Solutions Manual for DSP First
Second Edition

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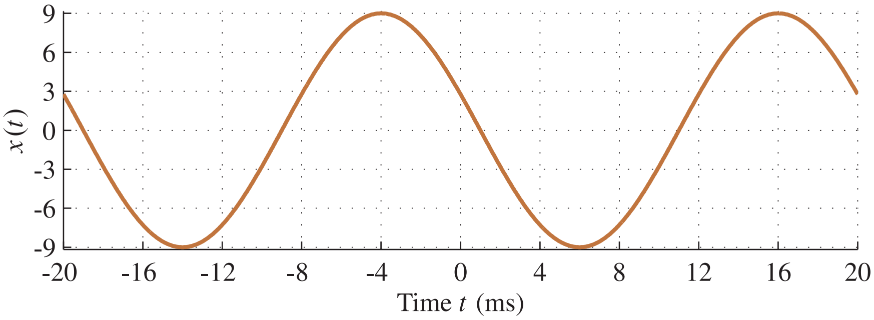
CHAPTER 2

Sinusoids

2-1 Problems

P-2.1 **DSP First 2e**

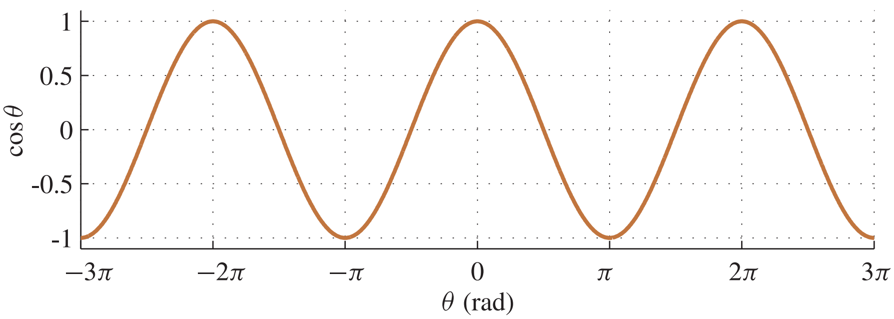


P-2.2 **DSP First 2e**

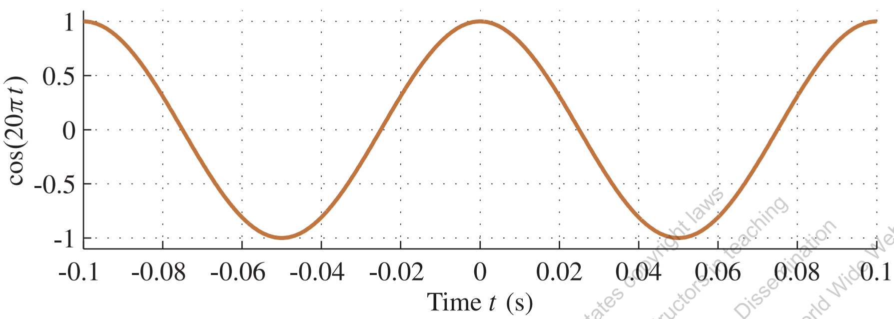
In the plot the period can be measured, $T = 12.5$ ms $\Rightarrow\ \omega_0 = 2\pi/(12.5\times10^{-3}) = 2\pi(80)$ rad.
Positive peak closest to $t = 0$ is at $t_1 = 2.5$ ms $\Rightarrow\ \varphi = -2\pi(2.5\times10^{-3})/(12.5\times10^{-3}) = 2\pi/5 = -0.4\pi$ rad.
Amplitude is $A = 8$.
$x(t) = 8\cos(160\pi t - 0.4\pi)$

P-2.3 **DSP First 2e**

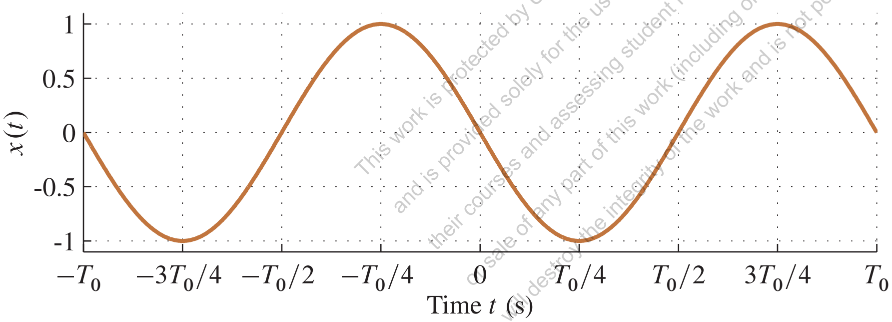
(a) Plot of $\cos\theta$



(b) Plot of $\cos(20\pi t)$



(c) Plot of $\cos(2\pi/T_0 + \pi/2)$



P-2.4 **DSP First 2e**

$$
\begin{aligned}
e^{j\theta} &= 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \cdots \\
&= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + \cdots \\
&= \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\right)}_{\cos\theta} + j\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots\right)}_{\sin\theta}
\end{aligned}
$$

Thus, $e^{j\theta} = \cos\theta + j\sin\theta$

P-2.5 **DSP First 2e**

(a) Real part of complex exponential is cosine.

$$\begin{aligned}\cos(\theta_1+\theta_2) &= \Re\left\{e^{j(\theta_1+\theta_2)}\right\} = \Re\left\{e^{j\theta_1}e^{j\theta_2}\right\} \\ &= \Re\left\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)\right\} \\ &= \Re\left\{(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + j(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)\right\}\end{aligned}$$

$$\cos(\theta_1+\theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

(b) Change the sign of θ_2.

$$\begin{aligned}\cos(\theta_1-\theta_2) &= \Re\left\{e^{j(\theta_1-\theta_2)}\right\} = \Re\left\{e^{j\theta_1}e^{-j\theta_2}\right\} \\ &= \Re\left\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)\right\} \\ &= \Re\left\{(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + j(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)\right\}\end{aligned}$$

$$\cos(\theta_1-\theta_2) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$$

P-2.6 **DSP First 2e**

$$(\cos\theta + j\sin\theta)^n = \left(e^{j\theta}\right)^n = e^{jn\theta} = \cos(n\theta) + j\sin(n\theta)$$

$$\begin{aligned}
\left(\tfrac{3}{5} + j\tfrac{4}{5}\right)^n &= \left(e^{j0.927}\right)^{100} = \left(e^{j0.295167\pi}\right)^{100} \\
&= e^{j29.5167\pi} \\
&= e^{j1.5167\pi}\cancelto{1}{e^{j28\pi}} \\
&= \cos(1.5167) + j\sin(1.5167) \\
&= 0.0525 - j0.9986
\end{aligned}$$

P-2.7 **DSP First 2e**

(a) $3e^{j\pi/3} + 4e^{-j\pi/6} = 5e^{j0.12} = 4.9641 + j0.5981$

(b) $(\sqrt{3} - j3)^{10} = \left(\sqrt{12}e^{-j\pi/3}\right)^{10} = 248,832\,\underbrace{e^{-j10\pi/3}}_{e^{+j2\pi/3}} = -124,416 + j215,494.83$

(c) $(\sqrt{3} - j3)^{-1} = \left(\sqrt{12}e^{-j\pi/3}\right)^{-1} = (1/\sqrt{12})e^{+j\pi/3} = 0.2887e^{+j\pi/3} = 0.14434 + j0.25$

(d) $(\sqrt{3} - j3)^{1/3} = \left(\sqrt{12}e^{-j\pi/3}e^{j2\pi\ell}\right)^{1/3} = \left((12)^{1/6}e^{-j\pi/9}e^{j2\pi\ell/3}\right)$ for $\ell = 0, 1, 2$.
There are 3 answers:
$1.513e^{-j\pi/9} = 1.422 - j0.5175$
$1.513e^{-j7\pi/9} = -1.159 - j0.9726$
$1.513e^{-j13\pi/9} = 1.513e^{+j5\pi/9} = -0.2627 + j1.49$

(e) $\Re\left\{je^{-j\pi/3}\right\} = \Re\left\{e^{j\pi/2}e^{-j\pi/3}\right\} = \Re\left\{e^{j\pi/6}\right\} = \cos(\pi/6) = \sqrt{3}/2 = 0.866$

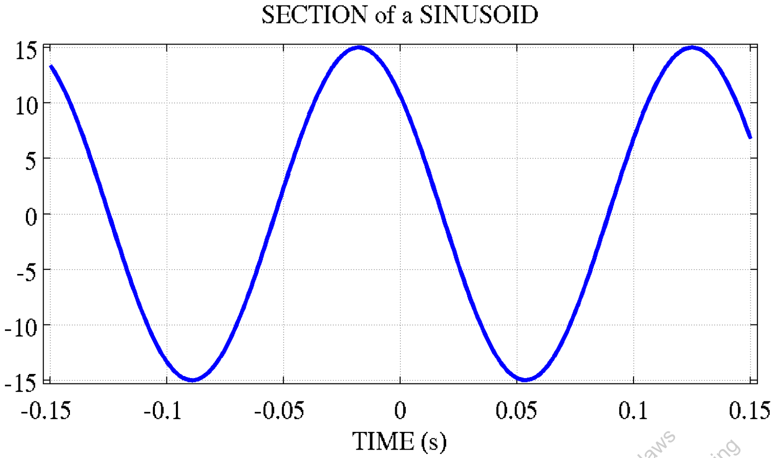
P-2.8 **DSP First 2e**

The variable `zz` defines $z(t)$, and `xx` defines $x(t) = \Re\{z(t)\}$.

$z(t) = 15e^{j(2\pi(7)(t+0.875))} \quad \Rightarrow \quad x(t) = 15\cos(2\pi(7)(t+0.875))$

The period of $x(t)$ is $1/7 = 0.1429$, so the time interval $-0.15 \le t \le 0.15$ is $(0.3)(7) = 2.1$ periods.

There will be positive peaks of the cosine wave at $t = -0.1607$ s and $t = -0.0179$ s.



P-2.9 **DSP First 2e**

$A = 9$

$T = 8 \times 10^{-3}\text{ s} \;\Rightarrow\; \omega_0 = 2000\pi/8 = 250\pi\text{ rad/s}$

$t_1 = -3 \times 10^{-3}\text{ s} \;\Rightarrow\; \varphi = -2\pi(-3/8) = 3\pi/4\text{ rad}$

$z(t) = 9e^{j(250\pi t + 0.75\pi)}$, $X = 9e^{j0.75\pi}$, and $x(t) = 9\cos(250\pi t + 0.75\pi)$

(a) Add complex amps: $3e^{-j2\pi/3} + 1 = 2.646e^{-j1.761} \quad \Rightarrow x(t) = 2.646\cos(\omega_0 t - 1.761)$

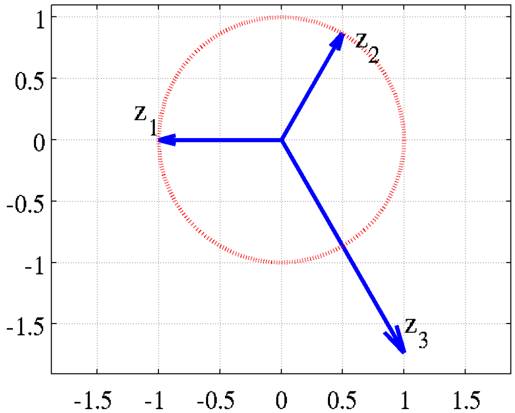
(b) $x(t) = \Re\{z(t)\} = \Re\{2.646e^{-j1.761}e^{j\omega_0 t}\}$

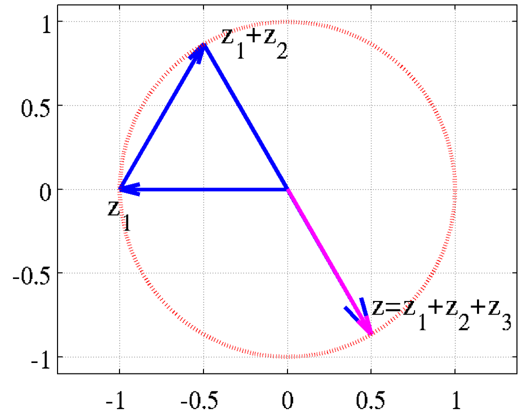
P-2.11 **DSP First 2e**

Add complex amps: $e^{-j\pi} + e^{j\pi/3} + 2e^{-j\pi/3} = \underbrace{e^{-j\pi} + e^{j\pi/3} + e^{-j\pi/3}}_{=0} + e^{-j\pi/3} = e^{-j\pi/3}$

$\Rightarrow x(t) = \cos(\omega t - \pi/3)$

Here is the MATLAB plot of the vectors.





P-2.12 **DSP First 2e**

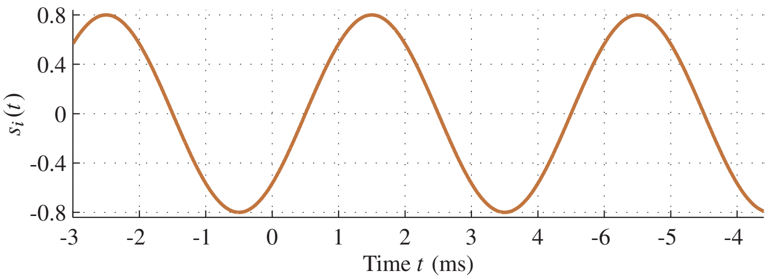
Find angles satisfying $-\pi < \theta \leq \pi$; all others are obtained by adding integer multiples of 2π.

$$
\begin{aligned}
\Re\{(1+j)e^{j\theta}\} &= 0 \\
\Re\{\sqrt{2}e^{j\pi/4}e^{j\theta}\} &= 0 \\
\Re\{\sqrt{2}e^{j(\theta+\pi/4)}\} &= 0 \\
\sqrt{2}\cos(\theta+\pi/4) &= 0 \\
\Rightarrow \quad \theta+\pi/4 = \begin{cases}\pi/2 \\ -\pi/2\end{cases} \qquad &\Rightarrow \quad \theta = \begin{cases}\pi/4 \\ -3\pi/4\end{cases} \qquad \Rightarrow \quad e^{j\theta} = \begin{cases}(1+j)/\sqrt{2} \\ (-1-j)/\sqrt{2}\end{cases}
\end{aligned}
$$

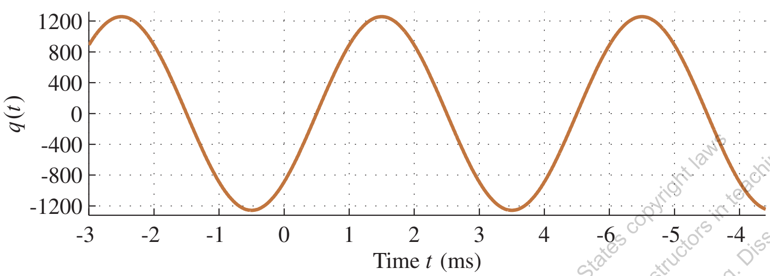
P-2.13 **DSP First 2e**

Three periods of the signal will be $3(1/250) = 12$ ms.

(a) Plot $s_i(t) = \Re\{js(t)\} = \Re\{0.8e^{j\pi/2}e^{j\pi/4}e^{j500\pi t}\} = 0.8\cos(2\pi(250)t + 3\pi/4)$.



(b) Plot $q(t) = \Re\{\frac{d}{dt}s(t)\} = \Re\{0.8e^{j\pi/4}(j500\pi)e^{j500\pi t}\} = \Re\{400\pi e^{j3\pi/4}e^{j500\pi t}\} = 400\pi\cos(500\pi t + 3\pi/4)$



P-2.14 **DSP First 2e**

(a) If $z_1(t) = \sqrt{5}e^{-j\pi/3}e^{j7t}$ then $x_1(t) = \Re\{z_1(t)\}$.

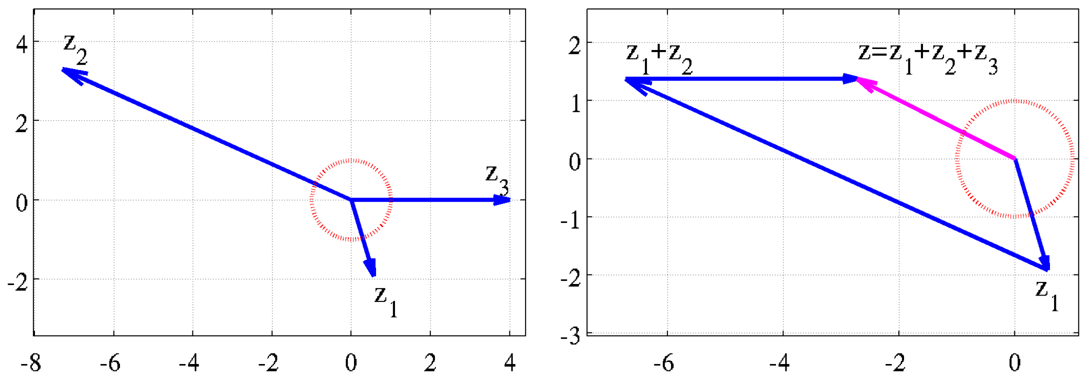
(b) If $z_2(t) = \sqrt{5}e^{j\pi}e^{j7t}$ then $x_2(t) = \Re\{z_2(t)\}$.

(c) If $z(t) = z_1(t) + z_2(t) = \sqrt{5}e^{j7t}\left(e^{-j\pi/3} + e^{j\pi}\right) = \sqrt{5}e^{-j2\pi/3}e^{j7t}$, then $x(t) = \Re\{z(t)\} = \sqrt{5}\cos(7t - 2\pi/3)$.

P-2.15 **DSP First 2e**

Need to add complex amps: $2e^{j5} + 8e^{j9} + 4e^{j0} = 3.051e^{j2.673}$

Here is the plot of vectors representing the complex amplitudes:



P-2.16 **DSP First 2e**

(a) $\varphi = -2\pi\dfrac{t_1}{T} = -2\pi\dfrac{(-2)}{8} = \dfrac{4\pi}{8} = \dfrac{\pi}{2} \quad \Rightarrow$ True.

(b) $\varphi = -2\pi\dfrac{t_1}{T} = -2\pi\dfrac{3}{8} = -\dfrac{3\pi}{4} \quad \Rightarrow$ False.

(c) In this case a multiple of 2π must be added.
$\varphi = -2\pi\dfrac{t_1}{T} = -2\pi\dfrac{7}{8} = \dfrac{-7\pi}{4} \rightarrow \dfrac{-7\pi}{4} + 2\pi = \dfrac{\pi}{4} \quad \Rightarrow$ True.

P-2.17 **DSP First 2e**

(a) Need to plot five vectors: $\{1, e^{j2\pi/5}, e^{j4\pi/5}, e^{j6\pi/5}, e^{j8\pi/5}\}$.

Note: one is NOT missing; these are the five "5$^{\text{th}}$ roots of unity."

(b) The sum is zero: $x(t) = \sum_{k=0}^{4} \cos(\omega t + \frac{2}{5}\pi k) = 0$.

If the upper limit were 3 instead of 4,

then $x(t) = \sum_{k=0}^{3} \cos(\omega t + \frac{2}{5}\pi k) = x(t) = \sum_{k=0}^{4} \cos(\omega t + \frac{2}{5}\pi k) - \cos(\omega t + \frac{8}{5}\pi) = -\cos(\omega t + \frac{8}{5}\pi)$

P-2.18 **DSP First 2e**

(a) Inverse Euler formula:

$\omega = 8$ rad/s, $A = 9/2$, $\varphi = -2\pi/3$

(b) 30-60-90 triangle:

$\omega = 9$ rad/s, $\varphi = 0$, $A = 8.66$

P-2.19 **DSP First 2e**

(a) $9e^{j0.5} = 3Ae^{j(-2+\varphi)} + 4$

(b) $9e^{j0.5} = 3\underbrace{Ae^{j\varphi}}_{z}e^{-j2} + 4$

(c) $z = \dfrac{9e^{j0.5} - 4}{3e^{-j2}} = (1/3)e^{j2}\left(9e^{j0.5} - 4\right) = 3e^{j2.5} - (4/3)e^{j2} = 1.938e^{j2.836}$

(d) $A = 1.938$ and $\varphi = 2.836$

P-2.20 **DSP First 2e**

(a) Convert to complex amplitudes (phasors):

$$\begin{aligned} 1 &= A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2} \\ e^{-j\pi/2} &= 2A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2} \end{aligned}$$

(b) Write complex amplitudes as z_1 and z_2:

$$\begin{aligned} 1 &= z_1 + z_2 \\ e^{-j\pi/2} &= 2z_1 + z_2 \end{aligned}$$

(c) $z_1 = e^{-j\pi/2} - 1 = \sqrt{2}e^{-j3\pi/4}$ and $z_2 = 2 - e^{-j\pi/2} = 2.236e^{j0.464}$

(d) $A_1 = \sqrt{2}$, $\varphi_1 = -0.75\pi$ rad, and $A_2 = 2.236 = \sqrt{5}$, $\varphi_2 = 0.148\pi = 0.464$ rad

P-2.21 DSP First 2e

(a) Convert to complex amplitudes (phasors):

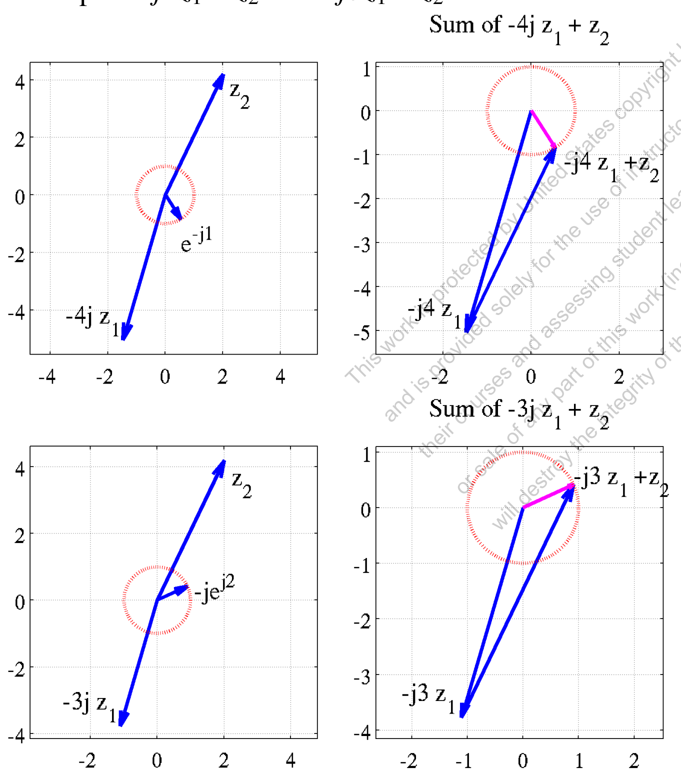
$$e^{-j1} = 4e^{-j\pi/2}A_1e^{j\varphi_1} + A_2e^{j\varphi_2}$$
$$e^{-j\pi/2+j2} = 3e^{-j\pi/2}A_1e^{j\varphi_1} + A_2e^{j\varphi_2}$$

$$e^{-j1} = 4e^{-j\pi/2}z_1 + z_2$$
$$e^{-j\pi/2+j2} = 3e^{-j\pi/2}z_1 + z_2$$

$$z_1 = 1.2576 - j0.3690 = 1.311e^{-j0.285}$$
$$z_2 = 2.0163 + j4.1890 = 4.649e^{j1.122}$$

$$A_1 = 1.311, \quad \varphi_1 = -0.285\,\text{rad}$$
$$A_2 = 4.649, \quad \varphi_2 = 1.122\,\text{rad}$$

(b) Should plot $-j4z_1 + z_2$ and $-j3z_1 + z_2$. Here is the MATLAB plot of the vectors.



P-2.22 **DSP First 2e**

Convert to phasors (complex amps): $Me^{j\pi/3} = 5e^{j\psi} - 4$

The lefthand side is a ray from the origin at the angle of $\pi/3$ rad, or 60° when $M > 0$; and at $-2\pi/3$ when $M < 0$.

The righthand side is the set $\{z : z = 5e^{j\psi} - 4\}$ which is a circle of radius 5 centered at $z = -4 + j0$. Since the origin is inside the circle, there must be two solutions.

$$\text{For } M > 0\text{, ray at } \pi/3\text{:}\quad M = 5e^{j(\psi-\pi/3)} - 4e^{-j\pi/3} \quad \text{must be purely real}$$
$$0 = \Im\{5e^{j(\psi-\pi/3)} - 4e^{-j\pi/3}\} = 5\sin(\psi - \pi/3) - 4(-\sqrt{3}/2)$$
$$\Rightarrow \sin(\psi - \pi/3) = -2\sqrt{3}/5 \;\Rightarrow\; \psi - \pi/3 = -0.7654 \;\Rightarrow\; \psi = 0.2818 \text{ (or } 16.1458^\circ)$$
$$\text{Then solve for } M \text{ via : } \Im\{Me^{j\pi/3} = 5e^{j\psi} - 4\}$$
$$\Rightarrow\; M(\sqrt{3}/2) = 5\sin\psi \;\Rightarrow\; M = (10/\sqrt{3})\sin\psi \;\Rightarrow\; M = 1.6056$$

$$\text{For } M < 0\text{, ray at } -2\pi/3\text{:}\quad M = 5e^{j(\psi+2\pi/3)} - 4e^{j2\pi/3} \quad \text{must be purely real}$$
$$0 = \Im\{5e^{j(\psi+2\pi/3)} - 4e^{j2\pi/3}\} = 5\sin(\psi + 2\pi/3) - 4(\sqrt{3}/2)$$
$$\Rightarrow \sin(\psi + 2\pi/3) = 2\sqrt{3}/5 \;\Rightarrow\; \psi + 2\pi/3 = 0.7654 \;\Rightarrow\; \psi = -1.329 \text{ (or } -76.146^\circ)$$
$$\text{Then solve for } M \text{ via : } \Im\{Me^{-j2\pi/3} = 5e^{j\psi} - 4\}$$
$$\Rightarrow\; M(-\sqrt{3}/2) = 5\sin\psi \;\Rightarrow\; M = (-10/\sqrt{3})\sin\psi \;\Rightarrow\; M = 5.6056$$

Another way to obtain M follows:

$$\begin{aligned}
Me^{j\pi/3} &= 5e^{j\psi} - 4 \\
\Rightarrow\; Me^{j\pi/3} + 4 &= 5e^{j\psi} \\
\Rightarrow\; |Me^{j\pi/3} + 4|^2 &= |5e^{j\psi}|^2 = 25 \\
M^2 + 8M\cos(\pi/3) + 16 &= 25 \\
M^2 + 4M - 9 &= 0 \quad \text{which has two roots: } M = 5.6056 \text{ and } M = 1.6056.
\end{aligned}$$

P-2.23 **DSP First 2e**

(a) $z(t-0.24) = Ze^{j10\pi(t-0.24)} = 7e^{j0.3\pi}e^{j10\pi t}e^{-j2.4\pi} = \underbrace{7e^{-j2.1\pi}}e^{j10\pi t} = \underbrace{7e^{-j0.1\pi}}_{W}e^{j10\pi t}$

(b) $z(t-t_d) = Ze^{j10\pi(t-t_d)} = 7e^{j0.3\pi}e^{j10\pi t}e^{-j10\pi t_d}$ must equal $y(t) = Ye^{j10\pi t} = 7e^{-j0.1\pi}e^{j10\pi t}$

$\Rightarrow 7e^{j0.3\pi - j10\pi t_d} = 7e^{-j0.1\pi} \quad \Rightarrow \quad 0.3\pi - 10\pi t_d = -0.1\pi \quad \Rightarrow \quad t_d = (0.4/10) = 0.04\text{ s}$

P-2.24 **DSP First 2e**

(a) The frequency is the same for all terms, so $\hat{\omega}_0 = 0.22\pi$ rad in the expression for $y[n]$.

(b) Perform phasor addition:

$$\begin{aligned} y[n] &= 7e^{j(0.22\pi(n+1)-0.25\pi)} - 14e^{j(0.22\pi n-0.25\pi)} + 7e^{j(0.22\pi(n-1)-0.25\pi)} \\ &= 7e^{j(0.22\pi n-0.03\pi)} - 14e^{j(0.22\pi n-0.25\pi)} + 7e^{j(0.22\pi n-0.47\pi)} \\ &= \underbrace{\left(7e^{-j0.03\pi} - 14e^{-j0.25\pi} + 7e^{-j0.47\pi}\right)}_{\text{Phasor Addition}} e^{j0.22\pi n} \\ &= 3.213\, e^{j0.75\pi} e^{j0.22\pi n} \quad \Rightarrow \quad A = 3.213, \quad \varphi = 0.75\pi \text{ rad} \end{aligned}$$

P-2.25 **DSP First 2e**

(a) $\frac{d}{dt}z(t) = \frac{d}{dt}Ze^{j2\pi t} = \underbrace{(j2\pi)Z}_{Q}\,e^{j2\pi t} \quad\Rightarrow\quad Q = (j2\pi)(e^{j\pi/4}) = 2\pi e^{j3\pi/4}$

(b) Need a plot. Angle of Q is greater by $\pi/2$ rad.

(c) Compare the interchange of derivative and real part, which is always true.

$$\Re\left\{\tfrac{d}{dt}z(t)\right\} = \Re\{2\pi e^{j3\pi/4}e^{j2\pi t}\} = 2\pi\cos(2\pi t + 3\pi/4)$$
$$\frac{d}{dt}\Re\{z(t)\} = \frac{d}{dt}\Re\left\{e^{j\pi/4}e^{j2\pi t}\right\} = \frac{d}{dt}\cos(2\pi t + \pi/4) = (2\pi)(-\sin(2\pi t + \pi/4)) = 2\pi\cos(2\pi t + 3\pi/4)$$

(d) Integrating a complex exponential over one period should give zero.

$$\int_{-0.5}^{0.5} e^{j\pi/4}e^{j2\pi t}\,dt = \left.\frac{e^{j\pi/4}e^{j2\pi t}}{j2\pi}\right|_{-0.5}^{0.5} = e^{j\pi/4}\,\frac{e^{j\pi} - e^{-j\pi}}{j2\pi} = 0$$

P-2.26 **DSP First 2e**

Try $x(t) = Ae^{j\omega t}$ and solve for ω

$$\frac{d}{dt}x(t) = j\omega Ae^{j\omega t} \quad \text{and} \quad \frac{d^2}{dt^2}x(t) = \underbrace{(j\omega)^2}_{-\omega^2} Ae^{j\omega t}$$

Plug $x(t)$ into differential equation

$$-\omega^2 Ae^{j\omega t} = -289\, Ae^{j\omega t}$$

$$\Rightarrow \ -\omega^2 = -289 \quad \Rightarrow \ \omega = \pm 17$$

Two solutions: $x(t) = Ae^{j17t}$ or $x(t) = Ae^{-j17t}$

P-2.27 **DSP First 2e**

(a) $v(t) = -L\dfrac{d}{dt}i(t) = -L\dfrac{d}{dt}\left(C\dfrac{dv}{dt}\right) = -LC\dfrac{d^2v(t)}{dt^2} \quad\Rightarrow\quad \dfrac{d^2v(t)}{dt^2} = -\dfrac{1}{LC}v(t)$

(b) The frequency of oscillation will be $\omega_0 = \dfrac{1}{\sqrt{LC}}$

(c) Starting with $v(t) = A\cos(\omega_0 t + \varphi)$, we obtain $\dfrac{d^2v(t)}{dt^2} = -\underbrace{\omega_0^2}_{1/LC}\underbrace{A\cos(\omega_0 t + \varphi)}_{v(t)} = -\dfrac{1}{LC}v(t)$

(d) $v(t) = 5\cos(\omega_0 t + \pi/3) \quad\Rightarrow\quad i(t) = C\frac{dv}{dt} = 5C\omega_0\sin(\omega_0 t + \pi/3) = 5C\omega_0\cos(\omega_0 t + \pi/3 - \pi/2)$

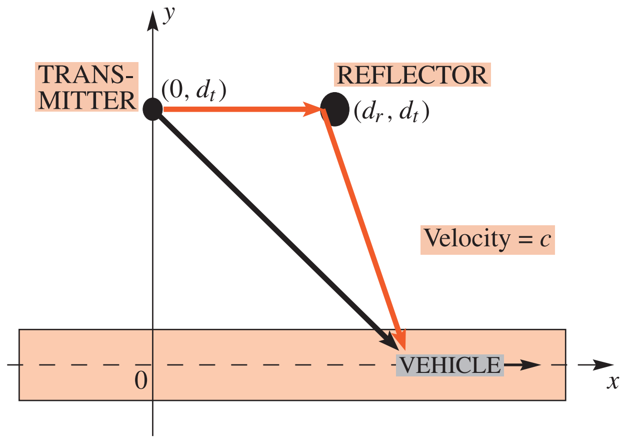
There is a 90° phase difference between the current and the voltage.

(e) This is true in general:

$$i(t) = C\frac{d}{dt}v(t) = C\frac{d}{dt}\left(-L\frac{di}{dt}\right) = -LC\frac{d^2i(t)}{dt^2} \quad\Rightarrow\quad \frac{d^2i(t)}{dt^2} = -\frac{1}{LC}i(t)$$

P-2.28 **DSP First 2e**

In a mobile radio system a transmitting tower sends a sinusoidal signal, and a mobile user receives not one but two copies of the transmitted signal: a direct-path transmission and a reflected-path signal (e.g., from a large building) as depicted in the following figure.



The received signal is the sum of the two copies, and since they travel different distances they have different time delays, i.e.,

$$r(t) = s(t - t_1) + s(t - t_2)$$

The distance between the mobile user in the vehicle at x and the transmitting tower is always changing. Suppose that the direct-path distance is

$$d_1 = \sqrt{x^2 + d_t^2} \quad \text{(meters)}$$

where $d_t = 1000$ meters, and where x is the position of the vehicle moving along the x-axis. Assume that the reflected-path distance is

$$d_2 = d_r + \sqrt{(x - d_r)^2 + d_t^2} \quad \text{(meters)}$$

where $d_r = 55$ meters.

(a) The amount of the delay (in seconds) can be computed for both propagation paths, by converting distance into time delay by dividing by the speed of light ($c = 3\times10^8$ m/s).

$$t_1 = d_1/c = \frac{\sqrt{x^2 + d_t^2}}{c} = \frac{\sqrt{x^2 + 10^6}}{3\times10^8} \quad \text{secs.}$$

$$t_2 = d_2/c = \frac{d_r + \sqrt{(x - d_r)^2 + d_t^2}}{c} = \frac{55 + \sqrt{(x - 55)^2 + 10^6}}{3\times10^8} \quad \text{secs.}$$

(b) When the transmitted signal is $s(t) = \cos(300\pi\times10^6 t)$, the general formula for the received signal is:

$$r(t) = s(t - t_1) + s(t - t_2) = \cos(300\pi\times10^6(t - t_1)) + \cos(300\pi\times10^6(t - t_2))$$

When $x = 0$ we can calculate t_1 and t_2, and then perform a phasor addition to express $r(t)$ as a sinusoid with a known amplitude, phase, and frequency. When $x = 0$, the time delays are

$$t_1 = \frac{\sqrt{0^2 + 10^6}}{3\times 10^8} = 3.3333\times 10^{-6} \text{ secs.}$$

$$t_2 = \frac{55 + \sqrt{(0-55)^2 + 10^6}}{3\times 10^8} = 3.5217\times 10^{-6} \text{ secs.}$$

Thus we must perform the following addition:

$$\begin{aligned} r(t) &= \cos(300\pi\times 10^6(t - 3.3333\times 10^{-6})) + \cos(300\pi\times 10^6(t - 3.5217\times 10^{-6})) \\ &= \cos(300\pi\times 10^6 t - 1000\pi)) + \cos(300\pi\times 10^6 t - 1056.5113579\pi) \end{aligned}$$

As a phasor addition, we carry out the following steps (since 1000π and 1056π are integer multiples of 2π):

$$\begin{aligned} R &= 1e^{j0} + 1e^{j0.5113579\pi} \\ &= 1 + j0 + (-0.035674 + j0.99936) \\ &= 0.9643 + j0.9994 = 1.389e^{j0.803} = 1.389e^{j0.256\pi} = 1.389\angle 46.02^\circ \end{aligned}$$

From the polar form of the phasor R, we can write $r(t)$ as a sinusoid:

$$r(t) = 1.389\cos(300\pi\times 10^6 t + 0.256\pi)$$

(c) In order to find the locations where the signal strength is zero, we note that the phase angles of the two delayed sinusoids must differ by an odd multiple of π in order to get cancellation. Thus,

$$\begin{aligned} (2\ell + 1)\pi &= \Delta\varphi = -\omega t_1 - (-\omega t_2) \\ &= -300\pi\times 10^6\left(\frac{\sqrt{x^2 + 10^6}}{3\times 10^8} - \frac{55 + \sqrt{(x-55)^2 + 10^6}}{3\times 10^8}\right) \\ &= -\pi\left(\sqrt{x^2 + 10^6} - 55 - \sqrt{(x-55)^2 + 10^6}\right) \end{aligned}$$

The general solution to this equation is difficult, involving a quartic. However, if we choose $\ell = 27$ so that the left hand side becomes 55π, then the 55π term on the right hand side will cancel, and we obtain an equation in which squaring both sides will produce the answer.

$$\begin{aligned} \pi\sqrt{x^2 + 10^6} &= -\pi\sqrt{(x-55)^2 + 10^6} \\ \Longrightarrow x^2 + 10^6 &= (x-55)^2 + 10^6 \\ \Longrightarrow x^2 &= x^2 - 110x + 55^2 \\ \Longrightarrow 110x &= 55^2 \\ \Longrightarrow x &= \left(\frac{55}{110}\right)55 = 27.5 \text{ meters} \end{aligned}$$

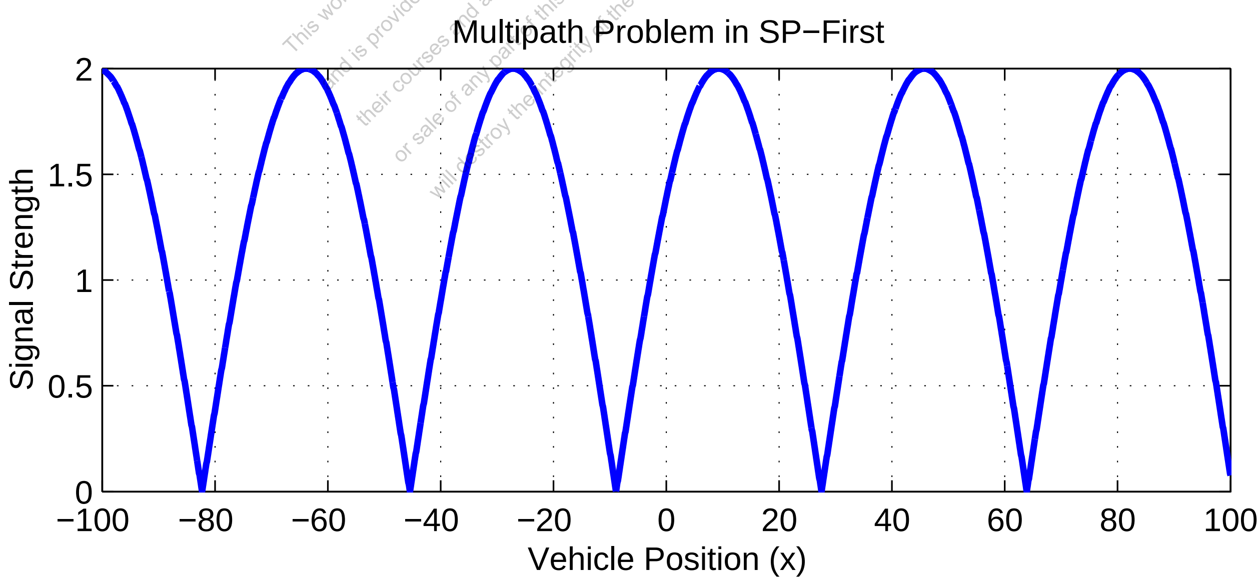
The general solution would be done in the following manner:

$$-(2\ell+1) = \sqrt{x^2+10^6} - 55 - \sqrt{(x-55)^2+10^6}$$
$$\Rightarrow 55-(2\ell+1) = \sqrt{x^2+10^6} - \sqrt{(x-55)^2+10^6}$$
$$\Rightarrow 55^2 - 110(2\ell+1) + (2\ell+1)^2 = x^2+10^6 - 2\sqrt{x^2+10^6}\sqrt{(x-55)^2+10^6} + (x-55)^2+10^6$$
$$\Rightarrow 2\sqrt{x^2+10^6}\sqrt{(x-55)^2+10^6} = -4\ell^2 + 216\ell + 109 - 55^2 + x^2 + 2\times10^6 + (x-55)^2$$

Squaring both sides would eliminate the square roots, but would produce a fourth-degree polynomial that would have to be solved for the vehicle position x.

(d) Here is a MATLAB script that will plot the signal strength versus vehicle position x, thus demonstrating that there are numerous locations where no signal is received (note the null at $x = 27.5$).

```
xx = -100:0.05:100;
d1 = sqrt(xx.*xx + 1e6);
d2 = 55 + sqrt((xx-55).*(xx-55)+1e6);
omeg = 300e6*pi;  c = 3e8;
phi1 = -omeg*d1/c;
phi2 = -omeg*d2/c;
RR = 1*exp(j*phi1) + 1*exp(j*phi2);
subplot('Position',[0.1,0.1,0.6,0.3]);
hp = plot(xx,abs(RR)); grid on,
xlabel('Vehicle Position (x)');
ylabel('Signal Strength');
title('Multipath Problem in SP-First');
set(hp,'LineWidth',2);
print -dpdf multipathResult.pdf
```

Over the range $-100 \le x \le 100$ the nulls appear to be equally spaced 36.4 meters apart, but they are not uniform. A plot over the range $0 \le x \le 1500$ would demonstrate the non-uniformity.