Differential Equations and Linear Algebra 4th Edition edwards Solutions Manual

Instructor's Solutions Manual

Differential Equations & Linear Algebra

Fourth Edition

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PREFACE

This is a solutions manual to accompany the textbook **DIFFERENTIAL EQUATIONS & LINEAR ALGEBRA** (4th edition, 2018) by C. Henry Edwards, David E. Penney, and David T. Calvis. We include solutions to most of the problems in the text. The corresponding **Student's Solutions Manual** contains solutions to most of the odd-numbered solutions in the text.

Our goal is to support teaching of the subject of differential equations with linear algebra in every way that we can. We therefore invite comments and suggested improvements for future printings of this manual, as well as advice regarding features that might be added to increase its usefulness in subsequent editions. Additional supplementary material can be found at the Expanded Applications website listed below.

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http://goo.gl/UYnW2g

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CHAPTER 1

FIRST-ORDER DIFFERENTIAL EQUATIONS

SECTION 1.1

DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELS

The main purpose of Section 1.1 is simply to introduce the basic notation and terminology of differential equations, and to show the student what is meant by a solution of a differential equation. Also, the use of differential equations in the mathematical modeling of real-world phenomena is outlined.

Problems 1-12 are routine verifications by direct substitution of the suggested solutions into the given differential equations. We include here just some typical examples of such verifications.

3. If $y_1 = \cos 2x$ and $y_2 = \sin 2x$, then $y_1' = -2\sin 2x$ $y_2' = 2\cos 2x$, so $y_1'' = -4\cos 2x = -4y_1$ and $y_2'' = -4\sin 2x = -4y_2$. Thus $y_1'' + 4y_1 = 0$ and $y_2'' + 4y_2 = 0$.

4. If $y_1 = e^{3x}$ and $y_2 = e^{-3x}$, then $y_1 = 3e^{3x}$ and $y_2 = -3e^{-3x}$, so $y_1'' = 9e^{3x} = 9y_1$ and $y_2'' = 9e^{-3x} = 9y_2$.

5. If $y = e^x - e^{-x}$, then $y' = e^x + e^{-x}$, so $y' - y = \left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right) = 2e^{-x}$. Thus $y' = y + 2e^{-x}$.

6. If $y_1 = e^{-2x}$ and $y_2 = xe^{-2x}$, then $y_1' = -2e^{-2x}$, $y_1'' = 4e^{-2x}$, $y_2' = e^{-2x} - 2xe^{-2x}$, and $y_2'' = -4e^{-2x} + 4xe^{-2x}$. Hence

$$y_1'' + 4y_1' + 4y_1 = \left(4e^{-2x}\right) + 4\left(-2e^{-2x}\right) + 4\left(e^{-2x}\right) = 0$$

and

$$y_2'' + 4y_2' + 4y_2 = \left(-4e^{-2x} + 4xe^{-2x}\right) + 4\left(e^{-2x} - 2xe^{-2x}\right) + 4\left(xe^{-2x}\right) = 0.$$

8. If $y_1 = \cos x - \cos 2x$ and $y_2 = \sin x - \cos 2x$, then $y_1' = -\sin x + 2\sin 2x$, $y_1'' = -\cos x + 4\cos 2x$, $y_2' = \cos x + 2\sin 2x$, and $y_2'' = -\sin x + 4\cos 2x$. Hence

$$y_1'' + y_1 = \left(-\cos x + 4\cos 2x\right) + \left(\cos x - \cos 2x\right) = 3\cos 2x$$

and

$$y_2'' + y_2 = \left(-\sin x + 4\cos 2x\right) + \left(\sin x - \cos 2x\right) = 3\cos 2x.$$

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11. If $y = y_1 = x^{-2}$, then $y' = -2x^{-3}$ and $y'' = 6x^{-4}$, so

$$x^2 y'' + 5x y' + 4y = x^2\left(6x^{-4}\right) + 5x\left(-2x^{-3}\right) + 4\left(x^{-2}\right) = 0.$$

If $y = y_2 = x^{-2}\ln x$, then $y' = x^{-3} - 2x^{-3}\ln x$ and $y'' = -5x^{-4} + 6x^{-4}\ln x$, so

$$\begin{aligned} x^2 y'' + 5x y' + 4y &= x^2\left(-5x^{-4} + 6x^{-4}\ln x\right) + 5x\left(x^{-3} - 2x^{-3}\ln x\right) + 4\left(x^{-2}\ln x\right) \\ &= \left(-5x^{-2} + 5x^{-2}\right) + \left(6x^{-2} - 10x^{-2} + 4x^{-2}\right)\ln x = 0. \end{aligned}$$

13. Substitution of $y = e^{rx}$ into $3y' = 2y$ gives the equation $3re^{rx} = 2e^{rx}$, which simplifies to $3r = 2$. Thus $r = 2/3$.

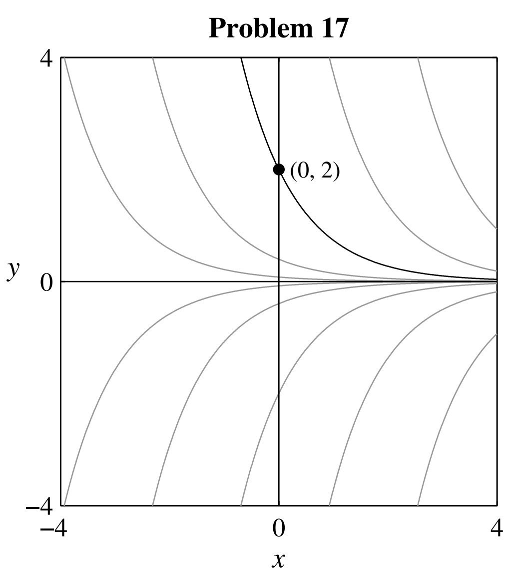
14. Substitution of $y = e^{rx}$ into $4y'' = y$ gives the equation $4r^2 e^{rx} = e^{rx}$, which simplifies to $4r^2 = 1$. Thus $r = \pm 1/2$.

15. Substitution of $y = e^{rx}$ into $y'' + y' - 2y = 0$ gives the equation $r^2e^{rx} + re^{rx} - 2e^{rx} = 0$, which simplifies to $r^2 + r - 2 = (r+2)(r-1) = 0$. Thus $r = -2$ or $r = 1$.

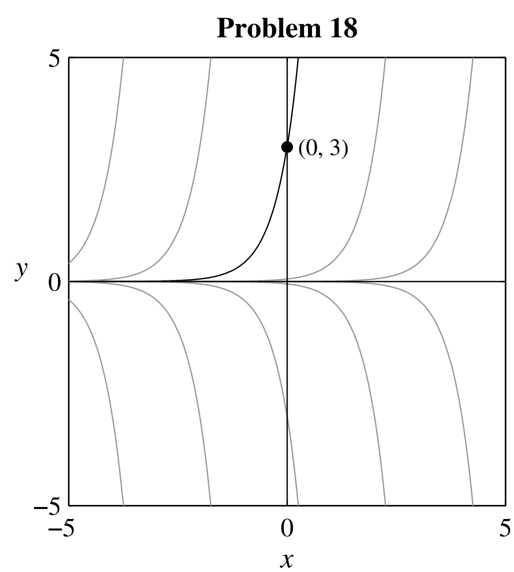
16. Substitution of $y = e^{rx}$ into $3y'' + 3y' - 4y = 0$ gives the equation $3r^2e^{rx} + 3re^{rx} - 4e^{rx} = 0$, which simplifies to $3r^2 + 3r - 4 = 0$. The quadratic formula then gives the solutions $r = \left(-3 \pm \sqrt{57}\right)/6$.

The verifications of the suggested solutions in Problems 17-26 are similar to those in Problems 1-12. We illustrate the determination of the value of *C* only in some typical cases. However, we illustrate typical solution curves for each of these problems.

17. $C = 2$



18. $C = 3$



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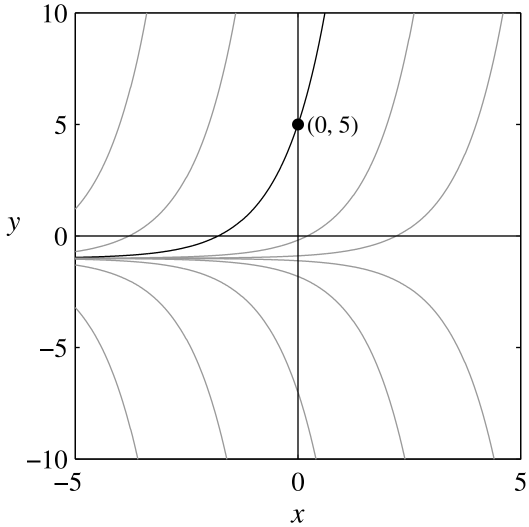
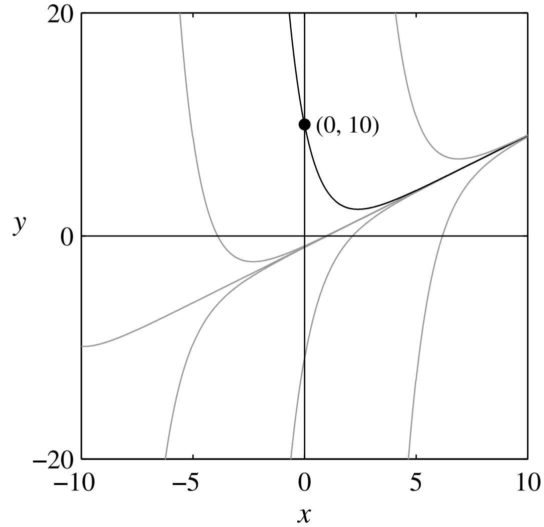
19. If $y(x) = Ce^x - 1$, then $y(0) = 5$ gives $C - 1 = 5$, so $C = 6$.

20. If $y(x) = C\,e^{-x} + x - 1$, then $y(0) = 10$ gives $C - 1 = 10$, or $C = 11$.

Problem 19



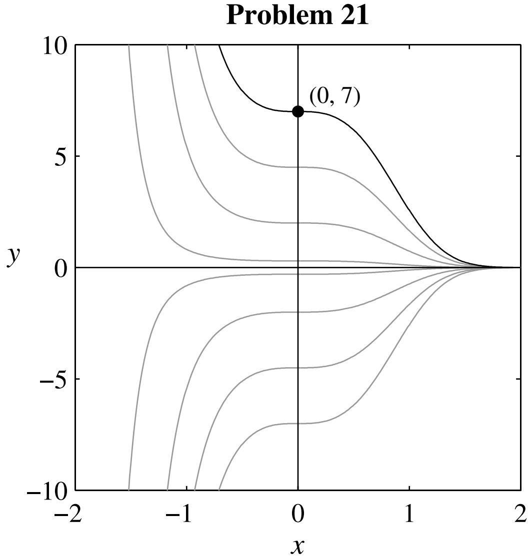
Problem 20



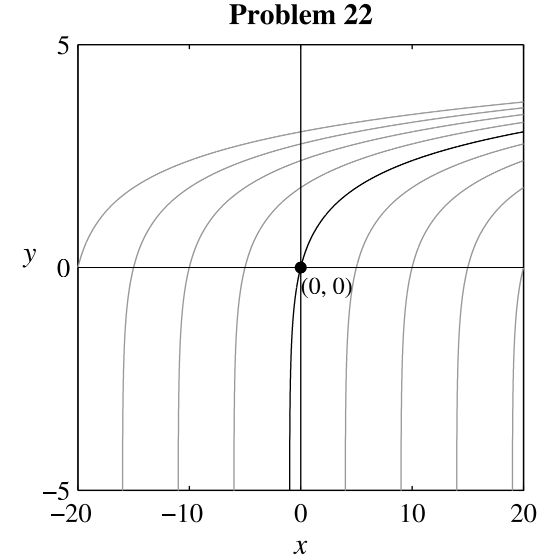
21. $C = 7$.

22. If $y(x) = \ln(x + C)$, then $y(0) = 0$ gives $\ln C = 0$, so $C = 1$.

Problem 21



Problem 22



23. If $y(x) = \frac{1}{4}x^5 + C\,x^{-2}$, then $y(2) = 1$ gives $\frac{1}{4} \cdot 32 + C \cdot \frac{1}{8} = 1$, or $C = -56$.

24. $C = 17$.

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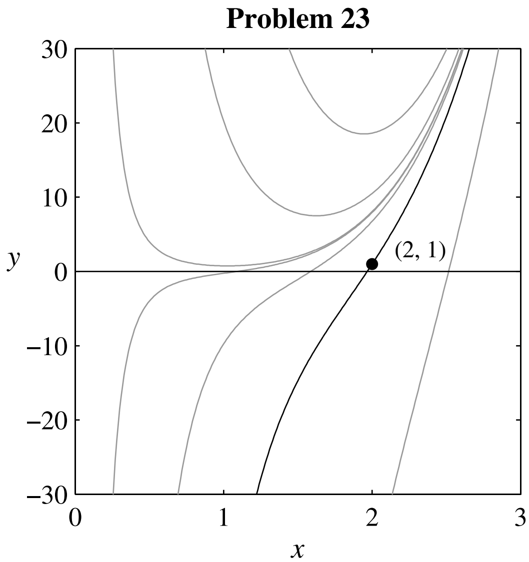
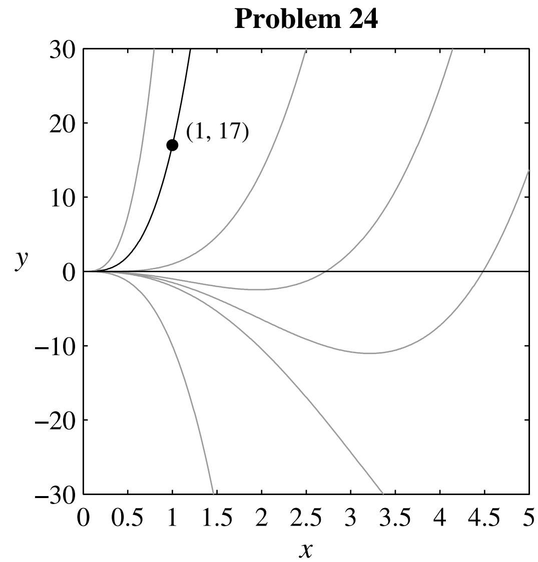
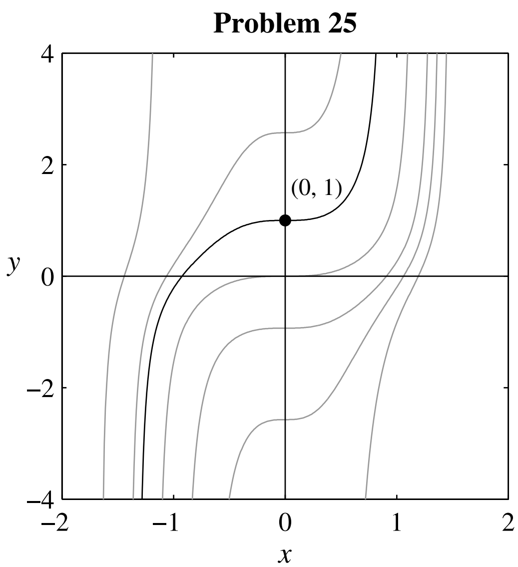
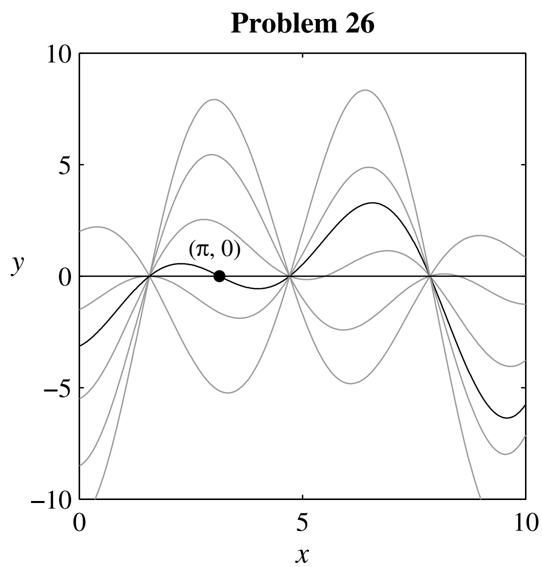




25. If $y = \tan\left(x^3 + C\right)$, then $y(0) = 1$ gives the equation $\tan C = 1$. Hence one value of C is $C = \pi/4$, as is this value plus any integral multiple of π.





26. Substitution of $x = \pi$ and $y = 0$ into $y = (x + C)\cos x$ yields $0 = (\pi + C)(-1)$, so $C = -\pi$.

27. $y' = x + y$

28. The slope of the line through (x, y) and $(x/2, 0)$ is $y' = \dfrac{y - 0}{x - x/2} = 2y/x$, so the differential equation is $xy' = 2y$.

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29. If $m = y'$ is the slope of the tangent line and m' is the slope of the normal line at (x, y), then the relation $m\,m' = -1$ yields $m' = -1/y' = (y-1)/(x-0)$. Solving for y' then gives the differential equation $(1-y)\,y' = x$.

30. Here $m = y'$ and $m' = D_x(x^2+k) = 2x$, so the orthogonality relation $m\,m' = -1$ gives the differential equation $2xy' = -1$.

31. The slope of the line through (x, y) and $(-y, x)$ is $y' = (x-y)/(-y-x)$, so the differential equation is $(x+y)y' = y-x$.

In Problems 32-36 we get the desired differential equation when we replace the "time rate of change" of the dependent variable with its derivative with respect to time *t*, the word "is" with the = sign, the phrase "proportional to" with *k*, and finally translate the remainder of the given sentence into symbols.

32. $dP/dt = k\sqrt{P}$

33. $dv/dt = kv^2$

34. $dv/dt = k(250-v)$

35. $dN/dt = k(P-N)$

36. $dN/dt = kN(P-N)$

37. The second derivative of any linear function is zero, so we spot the two solutions $y(x) \equiv 1$ and $y(x) = x$ of the differential equation $y'' = 0$.

38. A function whose derivative equals itself, and is hence a solution of the differential equation $y' = y$, is $y(x) = e^x$.

39. We reason that if $y = kx^2$, then each term in the differential equation is a multiple of x^2. The choice $k = 1$ balances the equation and provides the solution $y(x) = x^2$.

40. If *y* is a constant, then $y' \equiv 0$, so the differential equation reduces to $y^2 = 1$. This gives the two constant-valued solutions $y(x) \equiv 1$ and $y(x) \equiv -1$.

41. We reason that if $y = ke^x$, then each term in the differential equation is a multiple of e^x. The choice $k = \frac{1}{2}$ balances the equation and provides the solution $y(x) = \frac{1}{2}e^x$.

42. Two functions, each equaling the negative of its own second derivative, are the two solutions $y(x) = \cos x$ and $y(x) = \sin x$ of the differential equation $y'' = -y$.

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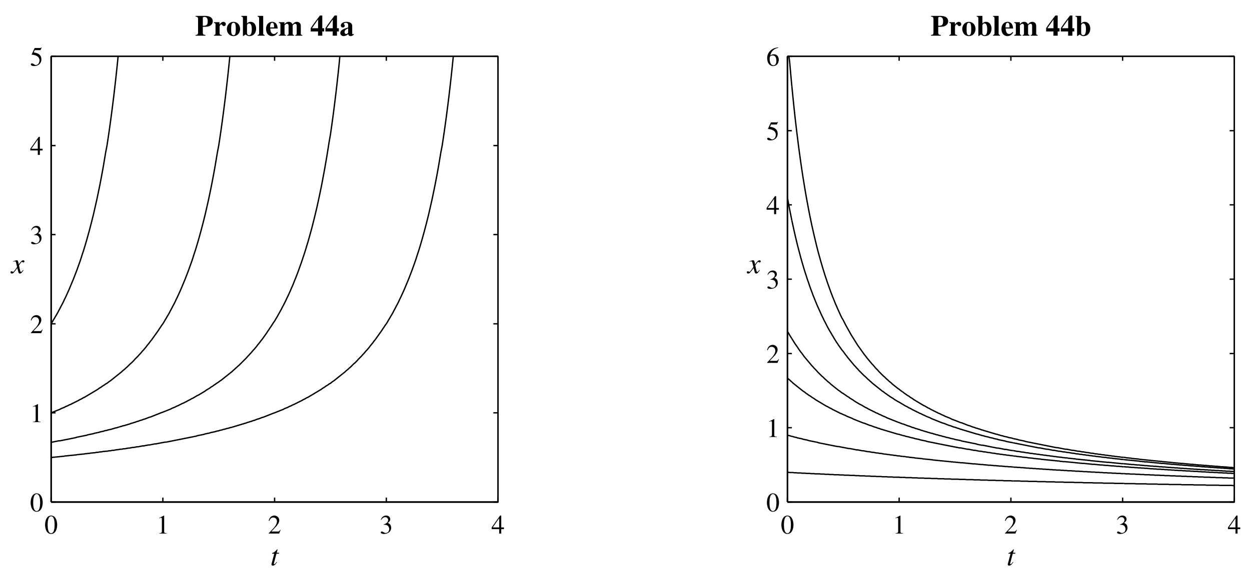
43. **(a)** We need only substitute $x(t) = 1/(C - kt)$ in both sides of the differential equation $x' = kx^2$ for a routine verification.

(b) The zero-valued function $x(t) \equiv 0$ obviously satisfies the initial value problem $x' = kx^2$, $x(0) = 0$.

44. **(a)** The figure shows typical graphs of solutions of the differential equation $x' = \frac{1}{2}x^2$.

(b) The figure shows typical graphs of solutions of the differential equation $x' = -\frac{1}{2}x^2$. We see that—whereas the graphs with $k = \frac{1}{2}$ appear to "diverge to infinity"—each solution with $k = -\frac{1}{2}$ appears to approach 0 as $t \to \infty$. Indeed, we see from the Problem 43(a) solution $x(t) = 1/(C - \frac{1}{2}t)$ that $x(t) \to \infty$ as $t \to 2C$. However, with $k = -\frac{1}{2}$ it is clear from the resulting solution $x(t) = 1/(C + \frac{1}{2}t)$ that $x(t)$ remains bounded on any bounded interval, but $x(t) \to 0$ as $t \to +\infty$.



45. Substitution of $P' = 1$ and $P = 10$ into the differential equation $P' = kP^2$ gives $k = \frac{1}{100}$, so Problem 43(a) yields a solution of the form $P(t) = 1/(C - \frac{1}{100}t)$. The initial condition $P(0) = 2$ now yields $C = \frac{1}{2}$, so we get the solution

$$P(t) = \frac{1}{\frac{1}{2} - \frac{t}{100}} = \frac{100}{50 - t}.$$

We now find readily that $P = 100$ when $t = 49$ and that $P = 1000$ when $t = 49.9$. It appears that P grows without bound (and thus "explodes") as t approaches 50.

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46. Substitution of $v' = -1$ and $v = 5$ into the differential equation $v' = kv^2$ gives $k = -\frac{1}{25}$, so Problem 43(a) yields a solution of the form $v(t) = 1/\left(C + t/25\right)$. The initial condition $v(0) = 10$ now yields $C = \frac{1}{10}$, so we get the solution

$$v(t) = \frac{1}{\frac{1}{10} + \frac{t}{25}} = \frac{50}{5 + 2t}.$$

We now find readily that $v = 1$ when $t = 22.5$ and that $v = 0.1$ when $t = 247.5$. It appears that v approaches 0 as t increases without bound. Thus the boat gradually slows, but never comes to a "full stop" in a finite period of time.

47. **(a)** $y(10) = 10$ yields $10 = 1/\left(C - 10\right)$, so $C = 101/10$.

(b) There is no such value of C, but the constant function $y(x) \equiv 0$ satisfies the conditions $y' = y^2$ and $y(0) = 0$.

(c) It is obvious visually (in Fig. 1.1.8 of the text) that one and only one solution curve passes through each point (a, b) of the xy-plane, so it follows that there exists a unique solution to the initial value problem $y' = y^2$, $y(a) = b$.

48. **(b)** Obviously the functions $u(x) = -x^4$ and $v(x) = +x^4$ both satisfy the differential equation $xy' = 4y$. But their derivatives $u'(x) = -4x^3$ and $v'(x) = +4x^3$ match at $x = 0$, where both are zero. Hence the given piecewise-defined function $y(x)$ is differentiable, and therefore satisfies the differential equation because $u(x)$ and $v(x)$ do so (for $x \le 0$ and $x \ge 0$, respectively).

(c) If $a \ge 0$ (for instance), then choose C_+ fixed so that $C_+ a^4 = b$. Then the function

$$y(x) = \begin{cases} C_- x^4 & \text{if } x \le 0 \\ C_+ x^4 & \text{if } x \ge 0 \end{cases}$$

satisfies the given differential equation for every real number value of C_-

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SECTION 1.2

INTEGRALS AS GENERAL AND PARTICULAR SOLUTIONS

This section introduces **general solutions** and **particular solutions** in the very simplest situation — a differential equation of the form $y' = f(x)$ — where only direct integration and evaluation of the constant of integration are involved. Students should review carefully the elementary concepts of velocity and acceleration, as well as the fps and mks unit systems.

1. Integration of $y' = 2x+1$ yields $y(x) = \int (2x+1)\,dx = x^2 + x + C$. Then substitution of $x = 0$, $y = 3$ gives $3 = 0+0+C = C$, so $y(x) = x^2 + x + 3$.

2. Integration of $y' = (x-2)^2$ yields $y(x) = \int (x-2)^2\,dx = \frac{1}{3}(x-2)^3 + C$. Then substitution of $x = 2$, $y = 1$ gives $1 = 0 + C = C$, so $y(x) = \frac{1}{3}(x-2)^3 + 1$.

3. Integration of $y' = \sqrt{x}$ yields $y(x) = \int \sqrt{x}\,dx = \frac{2}{3}x^{3/2} + C$. Then substitution of $x = 4$, $y = 0$ gives $0 = \frac{16}{3} + C$, so $y(x) = \frac{2}{3}\left(x^{3/2} - 8\right)$.

4. Integration of $y' = x^{-2}$ yields $y(x) = \int x^{-2}dx = -1/x + C$. Then substitution of $x = 1$, $y = 5$ gives $5 = -1 + C$, so $y(x) = -1/x + 6$.

5. Integration of $y' = (x+2)^{-1/2}$ yields $y(x) = \int (x+2)^{-1/2}\,dx = 2\sqrt{x+2} + C$. Then substitution of $x = 2$, $y = -1$ gives $-1 = 2\cdot 2 + C$, so $y(x) = 2\sqrt{x+2} - 5$.

6. Integration of $y' = x\left(x^2+9\right)^{1/2}$ yields $y(x) = \int x\left(x^2+9\right)^{1/2}\,dx = \frac{1}{3}\left(x^2+9\right)^{3/2} + C$. Then substitution of $x = -4$, $y = 0$ gives $0 = \frac{1}{3}(5)^3 + C$, so $y(x) = \frac{1}{3}\left[\left(x^2+9\right)^{3/2} - 125\right]$.

7. Integration of $y' = \dfrac{10}{x^2+1}$ yields $y(x) = \int \dfrac{10}{x^2+1}\,dx = 10\tan^{-1} x + C$. Then substitution of $x = 0$, $y = 0$ gives $0 = 10\cdot 0 + C$, so $y(x) = 10\tan^{-1} x$.

8. Integration of $y' = \cos 2x$ yields $y(x) = \int \cos 2x\,dx = \frac{1}{2}\sin 2x + C$. Then substitution of $x = 0$, $y = 1$ gives $1 = 0 + C$, so $y(x) = \frac{1}{2}\sin 2x + 1$.

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9. Integration of $y' = \dfrac{1}{\sqrt{1-x^2}}$ yields $y(x) = \int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$. Then substitution of $x = 0$, $y = 0$ gives $0 = 0 + C$, so $y(x) = \sin^{-1} x$.

10. Integration of $y' = xe^{-x}$ yields

$$y(x) = \int xe^{-x}\,dx = \int ue^{u}\,du = (u-1)e^{u} = -(x+1)e^{-x} + C,$$

using the substitution $u = -x$ together with Formula #46 inside the back cover of the textbook. Then substituting $x = 0$, $y = 1$ gives $1 = -1 + C$, so $y(x) = -(x+1)e^{-x} + 2$.

11. If $a(t) = 50$, then $v(t) = \int 50dt = 50t + v_0 = 50t + 10$. Hence

$$x(t) = \int (50t + 10)\,dt = 25t^2 + 10t + x_0 = 25t^2 + 10t + 20.$$

12. If $a(t) = -20$, then $v(t) = \int (-20)\,dt = -20t + v_0 = -20t - 15$. Hence

$$x(t) = \int (-20t - 15)\,dt = -10t^2 - 15t + x_0 = -10t^2 - 15t + 5.$$

13. If $a(t) = 3t$, then $v(t) = \int 3t\,dt = \frac{3}{2}t^2 + v_0 = \frac{3}{2}t^2 + 5$. Hence

$$x(t) = \int \left(\tfrac{3}{2}t^2 + 5\right)dt = \tfrac{1}{2}t^3 + 5t + x_0 = \tfrac{1}{2}t^3 + 5t.$$

14. If $a(t) = 2t + 1$, then $v(t) = \int (2t+1)\,dt = t^2 + t + v_0 = t^2 + t - 7$. Hence

$$x(t) = \int \left(t^2 + t - 7\right)dt = \tfrac{1}{3}t^3 + \tfrac{1}{2}t - 7t + x_0 = \tfrac{1}{3}t^3 + \tfrac{1}{2}t - 7t + 4.$$

15. If $a(t) = 4(t+3)^2$, then $v(t) = \int 4(t+3)^2\,dt = \frac{4}{3}(t+3)^3 + C = \frac{4}{3}(t+3)^3 - 37$ (taking $C = -37$ so that $v(0) = -1$). Hence

$$x(t) = \int \tfrac{4}{3}(t+3)^3 - 37\,dt = \tfrac{1}{3}(t+3)^4 - 37t + C = \tfrac{1}{3}(t+3)^4 - 37t - 26.$$

16. If $a(t) = \dfrac{1}{\sqrt{t+4}}$, then $v(t) = \int \dfrac{1}{\sqrt{t+4}}\,dt = 2\sqrt{t+4} + C = 2\sqrt{t+4} - 5$ (taking $C = -5$ so that $v(0) = -1$). Hence

$$x(t) = \int \left(2\sqrt{t+4} - 5\right)dt = \tfrac{4}{3}(t+4)^{3/2} - 5t + C = \tfrac{4}{3}(t+4)^{3/2} - 5t - \tfrac{29}{3}$$

(taking $C = -29/3$ so that $x(0) = 1$).

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17. If $a(t)=(t+1)^{-3}$, then $v(t)=\int (t+1)^{-3}\,dt=-\frac{1}{2}(t+1)^{-2}+C=-\frac{1}{2}(t+1)^{-2}+\frac{1}{2}$ (taking $C=\frac{1}{2}$ so that $v(0)=0$). Hence

$$x(t)=\int -\tfrac{1}{2}(t+1)^{-2}+\tfrac{1}{2}\,dt=\tfrac{1}{2}(t+1)^{-1}+\tfrac{1}{2}t+C=\tfrac{1}{2}\left[(t+1)^{-1}+t-1\right]$$

(taking $C=-\frac{1}{2}$ so that $x(0)=0$).

18. If $a(t)=50\sin 5t$, then $v(t)=\int 50\sin 5t\,dt=-10\cos 5t+C=-10\cos 5t$ (taking $C=0$ so that $v(0)=-10$). Hence

$$x(t)=\int -10\cos 5t\,dt=-2\sin 5t+C=-2\sin 5t+10$$

(taking $C=-10$ so that $x(0)=8$).

Students should understand that Problems 19-22, though different at first glance, are solved in the same way as the preceding ones, that is, by means of the fundamental theorem of calculus in the form $x(t)=x(t_0)+\int_{t_0}^{t} v(s)\,ds$ cited in the text. Actually in these problems $x(t)=\int_0^t v(s)\,ds,$ since t_0 and $x(t_0)$ are each given to be zero.

19. The graph of $v(t)$ shows that $v(t)=\begin{cases}5 & \text{if } 0\le t\le 5\\ 10-t & \text{if } 5\le t\le 10\end{cases}$, so that $x(t)=\begin{cases}5t+C_1 & \text{if } 0\le t\le 5\\ 10t-\frac{1}{2}t^2+C_2 & \text{if } 5\le t\le 10\end{cases}$. Now $C_1=0$ because $x(0)=0$, and continuity of $x(t)$ requires that $x(t)=5t$ and $x(t)=10t-\frac{1}{2}t^2+C_2$ agree when $t=5$. This implies that $C_2=-\frac{25}{2}$, leading to the graph of $x(t)$ shown.

Alternate solution for Problem 19 (and similarly for 20-22): The graph of $v(t)$ shows that $v(t)=\begin{cases}5 & \text{if } 0\le t\le 5\\ 10-t & \text{if } 5\le t\le 10\end{cases}$. Thus for $0\le t\le 5$, $x(t)=\int_0^t v(s)\,ds$ is given by $\int_0^t 5ds=5t$, whereas for $5\le t\le 10$ we have

$$x(t)=\int_0^t v(s)\,ds=\int_0^5 5\,ds+\int_5^t 10-s\,ds$$

$$=25+\left(10s-\frac{s^2}{2}\Bigg|_{s=5}^{s=t}\right)=25+10t-\frac{t^2}{2}-\frac{75}{2}=10t-\frac{t^2}{2}-\frac{25}{2}.$$

The graph of $x(t)$ is shown.

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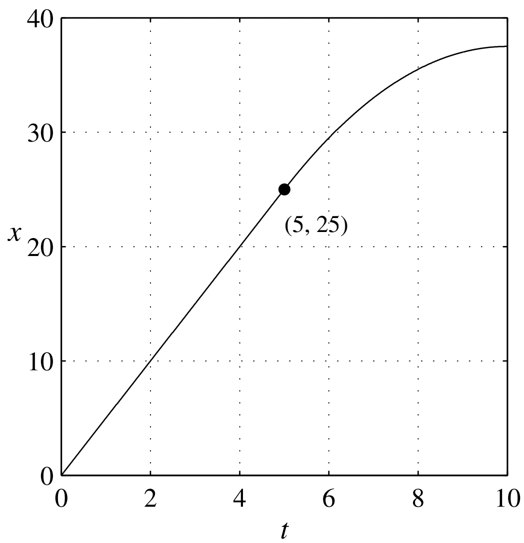
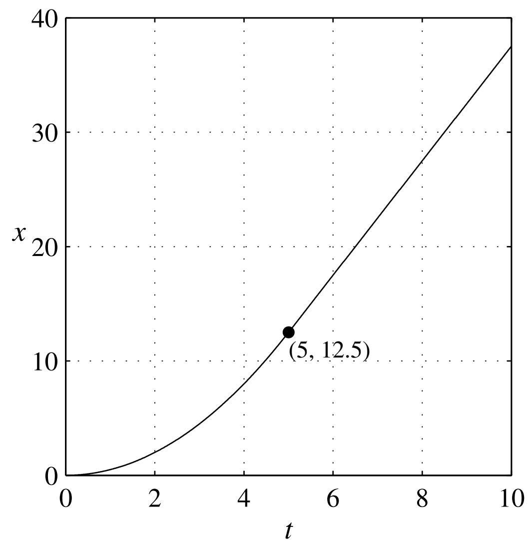
20. The graph of $v(t)$ shows that $v(t)=\begin{cases} t & \text{if } 0\le t\le 5 \\ 5 & \text{if } 5\le t\le 10 \end{cases}$, so that

$x(t)=\begin{cases} \frac{1}{2}t^2+C_1 & \text{if } 0\le t\le 5 \\ 5t+C_2 & \text{if } 5\le t\le 10 \end{cases}$. Now $C_1=0$ because $x(0)=0$, and continuity of $x(t)$ requires that $x(t)=\frac{1}{2}t^2$ and $x(t)=5t+C_2$ agree when $t=5$. This implies that $C_2=-\frac{25}{2}$, leading to the graph of $x(t)$ shown.

Problem 19



Problem 20



21. The graph of $v(t)$ shows that $v(t)=\begin{cases} t & \text{if } 0\le t\le 5 \\ 10-t & \text{if } 5\le t\le 10 \end{cases}$, so that

$x(t)=\begin{cases} \frac{1}{2}t^2+C_1 & \text{if } 0\le t\le 5 \\ 10t-\frac{1}{2}t^2+C_2 & \text{if } 5\le t\le 10 \end{cases}$. Now $C_1=0$ because $x(0)=0$, and continuity of $x(t)$ requires that $x(t)=\frac{1}{2}t^2$ and $x(t)=10t-\frac{1}{2}t^2+C_2$ agree when $t=5$. This implies that $C_2=-25$, leading to the graph of $x(t)$ shown.

22. For $0\le t\le 3$, $v(t)=\frac{5}{3}t$, so $x(t)=\frac{5}{6}t^2+C_1$. Now $C_1=0$ because $x(0)=0$, so $x(t)=\frac{5}{6}t^2$ on this first interval, and its right-endpoint value is $x(3)=\frac{15}{2}$.

For $3\le t\le 7$, $v(t)=5$, so $x(t)=5t+C_2$ Now $x(3)=\frac{15}{2}$ implies that $C_2=-\frac{15}{2}$, so $x(t)=5t-\frac{15}{2}$ on this second interval, and its right-endpoint value is $x(7)=\frac{55}{2}$.

For $7\le t\le 10$, $v-5=-\frac{5}{3}(t-7)$, so $v(t)=-\frac{5}{3}t+\frac{50}{3}$. Hence $x(t)=-\frac{5}{6}t^2+\frac{50}{3}t+C_3$, and $x(7)=\frac{55}{2}$ implies that $C_3=-\frac{290}{6}$. Finally, $x(t)=\frac{1}{6}(-5t^2+100t-290)$ on this third interval, leading to the graph of $x(t)$ shown.

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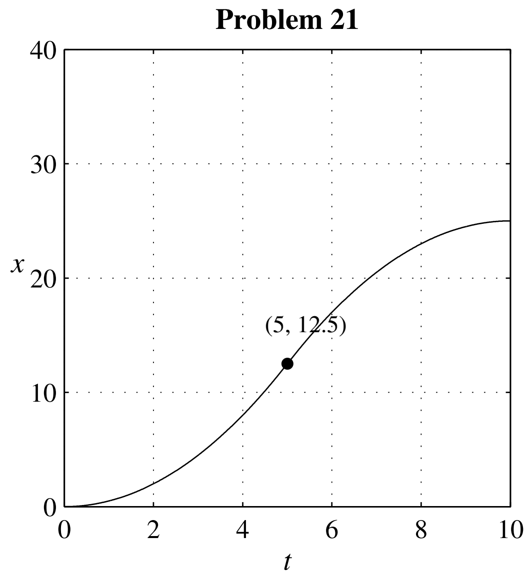

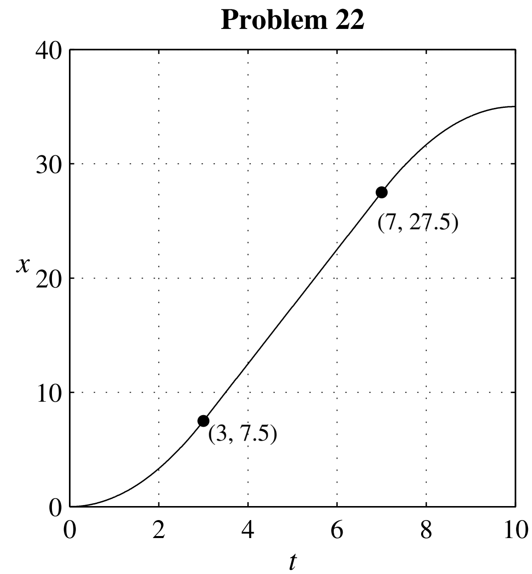

23. $v(t) = -9.8t + 49$, so the ball reaches its maximum height ($v = 0$) after $t = 5$ seconds. Its maximum height then is $y(5) = -4.9(5)^2 + 49(5) = 122.5$ meters.

24. $v = -32t$ and $y = -16t^2 + 400$, so the ball hits the ground ($y = 0$) when $t = 5$ sec, and then $v = -32(5) = -160$ ft/sec.

25. $a = -10\text{ m/s}^2$ and $v_0 = 100\text{ km/h} \approx 27.78\text{ m/s}$, so $v = -10t + 27.78$, and hence $x(t) = -5t^2 + 27.78t$. The car stops when $v = 0$, that is $t \approx 2.78$ s, and thus the distance traveled before stopping is $x(2.78) \approx 38.59$ meters.

26. $v = -9.8t + 100$ and $y = -4.9t^2 + 100t + 20$.

(a) $v = 0$ when $t = 100/9.8$ s, so the projectile's maximum height is $y(100/9.8) = -4.9(100/9.8)^2 + 100(100/9.8) + 20 \approx 530$ meters.

(b) It passes the top of the building when $y(t) = -4.9t^2 + 100t + 20 = 20$, and hence after $t = 100/4.9 \approx 20.41$ seconds.

(c) The roots of the quadratic equation $y(t) = -4.9t^2 + 100t + 20 = 0$ are $t = -0.20, 20.61$. Hence the projectile is in the air 20.61 seconds.

27. $a = -9.8\text{ m/s}^2$, so $v = -9.8t - 10$ and $y = -4.9t^2 - 10t + y_0$. The ball hits the ground when $y = 0$ and $v = -9.8t - 10 = -60$ m/s, so $t \approx 5.10$ s. Hence the height of the building is

$$y_0 = 4.9(5.10)^2 + 10(5.10) \approx 178.57 \text{ m}.$$

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28. $v=-32t-40$ and $y=-16t^2-40t+555$. The ball hits the ground ($y=0$) when $t\approx 4.77$ s, with velocity $v=v(4.77)\approx -192.64$ ft/s, an impact speed of about 131 mph.

29. Integration of $dv/dt=0.12t^2+0.6t$ with $v(0)=0$ gives $v(t)=0.04t^3+0.3t^2$. Hence $v(10)=70$ ft/s. Then integration of $dx/dt=0.04t^3+0.3t^2$ with $x(0)=0$ gives $x(t)=0.01t^4+0.1t^3$, so $x(10)=200$ ft. Thus after 10 seconds the car has gone 200 ft and is traveling at 70 ft/s.

30. Taking $x_0=0$ and $v_0=60$ mph $=88$ ft/s, we get $v=-at+88$, and $v=0$ yields $t=88/a$. Substituting this value of *t*, as well as $x=176$ ft, into $x=-at^2/2+88t$ leads to $a=22$ ft/s^2. Hence the car skids for $t=88/22=4$ s.

31. If $a=-20$ m/s^2 and $x_0=0$, then the car's velocity and position at time *t* are given by $v=-20t+v_0$ and $x=-10t^2+v_0t$. It stops when $v=0$ (so $v_0=20t$), and hence when $x=75=-10t^2+(20t)t=10t^2$. Thus $t=\sqrt{7.5}$ s, so

$$v_0=20\sqrt{7.5}\approx 54.77\text{ m/s}\approx 197\text{ km/hr}.$$

32. Starting with $x_0=0$ and $v_0=50$ km/h $=5\times10^4$ m/h, we find by the method of Problem 30 that the car's deceleration is $a=(25/3)\times10^7$ m/h^2. Then, starting with $x_0=0$ and $v_0=100$ km/h $=10^5$ m/h, we substitute $t=v_0/a$ into $x=-\frac{1}{2}at^2+v_0t$ and find that $x=60$ m when $v=0$. Thus doubling the initial velocity quadruples the distance the car skids.

33. If $v_0=0$ and $y_0=20$, then $v=-at$ and $y=-\frac{1}{2}at^2+20$. Substitution of $t=2$, $y=0$ yields $a=10$ ft/s^2. If $v_0=0$ and $y_0=200$, then $v=-10t$ and $y=-5t^2+200$. Hence $y=0$ when $t=\sqrt{40}=2\sqrt{10}$ s and $v=-20\sqrt{10}\approx -63.25$ ft/s.

34. **On Earth:** $v=-32t+v_0$, so $t=v_0/32$ at maximum height (when $v=0$). Substituting this value of *t* and $y=144$ in $y=-16t^2+v_0t$, we solve for $v_0=96$ ft/s as the initial speed with which the person can throw a ball straight upward.

On Planet Gzyx: From Problem 33, the surface gravitational acceleration on planet Gzyx is $a=10$ ft/s^2, so $v=-10t+96$ and $y=-5t^2+96t$. Therefore $v=0$ yields $t=9.6$ s and so $y_{max}=y(9.6)=460.8$ ft is the height a ball will reach if its initial velocity is 96 ft/s.

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35. If $v_0 = 0$ and $y_0 = h$, then the stone's velocity and height are given by $v = -gt$ and $y = -0.5gt^2 + h$, respectively. Hence $y = 0$ when $t = \sqrt{2h/g}$, so $v = -g\sqrt{2h/g} = -\sqrt{2gh}$.

36. The method of solution is precisely the same as that in Problem 30. We find first that, on Earth, the woman must jump straight upward with initial velocity $v_0 = 12$ ft/s to reach a maximum height of 2.25 ft. Then we find that, on the Moon, this initial velocity yields a maximum height of about 13.58 ft.

37. We use units of miles and hours. If $x_0 = v_0 = 0$, then the car's velocity and position after t hours are given by $v = at$ and $x = \frac{1}{2}at^2$, respectively. Since $v = 60$ when $t = 5/6$, the velocity equation yields . Hence the distance traveled by 12:50 pm is $x = \frac{1}{2} \cdot 72 \cdot (5/6)^2 = 25$ miles.

38. Again we have $v = at$ and $x = \frac{1}{2}at^2$. But now $v = 60$ when $x = 35$. Substitution of $a = 60/t$ (from the velocity equation) into the position equation yields $35 = \frac{1}{2}(60/t)t^2 = 30t$, whence $t = 7/6\,\text{h}$, that is, 1:10 pm.

39. Integration of $y' = (9/v_S)(1 - 4x^2)$ yields $y = (3/v_S)(3x - 4x^3) + C$, and the initial condition $y(-1/2) = 0$ gives $C = 3/v_S$. Hence the swimmer's trajectory is $y(x) = (3/v_S)(3x - 4x^3 + 1)$. Substitution of $y(1/2) = 1$ now gives $v_S = 6\,\text{mph}$.

40. Integration of $y' = 3(1 - 16x^4)$ yields $y = 3x - (48/5)x^5 + C$, and the initial condition $y(-1/2) = 0$ gives $C = 6/5$. Hence the swimmer's trajectory is

$$y(x) = (1/5)(15x - 48x^5 + 6),$$

and so his downstream drift is $y(1/2) = 2.4\,\text{miles}$.

41. The bomb equations are $a = -32$, $v = -32t$, and $s_B = s = -16t^2 + 800$ with $t = 0$ at the instant the bomb is dropped. The projectile is fired at time $t = 2$, so its corresponding equations are $a = -32$, $v = -32(t - 2) + v_0$, and $s_P = s = -16(t-2)^2 + v_0(t - 2)$ for $t \geq 2$ (the arbitrary constant vanishing because $s_P(2) = 0$). Now the condition $s_B(t) = -16t^2 + 800 = 400$ gives $t = 5$, and then the further requirement that $s_P(5) = 400$ yields $v_0 = 544/3 \approx 181.33$ ft/s for the projectile's needed initial velocity.

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