

## Chapter 2 Exercises

### 2.1

The electric field outside a charged sphere is the same as for a point source,

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2},$$

where  $Q$  is the charge on the inner surface of radius  $a$ . The potential drop is the integral

$$\Delta V = - \int_b^a \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0} \left( \frac{1}{a} - \frac{1}{b} \right).$$

The capacitance is therefore

$$C = \frac{Q}{\Delta V} = 4\pi\epsilon_0 \left( \frac{ab}{b-a} \right)$$

This is inversely proportional to the resistance found in Exercise 1.4.

### 2.2

The planar capacitor formula is

$$C = \frac{A\kappa\epsilon_0}{d}.$$

Solving for  $A$ ,

$$A = \frac{Cd}{\kappa\epsilon_0} = \frac{(1 \text{ F})(10^{-9} \text{ m})}{10(8.85 \times 10^{-12} \text{ C}^2/\text{N}^2 - \text{m}^2)} = 1.1 \text{ m}^2.$$

### 2.3

The solenoid inductor formula is

$$L = \frac{A\mu_0 N^2}{l}.$$

The loop area is  $A \approx \pi(5 \text{ cm}^2) = 76 \text{ cm}^2 = 7.6 \times 10^{-3} \text{ m}^2$ . The density of coils is  $N/l \approx (6^2 - 4^2)/(.05^2)(2\pi(.05 \text{ m})) = 2.5 \times 10^4/\text{m}$ .

Solving for  $l$ ,

$$l = \frac{L}{A\mu_0(N/l)^2} = \frac{1 \text{ H}}{(7.6 \times 10^{-3} \text{ m}^2)(4\pi \times 10^{-7} \text{ N/A}^2)(2.5 \times 10^4/\text{m})^2} = 17 \text{ cm}.$$

### 2.4

Integrate the flux over a rectangular section between the wires of length  $l$ . The magnetic field from a wire is

$$B = \frac{\mu_0 I}{2\pi r}.$$

The flux from one wire is

$$\Phi = l \int_b^a \frac{\mu_0 I}{2\pi r} dr = \frac{\mu_0 I l}{2\pi} \ln(a/b).$$

The second wire contributes flux in the same direction, so the total flux is

$$\Phi_{tot} = \frac{\mu_0 I I}{\pi} \ln(b/a).$$

The voltage drop is

$$\Delta V = \frac{\partial \Phi_{tot}}{\partial t} = \frac{\mu_0 l}{\pi} \ln(a/b) \frac{\partial I}{\partial t}.$$

Therefore

$$L = \frac{\mu_0 l}{\pi} \ln(a/b).$$

[Note: Technically we should account for the magnetic field inside each wire. The current inside the radius  $r$  is  $I r^2/b^2$ .  $B(2\pi r) = \mu_0 I r^2/b^2$ , so

$$B = \frac{\mu_0 r I}{2\pi b^2}$$

inside the wire. The integral of the flux is inside the wire is

$$\Phi = l \int_0^b \frac{\mu_0 r I}{2\pi b^2} = l \frac{\mu_0 I}{4\pi}.$$

The contribution to the inductance  $L$  from both wires is then

$$L' = \frac{\mu_0 l}{2\pi}.$$

which implies  $L/L' = 2 \ln(a/b)$ , which means the field inside the wire is negligible if  $a \gg b$ . ]

## 2.5

The Biot-Savart law is

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{R}}{R^3}.$$

For a loop of radius  $L$  centered at  $(0,0)$ , the unit vector along  $d\vec{l}$  is

$$\hat{\theta} = -\hat{x} \sin \theta + \hat{y} \cos \theta.$$

The distance from a point of the circle to a point  $r$  on the  $x$ -axis is

$$R = |L(\cos \theta, \sin \theta) - (r, 0)| = \sqrt{(L \cos \theta - r)^2 + L^2 \sin^2 \theta}$$

The direction of  $\vec{R}$  is

$$\hat{R} = -\frac{L(\cos \theta, \sin \theta) - (r, 0)}{R} = -\frac{1}{R}(\hat{x}(L \cos \theta - r) + \hat{y}L \sin \theta).$$

The cross product is

$$\hat{\theta} \times \hat{R} = -\frac{\hat{z}}{R}(r \cos \theta - L).$$

The integral of the Biot-Savart element is then

$$\vec{B} = -\frac{\mu_0 I \hat{z}}{4\pi} \int_0^{2\pi} L d\theta \frac{r \cos \theta - L}{[(L \cos \theta - r)^2 + L^2 \sin^2 \theta]^{3/2}}$$

This function involves elliptic integrals. Doing it numerically for  $r = 0$  (the center of the loop), gives

$$\vec{B} = \frac{\mu_0 I \hat{z}}{4L},$$

while at  $r = L/2$ , it is

$$\vec{B} = (3.91/\pi) \frac{\mu_0 I \hat{z}}{4L}.$$

The magnitude of  $B$  rises sharply near  $r = L$ .

## 2.6

$$V_o = IR = RC \frac{\partial}{\partial t}(V - V_o) = -RC \frac{\partial V_o}{\partial t}.$$

This has the solution

$$V_o(t) = V e^{-t/RC}.$$

## 2.7

Kirchhoff:

$$0 = RC \frac{\partial V_o}{\partial t} + V_o.$$

This has the general solution

$$V_o(t) = V_1 e^{-t/RC} + V_2.$$

Setting this to  $V$  at  $t = t_0$  gives

$$V = V_1 e^{-t_0/RC} + V_2.$$

The current through the capacitor is

$$I(t_0) = -\frac{V}{R} = C \frac{\partial}{\partial t}(V_1 e^{-t/RC} + V_2 R) \Big|_{t=t_0} = C \left( -\frac{V_1}{RC} e^{-t_0/RC} \right)$$

which implies  $V_1 = V e^{t_0/RC}$ . Plugging this into the above gives  $V_2 = 0$ , so the solution is

$$V_o(t) = V e^{-(t-t_0)/RC}.$$

This is decay to zero with the same time constant  $RC$ .

## 2.8

The integrator circuit of Figure 2.16(a) is governed by equation (2.4.13),

$$V_o + RC\dot{V}_o = V_i.$$

For  $V_o = \sin \omega t$ , this implies

$$\sin \omega t + \omega RC \cos \omega t = V_i.$$

When  $\omega RC \gg 1$ , the  $\sin \omega t$  term is negligible, and  $V_i$  is proportional to  $\dot{V}_o$ , i.e.,  $V_o$  is proportional to the antiderivative of  $V_i$ .

The differentiator circuit of Figure 2.16(b) is governed by equation (2.4.17),

$$\dot{V}_o + \frac{V_o}{RC} = \dot{V}_i.$$

For  $V_o = \sin \omega t$ , this implies

$$\omega \cos \omega t + \frac{1}{RC} \sin \omega t = \dot{V}_i.$$

When  $\omega RC \ll 1$ , the  $\cos \omega t$  term is negligible, and  $\dot{V}_i$  is proportional to  $V_o$ , i.e.,  $V_o$  is proportional to the derivative of  $V_i$ .

## 2.9

This is the circuit shown in Fig. 2.18(b). We have

$$\frac{V_o}{V_i} = \frac{R}{R + Z_C} = \frac{R}{R - i/\omega C}$$

$$\left| \frac{V_o}{V_i} \right|^2 = \frac{R}{R - i/\omega C} \frac{R}{R + i/\omega C} = \frac{R^2}{R^2 + 1/\omega^2 C^2} = \frac{1}{1 + 1/\omega^2 R^2 C^2}.$$

## 2.10

$$\frac{V_o}{V_i} = \frac{Z_L}{R + Z_L} = \frac{i\omega L}{R + i\omega L}$$

This is a high-pass filter.

**2.11** From Exercise 2.9, we have

$$\left| \frac{V_o}{V_i} \right|^2 = \frac{1}{1 + 1/\omega^2 R^2 C^2}$$

Solve for  $\omega$ :

$$\omega = \frac{1}{RC} \left( \frac{1}{|V_o/V_i|^2} - 1 \right)^{-1/2}$$

a)

$$\begin{aligned} -3 \text{ dB} &= 10 \log_{10} |V_o/V_i|^2 \\ |V_o/V_i|^2 &= 10^{-.3} = 0.5 \rightarrow \omega = 1/RC \end{aligned}$$

b)

$$|V_o/V_i|^2 = 10^{-1} = 0.1 \rightarrow \omega = 1/3RC$$

c)

$$|V_o/V_i|^2 = 10^{-2} = 0.01 \rightarrow \omega = 1/10RC$$

## 2.12

a)

$$\begin{aligned} 10 \text{ dBm} &= 20 \log_{10}(V/V_0) \\ \frac{V}{V_0} &= 10^{1/2} \\ V &= 3.1V_0 = 1 \text{ V} \\ \bar{P} &= \frac{V^2}{2R} = 10 \text{ mW}. \end{aligned}$$

We could also have done this just by noting that 10 dBm is 10 times greater than 0 dBm.

b)

$$\begin{aligned} 35 \text{ dB} &= 20 \log_{10} V_2/V_1 \\ \frac{V_2}{V_1} &= 10^{35/20} = 56 \end{aligned}$$

## 2.13

$$\sin \omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

From (2.7.11),

$$F(\omega') = 2\pi \frac{\delta(\omega' - \omega) - \delta(\omega' + \omega)}{2i}$$

## 2.14

$f(t)$  is given by (note typo in book)

$$f(t) = \begin{cases} 1, & (n-1)T < t < (n-1/2)T \\ 0, & (n-1/2)T < t < nT. \end{cases}$$

$$\begin{aligned}
c_n &= \frac{1}{T} \int_{T/2}^T e^{-i2\pi nt/T} dt \\
&= \frac{e^{-i2\pi n} - e^{-i\pi n}}{-i2\pi n} \\
&= \frac{e^{-3i\pi n/2}(e^{-i\pi n/2} - e^{i\pi n/2})}{-i2\pi n} = \frac{1}{\pi n} e^{-3i\pi n/2} \sin(\pi n/2)
\end{aligned}$$

$n$	$c_n$
0	$\frac{1}{2}$
$\pm 1$	$i\pi^{-1}$
$\pm 2$	0
$\pm 3$	$\frac{i}{3}\pi^{-1}$
$\pm 4$	0
$\pm 5$	$\frac{i}{5}\pi^{-1}$
$\pm 6$	0
$\pm 7$	$\frac{i}{7}\pi^{-1}$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i2\pi nt/T} = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{i}{\pi n} (e^{i2\pi nt/T} - e^{-i2\pi nt/T}) = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin(2\pi nt/T)$$

The sum of the terms up through  $n = 7$  (first five nonzero terms) is shown in Figure 6.

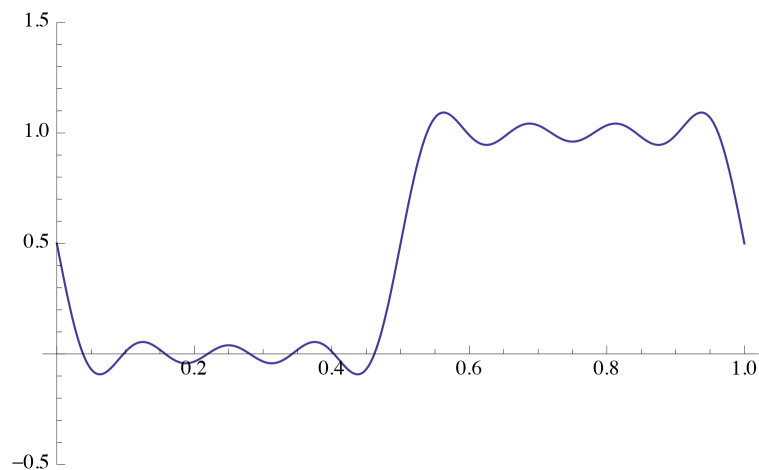


Figure 6: Fourier sum for Exercise 2.14.

## 2.15

The Fourier transform is

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt.$$

$f(t)$  is given by

$$f(t) = \begin{cases} 1, & (n-1)T < t < (n-1/2)T \\ 0, & (n-1/2)T < t < nT. \end{cases}$$

The transform is

$$\begin{aligned} F(\omega) &= \sum_{n=-\infty}^{\infty} \int_{(n-1)T}^{(n-1/2)T} e^{-i\omega t} dt = \sum_{n=-\infty}^{\infty} \frac{i}{\omega} (e^{-i\omega(n-1/2)T} - e^{-i\omega(n-1)T}) \\ &= \frac{i}{\omega} (e^{i\omega T/2} - e^{i\omega T}) \sum_{n=-\infty}^{\infty} e^{-i\omega n T}. \end{aligned}$$

The sum will be equal to infinity for  $\omega = 2\pi n'/T$  and zero otherwise (this is equivalent to a  $\delta$ -function). Thus we have

$$F(\omega) = \begin{cases} \frac{-2i}{2\pi n'/T} = \frac{-i}{\pi n'}, & \omega = 2\pi n'/T \\ 0, & \text{else.} \end{cases}$$

This is the same as the result of Exercise 2.14.

## 2.16

The Fourier transform is

$$F(\omega) = \int_{-\infty}^{\infty} \Theta(t)e^{-st}e^{-i\omega t} dt = \int_0^{\infty} e^{-st}e^{-i\omega t} dt = -\frac{1}{-s-i\omega} = \frac{-i}{\omega-is}$$

The response function (2.5.7) is

$$\frac{V_o}{V_i} = \frac{-i/\omega C}{R-i/\omega C} = \frac{-i/RC}{\omega-i/RC}$$

and the product is

$$F_o(\omega) = \left( \frac{-i/RC}{\omega-i/RC} \right) \left( \frac{-i}{\omega-is} \right)$$

The reverse Fourier transform is

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \frac{-i/RC}{\omega-i/RC} \right) \left( \frac{-i}{\omega-is} \right) e^{i\omega t} dt.$$

For  $t > 0$ ,  $\omega \rightarrow +i\infty$  converges, so this becomes

$$f(t) = i(-i/RC) \frac{-i}{i/RC-is} e^{i(i/RC)t} + i \frac{(-i/RC)}{is-i/RC} (-i) e^{i(is)t}.$$

Setting  $s = 0$  gives

$$f(t) = 1 - e^{-t/RC}.$$

### 2.17

Loops:

$$V_i = IR + I_C Z_C$$

$$V_i = IR + I_L Z_L$$

Node:

$$I = I_C + I_L$$

Solution:

$$I_C = \frac{Z_L}{RZ_C + RZ_L + Z_C Z_L} V_i = \frac{i\omega L}{-iR/\omega C + i\omega RL + L/C} V_i$$

$$V_C = I_C Z_C = \frac{L/C}{-iR/\omega C + i\omega RL + L/C} V_i$$

### 2.18

(2.8.2) is

$$\frac{V_o}{V_i} = \frac{R}{R - i/\omega C + i\omega L}.$$

(2.8.12) is

$$\dot{V}_i = \dot{I}R + \frac{I}{C} + L\ddot{I}.$$

For  $V_i = V_i(0)e^{i\omega t}$  and  $I = I_0 e^{i\omega t + \phi}$ , this becomes

$$i\omega V_i(0)e^{i\omega t} = i\omega I_0 R e^{i(\omega t + \phi)} + \frac{I_0}{C} e^{i(\omega t + \phi)} - L\omega^2 I_0 e^{i(\omega t + \phi)}.$$

The output  $V_o$  is across  $R$ , so  $V_o = I_0 R e^{i(\omega t + \phi)}$ . The above equation therefore becomes

$$i\omega V_i = i\omega V_o + \frac{V_o}{RC} - \frac{L}{R}\omega^2 V_o.$$

Solving for  $V_o/V_i$  gives

$$\frac{V_o}{V_i} = \frac{i\omega}{i\omega + 1/RC - \omega^2 L/R}$$

which is the same as (2.8.2).



### 2.19

We want a high-pass filter like that shown in Fig. 2.18(b), which has response (see Exercise 2.9)

$$\left| \frac{V_o}{V_i} \right|^2 = \frac{1}{1 + 1/\omega^2 R^2 C^2}$$

We solve for  $C$ :

$$C = \frac{\omega}{R} \sqrt{\frac{1}{|V_o/V_i|^2} - 1} = \frac{2\pi(100 \text{ Hz})}{50 \times 10^6 \Omega} \sqrt{1/(.95)^2 - 1} = 0.6 \mu\text{F}.$$

### 2.20

Solve for 10% value:

$$e^{-t_{10}^2/2\tau^2} = .1$$
$$t_{10} = \sqrt{-2\tau^2 \ln(0.1)}.$$

For 90% value,

$$t_{90} = \sqrt{-2\tau^2 \ln(0.9)}.$$

On the positive side,

$$t_{10} - t_{90} = \sqrt{-2\tau^2 \ln(0.9)} - \sqrt{-2\tau^2 \ln(0.1)} = \tau\sqrt{2}(\sqrt{-\ln(0.1)} - \sqrt{-\ln(0.9)}) = 1.2\sqrt{2}\tau.$$

### 2.21

The electric field is purely radial. For charge  $+Q$  on the inner sphere and  $-Q$  on the outer sphere, the electric field is

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

and the potential drop is

$$\Delta V(r) = - \int_{r_1}^{r_2} E(r) dr = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}.$$

Comparing to the definition  $Q = C\Delta V$ , this gives

$$C = 4\pi\epsilon_0 \left( \frac{1}{r_1} - \frac{1}{r_2} \right)^{-1}.$$

### 2.22

The electric field from a line charge is found from Gauss's law,

$$E = \frac{\sigma}{2\pi r \epsilon_0},$$

where  $\sigma = Q/l$  is the charge density. The voltage drop from wire of radius  $b$  to a distance  $a$  is

$$\Delta V = - \int_b^a E(r) dr = \frac{Q/l}{2\pi\epsilon_0} \ln(a/b).$$

By superposition, the other wire contributes the same, so we multiply by 2. This implies

$$C = \frac{\pi\epsilon_0 l}{\ln(a/b)}.$$

### 2.23

$$C = \frac{A\epsilon_0}{d} = \frac{lw\epsilon_0}{d}$$

$$L = \frac{\mu_0 ld}{w}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{\mu_0 ld}{w} \frac{lw\epsilon_0}{d}}} = \frac{1}{\sqrt{\mu_0\epsilon_0 l^2}} = \frac{c}{l}.$$

### 2.24

The high-pass filter of Fig. 2.18(b) has response (see Exercise 2.9)

$$\frac{V_o}{V_i} = \frac{R}{R - i/\omega C} = \frac{R(R + i/\omega C)}{R^2 + 1/\omega^2 C^2}.$$

From (2.5.9),

$$\begin{aligned} \tan \phi &= \frac{\text{Im } V_o/V_i}{\text{Re } V_o/V_i} \\ &= \frac{1/\omega C}{R} \end{aligned}$$

At high frequency,  $\phi = 0$ . At low frequency,  $\phi = \pi/2$ .

**2.25**

$$\frac{V_o}{V_i} = \frac{i\omega L}{i\omega L + R} = \frac{i\omega L/R}{i\omega L/R + 1}$$

Figure ?? shows the plot with frequency in units of  $R/L$ .

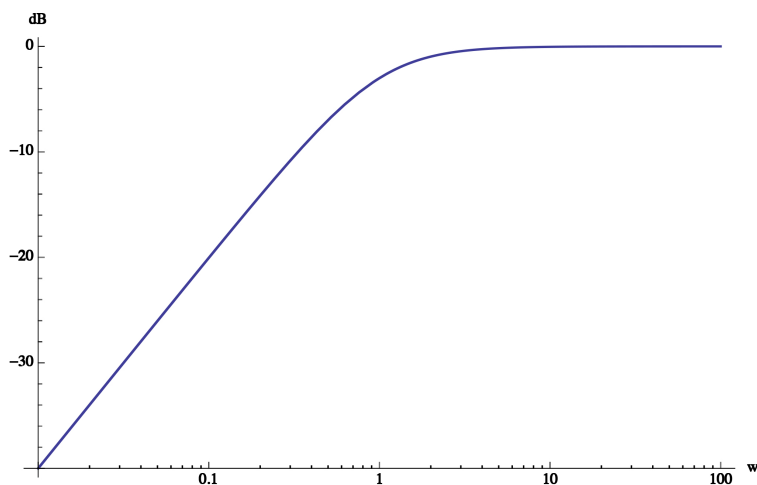


Figure 7: Response function for Exercise 2.25.

**2.26**

$|V_o|^2$  is increased by  $(3/2)^2 = 2.25$ .

$$10(\log_{10} 2.25) = 3.52 \text{ dB}$$

**2.27**

From Exercises 2.9 and 2.18:

$$\left| \frac{V_o}{V_i} \right|^2 = \frac{1}{1 + 1/\omega^2 R^2 C^2}$$

Solving for  $\omega$ ,

$$\omega = \frac{1}{RC} \left( \frac{1}{|V_o/V_i|^2} - 1 \right)^{-1/2} = \frac{0.1}{RC}$$

**2.28**

The low-pass filter response (2.5.7) is

$$\frac{V_o}{V_i} = \frac{-i/\omega C}{R - i/\omega C}$$

Power response is

$$\left| \frac{V_o}{V_i} \right|^2 = \frac{-i/\omega C}{R - i/\omega C} \frac{i/\omega C}{R + i/\omega C} = \frac{1/\omega^2 C^2}{R^2 + 1/\omega^2 C^2} = \frac{1}{\omega^2 (RC)^2 + 1}.$$

Solve for  $\omega$ :

$$\omega = \frac{1}{RC} \sqrt{\frac{1}{|V_o/V_i|^2} - 1}$$

10%:  
 $\omega = \frac{3}{RC}.$

90%:  
 $\omega = \frac{1}{3RC}.$

10-90 range:

$$\Delta\omega = \frac{2.667}{RC}.$$

## 2.29

$$\Delta\text{dB} = 20 \log_{10} V_2/V_1$$

$$\frac{V_2}{V_1} = 10^{\Delta\text{dB}/20} = 10^{13/20} = 4.5$$

The amplitude signal-to-noise ratio increases by a factor of 4.5, i.e. from 2 to 9.

It is also common to talk in terms of the signal-to-noise power ratio.

## 2.30

The circuit is shown in Figure 8.

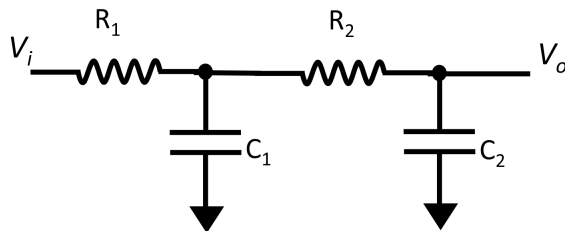


Figure 8: Circuit for Exercise 2.30.

Loops:

$$\begin{aligned} V_i &= I_1 R_1 + I_{C1} Z_{C1} \\ V_i &= I_1 R_1 + I_2 R_2 + V_o \\ V_o &= I_2 Z_{C2} \end{aligned}$$

Node:

$$I_1 = I_{C1} + I_2$$

Set  $R_1 = R_2 = R$ ,  $C_1 = C_2 = C$ . Solution:

$$\frac{V_o}{V_i} = \frac{Z_C^2}{R^2 + 3RZ_C + Z_C^2}$$

$$\left| \frac{V_o}{V_i} \right|^2 = \frac{1}{1 + 7C^2 R^2 \omega^2 + C^4 R^4 \omega^4}$$

In the Figure 9, the lower curve is this response function, while the upper curve is the single low-pass response, from Exercise 2.27.

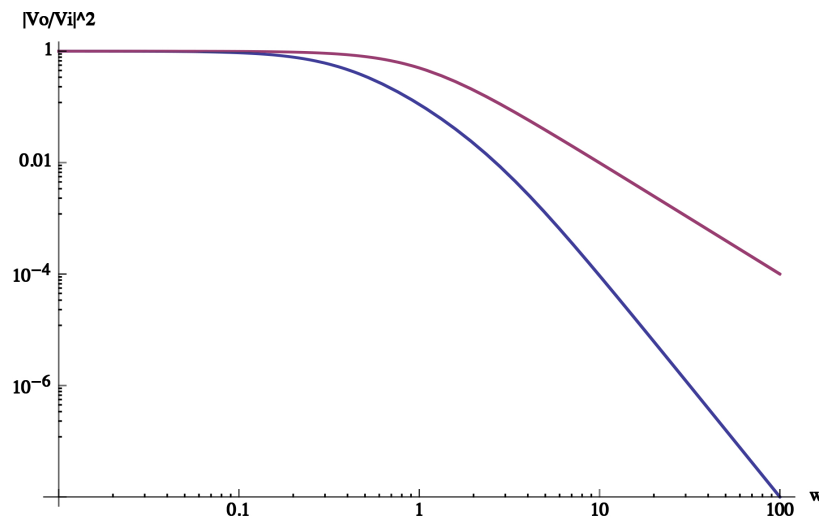


Figure 9: Response functions for Exercise 2.30.

### 2.31

Assuming perfect detector; no current flows into output;  $I_o$  is the current flowing from top to bottom in the right side of the circuit.

Loops:

$$V_i = I_{C1} Z_C + I_{R3} \frac{R}{2}$$

$$V_i = I_{R1}R + I_{C3} \frac{Z_C}{2}$$

$$V_i = I_{R1}R + I_o R + V_o$$

$$V_o = I_o Z_C + I_{R3} \frac{R}{2}$$

Nodes:

$$I_{R1} = I_{C3} + I_o$$

$$I_{C1} + I_o = I_{R3}$$

Solution:

$$\frac{V_o}{V_i} = \frac{(-1 + C^2 R^2 \omega^2)}{-1 - 4iCR\omega + C^2 R^2 \omega^2}$$

This is plotted in Figure 10. When  $\omega = 1/RC$ , this equals 0. When  $\omega \rightarrow 0$  or  $\omega \rightarrow \infty$ , it approaches unity.

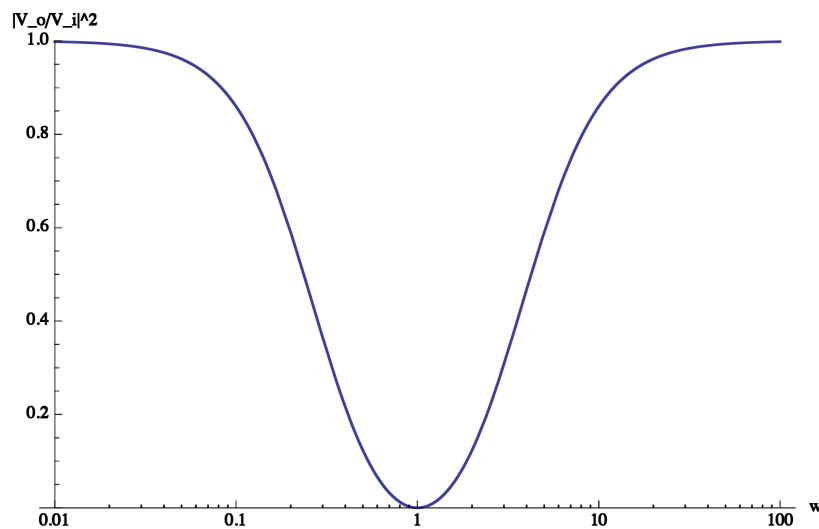


Figure 10: Response function for Exercise 2.31.

### 2.32

The circuit in the book is missing a  $50\text{-}\Omega$  resistor between  $V_i$  and the rest of the circuit—as drawn, the circuit will give  $V_o = V_i$  for all inputs. Putting a resistor there gives

Loops:

$$V_i = IR + V_o$$

$$V_o = I_1(Z_{C1} + Z_{L1})$$

$$V_o = I_2(Z_{C2} + Z_{L2})$$

Node:

$$I = I_1 + I_2$$

Solution:

$$\frac{V_o}{V_i} = \frac{(-1 + C_1 L_1 \omega^2)(-1 + C_2 L_2 \omega^2)}{1 + iC_1 R \omega + iC_2 R \omega - C_1 L_1 \omega^2 - C_2 L_2 \omega^2 - iC_1 C_2 L_1 R \omega^3 - iC_1 C_2 L_2 R \omega^3 + C_1 C_2 L_1 L_2 \omega^4}$$

This is a double notch filter, with zeroes where the two terms in the numerator vanish. Figure 11 shows a plot for the values given:

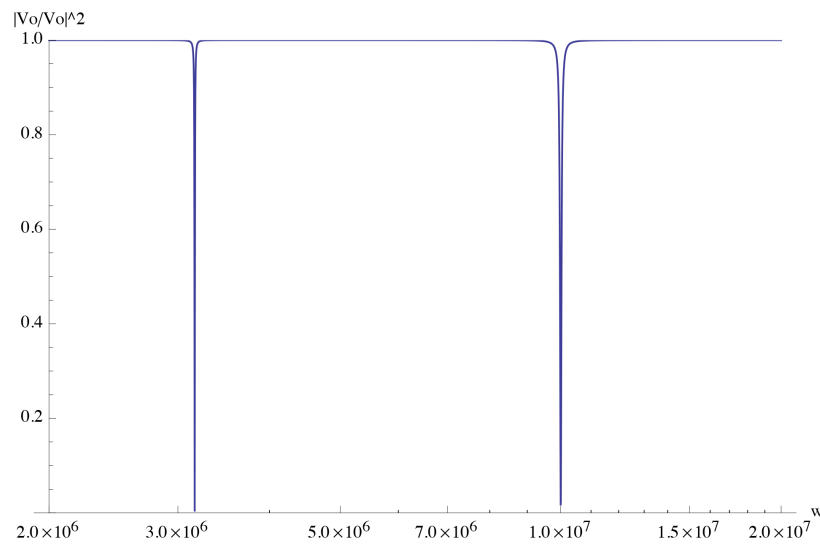


Figure 11: Double notch response function, for Exercise 2.32.

### 2.33

a) Capacitor relation:

$$I = C \frac{\partial \Delta V_C}{\partial t}$$

Loops:

$$V_s = I_1 R_1 + I_2 R_2$$

$$V_i = \Delta V_C + V_o \rightarrow \dot{V}_i = \frac{I_C}{C} + \dot{V}_o$$

$$V_o = I_2 R_2$$

These become

$$\begin{aligned}
V_s &= I_1 R_1 + I_2 R_2 \\
i\omega V_1 e^{i\omega t} &= \frac{I_C}{C} + i\omega V_{AC} e^{i(\omega t + \phi)} \\
V_{DC} + V_{AC} e^{i(\omega t + \phi)} &= I_2 R_2
\end{aligned}$$

Node:

$$I_C + I_1 = I_2.$$

Write  $I_1 = I_{1DC} + I_{1AC}$  and  $I_2 = I_{2DC} + I_{2AC}$  (note that there is no DC component through the capacitor), and set DC terms equal and AC terms equal:

$$\begin{aligned}
V_s &= I_{1DC} R_1 + I_{2DC} R_2 \\
0 &= I_{1AC} R_1 + I_{2AC} R_2 \\
i\omega V_1 e^{i\omega t} &= \frac{I_C}{C} + i\omega V_{AC} e^{i(\omega t + \phi)} \\
V_{DC} &= I_{2DC} R_2 \\
V_{AC} e^{i(\omega t + \phi)} &= I_{2AC} R_2 \\
I_C + I_{1AC} &= I_{2AC} \\
I_{1DC} &= I_{2DC}
\end{aligned}$$

We solve these for  $I_C, I_{1DC}, I_{1AC}, I_{2DC}, I_{2AC}, V_{DC}$ , and  $V_{AC} e^{i\phi}$ , which gives

$$\begin{aligned}
V_{DC} &= \frac{R_2 V_s}{R_1 + R_2}, \\
V_{AC} e^{i\phi} &= \frac{C R_1 R_2 \omega}{-i R_1 - i R_2 + C R_1 R_2 \omega} V_1 \\
&= \frac{C R_{\text{eff}} \omega}{-i + C R_{\text{eff}} \omega} V_1
\end{aligned}$$

where

$$R_{\text{eff}} = \frac{R_1 R_2}{R_1 + R_2}.$$

### 2.34

We want a low-pass filter that eliminates AC frequency of 60 Hz. We use the circuit shown in Fig. 2.18(a) with a polarized capacitor with the negative side grounded. Formula (2.5.11) gives us

$$|V_o/V_i|^2 = \frac{1}{1 + \omega^2 R^2 C^2}.$$

We would like low series  $R$  to prevent DC droop of the voltage supply. Pick  $R = 1 \Omega$ , which is small compared to a typical  $50 \Omega$  load impedance. Solve for  $C$ :

$$C = \frac{1}{\omega R} \sqrt{1/|V_o/V_i|^2 - 1}$$



To eliminate 99% of the ripple, pick  $|V_o/V_i|^2 = .01$ . Setting  $\omega = 2\pi(60 \text{ Hz}) = 377 \text{ s}^{-1}$ , we then have

$$C = \frac{1}{(377 \text{ s}^{-1})(50 \Omega)} \sqrt{99} = 0.00053 \text{ F} = 530 \mu\text{F}.$$