15. Proof 1. $\binom{n}{k}\binom{n-k}{j} = \frac{n!(n-k)!}{k!(n-k)!j!(n-k-j!)} = \frac{n!}{k!j!(n-k-j)!}$. Similarly, $\binom{n}{k}\binom{n-k}{j}$ equals the same expression.

    Proof 2. There are two ways of choosing two disjoint sets, one with k-elements and one with j-elements. We can pick the k element set first, then choose j elements from what is left, or we can pick the j element set first, then choose k elements from what is left.

16.

$n\backslash k$	3	4	5	6
6	20	15	6	1
7		35	21	7
8			56	28
9				84

17. The formula is simply the expansion of $(1-1)^n$.

18. $(1+x)^n = \sum_{i=0}^{n}\binom{n}{i}x^i$. Taking the derivative of both sides, we get

$$n(1+x)^{n-1} = \sum_{i=1}^{n} i\binom{n}{i}x^{i-1}\ .$$

    Thus, if we let $x=1$, we have $n2^{n-1} = \sum_{i=1}^{n} i\binom{n}{i}$.

19. False. $\binom{4}{2}$ is 6, but $\binom{2}{0}+\binom{2}{1}+\binom{2}{2}$ is 4. The correct statement is

$$\binom{n}{k} = \binom{n-2}{k-2} + 2\binom{n-2}{k-1} + \binom{n-2}{k}\ .$$

    The proof consists of applying the Pascal relationship to both $\binom{n-1}{k-1}$ and $\binom{n-1}{k}$ and adding the results.

1.4 Relations

1. If f is one-to-one then no two ordered pairs in R can have the same second element. If f is onto then the set of second elements of the ordered pairs in R must be equal to the range T of the function.
2. (a) No
   (b) Yes
   (c) No
3. The relation is reflexive ($x^2 = x^2$), symmetric ($x^2 = y^2 \Rightarrow y^2 = x^2$), and transitive ($x^2 = y^2$ and $y^2 = z^2 \Rightarrow x^2 = z^2$). The equivalence classes are the sets $\{k, -k\}$ for each integer k. When $k = 0$, the set is $\{0\}$.