

**2.1 EXERCISE SOLUTIONS**

- Organizing the data into a frequency distribution may make patterns within the data more evident. Sometimes it is easier to identify patterns of a data set by looking at a graph of the frequency distribution.
- If there are too few or too many classes, it may be difficult to detect patterns because the data are too condensed or too spread out.
- Class limits determine which numbers can belong to that class.  
Class boundaries are the numbers that separate classes without forming gaps between them.
- Relative frequency of a class is the portion, or percentage, of the data that falls in that class.  
Cumulative frequency of a class is the sum of the frequencies of that class and all previous classes.
- The sum of the relative frequencies must be 1 or 100% because it is the sum of all portions or percentages of the data.
- A frequency polygon displays frequencies or relative frequencies whereas an ogive displays cumulative frequencies.
- False. Class width is the difference between the lower (or upper limits) of consecutive classes.
- True
- False. An ogive is a graph that displays cumulative frequencies.
- True

$$11. \text{ Class width} = \frac{\text{Range}}{\text{Number of classes}} = \frac{64-9}{7} \approx 7.9 \cdot 8$$

Lower class limits: 9, 17, 25, 33, 41, 49, 57  
Upper class limits: 16, 24, 32, 40, 48, 56, 64

$$12. \text{ Class width} = \frac{\text{Range}}{\text{Number of classes}} = \frac{88-12}{6} \approx 12.7 \cdot 13$$

Lower class limits: 12, 25, 38, 51, 64, 77  
Upper class limits: 24, 37, 50, 63, 76, 89

$$13. \text{ Class width} = \frac{\text{Range}}{\text{Number of classes}} = \frac{135-17}{8} = 14.75 \cdot 15$$

Lower class limits: 17, 32, 47, 62, 77, 92, 107, 122  
Upper class limits: 31, 46, 61, 76, 91, 106, 121, 136

$$14. \text{ Class width} = \frac{\text{Range}}{\text{Number of classes}} = \frac{247-54}{10} = 19.3 \cdot 20$$

Lower class limits: 54, 74, 94, 114, 134, 154, 174, 194, 214, 234  
Upper class limits: 73, 93, 113, 133, 153, 173, 193, 213, 233, 253

15. (a) Class width =  $11 - 0 = 11$

(b) and (c)

$$\text{Midpoint} = \frac{(\text{Lower class limit}) + (\text{Upper class limit})}{2}$$

Find the class boundaries. Because the data entries are integers, subtract 0.5 from each lower limit to find the lower class boundaries and add 0.5 to each upper limit to find the upper class boundaries.

Class	Midpoint	Class boundaries
0 - 10	5	- 0.5 - 10.5
11 - 21	16	10.5 - 21.5
22 - 32	27	21.5 - 32.5
33 - 43	38	32.5 - 43.5
44 - 54	49	43.5 - 54.5
55 - 65	60	54.5 - 65.5
66 - 76	71	65.5 - 76.5

16. (a) Class width =  $33 - 25 = 8$

(b) and (c)

$$\text{Midpoint} = \frac{(\text{Lower class limit}) + (\text{Upper class limit})}{2}$$

Find the class boundaries. Because the data entries are integers, subtract 0.5 from each lower limit to find the lower class boundaries and add 0.5 to each upper limit to find the upper class boundaries.

Class	Midpoint	Class boundaries
25 - 32	28.5	24.5- 32.5
33 - 40	36.5	32.5- 40.5
41 - 48	44.5	40.5- 48.5
49 - 56	52.5	48.5- 56.5
57 - 64	60.5	56.5- 64.5
65 - 72	68.5	64.5- 72.5
73 - 80	76.5	72.5- 80.5

$$17. \text{Relative frequency} = \frac{\text{Class frequency}}{\text{Sample size}} = \frac{f}{n}$$

The cumulative frequency of a class is the sum of the frequencies of that class and all previous classes.

Class	Frequency $f$	Midpoint	Relative frequency	Cumulative frequency
0 - 10	188	5	0.15	188
11 - 21	372	16	0.30	560
22 - 32	264	27	0.22	824
33 - 43	205	38	0.17	1029
44 - 54	83	49	0.07	1112
55 - 65	76	60	0.06	1188
66 - 76	32	71	0.03	1220
	$\Sigma f = 1220$		$\Sigma \frac{f}{n} = 1$	

$$18. \text{Relative frequency} = \frac{\text{Class frequency}}{\text{Sample size}} = \frac{f}{n}$$

The cumulative frequency of a class is the sum of the frequencies of that class and all previous classes.

Class	Frequency, $f$	Midpoint	Relative frequency	Cumulative frequency
25 - 32	86	28.5	0.24	86
33 - 40	39	36.5	0.11	125
41 - 48	41	44.5	0.11	166
49 - 56	48	52.5	0.13	214
57 - 64	43	60.5	0.12	257
65 - 72	68	68.5	0.19	325
73 - 80	40	76.5	0.11	365
	$\Sigma f = 365$		$\Sigma \frac{f}{n} = 1$	

19. (a) Number of classes: 7

(b) Greatest frequency: about 300  
Least frequency: about 10

(c) Class width: 10

(d) *Sample answer:* About half of the employee salaries are between \$50,000 and \$69,000.

20. (a) Number of classes: 6

(b) Greatest frequency: 37  
Least frequency: 1

(c) Class width: 53

- (d) *Sample answer:* The heights of most roller coasters are less than 231 feet.
21. Identify the highest point and its respective class. Class with greatest frequency: 506 - 510  
Identify the lowest point (not including the points on the horizontal axis) and its respective class.  
Class with least frequency: 474 - 478
22. Identify the highest point and its respective class. Class with greatest frequency: 3.5 - 4.5 miles  
Identify the lowest point (not including the points on the horizontal axis) and its respective class.  
Class with least frequency: 0.5 - 1.5 miles
23. (a) Identify the tallest bar and its respective class. Class with greatest relative frequency: 35 - 36 centimeters  
Identify the shortest bar and its respective class. Class with least relative frequency: 39 - 40 centimeters
- (b) Greatest relative frequency  $\approx 0.25$   
Least relative frequency  $\approx 0.01$
- (c) *Sample answer:* From the graph, 0.25 or 25% of females have a fibula length between 35 and 36 centimeters.
24. (a) Identify the tallest bar and its respective class. Class with greatest relative frequency: 11 - 12 minutes  
Identify the shortest bar and its respective class. Class with least relative frequency: 14 - 15 minutes
- (b) Greatest relative frequency  $\approx 38\%$   
Least relative frequency  $\approx 4\%$
- (c) *Sample answer:* From the graph, about 0.75 or 75% of campus security response times are between 11 and 13 minutes.
25. (a) Locate the cumulative frequency of the highest (right-most) point. The number in the sample is 75.
- (b) Locate the neighboring points where the pitch between them is the steepest. The greatest increase in frequency is from 158.5 - 201.5 pounds.
26. (a) Locate the cumulative frequency of the highest (right-most) point. The number in the sample is 77.
- (b) Locate the neighboring points where the pitch between them is the steepest. The greatest increase in frequency is from 68 - 70 inches.
27. (a) Locate 201.5 on the horizontal axis and find the corresponding cumulative frequency at the point on the ogive: 47
- (b) Locate 68 on the vertical axis and find the corresponding weight at the point on the ogive: 287.5 pounds
- (c) Subtract the cumulative frequency for each weight:  $62 - 22 = 40$

(d) Subtract the cumulative frequency for bears weighing 330.5 pounds from the number in the sample:  $75 - 69 = 6$

28. (a) Locate 72 on the horizontal axis and find the corresponding cumulative frequency at the point on the ogive: 71

(b) Locate 15 on the vertical axis and find the corresponding height at the point on the ogive: 68 inches

(c) Subtract the cumulative frequency for each height:  $71 - 15 = 56$

(d) Subtract the cumulative frequency for adult males that are 70 inches tall from the number in the sample:  $77 - 47 = 30$

29. Class width =  $\frac{\text{Range}}{\text{Number of classes}} = \frac{39-0}{5} = 7.8 \cdot 8$

Class	Frequency, $f$	Midpoint	Relative frequency	Cumulative frequency
0 - 7	8	3.5	0.33	8
8 - 15	7	11.5	0.29	15
16 - 23	3	19.5	0.13	18
24 - 31	3	27.5	0.13	21
32 - 39	3	35.5	0.13	24
	$\Sigma f = 24$		$\Sigma \frac{f}{n} \approx 1$	

Class with greatest frequency: 0 - 7

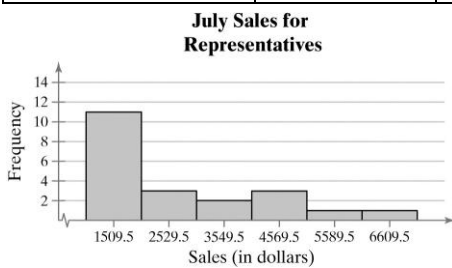
Classes with least frequency: 16 - 23, 24 - 31, 32 - 39

30. Class width =  $\frac{\text{Range}}{\text{Number of classes}} = \frac{530-30}{6} \approx 83.3 \cdot 84$

Class	Frequency, $f$	Midpoint	Relative frequency	Cumulative frequency
30 - 113	5	71.5	0.17	5
114 - 197	7	155.5	0.23	12
198 - 281	8	239.5	0.27	20
282 - 365	3	323.5	0.10	23
366 - 449	3	407.5	0.10	26
450 - 533	4	491.5	0.13	30
	$\Sigma f = 30$		$\Sigma \frac{f}{n} = 1$	

31. Class width =  $\frac{\text{Range}}{\text{Number of classes}} = \frac{7119-1000}{6} \approx 1019.8 \cdot 1020$

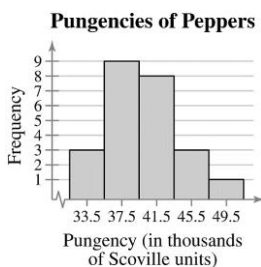
Class	Frequency, $f$	Mid-point	Relative frequency	Cumulative frequency
1000 - 2019	11	1509.5	0.52	11
2020 - 3039	3	2529.5	0.14	14
3040 - 4059	2	3549.5	0.10	16
4060 - 5079	3	4569.5	0.14	19
5080 - 6099	1	5589.5	0.05	20
6100 - 7119	1	6609.5	0.05	21
	$\Sigma f = 21$		$\Sigma \frac{f}{n} = 1$	



*Sample answer:* The graph shows that most of the sales representatives at the company sold from \$1000 to \$2019.

32. Class width =  $\frac{\text{Range}}{\text{Number of classes}} = \frac{51-32}{5} = 3.8 \cdot 4$

Class	Frequency, $f$	Midpoint	Relative frequency	Cumulative frequency
32-35	3	33.5	0.1250	3
36-39	9	37.5	0.3750	12
40-43	8	41.5	0.3333	20
44-47	3	45.5	0.1250	23
48-51	1	49.5	0.0417	24
	$\Sigma f = 24$		$\Sigma \frac{f}{n} = 1$	



*Sample answer:* The graph shows that most of the pungencies of the peppers were between 36,000 and 43,000 Scoville units.

33. Class width =  $\frac{\text{Range}}{\text{Number of classes}} = \frac{514-291}{8} = 27.875 \cdot 28$

