h}$$

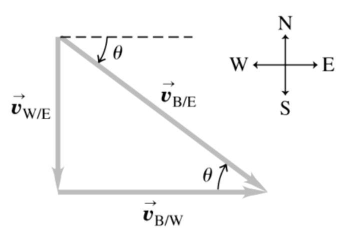
Solve: The length of the day is an order of magnitude greater than this, so the acceleration due to the rotation of the Earth (at the equator) is about two orders of magnitude less than that of gravity because the radial acceleration is inversely proportional to the square of the period of rotation.

***3.35. Set Up:** Apply the relative velocity relation. The relative velocities are $\vec{\upsilon}_{\text{C/E}}$, the canoe relative to the earth, $\vec{\upsilon}_{\text{R/E}}$, the velocity of the river relative to the earth and $\vec{\upsilon}_{\text{C/R}}$, the velocity of the canoe relative to the river.

Solve: $\vec{\upsilon}_{\text{C/E}} = \vec{\upsilon}_{\text{C/R}} + \vec{\upsilon}_{\text{R/E}}$ and therefore $\vec{\upsilon}_{\text{C/R}} = \vec{\upsilon}_{\text{C/E}} - \vec{\upsilon}_{\text{R/E}}$. The velocity components of $\vec{\upsilon}_{\text{C/R}}$ are $-0.53 \text{ m/s} + (0.31 \text{ m/s})/\sqrt{2}$, east and $(0.31 \text{ m/s})/\sqrt{2}$, south, for a velocity relative to the river of 0.38 m/s, at 54.8° south of west.

Reflect: The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.

3.36. Set Up: The relative velocities are the water relative to the earth, $\vec{\upsilon}_{\text{W/E}}$, the boat relative to the water, $\vec{\upsilon}_{\text{B/W}}$, and the boat relative to the earth, $\vec{\upsilon}_{\text{B/E}}$. $\vec{\upsilon}_{\text{B/W}}$ is due east and has magnitude 4.2 m/s. $\vec{\upsilon}_{\text{W/E}}$ is due south and has magnitude 2.0 m/s. $\vec{\upsilon}_{\text{B/E}} = \vec{\upsilon}_{\text{B/W}} + \vec{\upsilon}_{\text{W/E}}$. The relative velocity addition diagram is given in the figure below.



Solve: **(a)** $\upsilon_{\text{B/E}} = \sqrt{\upsilon_{\text{W/E}}^2 + \upsilon_{\text{B/W}}^2} = \sqrt{(2.0 \text{ m/s})^2 + (4.2 \text{ m/s})^2} = 4.65 \text{ m/s}$

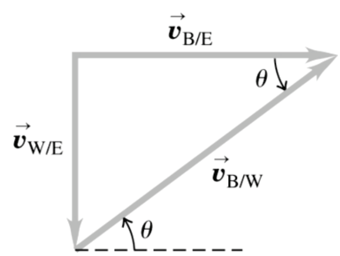
$$\tan\theta = \frac{\upsilon_{\text{W/E}}}{\upsilon_{\text{B/W}}} = \frac{2.0 \text{ m/s}}{4.2 \text{ m/s}}. \quad \theta = 25.5^\circ, \text{ south of east}$$

(b) To cross the river the boat must travel 800 m east, relative to the earth. The eastward component of $\vec{\upsilon}_{\text{B/E}}$ is $\upsilon_{\text{B/W}} = 4.2$ m/s.

$$t = \frac{800 \text{ m}}{4.2 \text{ m/s}} = 190 \text{ s}$$

(c) The component of $\vec{\upsilon}_{B/E}$ in the southward direction is $\upsilon_{W/E} = 2.0$ m/s. In the 190 s it takes him to cross the river he travels south a distance of $(2.0 \text{ m/s})(190 \text{ s}) = 380$ m.

***3.37. Set Up:** The relative velocities are the water relative to the earth, $\vec{\upsilon}_{W/E}$, the boat relative to the water, $\vec{\upsilon}_{B/W}$, and the boat relative to the earth, $\vec{\upsilon}_{B/E}$. $\vec{\upsilon}_{B/E}$ is due east, $\vec{\upsilon}_{W/E}$ is due south and has magnitude 2.0 m/s. $\vec{\upsilon}_{B/W} = 4.2$ m/s. $\vec{\upsilon}_{B/E} = \vec{\upsilon}_{B/W} + \vec{\upsilon}_{W/E}$. The velocity addition diagram is given in the figure below.



Solve: (a) Find the direction of $\vec{\upsilon}_{B/W}$. $\sin\theta = \dfrac{\upsilon_{W/E}}{\upsilon_{B/W}} = \dfrac{2.0 \text{ m/s}}{4.2 \text{ m/s}}$. $\theta = 28.4°$, north of east.

(b) $\upsilon_{B/E} = \sqrt{\upsilon_{B/W}{}^2 - \upsilon_{W/E}{}^2} = \sqrt{(4.2 \text{ m/s})^2 - (2.0 \text{ m/s})^2} = 3.7$ m/s

(c) $t = \dfrac{800 \text{ m}}{\upsilon_{B/E}} = \dfrac{800 \text{ m}}{3.7 \text{ m/s}} = 216$ s

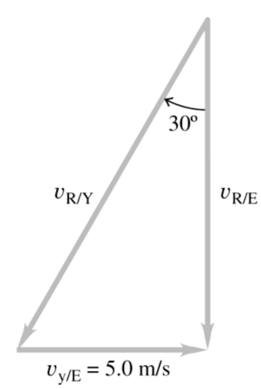
Reflect: It takes longer to cross the river in this problem than it did in Problem 3.40. In the direction straight across the river (east) the component of his velocity relative to the earth is less than 4.2 m/s.

3.38. Set Up: The relative velocities are the raindrops relative to the earth, $\vec{\upsilon}_{R/E}$, the raindrops relative to you, $\vec{\upsilon}_{R/Y}$, and you relative to the earth, $\vec{\upsilon}_{Y/E}$: $\vec{\upsilon}_{R/E} = \vec{\upsilon}_{R/Y} + \vec{\upsilon}_{Y/E}$. Suppose that you are running due east so that $\vec{\upsilon}_{Y/E}$ is due east and has magnitude 5.0 m/s. Thus, $\vec{\upsilon}_{R/Y}$ is 30° west of vertical and $\vec{\upsilon}_{R/E}$ is vertical (since there is no wind). The relative velocity addition diagram is shown below.

Solve: $\upsilon_{R/E} = \dfrac{\upsilon_{Y/E}}{\tan 30.0°} = \dfrac{4.9 \text{ m/s}}{\tan 30°} = 8.5$ m/s

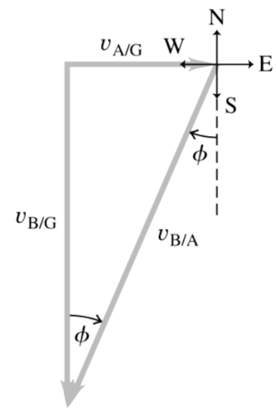
Reflect: We can also find the magnitude of the speed of the raindrops relative to you:

$$\upsilon_{R/Y} = \frac{\upsilon_{Y/E}}{\sin 30°} = \frac{4.9 \text{ m/s}}{\sin 30°} = 9.8 \text{ m/s}$$



3.39. Set Up: $\upsilon_{B/A} = 100$ km/h. $\vec{\upsilon}_{A/G} = 46$ km/h, east.

Solve: $\vec{\upsilon}_{B/G} = \vec{\upsilon}_{B/A} + \vec{\upsilon}_{A/G}$. We want $\vec{\upsilon}_{B/G}$ to be due south. The relative velocity addition diagram is shown in the figure below.



(a) $\sin\phi = \dfrac{\upsilon_{A/G}}{\upsilon_{B/A}} = \dfrac{46 \text{ km/h}}{100 \text{ km/h}}$, $\phi = 27°$, west of south.

(b) $\upsilon_{B/G} = \sqrt{\upsilon_{B/A}{}^2 - \upsilon_{A/G}{}^2} = 88.8$ km/h. $t = \dfrac{d}{\upsilon_{B/G}} = \dfrac{450 \text{ km}}{88.8 \text{ km/h}} = 5.1$ h

Reflect: The speed of the bird relative to the ground is less than its speed relative to the air. Part of its velocity relative to the air is directed to oppose the effect of the wind.

3.40. Set Up: Once the rocket leaves the incline it moves in projectile motion. The acceleration along the incline determines the initial velocity and initial position for the projectile motion. For motion along the incline let $+x$ be directed up the incline. $\upsilon_x^2 = \upsilon_{0x}^2 + 2a_x(x - x_0)$ gives $\upsilon_x = \sqrt{2(1.25 \text{ m/s}^2)(200 \text{ m})} = 22.36$ m/s. When the projectile motion begins the rocket has $\upsilon_0 = 22.36$ m/s at $35.0°$ above the horizontal and is at a vertical height of $(200.0 \text{ m})\sin 35.0° = 114.7$ m. For the projectile motion let $+x$ be horizontal to the right and let $+y$ be upward. Let $y = 0$ at the ground. Then $y_0 = 114.7$ m, $\upsilon_{0x} = \upsilon_0 \cos 35.0° = 18.32$ m/s, $\upsilon_{0y} = \upsilon_0 \sin 35.0° = 12.83$ m/s, $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$. Let $x = 0$ at point A, so $x_0 = (200.0 \text{ m})\cos 35.0° = 163.8$ m.

Solve: **(a)** At the maximum height we have $\upsilon_y = 0$. $\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{\upsilon_y^2 - \upsilon_{0y}^2}{2a_y} = \dfrac{0 - (12.83 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 8.40$ m and $y = 114.7 \text{ m} + 8.40 \text{ m} = 123$ m. The maximum height above ground is 123 m.

(b) The time in the air can be calculated from the vertical component of the projectile motion: $y - y_0 = -114.7$ m, $\upsilon_{0y} = 12.83$ m/s, $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ gives $(4.90 \text{ m/s}^2)t^2 - (12.83 \text{ m/s})t - 114.7$ m. The quadratic formula gives $t = \dfrac{1}{9.80}\left(12.83 \pm \sqrt{(12.83)^2 + 4(4.90)(114.7)}\right)$ s. The positive root is $t = 6.32$ s. Then $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (18.32 \text{ m/s})(6.32 \text{ s}) = 115.8$ m and $x = 163.8 \text{ m} + 115.8 \text{ m} = 280$ m. The horizontal range of the rocket is 280 m.

Reflect: The expressions for h and R derived in Example 3.5 do not apply here. They are only for a projectile that lands at the same elevation from which it was fired.

***3.41. Set Up:** For the motion of the football take +y upward. $\upsilon_{0x} = \upsilon_0 \cos 46.0° = 9.03$ m/s, $\upsilon_{0y} = \upsilon_0 \sin 46.0° = 9.35$ m/s, $a_x = 0$, and $a_y = -9.80$ m/s^2. Use the y component motion to find the time the football is in the air. This same time applies to the horizontal motion.

Solve: When the ball returns to the ground, $y - y_0 = 0$. $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ gives $0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ and

$$t = -\frac{2\upsilon_{0y}}{a_y} = -\frac{2(9.35 \text{ m/s})}{-9.80 \text{ m/s}^2} = 1.91 \text{ s}$$

Then $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (9.03 \text{ m/s})(1.91 \text{ s}) = 17.2$ m. In the 1.91 s that the ball is in the air the second player must travel $30.0 \text{ m} - 17.2 \text{ m} = 12.8$ m. Therefore, his (constant) velocity must be

$$\upsilon_x = \frac{x - x_0}{t} = \frac{12.8 \text{ m}}{1.91 \text{ s}} = 6.70 \text{ m/s}$$

***3.42. Set Up:** Take +y downward. $\upsilon_{0x} = 64.0$ m/s, $\upsilon_{0y} = 0$.

Solve: Use the vertical motion to find the time in the air: $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 90.0$ m gives

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(90.0 \text{ m})}{9.80 \text{ m/s}^2}} = 4.29 \text{ s}$$

Then $x - x_0 = (64.0 \text{ m/s})(4.29 \text{ s}) = 275$ m.

3.43. Set Up: Define the upward direction as the positive y direction. The y component of the initial velocity of each projectile is of equal magnitude, but in opposite directions. The magnitude is $\upsilon_{0y} = \upsilon_0 \sin\theta = (20 \text{ m/s})\sin 30° = 10.0$ m/s. The x component of the velocity for each is the same and is constant at $\upsilon_{0x} = \upsilon_0 \cos\theta = (20 \text{ m/s})\cos 30° = 17.3$ m/s. The vertical displacement of each projectile is $y - y_0 = -50$ m.

Solve: (a) The times at which each projectile hits the ground are

$$y - y_0 = \upsilon_{0y}t - \frac{1}{2}gt^2$$

$$t = \frac{\upsilon_{0y} \pm \sqrt{-2g(y - y_0) + \upsilon_{0y}^2}}{g}$$

For the projectile fired upward, $t_{\text{up}} = \dfrac{10 \text{ m/s} \pm \sqrt{-4(9.8 \text{ m/s}^2)(-50 \text{ m}) + (10 \text{ m/s})^2}}{2(9.8)} = 2.33 \text{ s}, 4.37 \text{ s}$

For the projectile fired downward, $t_{\text{down}} = \dfrac{-10 \text{ m/s} \pm \sqrt{-4(9.8 \text{ m/s}^2)(-50 \text{ m}) + (-10 \text{ m/s})^2}}{2(9.8)} = 2.33 \text{ s}, -4.37 \text{ s}$

The projectile fired down hits the ground before the projectile fire up, so $t_{\text{up}} = 4.4$ s and $t_{\text{down}} = 2.3$ s.

(b) To find the speed with which each projectile hits the ground, we first calculate the y component of the velocity of each (the x component is constant) by using $\upsilon_y = \upsilon_{0y} - gt$. This gives

$$\upsilon_{y,\text{up}} = 10 \text{ m/s} - (9.8 \text{ m/s}^2)(4.37 \text{ s}) = -32.9 \text{ m/s}$$

$$\upsilon_{y,\text{down}} = -10 \text{ m/s} - (9.8 \text{ m/s}^2)(2.33 \text{ s}) = -32.9 \text{ m/s}$$

Thus, the speed with which each projectile hits the ground is

$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{(17.3\text{ m/s})^2 + (-32.9\text{ m/s})^2} = 37\text{ m/s}$$

Solve: When the projectiles hit the ground, the *y* components of their velocities are the same. This is because, as projectile A descends past the 50 m mark, it has the same downward velocity as projectile B has initially. Thus, for a given angle with respect to the horizontal, the speed with which the projectile hits the ground is the same whether you fire the projectile upward or downward.

3.44. Set Up: Let +*y* be upward and +*x* be to the right. The missile has $\upsilon_{0x} = 30.0$ m/s, $\upsilon_{0y} = 40.0$ m/s, $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$. The cart has $a_x = 0$ and $\upsilon_{0x} = 30.0$ m/s.

Solve: (a) At the maximum height, $\upsilon_y = 0$.

$$\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0) \text{ gives } y - y_0 = \frac{\upsilon_y^2 - \upsilon_{0y}^2}{2a_y} = \frac{0 - (40.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 81.6\text{ m}$$

(b) Find *t* for $y - y_0 = 0$ (returns to initial level).

$$y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2 \text{ gives } t = -\frac{2\upsilon_{0y}}{a_y} = -\frac{2(40.0\text{ m/s})}{-9.80\text{ m/s}^2} = 8.16\text{ s}$$

Then $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (30.0\text{ m/s})(8.16\text{ s}) = 245\text{ m}$.

(c) The missile also travels horizontally 245 m so the missile lands in the cart.

3.45. Set Up: Let +*y* be upward. $y - y_0 = -1.0$ m when $x - x_0 = 188$ m. With $x_0 = y_0 = 0$, *x* and *y* are related by $y = (\tan\theta_0)x - \dfrac{g}{2\upsilon_0^2\cos^2\theta_0}x^2$.

Solve: (a) $\theta_0 = 45°$ so $\tan\theta_0 = 1$ and $\cos\theta_0 = \frac{1}{2}$. $y = x - \dfrac{gx^2}{\upsilon_0^2}$.

$$\upsilon_0 = \sqrt{\frac{gx^2}{x - y}} = \sqrt{\frac{(9.80\text{ m/s}^2)(188\text{ m})^2}{188\text{ m} + 1.0\text{ m}}} = 42.8\text{ m/s}$$

(b) For $x = 116$ m,

$$y = x - \frac{gx^2}{\upsilon_0^2} = 102\text{ m} - \frac{(9.80\text{ m/s}^2)(102\text{ m})^2}{(42.8\text{ m/s})^2} = 46.3\text{ m}.$$

The ball will be 46.3 m above the ground and therefore 43.5 m above the top of the fence.

***3.46. Set Up:** Example 3.5 gives $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$, were *R* is the range of a projectile that lands at the same elevation from which it was fired. The initial vertical velocity of the ball is $\upsilon_{0y} = \upsilon_0 \sin\theta_0$.

Solve: (a) Solving for the angle θ_0 gives

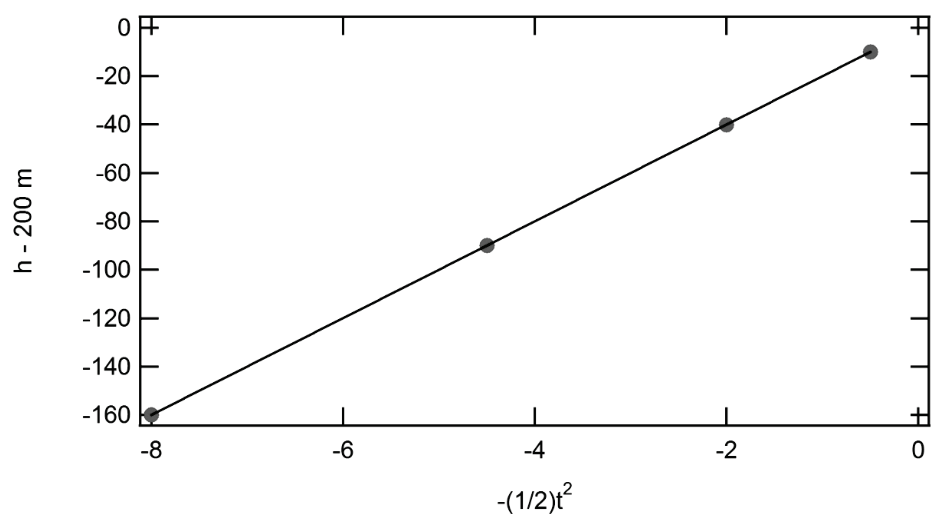
$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} \quad \Rightarrow \quad \theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{\upsilon_0^2}\right) = \frac{1}{2}\sin^{-1}\left(\frac{(60\text{ m})(9.8\text{ m/s}^2)}{(25\text{ m/s})^2}\right) = 35°, 55°$$

(b) The initial vertical velocity of the ball is $\upsilon_{0y} = \upsilon_0 \sin\theta_0 = (25\ \text{m/s})\sin(35° \text{ or } 55°) = 14.4\ \text{m/s or } 20.5\ \text{m/s}$. The time during which the ball remains in the air may be found from $y - y_0 = \upsilon_{0y}t - \frac{1}{2}gt^2$. When the ball lands, $y - y_0 = 0$, so $0 = \upsilon_{0y}t - \frac{1}{2}gt^2 \Rightarrow t = \frac{2\upsilon_{0y}}{g} = \frac{2(14.4\ \text{m/s or } 20.5\ \text{m/s})}{9.8\ \text{m/s}^2} = 2.9\ \text{s or } 4.2\ \text{s}$

Reflect: To find the time during which the ball remains in the air, we could have used $\upsilon_y = \upsilon_{0y} - gt$. When the ball lands, $\upsilon_y = -\upsilon_{0y}$, which gives $\upsilon_{0y} = \frac{1}{2}gt$, which is the same expression used in part (b).

***3.47. Set Up:** Let $+y$ be upward. The vertical displacement of the projectile is described by $y - y_0 = \upsilon_{0y}t - \frac{1}{2}gt^2$. For $\upsilon_{0y} = 0$ this gives $y - y_0 = -\frac{1}{2}gt^2$. Thus, if we plot $y - y_0$ vs $-\frac{1}{2}t^2$, the slope of the line will be g. Note that $y - y_0 = h - 200\ \text{m}$.

Solve: **(a)** The plot is shown below. The slope of the line gives $g = 20\ \text{m/s}^2$.



(b) The projectile reaches the bottom of the cliff at the time t that satisfies $y - y_0 = -\frac{1}{2}gt^2$, which is

$$t = \sqrt{\frac{2(y_0 - y)}{g}} = \sqrt{\frac{2(200\ \text{m})}{20\ \text{m/s}^2}} = 4.5\ \text{s}.$$

(c) The horizontal distance from the cliff to where the projectile lands is $x - x_0 = \upsilon_{0x}t = (15\ \text{m/s})(4.47\ \text{s}) = 67\ \text{m}$.

3.48. Set Up: Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. With $x_0 = y_0 = 0$, x and y are related by

$$y = (\tan\theta_0)x - \frac{g}{2\upsilon_0^2\cos^2\theta_0}x^2$$

$\theta_0 = 65.0°$. $y = 8.00\ \text{m}$ and $x = 18.0\ \text{m}$.

Solve: **(a)** $\upsilon_0 = \sqrt{\frac{gx^2}{2(\cos^2\theta_0)(x\tan\theta_0 - y)}} = 17.0\ \text{m/s}$

(b) $\upsilon_x = \upsilon_{0x} = \upsilon_0 \cos\theta_0 = 7.2$ m/s. $\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y(y - y_0)$ gives

$$\upsilon_y = -\sqrt{(\upsilon_0 \sin\theta_0)^2 + 2a_y(y - y_0)} = -\sqrt{(15.4 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(8.00 \text{ m})} = -9.0 \text{ m/s}$$

$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = 11.5 \text{ m/s}. \quad \tan\theta = \frac{|\upsilon_y|}{|\upsilon_x|} = \frac{9.0}{7.2} \text{ and } \theta = 51.2°, \text{ below the horizontal.}$$

Reflect: We can check our calculated v_0.

$$t = \frac{x - x_0}{\upsilon_{0x}} = \frac{18.0 \text{ m}}{7.2 \text{ m/s}} = 2.50 \text{ s}$$

Then $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2 = (15.4 \text{ m/s})(2.50 \text{ s}) - (4.9 \text{ m/s}^2)(2.50 \text{ s})^2 = 8 \text{ m},$ which checks.

***3.49. Set Up:** Use coordinates with the origin at the boy and with $+y$ downward. The ball has $\upsilon_{0y} = 0$, $\upsilon_{0y} = 0, \upsilon_{0x} = 6.50$ m/s, $a_x = 0$ and $a_y = 9.80$ m/s^2.

Solve: (a) The dog must travel horizontally the same distance the ball travels horizontally, so the dog must have a speed of 6.50 m/s.

(b) Use the vertical motion of the ball to find its time in the air. $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ gives

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(13.5 \text{ m})}{9.80 \text{ m/s}^2}} = 1.66 \text{ s}$$

Then $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (6.50 \text{ m/s})(1.66 \text{ s}) = 10.8$ m.

3.50. Set Up: The vertical component of the initial velocity is $\upsilon_{0y} = \upsilon_0 \sin\theta_0$. The height reached by the ball is $y - y_0 = 24$ m.

Solve: The height reached by the ball is $y - y_0 = \upsilon_{0y}t - \frac{1}{2}gt^2$. At the peak of its travel, the ball has zero vertical speed, so $\upsilon_y = 0 = \upsilon_{0y} - gt$. Solving for υ_{0y} gives

$$y - y_0 = \upsilon_{0y}t - \frac{1}{2}gt^2 = \upsilon_{0y}\left(\frac{\upsilon_{0y}}{g}\right) - \frac{1}{2}g\left(\frac{\upsilon_{0y}}{g}\right)^2 = \frac{\upsilon_{0y}^2}{2g}$$

$$\upsilon_{0y} = \sqrt{2g(y - y_0)}$$

Thus, the angle at which the ball was kicked is

$$\upsilon_{0y} = \upsilon_0 \sin\theta_0 = \sqrt{2g(y - y_0)}$$

$$\theta_0 = \sin^{-1}\frac{\sqrt{2g(y - y_0)}}{\upsilon_0} = \sin^{-1}\frac{\sqrt{2(9.8 \text{ m/s}^2)(24 \text{ m})}}{25 \text{ m/s}} = 60°$$

3.51. Set Up: Use coordinates with $+y$ upward. Once the water leaves the cannon it is in free fall and has $a_x = 0$ and $a_y = -9.80$ m/s^2. The water has $\upsilon_{0x} = \upsilon_0 \cos\theta_0 = 16.0$ m/s and $\upsilon_{0y} = \upsilon_0 \sin\theta_0 = 19.2$ m/s.

Solve: Use the vertical motion to find t that gives $y - y_0 = 11.0$ m: $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ gives

$$11.0 \text{ m} = (19.2 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

The quadratic formula gives $t = 1.955 \pm 1.255$ s, and $t = 0.70$ s or $t = 3.21$ s. Both answers are physical.

$$\text{For } t = 0.70 \text{ s}, \quad x - x_0 = \upsilon_{0x}t = (16.0 \text{ m/s})(0.70 \text{ s}) = 11.2 \text{ m.}$$
$$\text{For } t = 3.21 \text{ s}, \quad x - x_0 = \upsilon_{0x}t = (16.0 \text{ m/s})(3.21 \text{ s}) = 51.4 \text{ m.}$$

When the cannon is 11.2 m from the building the water hits this spot on the wall on its way up to its maximum height. When is it 51.4 m from the building it hits this spot after it has passed through its maximum height.

3.52. Set Up: Because the apple and the arrow are at the same elevation, we can use the formula for range to find the launch angle. From Example 3.5, the formula for range is $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$.

Solve: Solving for the launch angle and inserting the given quantities gives

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} \quad \Rightarrow \quad \theta_0 = \frac{1}{2}\sin^{-1}\frac{Rg}{\upsilon_0^2} = \frac{1}{2}\sin^{-1}\frac{(30 \text{ m})(9.8 \text{ m/s}^2)}{(75 \text{ m/s})^2} = 1.5°, 88.5°$$

The archer is unlikely to aim the arrow at 88.5° above the horizontal, although the arrow would hit the apple (assuming no air resistance), so the archer should aim at 1.5° above the horizontal.

***3.53. Set Up:** Let $+y$ be downward. $a_x = 0$, $a_y = +9.80 \text{ m/s}^2$. $\upsilon_{0x} = \upsilon_0 \cos\theta_0 = 5.36$ m/s, $\upsilon_{0y} = \upsilon_0 \sin\theta_0 = 4.50$ m/s.

Solve: Use the vertical motion to find the time in the air: $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 14.0$ m gives $14.0 \text{ m} = (4.50 \text{ m/s})t + (4.9 \text{ m/s}^2)t^2$. The quadratic formula gives

$$t = \frac{1}{2(4.9)}\left(-4.50 \pm \sqrt{(4.50)^2 - 4(4.9)(-14.0)}\right) \text{ s}$$

The positive root is $t = 1.29$ s. Then $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2 = (5.36 \text{ m/s})(1.29 \text{ s}) = 6.91$ m.

3.54. Set Up: The velocity has a horizontal tangential component and a vertical component. The vertical component of acceleration is zero and the horizontal component is $a_{\text{rad}} = \dfrac{\upsilon_x^2}{R}$. Let $+y$ be upward and $+x$ be in the direction of the tangential velocity at the instant we are considering.

Solve: **(a)** The bird's tangential velocity can be found from

$$\upsilon_x = \frac{\text{circumference}}{\text{time of rotation}} = \frac{2\pi(4.00 \text{ m})}{4.00 \text{ s}} = \frac{25.14 \text{ m}}{4.00 \text{ s}} = 6.28 \text{ m/s}$$

Thus its velocity consists of the components $\upsilon_x = 6.28$ m/s and $\upsilon_y = 4.00$ m/s. The speed relative to the ground is then $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = 7.4$ m/s.

(b) The bird's speed is constant, so its acceleration is strictly centripetal—entirely in the horizontal direction, toward the center of its spiral path—and has magnitude $a_{\text{rad}} = \dfrac{\upsilon_x^2}{r} = \dfrac{(6.28 \text{ m/s})^2}{4.00 \text{ m}} = 9.9 \text{ m/s}^2$.

(c) Using the vertical and horizontal velocity components $\theta = \tan^{-1}\dfrac{4.00 \text{ m/s}}{6.28 \text{ m/s}} = 32.5°$.

Reflect: The angle between the bird's velocity and the horizontal remains constant as the bird rises.

***3.55. Set Up:** For circular motion the acceleration has magnitude

$$a_{\text{rad}} = \frac{\upsilon^2}{R}$$

and is directed toward the center of the circle. The period T (time for one revolution) is

$$T = \frac{2\pi R}{\upsilon}$$

Solve: (a) $a_{\text{rad}} = \dfrac{\upsilon^2}{R} = \dfrac{(7.00\text{ m/s})^2}{14.0\text{ m}} = 3.50\text{ m/s}^2$, upward

(b) $a_{\text{rad}} = 3.50\text{ m/s}^2$, downward

(c) $T = \dfrac{2\pi R}{\upsilon} = \dfrac{2\pi(14.0\text{ m})}{7.00\text{ m/s}} = 12.6\text{ s}$

3.56. Set Up: Take $+y$ downward. $\upsilon_{0x} = \upsilon_0$, $\upsilon_{0y} = 0$. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$.

Solve: (a) Use the vertical motion to find the time for the boulder to reach the level of the lake: $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = +20\text{ m}$ gives

$$t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(20\text{ m})}{9.80\text{ m/s}^2}} = 2.02\text{ s}$$

The rock must travel horizontally 100 m during this time. $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2$ gives

$$\upsilon_0 = \upsilon_{0x} = \frac{x - x_0}{t} = \frac{100\text{ m}}{2.02\text{ s}} = 49\text{ m/s}$$

(b) In going from the edge of the cliff to the plain, the boulder travels downward a distance of $y - y_0 = 45\text{ m}$.

$$t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(45\text{ m})}{9.80\text{ m/s}^2}} = 3.03\text{ s}$$

and $x - x_0 = \upsilon_{0x}t = (49\text{ m/s})(3.03\text{ s}) = 148\text{ m}$. The rock lands $148\text{ m} - 100\text{ m} = 48\text{ m}$ beyond the foot of the dam.

Reflect: The boulder passes over the dam 2.02 s after it leaves the cliff and then travels an additional 1.01 s before landing on the plain. If the boulder has an initial speed that is less than 49 m/s, then it lands in the lake.

3.57. Set Up: Since $\Delta y = 0$ we may use the equations derived in Example 3.5 for the time of flight and range of the ball: $t = \dfrac{2\upsilon_0 \sin\theta_0}{g}$, $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g}$. The speed that the outfielder needs can be calculated from this time and the initial distance of the outfielder from the location where the ball lands.

Solve: The range of the ball is $R = \dfrac{\upsilon_0^2 \sin 2\theta_0}{g} = \dfrac{(34.5\text{ m/s})^2 \sin 130^\circ}{9.80\text{ m/s}^2} = \underline{93.04}6\text{ m}$. The time of flight for the ball is $t = \dfrac{2\upsilon_0 \sin\theta_0}{g} = \dfrac{2(34.5\text{ m/s})\sin 65^\circ}{(9.80\text{ m/s}^2)} = \underline{6.}382\text{ s}$. The distance that the outfielder must travel in this time is $\underline{95.76} - 72.0 = \underline{21.04}$ meters. Thus, the speed required is: $\text{speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{\underline{21.04}\text{ m}}{\underline{6.382}\text{ s}} = 3.30\text{ m/s}$.

Reflect: The time of flight can also be calculated directly from the range and the x component of velocity:

$$t = \frac{R}{\upsilon_0 \cos\theta} = \frac{\underline{93.04}\text{ m}}{(34.5\text{ m/s})\cos 65^\circ} = 6.38\text{ s}$$

Solutions to Passage Problems

3.58. Set Up: The maximum speed is the initial speed of the seeds. To ensure that we can capture the motion of the seeds, we use the upper limit of the range given for the initial speed. Thus, $\upsilon_{max} = 4.6$ m/s

Solve: The frame interval Δt must be long enough so that the speeds do not move more than $\Delta d = 0.20$ mm during this time. Thus, $\upsilon_{max} < \frac{\Delta d}{\Delta t} \Rightarrow \Delta t < \frac{\Delta d}{\upsilon_{max}}$. The frame rate *f* is number of frames captured per second, so it is the inverse of the frame time (which is the number of seconds per frame). Thus, the minimum frame rate is $f = \frac{1}{\Delta t} > \frac{\upsilon_{max}}{\Delta d} = \frac{4.6 \text{ m/s}}{0.20 \times 10^{-3} \text{ m}} = 23,000$ frames/s, so the correct answer is C.

***3.59. Set Up:** Let the positive *y* axis be in the upward direction. The seed is launched straight up, so it has no *x* component to its velocity. Because the *y* component of the velocity is zero at the maximum height, the total velocity of the seed is zero at the maximum height.

Solve: Inserting $\upsilon = 0$ into $\upsilon = \upsilon_0 - gt$ and solving for *t* gives $t = \frac{\upsilon_0}{g} = \frac{4.6 \text{ m/s}}{9.8 \text{ m/s}^2} = 0.47$ s, so the correct answer is B.

3.60. Set Up: Let the positive *y* axis be in the upward direction. The displacement of the seed is $y - y_0 = -20$ cm and the *y* component of its initial velocity is $\upsilon_{0y} = 0$. During its fall, the seed travels in the horizontal direction with constant speed $\upsilon_x = 4.6$ m/s.

Solve: The time for the seed to fall to the ground is

$$y - y_0 = \upsilon_{0y} t - \frac{1}{2} g t^2 = -\frac{1}{2} g t^2$$

$$t = \pm\sqrt{\frac{-2(y - y_0)}{g}}$$

During this time, the seed travels a horizontal distance

$$x - x_0 = \upsilon_{0x} t = \upsilon_{0x}\left(\pm\sqrt{\frac{-2(y - y_0)}{g}}\right) = (4.6 \text{ m/s})\left(\pm\sqrt{\frac{-2(-20 \times 10^{-2} \text{ m})}{9.8 \text{ m/s}^2}}\right) = \pm 0.93 \text{ m}$$

The correct answer is B.

Reflect: The plus-minus sign means that the seed can travel horizontally to the left or to the right.

***3.61. Set Up:** Consider each option in light of the data.

Solve: To maximize the height that the seeds attain above the plant, the seeds should be launched at +90°, which is not the case. Thus, hypothesis A is not correct. To drive the seeds into the ground with the maximum force or to minimize the time that the seeds spend in the air, the seeds should be launched at −90°, which is not the case. Thus, hypotheses B and D are not correct. Hypothesis C seems plausible given the data, so the correct answer is C.