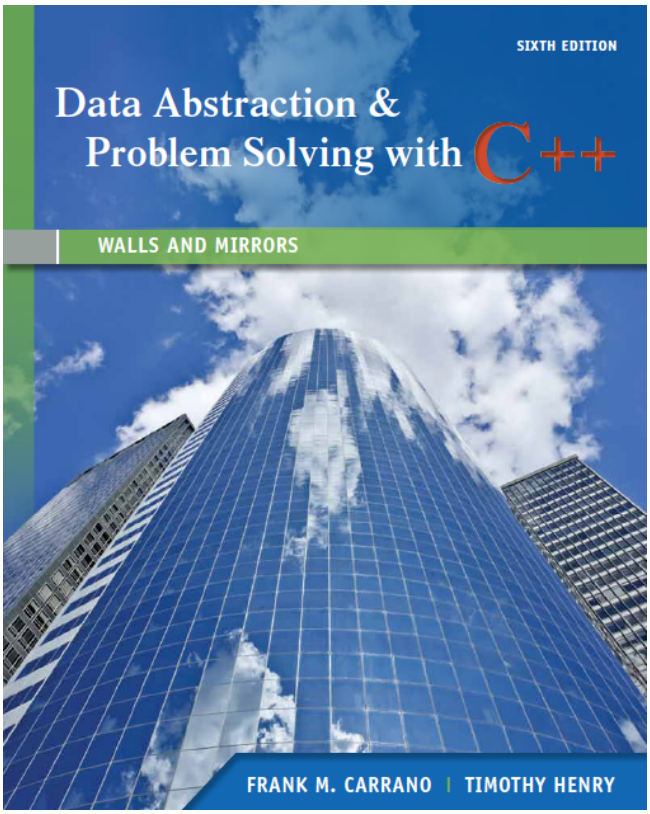
Solutions for Selected Exercises



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Chapter 1 Data Abstraction: The Walls

1

```
const CENTS_PER_DOLLAR = 100;

/** Computes the change remaining from purchasing an item costing
    dollarCost dollars and centsCost cents with d dollars and c cents.
    Precondition: dollarCost, centsCost, d and c are all nonnegative
    integers and centsCost and c are both less than CENTS_PER_DOLLAR.
    Postcondition: d and c contain the computed remainder values in
    dollars and cents respectively. If input value d < dollarCost, the
    proper negative values for the amount owed in d dollars and/or c
    cents is returned. */
void computeChange(int dollarCost, int centsCost, int& d, int& c);
```

2a

```
const MONTHS_PER_YEAR = 12;
const DAYS_PER_MONTH[] = {31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};

/** Increments the input Date values (month, day, year) by one day.
    Precondition: 1 <= month <= MONTHS_PER_YEAR,
                  1 <= day <= DAYS_PER_MONTH[month - 1], except
                  when month == 2, day == 29 and isLeapYear(year) is true.
    Postcondition: The valid numeric values for the succeeding month, day,
                   and year are returned. */
void incrementDate(int& month, int& day, int& year);

/** Determines if the input year is a leap year.
    Precondition: year > 0.
    Postcondition: Returns true if year is a leap year; false otherwise. */
bool isLeapYear(int year);
```

3a Change the purpose of an appointment:

```
changeAppointmentPurpose(apptDate: Date, apptTime: Time,
                         purpose: string): boolean
   if (isAppointment(apptDate, apptTime))
      cancelAppointment(apptDate, apptTime)

   return makeAppointment(apptDate, apptTime, purpose)
```

3b Display all the appointments for a given date:

```
displayAllAppointments(apptDate: Date)
   time = startOfDay

   while (time < endOfDay)
      if (isAppointment(apptDate, time))
         displayAppointment(apptDate, time)

      time = time + halfHour
```

This implementation requires the definition of a new operation, `displayAppointment`(), as well as definitions for the constants `startOfDay`, `endOfDay` and `halfHour`.

4

```
   Bag<string> fragileBag;

   while (storeBag.contains("eggs"))
   {
      storeBag.remove("eggs");
      fragileBag.add("eggs");
   }  // end while

   while (storeBag.contains("bread"))
   {
      storeBag.remove("bread");
      fragileBag.add("bread");
   }  // end while

   // Transfer remaining items from storeBag to groceryBag;
   Bag<string> groceryBag;
   v = storeBag.toVector();
   for (int i = 0; i < v.size(); i++)
      groceryBag.add(v.at(i));
```

5

```
/** Removes and counts all occurrences, if any, of a given string
 from a given bag of strings.
 @param bag  A given bag of strings.
 @param givenString  A string.
 @return  The number of occurrences of givenString that occurred
 and were removed from the given bag. */
int removeAndCount(ArrayBag<string>& bag, string givenString)
{
   int counter = 0;
   while (bag.contains(givenString))
   {
      counter++;
      bag.remove(givenString);
   }  // end while

   return counter;
}  // end removeAndCount
```

6

```
/** Creates a new bag that combines the contents of this bag and a
 second given bag without affecting the original two bags.
 @param anotherBag  The given bag.
 @return  A bag that is the union of the two bags. */
public BagInterface<ItemType> union(BagInterface<ItemType> anotherBag);
```

7

```
/** Creates a new bag that contains those objects that occur in both this
 bag and a second given bag without affecting the original two bags.
 @param anotherBag  The given bag.
 @return  A bag that is the intersection of the two bags. */
public BagInterface<ItemType> intersection(BagInterface<ItemType> anotherBag);
```

8

```
/** Creates a new bag of objects that would be left in this bag
 after removing those that also occur in a second given bag
 without affecting the original two bags.
 @param anotherBag  The given bag.
 @return  A bag that is the difference of the two bags. */
public BagInterface<T> difference(BagInterface<T> anotherBag);
```

9a

```
display(p.coefficient(p.degree()))
```

9b

```
p.changeCoefficient(p.coefficient(3) + 8, 3)
```

9c

```
for (power = 0; power < p.degree() || power < q.degree(); power++)
     // R is the sum of polynomials P and Q to degree power.
     r.changeCoefficient(p.coefficient(power) + q.coefficient(power), power)
```

Chapter 2 Recursion: The Mirrors

---

1 The problem is defined in terms of a smaller problem of the same type:
Here, the last value in the array is checked and then the remaining part of the array is passed to the function.

Each recursive call diminishes the size of the problem:
The recursive call to `getNumberEqual` subtracts 1 from the current value for `n`, passing this as the parameter `n` in the next call, effectively reducing the size of the unsearched remainder of the array by 1.

An instance of the problem serves as the base case:
Here, the case where the size of the array is 0 (i.e.: $n \leq 0$) results in the return of the value 0: an array of size 0 can have no instances of the `desiredValue`. This terminates the recursion.

As the problem size diminishes, the base case is reached:
`n` is an integer and is decremented by 1 with each recursive call. After `n` recursive calls, the parameter `n` in the *n*th call will have the value 0 and the base case will be reached.

2a The call `rabbit(5)` produces the following box trace:

```
n = 5
rabbit(4) = ?
rabbit(3) = ?
return ?
```

Follow the rabbit(4) call

```
n = 5
rabbit(4) = ?
rabbit(3) = ?
return ?
```

```
n = 4
rabbit(3) = ?
rabbit(2) = ?
return ?
```

Follow the rabbit(3) call

```
n = 5
rabbit(4) = ?
rabbit(3) = ?
return ?
```

```
n = 4
rabbit(3) = ?
rabbit(2) = ?
return ?
```

```
n = 3
rabbit(2) = ?
rabbit(1) = ?
return ?
```

Follow the rabbit(2) call

```
n = 5
rabbit(4) = ?
rabbit(3) = ?
return ?
```

```
n = 4
rabbit(3) = ?
rabbit(2) = ?
return ?
```

```
n = 3
rabbit(2) = ?
rabbit(1) = ?
return ?
```

```
n = 2

return 1
```

Base case: n = 2

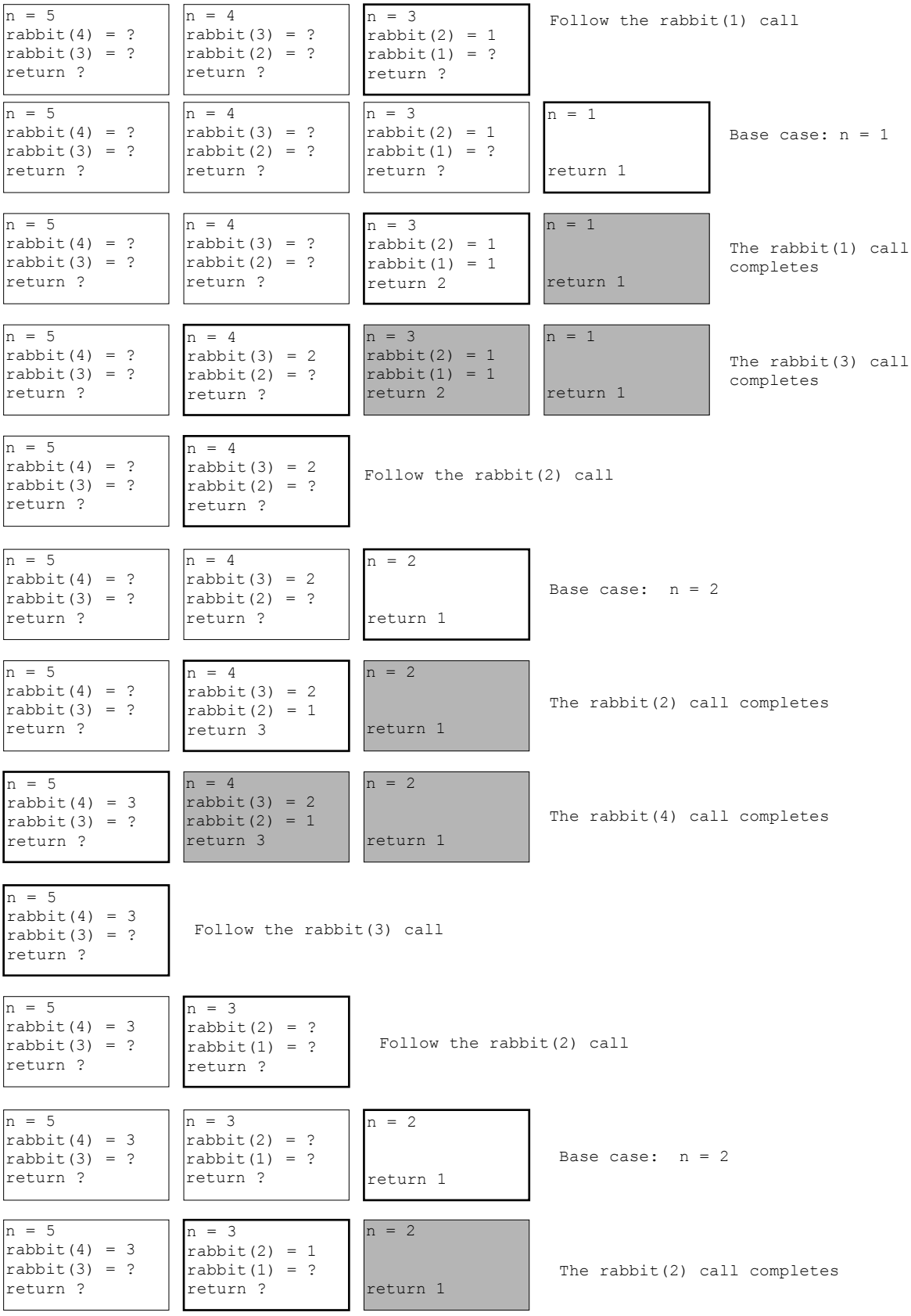
```
n = 5
rabbit(4) = ?
rabbit(3) = ?
return ?
```

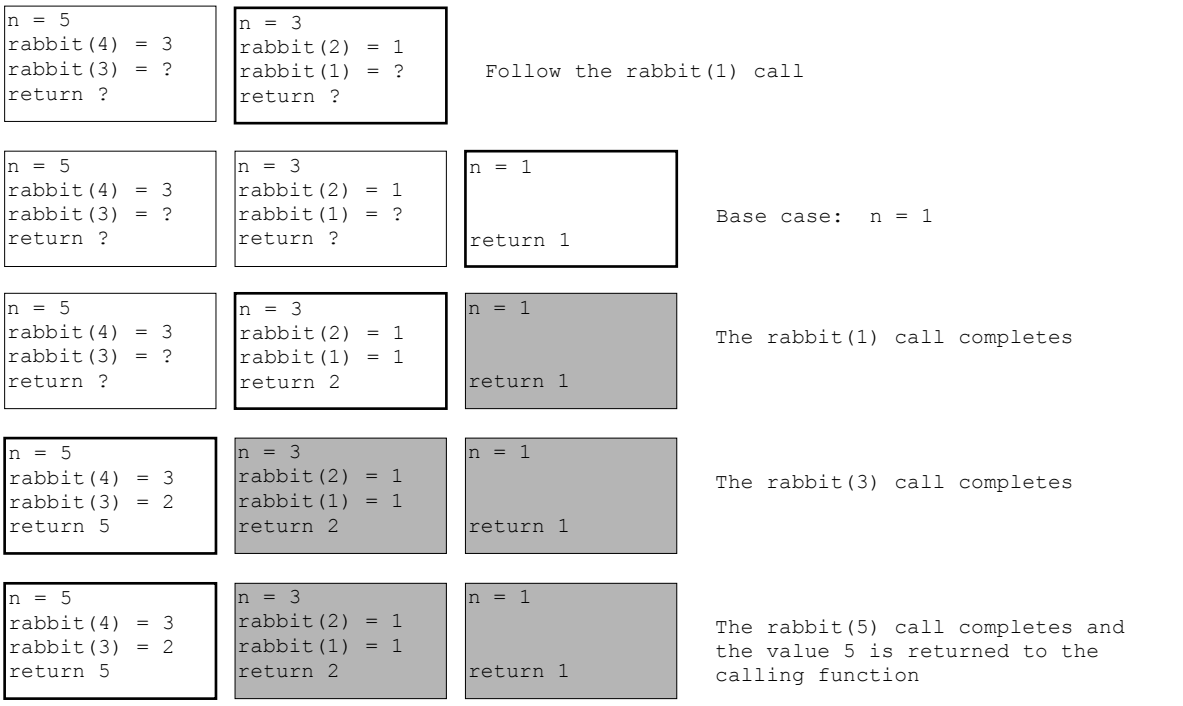
```
n = 4
rabbit(3) = ?
rabbit(2) = ?
return ?
```

```
n = 3
rabbit(2) = ?
rabbit(1) = ?
return ?
```

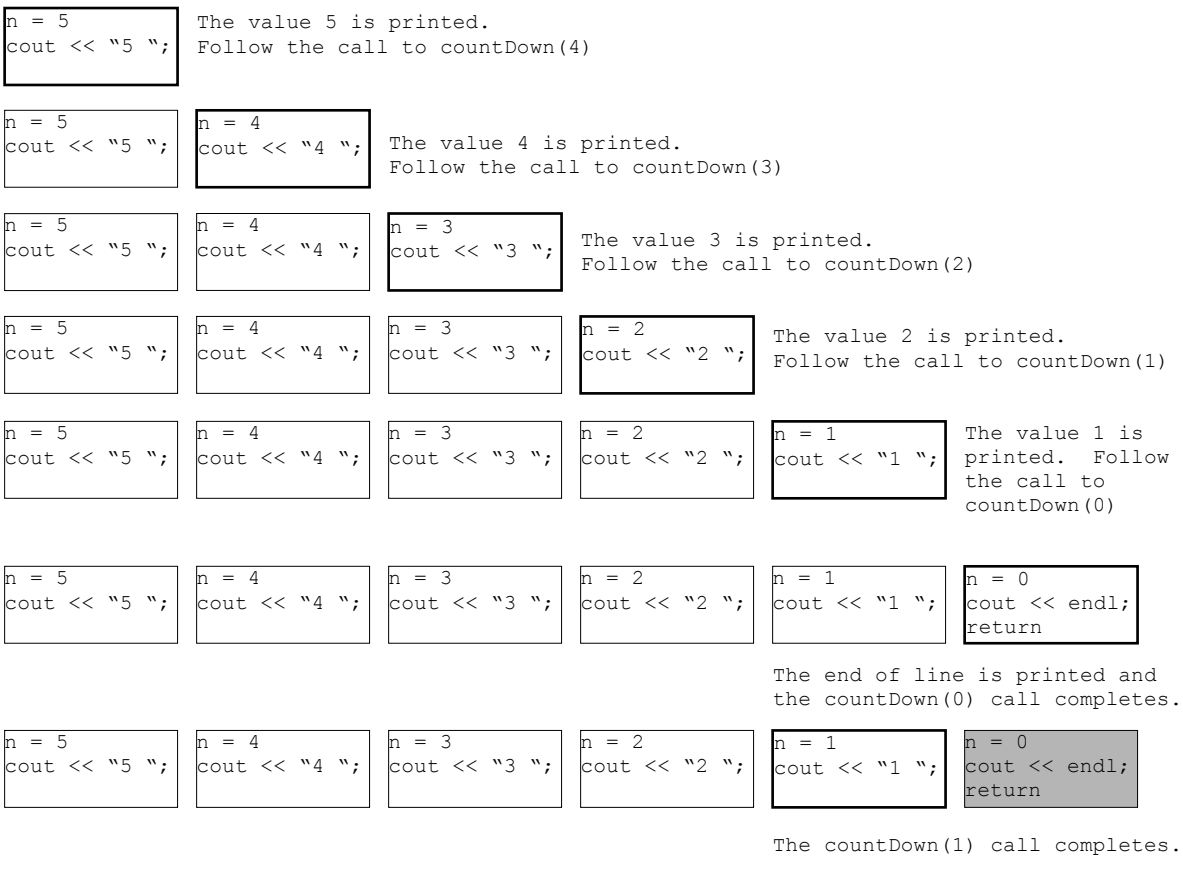
```
n = 2

return 1
```

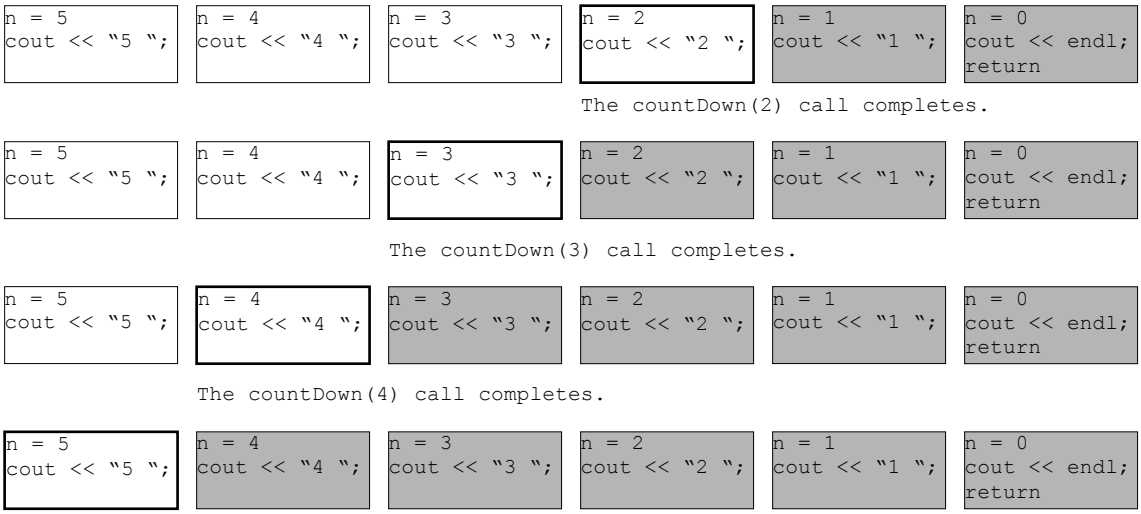
The rabbit(2) call completes





2b The call `countDown(5)` produces the following box trace:





3

```
/** Returns the sum of the first n integers in the array anArray.
    Precondition:  0 <= n <= size of anArray.
    Postcondition:  The Sum of the first n integers in the
                    array anArray are returned.  The contents of
                    anArray and the value of n are unchanged. */
int computeSum(const  int anArray[], int n)
{  // base case
   if (n <= 0)
      return 0;
   else  // reduce the problem size
      return anArray[n - 1] + computeSum(anArray, n - 1);
}  // end computeSum
```

4

```
int sum (int start, int end )
{
   if (start == end)
      return end;
   else
      return start + sum(start + 1, end);
}
```

5

```
#include <string>
using namespace std;

// ------------------------------------------------
// Writes a character string backward.
// Precondition:  The string str contains size characters,
//                where size >= 1.
// Postcondition:  str is written backward, but remains
//                 unchanged.
// ------------------------------------------------
void writeBackward(string str, int size)
{  // base case
   if (size == 1)
      cout << str[0];

   // else, write rest of string
   else if (size > 1)
   {
      cout << str[size - 1];
      writeBackward(str, size - 1);
   }
   // size <= 0  do nothing;
}  // end writeBackward
```

---

6 The recursive method does not have a base case. As such, it will never terminate.

7

```
// ------------------------------------------
// Prints out the integers from 1 through n as a
// comma separated list followed by a newline.
// Precondition:  n >= 0 and limit == n.
// Postcondition:  The integers from 1 through n
//                 are printed out followed by a
//                 newline.
// ------------------------------------------
void printIntegers(int n, int limit)
{
   if (n > 0)
   {  // print out the rest of the integers
      printIntegers(n - 1, limit);

      // now print out this integer
      cout << n;

      // test for end of string
      if (n != limit)
        cout << ", ";
      else
        cout << "." << endl;   // end of string
   } // end if

   // n <= 0 do nothing
}  // end printIntegers
```

8

```
int getSum(int n)
{
   int result;
   if (n == 1)
      result = 1;
   else
      result = n + sum (n-1);
   return result;
}  // end getSum
```

9

```
const int NUMBER_BASE = 10;

/** Displays the decimal digits of number in reverse order.
    Precondition:  number >= 0.
    Postcondition:  The decimal digits of number are printed in reverse order.
                    This function does not output a newline character at the
                    end of a string. */
void reverseDigits(int number)
{  // check for input bounds
   if (number >= 0)
   {  // base case
      if (number < NUMBER_BASE)
         cout << number;
      else
      {  // print out rightmost digit
         cout << number % NUMBER_BASE;

         // pass remainder of digits to next call
         reverseDigits(number / NUMBER_BASE);
      } // end if
   }  // end if
}  // end reverseDigits
```

10a

```
/** Displays a line of n characters, where ch is the character.
    Precondition:   n >= 0.
    Postcondition:  A line of n characters ch is output
                    followed by a newline. */
void writeLine(char ch, int n)
{  // base case
   if (n <= 0)
      cout << endl;

   // write rest of line
   else
   {
      cout << ch;

      writeLine(ch, n - 1);
   }  // end if
}  // end writeLine
```

10b

```
/** Displays a block of m rows by n columns of character ch.
    Precondition:  m >= 0 and n >= 0.
    Postcondition:  A block of m rows by n columns of
                    character ch is printed. */
void writeBlock(char ch, int m, int n)
{  if (m > 0)
   {
      writeLine(ch, n);              // write first line
      writeBlock(ch, m - 1, n);      // write rest of block
   }

   // base case:  m <= 0 do nothing.
}  // end writeBlock
```

---

11 Running the given program produces the following output:

```
Enter: a = 1 b = 7
Enter: a = 1 b = 3
Leave: a = 1 b = 3
Leave: a = 1 b = 7
2
```

---

12 Running the given program produces the following output:

```
Enter: first = 1 last = 30
Enter: first = 1 last = 14
Enter: first = 1 last = 6
Enter: first = 4 last = 6
Leave: first = 4 last = 6
Leave: first = 1 last = 6
Leave: first = 1 last = 14
Leave: first = 1 last = 30
5
```

---

13 The algorithm first checks to see if n is a positive number: if not it immediately terminates. Otherwise, an integer division of n by 8 is taken and if the result is greater than 0 (i.e.: if $n > 8$), the function is called again with $n/8$ as an argument. This call processes that portion of the number composed of higher powers of 8. After this call, the residue for the current power, n % 8, is printed.

The given function computes the number of times 8^0, 8^1, 8^2, ... will divide n. These values are stacked recursively and are printed out in the reverse of the order of computation. The following is the hand execution with $n = 100$:

```
n = 100
displayOctal(12)
      n = 12
      displayOctal(1)
            n = 1
            cout << 1
      cout << 4
cout << 4

Output:  144
```

---

14 Even though the precondition states that *n* is nonnegative, there is no actual code to keep a negative value for *n* from being used as the argument in the function.

A call to the function `f` will produce a further call to `f` with a negative argument when *n* = 3. Because 3 is not within the subrange of 0 to 2, the default case will execute, and the function will attempt to evaluate `f(1)` and `f(-1)`. Because the value for `f(n)` is based on the values for `f(n-2)` and `f(n-4)`, if *n* is even, its addends will be the next two smaller even integers; likewise, if *n* is odd, `f(n)`'s addends will be *n*'s next two smaller odd integers. Thus any odd nonnegative integer will eventually cause `f` to evaluate `f(3)`.

Theoretically, calling `f` with an odd integer will cause an infinite sequence of function calls. On the practical level, the computer's run-time stack will overflow, or an integer underflow will happen.

The following is the exact output of the program:

```
Function entered with n = 8
Function entered with n = 6
Function entered with n = 4
Function entered with n = 2
Function entered with n = 0
Function entered with n = 2
Function entered with n = 4
Function entered with n = 2
Function entered with n = 0
The value of f(8) is 27
```

---

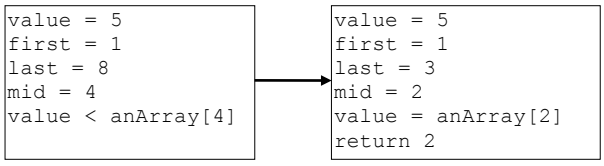
15 The following output is produced when `x` is a value argument:

```
6 2
7 1
8 0
8 0
7 1
6 2
```

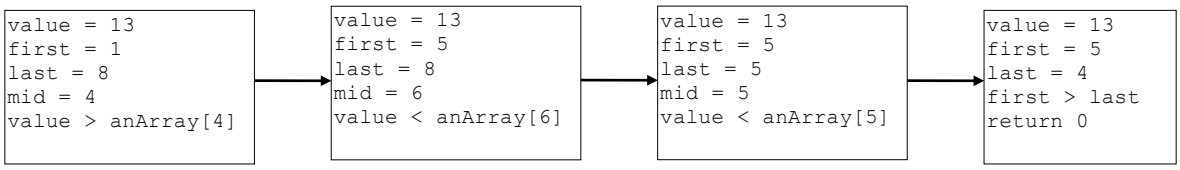
Changing `x` to a reference argument produces:

```
6 2
7 1
8 0
8 0
8 1
8 2
```

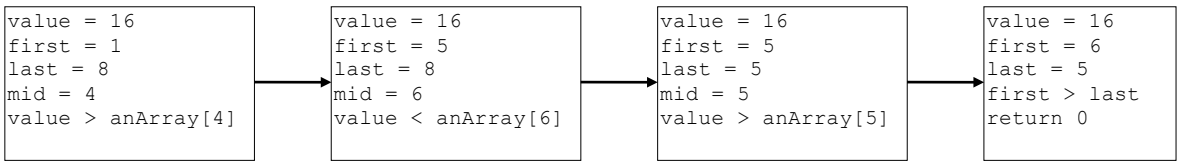
16a The call `binSearch(5)` produces the following box trace:



16b The call `binSearch(13)` produces the following box trace:



16c The call `binSearch(16)` produces the following box trace:



---

17 **a** For a binary search to work, the array must first be sorted in either ascending or descending order.

b The index is (0 + 101)/2 = 50.

c Number of comparisons = $\lfloor \lg 101 \rfloor = 6$.

18a

```
/** Returns the value of x raised to the nth power.
    Precondition:  n >= 0
    Postcondition:  The computed value is returned. */
double power1(double x, int n)
{  double result = 1;          // value of x^0

   while (n > 0)               // iterate until n == 0
   {  result *= x;
      n--;
   }
   return result;
}  // end power1
```

18b

```
/** Returns the value of x raised to the nth power.
    Precondition:  n >= 0
    Postcondition:  The computed value is returned. */
double power2(double x, int n)
{  // base case
   if (n == 0)
      return 1;

   // else, multiply x by rest of computation
   else
      return x * power2(x, n-1);
}  // end power2
```

18c

```
/** Returns the value of x raised to the xth power.
    Precondition:  n >= 0
    Postcondition:  The computed value is returned. */
double power3(double x, int n)
{
   if (n == 0)
      return 1;
   else
   {  // do this computation only once!!
      double halfPower = power3(x, n/2);

      // if n is even...
      if (n % 2 == 0)
        return halfPower * halfPower;

      // if n is odd...
      else
        return x * halfPower * halfPower;
   }
}  // end power3
```

18d The following table lists the number of multiplications performed by each of the algorithms for computing the values on the top line:

	3^{32}	3^{19}
power1	32	19
power2	32	19
power3	7	8

18e The following table lists the number of recursive calls made by each of the algorithms indicated in order to perform the computation on the inputs given on the top line:

	3^{32}	3^{19}
power2	32	19
power3	6	5

---

19 Maintain a count of the recursive depth of each call by passing this count as an additional parameter to the function call and print that many spaces or tabs in front of each message:

```
/** Computes a term in the Fibonacci sequence.
    Precondition:  n is a positive integer and tab = 0.
    Postcondition:  The progress of the recursive function call is displayed
                    as a sequence of increasingly nested blocks.  The function
                    returns the nth Fibonacci number. */
int rabbit(int n, int tab)
{
   int value;

   // Indent the proper distance for this block
   for (int i = 0; i < tab; i++)
      cout << '\t';

   // Display status of call
   cout << "Enter: n = " << n << endl;

   if (n <= 2)
      value = 1;

   else  // n > 2, so n-1 > 0 and n-2 > 0
      // indent by one for next call
      value = rabbit(n-1, tab+1) + rabbit(n-2, tab+1);

   // Indent the proper distance for this block
   for (i = 0; i < tab; i)
      cout << '\t';

   // Display status of call
   cout << "Leave: n = " << n << " value = " << value << endl;

   return value;
}
```

20a `f(6)` is 8; `f(7)` is 11; `f(12)` is 95; `f(15)` is 320.

20b Since we only need the five most recently computed values, we will maintain a "circular" five-element array indexed modulus 5.

```
// Pre: n > 0.
int fOfN(int n)
{
   int last5[5] = {1, 1, 1, 3, 5};

   for (int i = 5; i < n; i++)
   {
      int fi = last5[(i - 1) % 5] + 3 * last5[(i - 5) % 5];

      // Replace entry in last5
      last5[i % 5] = fi;  // f(i) = f(i - 1) + 3 x f(i - 5)
   }  // end for

   return last5[(n - 1) % 5];
}  // end fOfN
```

21a A function to compute n! iteratively:

```
long fact(int n)
{  int i;
   long result;

   if (n < 1) // base case
      result = 0;
   else
   {
      result = 1;
      for (i = 2; i <= n; i++)
         result *= i;
   }  // end if

   return result;
}  // end fact
```

21b A simple iterative solution to writing a string backwards:

```
#include <string>

void writeBackward(string  str)
{
   for (int i = str.size() - 1; i >= 0; i--)
      cout << str[i];

   cout << endl;
}  // end writeBackward
```

21c A function to perform an iterative binary search:

```
/** Searches a sorted array and returns the index in the
    array corresponding to the value key if key is in the
    array, -1 otherwise.
    Precondition:  high is sorted in ascending order.
                   low = 0 and high = the size of high - 1.
    Postcondition:  If key is found, its location index
                    in high is returned, else -1 is returned. */
int binarySearch(int  anArray[], int key, int low, int high)
{
   int mid, result;
   while (low < high)
   {  mid = (low + high)/2;

      if (anArray[mid] == key)
      {  low = mid;
         high = mid;
      }
      else if (anArray[mid] < key)
         low = mid + 1; // search the upper half
      else
         high = mid - 1; // search the lower half
   }  // end while

   if (low > high)
      result = -1;       // if not found, return -1
   else if (anArray[low] != key)
      result = -1;
```



```
   else
      result = low;

   return result;
}  // end binarySearch
```

21d We implement a function to find the kth smallest entry in an array using an integer array and a selection sort up to k times. We assume a standard integer swap function.

```
int kSmall(int k, int anArray[], int size)
{
   for (int i=0; i<k; i++)
      for (int j = i+1; j < size; j++)
         if (anArray[j] < anArray[i])
            swap(anArray[i], anArray[j]);

   return anArray[k-1];
}  // end kSmall
```

---

22 The for loop invariant is:

$3 \leq i \leq n$

So the sum = $\sum_{i=3}^{n} rabbit(i-1) + rabbit(i-2)$.

23a We must verify that the equation holds for both the base case and the recursive case.

For the base case, let $gcd(a, b) = b$. Then, $a \bmod b = 0$ and, since $0/n = 0$ for all n, then $gcd(b, 0) = b$. Hence, $gcd(b, a \bmod b) = b$.

For the recursive case, let $gcd(a, b) = d$, i.e.: $a = dj$ and $b = dk$ for integers d, j and k. Now there exists integer $n = a \bmod b$ such that $(n - a)/b = q$, where q is an integer. Then, $n-a = bq$ and, so $n - dj = dkq$ i.e. $n = d(kq + j)$. Then, $(n/d) = kq + j$, where $(kq + j)$ is an integer. So, d divides n i.e.: d divides $(a \bmod b)$.

To show that d is the greatest common divisor of b and $a \bmod b$, suppose for contradiction there exists integer $g > d$ such that $b = gr$ and $(a \bmod b) = gs$ for integers r and s. Then, $(gs - a)/gr = q'$ where q' is an integer. So $gs - a = grq'$ i.e.: $a = g(s - rq')$. Thus, g divides a and g divides b. But $gcd(a, b) = d$ by hypothesis. Therefore, $gcd(b, a \bmod b) = d$.

The proof is symmetrical where $gcd(b, a \bmod b) = d$ is taken for the hypothesis.

23b If $b > a$ in the call to *gcd*, $a \bmod b = a$ and so the recursive call effectively reverses the arguments.

23c When $a > b$, the argument associated with the parameter a decreases in the next recursive call. If $b > a$, the next recursive call will swap the arguments so that $a > b$. Thus, the first argument will eventually equal the second and so eventually $a \bmod b$ will be 0.

24a

$$c(n) = \begin{cases} 0 & if \quad n = 1 \\ 1 & if \quad n = 2 \\ \sum_{i=1}^{n-1} (c(n-i)+1) & if \quad n > 2 \end{cases}$$

24b

$$c(n) = \begin{cases} 0 & if \quad n = 1 \\ 1 & if \quad n = 2 \\ c(n-1)+c(n-2) & if \quad n > 2 \end{cases}$$

---

25 *Acker*(1, 2) = 4

```
int acker(int m, int n)
{
   int result;

   if (m == 0)
      result = n+1;

   else if (n == 0)
      result = acker(m-1, 1);

   else
      result = acker(m-1, acker(m, n-1));

   return result;
}  // end acker
```

Chapter 3 Array-Based Implementations

1

```
/** Computes the sum of the integers in the bag aBag.
 @param aBag  A bag of integers.
 @return  The sum of the integers in aBag. */
int sumOfBag(ArrayBag<int>& aBag)
{
   int sum = 0;
   int size = aBag.getCurrentSize();
   vector<int> bagContents = aBag.toVector();

   for (int i = 0; i < size; i++)
      sum += bagContents.at(i);

   return sum;
}  // end sumOfBag
```

2

```
/** Replaces one occurrence of a given item in a bag with another one.
 @param aBag  A bag.
 @param itemToReplace  The item to replace.
 @param replacement  The item that replaces itemToReplace.
 @return  True if the replacement is successful; otherwise returns false. */
bool replace(ArrayBag<string>& aBag, string itemToReplace, string replacement )
{
   bool success = aBag.remove(itemToReplace);
   if (success)
      success = aBag.add(replacement);

   return success;
} // end replace
```

3

a An advantage to defining such operations externally to the ADT is that they are independent of the ADT's implementation. The client simply uses ADT operations.

The disadvantage is that the client must use existing ADT operations. Certain operations might be impossible to define—either at all or efficiently— at the client level if the ADT does not provide sufficient access to the ADT's data. For `replace`, the client has no control over which occurrence of the item is replaced.

b Defining `replace` within the ADT obviously alleviates the disadvantage cited in part a. Replacing an item can be done more efficiently than first removing it from the bag and then adding another item to the bag.

However, the client cannot control how `replace` behaves. Its specification is at the discretion of the ADT designer.

4

Some ADT rectangle operations:

```
// Creates a rectangle and initializes its length and width to default values.
+Rectangle()

// Creates a rectangle and initilializes its length and width to the
// values received as parameters.
+Rectangle(length: double, width: double)

// Sets or modifies the length of this rectangle.
// Checks to make sure that the new length is greater than 0.
+setLength(length: double)

// Sets or modifies the width of this rectangle.
// Checks to make sure that the new width is greater than 0.
+setWidth(width:double)

// Returns this rectangle's length.
+getLength(): double

// Returns this rectangle's width.
+getWidth(): double

// Returns this rectangle's area.
+getArea(): double

// Returns this  rectangle's perimeter.
+getPerimeter(): double
```

```
/** Header file for the class Rectangle. */
class Rectangle
{
private:
   double length;
   double width;

public:
   /** Creates a rectangle and initializes its length and width to
       default values. */
   Rectangle();

   /** Creates a rectangle and initializes its length and width to given
       values. */
   Rectangle(double initialLength, double initialWidth);

   /** Sets or modifies the length of this rectangle.
       Checks to make sure that the new length is greater than 0. */
   void setLength(double newLength);

   /** Sets or modifies the width of this rectangle.
       Checks to make sure that the new width is greater than 0. */
   void setWidth(double newWidth);

   /** @return  This rectangle's length. */
   double getLength();

   /** @return  This rectangle's width. */
   double getWidth();

   /** @return  This rectangle's area. */
   double getArea();
```

