SOLUTIONS MANUAL

DIGITAL DESIGN

WITH AN INTRODUCTION TO THE VERILOG HDL
Fifth Edition

M. MORRIS MANO
Professor Emeritus
California State University, Los Angeles

MICHAEL D. CILETTI
Professor Emeritus

University of Colorado, Colorado Springs

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Digital Design With An Introduction to the Verilog HDL – Solution Manual. M. Mano. M.D. Ciletti,

CHAPTER 1

1.1

Base-10:	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Octal:	20	21	22	23	24	25	26	27	30	31	32	33	34	35	36	37	40
Hex:	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E	1F	20
Base-12	14	15	16	17	18	19	1A	1B	20	21	22	23	24	25	26	27	28

1.2 **(a)** 32,768 **(b)** 67,108,864 **(c)** 6,871,947,674

1.3 $(4310)_5 = 4 * 5^3 + 3 * 5^2 + 1 * 5^1 = 580_{10}$

$(198)_{12} = 1 * 12^2 + 9 * 12^1 + 8 * 12^0 = 260_{10}$

$(435)_8 = 4 * 8^2 + 3 * 8^1 + 5 * 8^0 = 285_{10}$

$(345)_6 = 3 * 6^2 + 4 * 6^1 + 5 * 6^0 = 137_{10}$

1.4 16-bit binary: 1111_1111_1111_1111
Decimal equivalent: 2^{16} -1 = $65{,}535_{10}$
Hexadecimal equivalent: FFFF_{16}

1.5 Let b = base

(a) 14/2 = (b + 4)/2 = 5, so b = 6

(b) 54/4 = (5*b + 4)/4 = b + 3, so 5 * b = 52 – 4, and b = 8

(c) (2 *b + 4) + (b + 7) = 4b, so b = 11

1.6 $(x – 3)(x – 6) = x^2 –(6 + 3)x + 6*3 = x^2 -11x + 22$

Therefore: 6 + 3 = b + 1m, so b = 8
Also, $6*3 = (18)_{10} = (22)_8$

1.7 $64CD_{16} = 0110\_0100\_1100\_1101_2$ = 110_010_011_001 _101 = $(62315\ )_8$

1.8 **(a)** Results of repeated division by 2 (quotients are followed by remainders):

431_{10} = 215(1); 107(1); 53(1); 26(1); 13(0); 6(1) 3(0) 1(1)
Answer: $1111\_1010_2 = FA_{16}$

(b) Results of repeated division by 16:

431_{10} = 26(15); 1(10) (Faster)
Answer: FA = 1111_1010

1.9 **(a)** 10110.0101_2 = 16 + 4 + 2 + .25 + .0625 = 22.3125

(b) 16.5_{16} = 16 + 6 + 5*(.0615) = 22.3125

(c) 26.24_8 = 2 * 8 + 6 + 2/8 + 4/64 = 22.3125

(d) $DADA.B_{16} = 14*16^3 + 10*16^2 + 14*16 + 10 + 11/16 = 60{,}138.6875$

(e) $1010.1101_2 = 8 + 2 + .5 + .25 + .0625 = 10.8125$

1.10 **(a)** $1.10010_2 = 0001.1001_2 = 1.9_{16} = 1 + 9/16 = 1.563_{10}$

(b) $110.010_2 = 0110.0100_2 = 6.4_{16} = 6 + 4/16 = 6.25_{10}$

Reason: 110.010_2 is the same as 1.10010_2 shifted to the left by two places.

1.11

$$\begin{array}{r} \underline{1011.11} \\ 101\,|\,111011.0000 \\ \underline{101} \\ 01001 \\ \underline{101} \\ 1001 \\ \underline{101} \\ 1000 \\ \underline{101} \\ 0110 \end{array}$$

The quotient is carried to two decimal places, giving 1011.11
Checking: $111011_2 / 101_2 = 59_{10} / 5_{10} \cong 1011.11_2 = 58.75_{10}$

1.12 **(a)** 10000 and 110111

$$\begin{array}{r} 1011 \\ \underline{+101} \\ 10000 = 16_{10} \end{array} \qquad \begin{array}{r} 1011 \\ \underline{x101} \\ 1011 \\ \underline{1011} \\ 110111 = 55_{10} \end{array}$$

(b) 62_h and 958_h

$$\begin{array}{rl} 2E_h & 0010\_1110 \\ \underline{+34_h} & \underline{0011\_0100} \\ 62_h & 0110\_0010 = 98_{10} \end{array} \qquad \begin{array}{r} 2E_h \\ \underline{x34_h} \\ B^38 \\ \underline{8^2A} \\ 9\ \ 5\ 8_h = 2392_{10} \end{array}$$

1.13 **(a)** Convert 27.315 to binary:

	Integer Quotient		Remainder	Coefficient
27/2 =	13	+	½	$a_0 = 1$
13/2	6	+	½	$a_1 = 1$
6/2	3	+	0	$a_2 = 0$
3/2	1	+	½	$a_3 = 1$
½	0	+	½	$a_4 = 1$

$27_{10} = 11011_2$

		Integer		Fraction	Coefficient
.315 x 2	=	0	+	.630	$a_{-1} = 0$
.630 x 2	=	1	+	.26	$a_{-2} = 1$
.26 x 2	=	0	+	.52	$a_{-3} = 0$
.52 x 2	=	1	+	.04	$a_{-4} = 1$

$.315_{10} \cong .0101_2 = .25 + .0625 = .3125$

$27.315 \cong 11011.0101_2$

(b) 2/3 $\cong$ .6666666667

		Integer		Fraction	Coefficient
.6666_6666_67 x 2	=	1	+	.3333_3333_34	$a_{-1} = 1$
.3333333334 x 2	=	0	+	.6666666668	$a_{-2} = 0$
.6666666668 x 2	=	1	+	.3333333336	$a_{-3} = 1$
.3333333336 x 2	=	0	+	.6666666672	$a_{-4} = 0$
.6666666672 x 2	=	1	+	.3333333344	$a_{-5} = 1$
.3333333344 x 2	=	0	+	.6666666688	$a_{-6} = 0$
.6666666688 x 2	=	1	+	.3333333376	$a_{-7} = 1$
.3333333376 x 2	=	0	+	.6666666752	$a_{-8} = 0$

$.6666666667_{10} \cong .10101010_2 = .5 + .125 + .0313 + ..0078 = .6641_{10}$

.101010102 = $.1010\_1010_2 = .AA_{16} = 10/16 + 10/256 = .6641_{10}$ (Same as (b)).

1.14

(a)	0001_0000	**(b)**	0000_0000	**(c)**	1101_1010
1s comp:	1110_1111	1s comp:	1111_1111	1s comp:	0010_0101
2s comp:	1111_0000	2s comp:	0000_0000	2s comp:	0010_0110
(d)	1010_1010	**(e)**	1000_0101	**(f)**	1111_1111
1s comp:	0101_0101	1s comp:	0111_1010	1s comp:	0000_0000
2s comp:	0101_0110	2s comp:	0111_1011	2s comp:	0000_0001

1.15

(a)	25,478,036	**(b)**	63,325,600
9s comp:	74,521,963	9s comp:	36,674,399
10s comp:	74,521,964	10s comp:	36,674,400
(c)	25,000,000	**(d)**	00000000
9s comp:	74,999,999	9s comp:	99999999
10s comp:	75,000,000	10s comp:	100000000

1.16

	C3DF	C3DF:	1100_0011_1101_1111
15s comp:	3C20	1s comp:	0011_1100_0010_0000
16s comp:	3C21	2s comp:	0011_1100_0010_0001 = 3C21

1.17 **(a)** 2,579 → 02,579 →97,420 (9s comp) → 97,421 (10s comp)
4637 – 2,579 = 2,579 + 97,421 = 2058$_{10}$

(b) 1800 → 01800 → 98199 (9s comp) → 98200 (10 comp)
125 – 1800 = 00125 + 98200 = 98325 (negative)
Magnitude: 1675
Result: 125 – 1800 = 1675

(c) 4,361 → 04361 → 95638 (9s comp) → 95639 (10s comp)
2043 – 4361 = 02043 + 95639 = 97682 (Negative)
Magnitude: 2318
Result: 2043 – 6152 = -2318

(d) 745 → 00745 → 99254 (9s comp) → 99255 (10s comp)
1631 -745 = 01631 + 99255 = 0886 (Positive)
Result: 1631 – 745 = 886

1.18 Note: Consider sign extension with 2s complement arithmetic.

(a) 0_10010
1s comp: 1_01101
2s comp: 1_01110
$\underline{0\_10011}$
Diff: 0_00001 (Positive)
Check:19-18 = +1

(**b**) 0_100110
1s comp: 1_011001 with sign extension
2s comp: 1_011010
$\underline{0\_100010}$
1_111100 sign bit indicates that the result is negative
0_000011 1s complement
0_000100 2s complement
000100 magnitude
Result: -4
Check: 34 -38 = -4

(c) 0_110101
1s comp: 1_001010
2s comp: 1_001011
$\underline{0\_001001}$
Diff: 1_010100 (negative)
0_101011 (1s comp)
0_101100 (2s complement)
101100 (magnitude)
-44_{10} (result)

(**d**) 0_010101
1s comp: 1_101010 with sign extension
2s comp: 1_101011
$\underline{0\_101000}$
0_010011 sign bit indicates that the result is positive
Result: 19_{10}
Check: 40 – 21 = 19_{10}

1.19 +9286 → 009286; +801 → 000801; -9286 → 990714; -801 → 999199

(a) (+9286) + (_801) = 009286 + 000801 = 010087

(b) (+9286) + (-801) = 009286 + 999199 = 008485

(c) (-9286) + (+801) = 990714 + 000801 = 991515

(d) (-9286) + (-801) = 990714 + 999199 = 989913

1.20 +49 → 0_110001 (Needs leading zero extension to indicate + value);
+29 → 0_011101 (Leading 0 indicates + value)
-49 → 1_001110 + 0_000001→ 1_001111
-29 → 1_100011 (sign extension indicates negative value)

(a) (+29) + (-49) = 0_011101 + 1_001111 = 1_101100 (1 indicates negative value.)
Magnitude = 0_010011 + 0_000001 = 0_010100 = 20; Result (+29) + (-49) = -20

(b) (-29) + (+49) = 1_100011 + 0_110001 = 0_010100 (0 indicates positive value)
(-29) + (+49) = +20

(c) Must increase word size by 1 (sign extension) to accomodate overflow of values:
(-29) + (-49) = 11_100011 + 11_001111 = 10_110010 (1 indicates negative result)
Magnitude: 01_001110 = 78_{10}
Result: (-29) + (-49) = -78_{10}

1.21 +9742 → 009742 → 990257 (9's comp) → 990258 (10s) comp
+641 → 000641 → 999358 (9's comp) → 999359 (10s) comp

(a) (+9742) + (+641) → 010383

(b) (+9742) + (-641) →009742 + 999359 = 009102
Result: (+9742) + (-641) = 9102

(c) -9742) + (+641) = 990258 + 000641 = 990899 (negative)
Magnitude: 009101
Result: (-9742) + (641) = -9101

(d) (-9742) + (-641) = 990258 + 999359 = 989617 (Negative)
Magnitude: 10383
Result: (-9742) + (-641) = -10383

1.22 6,514
BCD**:** 0110_0101_0001_0100
ASCII: 0_011_0110_0_011_0101_1_011_0001_1_011_0100
ASCII: 0011_0110_0011_0101_1011_0001_1011_0100

1.23

$$\begin{array}{lllll}
 & 0111 & 1001 & 0001 & (\ \ 791) \\
 & \underline{0110} & \underline{0101} & \underline{1000} & (+658) \\
 & 1101 & 1110 & 1001 & \\
 & \underline{0110} & \underline{0110} & & \\
0001 & 0011 & 0100 & & \\
0001 & 0001 & & & \\
0001 & 0100 & 0100 & 1001 & (1{,}449)
\end{array}$$

1.24 **(a)**

6	3	1	1	Decimal
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	1	0	0	3
0	1	1	0	4 (*or* 0101)
0	1	1	1	5
1	0	0	0	6
1	0	1	0	7 (*or* 1001)
1	0	1	1	8
1	1	0	0	9

(b)

6	4	2	1	Decimal
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
1	0	0	0	6 (*or* 0110)
1	0	0	1	7
1	0	1	0	8
1	0	1	1	9

1.25 **(a)** $6{,}248_{10}$ BCD: 0110_0010_0100_1000
(b) Excess-3: 1001_0101_0111_1011

(c) 2421: 0110_0010_0100_1110
(d) 6311: 1000_0010_0110_1011

1.26 6,248 9s Comp: 3,751
2421 code: 0011_0111_0101_0001
1s comp c: 1001_1101_1011_0001 (2421 code alternative #1)

$6,248_{2421}$ 0110_0010_0100_1110 (2421 code alternative #2)
1s comp c 1001_1101_1011_0001 Match

1.27 For a deck with 52 cards, we need 6 bits ($2^5 = 32 < 52 < 64 = 2^6$). Let the msb's select the suit (e.g., diamonds, hearts, clubs, spades are encoded respectively as 00, 01, 10, and 11. The remaining four bits select the "number" of the card. Example: 0001 (ace) through 1011 (9), plus 101 through 1100 (jack, queen, king). This a jack of spades might be coded as 11_1010. (Note: only 52 out of 64 patterns are used.)

1.28 G (dot) (space) B o o l e
11000111_11101111_01101000_01101110_00100000_11000100_11101111_11100101

1.29 Steve Jobs

1.30 73 F4 E5 76 E5 4A EF 62 73

73:	0_111_0011	s
F4:	1_111_0100	t
E5:	1_110_0101	e
76:	0_111_0110	v
E5:	1_110_0101	e
4A:	0_100_1010	j
EF:	1_110_1111	o
62:	0_110_0010	b
73:	0_111_0011	s

1.31 62 + 32 = 94 printing characters

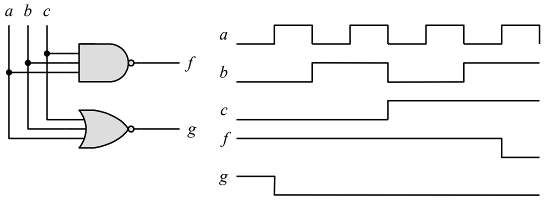
1.32 bit 6 from the right

1.33 **(a)** 897 **(b)** 564 **(c)** 871 **(d)** 2,199

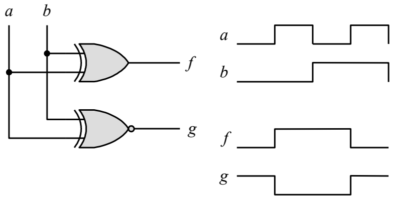
1.34 ASCII for decimal digits with even parity:

(0):	00110000	(1):	10110001	(2):	10110010	(3):	00110011
(4):	10110100	(5):	00110101	(6):	00110110	(7):	10110111
(8):	10111000	(9):	00111001				

1.35 **(a)**



1.36



CHAPTER 2

2.1 **(a)**

x y z	$x + y + z$	$(x + y + z)'$	x'	y'	z'	$x'y'z'$
0 0 0	0	1	1	1	1	1
0 0 1	1	0	1	1	0	0
0 1 0	1	0	1	0	1	0
0 1 1	1	0	1	0	0	0
1 0 0	1	0	0	1	1	0
1 0 1	1	0	0	1	0	0
1 1 0	1	0	0	0	1	0
1 1 1	1	0	0	0	0	0

$x\,y\,z$	(xyz)	$(xyz)'$	x'	y'	z'	$x' + y' + z'$
0 0 0	0	1	1	1	1	1
0 0 1	0	1	1	1	0	1
0 1 0	0	1	1	0	1	1
0 1 1	0	1	1	0	0	1
1 0 0	0	1	0	1	1	1
1 0 1	0	1	0	1	0	1
1 1 0	0	1	0	0	1	1
1 1 1	1	0	0	0	0	0

(b)

$x\,y\,z$	$x + yz$	$(x + y)$	$(x + z)$	$(x + y)(x + z)$
0 0 0	0	0	0	0
0 0 1	0	0	1	0
0 1 0	0	1	0	0
0 1 1	1	1	1	1
1 0 0	1	1	1	1
1 0 1	1	1	1	1
1 1 0	1	1	1	1
1 1 1	1	1	1	1

(c)

$x\,y\,z$	$x(y + z)$	xy	xz	$xy + xz$
0 0 0	0	0	0	0
0 0 1	0	0	0	0
0 1 0	0	0	0	0
0 1 1	0	0	0	0
1 0 0	0	0	0	0
1 0 1	1	0	1	1
1 1 0	1	1	0	1
1 1 1	1	1	1	1

(c)

$x\,y\,z$	x	$y + z$	$x + (y + z)$	$(x + y)$	$(x + y) + z$
0 0 0	0	0	0	0	0
0 0 1	0	1	1	0	1
0 1 0	0	1	1	1	1
0 1 1	0	1	1	1	1
1 0 0	1	0	1	1	1
1 0 1	1	1	1	1	1
1 1 0	1	1	1	1	1
1 1 1	1	1	1	1	1

(d)

$x\,y\,z$	yz	$x(yz)$	xy	$(xy)z$
0 0 0	0	0	0	0
0 0 1	0	0	0	0
0 1 0	0	0	0	0
0 1 1	1	0	0	0
1 0 0	0	0	0	0
1 0 1	0	0	0	0
1 1 0	0	0	1	0
1 1 1	1	1	1	1

2.2 **(a)** $xy + xy' = x(y + y') = x$

(b) $(x + y)(x + y') = x + yy' = x(x + y') + y(x + y') = xx + xy' + xy + yy' = x$

(c) $xyz + x'y + xyz' = xy(z + z') + x'y = xy + x'y = y$

(d) $(A + B)'(A' + B')' = (A'B')(A B) = (A'B')(BA) = A'(B'B)A = 0$

(e) $(a + b + c')(a'b' + c) = aa'b' + ac + ba'b' + bc + c'a'b' + c'c = ac + bc + a'b'c'$

(f) $a'bc + abc' + abc + a'bc' = a'b(c + c') + ab(c + c') = a'b + ab = (a' + a)b = b$

2.3 **(a)** $ABC + A'B + ABC' = AB + A'B = B$

(b) $x'yz + xz = (x'y + x)z = z(x + x')(x + y) = z(x + y)$

(c) $(x + y)'(x' + y') = x'y'(x' + y') = x'y'$

(d) $xy + x(wz + wz') = x(y + wz + wz') = x(w + y)$

(e) $(BC' + A'D)(AB' + CD') = BC'AB' + BC'CD' + A'DAB' + A'DCD' = 0$

(f) $(a' + c')(a + b' + c') = a'a + a'b' + a'c' + c'a + c'b' + c'c' = a'b' + a'c' + ac' + b'c' = c' + b'(a' + c')$
$= c' + b'c' + a'b' = c' + a'b'$

2.4 **(a)** $A'C' + ABC + AC' = C' + ABC = (C + C')(C' + AB) = AB + C'$

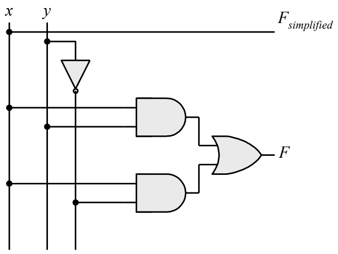
(b) $(x'y' + z)' + z + xy + wz = (x'y')'z' + z + xy + wz = [(x + y)z' + z] + xy + wz =$
$= (z + z')(z + x + y) + xy + wz = z + wz + x + xy + y = z(1 + w) + x(1 + y) + y = x + y + z$

(c) $A'B(D' + C'D) + B(A + A'CD) = B(A'D' + A'C'D + A + A'CD)$
$= B(A'D' + A + A'D(C + C') = B(A + A'(D' + D)) = B(A + A') = B$

(d) $(A' + C)(A' + C')(A + B + C'D) = (A' + CC')(A + B + C'D) = A'(A + B + C'D)$
$= AA' + A'B + A'C'D = A'(B + C'D)$

(e) $ABC'D + A'BD + ABCD = AB(C + C')D + A'BD = ABD + A'BD = BD$

2.5 **(a)**



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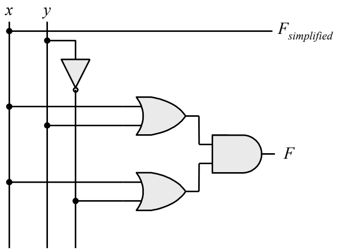

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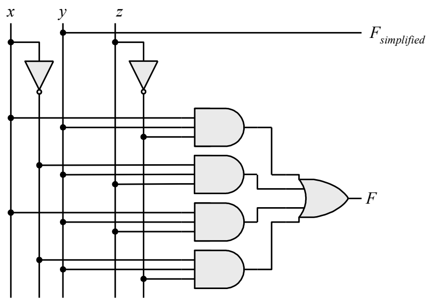

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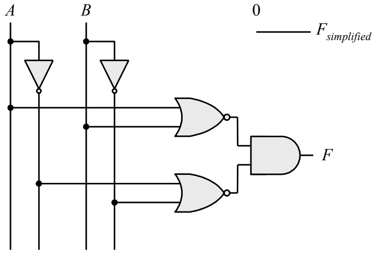

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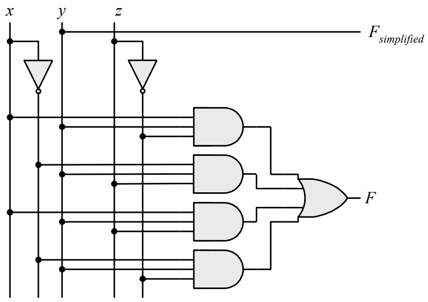

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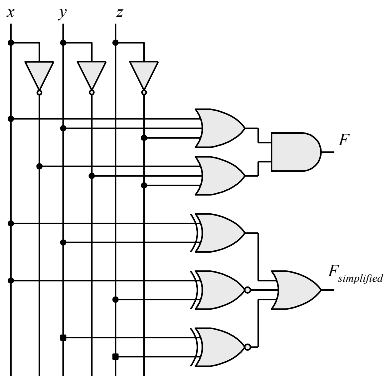
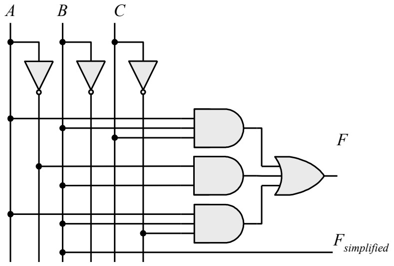
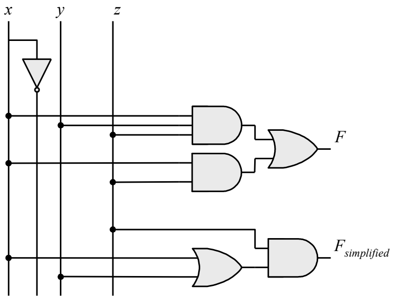

x
y
z
F
$F_{simplified}$

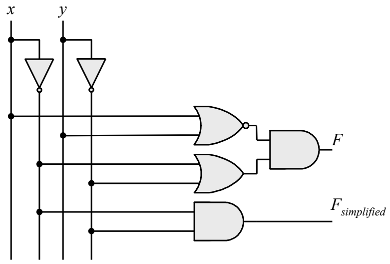
2.6 **(a)**


A
B
C
F
$F_{simplified}$

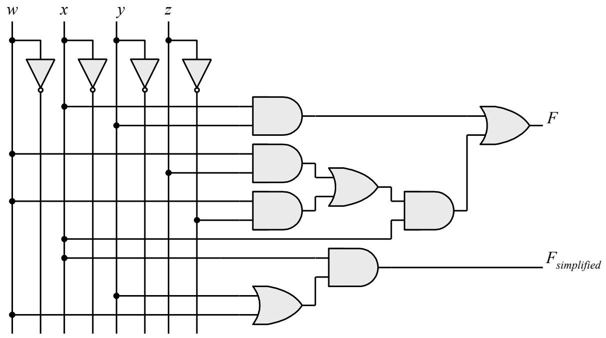
(b)


x
y
z
F
$F_{simplified}$

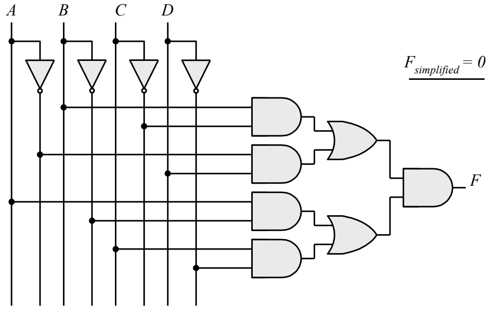
(c)


x
y
F
$F_{simplified}$

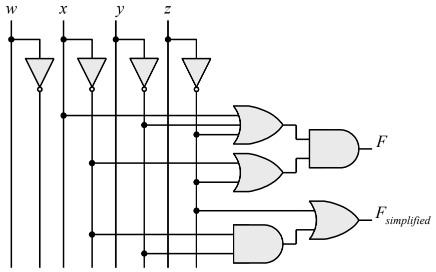
(d)


w
x
y
z
F
$F_{simplified}$

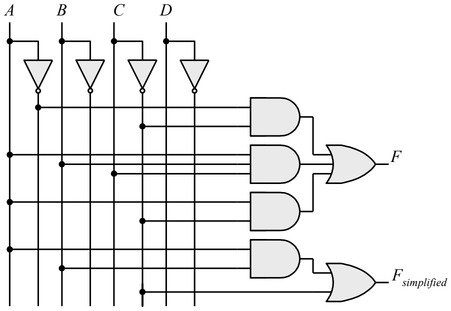
(e)


A
B
C
D
$F_{simplified} = 0$
F

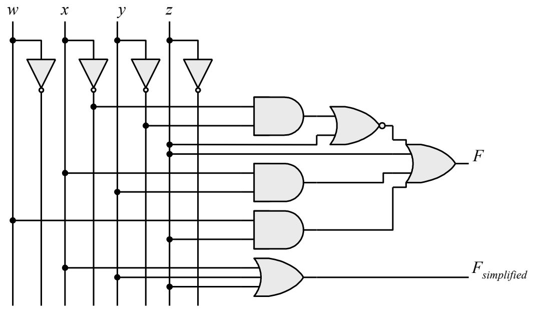
(f)


w
x
y
z
F
$F_{simplified}$

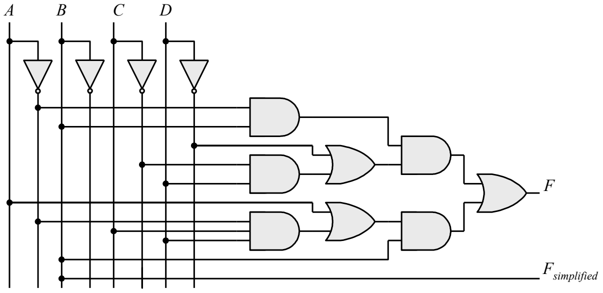
2.7 **(a)**


A
B
C
D
F
$F_{simplified}$

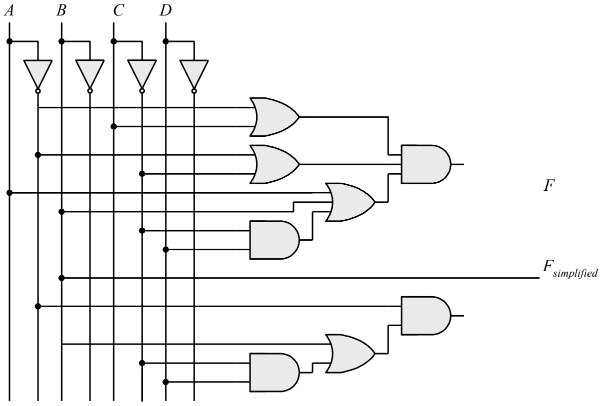
(b)



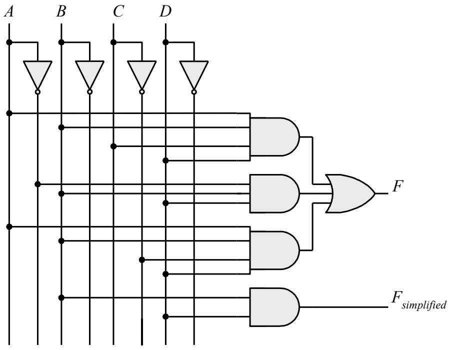
(c)



(d)



(e)



2.8 $F' = (wx + yz)' = (wx)'(yz)' = (w' + x')(y' + z')$

$FF' = wx(w' + x')(y' + z') + yz(w' + x')(y' + z') = 0$
$F + F' = wx + yz + (wx + yz)' = A + A' = 1 \text{ with } A = wx + yz$

2.9 **(a)** $F' = (xy' + x'y)' = (xy')'(x'y)' = (x' + y)(x + y') = xy + x'y'$

(b) $F' = [(a + c)\ (a + b')(a' + b + c')]' = (a + c)' + (a + b')' + (a' + b + c')'$
$=a'c' + a'b + ab'c$

(c) $F' = [z + z'(v'w + xy)]' = z'[z'(v'w + xy)]' = z'[z'v'w + xyz']'$
$= z'[(z'v'w)'(xyz')'] = z'[(z + v + w') +(x' + y' + z)]$
$= z'z + z'v + z'w' + z'x' + z'y' +z' z = z'(v + w' + x' + y')$

2.10 **(a)** $F_1 + F_2 = \Sigma\ m_{1i} + \Sigma m_{2i} = \Sigma\ (m_{1i} + m_{2i})$

(b) $F1\ F2 = \Sigma\ m_i\ \Sigma m_j$ where $m_i\ m_j = 0$ if $i \neq j$ and $m_i\ m_j = 1$ if $i = j$

2.11 **(a)** $F(x, y, z) = \Sigma(1, 4, 5, 6, 7)$

(b) $F(a, b, c) = \Sigma(0, 2, 3, 7)$

$F = xy + xy' + y'z$

x y z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

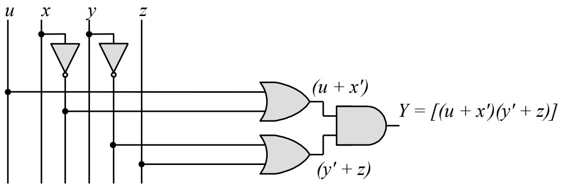
$F = bc + a'c'$

a b c	F
0 0 0	1
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	0
1 1 1	1

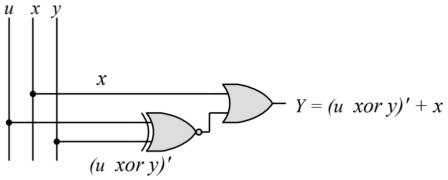
2.12 $A = 1011\_0001$
$B = 1010\_1100$

(a) *A AND B = 1010_0000*
(b) *A OR B = 1011_1101*
(c) *A XOR B = 0001_1101*
(d) *NOT A = 0100_1110*
(e) *NOT B = 0101_0011*

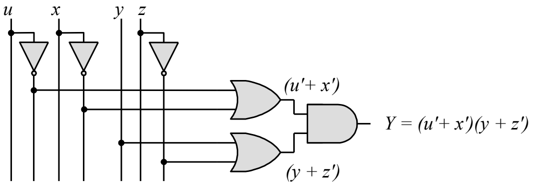
2.13 **(a)**



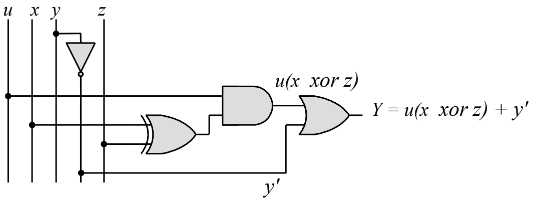
(b)



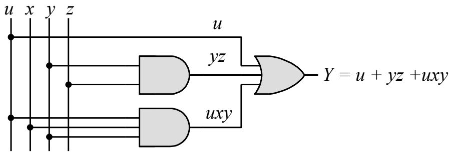
(c)



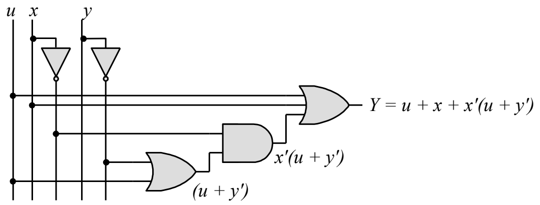
(d)



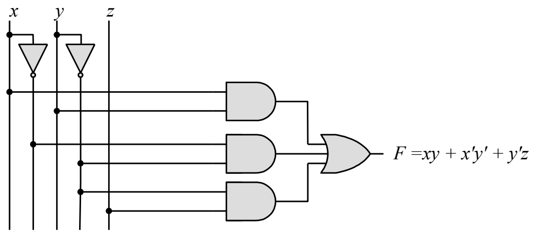
(e)



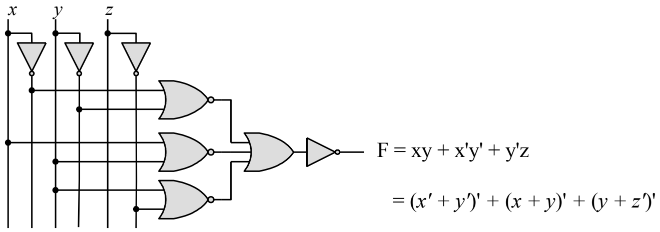
(f)


u
x
y
Y = u + x + x′(u + y′)
x′(u + y′)
(u + y′)

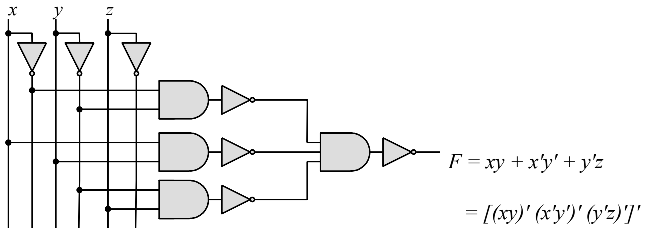
2.14 **(a)**


x
y
z
F =xy + x′y′ + y′z

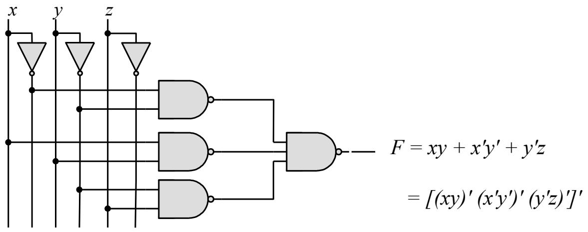
(b)


x
y
z
F = xy + x'y' + y'z
= (x′ + y′)' + (x + y)' + (y + z′)'

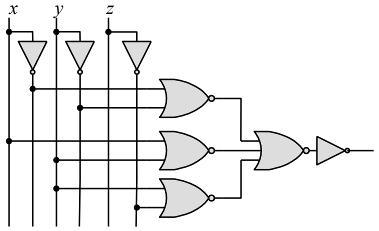
(c)


x
y
z
F = xy + x′y′ + y′z
= [(xy)′ (x′y′)′ (y′z)′]′

(d)


x
y
z
F = xy + x′y′ + y′z
= [(xy)′ (x′y′)′ (y′z)′]′

(e)



$F = xy + x'y' + y'z$

$= (x' + y')' + (x + y)' + (y + z')'$

2.15 **(a)** $T_1 = A'B'C' + A'B'C + A'BC' = A'B'(C' + C) + A'C'(B' + B) = A'B' + A'C' = A'(B' + C')$

(b) $T_2 = T_1' = A'BC + AB'C' + AB'C + ABC' + ABC$

$= BC(A' + A) + AB'(C' + C) + AB(C' + C)$

$= BC + AB' + AB = BC + A(B' + B) = A + BC$

$\sum(3, 5, 6, 7) = \Pi(0, 1, 2, 4)$

$T_1 = A'B'C' + A'B'C + A'BC'$

$A'B' \quad A'C'$

$T_1 = A'B' \ A'C' = A'(B' + C')$

$T_2 = A'BC + AB'C' + AB'C + ABC' + ABC$

$AC' \quad AC$

BC

$T_2 = AC' + BC + AC = A + BC$

2.16 **(a)** $F(A, B, C) = A'B'C' + A'B'C + A'BC' + A'BC + AB'C' + AB'C + ABC' + ABC$

$= A'(B'C' + B'C + BC' + BC) + A((B'C' + B'C + BC' + BC)$

$= (A' + A)(B'C' + B'C + BC' + BC) = B'C' + B'C + BC' + BC$

$= B'(C' + C) + B(C' + C) = B' + B = 1$

(b) $F(x_1, x_2, x_3, \ldots, x_n) = \Sigma m_i$ has $2^n/2$ minterms with x_1 and $2^n/2$ minterms with x'_1, which can be factored and removed as in (a). The remaining 2^{n-1} product terms will have $2^{n-1}/2$ minterms with x_2 and $2^{n-1}/2$ minterms with x'_2, which and be factored to remove x_2 and x'_2. continue this process until the last term is left and $x_n + x'_n = 1$. Alternatively, by induction, F can be written as $F = x_nG + x'_nG$ with $G = 1$. So $F = (x_n + x'_n)G = 1$.

2.17 **(a)** $F = (b + cd)(c + bd) \quad bc + bd + cd + bcd = \Sigma(3, 5, 6, 7, 11, 14, 15)$
$F' = \Sigma(0, 1, 2, 4, 8, 9, 10, 12, 13)$
$F = \Pi(0, 1, 2, 4, 8, 9, 10, 12, 13)$

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(b) $(cd + b'c + bd')(b + d) = bcd + bd' + cd + b'cd = cd + bd'$
$= \Sigma (3, 4, 7, 11, 12,14, 15)$
$= \Pi (0, 1, 2, 5, 6, 8, 9, 10, 13)$

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

(c) $(c' + d)(b + c') = bc' + c' + bd + c'd = (c' + bd)$
$= \Sigma(0, 1, 4, 5, 7, 8, 12, 13, 15)$
$F = \Pi (2, 3, 6, 9, 10, 11, 14)$

(d) $bd' + acd' + ab'c + a'c' = \Sigma(0, 1, 4, 5, 10, 11, 14)$

$F' = \Sigma(2, 3, 6, 7, 8, 9, 12, 13, 15)$

$F = \Pi(02, 3, 6, 7, 8, 12, 13, 15)$

a b c d	F
0 0 0 0	1
0 0 0 1	1
0 0 1 0	0
0 0 1 1	0
0 1 0 0	1
0 1 0 1	1
0 1 1 0	0
0 1 1 1	0
1 0 0 0	0
1 0 0 1	0
1 0 1 0	1
1 0 1 1	1
1 1 0 0	1
1 1 0 1	0
1 1 1 0	1
1 1 1 1	0

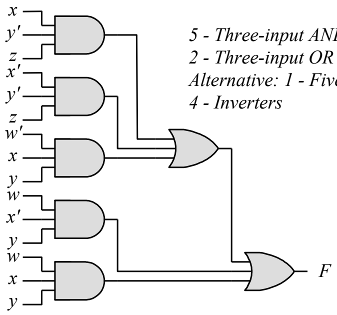
2.18 (a)

wx y z	F
00 0 0	0
00 0 1	1
00 1 0	0
00 1 1	0
01 0 0	0
01 0 1	1
01 1 0	1
01 1 1	1
10 0 0	0
10 0 1	1
10 1 0	1
10 1 1	1
11 0 0	0
11 0 1	1
11 1 0	1
11 1 1	1

$F = xy'z + x'y'z + w'xy + wx'y + wxy$

$F = \Sigma(1, 5, 6, 7, 9, 10 11, 13, 14, 15)$

(b)

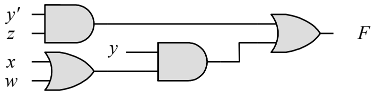


5 - Three-input AND gates
2 - Three-input OR gates
Alternative: 1 - Five-input OR gate
4 - Inverters

(c) $F = xy'z + x'y'z + w'xy + wx'y + wxy = y'z + xy + wy = y'z + y(w + x)$

(d) $F = y'z + yw + yx) = \Sigma(1, 5, 9, 13 , 10, 11, 13, 15, 6, 7, 14, 15)$
$= \Sigma(1, 5, 6, 7, 9, 10, 11, 13, 14, 15)$

(e)



1 – Inverter, 2 – Two-input AND gates, 2 – Two-input OR gates

2.19 $F = B'D + A'D + BD$

ABCD	ABCD	ABCD
-B'-D	A'--D	-B-D
0001 = 1	0001 = 1	0101 = 5
0011 = 3	0011 = 3	0111 = 7
1001 = 9	0101 = 5	1101 = 13
1011 = 11	0111 = 7	1111 = 15

$F = \Sigma(1, 3, 5, 7, 9, 11,13, 15) = \Pi(0, 2, 4, 6, 8, 10, 12, 14)$

2.20 **(a)** $F(A, B, C, D) = \Sigma(2, 4, 7, 10, 12, 14)$
$F'(A, B, C, D) = \Sigma(0, 1, 3, 5, 6, 8, 9, 11, 13, 15)$

(b) $F(x, y, z) = \Pi(3, 5, 7)$
$F' = \Sigma(3, 5, 7)$

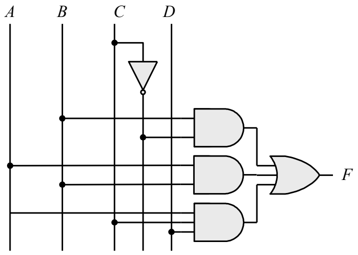
2.21 ***(a)*** $F(x, y, z) = \Sigma(1, 3, 5) = \Pi(0, 2, 4, 6, 7)$

(b) $F(A, B, C, D) = \Pi(3, 5, 8, 11) = \Sigma(0, 1, 2, 4, 6, 7, 9, 10, 12, 13, 14, 15)$

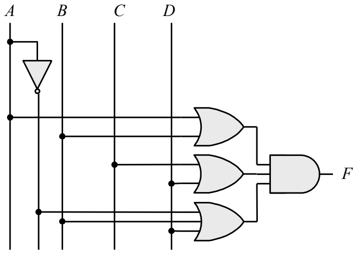
2.22 **(a)** $(u + xw)(x + u'v) = ux + uu'v + xxw + xwu'v = ux + xw + xwu'v$
$= ux + xw = x(u + w)$
$= ux + xw$ (SOP form)
$= x(u + w)$ (POS form)

(b) $x' + x(x + y')(y + z') = x' + x(xy + xz' + y'y + y'z')$
$= x' + xy + xz' + xy'z' = x' + xy + xz'$ *(SOP form)*
$= (x' + y + z')$ *(POS form)*

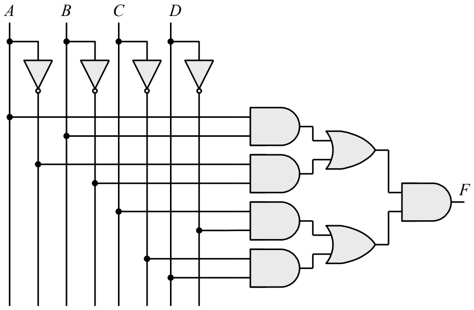
2.23 **(a)** $B'C + AB + ACD$



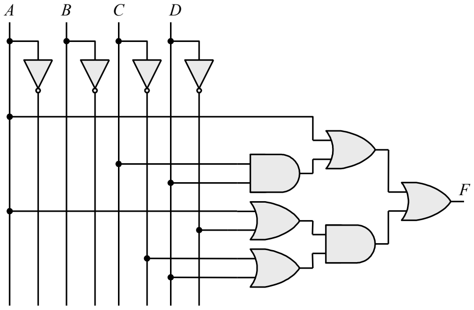
(b) $(A + B)(C + D)(A' + B + D)$



(c) $(AB + A'B')(CD' + C'D)$



(d) $A + CD + (A + D')(C' + D)$



2.24 $x \oplus y = x'y + xy'$ and $(x \oplus y)' = (x + y')(x' + y)$

Dual of $x'y + xy' = (x' + y)(x + y') = (x \oplus y)'$

2.25 **(a)** $x | y = xy' \neq y | x = x'y$ Not commutative
$(x | y) | z = xy'z' \neq x | (y | z) = x(yz')' = xy' + xz$ Not associative

(b) $(x \oplus y) = xy' + x'y = y \oplus x = yx' + y'x$ Commutative

$(x \oplus y) \oplus z = \sum(1, 2, 4, 7) = x \oplus (y \oplus z)$ Associative

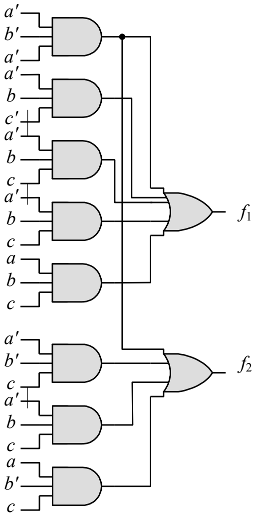
2.26

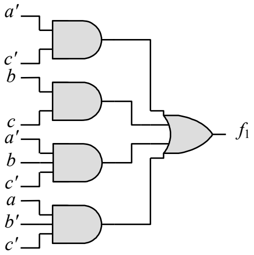
Gate		NAND (Positive logic)		NOR (Negative logic)	
x y	z	x y	z	x y	z
L L	H	0 0	1	1 1	0
L H	H	0 1	1	1 0	0
H L	H	1 0	1	0 1	0
H H	L	1 1	0	0 0	1

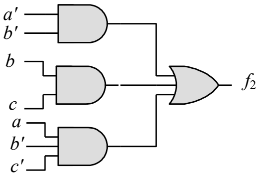
Gate		NOR (Positive logic)		NAND (Negative logic)	
x y	z	x y	z	x y	z
L L	H	0 0	1	1 1	0
L H	L	0 1	0	1 0	1
H L	L	1 0	0	0 1	1
H H	L	1 1	0	0 0	1

2.27 $f_1 = a'b'c' + a'bc' + a'bc + ab'c' + abc = a'c' + bc + a'bc' + ab'c'$

$f_2 = a'b'c' + a'b'c + a'bc + ab'c' + abc = a'b' + bc + ab'c'$







2.28 **(a)** $y = a(bcd)'e = a(b' + c' + d')e$

$$y = a(b' + c' + d')e = ab'e + ac'e + ad'e$$
$$= \Sigma(17, 19, 21, 23, 25, 27, 29)$$

a bcde	y	a bcde	y
0 0000	0	1 0000	0
0 0001	0	**1 0001**	1
0 0010	0	1 0010	0
0 0011	0	**1 0011**	1
0 0100	0	1 0100	0
0 0101	0	**1 0101**	1
0 0110	0	1 0110	0
0 0111	0	**1 0111**	1
	0		0
0 1000	0	1 1000	0
0 1001	0	**1 1001**	1
0 1010	0	1 1010	0
0 1011	0	**1 1011**	1
0 1100	0	1 1100	0
0 1101	0	**1 1101**	1
0 1110	0	1 1110	0
0 1111	0	1 1111	0

(b) $y_1 = a \oplus (c + d + e)= a'(c + d +e) + a(c'd'e') = a'c + a'd + a'e + ac'd'e'$

$y_2 = b'(c + d + e)f = b'cf + b'df + b'ef$

$y_1 = a\ \ (c + d + e) = a'(c + d +e) + a(c'd'e') = a'c + a'd + a'e + ac'd'e'$

$y_2 = b'(c + d + e)f = b'cf + b'df + b'ef$

a'-c---
001000 = 8
001001 = 9
001010 = 10
001011 = 11

001100 = 12
001101 = 13
001110 = 14
001111 = 15

011000 = 24
011001 = 25
011010 = 26
011011 = 27

011100 = 28
011101 = 29
011110 = 30
011111 = 31

a'--d--
000100 = 8
000101 = 9
000110 = 10
000111 = 11

001100 = 12
001101 = 13
001110 = 14
001111 = 15

010100 = 20
010101 = 21
010110 = 22
010111 = 23

011100 = 28
011101 = 29
011110 = 30
011111 = 31

a'---e-
000010 = 2
000011 = 3
000110 = 6
000111 = 7

001010 = 10
001011 = 11
001110 = 14
001111 = 15

010010 = 18
010011 = 19
010110 = 22
010111 = 23

011010 = 26
011001 = 27
011110 = 30
011111 = 31

a-c'd'e'-
100000 = 32
100001 = 33
110000 = 34
110001 = 35

-b' c--f

001001 = 9
001011 = 11
001101 = 13
001111 = 15
101001 = 41
101011 = 43
101101 = 45
101111 = 47

-b' -d-f

001001 = 9
001011 = 11
001101 = 13
001111 = 15
101001 = 41
101011 = 43
101101 = 45
101111 = 47

-b' --ef

000011 = 3
000111 = 7
001011 = 11
001111 = 15
100011 = 35
100111 = 39
101011 = 51
101111 = 55

$y_1 = \Sigma$ (2, 3, 6, 7, 8, 9, 10 ,11, 12, 13, 14, 15, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35)

$y_2 = \Sigma$ (3, 7, 9, 13, 15, 35, 39, 41, 43, 45, 47, 51, 55)

ab cdef	y_1 y_2	*ab cdef*	y_1 y_2	*ab cdef*	y_1 y_2	*ab cdef*	y_1 y_2
00 0000	0 0	01 0000	0 0	10 0000	1 0	11 0000	0 0
00 0001	0 0	01 0001	0 0	10 0001	1 0	11 0001	0 0
00 0010	1 0	01 0010	1 0	10 0010	1 0	11 0010	0 0
00 0011	1 1	01 0011	1 0	10 0011	1 1	11 0011	0 1
00 0100	0 0	01 0100	0 0	10 0100	0 0	11 0100	0 0
00 0101	0 0	01 0101	0 0	10 0101	0 0	11 0101	0 0
00 0110	1 0	01 0110	1 0	10 0110	0 0	11 0110	0 0
00 0111	1 1	01 0111	1 0	10 0111	0 1	11 0111	0 1
00 1000	1 0	01 1000	1 0	10 1000	0 0	11 1000	0 0
00 1001	1 1	01 1001	1 0	10 1001	0 1	11 1001	0 0
00 1010	1 0	01 1010	1 0	10 1010	0 0	11 1010	0 0
00 1011	1 0	01 1011	1 0	10 1011	0 1	11 1011	0 0
00 1100	1 0	01 1100	1 0	10 1100	0 0	11 1100	0 0
00 1101	1 1	01 1101	1 0	10 1101	0 1	11 1101	0 0
00 1110	1 0	01 1110	1 0	10 1110	0 0	11 1110	0 0
00 1111	1 1	01 1111	1 0	10 1111	0 1	11 1111	0 0