SOLUTIONS MANUAL

Engineering Materials I
An Introduction to Properties, Applications and Design, Fourth Edition

Solutions to Examples

2. 1. (a) For commodity A $\qquad P(t) = C_A \exp\dfrac{r_A}{100}t,$

and for commodity B $Q(t) = C_B \exp\dfrac{r_B}{100}t,$

where C_A and C_B are the current rates of consumption ($t = t_0$) and $P(t)$ and $Q(t)$ are the values at $t = t$. Equating and solving for t gives

$$t = \frac{100}{r_B - r_A}\ln\left(\frac{C_A}{C_B}\right).$$

(b) The doubling time, t_D, is calculated by setting $C(t = t) = 2C_0$, giving

$$t_D = \frac{100}{r}\ln 2 \approx \frac{70}{r}.$$

Substitution of the values given for r in the table into this equation gives the doubling times as 35, 23 and 18 years respectively.

(c) Using the equation of Answer (a) we find that aluminium overtakes steel in 201 years; polymers overtake steel in 55 years.

2.2. Principal conservation measures (see Section 2.7):

Substitution

Examples: aluminium for copper as a conductor; reinforced concrete for wood, stone or cast-iron in construction; plastics for glass or metals as containers. For many applications, substitutes are easily found at small penalty of cost. But in certain specific uses, most elements are not easily replaced. Examples: tungsten in cutting tools and lighting (a fluorescent tube contains more tungsten, as a starter filament, than an incandescent bulb!); lead in lead-acid batteries; platinum as a catalyst in chemical processing; etc. A long development time (up to 25 years) may be needed to find a replacement.

Recycling

The fraction of material recycled is obviously important. Products may be re-designed to make recycling easier, and new recycling processes developed, but development time is again important.

More Economic Design

Design to use proportionally smaller amounts of scarce materials, for example, by building large plant (economy of scale); using high-strength materials; use of surface coatings to prevent metal loss by corrosion (e.g. in motor cars).

2.3. (a) If the current rate of consumption in tonnes per year is C then exponential growth means that

$$\frac{dC}{dt} = \frac{r}{100}C,$$

where r is the fractional rate of growth in % per year. Integrating gives

$$C = C_0 \exp\left\{\frac{r(t - t_0)}{100}\right\},$$

where C_0 was the consumption rate at time $t = t_0$.

(b) Set

$$\frac{Q}{2} = \int_0^{t_{1/2}} C dt,$$

where

$$C = C_0 \exp\left\{\frac{rt}{100}\right\}.$$

Then

$$\frac{Q}{2} = \left[C_0 \frac{100}{r} \exp\left\{\frac{rt}{100}\right\}\right]_0^{t_{1/2}},$$

which gives the desired result.

2.4. See Chapter 2 for discussion with examples.

3.1. Refer to the results at the end of Chapter 3 for the elastic buckling of struts (pp 52 and 53), and second moments of area (pp 49 and 50). Appropriate situation is probably Case 2 (left hand side drawing).

$$F_{cr} = 9.87\left(\frac{EI}{l^2}\right), \quad I = \frac{\pi r^4}{4},$$

$$F_{cr} = \frac{9.87\pi E}{4}\left(\frac{r}{l}\right)^2 r^2 = \frac{9.87\pi \times 2 \times 10^4 \text{ N mm}^{-2}}{4}\left(\frac{8.5 \text{ mm}}{750 \text{ mm}}\right)^2 8.5^2 \text{ mm}^2$$

$$= 1439 \text{ N} = 148 \text{ kgf}.$$

This gives a factor of safety of about $148/90 = 1.65$, so he should be OK.

3.2. Refer to the results at the end of Chapter 3 for the mode 1 natural vibration frequencies of beams (pp 50 and 51), and second moments of area (pp 49 and 50).

The appropriate situation is Case 2.

$$f = 0.560\sqrt{\frac{EI}{Ml^3}}, \quad I = \frac{bd^3}{12}, \quad M = \rho lbd.$$

$$f = 0.560\left(\frac{Ed^2}{12\rho l^4}\right)^{1/2} = 0.1617\left(\frac{d}{l^2}\right)\left(\frac{E}{\rho}\right)^{1/2},$$

$$E = \left(\frac{f}{0.1617}\right)^2\left(\frac{l^2}{d}\right)^2\rho.$$

Because the frequency of natural vibration involves a force acting on a mass to give it an acceleration, it is crucial when real numbers are put into the governing equation that the basic SI units are used as follows

$$E = \left(\frac{440}{0.1617}\right)^2\left(\frac{0.085^2}{0.00386}\right)^2 \times 7.85 \times 10^3 = 204 \times 10^9 \text{ N m}^{-2} = 204 \text{ GN m}^{-2}.$$

3.3. This is a consequence of the equations of static equilibrium.

3.4. Principal planes have no components of shear stress acting on them. Principal directions are normal to principal planes. Principal stresses are normal stresses acting on principal planes. The shear stress components all vanish.

3.5. (a) $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, (b) $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, (c) $\begin{pmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{pmatrix}$, (d) $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$.

3.6. $\begin{pmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. No.

3.7. $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0.5\sigma_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. There are no shear stress components normal to these axes.

3.8. Because the two shear strain terms on any given axis plane are defined so there is no rotation.

3.9. Principal strains are axial strains. The shear strain components all vanish.

3.10. (a) $\begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & -\upsilon\varepsilon_1 & 0 \\ 0 & 0 & -\upsilon\varepsilon_1 \end{pmatrix}$. (b) $\begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$.

3.11. Compare the right hand diagram on p 44 with the centre diagram in Fig. 3.5.

$$e_{13} = \gamma \text{ and } e_{31} = 0.$$

$$\varepsilon_{13} = \varepsilon_{31} = \frac{1}{2}(e_{13} + e_{31}) = \frac{\gamma}{2}.$$

3.12. From the bottom diagram in Fig. 3.5, the dilatation is defined as $\Delta = \dfrac{\Delta V}{V}$.

Consider a cube of material of unit side. For each of the three principal (axial) strains, the increase in the volume of the cube is equal to the strain. For example, a strain ε_1 makes the cube longer in the 1 direction by ε_1. The increase in the volume of the cube is ε_1 x 1 x 1 = ε_1. The dilatation produced by ε_1 is therefore (ε_1 x 1 x 1)/(1 x 1 x 1) = ε_1. Therefore,

$$\Delta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3.$$

3.13. Take a cube of the material having its faces normal to the principal directions, and apply a stress σ_1 along the 1 direction. Since $E = \dfrac{\sigma_1}{\varepsilon_1}$, $\varepsilon_1 = \dfrac{\sigma_1}{E}$. Now apply a stress σ_2 along the 2 direction. This will produce a strain along the 2 direction of $\varepsilon_2 = \dfrac{\sigma_2}{E}$. In turn, this strain will produce a strain along the 1 direction of $\varepsilon_1 = -\upsilon\varepsilon_2 = -\upsilon\dfrac{\sigma_2}{E}$. The net strain along the 1 direction now becomes $\varepsilon_1 = \dfrac{\sigma_1}{E} - \upsilon\dfrac{\sigma_2}{E}$. Next apply a stress σ_3 along the 3 direction, and repeat as before to find the total net value of ε_1, as given by the first equation. Repeat this procedure to find the equations for ε_2 and ε_3.

Sum the three principal strains using the three equations. It is then straightforward to show that

$$\Delta = \frac{(1-2\upsilon)}{E}(\sigma_1 + \sigma_2 + \sigma_3).$$

The dilatation is zero when $\upsilon = 1/2$.

3.14. For uniaxial tension, $\Delta = \dfrac{(1-2\upsilon)}{E}\sigma_1 = (1-2\upsilon)\varepsilon_1.$

The volume changes are therefore $0.4\varepsilon_1$, ε_1, and 0.

3.15. From Eqn (3.8), $K = -\dfrac{p}{\Delta}$. For hydrostatic pressure loading, the equation

$$\Delta = \frac{(1-2\upsilon)}{E}(\sigma_1 + \sigma_2 + \sigma_3) \qquad \text{becomes} \qquad \Delta = -\frac{(1-2\upsilon)}{E}(3p). \qquad \text{Then}$$

$$K = -p \times \frac{E}{-(1-2\upsilon)3p} = \frac{E}{3(1-2\upsilon)}.$$

This shows that $K = E$ when $\upsilon = 1/3$.

3.16. The solid rubber sole is very resistant to being compressed, because it is restrained against lateral Poisson's ratio expansion by being glued to the relatively stiff sole. However, the moulded surface has a much lower resistance to being compressed, because the lateral Poisson's ratio expansion of each separate rubber cube can occur without constraint (provided the gaps between adjacent cubes do not close-us completely). So your colleague is correct.

3.17. The axial force applied to the cork to push it into the bottle results in a zero lateral Poisson's ratio expansion, so it does not become any harder to push the cork into the neck of the bottle. However, the axial force applied to the rubber bung results in a large lateral Poisson's ratio expansion, which can make it almost impossible to force the bung into the neck of the bottle.

4. 1. Refer to Fig. 4.11.

$$\text{Force } F \text{ between atoms} = \frac{dU}{dr}.$$

At the equilibrium distance, r_o, the energy U is a minimum (i.e. F is zero, and U is the "dissociation energy" U_o).

$$\frac{dU}{dr} = \frac{mA}{r^{m+1}} - \frac{nB}{r^{n+1}} = 0,$$

$$\text{or } B = \frac{m}{n} r_o^{n-m} A.$$

$$U_o = -\frac{A}{r_o^m} + \frac{1}{r_o^n} \cdot \frac{m}{n} r_o^{n-m} A$$

$$= -\frac{A}{r_o^m}\left(1 - \frac{m}{n}\right).$$

Now, for $r_o = 0.3$ nm, $U_o = -4$ eV.

$$A = 4 \cdot \frac{5}{4}(0.3)^2 = \underline{0.45\,\text{eV nm}^2}$$

$$= \underline{7.2 \times 10^{-20}\,\text{J nm}^2}.$$

$B = \frac{1}{5}(0.3)^8 \times 0.45 = 0.59 \times 10^{-5}\,\text{eV nm}^{10} = \underline{9.4 \times 10^{-25}\,\text{J nm}^{10}}$. Max force is at $\frac{d^2U}{dr^2} = 0$.

i.e. at value of r given by

$$-\frac{m(m+1)A}{r^{m+2}} + \frac{n(n+1)B}{r^{n+2}} = 0 \text{ which is } r = \left\{ \frac{B}{A} \frac{n}{m} \frac{(n+1)}{(m+1)} \right\}^{\frac{1}{n-m}}$$

$$= \left\{ \frac{n+1}{m+1} \right\}^{\frac{1}{n-m}} \quad r_0 = \left(\frac{11}{3} \right)^{\frac{1}{8}} \times 0.3 = \underline{0.352\,\text{nm}}.$$

and Force $= \dfrac{\mathrm{d}U}{\mathrm{d}r} = \dfrac{mA}{r^{m+1}} \left\{ 1 - \dfrac{r_0^{n-m}}{r^{n-m}} \right\}$

$$= \frac{2 \times 0.45}{(0.352)^3} \left\{ 1 - \frac{(0.3)^8}{(0.352)^8} \right\} = 14.9\,\text{eV nm}^{-1}$$

$$= \frac{14.9 \times 1.602 \times 10^{-19}}{10^{-9}} \,\text{J m}^{-1}$$

$$= \underline{2.39 \times 10^{-9}\,\text{N}.}$$

4.2. The term $-A/r^m$ is an attractive potential which depends on the type of bonding. The B/r^n term is a repulsive potential due to charge-cloud overlap and diminished screening of the nuclei (see Section 4.2).

4.3. The values of $\tilde{A}$ are shown below. The mean is 88. The calculated values of the moduli are

Material	Calculated from $88\,kT_M/\Omega$	Measured
Ice	$1.0 \times 10^{10}\,\text{N m}^{-2}$	$7.7 \times 10^9\,\text{N m}^{-2}$
Diamond	$9.0 \times 10^{11}\,\text{N m}^{-2}$	$1.0 \times 10^{12}\,\text{N m}^{-2}$

The calculated values, for these extremes of elastic behaviour, are close to the measured values. The important point is that the moduli are roughly propertional to absolute melting temperatures.

$\tilde{A}$ values: Ni, 98; Cu, 78; Ag, 76; Al, 89; Pb, 51; Fe, 96; V, 61; Cr, 116; Nb, 48; Mo, 138; Ta, 72; W, 127.

4.4. From eqn (4.3), $\dfrac{\mathrm{d}U}{\mathrm{d}r} = \dfrac{q^2}{4\pi\varepsilon_0 r^2} - \dfrac{nB}{r^{n+1}}$.

Setting $\left(\dfrac{\mathrm{d}U}{\mathrm{d}r} \right)_{r=r_0} = 0$ gives $\dfrac{q^2}{4\pi\varepsilon_0 r_0^2} = \dfrac{nB}{r_0^{n+1}}$.

Therefore, $B = \dfrac{q^2 r_0^{n-1}}{4\pi n \varepsilon_0}$ as required.

From eqn (4.8), $S_0 = \left(\dfrac{\mathrm{d}^2 U}{\mathrm{d}r^2} \right)_{r=r_0}$.

From eqn (4.3) and the result derived above for B,

$$U(r) = U_i - \frac{q^2}{4\pi\varepsilon_0 r} + \frac{q^2 r_0^{n-1}}{4\pi n \varepsilon_0} \times \frac{1}{r^n} .$$

$$\frac{dU}{dr} = \frac{q^2}{4\pi\varepsilon_0 r^2} - \frac{q^2 r_0^{n-1}}{4\pi\varepsilon_0 r^{n+1}}.$$

$$\frac{d^2U}{dr^2} = -\frac{2q^2}{4\pi\varepsilon_0 r^3} + \frac{q^2(n+1)r_0^{n-1}}{4\pi\varepsilon_0 r^{n+2}}.$$

Therefore, $S_0 = -\dfrac{2q^2}{4\pi\varepsilon_0 r_0^3} + \dfrac{q^2(n+1)}{4\pi\varepsilon_0 r_0^3} = \left(\dfrac{q^2}{4\pi\varepsilon_0 r_0^3}\right)(-2+n+1).$

Finally, $S_0 = \dfrac{(n-1)q^2}{4\pi\varepsilon_0 r_0^3} = \dfrac{\alpha q^2}{4\pi\varepsilon_0 r_0^3}.$

4.5. See Section 4.2, last paragraph.

4.6. See Fig. 4.10.

4.7. See Section 4.5, paragraphs 1 to 3.

5. 1. (a) Let the spheres have a diameter of 1. Then (referring to Fig. 5.3) the unit cell has an edge length $\sqrt{2}$, and a volume $2\sqrt{2}$. It contains 4 atoms, with a total volume $4\pi/6$. Hence the density, ρ, is given by

$$\rho = \frac{4\pi}{6.2\sqrt{2}} = \underline{0.740}.$$

(b) Glassy nickel is less dense than crystalline nickel by the factor 0.636/0.740. The density is therefore $8.90(0.636/0.740) = \underline{7.65 \text{ Mg m}^{-3}}$.

5.2. (a)

(100)   (110)

(111)   (210)