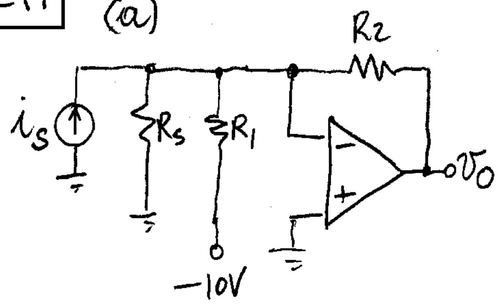
CH. 2 – PROBLEM SOLUTIONS

2.1 (a)



$$R_2 = \frac{\Delta v_O}{\Delta i_S} = \frac{5-(-5)}{(1-0)\,\text{mA}} = 10\ \text{k}\Omega.$$

$$i_S = 0 \Rightarrow v_O = +5\text{V} = -\frac{R_2}{R_1}(-10)$$

$$\Rightarrow R_1 = 2R_2 = 20\ \text{k}\Omega.$$

(b) $\beta = \dfrac{R_s /\!/ R_1}{R_s /\!/ R_1 + R_2}$. $R_s = \infty \Rightarrow \beta = \dfrac{20}{20+10} = \dfrac{2}{3}$, $T = a\beta = \dfrac{2}{3}a$.

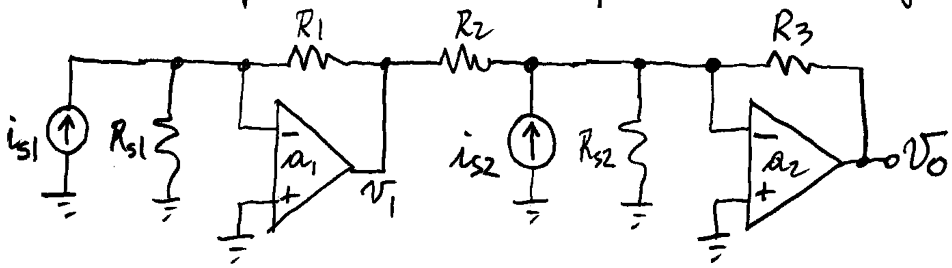
0.01% max deviation $\Rightarrow \dfrac{1}{T} \le \dfrac{0.01}{100} \Rightarrow T > 10^4 \Rightarrow \dfrac{2}{3}a > 10^4 \Rightarrow a > 15{,}000$ V/V.

(c) 0.025% max deviation with $a = 15{,}000$ V/V and R_s in place

$$\Rightarrow \frac{1}{15{,}000\beta} \le \frac{0.025}{100} \Rightarrow \beta \ge \frac{1}{3.75} \Rightarrow \frac{1}{1 + R_2/R_s + R_2/R_1} > \frac{1}{3.75} \Rightarrow$$

$$\frac{1}{1 + 10/R_s + 0.5} > \frac{1}{3.75} \Rightarrow \frac{10}{R_s} < 2.25 \Rightarrow R_s \ge 4.444\ \text{k}\Omega.$$

2.2 (a) Input sources must be fed to "virtual grounds":



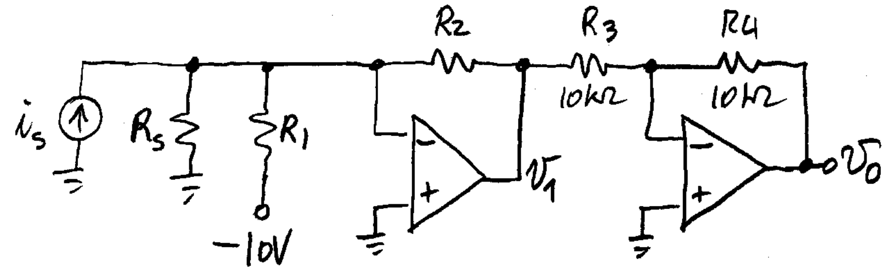
$v_O = -R_3 i_{S2} - \frac{R_3}{R_2} v_1 = -R_3 i_{S2} - \frac{R_2}{R_3}(-R_1 i_{S1}) = A_1 i_{S1} + A_2 i_{S2}$, $A_2 = R_3$,
$A_1 = \frac{R_2 R_3}{R_1}$. $A_1 = A_2 = 10\ V/mA \Rightarrow R_3 = 10\ k\Omega$. Need $R_2/R_1 = 1$. Pick also $R_1 = R_2 = 10\ k\Omega$.

(b) $\beta_1 = \frac{R_{S1}}{R_{S1}+R_1} = \frac{30}{30+10} = 0.75 \Rightarrow T_1 = a_1\beta_1 = 10^3 \times 0.75 = 750.$

$\beta_2 = \frac{R_2 // R_{S2}}{R_2 // R_{S2} + R_3} = \frac{1}{1 + R_3/R_2 + R_3/R_{S2}} = \frac{1}{1+1+1/3} = \frac{3}{7} \Rightarrow T_2 = 428.6.$

$v_O = 10^4\left(1 - \frac{1}{428.6}\right) i_{S2} - \frac{10^4}{10^4}\left(1 - \frac{1}{428.6}\right)(-10^4)\left(1 - \frac{1}{750}\right) i_{S1}$. $A_2 \cong 9997\ A/V$,
$A_1 = 9963\ A/V$.

2.3 Need an I-V converter stage with offset, followed by an inverting stage to ensure proper output polarity.

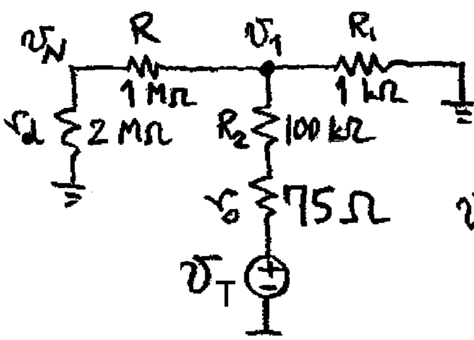


$\Delta i_S = (20-4)\,\text{mA} = 16\,\text{mA}$, $\Delta v_1 = -\Delta v_O = -8\,\text{V}$, $R_2 = \left|\dfrac{\Delta V_1}{\Delta i_S}\right| = \dfrac{8}{16} = 500\,\Omega$.

$i_S = 4\,\text{mA} \Rightarrow v_O = 0 \Rightarrow v_1 = 0 \Rightarrow i_{R_2} = 0 \Rightarrow i_{R_1} = i_S = 4\,\text{mA} \Rightarrow$

$R_1 = 10/4 = 2.5\,\text{k}\Omega$.

2.4



$v_N = \dfrac{r_d}{r_d + R} v_1 = \dfrac{2}{3} v_1$;

$$v_1 = \frac{(r_d + R) \| R_1}{[(r_d + R) \| R_1] + R_2 + r_o} v_T$$

$$\cong \frac{1}{1+100} v_T = \frac{v_T}{101};$$

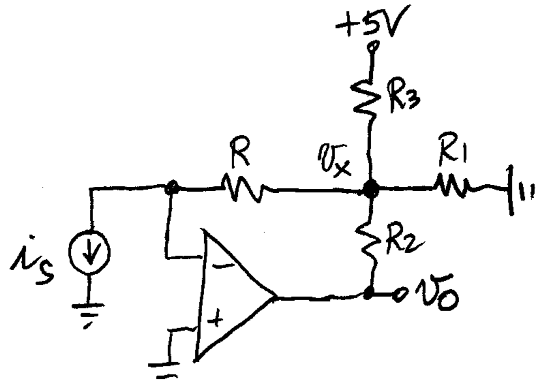
$\beta \cong \dfrac{2}{3} \times \dfrac{1}{101} = \dfrac{2}{303}\,\text{V/V}$; $T = a\beta = 2 \times 10^5 \times \dfrac{2}{303} = 1320.$

$A \cong A_{ideal}(1 - 1/T) = A_{ideal} \times (-0.9992)$;

$R_i \cong \dfrac{R \| r_d}{1+T} = 505\,\Omega$; $R_o \cong \dfrac{r_o}{1+T} = 57\,\text{m}\Omega$.

2.5

(a) $A = \Delta v_O/\Delta i_S = [4-(-4)]/(10\,\mu A) = 800$ kΩ. Use a high-sensitivity I-V converter with R_3 and the +5V source to shift v_O downward.



Let $R = 100$ kΩ. When $i_S = 0$ we have $i_R = 0 \Rightarrow v_X = v_N = 0 \Rightarrow i_{R1} = 0 \Rightarrow v_O = -(R_2/R_3)5 = -4$ V $\Rightarrow R_2/R_3 = 4/5 = 0.8$.

When $i_S = 10\,\mu A$ we have $v_X = v_N + Ri_S = 0 + 10^5 \times 10 \times 10^{-6} = 1$ V & we want $v_O = +4$V. KCL @ v_X: $\frac{5-1}{R_3} + \frac{4-1}{R_2} = 0.01 + \frac{1}{R_1}$. Let $R_1 = 1$ kΩ.

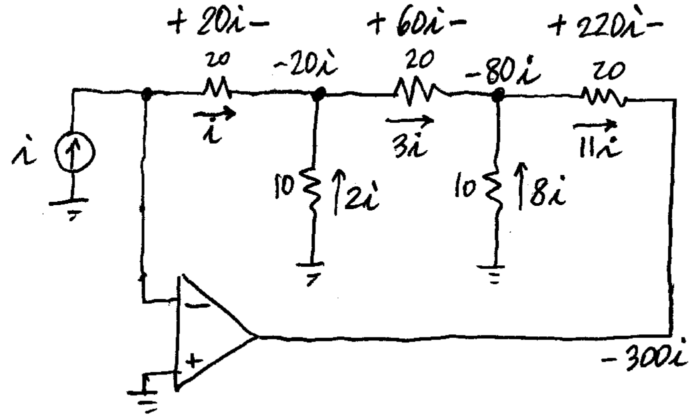
Substituting, $\frac{4}{R_3} + \frac{3}{0.8R_3} = 1.01 \Rightarrow R_3 = 7.673$ kΩ, $R_2 = 6.138$ kΩ.

(b)

$$\beta = \frac{R_3 /\!/ R_1}{R_3 /\!/ R_1 + R_2} = \frac{1}{1 + R_2/R_3 + R_2/R_1} = \frac{1}{1 + 0.8 + 6.138} \cong \frac{1}{8}.\quad \frac{1}{T} \le \frac{0.1}{100} = 10^{-3}$$

$\Rightarrow T = a\beta \Rightarrow 10^3 \le a/8 \Rightarrow a \ge 8{,}000$ V/V.

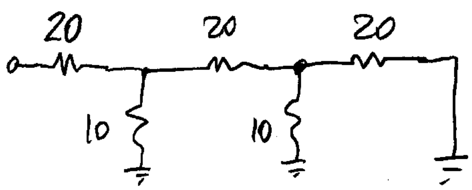
2.6



Start out @ left, and work your way toward the right via repeated application of Ω, KVL, and KCL to get $A = \frac{v_O}{i_I} = -300$ V/mA.

Shunt @ input ⇒ $R_i = \frac{r_{i0}}{1+T}$.

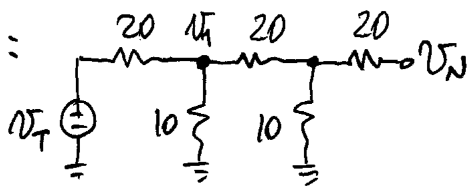
r_{i0}:



Start @ right and work your way toward the left:

$r_{i0} = \{[(20\|10)+20]\|10\}+20 = 27.3\ \text{k}\Omega$.

β:



$$v_N = \frac{10}{20+10} v_1 = \frac{1}{3}\frac{10\|30}{20+(10\|30)} v_T$$

$$= \frac{1}{3}\frac{1}{20/10+20/30+1} v_T = \frac{1}{11} v_T.$$

$T = a\beta = 10^4/11 = 909$; $R_i = (27.3\times10^3)/909 = 30\ \text{m}\Omega$.

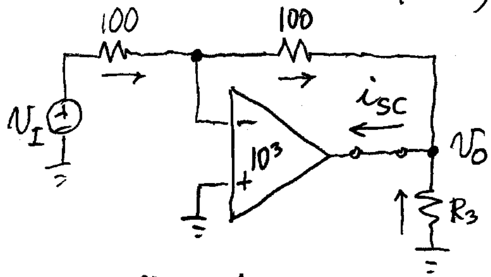
$\boxed{2.7}$ (a) $i_O = i_{R_2} + i_{R_3} = i_{R_1} + i_{R_3} = \frac{v_I}{R_1} + \left(\frac{R_2}{R_1} v_I\right)\frac{1}{R_3} = \frac{v_I}{R_1}\left(1 + \frac{R_2}{R_3}\right) = \frac{v_I}{R}$, $R = \frac{R_1}{1 + R_2/R_3}$.

(b) $R_i = R_1 \Rightarrow R_1 = 1\ M\Omega$. Let $R_2 = R_1 = 1\ M\Omega$. Then, for $i_O/v_I = (2\ mA)/(1\ V) = 2\times10^{-3}$ A/V, we need $\frac{1}{2\times10^{-3}} = 10^6/(1 + 10^6/R_3) \Rightarrow R_3 = 500.25\ \Omega$.

(c) $|v_L| \le 10 - |v_I|$ V.

2.8

(a)

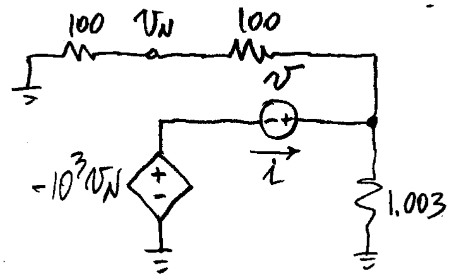


$\beta = 1/2,\ T = a\beta = 500,\ v_O = -\frac{100}{100}\frac{1}{1+1/500}v_I = -\frac{500}{501}v_I.$

$i_{SC} = \frac{v_I - v_O}{100+100} + \frac{0 - v_O}{R_3} = \frac{v_I}{200} - v_O\left(\frac{1}{200} + \frac{1}{R_3}\right)$

$i_{SC} = v_I\left[\frac{1}{200} + \frac{500}{501}\left(\frac{1}{200} + \frac{1}{R_3}\right)\right].$

$i_{SC} = \frac{v_I}{1\,\text{k}\Omega} \Rightarrow R_3 = \frac{50{,}000}{49{,}849} = 1.00303\,\text{k}\Omega.$



$i = \frac{-10^3 v_N + v}{200 /\!/ R_3},\ v_N = \frac{1}{2}\left(-10^3 v_N + v\right) \Rightarrow$

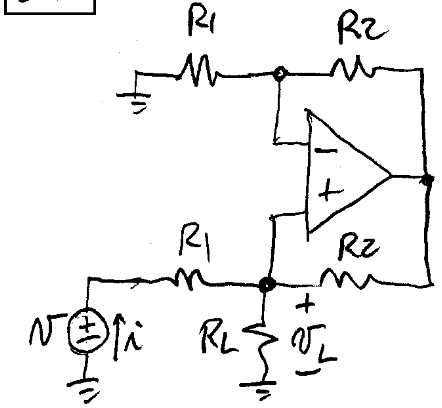
$v_N = \frac{v}{1002} \Rightarrow i = \frac{-1000/1002 + 1}{200 /\!/ R_3}v$

$R_o = v/i = 501\left(200 /\!/ 1.00303\right) = 500\ \text{k}\Omega.$

(b) $v_I = 2\text{V} \Rightarrow i_{SC} = \frac{2}{1} = 2.0\,\text{mA};\ i_O(v_L = 5\text{V}) = i_{SC} + \frac{5}{500\,\text{k}\Omega} =$
$2.0 + 0.01 = 2.01\,\text{mA}.\ i_L(v_L = -4\text{V}) = 2.0 - \frac{4}{500} = 1.992\,\text{mA}.$

2.9 Eq. (2.7) gives $\lim_{a\to\infty} R_o = \infty$, so (c) is correct. (a) is wrong because it ignores negative feedback. (b) is wrong because the op amp keeps a virtual short between v_N and v_P, not between v_N and v_O.

2.10



(a) Apply a test voltage v.

$$v_L = R_L i_L = R_L \frac{v}{R_1}$$

$$i = \frac{v - v_L}{R_1} = \frac{1 - R_L/R_1}{R_1} v$$

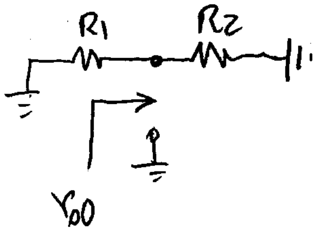
$$R_i = v/i = R_1 / (1 - R_L/R_1).$$

(b) $R_L = R_1 \Rightarrow v_L = R_L \frac{v}{R_1} = v \Rightarrow i = 0 \Rightarrow R_i = \infty$. $R_L < R_1 \Rightarrow v_L < v \Rightarrow i$ flowing toward the right $\Rightarrow R_i > 0$

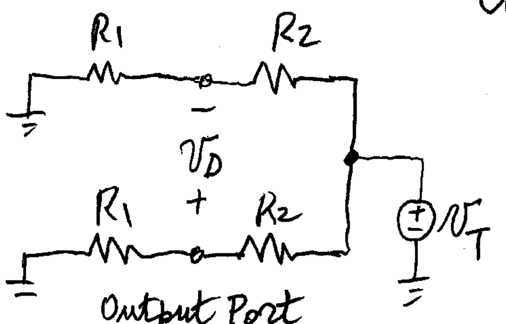
$R_L > R_1 \Rightarrow v_L > v \Rightarrow i$ flowing toward the left $\Rightarrow$ negative R_i.

2.11

(a) $R_o = r_{o0}\dfrac{1+T_{sc}}{1+T_{oc}}$, $r_{o0} = \lim_{a\to 0} R_o = R_1 // R_2$.



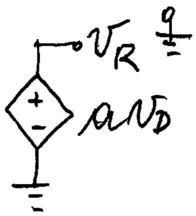
Output port open-circuited ⇒ $v_D = 0$ ⇒ $v_R = 0$ ⇒ $T_{oc} = -v_R/v_T = 0$. Output port short-circuited ⇒ $v_D = -\dfrac{R_1}{R_1+R_2} v_T$



$$v_R = a v_D = -\frac{a}{1+R_2/R_1} v_T$$

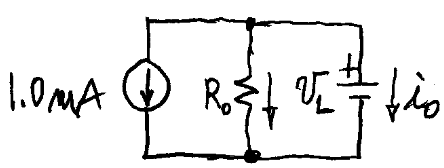
$$T_{sc} = -\frac{v_R}{v_T} = \frac{a}{1+R_2/R_1}.$$

$$R_o = (R_1//R_2)\left(1+\frac{a}{1+R_2/R_1}\right).$$



(b) $i_O(v_L=0) = \dfrac{v_I}{R_1} = -1.0$ mA.

$R_o = (10^3//10^3)\left(1+\dfrac{10^4}{1+1}\right) = 2.5$ MΩ. Use Norton equivalent: $1.0 + \dfrac{v_L}{R_o} + i_O = 0$

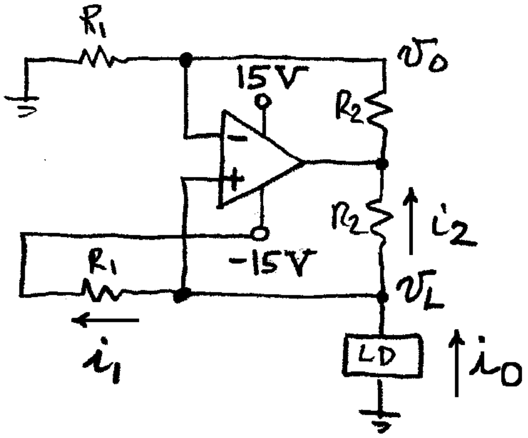


$$\Rightarrow i_O = -1.0\,\text{mA} - \frac{v_L}{2.5\,\text{M}\Omega}.$$

$$i_O(v_L = +5\,\text{V}) = -10^{-3} - \frac{5}{2.5\times10^{6}} = -1.002\,\text{mA}$$

$$i_O(v_L = -2.5\,\text{V}) = -10^{-3} + \frac{2.5}{2.5\times10^{-6}} = -0.999\,\text{mA}.$$

2.12 $R_1 = 15/1.5 = 10.0\ k\Omega$, 1%; $R_2 \le 0.3R_1$.



Use $R_2 = 2.00\ k\Omega$, 1%.

Then, $v_O = (1+2/10)v_L = 1.2\,v_L$.

(a) $v_L = -2\times1.5 = -3\text{ V}$; $v_O = -3.6\text{ V}$; $i_1 = [3-(-15)]/10 = 1.2\text{ mA}$; $i_2 = [-3-(-3.6)]/2 = 0.3\text{ mA}$; clearly, $i_O = i_1 + i_2 = 1.2+0.3 = 1.5\text{ mA}$.

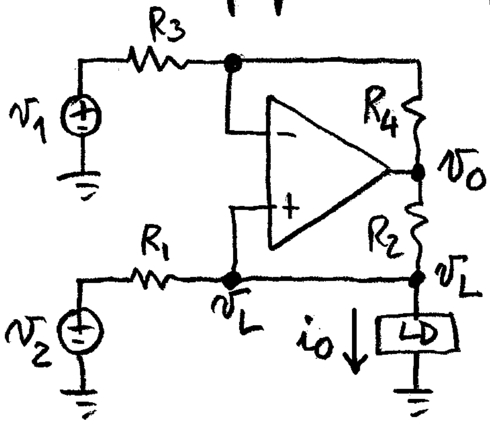
(b) $v_L = -9\text{ V}$, $v_O = -10.8\text{ V}$, $i_1 = 0.6\text{ mA}$, $i_2 = 0.9\text{ mA}$.

(c) With the cathode at ground, the zener gives $v_L = -5\text{ V}$, so $v_O = -6\text{ V}$, $i_1 = 1\text{ mA}$, $i_2 = 0.5\text{ mA}$.

(d) $v_O = v_L = 0$, $i_1 = 1.5\text{ mA}$, $i_2 = 0$.

(e) With a 10-kΩ load the op amp saturates at −13 V. By KCL, $(0-v_L)/10 = (v_L+15)/10 + (v_L+13)/2$, or $v_L = -80/7\text{ V}$. So, $i_O = 1.143\text{ mA}$, $i_1 = 0.357\text{ mA}$, $i_2 = 0.786\text{ mA}$. Because of saturation we have $i_1 + i_2 = i_O \ne 1.5\text{ mA}$.

$\boxed{2.13}$ Superposition: $v_O = -\frac{R_4}{R_3}v_1 + \left(1+\frac{R_4}{R_3}\right)v_L$; KCL:



$$i_O = \frac{v_2 - v_L}{R_1} + \frac{v_O - v_L}{R_2}$$

$$= \frac{v_2}{R_1} + \frac{v_O}{R_2} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_L$$

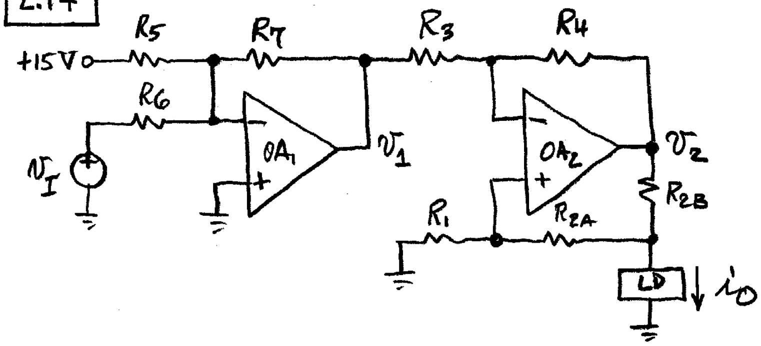
$$= \frac{v_2}{R_1} - \frac{R_4}{R_2R_3}v_1 - v_L\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2} - \frac{R_4}{R_2R_3}\right)$$

$$= \frac{1}{R_1}\left(v_2 - \frac{R_1}{R_2}\frac{R_4}{R_3}v_1\right) - \frac{v_L}{R_2}\left(\frac{R_2}{R_1} - \frac{R_4}{R_3}\right)$$

$$i_O = \frac{1}{R_1}\left(v_2 - \frac{R_4/R_3}{R_2/R_1}v_1\right) - \frac{v_L}{R_o}, \quad R_o = \frac{R_2}{R_2/R_1 - R_4/R_3}.$$

If $R_4/R_3 = R_2/R_1$, then $i_O = \frac{1}{R_1}(v_2 - v_1)$, and $R_o = \infty$.

2.14

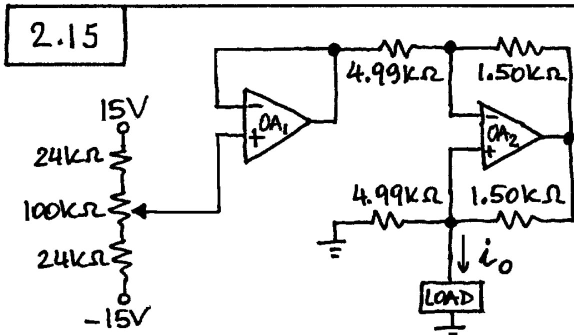


Let $R_1 = R_3 = R_4 = 10\ \mathrm{k\Omega}$. Assume a maximum drop of 2 V across R_{2B}, so $R_{2B} = 2/20 = 100\ \Omega$. Then, $R_{2A} = 10\ \mathrm{k\Omega} - 100\ \Omega = 9.9\ \mathrm{k\Omega}$.

$v_I = 0 \Rightarrow v_1 = -(R_7/R_5)15 = -0.4 \Rightarrow R_5/R_7 = 37.5$

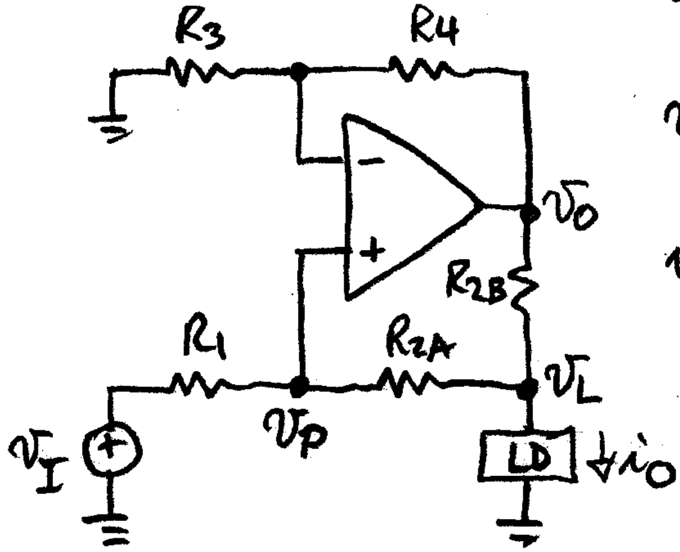
$v_I = 10\ \mathrm{V} \Rightarrow v_1 = -0.4 - (R_7/R_6)10 = -2 \Rightarrow R_6/R_7 = 6.25$. Use $R_7 = 2\ \mathrm{k\Omega}$, $R_6 = 12.5\ \mathrm{k\Omega}$, $R_5 = 75\ \mathrm{k\Omega}$.

2.15



OA_1 provides a variable voltage from −10V to +10V, which OA_2 converts to a variable current from −2mA to +2mA.

2.16 (a)



$$v_O = \left(1+\frac{R_4}{R_3}\right) v_P$$

$$v_P = \frac{R_{2A} v_I + R_1 v_L}{R_1 + R_{2A}}$$

$$i_O = \frac{v_I - v_L}{R_1 + R_{2A}} + \frac{v_O - v_L}{R_{2B}}$$

Eliminating v_O and v_P gives

$$i_O = \frac{1}{R} v_I - \frac{1}{R_o} v_L, \text{ where}$$

$$R = \frac{R_3 R_{2B} (R_1 + R_{2A})}{R_3 (R_{2A} + R_{2B}) + R_4 R_{2A}}, \quad R_o = \frac{R_{2B} (1 + R_{2A}/R_1)}{R_4/R_3 - (R_{2A}+R_{2B})/R_1}.$$

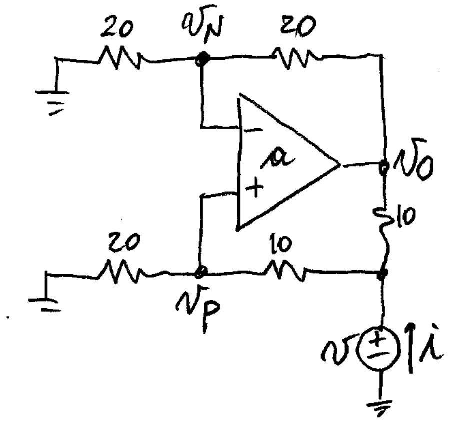
To make $R_o = \infty$ impose $R_4/R_3 = R_2/R_1$, where $R_2 = R_{2A} + R_{2B}$. This gives

$$R = \frac{R_3 R_{2B} (R_1 + R_{2A})}{R_4 (R_1 + R_{2A})} = \frac{R_3}{R_4} R_{2B} \Rightarrow \frac{1}{R} = \frac{R_4/R_3}{R_{2B}}.$$

(b) Imposing $10 = 13 - (R_4/R_3)10$ gives $R_4/R_3 = 0.3$. Let $R_1 = R_3 = 100\ \text{k}\Omega$, $R_4 = R_{2A} + R_{2B} = 30.1\ \text{k}\Omega$. Then, imposing $(R_4/R_3)/R_{2B} = 0.301/R_{2B} = 1\ \text{mA/V}$ gives $R_{2B} = 301\ \Omega$. Finally, $R_{2A} = 30.1 - 0.301 = 29.8\ \text{k}\Omega$ (use $30.1\ \text{k}\Omega$, 1%).

$\boxed{2.17}$

Test method:



$$v_P = \frac{2}{3}v, \quad v_N = \frac{1}{2}v_O, \quad v_O = a(v_P - v_N)$$

$$v_O = a\left(\frac{2}{3}v - \frac{1}{2}v_O\right) \Rightarrow v_O = \frac{4a}{6+3a}v$$

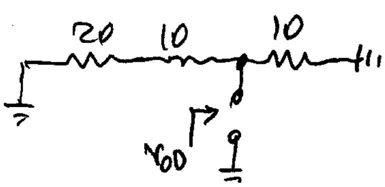
$$i = \frac{v}{20+10} + \frac{v - v_O}{10} = v\left(\frac{1}{30} + \frac{1}{10} - \frac{1}{10}\frac{4a}{6+3a}\right)$$

$$i = \frac{v}{30}\left[1 + 3 - \frac{12a}{6+3a}\right] = \frac{24v}{30(6+3a)}$$

$$R_o = \frac{v}{i} = \frac{30(6+3a)}{24} = (7.5\text{ k}\Omega)\left(1 + \frac{a}{2}\right)$$

$$= (7.5\text{ k}\Omega)\left(1 + \frac{10^4}{2}\right) = 37.5\text{ M}\Omega.$$

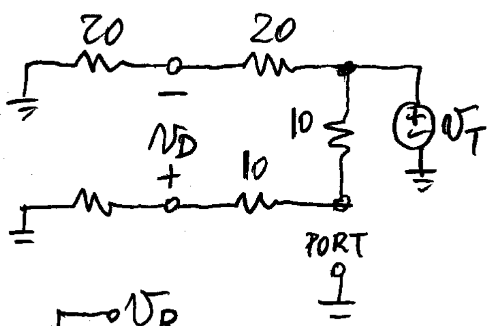
Blackman: $R_o = r_{oo}\dfrac{1+T_{sc}}{1+T_{oc}}$.

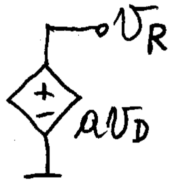


$$r_{oo} = (20+10)//10 = 7.5\text{ k}\Omega.$$

PORT open-circuited $\Rightarrow v_D = 0 \Rightarrow v_R = 0$

$$\Rightarrow T_{oc} = -\frac{v_R}{v_T} = 0$$





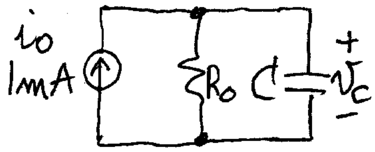
PORT short-circuited $\Rightarrow$

$$v_D = -\frac{1}{2}v_T \Rightarrow v_R = -\frac{a}{2}v_T \Rightarrow$$

$$T_{sc} = -\frac{v_R}{v_T} = \frac{a}{2}$$

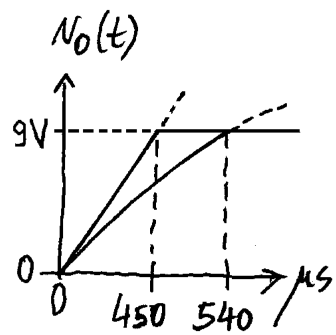
$$R_o = (7.5\text{ k}\Omega)\frac{1+T_{sc}}{1+T_{oc}} = (7.5\text{ k}\Omega)\left(1 + \frac{10^4}{2}\right).$$

$\boxed{2.18}$ Work with Norton equivalent.



(a) $R_o = \infty$, $v_C = \frac{i_O}{C} t = \frac{10^{3}}{10^{-7}} t = 10^4 t$

$v_O(t) = \left(1 + \frac{R_4}{R_3}\right) v_C(t) = 2 \times 10^4 t$, ramp till op amp saturates at 9 V. Impose $9 = 2 \times 10^4 t$ and get $t = 450\,\mu s$.



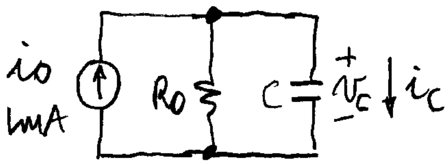
(b) $R_4 = 1.8\text{ k}\Omega \Rightarrow R_o = \frac{R_2}{R_2/R_1 - R_4/R_3} = \frac{2}{1 - 0.9} = 20\text{ k}\Omega$.

$\Rightarrow$ exponential xsient with $\tau = R_o C = 2\text{ ms}$ and $v_C(\infty) = R_o i_O = 20\text{ V} \Rightarrow v_O(\infty) = (1 + 1.8/2) v_C(\infty) = 38\text{ V}$. $v_O(t) = (38\text{ V})\left[1 - \exp{-t/2\text{ms}}\right] = 38\left[1 - \exp^{-500t}\right]$

Impose $9 = 38\left(1 - \exp^{-500t}\right) \Rightarrow t \cong 540\,\mu s$

$\boxed{2.19}$ Work with Norton equivalent. $R_4 = 2.2\,k\Omega \Rightarrow$

$$R_0 = \frac{R_2}{R_2/R_1 - R_4/R_3} = \frac{2}{1-1.1} = -20\ k\Omega.$$



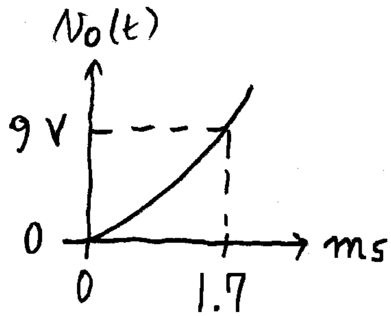
$$i_C = i_O - \frac{v_C}{R_O} = 10^{-3} - \frac{v_C}{-20\times10^3} = C\frac{dv_C}{dt} \Rightarrow$$

$$20 + v_C = 20\times10^3\times10^{-6}\frac{dv_C}{dt} = (20\text{ms})\frac{dv_C}{dt}$$

Assume solution of the type $v_C = Ae^{st} + B$:

$20 + Ae^{st} + B = (2\text{ms})sAe^{st} \Rightarrow B = -20,\ s = 1/(20\text{ ms}) = 50$, so

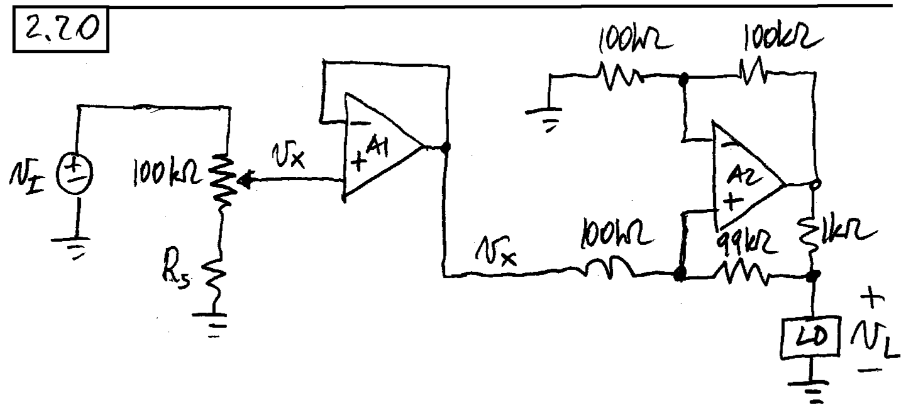
$v_C(t) = Ae^{50t} - 20$. $v_C(0) = 0 \Rightarrow A = 20 \Rightarrow v_C = (20\text{V})(e^{50t} - 1)$.



$v_O(t) = (1 + R_4/R_3)v_C(t) = 42(e^{50t} - 1)$. Impose

$9 = 42(e^{50t} - 1) \Rightarrow t = 1.7\text{ ms}$

2.20



Use variable input attenuator to implement $0.1 v_I \le v_X \le v_I$, and then use follower A_1 to buffer v_X to the Howland pump with zero resistance to avoid disturbing the resistance ratios. Wiper down $\Rightarrow 0.1 = R_s/(100+R_s) \Rightarrow R_s = 11.1$ kΩ. $i_O = \frac{v_X}{1\text{ k}\Omega}$.

$\boxed{2.21}$ (a) Denote the output of OA_1 as v_{O1}, and that of OA_2 as v_{O2}. By inspection, we have $v_{O2} = v_L$. By the superposition principle,

$$v_{O1} = -\frac{R_4}{R_3}v_1 + \left(1+\frac{R_4}{R_3}\right)\frac{R_2 v_2 + R_1 v_L}{R_1 + R_2}$$

$$= \frac{1+R_4/R_3}{1+R_1/R_2}v_2 - \frac{R_4}{R_3}v_1 + \frac{1+R_4/R_3}{1+R_2/R_1}v_L.$$

$$i_O = \frac{v_{O1} - v_L}{R_5} = A_2 v_2 - A_1 v_1 - \frac{1}{R_o}v_L, \text{ where}$$

$$A_2 = \frac{1+R_4/R_3}{1+R_1/R_2}\frac{1}{R_5}, \quad A_1 = \frac{R_4}{R_3}\frac{1}{R_5}, \text{ and}$$

$$\frac{1}{R_o} = \frac{1}{R_5}\left(1 - \frac{1+R_4/R_3}{1+R_2/R_1}\right) = \frac{1}{(1+R_2/R_1)R_5}\left(\frac{R_2}{R_1} - \frac{R_4}{R_3}\right)$$

To make $R_o \to \infty$ impose $R_4/R_3 = R_2/R_1$, after which it is readily seen that $A_1 = A_2 = \frac{R_2/R_1}{R_5}$.

In summary, imposing $R_4/R_3 = R_2/R_1$ gives

$$i_O = Av_I - \frac{1}{R_o}v_L, \; A = \frac{R_2/R_1}{R_5}, \; v_I = v_2 - v_1, \; R_o = \infty.$$

(b) If the resistances are mismatched, A_1 and A_2 will also be mismatched, so we no longer have true difference operation.

Writing $R_o = \dfrac{(1+R_2/R_1)R_5}{R_2/R_1 - (R_2/R_1)(1-\varepsilon)} = \left(1+\dfrac{R_2}{R_1}\right)\dfrac{R_5}{\varepsilon}$

gives, for 1% resistors, $|R_o| \geq 25(1+R_2/R_1)R_5$.

$\boxed{2.22}$ (a) Denote the output of OA_1 as v_{O1}, that of OA_2 as v_{O2}. By OA_2's action, $v_{O1} = v_L$ and $v_{O2} = v_L + R_5 i_O$. By the superposition principle,

$$v_{O1} = -\frac{R_4}{R_3} v_I + \left(1 + \frac{R_4}{R_3}\right) \frac{R_1}{R_1 + R_2} \left(v_L + R_5 i_O\right) = v_L.$$

Solving for i_O gives $i_O = A v_I - (1/R_o) v_L$,

$$A = \frac{1 + R_2/R_1}{1 + R_4/R_3} \frac{R_4/R_3}{R_5}, \quad R_o = \frac{(1 + R_4/R_3) R_5}{R_2/R_1 - R_4/R_3}.$$

Imposing $R_4/R_3 = R_2/R_1$ gives $R_o = \infty$ and $A = (R_2/R_1)/R_5$.

(b) Writing $R_o = \dfrac{(1 + R_2/R_1) R_5}{R_2/R_1 - (R_2/R_1)(1 - \varepsilon)}$

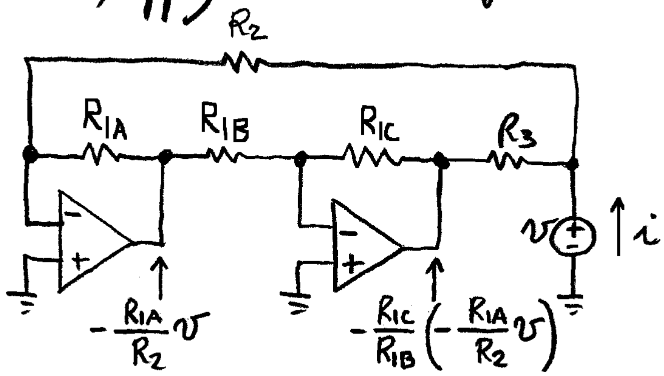
$= (1 + R_1/R_2) R_5 / \varepsilon$. With 1% resistors we can expect $|R_o| \geq 25 (1 + R_1/R_2) R_5$.

$\boxed{2.23}$ (a) Denote the outputs of OA_1 and OA_2 as v_{O1} and v_{O2}. We have $v_{O2} = -v_{O1} = -[-v_I - (R_1/R_2)v_L] = v_I + (R_1/R_2)v_L$; $i_O = \dfrac{v_{O2}-v_L}{R_3} - \dfrac{v_L}{R_2} = \dfrac{v_I}{R_3} - v_L\left[\dfrac{1}{R_3} + \dfrac{1}{R_2} - \dfrac{R_1/R_2}{R_3}\right]$,

or $i_O = Av_I - \dfrac{1}{R_o}v_L$, $A = \dfrac{1}{R_3}$, $R_o = \dfrac{R_2R_3}{R_2+R_3-R_1}$.

To achieve $R_o = \infty$, impose $\underline{R_2 + R_3 = R_1}$.

(b) To find the effect of mismatches upon R_o, apply a test voltage at the output:



$$i = \frac{v}{R_2} + \frac{v - (R_{1A}R_{1C}/R_2R_{1B})v}{R_3}$$

$$= v\left[\frac{1}{R_2} + \frac{1}{R_3} - \frac{R_{1A}(R_{1C}/R_{1B})}{R_2R_3}\right]$$

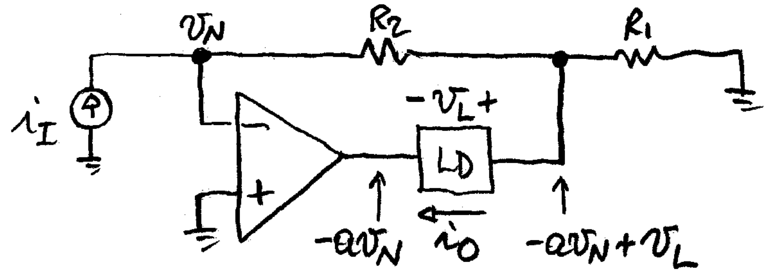
$$R_o = \frac{v}{i} = \frac{R_2R_3}{R_2+R_3-R_{1A}(R_{1C}/R_{1B})}.$$

R_o is maximized when R_2, R_3, and R_{1B} are maximized, and R_{1A} and R_{1C} are minimized. For 1% resistors, rewrite as

$$R_{o(max)} = \frac{(R_2 \times R_3)1.01^2}{(R_2+R_3)1.01 - (R_2+R_3)0.99(0.99/1.01)}$$

$$\cong 25\frac{R_3}{1+R_3/R_2}.$$

$\boxed{2.24}$ (a)



Ω: $v_N - (-av_N + v_L) = R_2 i_I$

$\Rightarrow$ $v_N = (R_2 i_I + v_L)/(1+a)$. KCL:

$$i_O = i_I + \frac{av_N - v_L}{R_1} = i_I + \frac{a(R_2 i_I + v_L)}{(1+a)R_1} - \frac{v_L}{R_1}$$

$$= i_I\left(1 + \frac{R_2/R_1}{1+1/a}\right) - \frac{1}{R_1}v_L\left(1 - \frac{1}{1+a}\right) = Ai_I - \frac{v_L}{R_o},$$

$$A = 1 + \frac{R_2/R_1}{1+1/a},\ R_o = R_1(1+a)$$

(b) Use $R_1 = 2\ k\Omega$, $R_2 = 18\ k\Omega$.

$A_{ideal} = 10$ A/A; $A_{actual} = 1 + 9/(1 + 1/200{,}000)$

$= 9.999955$; gain error $= -0.00045\%$.

$R_o \cong 2 \times 10^3 \times (1 + 200{,}000) = 400\ M\Omega$.

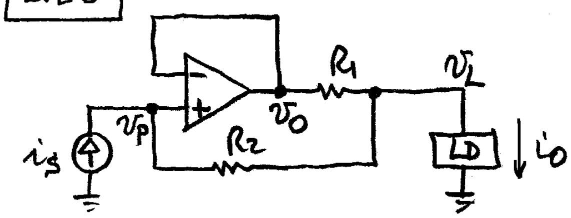
2.25 The op amp keeps $v_O = v_N = v_P$. By the superposition principle, $v_P = (R_s \| R_2) i_S + \frac{R_s}{R_s+R_2} v_L$. By KCL, $i_O = (v_P - v_L)/(R_1 \| R_2)$. Substituting,

$$i_O = \frac{R_s \| R_2}{R_1 \| R_2} i_S - \frac{v_L}{R_1 \| R_2}\left[1 - \frac{R_s}{R_s+R_2}\right] = A i_S - \frac{v_L}{R_o},$$

$$A = \frac{1+R_2/R_1}{1+R_2/R_s}, \quad R_o = \frac{R_s+R_2}{1+R_2/R_1}.$$

For $R_s \to \infty$ we get $A = 1 + R_2/R_1$ and $R_o = \infty$.

2.26



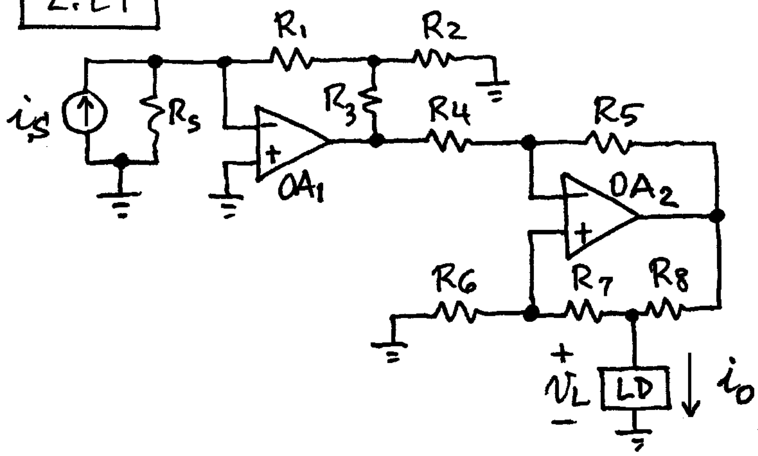
$v_P = v_L + R_2 i_S$; $v_O = a(v_P - v_O) \Rightarrow v_O = \frac{a}{1+a} v_P$

$v_O = \frac{a}{1+a}(v_L + R_2 i_S)$. $i_O = i_S + \frac{v_O - v_L}{R_1} \Rightarrow$

$$i_O = i_S + \frac{1}{R_1}\left[\frac{a}{1+a} v_L - v_L + \frac{a}{1+a} R_2 i_S\right] = A i_S - \frac{1}{R_o} v_L,$$

$A = 1 + (R_2/R_1)/(1 + 1/a)$, $R_o = R_1(1+a)$.

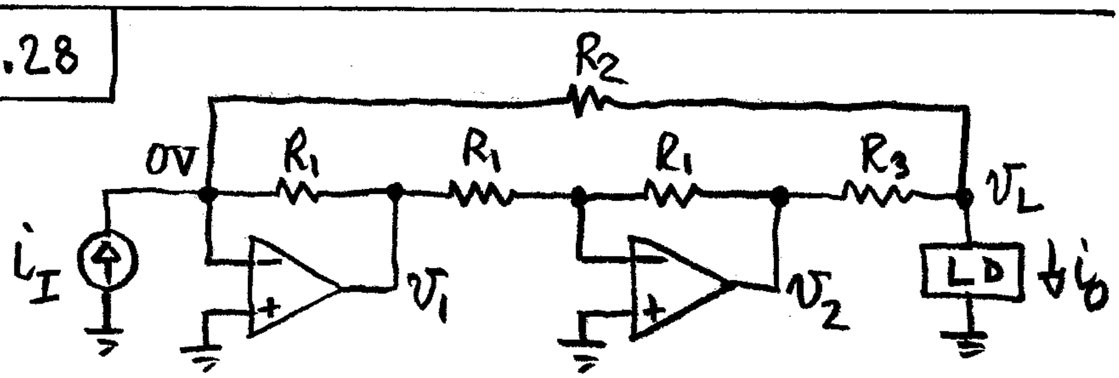
2.27



Choose the I-V converter components for a 10-V full scale at OA_1's output. Thus, let $R_1 = 1\,\text{M}\Omega$, $R_2 = 1\,\text{k}\Omega$, $R_3 = 100\,\text{k}\Omega$.

$i_{O(max)} = 10^5 \times 100 \times 10^{-9} = 10\,\text{mA}$. Imposing $R_8 = 500\,\Omega$ yields a voltage compliance of $10 - 0.5 \times 10 = 5\,\text{V}$. Finally, let $R_4 = R_5 = R_6 = 100\,\text{k}\Omega$, $R_7 = 99.5\,\text{k}\Omega$.

2.28



$$v_1 = -R_1 i_I - (R_1/R_2)v_L\,; \quad v_2 = -v_1 = R_1 i_I + \frac{R_1}{R_2} v_L .$$

$$i_O = \frac{v_2 - v_L}{R_3} - \frac{v_L}{R_2} = \frac{R_1}{R_3} i_I - v_L\left[\frac{1}{R_2} + \frac{1}{R_3} - \frac{R_1}{R_2 R_3}\right].$$

Impose $R_2 + R_3 = R_1$ to achieve $R_o = \infty$, and $R_1/R_3 = 10$ to achieve the desired gain. Moreover, $R_i = 0$ because of the input virtual ground. Use $R_1 = 10.0\ \mathrm{k\Omega}$, $R_3 = 1.00\ \mathrm{k\Omega}$, and $R_2 = 9.09\ \mathrm{k\Omega}$.

$\boxed{2.29}$ The output of OA_1 is $v_1 = -\frac{R}{R_2}v_2 - \frac{R}{R_4}v_4$.

By the superposition principle,

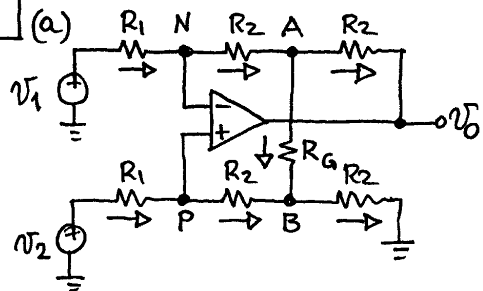
$$v_O = -\frac{R_F}{R_1}v_1 - \frac{R_F}{R_3}v_3 - \frac{R_F}{R}v_1 =$$

$$= \frac{R_F}{R_2}v_2 + \frac{R_F}{R_4}v_4 - \frac{R_F}{R_1}v_1 - \frac{R_F}{R_3}v_3 \,.$$

The circuit sums the even-numbered inputs with positive gains, and the odd-numbered inputs with negative gains. Since the summing junctions of both op amps are at virtual ground, leaving an input floating has no effect. By contrast, leaving any input floating in Fig. P1.31 affects the output because in general $v_N = v_P \neq 0$.

$\boxed{2.30}$ Applying a test voltage v between the inputs of Fig. 2.14a yields, by the virtual short concept,
$i = v/(R_1+0+R_1) = v/2R_1$. So, $R_{id} = 2R_1$.
In response to an input test voltage v, the R_1 resistances in Fig. 2.14(b) will draw the same current $v/(R_1+R_2)$, so $i = 2v/(R_1+R_2)$, or
$R_{ic} = v/i = (R_1+R_2)/2$.

2.31 (a)



KCL at N: $\frac{v_1 - v_N}{R_1} = \frac{v_N - v_A}{R_2}$

KCL at P: $\frac{v_2 - v_P}{R_1} = \frac{v_P - v_B}{R_2}$.

Letting $v_N = v_P$ and subtracting,

$v_A - v_B = (R_2/R_1)(v_2 - v_1)$ .... (1)

KCL at A: $\frac{v_1 - v_A}{R_1 + R_2} = \frac{v_A - v_B}{R_G} + \frac{v_A - v_O}{R_2}$.

KCL at B: $\frac{v_2 - v_B}{R_1 + R_2} + \frac{v_A - v_B}{R_G} = \frac{v_B}{R_2}$. Subtracting,

$$\frac{(v_2 - v_1) + (v_A - v_B)}{R_1 + R_2} + 2\frac{v_A - v_B}{R_G} = \frac{(v_B - v_A) + v_O}{R_2}.$$

Combining with Eq. (1),

$$(v_2 - v_1)\left(\frac{1 + R_2/R_1}{R_1 + R_2} + 2\frac{R_2/R_1}{R_G} + \frac{1}{R_1}\right) = \frac{v_O}{R_2}.$$

Solving for v_O and simplifying,

$$v_O = 2\frac{R_2}{R_1}\left(1 + \frac{R_2}{R_G}\right)(v_2 - v_1).$$

(b) Let R_G = 100-kΩ pot in series with a 5-kΩ resistor. Then, $100 = 2(R_2/R_1)(1 + R_2/5)$ and $10 = 2(R_2/R_1)[1 + R_2/(100+5)]$. Dividing, $100/10 = (1 + R_2/5)/(1 + R_2/105)$. Solving, $R_2 = 85.9$ kΩ. Back substituting yields $R_1 = 31.24$ kΩ. Use $R_1 = 31.6$ kΩ, $R_2 = 86.6$ kΩ, R_G = 100-kΩ pot + 4.99 kΩ, all 1%.

$\boxed{2.32}$ (a) $v_{O2} = -(R_3/R_G)v_O$. Superposition: $v_{P1} = \frac{R_2 v_2 + R_1 v_{O2}}{R_1 + R_2}$. Voltage divider: $v_{N1} = [R_2/(R_1+R_2)]v_1$. Eliminating v_{O2} and letting $v_{N1} = v_{P1}$ gives $v_O = \frac{R_2}{R_1}\frac{R_G}{R_3}(v_2 - v_1)$.

(b) Let $R_1 = R_2 = 10\ \text{k}\Omega$. Then, $A = R_G/R_3$. Let $R_3 = 1\ \text{k}\Omega$ and let R_G be a 100-kΩ pot in series with a 1-kΩ resistor. Then,
$A_{(min)} = 1$ V/V, $A_{(max)} = (1+100)/1 \cong 100$ V/V.

$\boxed{2.33}$ (a) $(v_1+v_2)/2 = 10\cos 2\pi 60t$ V; $v_2 - v_1 = 0.01\cos 2\pi 10^3 t$ V; $A_{dm} = 2/0.01 = 200$ V/V; $A_{cm} = 0.1/10 = 0.01$ V/V; CMRR = $20\log_{10}(200/0.01) = 86$ dB.

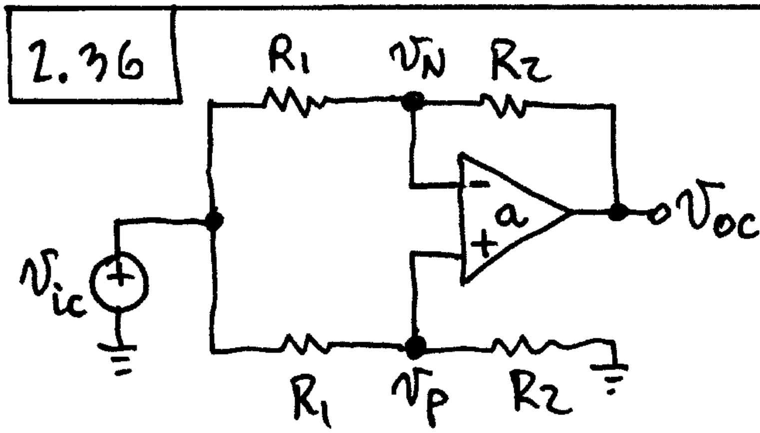
(b) $(v_1+v_2)/2 = 10.005\cos 2\pi 60t$ V; $v_2 - v_1 = -0.01\sin 2\pi 60t + 0.01\sin 2\pi 10^3 t$ V; $A_{dm} = 2.5/0.01 = 250$ V/V. At 60 Hz, we have $0.5 = 250\times(-0.01) + A_{cm}\times 10.005$, or $A_{cm} \cong 0.3$ V/V; CMRR = $20\log_{10}(250/0.3) = 58.4$ dB.

$\boxed{2.34}$ $A_{dm} \cong (100\ \text{k}\Omega)/(1\ \text{k}\Omega) = 100\ \text{V/V} = 40$ dB.
To find A_{cm}, tie the inputs together and apply a common signal. Then,
$A_{cm} = -\frac{99.7}{1.01} + \left(1 + \frac{99.7}{1.01}\right)\frac{102}{102 + 0.995} = 0.0367$ V/V $= -28.7$ dB. CMRR $\cong 40 - (-28.7) = 68.7$ dB.

2.35 $|A_{dm}| = 10^3$ V/V & CMRR $= 10^5 \Rightarrow |A_{cm}| = 10^{-2}$ V/V.

$v_{id} = v_2 - v_1 = 2$ mV; $v_{ic} = (v_1 + v_2)/2 = 4$ V;
$|v_{od}| = 10^3 \times 2 \times 10^{-3} = 2$ V; $|v_{oc}| = 10^{-2} \times 4 = 0.04$ V.
Error $= 100|v_{oc}|/|v_{od}| = 2\%$.

2.36



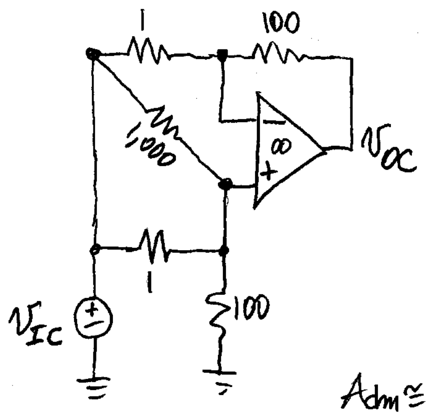
$$v_{oc} = a(v_P - v_N) = a\left[\frac{R_2}{R_1+R_2}v_{ic} - \frac{R_2 v_{ic} + R_1 v_{oc}}{R_1+R_2}\right]$$

$$= a\left[\cancel{\frac{R_2}{R_1+R_2}v_{ic}} - \cancel{\frac{R_2}{R_1+R_2}v_{ic}} - \frac{R_1}{R_1+R_2}v_{oc}\right]$$

$\Rightarrow v_{oc}(1+a\beta) = 0 \Rightarrow v_{oc} = 0$ regardless of v_{ic}
$\Rightarrow$ CMRR $= \infty$. Intuitively: v_{oc} can only be zero. Suppose v_{oc} was positive. Then, v_N would be $> v_P$, implying that $v_O = a(v_P - v_N)$ would have to swing negative, a contradiction.

2.37

Tie inputs together and drive them with a common-mode voltage v_{IC}, and find v_{OC}.

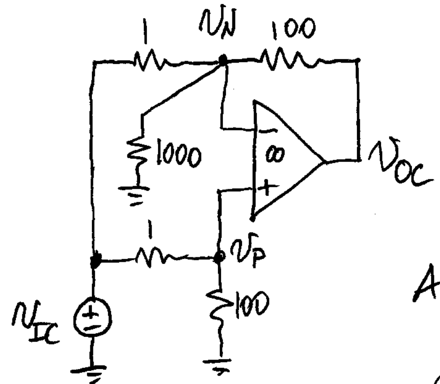


(a) $R_1 = 1\,k\Omega$, $R_2 = R_4 = 100\,k\Omega$, $R_3 = 1 // 1{,}000 = \frac{1000}{1001}\,k\Omega$. Superposition:

$$v_{OC} = -\frac{100}{1}v_{IC} + \left(1+\frac{100}{1}\right)\frac{100}{100+1000/1001}v_{IC}$$

$$v_{OC} = v_{IC}\left(-100+101\,\frac{1001}{1011}\right) = \frac{v_{IC}}{1011} \Rightarrow A_{cm} = \frac{1}{1011}.$$

$A_{dm} \cong 100$ V/V; CMRR $= 20 \log 100/(1/1011) = 100.1$ dB.

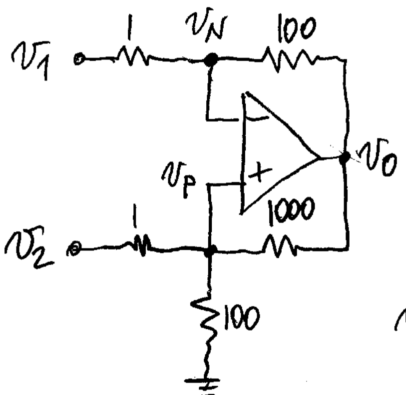


(b) Superposition:

$$v_{OC} = -\frac{100}{1}v_{IC} + \left(1+\frac{100}{1000/1001}\right)v_{IC}$$

$$= v_{IC}(-100+101.1)v_{IC} = 1.1\,v_{IC} \Rightarrow$$

$A_{cm} = 1.1$ V/V. CMRR $\cong 20 \log 100/1.1 = 39.17$ dB



(c) $v_N = v_P$. $v_N = \frac{100 v_1 + 1 v_O}{101}$. KCL:

$$\frac{v_2 - v_P}{1} = \frac{v_P}{100} + \frac{v_P - v_O}{1000}$$

$$\Rightarrow 1000 v_2 + v_O = 1011 v_P = \frac{1011}{101}(100 v_1 + v_O)$$

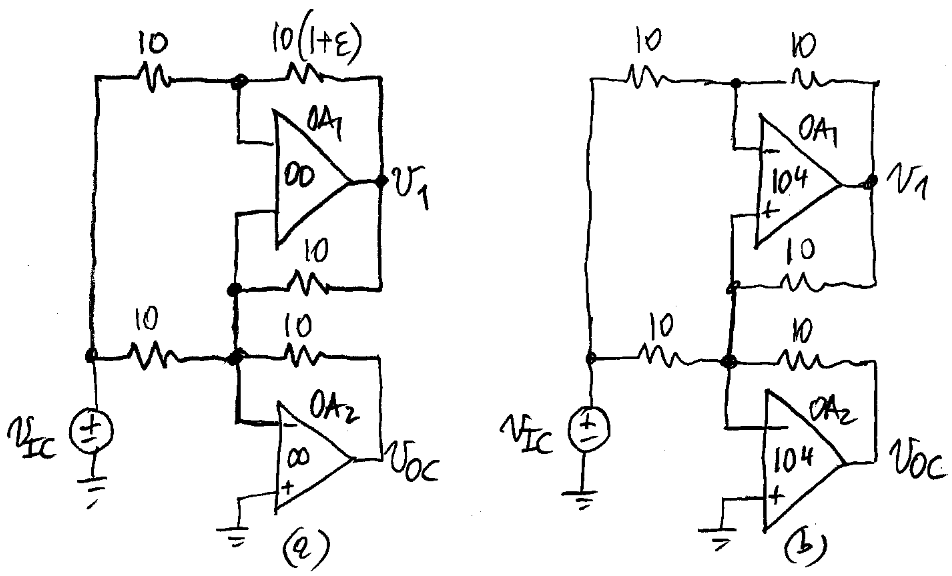
$$1000\times 101\, v_2 - 1011 \times 100\, v_1 = 910\, v_O$$

$$v_O = \frac{10100 v_2 - 10110 v_1}{91} \Rightarrow A_{dm} = \frac{10100+10110}{2\times 91} \Rightarrow$$

$A_{dm} = 111$ V/V, $A_{cm} = \frac{10100-10110}{91} = -\frac{1}{9.1}$ V/V

CMRR $= 20 \log 111/(1/9.1) = 60.1$ dB.

$\boxed{2.38}$ The imbalances are small enough to keep $A_{dm} \cong 1$ V/V both in (a) and (b). Consequently, we only need to find the worst case value of A_{cm}. Tie the inputs together, apply v_{IC}, and find v_{OC}.



(a) Superposition: $v_{OC} = -\frac{10}{10} v_{IC} - \frac{10}{10} v_1$, $v_1 = -\frac{10(1+\varepsilon)}{10} v_{IC}$

$\therefore v_{OC} = -v_I + (1+\varepsilon) v_{IC} = \varepsilon v_{IC} \Rightarrow A_{cm} = \varepsilon = 4\frac{p}{100} = 4\,\frac{0.1}{100} = \frac{1}{250}$ V/V.

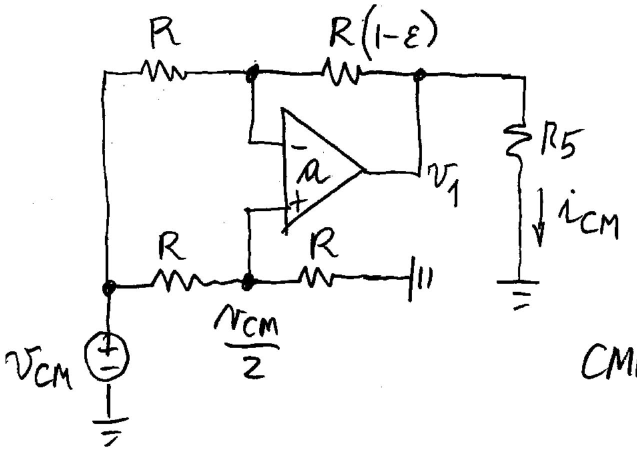
$\mathrm{CMRR} = 20 \log 1/(1/250) \cong 48$ dB

(b) Only the error due to OA_1 matters, since that due to OA_2 affects both inputs equally. OA_1: $\beta_1 = 0.5$, $T_1 = 0.5 \times 10^4$, $v_1 =$

$-\frac{10}{10} \frac{1}{1+1/T_1} v_{IC} \cong -1\left(1 - \frac{1}{T_1}\right) v_{IC} = -v_{IC} + \frac{v_{IC}}{T_1}$. $v_{OC} = v_{IC}\left(-1 + 1 + \frac{1}{T_1}\right)$

$\Rightarrow A_{cm} = 1/T_1 = 1/5000$ V/V. $\mathrm{CMRR} = 20 \log 1/(1/5000) \cong 74$ dB.

2.39 The imbalances of (a) and (b) are small enough to ensure $A_{dm} \cong 1/R_5$ in both cases. We therefore need only to find the worst case value of A_{cm}. Tie the inputs together, apply v_{CM}, and find i_{CM}. With a short-circuit load, OA_2 will return 0 V regardless of whether $a_2 = \infty$ or $a_2 = 10^3$ V/V, so we can replace it by a mere wire.



(a) $a = \infty$, $\varepsilon = 4\frac{p}{100} = 0.04$.

Superposition: $v_1 = -\frac{R(1-\varepsilon)}{R} v_{CM} + \left[1 + \frac{R(1-\varepsilon)}{R}\right]\frac{1}{2} v_{CM} = \frac{\varepsilon}{2} v_{CM}$,

$i_{CM} = \frac{v_1}{R_5} = \frac{\varepsilon/2}{R_5}$.

$$CMRR = 20\log\frac{1/R_5}{(\varepsilon/2)/R_5} = 20\log 50 \cong 34\text{ dB}.$$

(b) $\varepsilon = 0$, $a = 10^3$ V/V. $\beta = \frac{1}{2}$, $T = a\beta = 500$,

$$v_1 = \left[-\frac{R}{R} v_{CM} + \left(1 + \frac{R}{R}\right)\frac{1}{2} v_{CM}\right]\frac{1}{1+1/T} = 0 v_{CM}\frac{1}{1+1/T} \Rightarrow A_{cm} = 0 \Rightarrow CMRR = \infty.$$

$\boxed{2.40}$ $v_{N1} = v_{P1} = v_1 = 5\text{ V} - 5\sin\omega t\text{ mV};$

$v_{N2} = v_{P2} = v_2 = 5\text{ V} + 5\sin\omega t\text{ mV};$

$$v_{O1} = v_{N1} + R_3\frac{v_{N1} - v_{N2}}{R_G} = 5\text{ V} - 5\sin\omega t\text{ mV} + \frac{10^6}{2\times10^3}(-10\sin\omega t\text{ mV}) = 5\text{ V} - 5.005\sin\omega t\text{ V};$$

$v_{O2} = 5\text{ V} + 5.005\sin\omega t\text{ V};$

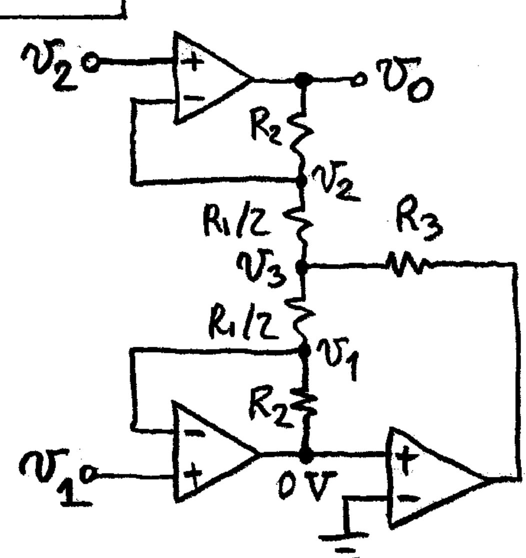
$$v_{N3} = v_{P3} = \frac{R_2}{R_1 + R_2}v_{O2} = 2.5\text{ V} + 2.5025\sin\omega t\text{ V};$$

$$v_O = \frac{R_2}{R_1}(v_{O2} - v_{O1}) = 10.01\sin\omega t\text{ V}.$$

2.41 $v_O = v_{O1} - v_{O2} = a_1(v_{P1} - v_{N1}) - a_2(v_{P2} - v_{N2}) = a[(v_{P1} - v_{P2}) - (v_{N1} - v_{N2})] = a[v_I - R_G v_O/(R_G + 2R_3)]$. This is of the type $v_O = a(v_I - \beta v_O)$, $\beta = R_G/(R_G + 2R_3)$.

2.42 From Problem 2.34, $\beta_I = 1/A_I = 1/50$ V/V; moreover, $\beta_{II} = 1/A_{II} = 1/20$ V/V. We can guarantee a 0.1% maximum deviation of $A = A_I \times A_{II}$ from ideality by imposing a 0.05% maximum deviation of A_I and A_{II}. Thus, $100/a_I\beta_I \le 0.05 \Rightarrow a_I \ge 100 \times 50/0.05 = 10^5$ V/V; likewise, $a_{II} \ge 4 \times 10^4$ V/V.

2.43



$v_1 = \frac{R_2}{R_2 + R_1/2} v_3 \Rightarrow$

$v_3 = \left(1 + \frac{R_1}{2R_2}\right) v_1$;

$\frac{v_O - v_2}{R_2} = \frac{v_2 - v_3}{R_1/2}$.

Eliminating v_3,

$v_O = \left(1 + \frac{2R_2}{R_1}\right)(v_2 - v_1)$.

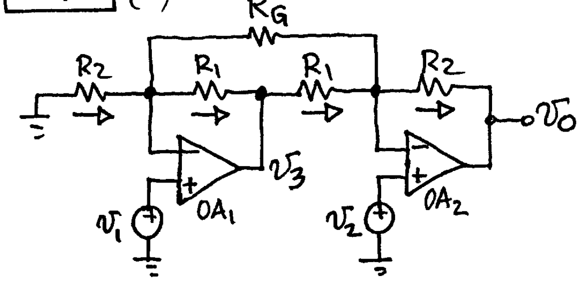
2.44 (a) Superposition:

$$v_O = \left[1+\frac{R_2}{R_1}\right]\left[v_{CM}+\frac{v_{DM}}{2}\right] - \frac{R_2}{R_1}\left[1+\frac{R_1}{R_2}(1-\varepsilon)\right]\left[v_{CM}-\frac{v_{DM}}{2}\right]$$

$$= \left(1+\frac{R_2}{R_1}-\frac{\varepsilon}{2}\right)v_{DM} + \varepsilon v_{CM}$$

(b) With 1% resistors, ε can be as large as 0.04. Since this is much less than 100, we can write CMRR $\geq 20\log_{10}(100/0.04) = 68$ dB.

2.45 (a)



$v_{N1} = v_{P1} = v_1$, $v_{N2} = v_{P2} = v_2$. Applying KCL:

$$\frac{0-v_1}{R_2} = \frac{v_1-v_2}{R_G}+\frac{v_1-v_3}{R_1};\quad \frac{v_2-v_O}{R_2} = \frac{v_1-v_2}{R_G}+\frac{v_3-v_2}{R_1}.$$

Adding the two equations pairwise gives

$$\frac{v_2-v_1}{R_2} - \frac{v_O}{R_2} = 2\,\frac{v_1-v_2}{R_G}+\frac{v_1-v_2}{R_1}.$$ Solving for v_O yields $v_o = \left(1+\frac{R_2}{R_1}+2\frac{R_2}{R_G}\right)(v_2-v_1)$.

(b) Let $R_G = R_{GA}+R_{GB}$, where R_{GA} = 10-kΩ pot. Arbitrarily impose $R_2/R_1 = 1$, so that $A = 2(1+R_2/R_G)$. $10 \leq A \leq 100 \Rightarrow$ $5 \leq (1+R_2/R_G) \leq 50 \Rightarrow 4 \leq R_2/R_G \leq 49$. $R_G = 0+R_{GB} \Rightarrow R_2/R_{GB} = 49$; $R_G = 10+R_{GB}$ $\Rightarrow R_2/(10+R_{GB}) = 4$. Solving, $R_{GB} = 889\ \Omega$ (use 887 Ω, 1%); $R_2 = 49R_{GB} = 43.5$ kΩ = R_1 (use $R_1 = R_2 = 43.2$ kΩ, 1%).

2.46 (a) The op amps keep $v_{P1} = v_{N1} = v_1$, $v_{N2} = v_{P2} = v_2$. Let v_3 be the output of OA_2. Summing currents at v_{P1} and v_{N2} gives

$$\frac{v_O - v_2}{R} + \frac{v_1 - v_2}{R_G} + \frac{v_3 - v_2}{R} = 0$$

$$\frac{v_3 - v_1}{R} + \frac{v_2 - v_1}{R_G} + \frac{0 - v_1}{R} = 0$$

Eliminating v_3 and collecting gives

$$v_O = 2\left(1 + \frac{R}{R_G}\right)(v_2 - v_1).$$

(b) Let R_G be a 10-kΩ pot in series with a resistance R_s. Then,

$$2\left(1 + \frac{R}{R_s}\right) = 100 \Rightarrow R = 49R_s$$

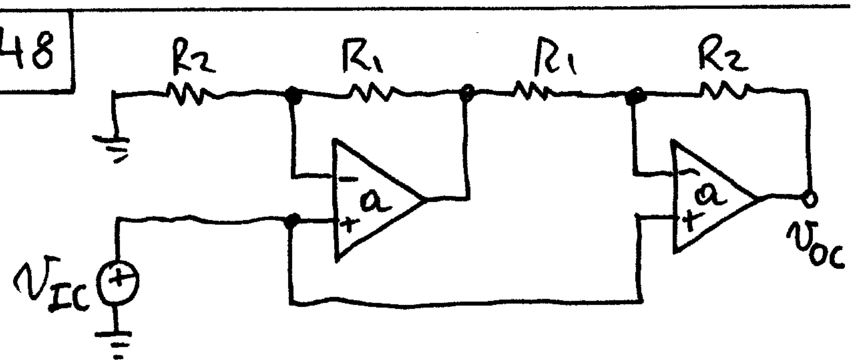
$$2\left(1 + \frac{R}{10 + R_s}\right) = 10 \Rightarrow R = 4(10 + R_s).$$ Solving,

$R_s = 888\ \Omega$ (use $887\ \Omega$, 1%), $R = 43.5$ kΩ (use 43.2 kΩ, 1%).

2.47 Regard the capacitor as an open circuit in dc analysis. By op amp action, $v_{P1} = v_{N1} = v_1$, $v_{N2} = v_{P2} = v_2$. Moreover, the output of OA_2 is $v_3 = (1 + R_1/R_2)v_1 = -\frac{R_1}{R_2}v_O + \left(1 + \frac{R_1}{R_2}\right)v_2$.

Thus, $v_O = \left(1 + \frac{R_2}{R_1}\right)(v_2 - v_1)$.

2.48



$$A = 1 + R_2/R_1 \Rightarrow R_2/R_1 = A-1, \quad R_1/R_2 = 1/(A-1)$$

$$v_1 = \frac{1 + R_1/R_2}{1 + \frac{1+R_1/R_2}{a}} v_{IC} = \frac{A}{A-1+A/a} v_{IC}$$

$$v_{OC} = \frac{1}{1 + \frac{1+R_2/R_1}{a}}\left[\left(1+\frac{R_2}{R_1}\right)v_{IC} - \frac{R_2}{R_1}v_1\right]$$

$$= \frac{A}{1+A/a}\left[1 - \frac{A-1}{A-1+A/a}\right] v_{IC}$$

$$= \frac{A}{A(1+1/a) + a(1-1/A)} v_{IC} = A_{cm} v_{IC}$$

$$CMRR = \frac{A}{A_{cm}} = a\left(1-\frac{1}{A}\right) + A\left(1+\frac{1}{a}\right) \cong \frac{A-1}{A} a$$

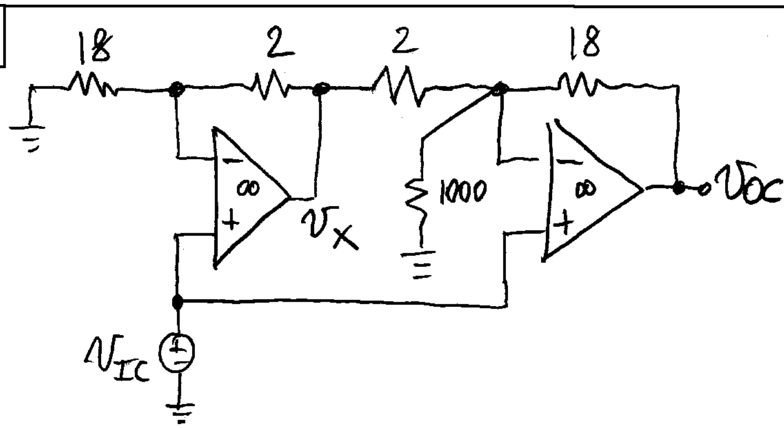
Since in general $A \ll a$. We readily see that for sufficiently large gains, or $A \gg 1$, we have $CMRR \cong a$, regardless of A.

$$A = 10^3 \text{ V/V} \Rightarrow CMRR_{dB} = \left|\frac{999}{1000} 10^5\right|_{dB} = 99.99 \text{ dB}$$

$$A = 10 \text{ V/V} \Rightarrow CMRR_{dB} = \left|\frac{9}{10} 10^5\right|_{dB} = 99.08 \text{ dB},$$

indicating an insignificant change.

2.49



(a) Since $1\,M\Omega \gg 18\,k\Omega > 2\,k\Omega$, we expect the 1-MΩ resistor to have little effect on A_{dm}, so $A_{dm} \cong 1+18/2 = 10$ V/V.

To find A_{cm}, tie the inputs together and drive them with v_{IC}. Then, using the superposition principle,

$$v_{OC} = -\frac{18}{2}v_X + \left(1+\frac{18}{2/\!/1000}\right)v_{IC} = -9\left(1+\frac{2}{18}\right)v_{IC} + \left(1+\frac{18}{2}+\frac{18}{1000}\right)v_{IC}$$

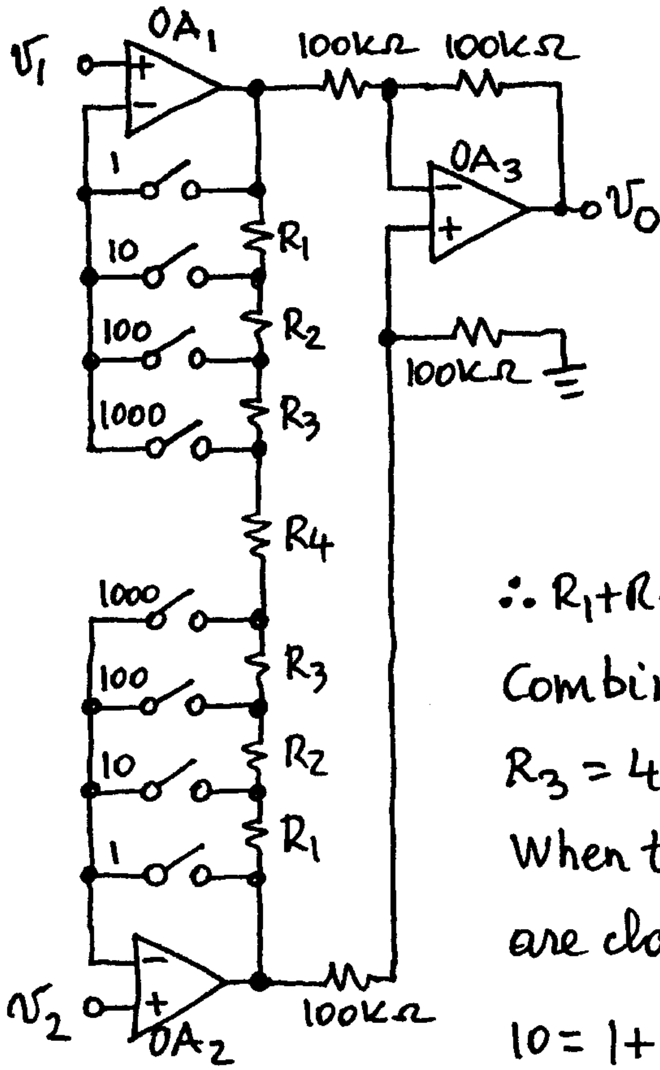
$$= \left(-10+10+\frac{9}{500}\right)v_{IC} \Rightarrow A_{cm} = \frac{v_{OC}}{v_{IC}} = \frac{9}{500}.$$

$CMRR = 20 \log 10/(9/500) = 54.9$ dB.

(b) $v_{N2} = v_{P2} = v_{IC} \Rightarrow i_{1M\Omega} = 0$, so the presence of the 1-MΩ resistance has no effect on the CMRR in this case.

2.50

When the "1000" switches are closed,



$$1000 = 1 + 2\frac{R_1+R_2+R_3}{1}.$$

$\therefore R_1+R_2+R_3 = 499.5\,\text{k}\Omega.$

When the "100" switches are closed,

$$100 = 1 + 2\frac{R_1+R_2}{2R_3+1}.$$

$\therefore R_1+R_2 = 99R_3 + 49.5\,\text{k}\Omega.$

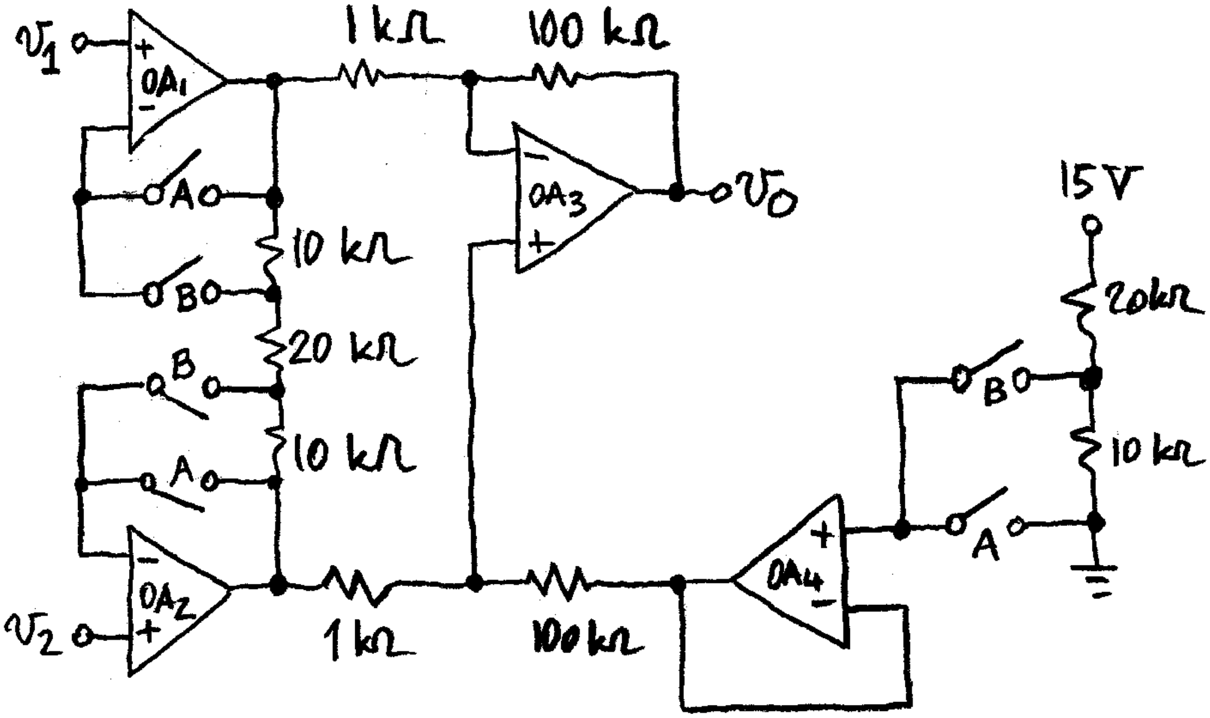
Combining yields $R_3 = 4.5\,\text{k}\Omega$.

When the "10" switches are closed,

$$10 = 1 + 2\frac{R_1}{2R_2+2R_3+1}.$$

$\therefore R_1 = 9R_2 + 45\,\text{k}\Omega$. Combining yields $R_2 = 45\,\text{k}\Omega$ and $R_1 = 450\,\text{k}\Omega$. Summarizing, $R_1 = 450\,\text{k}\Omega$, $R_2 = 45\,\text{k}\Omega$, $R_3 = 4.5\,\text{k}\Omega$, $R_4 = 1\,\text{k}\Omega$. All other resistors $= 100\,\text{k}\Omega$.

2.51



"A" switches closed $\Rightarrow$ $v_O = 1 \times 100\,(v_2 - v_1) + 0\ \text{V}$.

"B" switches closed $\Rightarrow$ $v_O = \left(1 + 2\frac{10}{20}\right) \times 100\,(v_2 - v_1) + 5\ \text{V}$.

$\boxed{2.52}$ (a) Let the outputs of OA_1 and OA_2 be v_{O1} and v_{O2}. Superposition:

$$v_{O1} = \left(1+\frac{R_1}{R_3}\right)v_1 - \frac{R_1}{R_3}v_L$$

$$v_{O2} = \left(1+\frac{R_5}{R_4}\right)v_2 - \frac{R_5}{R_4}\left[\left(1+\frac{R_1}{R_3}\right)v_1 - \frac{R_1}{R_3}v_L\right]$$

KCL: $i_O = \dfrac{v_1 - v_L}{R_3} + \dfrac{v_{O2} - v_L}{R_2}$. Eliminating v_{O2},

$$i_O = \frac{v_2}{R_2}\left[1+\frac{R_5}{R_4}\right] - \frac{v_1}{R_2}\left[\frac{R_5}{R_4}\left(1+\frac{R_1}{R_3}\right) - \frac{R_2}{R_3}\right] - v_L \times \frac{R_2+R_3-R_1R_5/R_4}{R_2R_3}.$$

It is readily seen that imposing $R_2 + R_3 = R_1R_5/R_4$ gives

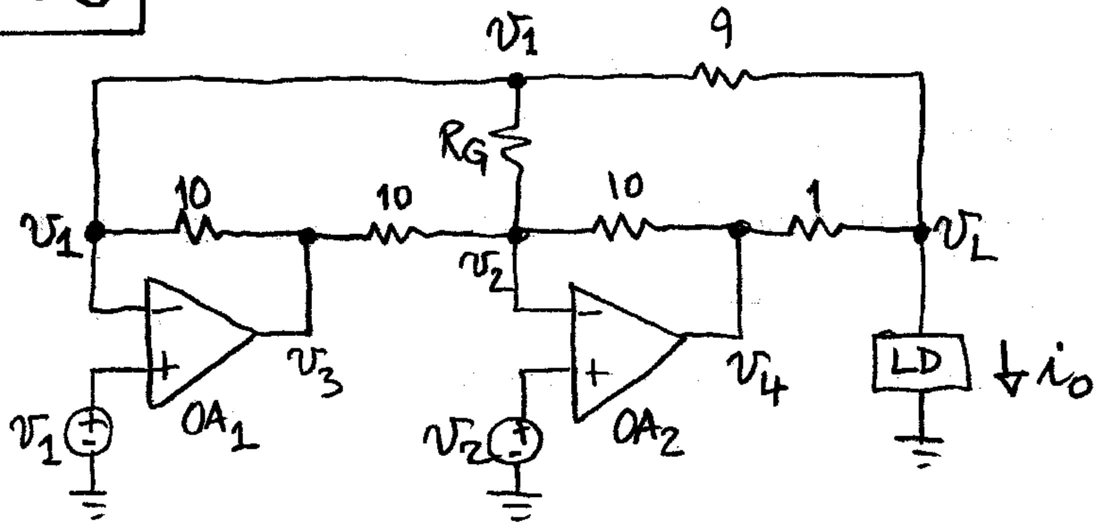
$$i_O = \frac{1}{R}(v_2 - v_1),\quad \frac{1}{R} = \frac{1+R_5/R_4}{R_2}.$$

(b) Use $R_1 = R_4 = R_5 = 100\ \text{k}\Omega$, $R_2 = 2.00\ \text{k}\Omega$, and $R_3 = 100 - 2 = 98.0\ \text{k}\Omega$.

(c) If the resistances are mismatched, the gains with which the circuit processes v_1 and v_2 will also be mismatched. Moreover, $R_o \neq \infty$. R_o is minimized when R_2, R_3, and R_4 are maximized, R_1 and R_5 are minimized.

$$R_{o(\min)} \cong \frac{2\times 10^3 \times 98 \times 10^3}{10^5 \times 1.001 - (10^5 \times 0.999)^2/(10^5 \times 1.001)} = 490\ \text{k}\Omega.$$

2.53



Summing currents at the inverting inputs of the

$$\frac{v_L - v_1}{9} + \frac{v_2 - v_1}{R_G} + \frac{v_3 - v_1}{10} = 0$$

$$\frac{v_1 - v_2}{R_G} + \frac{v_3 - v_2}{10} + \frac{v_4 - v_2}{10} = 0$$

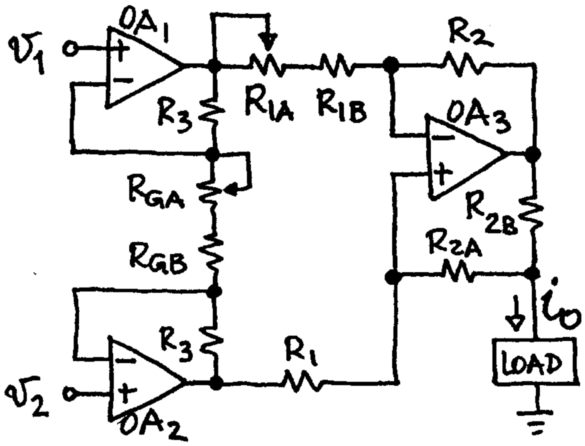
Solving for v_4 gives

$$v_4 = \frac{10}{9} v_L + v_2\left(\frac{20}{R_G} + 2\right) - v_1\left(\frac{20}{R_G} + \frac{19}{9}\right).$$ KCL:

$$i_O = \frac{v_1 - v_L}{9} + \frac{v_4 - v_L}{1}.$$ Substituting v_4 gives

$$i_O = 2\left(1 + \frac{10}{R_G}\right)(v_2 - v_1).$$

$\boxed{2.54}$ (a) $1/R = A_1/R_1$, $A_1 = 1 + 2R_3/R_G$. Since



$1/R$ must vary over a 100 : 1 range and since $A_1 > 1$, impose $2 \le A_1 \le 200$. Then,

$$200 = 1 + 2\frac{R_3}{0 + R_{GB}},$$

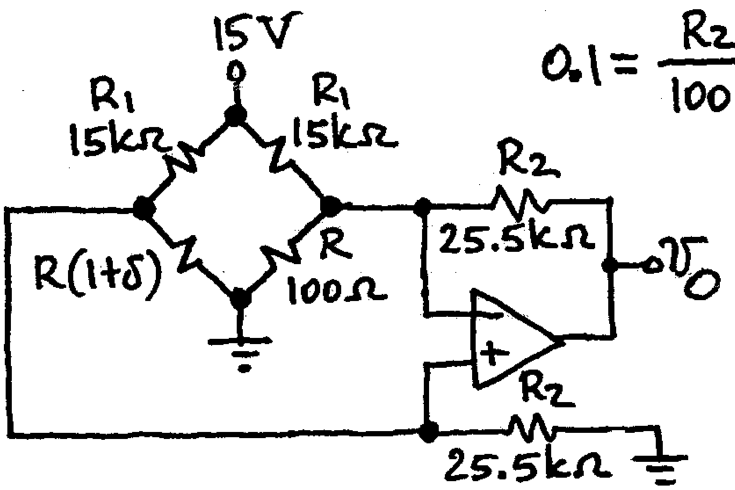
$$2 = 1 + 2\frac{R_3}{100 + R_{GB}}.$$

Solving yields $R_3 = 50.25\ \text{k}\Omega$ (use $49.9\ \text{k}\Omega$), and $R_{GB} = 0.505\ \text{k}\Omega$ (use $499\ \Omega$). When $A_1 = 2$ we want $1/R = 2/R_1 = 1\ \text{mA/V} \Rightarrow R_1 = 2\ \text{k}\Omega$. Use the improved Howland circuit with $R_1 = R_2 = 100\ \text{k}\Omega$ and $R_{2B} = 2\ \text{k}\Omega$. Then, $R_{2A} = 100 - 2 = 98\ \text{k}\Omega$ (use $97.6\ \text{k}\Omega$). Now 4% of $100\ \text{k}\Omega$ is $4\ \text{k}\Omega$. Use $R_{1A} = 10\ \text{k}\Omega$ to be on the safe side, and $R_{1B} = 95.3\ \text{k}\Omega$. Summarizing, $R_1 = R_2 = 100\ \text{k}\Omega$, $R_{1A} = 10\ \text{k}\Omega$ pot, $R_{1B} = 95.3\ \text{k}\Omega$, $R_{2A} = 97.6\ \text{k}\Omega$, $R_{2B} = 2.00\ \text{k}\Omega$, $R_3 = 49.9\ \text{k}\Omega$, $R_{GA} = 100\ \text{k}\Omega$ pot, $R_{GB} = 499\ \Omega$.

(b) Let $v_1 = v_2 = 0\ \text{V}$ and adjust R_{1A} as in Fig. 2.9.

$\boxed{2.55}$ With reference to Fig. 2.34, we want $2R_2R_3/R_1 = 10$ V/mA $= 10$ kΩ. Let $R_1 = R_2 =$ 10.0 kΩ. Then, $R_3 = 10/2 = 5$ kΩ (use 4.99 kΩ, 1%). Moreover, $R_4 = 4.99$ kΩ, 1%.

$\boxed{2.56}$ (a) Let $R_1 = 15$ kΩ. Then,



$$0.1 = \frac{R_2}{100}15\frac{0.00392}{1+\frac{15{,}000}{100}+\frac{15{,}000}{R_2}}.$$

This yields $R_2 = 170.7\left(151+\frac{15000}{R_2}\right)$. Starting out with $R_2 = 10$ kΩ and solving by iteration yields $R_2 = 25.8$ kΩ.

(b) $$v_O = \frac{25.5}{0.1}15\frac{0.392}{\frac{15}{0.1}+\left(1+\frac{15}{25.5}\right)(1+0.392)} =$$

9.96V, which corresponds to a 0.4°C error.

2.57 (a) KCL at the op amp input nodes:

$$\frac{V_{REF}-v_N}{R_1}=\frac{v_N}{R_2}+\frac{v_N-v_O}{R_2} \text{ and } \frac{V_{REF}-v_P}{R_1}=\frac{v_P}{R(1+\delta)}+\frac{v_O}{R_2}.$$

Letting $v_N=v_P$ and solving for v_O yields $v_O=(R_2/R)[\delta/(1+\delta)]v_P$. Voltage divider:

$$\frac{v_P}{V_{REF}}=\frac{[R(1+\delta)]//R_2}{[R(1+\delta)]//R_2+R_1}=\frac{1}{1+\dfrac{R_1}{[R(1+\delta)]//R_2}}=$$

$$\frac{1}{1+R_1\dfrac{R(1+\delta)+R_2}{R(1+\delta)R_2}}=\frac{1}{1+\dfrac{R_1}{R_2}\left(1+\dfrac{R_2}{R}\dfrac{1}{1+\delta}\right)}=$$

$$\frac{1+\delta}{1+\delta\left(1+\dfrac{R_1}{R_2}\right)+\dfrac{R_1}{R}}.$$ Eliminating v_P yields

$$v_O=\frac{R_2}{R}V_{REF}\frac{\delta}{\dfrac{R_1}{R}+\left(1+\dfrac{R_1}{R_2}\right)(1+\delta)};$$

$$\lim_{\delta\to 0} v_O=\frac{R_2}{R}V_{REF}\frac{\delta}{1+R_1/R+R_1/R_2}.$$

(b) The output of OA_1 is $v_1=-\dfrac{R(1+\delta)}{R_1}V_{REF}$.

Superposition: $v_O=-(R_2/R)v_1-(R_2/R_1)V_{REF}$.

Eliminating v_1, $v_O=(R_2/R_1)V_{REF}\,\delta$.

$\boxed{2.58}$ Impose 1mA through each side of the bridge. Thus, $R_1 = 2.5/2 = 1.25\,k\Omega$. Let $R_2 = 30\,k\Omega$ and $R = 100\,\Omega$, both 1%. Then,

$$0.1 = A\frac{100}{2\times 1250}2.5\times 0.00392 \Rightarrow A = 255\text{ V/V.}$$

$\boxed{2.59}$ (a) Let $i_{RTD} = 1\,mA$, So $R_1 = 15\,k\Omega$. Then,

$$0.1 = \frac{R_2}{15{,}000}15\times 0.00392 \Rightarrow R_2 = 25.5\,k\Omega.$$

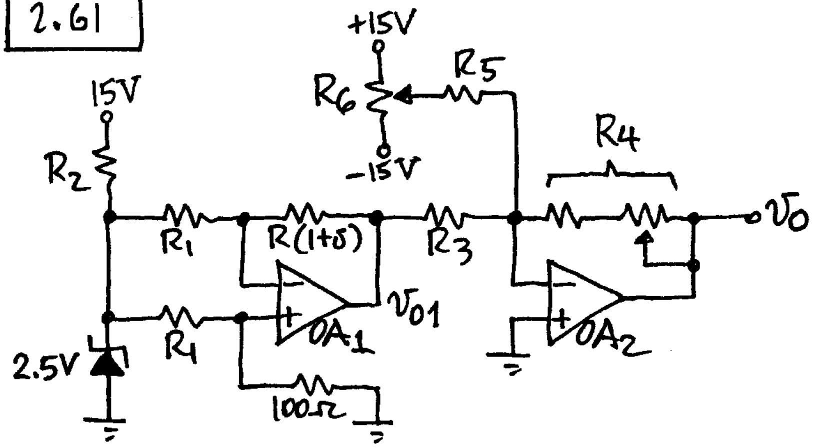
(b) Use the same topology, components, and calibration procedure as in Example 2.13.

$\boxed{2.60}$ Since $v_N = v_P$, it follows that the two legs of the bridge must conduct identical currents,

$$\frac{V_{REF} - v_O}{R_1 + R(1+\delta)} = \frac{V_{REF}}{R_1 + R}. \text{ Thus, } v_O = -\frac{R}{R_1+R}V_{REF}\delta.$$

The disadvantage is very low sensitivity, thus requiring an additional gain stage.

2.61



Let $R_1 = 2.49\,\text{k}\Omega$. Then, $\Delta T = 1°\text{C} \Rightarrow \Delta v_{O1} = [100/(100+2490)]\times 2.5 \times 0.00392 = 378.38\,\mu\text{V}$.
$\Delta v_O = (R_4/R_3)\Delta v_{O1} = 0.1\,\text{V} \Rightarrow R_4/R_3 = 264.3$.
Use $R_3 = 1\,\text{k}\Omega$, $R_4 = 237\,\text{k}\Omega$ in series with a 50-kΩ pot. Let $R_5 = 3.3\,\text{M}\Omega$, $R_6 = 100$-kΩ pot, $R_2 = 3.9\,\text{k}\Omega$. To calibrate:
With $T = 0°\text{C}$, adjust R_6 for $v_O = 0\,\text{V}$.
With $T = 100°\text{C}$, adjust R_4 for $v_O = 10.0\,\text{V}$.

2.62 $v_{N1} = v_{P1} = v_{N2} = v_{P2} = 0\,\text{V}$.
$v_{O1} = -[R(1+\delta)/R_1]V_{REF}$. $v_O = -R_2[V_{REF}/R_1 + v_{O1}/R] = -R_2\{V_{REF}/R_1 - [(1+\delta)/R_1]V_{REF}\}$, i.e.
$v_O = (R_2/R_1)V_{REF}\,\delta$.

