SOLUTION MANUAL CONTENTS

12–1.

A baseball is thrown downward from a 50-ft tower with an initial speed of 18 ft/s. Determine the speed at which it hits the ground and the time of travel.

SOLUTION

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$v_2^2 = (18)^2 + 2(32.2)(50 - 0)$

$v_2 = 59.532 = 59.5\text{ ft/s}$ **Ans.**

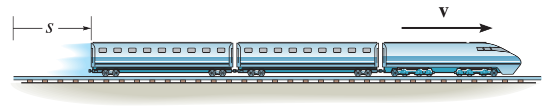
$v_2 = v_1 + a_c t$

$59.532 = 18 + 32.2(t)$

$t = 1.29\text{ s}$ **Ans.**

12–2.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\,v^{-4})\ \text{m/s}^2$, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.



SOLUTION

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}(v^5 - 32)$$

$$v = 3.925\ \text{m/s} = 3.93\ \text{m/s} \qquad \textbf{Ans.}$$

$$ads = vdv$$

$$ds = \frac{vdv}{a} = \frac{1}{60}v^5\,dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\,dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg|_2^{3.925}$$

$$= 9.98\ \text{m} \qquad \textbf{Ans.}$$

12–3.

From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/s (55 mi/h) when it hits the ground? Each floor is 12 ft higher than the one below it. (*Note:* You may want to remember this when traveling 55 mi/h.)

SOLUTION

$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$80.7^2 = 0 + 2(32.2)(s - 0)$$

$$s = 101.13 \text{ ft}$$

$$\text{\# of floors} = \frac{101.13}{12} = 8.43$$

The car must be dropped from the 9th floor. **Ans.**

***12–4.**

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})\text{ hr} = 30\text{ s}$ **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792\text{ km} = 792\text{ m}$ **Ans.**

12–5.

A bus starts from rest with a constant acceleration of 1 m/s^2. Determine the time required for it to attain a speed of 25 m/s and the distance traveled.

SOLUTION

***Kinematics*:**

$v_0 = 0$, $v = 25$ m/s, $s_0 = 0$, and $a_c = 1 \text{ m/s}^2$.

$(\overset{+}{\rightarrow})$ $$v = v_0 + a_c t$$

$$25 = 0 + (1)t$$

$$t = 25 \text{ s}$$ **Ans.**

$(\overset{+}{\rightarrow})$ $$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$25^2 = 0 + 2(1)(s - 0)$$

$$s = 312.5 \text{ m}$$ **Ans.**

12–6.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$$+\downarrow s = s_1 + v_1 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

$$s_A = 64.4 \text{ ft}$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(1)^2$$

$$s_B = 16.1 \text{ ft}$$

$$\Delta s = 64.4 - 16.1 = 48.3 \text{ ft}$$ **Ans.**

12–7.

A bicyclist starts from rest and after traveling along a straight path a distance of 20 m reaches a speed of 30 km/h. Determine his acceleration if it is *constant*. Also, how long does it take to reach the speed of 30 km/h?

SOLUTION

$v_2 = 30 \text{ km/h} = 8.33 \text{ m/s}$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(8.33)^2 = 0 + 2\,a_c(20 - 0)$

$a_c = 1.74 \text{ m/s}^2$ **Ans.**

$v_2 = v_1 + a_c\,t$

$8.33 = 0 + 1.74(t)$

$t = 4.80 \text{ s}$ **Ans.**

*■12–8.

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})\ \text{m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$v = 1.29$ m/s **Ans.**

12–9.

If it takes 3 s for a ball to strike the ground when it is released from rest, determine the height in meters of the building from which it was released. Also, what is the velocity of the ball when it strikes the ground?

SOLUTION

Kinematics:

$v_0 = 0,\ a_c = g = 9.81\ \text{m/s}^2,\ t = 3\ \text{s, and } s = h.$

$(+\downarrow)\qquad v = v_0 + a_c t$

$= 0 + (9.81)(3)$

$= 29.4\ \text{m/s}$ **Ans.**

$(+\downarrow)\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$h = 0 + 0 + \frac{1}{2}(9.81)(3^2)$

$= 44.1\ \text{m}$ **Ans.**

12–10.

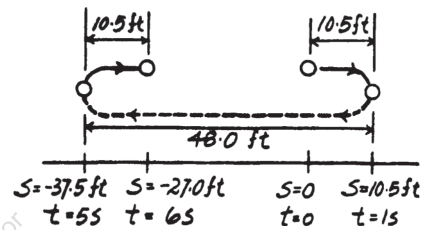
The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

SOLUTION

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft}$$ **Ans.**

Total DistanceTraveled: The velocity of the particle can be determined by applying Eq. 12–1.



$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{\text{tot}} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft}$$ **Ans.**

12–11.

If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2\text{ ft/s}^2$ to the left.

SOLUTION

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20\text{ ft}$$ **Ans.**

***12–12.**

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

SOLUTION

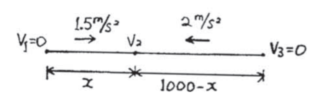
Using formulas of constant acceleration:

$v_2 = 1.5\, t_1$

$x = \frac{1}{2}(1.5)(t_1^2)$

$0 = v_2 - 2\, t_2$

$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$



Combining equations:

$t_1 = 1.33\, t_2; \quad v_2 = 2\, t_2$

$x = 1.33\, t_2^2$

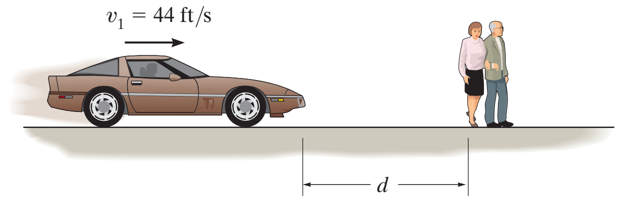
$1000 - 1.33\, t_2^2 = 2\, t_2^2 - t_2^2$

$t_2 = 20.702 \text{ s}; \qquad t_1 = 27.603 \text{ s}$

$t = t_1 + t_2 = 48.3 \text{ s}$ **Ans.**

12–13.

Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



SOLUTION

Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

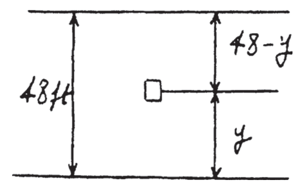
$$0^2 = 44^2 + 2(-2)(d - 132)$$

$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

12–14.

A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at $0.6\ \text{ft/s}^2$, decelerate at $0.3\ \text{ft/s}^2$, and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.

SOLUTION



$+\uparrow \quad v^2 = v_0^2 + 2\,a_c\,(s - s_0)$

$v_{max}^2 = 0 + 2(0.6)(y - 0)$

$0 = v_{max}^2 + 2(-0.3)(48 - y)$

$0 = 1.2\,y - 0.6(48 - y)$

$y = 16.0\ \text{ft}, \qquad v_{max} = 4.382\ \text{ft/s} < 8\ \text{ft/s}$

$+\uparrow \quad v = v_0 + a_c\,t$

$4.382 = 0 + 0.6\,t_1$

$t_1 = 7.303\ \text{s}$

$0 = 4.382 - 0.3\,t_2$

$t_2 = 14.61\ \text{s}$

$t = t_1 + t_2 = 21.9\ \text{s}$ **Ans.**

12–15.

A train starts from rest at station A and accelerates at 0.5 m/s^2 for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at 1 m/s^2 until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5 \text{ m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900 \text{ m}$$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30 \text{ m/s}$$

For stage (2) motion, $v_0 = 30$ m/s, $s_0 = 900$ m, $a_c = 0$ and $t = 15(60) = 900$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900 \text{ m}$$

For stage (3) motion, $v_0 = 30$ m/s, $v = 0$, $s_0 = 27\,900$ m and $a_c = -1 \text{ m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30 \text{ s}$$

$$\overset{+}{\rightarrow} \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350 \text{ m} = 28.4 \text{ km}$$ **Ans.**

***12–16.**

A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.

SOLUTION

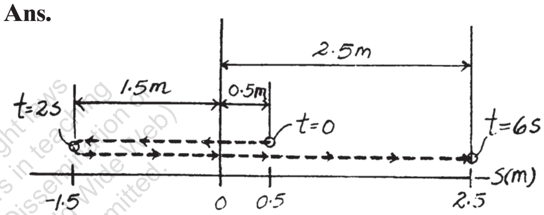
$$\Delta s = (s_C - s_A) = 2 \text{ m}$$

$$s_T = (0.5 + 1.5 + 1.5 + 2.5) = 6 \text{ m}$$

$$t = (2 + 4) = 6 \text{ s}$$

$$v_{avg} = \frac{\Delta s}{t} = \frac{2}{6} = 0.333 \text{ m/s}$$ **Ans.**

$$(v_{sp})_{avg} = \frac{s_T}{t} = \frac{6}{6} = 1 \text{ m/s}$$ **Ans.**



12–17.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_2^v dv = \int_0^t (2\,t - 1)\,dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2\,t + 1$$

When $t = 6$ s,

$$v = 32\ \text{m/s}$$ **Ans.**

$$s = 67\ \text{m}$$ **Ans.**

Since $v \neq 0$ then

$$d = 67 - 1 = 66\ \text{m}$$ **Ans.**

12–18.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

SOLUTION

$v = 60(1 - e^{-t})$

$$\int_0^s ds = \int v\,dt = \int_0^3 60(1 - e^{-t})dt$$

$s = 60(t + e^{-t})\big|_0^3$

$s = 123$ ft **Ans.**

$$a = \frac{dv}{dt} = 60(e^{-t})$$

At $t = 3$ s

$a = 60e^{-3} = 2.99\ \text{ft/s}^2$ **Ans.**

12–19.

A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its position when $t = 6$ s if $s = 5$ m when $t = 0$.

SOLUTION

$$\frac{ds}{dt} = \frac{5}{4 + s}$$

$$\int_5^s (4 + s)\, ds = \int_0^t 5\, dt$$

$$4\,s + 0.5\,s^2 - 32.5 = 5\,t$$

When $t = 6$ s,

$$s^2 + 8\,s - 125 = 0$$

Solving for the positive root

$$s = 7.87 \text{ m}$$ **Ans.**

***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

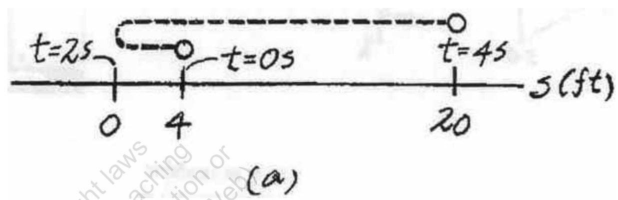
Position: The position of the particle can be determined by integrating the kinematic equation $ds = v\,dt$ using the initial condition $s = 4$ ft when $t = 0$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{4\text{ ft}}^{s} ds = \int_0^t \left(3t^2 - 6t\right)dt$$

$$s\Big|_{4\text{ ft}}^{s} = (t^3 - 3t^2)\Big|_0^t$$

$$s = \left(t^3 - 3t^2 + 4\right)\text{ ft}$$



When $t = 4$ s,

$$s|_{4\text{ s}} = 4^3 - 3(4^2) + 4 = 20\text{ ft} \qquad \textbf{Ans.}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = 3t^2 - 6t = 0$$

$$t(3t - 6) = 0$$

$$t = 0 \text{ and } t = 2\text{ s}$$

The position of the particle at $t = 0$ and 2 s is

$$s|_{0\text{ s}} = 0 - 3\left(0^2\right) + 4 = 4\text{ ft}$$

$$s|_{2\text{ s}} = 2^3 - 3\left(2^2\right) + 4 = 0$$

Using the above result, the path of the particle shown in Fig. *a* is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24\text{ ft} \qquad \textbf{Ans.}$$

Acceleration:

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 6t)$$

$$a = (6t - 6)\text{ ft/s}^2$$

When $t = 2$ s,

$$a|_{t=2\text{ s}} = 6(2) - 6 = 6\text{ ft/s}^2 \rightarrow \qquad \textbf{Ans.}$$

12–21.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \to \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \to \infty$, $\frac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \to 1$. Then, from Eq. (1)

$$v_{max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

12–22.

The position of a particle on a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint*: Plot the path to determine the total distance traveled.

SOLUTION

$s = t^3 - 9t^2 + 15t$

$v = \dfrac{ds}{dt} = 3t^2 - 18t + 15$

$v = 0$ when $t = 1$ s and $t = 5$ s

$t = 0, \; s = 0$

$t = 1$ s, $\quad s = 7$ ft

$t = 5$ s, $s = -25$ ft

$t = 6$ s, $s = -18$ ft **Ans.**

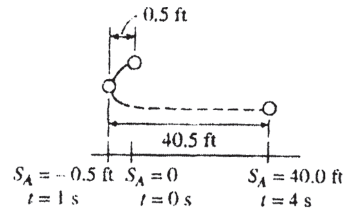
$s_T = 7 + 7 + 25 + (25 - 18) = 46$ ft **Ans.**

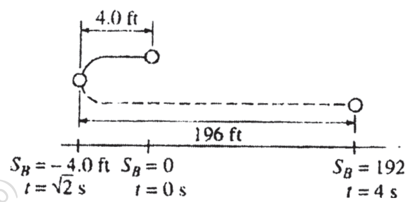
12–23.

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)\ \text{ft/s}^2$ and $a_B = (12t^2 - 8)\ \text{ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

SOLUTION

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.





$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determined using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$

$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\,s} = 1^3 - \frac{3}{2}(1^2) = -0.500\ \text{ft}$$

$$s_A|_{t=4\,s} = 4^3 - \frac{3}{2}(4^2) = 40.0\ \text{ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0\ \text{ft}$$ **Ans.**

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4\ \text{ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192\ \text{ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200\ \text{ft}$$ **Ans.**

At $t = 4$ s the distance beween A and B is

$$\Delta s_{AB} = 192 - 40 = 152\ \text{ft}$$ **Ans.**

***12–24.**

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\,ds = \int_0^t -4\,dt$$

$$-s^{-1}\big|_2^s = -4t\big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4\left(\frac{2}{8t + 1}\right)^2 = -\frac{16}{(8t + 1)^2} \text{ m/s}$$ **Ans.**

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$ **Ans.**

12–25.

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$(+\downarrow)$

$$dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$(+\downarrow)$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

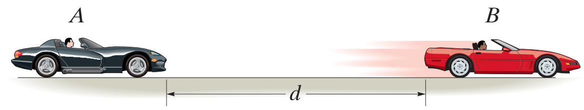
$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m} \qquad \textbf{Ans.}$$

12–26.

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



SOLUTION

Motion of car A:

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right| \qquad \textbf{Ans.}$$

12–27.

A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of $a = (-1.5v^{1/2})\ \text{m/s}^2$, where v is in m/s, determine the distance it travels before it stops.

SOLUTION

$$a = \frac{dv}{dt} = -1.5v^{\frac{1}{2}}$$

$$\int_4^v v^{-\frac{1}{2}}\,dv = \int_0^t -1.5\,dt$$

$$2v^{\frac{1}{2}}\Big|_4^v = -1.5t\Big|_0^t$$

$$2\left(v^{\frac{1}{2}} - 2\right) = -1.5t$$

$$v = (2 - 0.75t)^2\ \text{m/s} \qquad \textbf{(1)}$$

$$\int_0^s ds = \int_0^t (2 - 0.75t)^2\,dt = \int_0^t (4 - 3t + 0.5625t^2)\,dt$$

$$s = 4t - 1.5t^2 + 0.1875t^3 \qquad \textbf{(2)}$$

From Eq. (1), the particle will stop when

$$0 = (2 - 0.75t)^2$$

$$t = 2.667\ \text{s}$$

$$s\big|_{t=2.667} = 4(2.667) - 1.5(2.667)^2 + 0.1875(2.667)^3 = 3.56\ \text{m} \qquad \textbf{Ans.}$$

***12–28.**

A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its deceleration when $s = 2$ m.

SOLUTION

$$v = \frac{5}{4 + s}$$

$$v\,dv = a\,ds$$

$$dv = \frac{-5\,ds}{(4 + s)^2}$$

$$\frac{5}{(4 + s)}\left(\frac{-5\,ds}{(4 + s)^2}\right) = a\,ds$$

$$a = \frac{-25}{(4 + s)^3}$$

When $s = 2$ m

$$a = -0.116 \text{ m/s}^2$$ **Ans.**

12–29.

A particle moves along a straight line with an acceleration $a = 2v^{1/2}\ \text{m/s}^2$, where v is in m/s. If $s = 0$, $v = 4$ m/s when $t = 0$, determine the time for the particle to achieve a velocity of 20 m/s. Also, find the displacement of particle when $t = 2$ s.

SOLUTION

Velocity:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{2v^{1/2}}$$

$$t\Big|_0^t = v^{1/2}\Big|_4^v$$

$$t = v^{1/2} - 2$$

$$v = (t + 2)^2$$

When $v = 20$ m/s,

$$20 = (t + 2)^2$$

$$t = 2.47 \text{ s}$$ **Ans.**

Position:

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t (t + 2)^2\,dt$$

$$s\Big|_0^s = \frac{1}{3}(t + 2)^3\Big|_0^t$$

$$s = \frac{1}{3}[(t + 2)^3 - 2^3]$$

$$= \frac{1}{3}t(t^2 + 6t + 12)$$

When $t = 2$ s,

$$s = \frac{1}{3}(2)[(2)^2 + 6(2) + 12]$$

$$= 18.7 \text{ m}$$ **Ans.**

12–30.

As a train accelerates uniformly it passes successive kilometer marks while traveling at velocities of 2 m/s and then 10 m/s. Determine the train's velocity when it passes the next kilometer mark and the time it takes to travel the 2-km distance.

SOLUTION

Kinematics: For the first kilometer of the journey, $v_0 = 2$ m/s, $v = 10$ m/s, $s_0 = 0$, and $s = 1000$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$10^2 = 2^2 + 2a_c(1000 - 0)$$

$$a_c = 0.048 \text{ m/s}^2$$

For the second kilometer, $v_0 = 10$ m/s, $s_0 = 1000$ m, $s = 2000$ m, and $a_c = 0.048$ m/s^2. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 10^2 + 2(0.048)(2000 - 1000)$$

$$v = 14 \text{ m/s}$$ **Ans.**

For the whole journey, $v_0 = 2$ m/s, $v = 14$ m/s, and $a_c = 0.048$ m/s^2. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

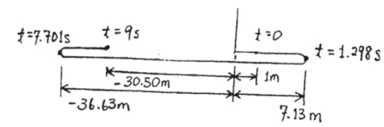
$$14 = 2 + 0.048t$$

$$t = 250 \text{ s}$$ **Ans.**

12–31.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION



$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

$v - 10 = t^2 - 9t$

$v = t^2 - 9t + 10$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$$

$$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$$

Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $s = 7.13$ m

When $t = 7.701$ s, $s = -36.63$ m

When $t = 9$ s, $s = -30.50$ m

(a) $s = -30.5$ m **Ans.**

(b) $s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$s_{Tot} = 56.0$ m **Ans.**

(c) $v = 10$ m/s **Ans.**

***12–32.**

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4}s^{1/2}ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\frac{1}{6}s^{3/2}\right|_1^s$$

$$v = \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2} \text{ m/s}$$

When $s = 2$ m, $v = 0.781$ m/s. **Ans.**

12–33.

At $t = 0$ bullet A is fired vertically with an initial (muzzle) velocity of 450 m/s. When $t = 3$ s, bullet B is fired upward with a muzzle velocity of 600 m/s. Determine the time t, after A is fired, as to when bullet B passes bullet A. At what altitude does this occur?

SOLUTION

$$+\uparrow s_A = (s_A)_0 + (v_A)_0\, t + \frac{1}{2} a_c\, t^2$$

$$s_A = 0 + 450\, t + \frac{1}{2}(-9.81)\, t^2$$

$$+\uparrow s_B = (s_B)_0 + (v_B)_0\, t + \frac{1}{2} a_c\, t^2$$

$$s_B = 0 + 600(t - 3) + \frac{1}{2}(-9.81)(t - 3)^2$$

Require $s_A = s_B$

$$450\, t - 4.905\, t^2 = 600\, t - 1800 - 4.905\, t^2 + 29.43\, t - 44.145$$

$t = 10.3$ s **Ans.**

$h = s_A = s_B = 4.11$ km **Ans.**

12–34.

A boy throws a ball straight up from the top of a 12-m high tower. If the ball falls past him 0.75 s later, determine the velocity at which it was thrown, the velocity of the ball when it strikes the ground, and the time of flight.

SOLUTION

***Kinematics*:** When the ball passes the boy, the displacement of the ball in equal to zero.

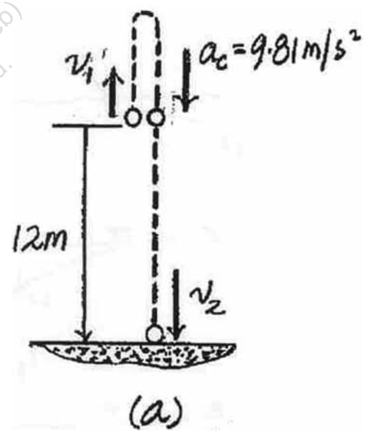
Thus, $s = 0$. Also, $s_0 = 0$, $v_0 = v_1$, $t = 0.75$ s, and $a_c = -9.81\text{ m/s}^2$.

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 0 + v_1(0.75) + \frac{1}{2}(-9.81)(0.75^2)$$

$$v_1 = 3.679\text{ m/s} = 3.68\text{ m/s} \qquad \textbf{Ans.}$$

When the ball strikes the ground, its displacement from the roof top is $s = -12$ m. Also, $v_0 = v_1 = 3.679$ m/s, $t = t_2$, $v = v_2$, and $a_c = -9.81\text{ m/s}^2$.



$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-12 = 0 + 3.679t_2 + \frac{1}{2}(-9.81)t_2^2$$

$$4.905t_2^2 - 3.679t_2 - 12 = 0$$

$$t_2 = \frac{3.679 \pm \sqrt{(-3.679)^2 - 4(4.905)(-12)}}{2(4.905)}$$

Choosing the positive root, we have

$$t_2 = 1.983\text{ s} = 1.98\text{ s} \qquad \textbf{Ans.}$$

Using this result,

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$v_2 = 3.679 + (-9.81)(1.983)$$

$$= -15.8\text{ m/s} = 15.8\text{ m/s}\downarrow \qquad \textbf{Ans.}$$

12–35.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

***12–36.**

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^{v} v^{-3}\,dv = \int_0^t -k\,dt$$

$$-\frac{1}{2}(v^{-2} - v_0^{-2}) = -kt$$

$$v = \left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{-\frac{1}{2}}$$ **Ans.**

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \frac{2\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}{2k}\Bigg|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{v_0}\right]$$ **Ans.**

12–37.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\ \text{m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0R^2\int_0^\infty \frac{dy}{(R + y)^2}$$

$$\frac{v^2}{2}\bigg|_v^0 = \frac{g_0R^2}{R + y}\bigg|_0^\infty$$

$$v = \sqrt{2g_0R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\ \text{m/s} = 11.2\ \text{km/s}$$ **Ans.**

12–38.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–37), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–37.

SOLUTION

From Prob. 12–37,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R+y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow$$ **Ans.**

12–39.

A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

SOLUTION

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$

The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{(1)}$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. (1)). Thus, the distance for this part of motion is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{\text{Tot}} = 2000$, then

$$s_{\text{Tot}} = s_1 + s_2$$

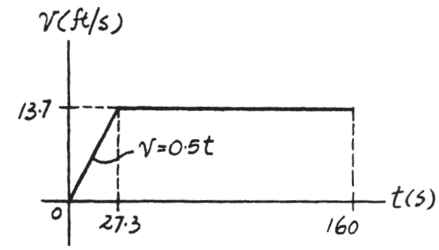
$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

Choose a root that is less than 160 s, then

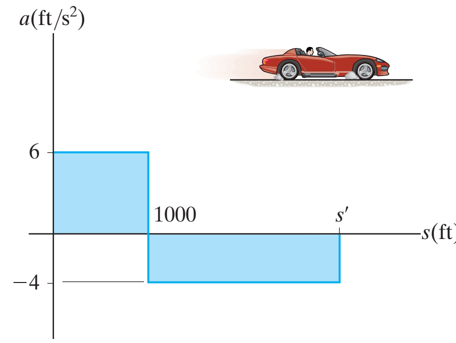
$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

v − t Graph: The equation for the velocity is given by Eq. (1). When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.



***12–40.**

A sports car travels along a straight road with an acceleration-deceleration described by the graph. If the car starts from rest, determine the distance s' the car travels until it stops. Construct the $v-s$ graph for $0 \leq s \leq s'$.



SOLUTION

***v* – *s* Graph:** For $0 \leq s < 1000$ ft, the initial condition is $v = 0$ at $s = 0$.

$(\stackrel{+}{\rightarrow})$ $$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 6\,ds$$

$$\frac{v^2}{2} = 6s$$

$$v = \left(\sqrt{12}s^{1/2}\right)\text{ ft/s}$$

When $s = 1000$ ft,

$$v = \sqrt{12}(1000)^{1/2} = 109.54\text{ ft/s} = 110\text{ ft/s}$$

For $1000\text{ ft} < s \leq s'$, the initial condition is $v = 109.54$ ft/s at $s = 1000$ ft.

$(\stackrel{+}{\rightarrow})$ $$v\,dv = a\,ds$$

$$\int_{109.54\text{ ft/s}}^v v\,dv = \int_{1000\text{ ft}}^s -4\,ds$$

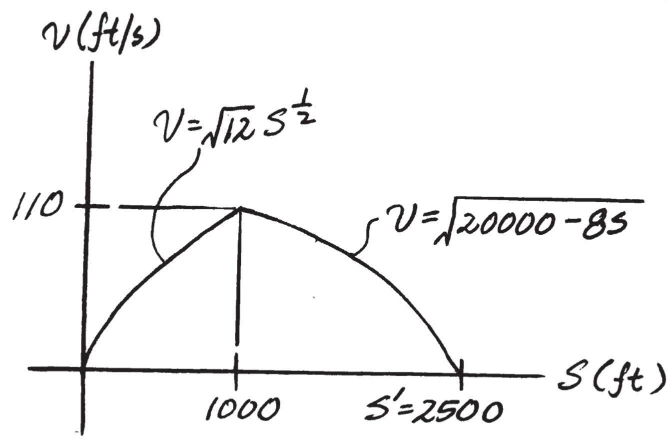
$$\left.\frac{v^2}{2}\right|_{109.54\text{ ft/s}}^v = -4s\Big|_{1000\text{ ft}}^s$$

$$v = \left(\sqrt{20\,000 - 8s}\right)\text{ ft/s}$$

When $v = 0$,

$$0 = \sqrt{20\,000 - 8s'} \qquad s' = 2500\text{ ft}$$ **Ans.**

The v–s graph is shown in Fig. *a*.



(a)

12–41.

A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the $v-t$ graph and determine the maximum speed of the train.

SOLUTION

For stage (1) motion,

$$(\overset{+}{\rightarrow}) \qquad v_1 = v_0 + (a_c)_1 t$$

$$v_{max} = 0 + (a_c)_1 t_1$$

$$v_{max} = (a_c)_1 t_1 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad v_1^2 = v_0^2 + 2(a_c)_1(s_1 - s_0)$$

$$v_{max}{}^2 = 0 + 2(a_c)_1(1000 - 0)$$

$$(a_c)_1 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(2)}$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

$$t_1 = \frac{2000}{v_{max}} \qquad \textbf{(3)}$$

For stage (2) motion, the train travels with the constant velocity of v_{max} for $t = (t_2 - t_1)$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s_2 = s_1 + v_1 t + \frac{1}{2}(a_c)_2 t^2$$

$$1000 + 2000 = 1000 + v_{max}(t_2 - t_1) + 0$$

$$t_2 - t_1 = \frac{2000}{v_{max}} \qquad \textbf{(4)}$$

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v_3 = v_2 + (a_c)_3 t$$

$$0 = v_{max} - (a_c)_3(360 - t_2)$$

$$v_{max} = (a_c)_3(360 - t_2) \qquad \textbf{(5)}$$

$$(\overset{+}{\rightarrow}) \qquad v_3^2 = v_2^2 + 2(a_c)_3(s_3 - s_2)$$

$$0 = v_{max}{}^2 + 2[-(a_c)_3](4000 - 3000)$$

$$(a_c)_3 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(6)}$$

Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

$$360 - t_2 = \frac{2000}{v_{max}} \qquad \textbf{(7)}$$

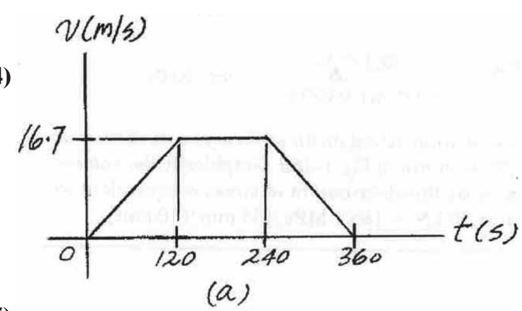
Solving Eqs. (3), (4), and (7), we have

$$t_1 = 120 \text{ s} \qquad t_2 = 240 \text{ s}$$

$$v_{max} = 16.7 \text{ m/s} \qquad \textbf{Ans.}$$

Based on the above results, the $v-t$ graph is shown in Fig. a.

$v = v_{max}$ for 2 min < t < 4 min



12–42.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \le t \le 4$ s.

SOLUTION

a–t Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$
$$a = (2t - 4)\ \text{m/s}^2$$

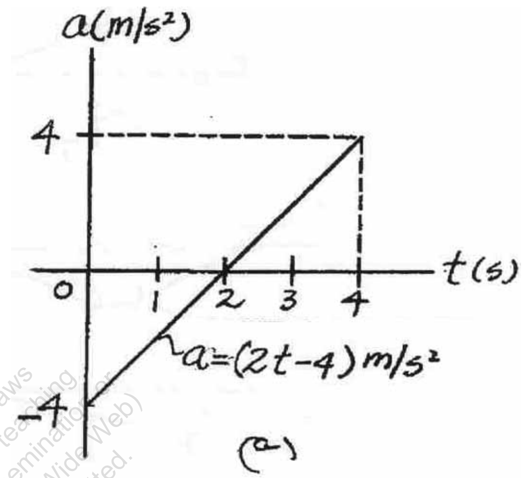
Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \text{m/s}^2$$

$$a|_{t=2} = 0$$

$$a|_{t=4\text{ s}} = 2(4) - 4 = 4\ \text{m/s}^2$$

The $a-t$ graph is shown in Fig. a.



v–t Graph: The slope of the $v-t$ graph is zero when $a = \frac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad t = 2\text{ s}$$

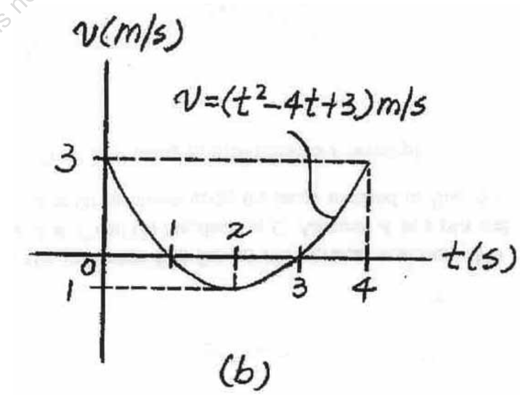
The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\text{ s}} = 0^2 - 4(0) + 3 = 3\ \text{m/s}$$

$$v|_{t=2\text{ s}} = 2^2 - 4(2) + 3 = -1\ \text{m/s}$$

$$v|_{t=4\text{ s}} = 4^2 - 4(4) + 3 = 3\ \text{m/s}$$

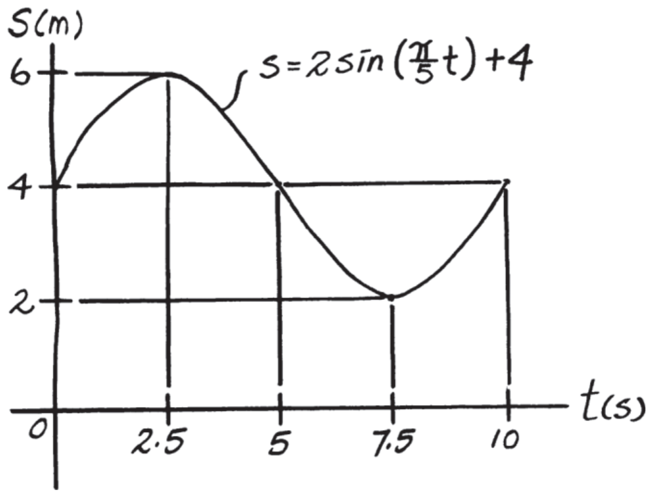
The $v-t$ graph is shown in Fig. b.

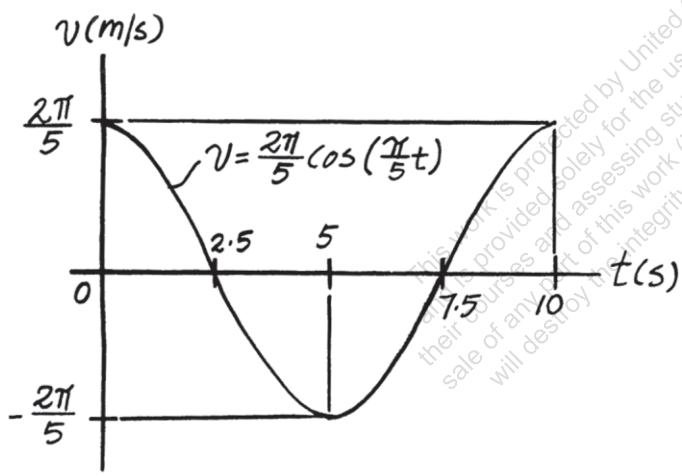


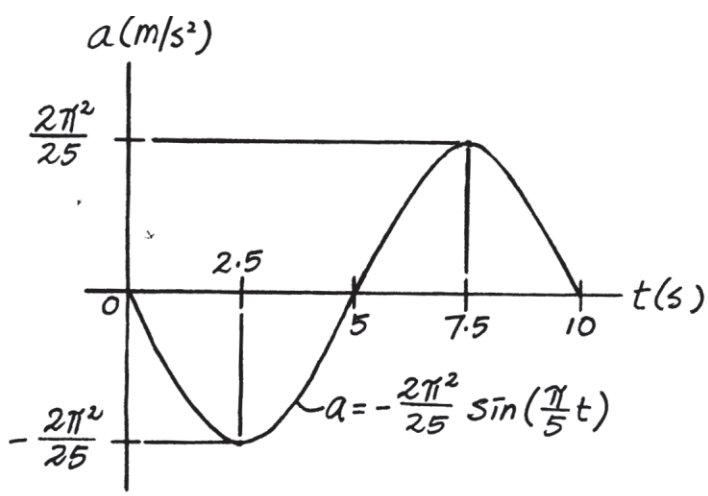
12–43.

If the position of a particle is defined by $s = [2 \sin [(\pi/5)t] + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \le t \le 10$ s.

SOLUTION







***12–44.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

SOLUTION

$v_1 = 0$

$$v_2 = 162\frac{\text{mi}}{\text{h}}\frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 237.6\ \text{ft/s}$$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$

$a_c = 5.64538\ \text{ft/s}^2$

$v_2 = v_1 + a_c t$

$237.6 = 0 + 5.64538\,t$

$t = 42.09 = 42.1\ \text{s}$

$$v_3 = 220\,\frac{\text{mi}}{\text{h}}\frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 322.67\ \text{ft/s}$$

$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$

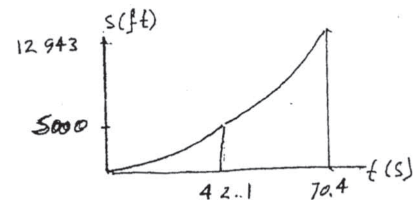
$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$

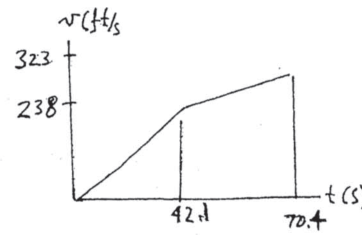
$s = 12\,943.34\ \text{ft}$

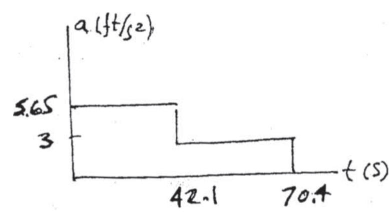
$v_3 = v_2 + a_c t$

$322.67 = 237.6 + 3\,t$

$t = 28.4\ \text{s}$

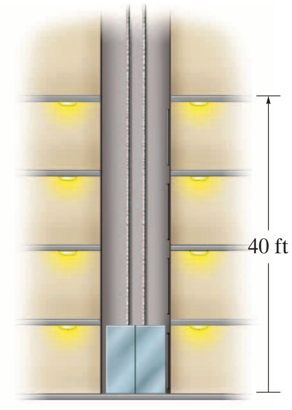






12–45.

The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



SOLUTION

$+\uparrow\; v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5\,t_1$

$+\uparrow\; v_3 = v_2 + a_c\,t$

$0 = v_{max} - 2\,t_2$

Thus

$t_1 = 0.4\,t_2$

$+\uparrow\; s_2 = s_1 + v_1 t_1 + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\,t_1^2$

$+\uparrow\; 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\,t_2^2$

$+\uparrow\; v^2 = v_1^2 + 2\,a_c(s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4h$

Thus,

$10\,h = 160 - 4h$

$h = 11.429$ ft

$v_{max} = 10.69$ ft/s

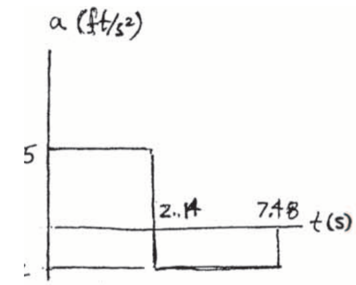
$t_1 = 2.138$ s

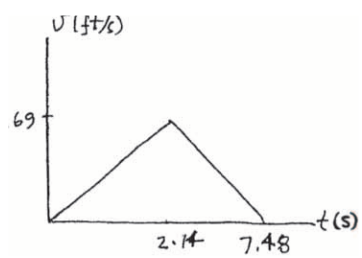
$t_2 = 5.345$ s

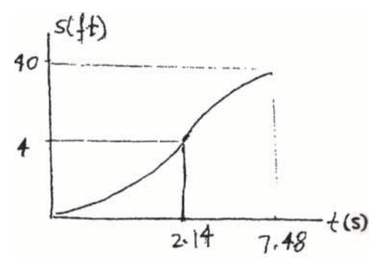
$t = t_1 + t_2 = 7.48$ s **Ans.**

When $t = 2.145$, $v = v_{max} = 10.7$ ft/s

and $h = 11.4$ ft.

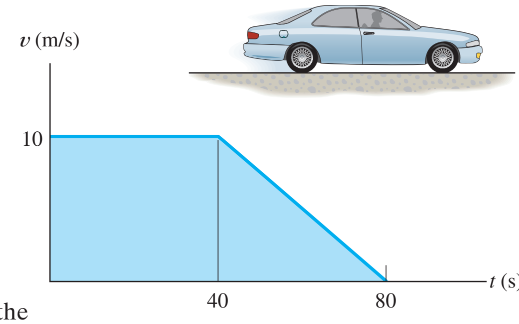






12–46.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v - t$ graph.

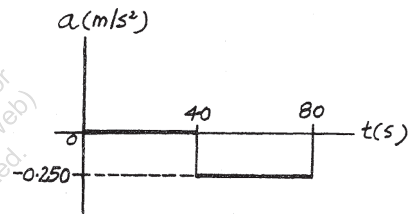
$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m} \qquad \textbf{Ans.}$$

a – t Graph: The acceleration in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 40 \text{ s}$,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80 \text{ s}$, $\dfrac{v - 10}{t - 40} = \dfrac{0 - 10}{80 - 40}$, $v = \left(-\dfrac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$

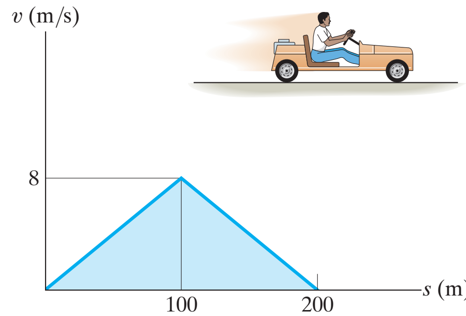


For $0 \leq t < 40$ s, $a = 0$.

For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

12–47.

The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



SOLUTION

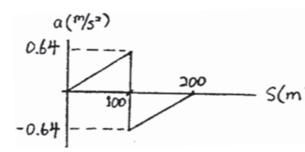
For $0 \leq s < 100$

$v = 0.08\,s, \qquad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\qquad a = 0.32\ \text{m/s}^2$ **Ans.**



For $100 < s < 200$

$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\qquad a = -0.32\ \text{m/s}^2$ **Ans.**

Also,

$v\,dv = a\,ds$

$a = v\left(\frac{dv}{ds}\right)$

At $s = 50$ m,

$a = 4\left(\frac{8}{100}\right) = 0.32\ \text{m/s}^2$ **Ans.**

At $s = 150$ m,

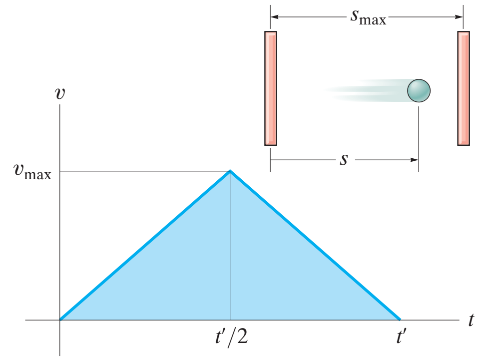
$a = 4\left(\frac{-8}{100}\right) = -0.32\ \text{m/s}^2$ **Ans.**

At $s = 100$ m, a changes from $a_{max} = 0.64\ \text{m/s}^2$

to $a_{min} = -0.64\ \text{m/s}^2$.

***12–48.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of $4\ \text{m/s}^2$. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

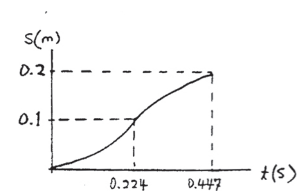
$a_c = 4\ \text{m/s}^2$

$\frac{s}{2} = 100\ \text{mm} = 0.1\ \text{m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\ \text{m/s} \qquad = 0.894\ \text{m/s}$ **Ans.**



$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4(\frac{t'}{2})$

$t' = 0.44721\ \text{s} \qquad = 0.447\ \text{s}$ **Ans.**

$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \frac{1}{2}(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224$ s,

$s = 0.1$ m

$\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$

$v = -4\,t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

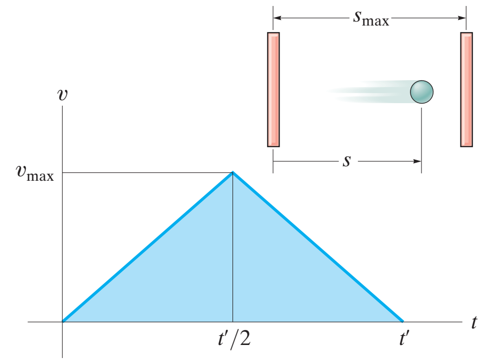
$s = -2\,t^2 + 1.788\,t - 0.2$

When $t = 0.447$ s,

$s = 0.2$ m

12–49.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1$ s,

$v = 100\,t$

$a = \dfrac{dv}{dt} = 100$

$ds = v\,dt$

$\int_0^s ds = \int_0^t 100\,t\,dt$

$s = 50\,t^2$

When $t = 0.1$ s,

$s = 0.5$ m

For $0.1\text{ s} < t < 0.2$ s,

$v = -100\,t + 20$

$a = \dfrac{dv}{dt} = -100$

$ds = v\,dt$

$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)dt$

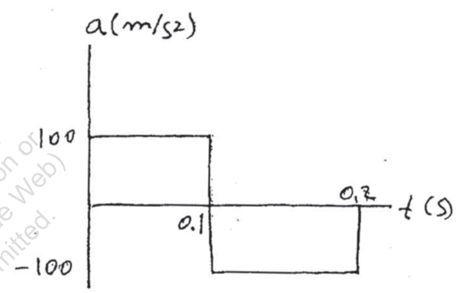
$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$

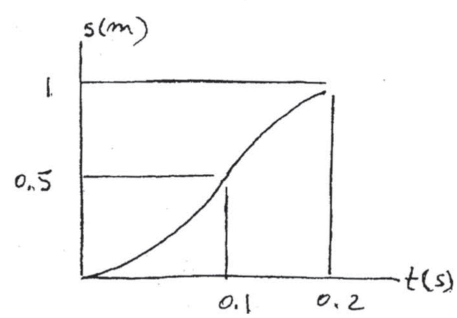
$s = -50\,t^2 + 20\,t - 1$

When $t = 0.2$ s,

$s = 1$ m

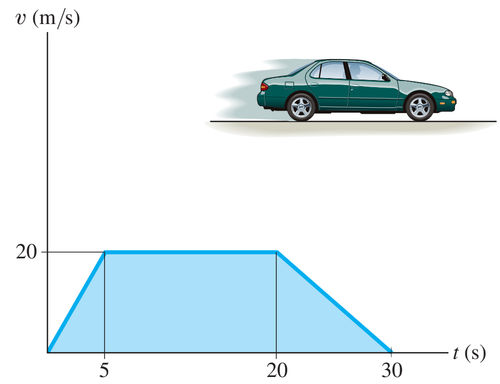
When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.





12–50.

The $v-t$ graph of a car while traveling along a road is shown. Draw the $s-t$ and $a-t$ graphs for the motion.



SOLUTION

$0 \le t \le 5 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{5} = 4 \text{ m/s}^2$

$5 \le t \le 20 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20 - 20}{20 - 5} = 0 \text{ m/s}^2$

$20 \le t \le 30 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0 - 20}{30 - 20} = -2 \text{ m/s}^2$

From the $v-t$ graph at $t_1 = 5$ s, $t_2 = 20$ s, and $t_3 = 30$ s,

$s_1 = A_1 = \dfrac{1}{2}(5)(20) = 50 \text{ m}$

$s_2 = A_1 + A_2 = 50 + 20(20 - 5) = 350 \text{ m}$

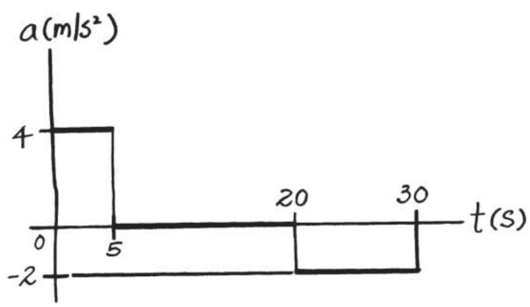
$s_3 = A_1 + A_2 + A_3 = 350 + \dfrac{1}{2}(30 - 20)(20) = 450 \text{ m}$

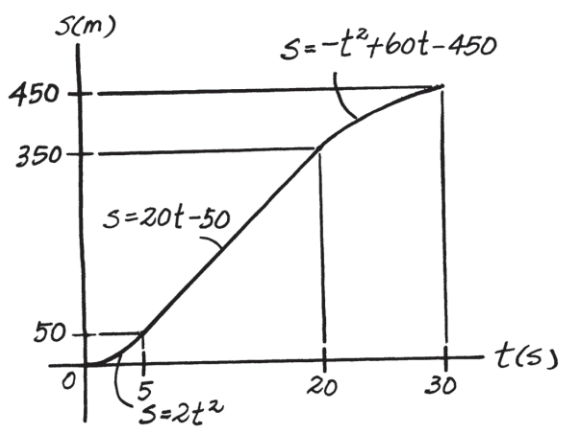
The equations defining the portions of the $s–t$ graph are

$0 \le t \le 5 \text{ s} \qquad v = 4t; \qquad ds = v\,dt; \qquad \int_0^s ds = \int_0^t 4t\,dt; \qquad s = 2t^2$

$5 \le t \le 20 \text{ s} \qquad v = 20; \qquad ds = v\,dt; \qquad \int_{50}^s ds = \int_5^t 20\,dt; \qquad s = 20t - 50$

$20 \le t \le 30 \text{ s} \qquad v = 2(30 - t); \qquad ds = v\,dt; \qquad \int_{350}^s ds = \int_{20}^t 2(30 - t)\,dt; \qquad s = -t^2 + 60t - 450$





For $0 \le t < 5$ s, $a = 4 \text{ m/s}^2$.

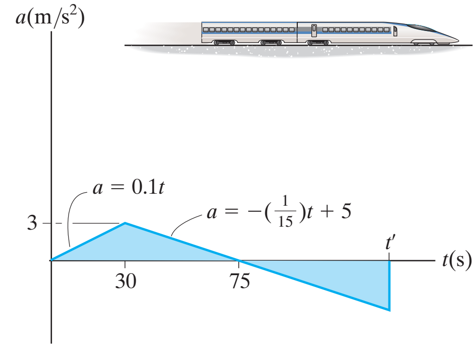
For $20 \text{ s} < t \le 30$ s, $a = -2 \text{ m/s}^2$.

At $t = 5$ s, $s = 50$ m. At $t = 20$ s, $s = 350$ m.

At $t = 30$ s, $s = 450$ m.

12–51.

The a–t graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the v–t and s–t graphs.



SOLUTION

$v-t$ Graph: For the time interval $0 \le t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.1t\,dt$$

$$v = \left(0.05t^2\right)\ \text{m/s}$$

When $t = 30$ s,

$$v\big|_{t=30\text{ s}} = 0.05\left(30^2\right) = 45\ \text{m/s}$$

or the time interval $30\text{ s} < t \le t'$, the initial condition is $v = 45$ m/s at $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_{45\text{ m/s}}^v dv = \int_{30\text{ s}}^t \left(-\frac{1}{15}t + 5\right)dt$$

$$v = \left(-\frac{1}{30}t^2 + 5t - 75\right)\ \text{m/s}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09\text{ s} = 133\text{ s} \qquad \textbf{Ans.}$$

Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a\,dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

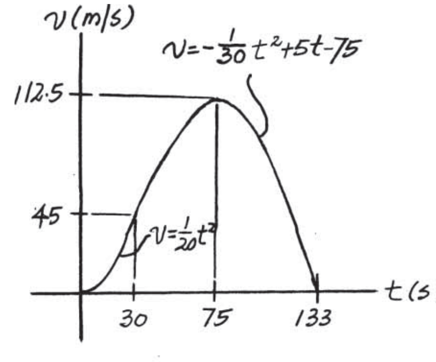
This equation is the same as the one obtained previously.

The slope of the v–$t$graph is zero when $t = 75$ s, which is the instant $a = \dfrac{dv}{dt} = 0$. Thus,

$$v\big|_{t=75\text{ s}} = -\frac{1}{30}\left(75^2\right) + 5(75) - 75 = 112.5\ \text{m/s}$$

12–51. continued

The v–t graph is shown in Fig. *a*.



(a)

$s - t$ Graph: Using the result of v, the equation of the s–t graph can be obtained by integrating the kinematic equation $ds = v dt$. For the time interval $0 \leq t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$$(\overset{+}{\rightarrow}) \qquad ds = v dt$$

$$\int_0^s ds = \int_0^t 0.05t^2\, dt$$

$$s = \left(\frac{1}{60}t^3\right) \text{m}$$

When $t = 30$ s,

$$s\Big|_{t=30\text{ s}} = \frac{1}{60}\left(30^3\right) = 450 \text{ m}$$

For the time interval $30 \text{ s} < t \leq t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v dt$$

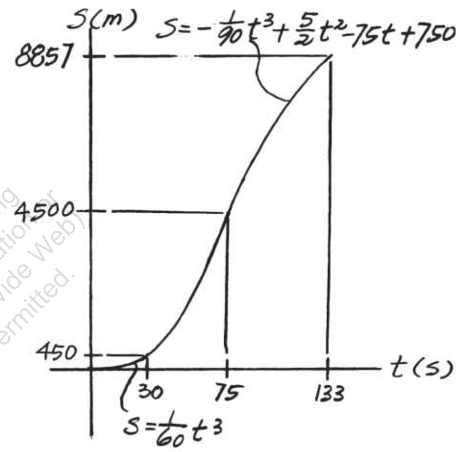
$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right) dt$$

$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right) \text{m}$$

When $t = 75$ s and $t' = 133.09$ s,

$$s\Big|_{t=75\text{ s}} = -\frac{1}{90}\left(75^3\right) + \frac{5}{2}\left(75^2\right) - 75(75) + 750 = 4500 \text{ m}$$

$$s\Big|_{t=133.09\text{ s}} = -\frac{1}{90}\left(133.09^3\right) + \frac{5}{2}\left(133.09^2\right) - 75(133.09) + 750 = 8857 \text{ m} \qquad \textbf{Ans.}$$



(b)

The s–t graph is shown in Fig. *b*.

When $t = 30$ s,

$v = 45$ m/s and $s = 450$ m.

When $t = 75$ s,

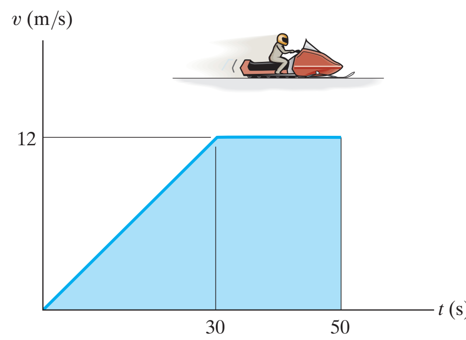
$v = v_{max} = 112.5$ m/s and $s = 4500$ m.

When $t = 133$ s,

$v = 0$ and $s = 8857$ m.

***12–52.**

The snowmobile moves along a straight course according to the v–t graph. Construct the s–t and a–t graphs for the same 50-s time interval. When $t = 0, s = 0$.



SOLUTION

***s–t* Graph:** The position function in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $0 \text{ s} \le t < 30 \text{ s}$, $v = \dfrac{12}{30}t = \left(\dfrac{2}{5}t\right)$ m/s.

$$ds = v dt$$

$$\int_0^s ds = \int_0^t \frac{2}{5} t dt$$

$$s = \left(\frac{1}{5}t^2\right) \text{ m}$$

At $t = 30$ s, $$s = \frac{1}{5}\left(30^2\right) = 180 \text{ m}$$

For time interval **$30 \text{ s} < t \le 50 \text{ s}$**,

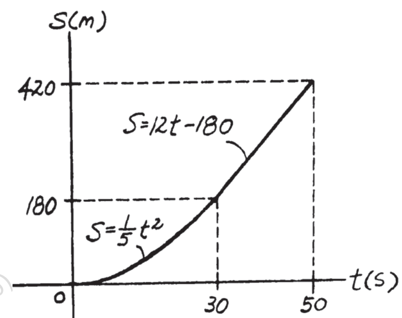
$$ds = v dt$$

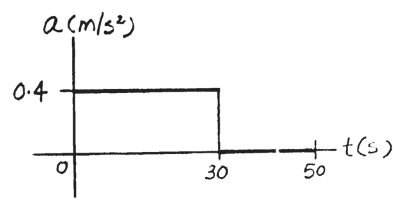
$$\int_{180 \text{ m}}^s ds = \int_{30 \text{ s}}^t 12 dt$$

$$s = (12t - 180) \text{ m}$$

At $t = 50$ s, $$s = 12(50) - 180 = 420 \text{ m}$$

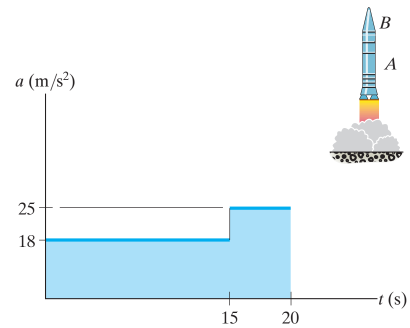
***a–t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval **$0 \text{ s} \le t < 30 \text{ s}$** and **$30 \text{ s} < t \le 50 \text{ s}$**, $a = \dfrac{dv}{dt} = \dfrac{2}{5} = 0.4 \text{ m/s}^2$ and $a = \dfrac{dv}{dt} = 0$, respectively.





12–53.

A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the two-stage motion of the missile for $0 \le t \le 20$ s.



SOLUTION

Since $v = \int a\,dt$, the constant lines of the a–t graph become sloping lines for the v–t graph.

The numerical values for each point are calculated from the total area under the a–t graph to the point.

At $t = 15$ s, $\quad v = (18)(15) = 270$ m/s

At $t = 20$ s, $\quad v = 270 + (25)(20 - 15) = 395$ m/s

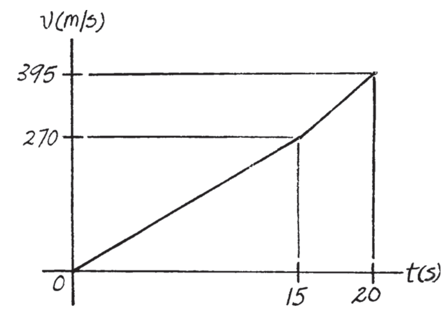
Since $s = \int v\,dt$, the sloping lines of the v–t graph become parabolic curves for the s–t graph.

The numerical values for each point are calculated from the total area under the v–t graph to the point.

At $t = 15$ s, $\quad s = \frac{1}{2}(15)(270) = 2025$ m

At $t = 20$ s, $\quad s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5 \text{ m} = 3.69$ km

Also:



$0 \le t \le 15$:

$$a = 18 \text{ m/s}^2$$

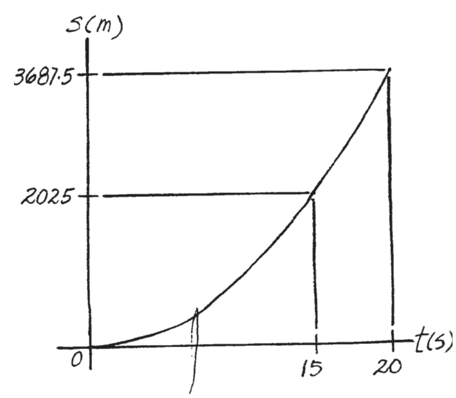
$$v = v_0 + a_c t = 0 + 18t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 0 + 0 + 9t^2$$

When $t = 15$:

$$v = 18(15) = 270 \text{ m/s}$$

$$s = 9(15)^2 = 2025 \text{ m} = 2.025 \text{ km}$$



$15 \le t \le 20$:

$$a = 25 \text{ m/s}^2$$

$$v = v_0 + a_c t = 270 + 25(t - 15)$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 2025 + 270(t - 15) + \frac{1}{2}(25)(t - 15)^2$$

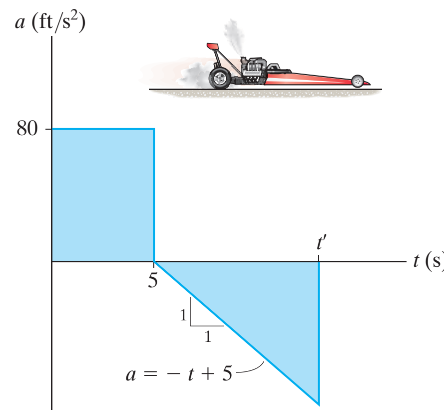
When $t = 20$:

$$v = 395 \text{ m/s}$$

$$s = 3687.5 \text{ m} = 3.69 \text{ km}$$

12–54.

The dragster starts from rest and has an acceleration described by the graph. Determine the time t' for it to stop. Also, what is its maximum speed? Construct the $v-t$ and $s-t$ graphs for the time interval $0 \le t \le t'$.



SOLUTION

v–t Graph: For the time interval $0 \le t < 5$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 80\,dt$$

$$v = (80t)\text{ ft/s}$$

The maximum speed occurs at the instant when the acceleration changes sign when $t = 5$ s. Thus,

$$v_{max} = v|_{t=5\,s} = 80(5) = 400\text{ ft/s} \qquad \textbf{Ans.}$$

For the time interval $5 < t \le t'$, the initial condition is $v = 400$ ft/s when $t = 5$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_{400\text{ ft/s}}^v dv = \int_{5\text{s}}^t (-t + 5)\,dt$$

$$v = \left(-\frac{t^2}{2} + 5t + 387.5\right)\text{ft/s}$$

Thus when $v = 0$,

$$0 = -\frac{t'^2}{2} + 5t' + 387.5$$

Choosing the positive root,

$$t' = 33.28\text{ s} = 33.3\text{ s} \qquad \textbf{Ans.}$$

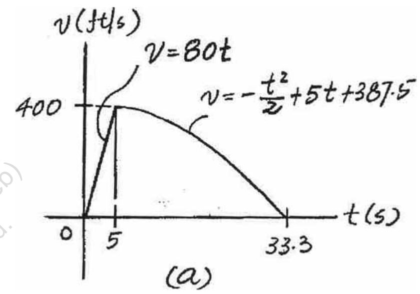
Also, the change in velocity is equal to the area under the $a-t$ graph. Thus

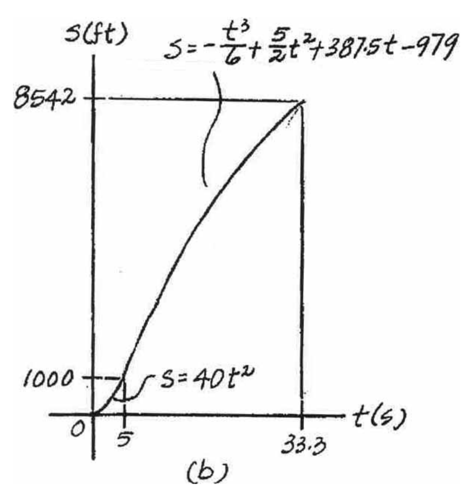
$$\Delta v = \int a\,dt$$

$$0 = 80(5) + \left\{\frac{1}{2}[(-t' + 5)(t' - 5)]\right\}$$

$$0 = -\frac{t'^2}{2} + 5t' + 387.5$$

This quadratic equation is the same as the one obtained previously. The $v-t$ graph is shown in Fig. a.





12–54. continued

s–t Graph: For the time interval $0 \leq t < 5\text{ s}$, the initial condition is $s = 0$ when $t = 0\text{ s}$.

$$\left(\xrightarrow{+}\right) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 80\,dt$$

$$s = (40t^2)\text{ ft}$$

When $t = 5\text{ s}$,

$$s|_{t=5\text{ s}} = 40(5^2) = 1000\text{ ft}$$

For the time interval $5\text{ s} < t \leq t' = 45\text{ s}$, the initial condition is $s = 1000\text{ ft}$ when $t = 5\text{ s}$.

$$\left(\xrightarrow{+}\right) \qquad ds = v\,dt$$

$$\int_{1000\text{ ft}}^s ds = \int_{5\text{ s}}^t \left(-\frac{t^2}{2} + 5t + 387.5\right)dt$$

$$s = \left(-\frac{t^3}{6} + \frac{5}{2}t^2 + 387.5t - 979.17\right)\text{ft}$$

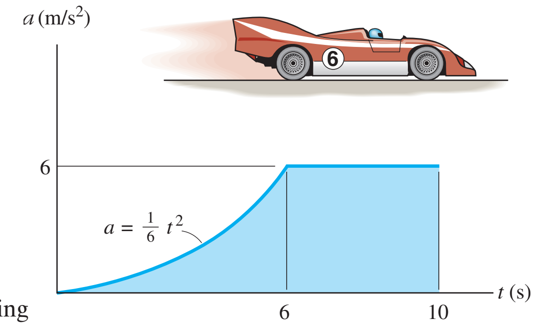
When $t = t' = 33.28\text{ s}$,

$$s|_{t=33.28\text{ s}} = -\frac{33.28^3}{6} + \frac{5}{2}(33.28^2) + 387.5(33.28) - 979.17 = 8542\text{ ft}$$

The $s–t$ graph is shown in Fig. b.

12–55.

A race car starting from rest travels along a straight road and for 10 s has the acceleration shown. Construct the v–t graph that describes the motion and find the distance traveled in 10 s.



SOLUTION

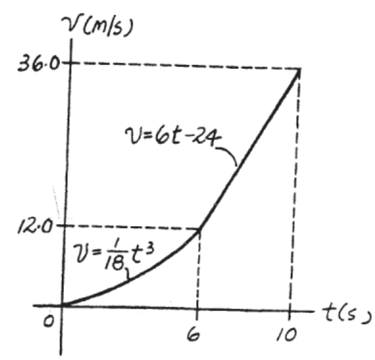
***v*–*t* Graph:** The velocity function in terms of time t can be obtained by applying formula $a = \frac{dv}{dt}$. For time interval $0\text{ s} \le t < 6\text{ s}$,

$$dv = adt$$

$$\int_0^v dv = \int_0^t \frac{1}{6}t^2 dt$$

$$v = \left(\frac{1}{18}t^3\right)\text{ m/s}$$

At $t = 6\text{ s}$, $\quad v = \frac{1}{18}\left(6^3\right) = 12.0\text{ m/s},$



For time interval $6\text{ s} < t \le 10\text{ s}$,

$$dv = adt$$

$$\int_{12.0\text{m/s}}^v dv = \int_{6s}^t 6dt$$

$$v = (6t - 24)\text{ m/s}$$

At $t = 10\text{ s}$, $\quad v = 6(10) - 24 = 36.0\text{ m/s}$

Position: The position in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $0\text{ s} \le t < 6\text{ s}$,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t \frac{1}{18}t^3\, dt$$

$$s = \left(\frac{1}{72}t^4\right)\text{m}$$

When $t = 6\text{ s}$, $v = 12.0\text{ m/s}$ and $s = \frac{1}{72}\left(6^4\right) = 18.0\text{ m}$.

For time interval $6\text{ s} < t \le 10\text{ s}$,

$$ds = vdt$$

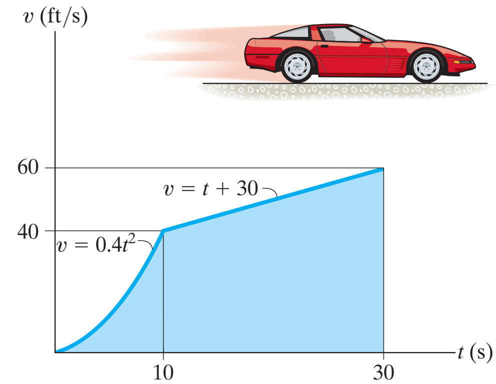
$$\int_{18.0\text{ m}}^s dv = \int_{6s}^t (6t - 24)dt$$

$$s = \left(3t^2 - 24t + 54\right)\text{ m}$$

When $t = 10\text{ s}$, $v = 36.0\text{ m/s}$ and $s = 3\left(10^2\right) - 24(10) + 54 = 114\text{ m}$ **Ans.**

***12–56.**

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the a–t graph and determine the maximum acceleration during the 30-s time interval. The car starts from rest at $s = 0$.



SOLUTION

For $t < 10$ s:

$$v = 0.4t^2$$

$$a = \frac{dv}{dt} = 0.8t$$

At $t = 10$ s:

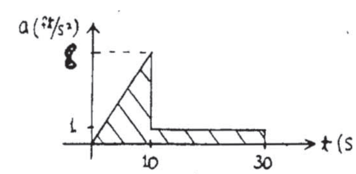
$$a = 8\ \text{ft/s}^2$$

For $10 < t \leq 30$ s:

$$v = t + 30$$

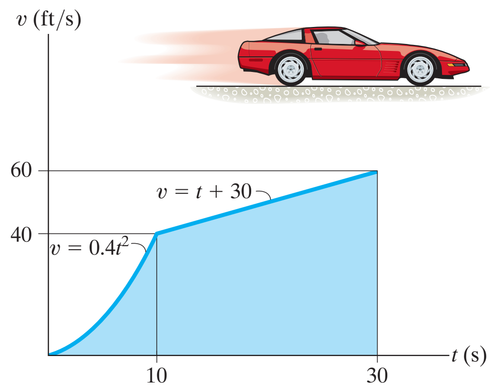
$$a = \frac{dv}{dt} = 1$$

$$a_{max} = 8\ \text{ft/s}^2$$ **Ans.**



12–57.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.



SOLUTION

For $t < 10$ s,

$$v = 0.4t^2$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2\,dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 < t < 30$ s,

$$v = t + 30$$

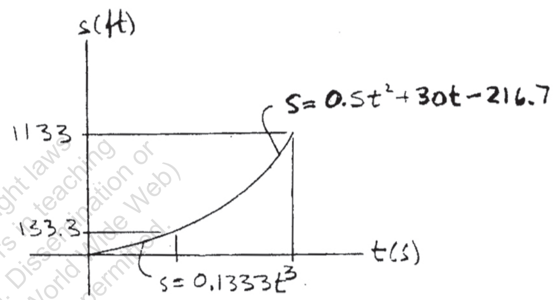
$$ds = v\,dt$$

$$\int_{133.3}^s ds = \int_{10}^t (t + 30)\,dt$$

$$s = 0.5t^2 + 30t - 216.7$$

At $t = 30$ s,

$$s = 1133 \text{ ft}$$



$$(v_{sp})_{Avg} = \frac{\Delta s}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s}$$ **Ans.**

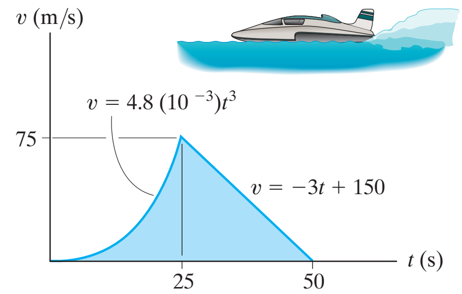
$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft}$$ **Ans.**

When $t = 0$ s, $s = 133$ ft.

When $t = 30$ s, $s = s_I = 1.33\ (10^3)$ ft

12–58.

The jet-powered boat starts from rest at $s = 0$ and travels along a straight line with the speed described by the graph. Construct the $s-t$ and $a-t$ graph for the time interval $0 \le t \le 50$ s.



SOLUTION

s–t Graph: The initial condition is $s = 0$ when $t = 0$.

$$\left(\overset{+}{\rightarrow}\right) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4.8(10^{-3})t^3\,dt$$

$$s = \left[1.2(10^{-3})t^4\right]\text{m}$$

At $t = 25$ s,

$$s|_{t=25\text{ s}} = 1.2(10^{-3})(25^4) = 468.75\text{ m}$$

For the time interval $25\text{ s} < t \le 50$ s, the initial condition $s = 468.75$ m when $t = 25$ s will be used.

$$\left(\overset{+}{\rightarrow}\right) \qquad ds = v\,dt$$

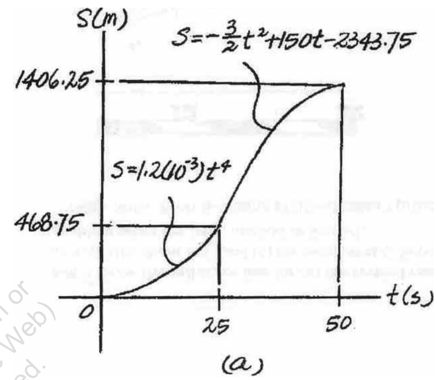
$$\int_{468.75\text{ m}}^s ds = \int_{25\text{ s}}^t (-3t + 150)\,dt$$

$$s = \left(-\frac{3}{2}t^2 + 150t - 2343.75\right)\text{m}$$

When $t = 50$ s,

$$s|_{t=50\text{ s}} = -\frac{3}{2}(50^2) + 150(50) - 2343.75 = 1406.25\text{ m}$$

The $s-t$ graph is shown in Fig. *a*.



a–t Graph: For the time interval $0 \le t < 25$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[4.8(10^{-3})t^3\right] = (0.0144t^2)\text{ m/s}^2$$

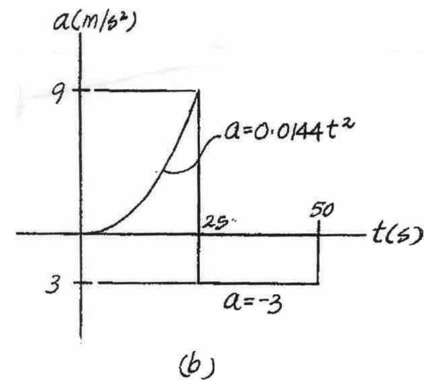
When $t = 25$ s,

$$a|_{t=25\text{ s}} = 0.0144(25^2)\text{ m/s}^2 = 9\text{ m/s}^2$$

For the time interval $25\text{ s} < t \le 50$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(-3t + 150) = -3\text{ m/s}^2$$

The $a-t$ graph is shown in Fig. *b*.



When $t = 25$ s,

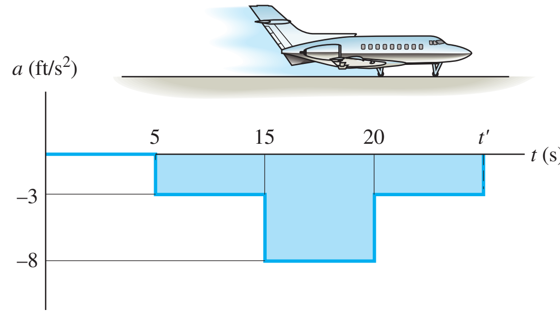
$$a = a_{max} = 9\text{ m/s}^2 \text{ and } s = 469\text{ m}.$$

When $t = 50$ s,

$$s = 1406\text{ m}.$$

12–59.

An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



SOLUTION

$v_0 = 110$ ft/s

$\Delta v = \int a\, dt$

$0 - 110 = -3(15 - 5) - 8(20 - 15) - 3(t' - 20)$

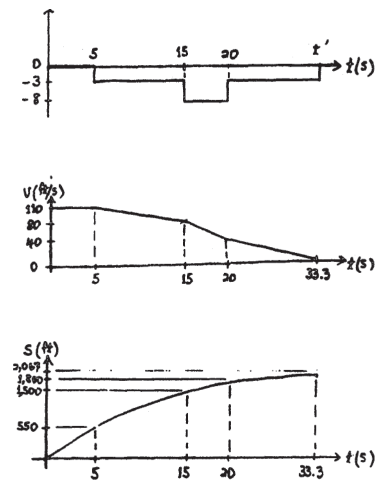
$t' = 33.3$ s **Ans.**

$s\big|_{t = 5s} = 550$ ft

$s\big|_{t = 15s} = 1500$ ft

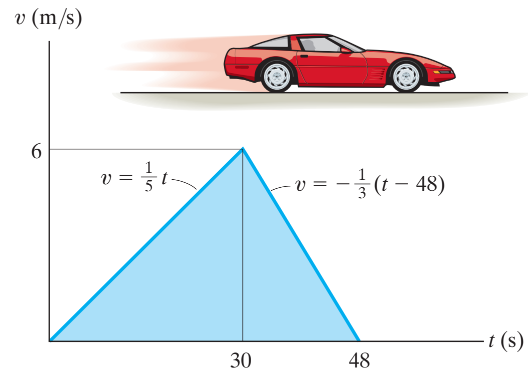
$s\big|_{t = 20s} = 1800$ ft

$s\big|_{t = 33.3s} = 2067$ ft



***12–60.**

A car travels along a straight road with the speed shown by the $v-t$ graph. Plot the $a-t$ graph.



SOLUTION

a–t Graph: For $0 \leq t < 30\text{s}$,

$$v = \frac{1}{5}t$$

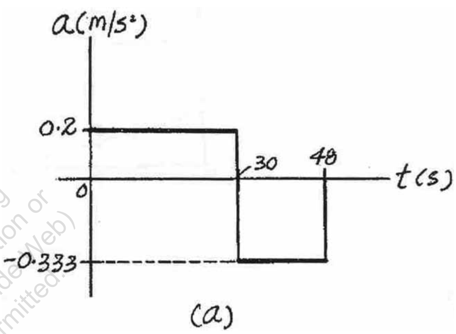
$$a = \frac{dv}{dt} = \frac{1}{5} = 0.2\ \text{m/s}^2$$

For $30\ \text{s} < t \leq 48\ \text{s}$

$$v = -\frac{1}{3}(t - 48)$$

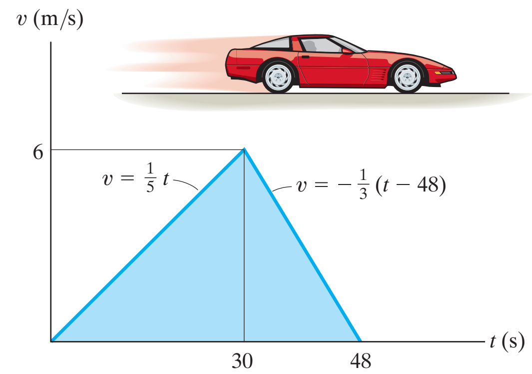
$$a = \frac{dv}{dt} = -\frac{1}{3}(1) = -0.333\ \text{m/s}^2$$

Using these results, $a-t$ graph shown in Fig. a can be plotted.



12–61.

A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops when $t = 48$ s. Also plot the s–t and a–t graphs.



SOLUTION

For $0 \leq t \leq 30$ s,

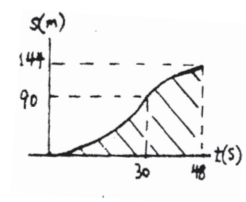
$$v = \frac{1}{5}t$$

$$a = \frac{dv}{dt} = \frac{1}{5}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{1}{5}t\,dt$$

$$s = \frac{1}{10}t^2$$



When $t = 30$ s, $\quad s = 90$ m,

$$v = -\frac{1}{3}(t - 48)$$

$$a = \frac{dv}{dt} = -\frac{1}{3}$$

$$ds = v\,dt$$

$$\int_{90}^s ds = \int_{30}^t -\frac{1}{3}(t - 48)dt$$

$$s = -\frac{1}{6}t^2 + 16t - 240$$

When $t = 48$ s,

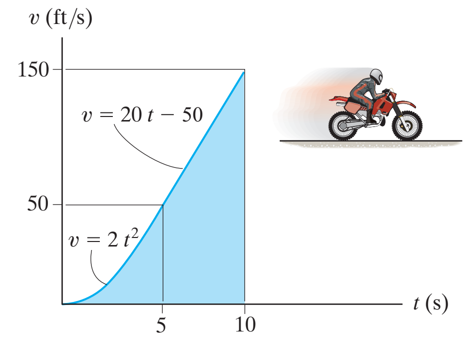
$$s = 144 \text{ m}$$ **Ans.**

Also, from the v–t graph

$$\Delta s = \int v\,dt \quad s - 0 = \frac{1}{2}(6)(48) = 144 \text{ m}$$ **Ans.**

12–62.

A motorcyclist travels along a straight road with the velocity described by the graph. Construct the $s-t$ and $a-t$ graphs.



SOLUTION

***s–t* Graph:** For the time interval $0 \leq t < 5\text{s}$, the initial condition is $s = 0$ when $t = 0$.

$$\left(\overset{+}{\rightarrow}\right) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 2t^2 dt$$

$$s = \left(\frac{2}{3}t^3\right)\text{ft}$$

When $t = 5\text{s}$,

$$s = \frac{2}{3}(5^3) = 83.33\text{ ft} = 83.3\text{ ft and } a = 20\text{ ft/s}^2$$

For the time interval $5\text{s} < t \leq 10\text{s}$, the initial condition is $s = 83.33$ ft when $t = 5$ s.

$$\left(\overset{+}{\rightarrow}\right) \qquad ds = v\,dt$$

$$\int_{83.33\text{ft}}^{s} ds = \int_{5\text{s}}^{t} (20t - 50)dt$$

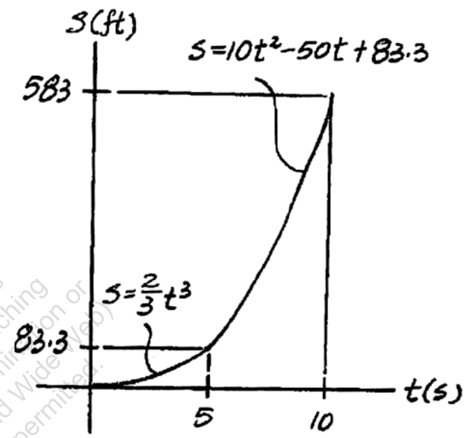
$$s\Big|_{83.33\text{ft}}^{s} = (10t^2 - 50t)\Big|_{5\text{s}}^{t}$$

$$s = (10t^2 - 50t + 83.33)\text{ ft}$$

When $t = 10$ s,

$$s|_{t=10\text{s}} = 10(10^2) - 50(10) + 83.33 = 583\text{ ft}$$

The $s-t$ graph is shown in Fig. a.



(a)

***a–t* Graph:** For the time interval $0 \leq t < 5$ s,

$$\left(\overset{+}{\rightarrow}\right) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(2t^2) = (4t)\text{ ft/s}^2$$

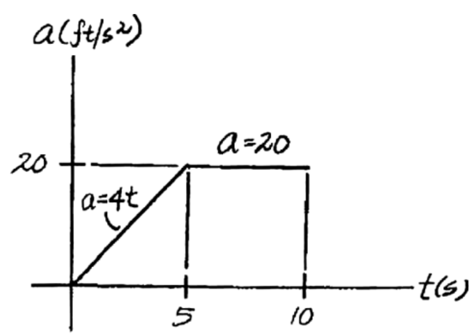
When $t = 5\text{s}$,

$$a = 4(5) = 20\text{ ft/s}^2$$

For the time interval $5\text{s} < t \leq 10$ s,

$$\left(\overset{+}{\rightarrow}\right) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(20t - 50) = 20\text{ ft/s}^2$$

The $a-t$ graph is shown in Fig. b.



(b)

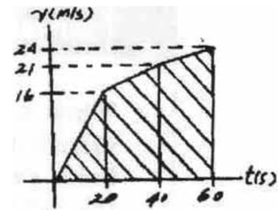
12–63.

The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

SOLUTION



The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m}$$ **Ans.**

***12–64.**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

SOLUTION

For package:

$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s_2 - s_0)$

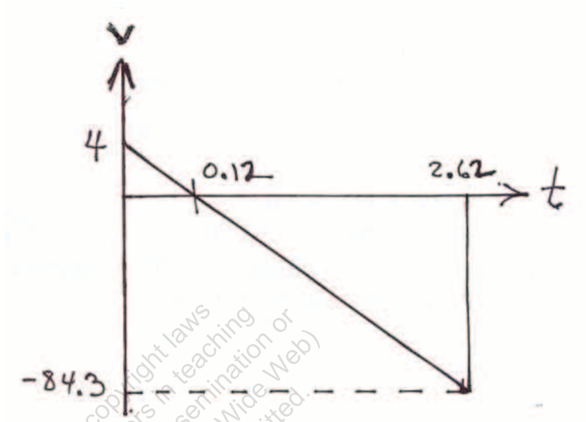
$v^2 = (4)^2 + 2(-32.2)(0 - 100)$

$v = 80.35 \text{ ft/s} \downarrow$

$(+\uparrow)\quad v = v_0 + a_c t$

$-80.35 = 4 + (-32.2)t$

$t = 2.620 \text{ s}$



For elevator:

$(+\uparrow)\quad s_2 = s_0 + vt$

$s = 100 + 4(2.620)$

$s = 110 \text{ ft}$ **Ans.**

12–65.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

SOLUTION

Car A:

$$v = v_0 + a_c t$$

$$v_A = 0 + 4t$$

At $t = 10$ s, $\quad v_A = 40$ m/s

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$$

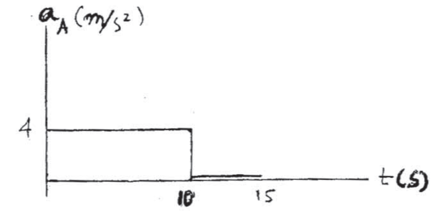
At $t = 10$ s, $\qquad s_A = 200$ m

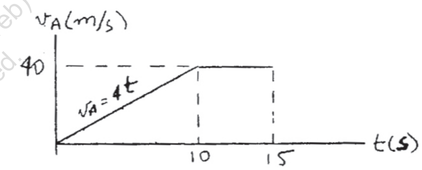
$t > 10$ s, $\qquad ds = v\,dt$

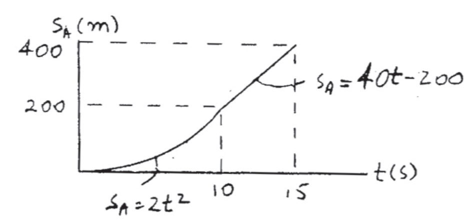
$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$$s_A = 40t - 200$$

At $t = 15$ s, $\qquad s_A = 400$ m







Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\qquad t = \dfrac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

12–65. continued

At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_{5}^{t} 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

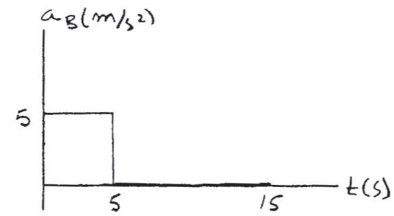
$$s_B = 25t - 62.5$$

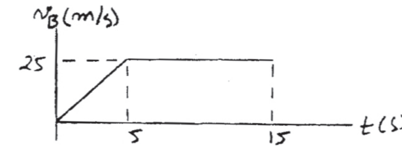
When $t = 15$ s, $\quad s_B = 312.5$

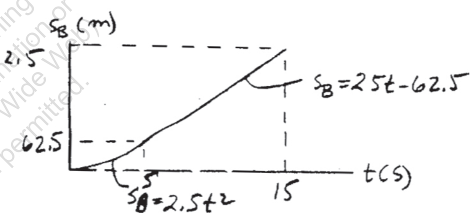
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.

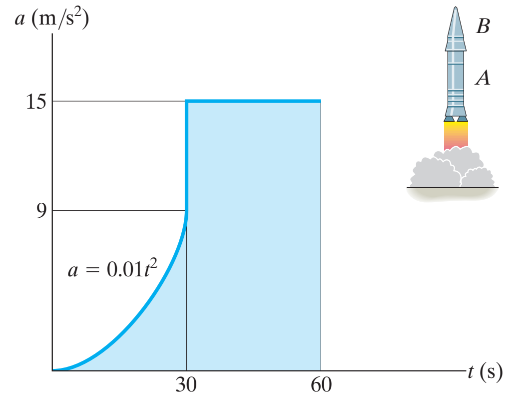






12–66.

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \le t \le 60$ s.



SOLUTION

For $0 \le t \le 30$ s

$$\int_0^v dv = \int_0^l 0.01\, t^2\, dt$$

$$v = 0.00333t^3$$

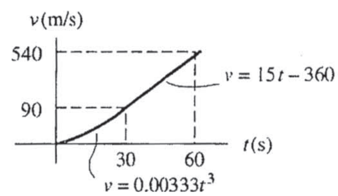
When $t = 30$ s, $\quad v = 90$ m/s

For $30\text{ s} \le t \le 60$ s

$$\int_{90}^v dv = \int_{30}^l 15dt$$

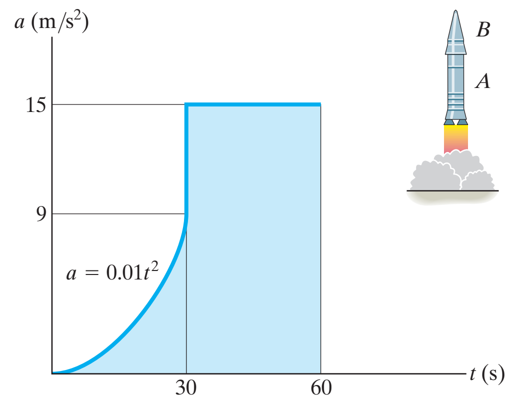
$$v = 15t - 360$$

When $t = 60$ s, $\quad v = 540$ m/s



12–67.

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the s–t graph which describes the motion of the second stage for $0 \le t \le 60$ s.



SOLUTION

v–t Graph: When $t = 0$, $v = 0$. For $0 \le t \le 30$ s,

$(+\uparrow) \qquad dv = a\,dt$

$$\int_0^v dv = \int_0^t 0.01t^2 dt$$

$$v\Big|_0^v = \frac{0.01}{3}t^3\Big|_0^t$$

$$v = \{0.003333t^3\}\text{ m/s}$$

When $t = 30$ s, $v = 0.003333(30^3) = 90$ m/s

For $30\text{ s} < t \le 60$ s,

$(+\uparrow) \qquad dv = a\,dt$

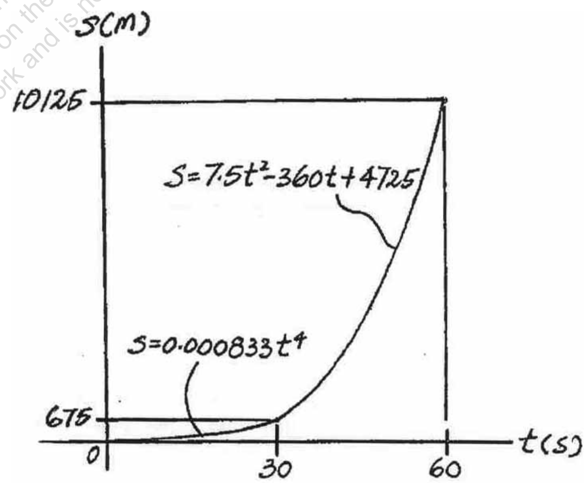
$$\int_{90\text{ m/s}}^v dv = \int_{30\text{ s}}^t 15\,dt$$

$$v\Big|_{90\text{ m/s}}^v = 15t\Big|_{30\text{ s}}^t$$

$$v - 90 = 15t - 450$$

$$v = \{15t - 360\}\text{ m/s}$$

When $t = 60$ s, $v = 15(60) - 360 = 540$ m/s



(a)

s–t Graph: When $t = 0$, $s = 0$. For $0 \le t \le 30$ s,

$(+\uparrow) \qquad ds = v dt$

$$\int_0^s ds = \int_0^t 0.003333t^3 dt$$

$$s\Big|_0^s = 0.0008333t^4\Big|_0^t$$

$$s = \{0.0008333\,t^4\}\text{ m}$$

When $t = 30$ s, $s = 0.0008333(30^4) = 675$ m

For $30\text{ s} < t \le 60$ s,

$(+\uparrow) \qquad ds = v dt$

$$\int_{675\text{ m}}^s ds = \int_{30\text{ s}}^t (15t - 360)\,dt$$

12–67. continued

$$s\Big|_{675\text{ m}}^{s} = (7.5t^2 - 360t)\Big|_{30\text{ s}}^{t}$$

$$s - 675 = (7.5t^2 - 360t) - [7.5(30^2) - 360(30)]$$

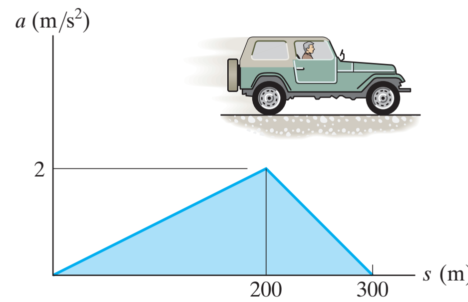
$$s = \{7.5t^2 - 360t + 4725\}\text{ m}$$

When $t = 60$ s, $s = 7.5(60^2) - 360(60) + 4725 = 10125$ m

Using these results, the s–t graph shown in Fig. *a* can be plotted.

***12–68.**

The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0, v = 0$.



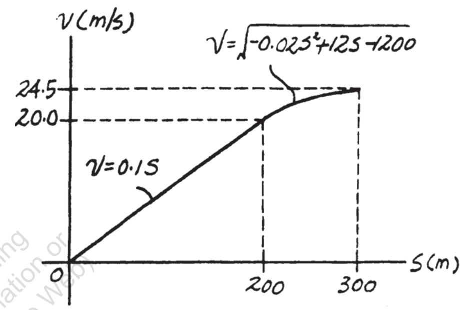
SOLUTION

***a–s* Graph**: The function of acceleration a in terms of s for the interval $0 \text{ m} \le s < 200 \text{ m}$ is

$$\frac{a - 0}{s - 0} = \frac{2 - 0}{200 - 0} \qquad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \le 300 \text{ m}$,

$$\frac{a - 2}{s - 200} = \frac{0 - 2}{300 - 200} \qquad a = (-0.02s + 6) \text{ m/s}^2$$



***v–s* Graph**: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0 \text{ m} \le s < 200 \text{ m}}$,

$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 0.01sds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0 \text{ m/s}$

For the interval $\mathbf{200 \text{ m} < s \le 300 \text{ m}}$,

$$vdv = ads$$

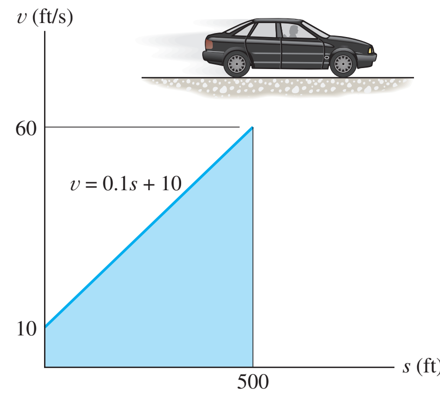
$$\int_{20.0 \text{ m/s}}^{v} vdv = \int_{200 \text{ m}}^{s} (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right) \text{ m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$

12–69.

The v–s graph for the car is given for the first 500 ft of its motion. Construct the a–s graph for $0 \le s \le 500$ ft. How long does it take to travel the 500-ft distance? The car starts at $s = 0$ when $t = 0$.

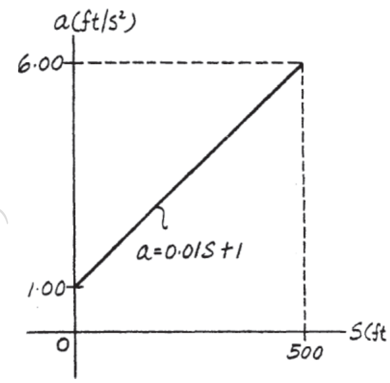


SOLUTION

a – s Graph: The acceleration a in terms of s can be obtained by applying $v dv = a ds$.

$$a = v\frac{dv}{ds} = (0.1s + 10)(0.1) = (0.01s + 1)\ \text{ft/s}^2$$

At $s = 0$ and $s = 500$ ft, $a = 0.01(0) + 1 = 1.00\ \text{ft/s}^2$ and $a = 0.01(500) + 1 = 6.00\ \text{ft/s}^2$, respectively.



Position: The position s in terms of time t can be obtained by applying $v = \frac{ds}{dt}$.

$$dt = \frac{ds}{v}$$

$$a\big|_{s=0} = 100\ \text{ft/s}^2$$

$$a\big|_{s=500\text{ft}} = 6.00\ \text{ft/s}^2$$

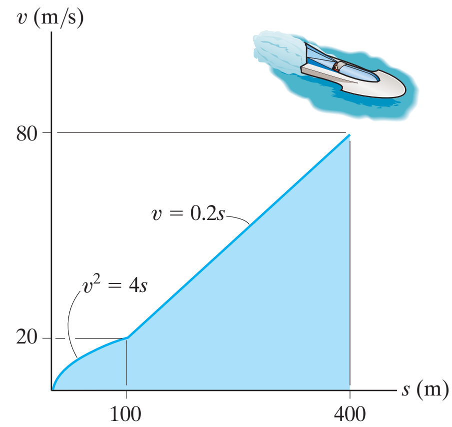
$$\int_0^t dt = \int_0^s \frac{ds}{0.1s + 10}$$

$$t = 10\ln(0.01s + 1)$$

When $s = 500$ ft, $\quad t = 10\ln[0.01(500) + 1] = 17.9\ \text{s}$ **Ans.**

12–70.

The boat travels along a straight line with the speed described by the graph. Construct the s–t and $a-s$ graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



SOLUTION

***s–t* Graph:** For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right) \text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10 \text{ s}$$

For $100 \text{ m} < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

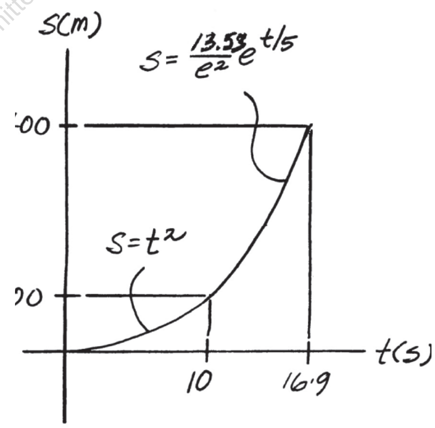
$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

$$s = \left(13.53e^{t/5}\right) \text{ m}$$

When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93 \text{ s} = 16.9 \text{ s} \qquad \textbf{Ans.}$$



(a)

The s–t graph is shown in Fig. a.

***a–s* Graph:** For $0 \text{ m} \leq s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2 \text{ m/s}^2$$

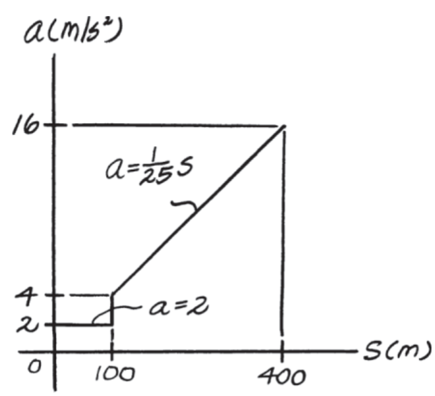
For $100 \text{ m} < s \leq 400$ m,

$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

When $s = 100$ m and 400 m,

$$a\big|_{s=100\text{ m}} = 0.04(100) = 4 \text{ m/s}^2$$

$$a\big|_{s=400\text{ m}} = 0.04(400) = 16 \text{ m/s}^2$$

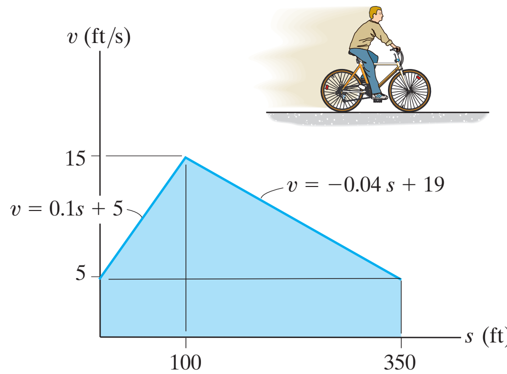


(b)

The a–s graph is shown in Fig. b.

12–71.

The $v-s$ graph of a cyclist traveling along a straight road is shown. Construct the $a-s$ graph.



SOLUTION

***a–s* Graph:** For $0 \le s < 100$ ft,

$$\left(\overset{+}{\rightarrow}\right) \qquad a = v\frac{dv}{ds} = (0.1s + 5)(0.1) = (0.01s + 0.5)\ \text{ft/s}^2$$

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\ \text{ft/s}^2$$

$$a|_{s=100\ \text{ft}} = 0.01(100) + 0.5 = 1.5\ \text{ft/s}^2$$

For $100\ \text{ft} < s \le 350$ ft,

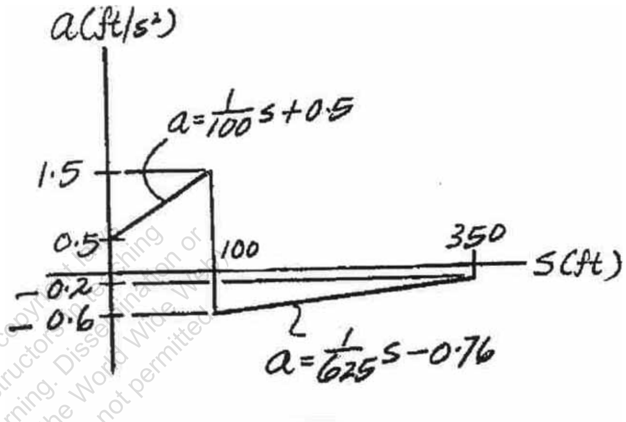
$$\left(\overset{+}{\rightarrow}\right) \qquad a = v\frac{dv}{ds} = (-0.04s + 19)(-0.04) = (0.0016s - 0.76)\ \text{ft/s}^2$$

Thus at $s = 100$ ft and 350 ft

$$a|_{s=100\ \text{ft}} = 0.0016(100) - 0.76 = -0.6\ \text{ft/s}^2$$

$$a|_{s=350\ \text{ft}} = 0.0016(350) - 0.76 = -0.2\ \text{ft/s}^2$$

The $a-s$ graph is shown in Fig. a.



(a)

Thus at $s = 0$ and 100 ft

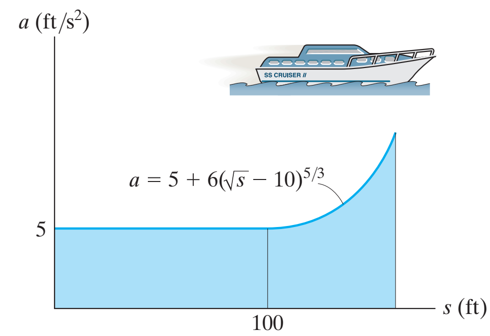
$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\ \text{ft/s}^2$$

$$a|_{s=100\ \text{ft}} = 0.01(100) + 0.5 = 1.5\ \text{ft/s}^2$$

At $s = 100$ ft, a changes from $a_{max} = 1.5\ \text{ft/s}^2$ to $a_{min} = -0.6\ \text{ft/s}^2$.

*■ **12–72.**

The a–s graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



SOLUTION

Velocity: The velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0\ ft \leq s < 100\ ft}$,

$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 5ds$$

$$v = \sqrt{10s} = \text{ft/s}$$

At $s = 75$ ft, $\quad v = \sqrt{10(75)} = 27.4$ ft/s **Ans.**

At $s = 100$ ft, $\quad v = \sqrt{10(100)} = 31.62$ ft/s **Ans.**

For the interval $\mathbf{100\ ft < s \leq 125\ ft}$,

$$vdv = ads$$

$$\int_{31.62\text{ ft/s}}^{v} vdv = \int_{100\text{ ft}}^{125\text{ ft}} [5 + 6(\sqrt{s} - 10)^{5/3}]ds$$

Evaluating the integral on the right using Simpson's rule, we have

$$\left.\frac{v^2}{2}\right|_{31.62\text{ ft/s}}^{v} = 201.032$$

At $s = 125$ ft, **Ans.**

$$v = 37.4 \text{ ft/s}$$

12–73.

The position of a particle is defined by $\mathbf{r} = \{5 \cos 2t\, \mathbf{i} + 4 \sin 2t\, \mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2t\mathbf{i} + 8 \cos 2t\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $v = -10 \sin 2(1)\mathbf{i} + 8 \cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$\mathbf{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration expressed in Cartesian vector from can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2t\mathbf{i} - 16 \sin 2t\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1)\mathbf{i} - 16 \sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Traveling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{(1)}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{(2)}$$

Adding Eqs (1) and (2) yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \qquad \textbf{(Equation of an Ellipse) (Q.E.D.)}$$

12–74.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \mathbf{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \mathbf{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}]\text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}]\text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= \{6\mathbf{i} + 4\mathbf{j}\}\text{ m}$$ **Ans.**

12–75.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}\ \text{ft/s}^2$. Determine the particle's position (x, y, z) at $t = 1$ s.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$dv = \text{a}dt$$

$$\int_0^v dv = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k})\, dt$$

$$v = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\}\ \text{ft/s}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$dr = \text{v}dt$$

$$\int_{\text{r}_1}^{\text{r}} dr = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k})\, dt$$

$$\text{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\text{r} = \{(t^3 + 3)\,\mathbf{i} + 2\mathbf{j} + (t^4 + 5)\mathbf{k}\}\ \text{ft}$$

When $t = 1\ s$, $\text{r} = (1^3 + 3)\mathbf{i} + 2\mathbf{j} + (1^4 + 5)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

(4 ft, 2 ft, 6 ft) **Ans.**

***12–76.**

The velocity of a particle is given by $\boldsymbol{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\boldsymbol{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \boldsymbol{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right] \text{m}$$

When $t = 2$ s,

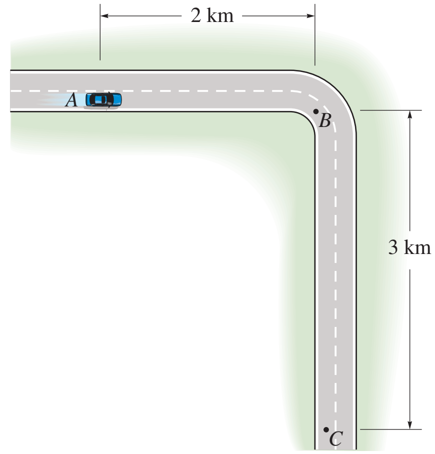
$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

$$(42.7, 16.0, 14.0)\ \text{m} \qquad \textbf{Ans.}$$

12–77.

The car travels from A to B, and then from B to C, as shown in the figure. Determine the magnitude of the displacement of the car and the distance traveled.



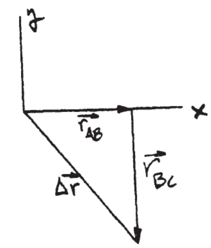
SOLUTION

Displacement: $\Delta \mathbf{r} = \{2\mathbf{i} - 3\mathbf{j}\}$ km

$\Delta r = \sqrt{2^2 + 3^2} = 3.61$ km **Ans.**

Distance Traveled:

$d = 2 + 3 = 5$ km **Ans.**



12–78.

A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes, and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

SOLUTION

Total Distance Traveled and Displacement: The total distance traveled is

$$s = 2 + 3 + 4 = 9 \text{ km} \qquad \textbf{Ans.}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{(2 - 4)^2 + 3^2} = 3.606 \text{ km} = 3.61 \text{ km} \qquad \textbf{Ans.}$$

Average Velocity and Speed: The total time is $\Delta t = 5 + 8 + 10 = 23 \text{ min} = 1380 \text{ s}$
The magnitude of average velocity is

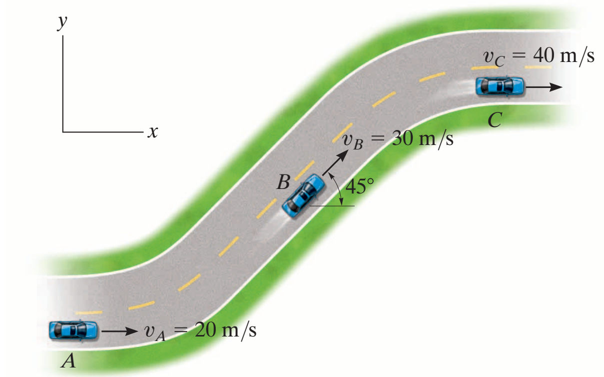
$$v_{\text{avg}} = \frac{\Delta r}{\Delta t} = \frac{3.606\left(10^3\right)}{1380} = 2.61 \text{ m/s} \qquad \textbf{Ans.}$$

and the average speed is

$$\left(v_{sp}\right)_{\text{avg}} = \frac{s}{\Delta t} = \frac{9\left(10^3\right)}{1380} = 6.52 \text{ m/s} \qquad \textbf{Ans.}$$

12–79.

A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3 s to go from A to B, and then 5 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



SOLUTION

$\boldsymbol{v}_A = 20\,\mathbf{i}$

$\boldsymbol{v}_B = 21.21\,\mathrm{i} + 21.21\mathbf{j}$

$\boldsymbol{v}_C = 40\mathbf{i}$

$$\mathbf{a}_{AB} = \frac{\Delta\,\boldsymbol{v}}{\Delta\,t} = \frac{21.21\,\mathbf{i} + 21.21\,\mathbf{j} - 20\,\mathbf{i}}{3}$$

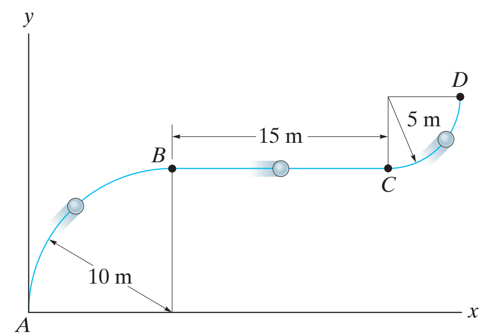
$\mathbf{a}_{AB} = \{\,0.404\,\mathbf{i} + 7.07\,\mathbf{j}\,\}\ \mathrm{m/s^2}$ **Ans.**

$$\mathbf{a}_{AC} = \frac{\Delta\,\boldsymbol{v}}{\Delta\,t} = \frac{40\,\mathbf{i} - 20\,\mathbf{i}}{8}$$

$\mathbf{a}_{AC} = \{\,2.50\,\mathbf{i}\,\}\ \mathrm{m/s^2}$ **Ans.**

***12–80.**

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sP} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

12–81.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x, y, and z, we obtain the x, y, and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}\left(0.25t^3\right) = \left(0.75t^2\right) \text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}\left(1.5t^2\right) = \left(3t\right) \text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}\left(6 - 0.75t^{5/2}\right) = \left(-1.875t^{3/2}\right) \text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75\left(2^2\right) = 3 \text{ m/s} \qquad v_y = 3(2) = 6 \text{ m/s} \qquad v_z = -1.875\left(2\right)^{3/2} = -5.303 \text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551 \text{ ft/s} = 8.55 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: The x, y, and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x, v_y, and v_z, respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}\left(0.75t^2\right) = (1.5t) \text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3 \text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}\left(-1.875t^{3/2}\right) = \left(-2.815t^{1/2}\right) \text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3 \text{ m/s}^2 \qquad a_y = 3 \text{ m/s}^2 \qquad a_z = -2.8125\left(2^{1/2}\right) = -3.977 \text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815 \text{ m/s}^2 = 5.82 \text{ m/s} \qquad \textbf{Ans.}$$

12–82.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s², where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{ m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{ m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{ m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{ m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

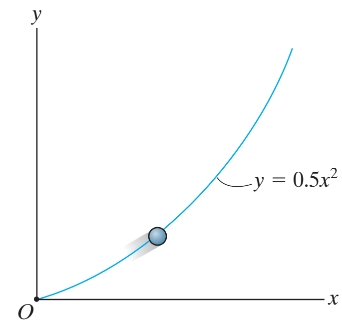
$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2$$ **Ans.**

12–83.

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t dt$$

$$x = \left(2.50t^2\right) \text{ ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s $a_x = \dfrac{dv_x}{dt} = 5$ ft/s^2, and $x = 2.50$ ft. Then, from Eq. (2)

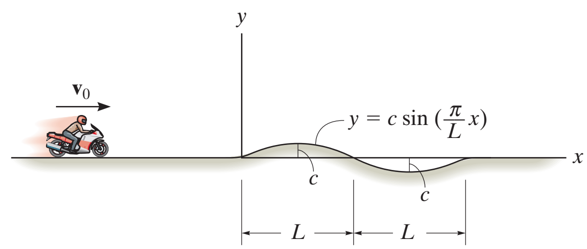
$$a_y = 5^2 + 2.50(5) = 37.5 \text{ ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

***12–84.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c\sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\,v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

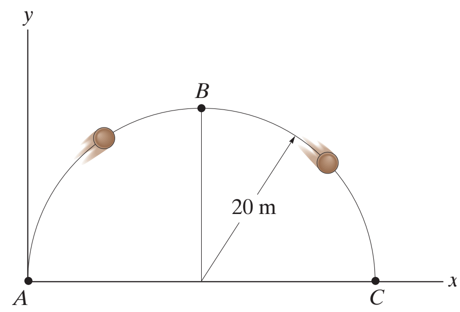
$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

12–85.

A particle travels along the curve from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



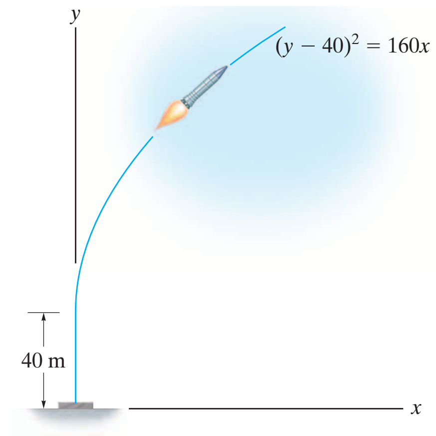
SOLUTION

Time from B to C is $3 - 1 = 2$ s

$$\mathbf{v}_{avg} = \frac{\Delta r}{\Delta t} = \frac{(\mathbf{r}_{AC} - \mathbf{r}_{AB})}{\Delta t} = \frac{40\mathbf{i} - (20\mathbf{i} + 20\mathbf{j})}{2} = \{10\mathbf{i} - 10\mathbf{j}\}\ \text{m/s}$$ **Ans.**

12–86.

When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



SOLUTION

$v_y = 180$ m/s

$(y - 40)^2 = 160\,x$

$2(y - 40)v_y = 160 v_x$ **(1)**

$2(80 - 40)(180) = 160 v_x$

$v_x = 90$ m/s

$v = \sqrt{90^2 + 180^2} = 201$ m/s **Ans.**

$a_y = \dfrac{d\,v_y}{dt} = 0$

From Eq. 1,

$2\,v_y^2 + 2(y - 40)a_y = 160\,a_x$

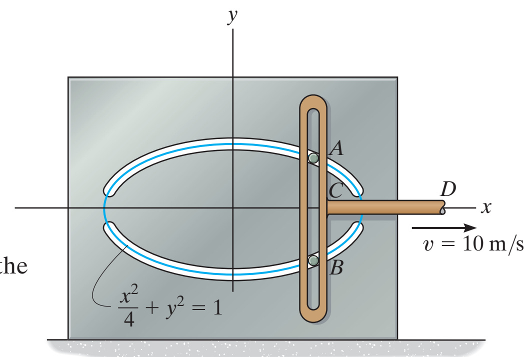
$2(180)^2 + 0 = 160\,a_x$

$a_x = 405$ m/s^2

$a = 405$ m/s^2 **Ans.**

12–87.

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s}\downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left({v_x}^2 + xa_x\right) + 2\left({v_y}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

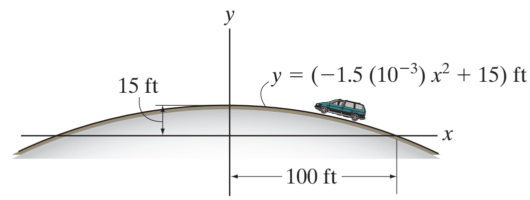
$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2\downarrow$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

***12–88.**

The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



SOLUTION

Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5\left(10^{-3}\right)x^2 + 15$$

$$\dot{y} = -3\left(10^{-3}\right)x\dot{x}$$

or

$$v_y = -3\left(10^{-3}\right)xv_x$$

When $x = 50$ ft,

$$v_y = -3\left(10^{-3}\right)(50)v_x = -0.15v_x \qquad \textbf{(1)}$$

The magnitude of the van's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \textbf{(2)}$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

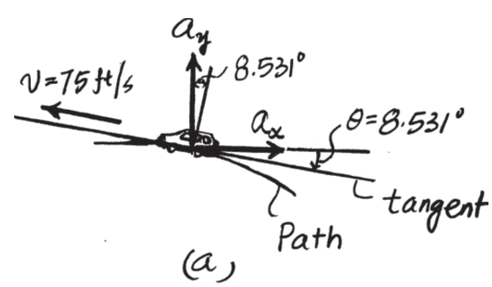
$$75 = \sqrt{{v_x}^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.



$$\ddot{y} = -3\left(10^{-3}\right)(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3\left(10^{-3}\right)\left({v_x}^2 + xa_x\right)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3\left(10^{-3}\right)\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \qquad \textbf{(3)}$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 50$ ft is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\Big|_{x=50\text{ ft}} = \tan^{-1}\left[-3\left(10^{-3}\right)x\right]\Big|_{x=50\text{ ft}} = \tan^{-1}(-0.15) = -8.531°$.

Thus, from the diagram shown in Fig. a,

$$a_x \cos 8.531° - a_y \sin 8.531° = 0 \qquad \textbf{(4)}$$

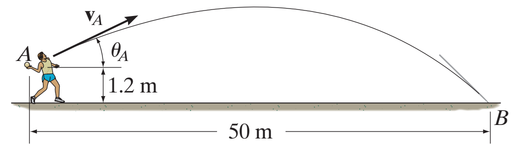
Solving Eqs. (3) and (4) yields

$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

12–89.

It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

x-Motion: Here, $(v_A)_x = v_A \cos\theta_A$, $x_A = 0$, $x_B = 50$ m, and $t = 2.5$ s. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad x_B = x_A + (v_A)_x t$$

$$50 = 0 + v_A \cos\theta_A (2.5)$$

$$v_A \cos\theta_A = 20 \qquad \textbf{(1)}$$

y-Motion: Here, $(v_A)_y = v_A \sin\theta_A$, $y_A = 0$, $y_B = -1.2$ m, and $a_y = -g = -9.81\ \text{m/s}^2$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.2 = 0 + v_A \sin\theta_A (2.5) + \frac{1}{2}(-9.81)\left(2.5^2\right)$$

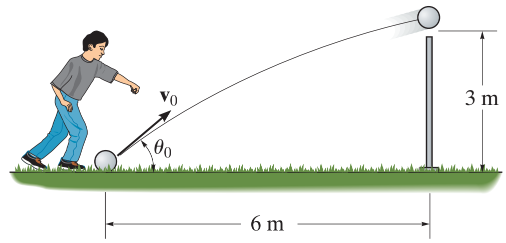
$$v_A \sin\theta_A = 11.7825 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta_A = 30.5^\circ \qquad v_A = 23.2\ \text{m/s} \qquad \textbf{Ans.}$$

12–90.

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

x-Motion: Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + (v_0)_x t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2} a_y t^2$$

$$3 = 0 + v_0 (\sin\theta) t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0 (\sin\theta) t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\frac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57°$$

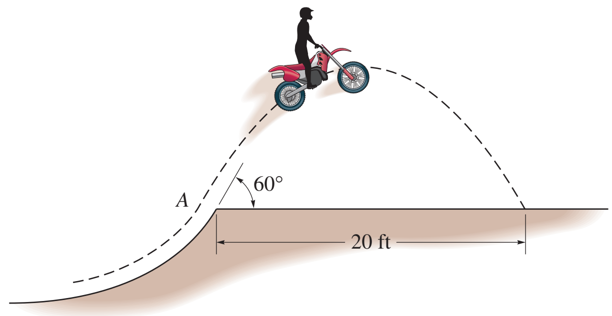
$$\theta = 58.28° = 58.3° \qquad \textbf{Ans.}$$

Substituting the result of θ into Eq. (2), we have

$$(v_0)_{min} = \sqrt{\frac{58.86}{\sin 116.57° - \cos^2 58.28°}} = 9.76 \text{ m/s} \qquad \textbf{Ans.}$$

12–91.

During a race the dirt bike was observed to leap up off the small hill at A at an angle of 60° with the horizontal. If the point of landing is 20 ft away, determine the approximate speed at which the bike was traveling just before it left the ground. Neglect the size of the bike for the calculation.



SOLUTION

$(\overset{+}{\rightarrow})\; s = s_0 + v_0 t$

$$20 = 0 + v_A \cos 60°\, t$$

$(+\uparrow)\; s = s_0 + v_0 + \frac{1}{2} a_c t^2$

$$0 = 0 + v_A \sin 60°\, t + \frac{1}{2}(-32.2)\, t^2$$

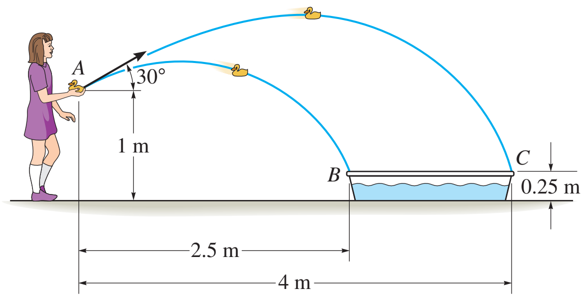
Solving

$t = 1.4668 \text{ s}$

$v_A = 27.3 \text{ ft/s}$ **Ans.**

***12–92.**

The girl always throws the toys at an angle of 30° from point A as shown. Determine the time between throws so that both toys strike the edges of the pool B and C at the same instant. With what speed must she throw each toy?



SOLUTION

To strike B:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$2.5 = 0 + v_A \cos 30°\, t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0.25 = 1 + v_A \text{ siv } 30°\, t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.6687 \text{ s}$

$(v_A)_B = 4.32 \text{ m/s}$ **Ans.**

To strike C:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$4 = 0 + v_A \cos 30°\, t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0.25 = 1 + v_A \text{ siv } 30°\, t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.790 \text{ s}$

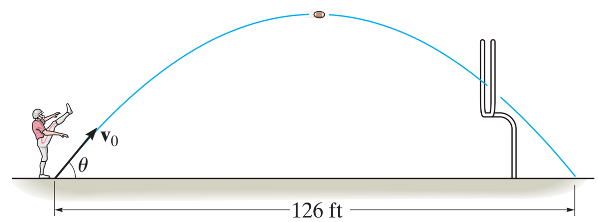
$(v_A)_C = 5.85 \text{ m/s}$ **Ans.**

Time between throws:

$\Delta t = 0.790 \text{ s} - 0.6687 \text{ s} = 0.121 \text{ s}$ **Ans.**

12–93.

The player kicks a football with an initial speed of $v_0 = 90$ ft/s. Determine the time the ball is in the air and the angle θ of the kick.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set with its origin coinciding with starting point of the football.

x-motion: Here, $x_0 = 0$, $x = 126$ ft, and $(v_0)_x = 90\cos\theta$

$$(\overset{+}{\rightarrow})\qquad x = x_0 + (v_0)_x t$$

$$126 = 0 + (90\cos\theta)t$$

$$t = \frac{126}{90\cos\theta} \qquad \textbf{(1)}$$

y-motion: Here, $y_0 = y = 0$, $(v_0)_y = 90\sin\theta$, and $a_y = -g = -32.2$ ft. Thus,

$$(+\uparrow)\qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$O = 0 + (90\sin\theta)t + \frac{1}{2}(-32.2)t^2$$

$$O = (90\sin\theta)t - 16.1t^2 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yields

$$O = 90\sin\theta\left(\frac{126}{90\cos\theta}\right) - 16.1\left(\frac{126}{90\cos\theta}\right)^2$$

$$O = \frac{126\sin\theta}{\cos\theta} - \frac{31.556}{\cos^2\theta}$$

$$O = 126\sin\theta\cos\theta - 31.556 \qquad \textbf{(3)}$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$, Eq. (3) becomes

$$63\sin 2\theta = 31.556$$

$$\sin 2\theta = 0.5009$$

$$2\theta = 30.06 \text{ or } 149.94$$

$$\theta = 15.03° = 15.0° \text{ or } \theta = 74.97° = 75.0° \qquad \textbf{Ans.}$$

If $\theta = 15.03°$,

$$t = \frac{126}{90\cos 15.03°} = 1.45\text{ s} \qquad \textbf{Ans.}$$

If $\theta = 74.97°$,

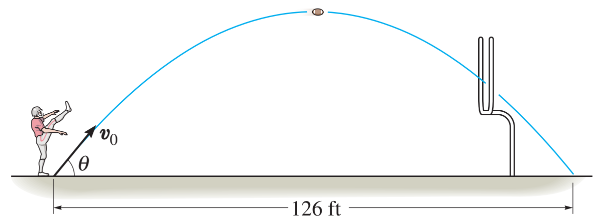
$$t = \frac{126}{90\cos 74.97°} = 5.40\text{ s} \qquad \textbf{Ans.}$$

Thus, $\theta = 15.0°,\ t = 1.45$ s

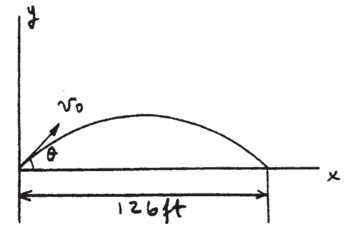
$\theta = 75.0°,\ t = 5.40$ s

12–94.

From a videotape, it was observed that a pro football player kicked a football 126 ft during a measured time of 3.6 seconds. Determine the initial speed of the ball and the angle θ at which it was kicked.



SOLUTION



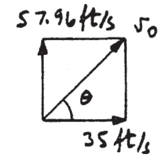
$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$126 = 0 + (v_0)_x (3.6)$$

$$(v_0)_x = 35 \text{ ft/s}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$O = 0 + (v_0)_y (3.6) + \frac{1}{2}(-32.2)(3.6)^2$$



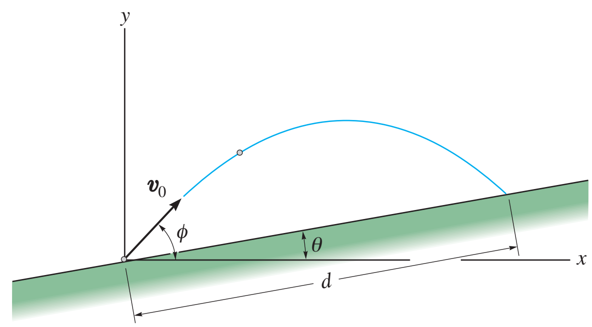
$$(v_0)_y = 57.96 \text{ ft/s}$$

$$v_0 = \sqrt{(35)^2 + (57.96)^2} = 67.7 \text{ ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{57.96}{35}\right) = 58.9°$$ **Ans.**

12–95.

A projectile is given a velocity $\mathbf{v}_0$ at an angle ϕ above the horizontal. Determine the distance d to where it strikes the sloped ground. The acceleration due to gravity is g.



SOLUTION

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

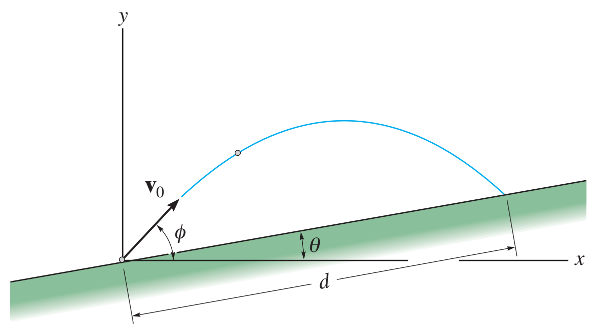
$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$ **Ans.**

***12–96.**

A projectile is given a velocity $\mathbf{v}_0$. Determine the angle ϕ at which it should be launched so that d is a maximum. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s_x = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)\qquad s_y = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

Require:

$$\frac{d(d)}{d\phi} = \frac{v_0^2}{g\cos\theta}\left[\cos 2\phi(2) - 2\tan\theta(2\cos\phi)(-\sin\phi)\right] = 0$$

$$\cos 2\phi + \tan\theta\sin 2\phi = 0$$

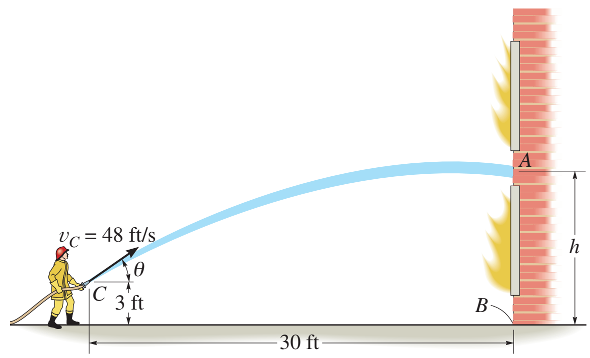
$$\frac{\sin 2\phi}{\cos 2\phi}\tan\theta + 1 = 0$$

$$\tan 2\phi = -\text{ctn}\,\theta$$

$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta) \qquad \textbf{Ans.}$$

12–97.

Determine the maximum height on the wall to which the firefighter can project water from the hose, if the speed of the water at the nozzle is $v_C = 48$ ft/s.



SOLUTION

$(+\uparrow)\ v = v_0 + a_c t$

$0 = 48 \sin\theta - 32.2\,t$

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$30 = 0 + 48\,(\cos\theta)(t)$

$48 \sin\theta = 32.2 \dfrac{30}{48 \cos\theta}$

$\sin\theta \cos\theta = 0.41927$

$\sin 2\theta = 0.83854$

$\theta = 28.5°$

$t = 0.7111$ s

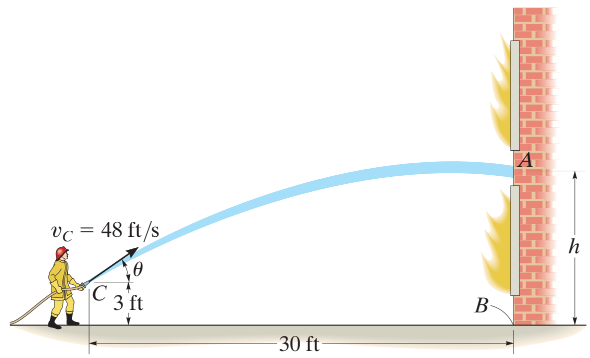
$(+\uparrow)\ s = s_0 + v_0 t + \dfrac{1}{2} a_c t^2$

$h - 3 = 0 + 48 \sin 28.5°\,(0.7111) + \dfrac{1}{2}(-32.2)(0.7111)^2$

$h = 11.1$ ft **Ans.**

■ **12–98.**

Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is $v_C = 48$ ft/s.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$30 = 0 + 48 \cos\theta\, t$$

$$t = \frac{30}{48 \cos\theta}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0 = 3 + 48 \sin\theta\, t + \frac{1}{2}(-32.2)t^2$$

$$0 = 3 + \frac{48 \sin\theta\,(30)}{48 \cos\theta} - 16.1\left(\frac{30}{48\cos\theta}\right)^2$$

$$0 = 3\cos^2\theta + 30 \sin\theta\cos\theta - 6.2891$$

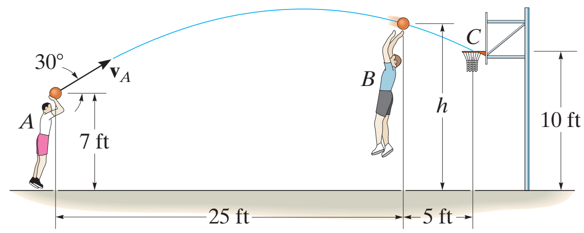
$$3\cos^2\theta + 15 \sin 2\theta = 6.2891$$

Solving

$\theta = 6.41°$ or $77.9°$ **Ans.**

12–99.

Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$$30 = 0 + v_A \cos 30°\, t_{AC}$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$10 = 7 + v_A \sin 30°\, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$$

Solving

$v_A = 36.73 = 36.7 \text{ ft/s}$ **Ans.**

$t_{AC} = 0.943 \text{ s}$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$$25 = 0 + 36.73 \cos 30°\, t_{AB}$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$h = 7 + 36.73 \sin 30°\, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$$

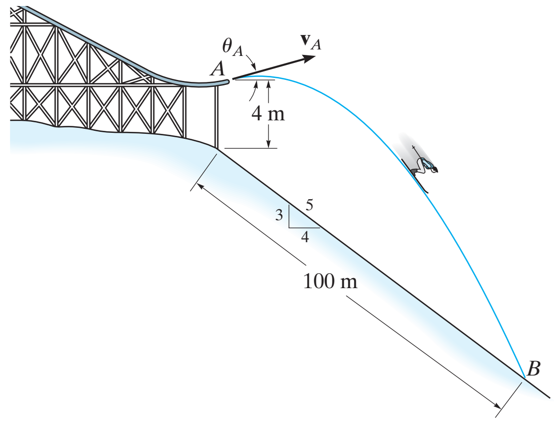
Solving

$t_{AB} = 0.786 \text{ s}$

$h = 11.5 \text{ ft}$ **Ans.**

***12–100.**

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

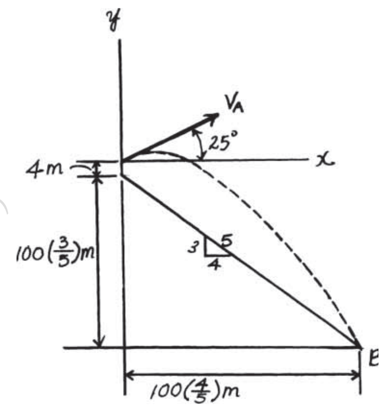
$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$

Solving,

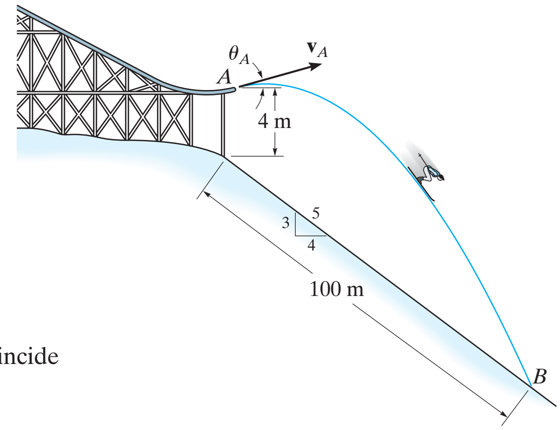
$$v_A = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

$$t_{AB} = 4.54 \text{ s} \qquad \textbf{Ans.}$$



12–101.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



SOLUTION

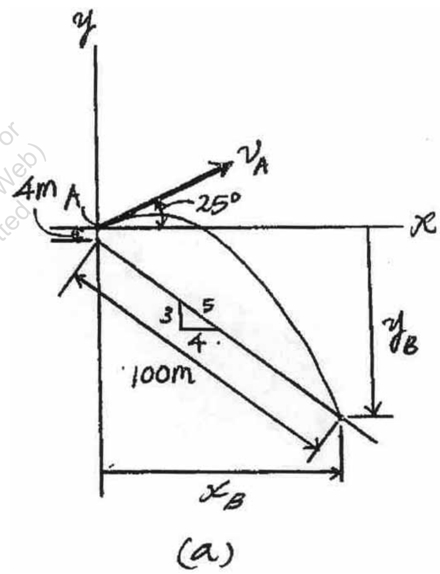
Coordinate System: $x-y$ coordinate system will be set with its origin to coincide with point A as shown in Fig. a.

x-motion: Here, $x_A = 0,\ x_B = 100\left(\frac{4}{5}\right) = 80$ m and $(v_A)_x = v_A \cos 25°$.

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$80 = 0 + (v_A \cos 25°)t$$

$$t = \frac{80}{v_A \cos 25°} \qquad \textbf{(1)}$$



y-motion: Here, $y_A = 0,\ y_B = -[4 + 100\left(\frac{3}{5}\right)] = -64$ m and $(v_A)_y = v_A \sin 25°$ and $a_y = -g = -9.81\ \text{m/s}^2$.

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-64 = 0 + v_A \sin 25°\ t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 25°\ t = 64 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yieldS

$$4.905\left(\frac{80}{v_A \cos 25°}\right)^2 = v_A \sin 25°\left(\frac{80}{v_A \cos 25°}\right) = 64$$

$$\left(\frac{80}{v_A \cos 25°}\right)^2 = 20.65$$

$$\frac{80}{v_A \cos 25°} = 4.545$$

$$v_A = 19.42\ \text{m/s} = 19.4\ \text{m/s} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25°} = 4.54465$$

12–101. continued

Using this result,

$(+\uparrow)$ $\quad (v_B)_y = (v_A)_y + a_y t$

$= 19.42 \sin 25° + (-9.81)(4.5446)$

$= -36.37 \text{ m/s} = 36.37 \text{ m/s} \downarrow$

And

$(\overset{+}{\rightarrow})$ $\quad (v_B)_x = (v_A)_x = v_A \cos 25° = 19.42 \cos 25° = 17.60 \text{ m/s} \rightarrow$

Thus,

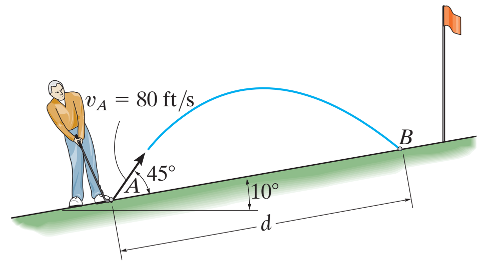
$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2}$

$= \sqrt{36.37^2 + 17.60^2}$

$= 40.4 \text{ m/s}$ **Ans.**

12–102.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 55° = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d \cos 10°$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d \cos 10° = 0 + 45.89t \qquad \textbf{(1)}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 55° = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d \sin 10°$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$d \sin 10° = 0 + 65.53t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{(2)}$$

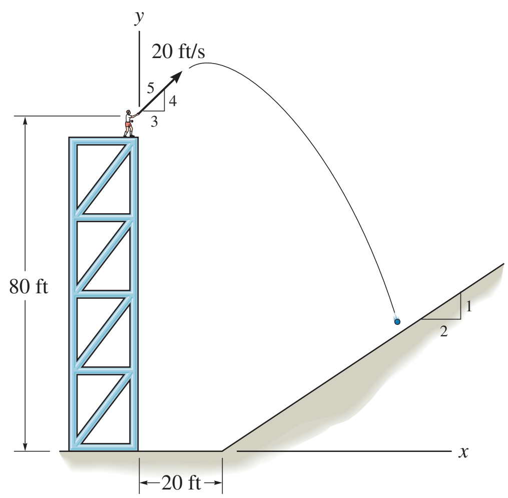
Solving Eqs. (1) and (2) yields

$$d = 166 \text{ ft} \qquad \textbf{Ans.}$$

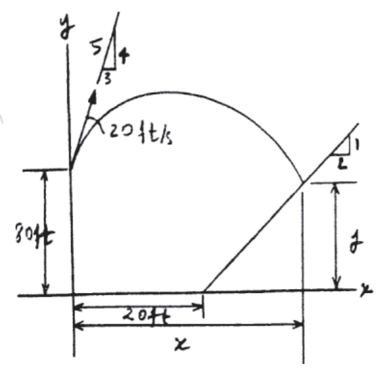
$$t = 3.568 \text{ s}$$

12–103.

The ball is thrown from the tower with a velocity of 20 ft/s as shown. Determine the x and y coordinates to where the ball strikes the slope. Also, determine the speed at which the ball hits the ground.



SOLUTION



Assume ball hits slope.

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$x = 0 + \frac{3}{5}(20)t = 12t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 80 + \frac{4}{5}(20)t + \frac{1}{2}(-32.2)t^2 = 80 + 16t - 16.1t^2$$

Equation of slope: $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{1}{2}(x - 20)$$

$$y = 0.5x - 10$$

Thus,

$$80 + 16t - 16.1t^2 = 0.5(12t) - 10$$

$$16.1t^2 - 10t - 90 = 0$$

Choosing the positive root:

$t = 2.6952$ s

$x = 12(2.6952) = 32.3$ ft **Ans.**

Since 32.3 ft $>$ 20 ft, assumption is valid.

$y = 80 + 16(2.6952) - 16.1(2.6952)^2 = 6.17$ ft **Ans.**

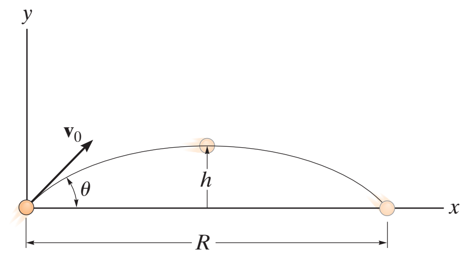
$(\xrightarrow{+})$ $\quad v_x = (v_0)_x = \frac{3}{5}(20) = 12$ ft/s

$(+\uparrow)$ $\quad v_y = (v_0)_y + a_c t = \frac{4}{5}(20) + (-32.2)(2.6952) = -70.785$ ft/s

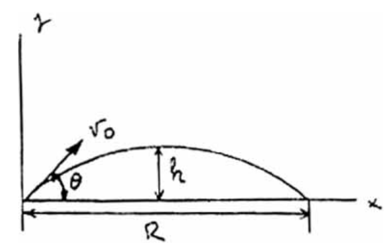
$v = \sqrt{(12)^2 + (-70.785)^2} = 71.8$ ft/s **Ans.**

***12–104.**

The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.



SOLUTION



$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t$$

$$R = 0 + (v_0 \cos\theta)t$$

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$0 = 0 + (v_0 \sin\theta)\, t + \frac{1}{2}(-g)t^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0 \cos\theta}\right)$$

$$R = \frac{v_0^2}{g} \sin 2\theta$$ **Ans.**

$$t = \frac{R}{v_0 \cos\theta} = \frac{v_0^2\,(2 \sin\theta \cos\theta)}{v_0\, g \cos\theta}$$

$$= \frac{2v_0}{g} \sin\theta$$ **Ans.**

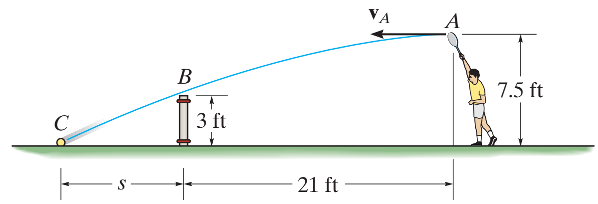
$(+\uparrow)$ $$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (v_0 \sin\theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g} \sin^2\theta$$ **Ans.**

12–105.

Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$3 = 7.5 + 0 + \frac{1}{2}(-32.2)t_1^2$$

$$t_1 = 0.5287 \text{ s}$$

For the ball to travel from A to C, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 7.5 + 0 + \frac{1}{2}(-32.2)t_2^2$$

$$t_2 = 0.6825 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A$. For the ball to travel from A to B, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 = 0 + v_A(0.5287)$$

$$v_A = 39.72 \text{ ft/s} = 39.7 \text{ ft/s}$$ **Ans.**

For the ball to travel from A to C, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

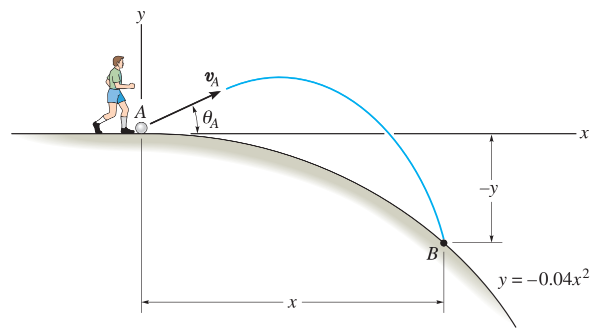
$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 + s = 0 + 39.72(0.6825)$$

$$s = 6.11 \text{ ft}$$ **Ans.**

12–106.

The ball at A is kicked with a speed $v_A = 8$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$(v_A)_x = 8 \cos 30° = 6.928$ ft/s

$(v_A)_y = 8 \sin 30° = 4$ ft/s

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$x = 0 + 6.928\, t$ **(1)**

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$y = 0 + 4t + \frac{1}{2}(-32.2)t^2$ **(2)**

$y = -0.04\, x^2$

From Eqs. (1) and (2):

$y = 0.5774\, x - 0.3354\, x^2$

$-0.04\, x^2 = 0.5774x - 0.3354\, x^2$

$0.2954\, x^2 = 0.5774x$

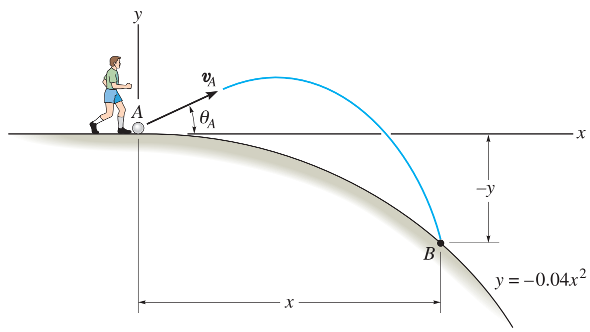
$x = 1.95$ ft **Ans.**

Thus,

$y = -0.04(1.954)^2 = -0.153$ ft **Ans.**

12–107.

The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



SOLUTION

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$$15 = 0 + v_A \cos 30°\, t$$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-9 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-32.2)t^2$$

$v_A = 16.5$ ft/s **Ans.**

$t = 1.047$ s

$(\xrightarrow{+})\ (v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

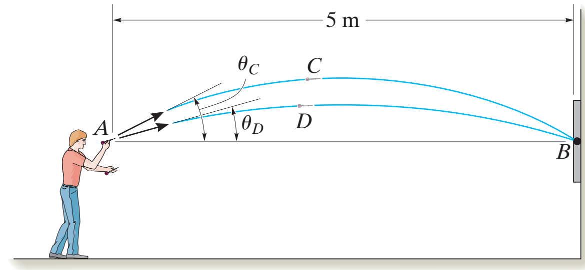
$(+\uparrow)\ v = v_0 + a_c t$

$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans.**

***12–108.**



The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C $(>\theta_D)$, then the second dart is thrown at θ_D.

SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$5 = 0 + (10\cos\theta)\,t \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$-10\sin\theta = 10\sin\theta - 9.81\,t$$

$$t = \frac{2(10\sin\theta)}{9.81} = 2.039\sin\theta$$

From Eq. (1),

$$5 = 20.39\sin\theta\cos\theta$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$

$$\sin 2\theta = 0.4905$$

The two roots are $\theta_D = 14.7^\circ$ **Ans.**

$\theta_C = 75.3^\circ$ **Ans.**

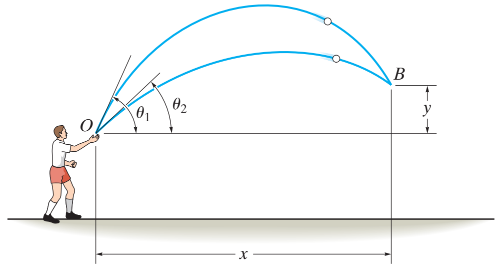
From Eq. (1): $t_D = 0.517$ s

$t_C = 1.97$ s

So that $\Delta t = t_C - t_D = 1.45$ s **Ans.**

12–109.

A boy throws a ball at O in the air with a speed v_0 at an angle θ_1. If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in mid air at B.



SOLUTION

Vertical Motion: For the first ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin \theta_1$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin \theta_1 t_1 + \frac{1}{2}(-g)t_1^2 \qquad \textbf{(1)}$$

For the second ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin \theta_2$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin \theta_2 t_2 + \frac{1}{2}(-g)t_2^2 \qquad \textbf{(2)}$$

Horizontal Motion: For the first ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos \theta_1$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos \theta_1 \, t_1 \qquad \textbf{(3)}$$

For the second ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos \theta_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos \theta_2 \, t_2 \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4), we have

$$t_2 = \frac{\cos \theta_1}{\cos \theta_2} t_1 \qquad \textbf{(5)}$$

Equating Eqs. (1) and (2), we have

$$v_0 t_1 \sin \theta_1 - v_0 t_2 \sin \theta_2 = \frac{1}{2} g\left(t_1^2 - t_2^2\right) \qquad \textbf{(6)}$$

Solving Eq. [5] into [6] yields

$$t_1 = \frac{2v_0 \cos \theta_2 \sin(\theta_1 - \theta_2)}{g(\cos^2 \theta_2 - \cos^2 \theta_1)}$$

$$t_2 = \frac{2v_0 \cos \theta_1 \sin(\theta_1 - \theta_2)}{g(\cos^2 \theta_2 - \cos^2 \theta_1)}$$

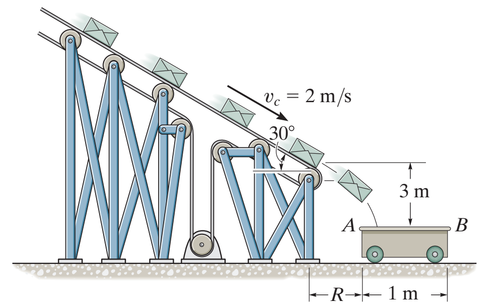
Thus, the time between the throws is

$$\Delta t = t_1 - t_2 = \frac{2v_0 \sin(\theta_1 - \theta_2)(\cos \theta_2 - \cos \theta_1)}{g(\cos^2 \theta_2 - \cos^2 \theta_1)}$$

$$= \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos \theta_2 + \cos \theta_1)} \qquad \textbf{Ans.}$$

12–110.

Small packages traveling on the conveyor belt fall off into a l-m-long loading car. If the conveyor is running at a constant speed of $v_C = 2$ m/s, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 2 \sin 30° = 1.00$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 3$ m, respectively.

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$3 = 0 + 1.00(t) + \frac{1}{2}(9.81)\left(t^2\right)$$

Choose the positive root $t = 0.6867$ s

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 2 \cos 30° = 1.732$ m/s and the initial horizontal position is $(s_0)_x = 0$. If $s_x = R$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$R = 0 + 1.732(0.6867) = 1.19 \text{ m}$$ **Ans.**

If $s_x = R + 1$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

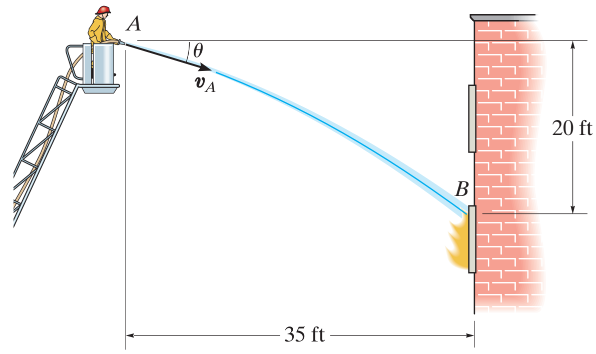
$$R + 1 = 0 + 1.732(0.6867)$$

$$R = 0.189 \text{ m}$$ **Ans.**

Thus, $R_{min} = 0.189$ m, $R_{max} = 1.19$ m **Ans.**

12–111.

The fireman wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80$ ft/s.



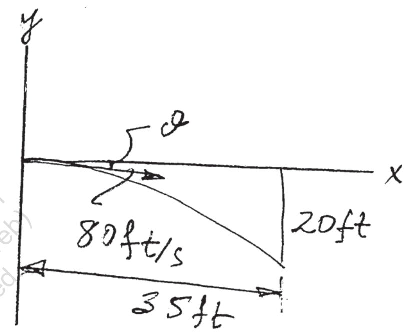
SOLUTION

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t$$

$$35 = 0 + (80)(\cos\theta)t$$

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-20 = 0 - 80\,(\sin\theta)t + \frac{1}{2}(-32.2)t^2$$



Thus,

$$20 = 80\sin\theta\,\frac{0.4375}{\cos\theta}t + 16.1\left(\frac{0.1914}{\cos^2\theta}\right)$$

$$20\cos^2\theta = 17.5\sin 2\theta + 3.0816$$

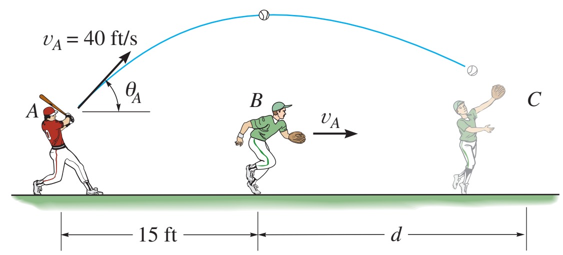
Solving,

$\theta_1 = 24.9°$ (below the horizontal) **Ans.**

$\theta_2 = 85.2°$ (above the horizontal) **Ans.**

***12–112.**

The baseball player A hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60°$ from the horizontal. When the ball is directly overhead of player B he begins to run under it. Determine the constant speed at which B must run and the distance d in order to make the catch at the same elevation at which the ball was hit.



SOLUTION

Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60° = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2}(-32.2)\,t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60° = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player B must travel in order to catch the baseball is

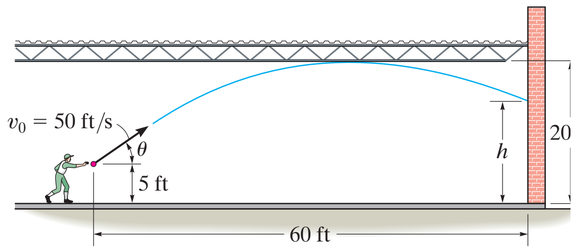
$d = R - 15 = 43.03 - 15 = 28.0$ ft **Ans.**

Player B is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$v_B = 40 \cos 60° = 20.0$ ft/s **Ans.**

12–113.

The man stands 60 ft from the wall and throws a ball at it with a speed $v_0 = 50\text{ ft/s}$. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height of 20 ft.



SOLUTION

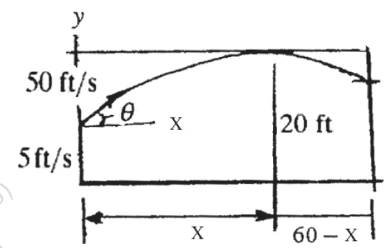
$v_x = 50\cos\theta$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$x = 0 + 50\cos\theta\, t \qquad$ **(1)**

$(+\uparrow)\qquad v = v_0 + a_c t$

$v_y = 50\sin\theta - 32.2\,t \qquad$ **(2)**



$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$y = 0 + 50\sin\theta\, t - 16.1\,t^2 \qquad$ **(3)**

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$v_y^2 = (50\sin\theta)^2 + 2(-32.2)(s - 0)$

$v_y^2 = 2500\sin^2\theta - 64.4\,s \qquad$ **(4)**

Require $v_y = 0$ at $s = 20 - 5 = 15$ ft

$0 = 2500\sin^2\theta - 64.4\,(15)$

$\theta = 38.433° = 38.4° \qquad$ **Ans.**

From Eq. (2)

$0 = 50\sin 38.433° - 32.2\,t$

$t = 0.9652$ s

From Eq. (1)

$x = 50\cos 38.433°(0.9652) = 37.8$ ft

Time for ball to hit wall

From Eq. (1),

$60 = 50(\cos 38.433°)t$

$t = 1.53193$ s

From Eq. (3)

$y = 50\sin 38.433°(1.53193) - 16.1(1.53193)^2$

$y = 9.830$ ft

$h = 9.830 + 5 = 14.8$ ft $\qquad$ **Ans.**

12–114.

A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

SOLUTION

$v = 16 \text{ m/s}$

$a_t = 8 \text{ m/s}^2$

$r = 50 \text{ m}$

$$a_n = \frac{v^2}{\rho} = \frac{(16)^2}{50} = 5.12 \text{ m/s}^2$$

$$a = \sqrt{(8)^2 + (5.12)^2} = 9.50 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–115.

Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

SOLUTION

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$ **Ans.**

***12–116.**

A car moves along a circular track of radius 250 ft such that its speed for a short period of time, $0 \le t \le 4$ s, is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 3$ s. How far has it traveled in $t = 3$ s?

SOLUTION

$v = 3(t + t^2)$

$$a_t = \frac{dv}{dt} = 3 + 6\,t$$

When $t = 3$ s, $\quad a_t = 3 + 6(3) = 21\ \text{ft/s}^2$

$$a_n = \frac{[3(3 + 3^2)]^2}{250} = 5.18\ \text{ft/s}^2$$

$$a = \sqrt{(21)^2 + (5.18)^2} = 21.6\ \text{ft/s}^2$$ **Ans.**

$$\int ds = \int_0^3 3(t + t^2)dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3\Big|_0^3$$

$$\Delta s = 40.5\ \text{ft}$$ **Ans.**

12–117.

A car travels along a horizontal circular curved road that has a radius of 600 m. If the speed is uniformly increased at a rate of $2000\ \text{km/h}^2$, determine the magnitude of the acceleration at the instant the speed of the car is 60 km/h.

SOLUTION

$$a_t = \left(\frac{2000\ \text{km}}{\text{h}^2}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\text{h}}{3600\ \text{s}}\right)^2 = 0.1543\ \text{m/s}^2$$

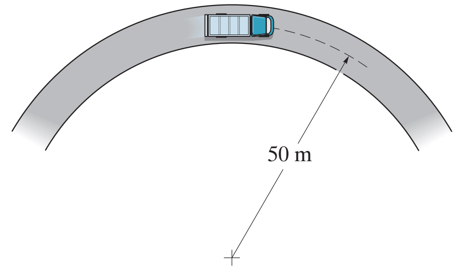
$$v = \left(\frac{60\ \text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\text{h}}{3600\ \text{s}}\right) = 16.67\ \text{m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{600} = 0.4630\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.1543^2 + 0.4630^2} = 0.488\ \text{m/s}^2$$ **Ans.**

12–118.

The truck travels in a circular path having a radius of 50 m at a speed of $v = 4$ m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



SOLUTION

$$v\,dv = a_t\,ds$$

$$\int_4^v v\,dv = \int_0^{10} 0.05s\,ds$$

$$0.5v^2 - 8 = \frac{0.05}{2}(10)^2$$

$$v = 4.583 = 4.58 \text{ m/s}$$ **Ans.**

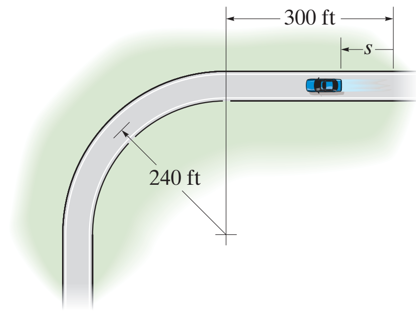
$$a_n = \frac{v^2}{\rho} = \frac{(4.583)^2}{50} = 0.420 \text{ m/s}^2$$

$$a_t = 0.05(10) = 0.5 \text{ m/s}^2$$

$$a = \sqrt{(0.420)^2 + (0.5)^2} = 0.653 \text{ m/s}^2$$ **Ans.**

12–119.

The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)\ \text{ft/s}^2$, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.



SOLUTION

$a_t = 0.05t^2$

$$\int_0^v dv = \int_0^t 0.05\, t^2\, dt$$

$v = 0.0167\, t^3$

$$\int_0^s ds = \int_0^t 0.0167\, t^3\, dt$$

$s = 4.167(10^{-3})\, t^4$

When $t = 18$ s, $\quad s = 437.4$ ft

Therefore the car is on a curved path.

$v = 0.0167(18^3) = 97.2$ ft/s **Ans.**

$$a_n = \frac{(97.2)^2}{240} = 39.37\ \text{ft/s}^2$$

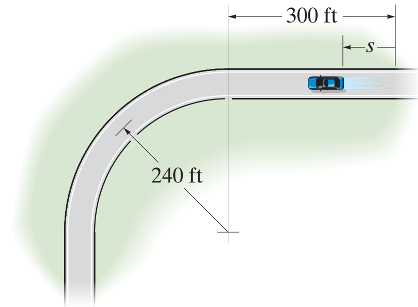
$a_t = 0.05(18^2) = 16.2\ \text{ft/s}^2$

$a = \sqrt{(39.37)^2 + (16.2)^2}$

$a = 42.6\ \text{ft/s}^2$ **Ans.**

***12–120.**

The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)\ \text{ft/s}^2$, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



SOLUTION

The car is on the curved path.

$$a_t = 0.05\,t^2$$

$$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$$

$$v = 0.0167\,t^3$$

$$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$$

$$s = 4.167(10^{-3})\,t^4$$

$$550 = 4.167(10^{-3})\,t^4$$

$$t = 19.06\ \text{s}$$

So that

$$v = 0.0167(19.06)^3 = 115.4$$

$$v = 115\ \text{ft/s} \qquad \textbf{Ans.}$$

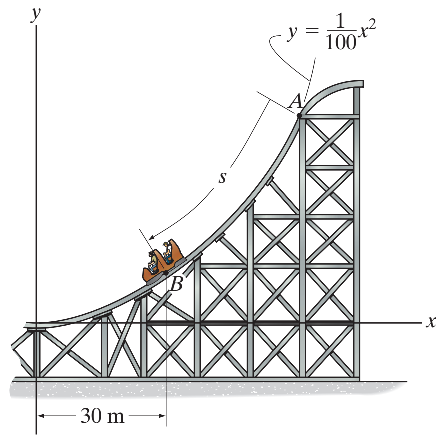
$$a_n = \frac{(115.4)^2}{240} = 55.51\ \text{ft/s}^2$$

$$a_t = 0.05(19.06)^2 = 18.17\ \text{ft/s}^2$$

$$a = \sqrt{(55.51)^2 + (18.17)^2} = 58.4\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–121.

When the roller coaster is at B, it has a speed of $25\ \text{m/s}$, which is increasing at $a_t = 3\ \text{m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



SOLUTION

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

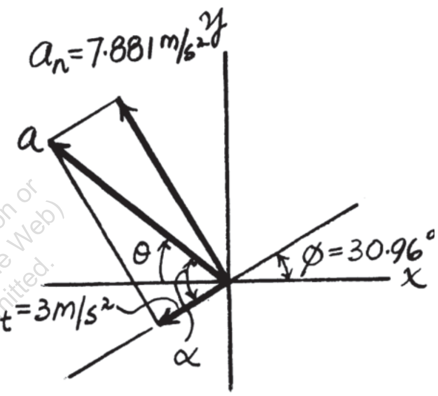
$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\text{ m}$$

Acceleration:

$$a_t = \dot{v} = 3\ \text{m/s}^2$$

$$a_n = \frac{v_B{}^2}{\rho} = \frac{25^2}{79.30} = 7.881\ \text{m/s}^2$$



(a)

The magnitude of the roller coaster's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{3^2 + 7.881^2} = 8.43\ \text{m/s}^2 \qquad \textbf{Ans.}$$

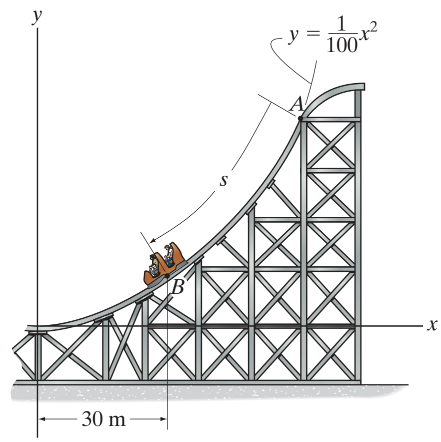
The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=30\text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96°$.

As shown in Fig. a, $\mathbf{a}_n$ is always directed towards the center of curvature of the path. Here, $\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16°$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

$$\theta = \alpha - \phi = 38.2° \qquad \textbf{Ans.}$$

12–122.

If the roller coaster starts from rest at A and its speed increases at $a_t = (6 - 0.06s)\ \text{m/s}^2$, determine the magnitude of its acceleration when it reaches B where $s_B = 40$ m.



SOLUTION

Velocity: Using the initial condition $v = 0$ at $s = 0$,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^s (6 - 0.06s)\,ds$$

$$v = \left(\sqrt{12s - 0.06s^2}\right)\ \text{m/s} \qquad \textbf{(1)}$$

Thus,

$$v_B = \sqrt{12(40) - 0.06(40)^2} = 19.60\ \text{m/s}$$

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 6 - 0.06(40) = 3.600\ \text{m/s}^2$$

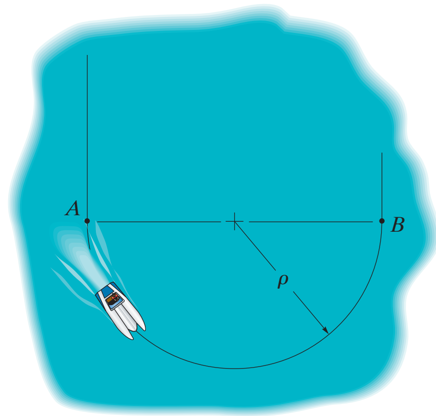
$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842\ \text{m/s}^2$$

The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{3.600^2 + 4.842^2} = 6.03\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–123.

The speedboat travels at a constant speed of 15 m/s while making a turn on a circular curve from A to B. If it takes 45 s to make the turn, determine the magnitude of the boat's acceleration during the turn.



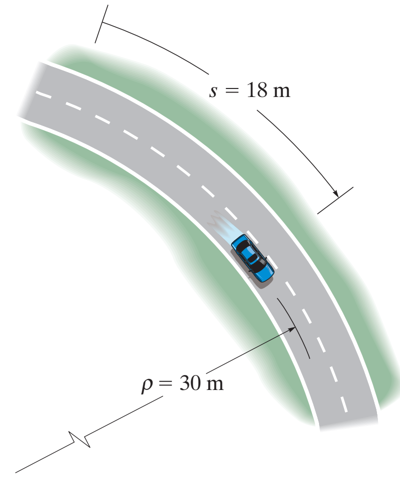
SOLUTION

Acceleration: During the turn, the boat travels $s = vt = 15(45) = 675$ m. Thus, the radius of the circular path is $\rho = \dfrac{s}{\pi} = \dfrac{675}{\pi}$ m. Since the boat has a constant speed, $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho} = \frac{15^2}{\left(\dfrac{675}{\pi}\right)} = 1.05 \text{ m/s}^2$$ **Ans.**

***12–124.**

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



SOLUTION

$$\int_0^v dv = \int_0^t 0.5e^t\, dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064 \text{ s}$$

$$v = 0.5(e^{3.7064} - 1) = 19.85 \text{ m/s} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

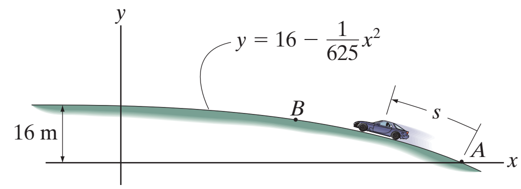
$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\text{ s}} = 20.35 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–125.

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B{}^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

When the car is at B $(s = 51.5 \text{ m})$

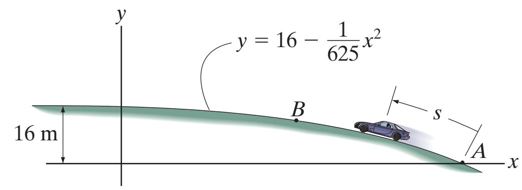
$$a_t = \left[0.225(51.5) - 3.75\right] = -2.591 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2$$ **Ans.**

12–126.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 100$ m.



SOLUTION

Velocity: The speed of the car at C is

$$v_C^{\,2} = v_A^{\,2} + 2a_t\,(s_C - s_A)$$

$$v_C^{\,2} = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361\ \text{m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}$$

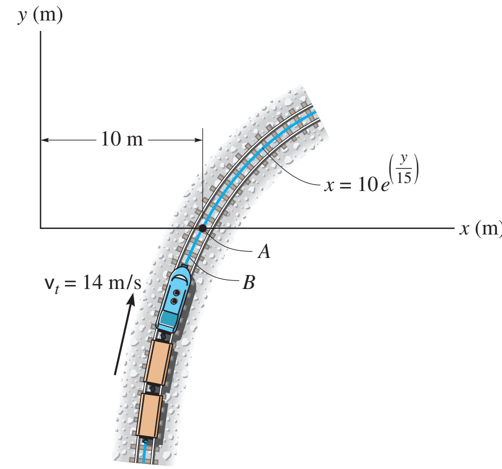
$$a_n = \frac{v_C^{\,2}}{\rho} = \frac{22.361^2}{312.5} = 1.60\ \text{m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68\ \text{m/s}^2$$ **Ans.**

12–127.

A train is traveling with a constant speed of 14 m/s along the curved path. Determine the magnitude of the acceleration of the front of the train, B, at the instant it reaches point A $(y = 0)$.



SOLUTION

$$x = 10e^{\left(\frac{y}{15}\right)}$$

$$y = 15\ln\left(\frac{x}{10}\right)$$

$$\frac{dy}{dx} = 15\left(\frac{10}{x}\right)\left(\frac{1}{10}\right) = \frac{15}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}$$

At $x = 10$,

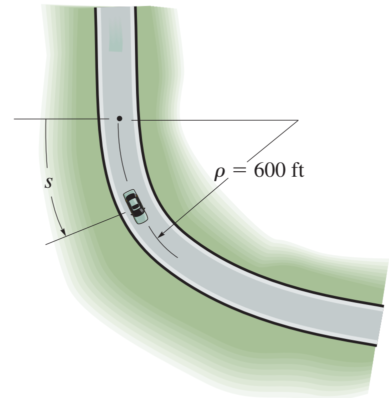
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (1.5)^2\right]^{\frac{3}{2}}}{|-0.15|} = 39.06 \text{ m}$$

$$a_t = \frac{dv}{dt} = 0$$

$$a_n = a = \frac{v^2}{\rho} = \frac{(14)^2}{39.06} = 5.02 \text{ m/s}^2$$ **Ans.**

***12–128.**

When a car starts to round a curved road with the radius of curvature of 600 ft, it is traveling at 75 ft/s. If the car's speed begins to decrease at a rate of $\dot{v} = (-0.06t^2)$ ft/s^2, determine the magnitude of the acceleration of the car when it has traveled a distance of $s = 700$ ft.



SOLUTION

Velocity: Using the initial condition $v = 75$ ft/s when $t = 0$ s,

$$\int dt = \int a_t dt$$

$$\int_{v=75\text{ ft/s}}^{v} dv = \int_0^t -0.06t^2 dt$$

$$v = (75 - 0.02t^3)\text{ ft/s}$$

Position: Using the initial condition $s = 0$ at $t = 0$ s,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t (75 - 0.02t^3)dt$$

$$s = [75t - 0.005t^4]\text{ ft}$$

At $s = 700$ ft,

$$700 = 75t - 0.005t^4$$

Solving the above equation by trial and error,

$t = 10$ s and $t = 20$ s. Pick the first solution.

Acceleration: When $t = 10$ s, $a_t = \dot{v} = -0.06(10^2) = -6$ ft/s^2 and $v = 75 - 0.02(10^3) = 55$ ft/s

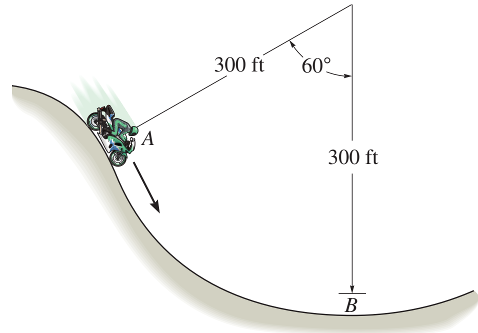
$$a_n = \frac{v^2}{\rho} = \frac{55^2}{600} = 5.042\text{ ft/s}^2$$

Thus, the magnitude of the truck's acceleration is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{(-6)^2 + 5.042^2} = 7.84\text{ ft/s}^2$$ **Ans.**

12–129.

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.3t)\ \text{ft/s}^2$, where t is in seconds. If he starts from rest at A, determine the magnitudes of his velocity and acceleration when he reaches B.



SOLUTION

$$\int_0^v dv = \int_0^t 0.3t\,dt$$

$$v = 0.15t^2$$

$$\int_0^s ds = \int_0^t 0.15t^2\,dt$$

$$s = 0.05t^3$$

When $s = \frac{\pi}{3}(300)$ ft, $\quad \frac{\pi}{3}(300) = 0.05t^3 \quad t = 18.453\ \text{s}$

$v = 0.15(18.453)^2 = 51.08\ \text{ft/s} = 51.1\ \text{ft/s}$ **Ans.**

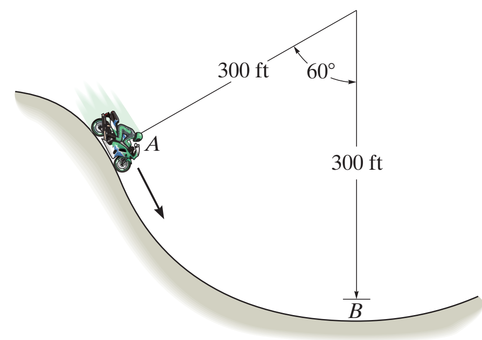
$a_t = \dot{v} = 0.3t|_{t=18.453\ \text{s}} = 5.536\ \text{ft/s}^2$

$$a_n = \frac{v^2}{\rho} = \frac{51.08^2}{300} = 8.696\ \text{ft/s}^2$$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(5.536)^2 + (8.696)^2} = 10.3\ \text{ft/s}^2$ **Ans.**

12–130.

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.04s)\ \text{ft/s}^2$ where s is in ft. If he starts at $v_A = 2\ \text{ft/s}$ where $s = 0$ at A, determine the magnitude of his velocity when he reaches B. Also, what is his initial acceleration?



SOLUTION

Velocity: At $s = 0, v = 2$. Here, $a_c = \dot{v} = 0.04s$. Then

$$\int v\,dv = \int a_t\,ds$$

$$\int_2^v v\,dv = \int_0^s 0.04s\,ds$$

$$\left.\frac{v^2}{2}\right|_2^v = 0.02s^2\Big|_0^s$$

$$\frac{v^2}{2} - 2 = 0.02s^2$$

$$v^2 = 0.04s^2 + 4 = 0.04(s^2 + 100)$$

$$v = 0.2\sqrt{s^2 + 100}$$

At B, $s = r\theta = 300\left(\frac{\pi}{3}\right) = 100\pi$ ft. Thus

$$v\Big|_{s=100\pi\ \text{ft}} = 0.2\sqrt{(100\pi)^2 + 100} = 62.9\ \text{ft/s}$$ **Ans.**

Acceleration: At $t = 0, s = 0$, and $v = 2$.

$$a_t = \dot{v} = 0.04\ \text{s}$$

$$a_t\Big|_{s=0} = 0$$

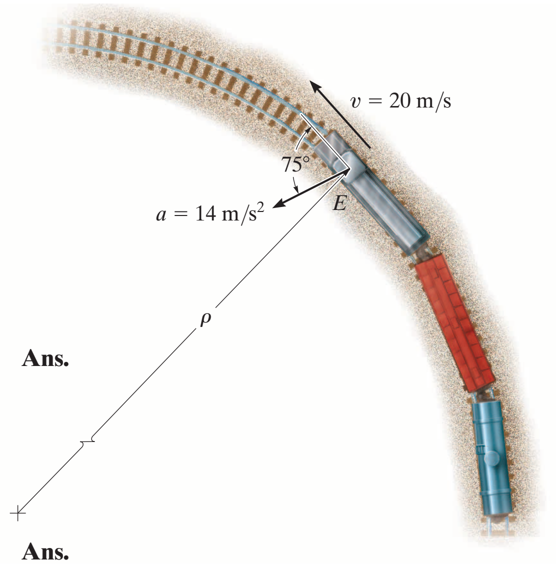
$$a_n = \frac{v^2}{\rho}$$

$$a_n\Big|_{s=0} = \frac{(2)^2}{300} = 0.01333\ \text{ft/s}^2$$

$$a = \sqrt{(0)^2 + (0.01333)^2} = 0.0133\ \text{ft/s}^2$$

12–131.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



SOLUTION

$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$ **Ans.**

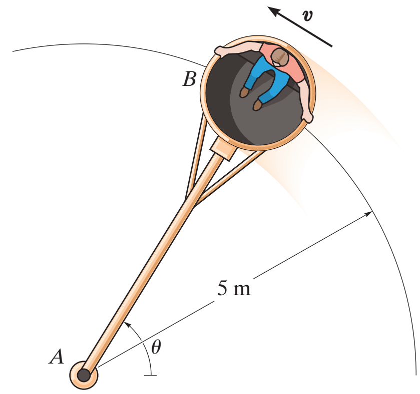
$a_n = 14 \sin 75°$

$a_n = \dfrac{(20)^2}{\rho}$

$\rho = 29.6 \text{ m}$ **Ans.**

***12–132.**

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right) \qquad \textbf{(1)}$$

When $\theta = 30°$, the car has traveled a distance of $s = r\theta = 5\left(\dfrac{30°}{180°}\pi\right) = 2.618\ \text{m}$. The time required for the car to travel this distance can be obtained by applying $v = \dfrac{ds}{dt}$.

$$ds = v\,dt$$

$$\int_0^{2.618\ \text{m}} ds = \int_0^t 0.5\left(e^t - 1\right) dt$$

$$2.618 = 0.5\left(e^t - t - 1\right)$$

Solving by trial and error $\quad t = 2.1234\ \text{s}$

Substituting $t = 2.1234\ \text{s}$ into Eq. (1) yields

$$v = 0.5\left(e^{2.1234} - 1\right) = 3.680\ \text{m/s} = 3.68\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for the car at $t = 2.1234\ \text{s}$ is $a_t = 0.5e^{2.1234} = 4.180\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

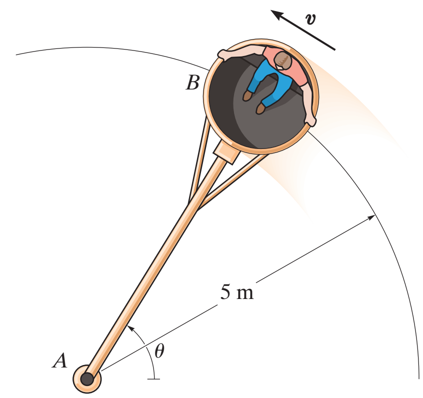
$$a_n = \frac{v^2}{\rho} = \frac{3.680^2}{5} = 2.708\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.180^2 + 2.708^2} = 4.98\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–133.

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right)$$

When $t = 2$ s, $\quad v = 0.5\left(e^2 - 1\right) = 3.195\ \text{m/s} = 3.19\ \text{m/s}$ **Ans.**

Acceleration: The tangential acceleration of the car at $t = 2$ s is $a_t = 0.5e^2 = 3.695\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.195^2}{5} = 2.041\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3.695^2 + 2.041^2} = 4.22\ \text{m/s}^2$$ **Ans.**

12–134.

A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)\ \text{ft/s}^2$, determine the magnitude of its acceleration at the instant $t = 5$ s.

SOLUTION

$$\int_{15}^{v} dv = \int_{0}^{5} 0.8t\,dt$$

$$v = 25\ \text{ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25\ \text{ft/s}^2$$

At $t = 5$ s, $\quad a_t = \dot{v} = 0.8(5) = 4\ \text{ft/s}^2$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–135.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2\ \text{m/s}^2$.

SOLUTION

$$a_t = 2\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36\ \text{m/s}^2$$ **Ans.**

***■12–136.**

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s} \qquad \textbf{Ans.}$$

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–137.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\,v)\ \text{m/s}^2$, determine the magnitude of the particle's acceleraton four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10$ m/s at $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10\,\text{m/s}}^{v} \frac{dv}{0.05v}$$

$$t = 20 \ln \frac{v}{10}$$

$$v = (10e^{t/20})\ \text{m/s}$$

When $t = 4$ s,

$$v = 10e^{4/20} = 12.214\ \text{m/s}$$

Acceleration: When $v = 12.214$ m/s ($t = 4$ s),

$$a_t = 0.05(12.214) = 0.6107\ \text{m/s}^2$$

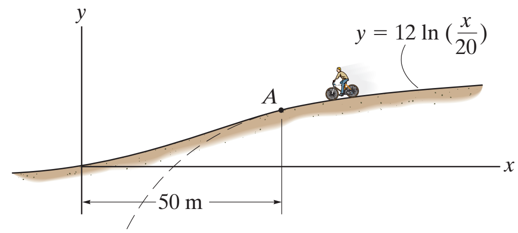
$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984\ \text{m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6107^2 + 2.984^2} = 3.05\ \text{m/s}^2$$ **Ans.**

12–138.

When the bicycle passes point A, it has a speed of 6 m/s, which is increasing at the rate of $\dot{v} = (0.5)\ \text{m/s}^2$. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = 12 \ln\left(\frac{x}{20}\right)$$

$$\frac{dy}{dx} = 12\left(\frac{1}{x/20}\right)\left(\frac{1}{20}\right) = \frac{12}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{12}{x^2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{12}{x}\right)^2\right]^{3/2}}{\left|-\frac{12}{x^2}\right|}\right|_{x=50\text{ m}} = 226.59\text{ m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}^2$$

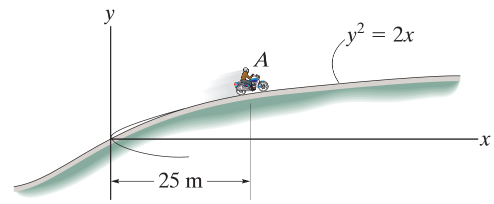
$$a_n = \frac{v^2}{\rho} = \frac{6^2}{226.59} = 0.1589\ \text{m/s}^2$$

The magnitude of the bicycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 0.1589^2} = 0.525\ \text{m/s}^2$$ **Ans.**

12–139.

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21 \text{ m}$$

Acceleration: The speed of the motorcycle at a is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67 \text{ m/s}$$

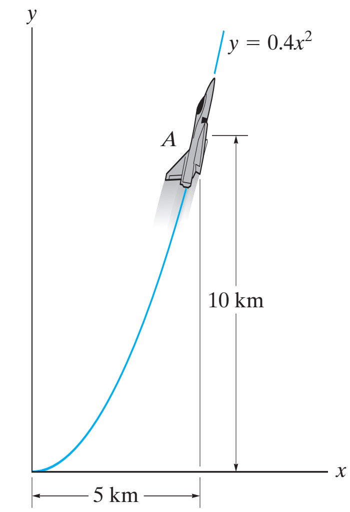
$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627 \text{ m/s}^2$$

Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 0.7627^2} = 0.763 \text{ m/s}^2$$ **Ans.**

***12–140.**

The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of $0.8\ \text{m/s}^2$. Determine the magnitude of acceleration of the plane when it is at point A.



SOLUTION

$$y = 0.4x^2$$

$$\frac{dy}{dx} = 0.8x\Big|_{x=5\text{ km}} = 4$$

$$\frac{d^2y}{dx^2} = 0.8$$

$$\rho = \frac{[1 + (4)^2]^{3/2}}{0.8} = 87.62\text{ km}$$

$$a_t = 0.8\ \text{m/s}^2$$

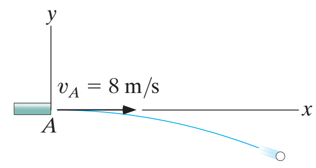
$$a_n = \frac{(0.200)^2}{87.62} = 0.457(10^{-3})\ \text{km/s}^2$$

$$a_n = 0.457\ \text{km/s}^2$$

$$a = \sqrt{(0.8)^2 + (0.457)^2} = 0.921\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–141.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.

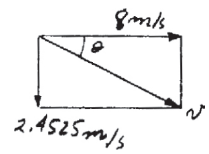


SOLUTION

$v_x = 8$ m/s

$(\xrightarrow{+})\quad s = v_0 t$

$x = 8t$

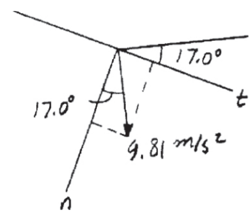


$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$

$y = -4.905t^2$

$y = -4.905\left(\frac{x}{8}\right)^2$



$y = -0.0766x^2$ (Parabola) **Ans.**

$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans.**

$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$

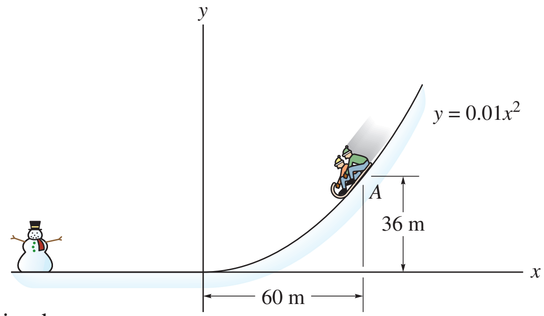
$a_x = 0 \quad a_y = 9.81\text{ m/s}^2$

$a_n = 9.81 \cos 17.04° = 9.38\text{ m/s}^2$ **Ans.**

$a_t = 9.81 \sin 17.04° = 2.88\text{ m/s}^2$ **Ans.**

12–142.

A toboggan is traveling down along a curve which can be approximated by the parabola $y = 0.01x^2$. Determine the magnitude of its acceleration when it reaches point A, where its speed is $v_A = 10$ m/s, and it is increasing at the rate of $\dot{v}_A = 3$ m/s^2.



SOLUTION

Acceleration: The radius of curvature of the path at point A must be determined first. Here, $\dfrac{dy}{dx} = 0.02x$ and $\dfrac{d^2y}{dx^2} = 0.02$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (0.02x)^2]^{3/2}}{|0.02|}\right|_{x=60\text{ m}} = 190.57 \text{ m}$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{10^2}{190.57} = 0.5247 \text{ m/s}^2$$

Here, $a_t = \dot{v}_A = 3$ m/s. Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 0.5247^2} = 3.05 \text{ m/s}^2$$ **Ans.**

12–143.

A particle P moves along the curve $y = (x^2 - 4)$ m with a constant speed of 5 m/s. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

SOLUTION

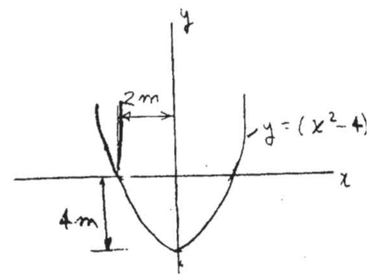
$y = (x^2 - 4)$

$a_t = \frac{dv}{dt} = 0,$

To obtain maximum $a = a_n$, ρ must be a minimum.

This occurs at:

$x = 0, \quad y = -4\text{ m}$ **Ans.**



Hence,

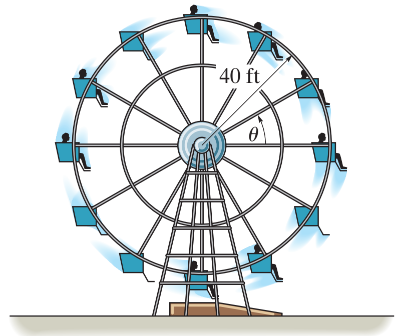
$$\left.\frac{dy}{dx}\right|_{x=0} = 2x = 0; \quad \frac{d^2y}{dx^2} = 2$$

$$\rho_{min} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+0]^{\frac{3}{2}}}{|2|} = \frac{1}{2}$$

$$(a)_{max} = (a_n)_{max} = \frac{v^2}{\rho_{min}} = \frac{5^2}{\frac{1}{2}} = 50\text{ m/s}^2$$ **Ans.**

***12–144.**

The Ferris wheel turns such that the speed of the passengers is increased by $\dot{v} = (4t)\ \text{ft/s}^2$, where t is in seconds. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = 30°$.



SOLUTION

$$\int_0^v dv = \int_0^t 4t\,dt$$

$$v = 2t^2$$

$$\int_0^s ds = \int_0^t 2t^2\,dt$$

$$s = \frac{2}{3}t^3$$

When $s = \frac{\pi}{6}(40)$ ft, $\quad \frac{\pi}{6}(40) = \frac{2}{3}t^3 \quad t = 3.1554$ s

$v = 2(3.1554)^2 = 19.91\ \text{ft/s} = 19.9\ \text{ft/s}$ **Ans.**

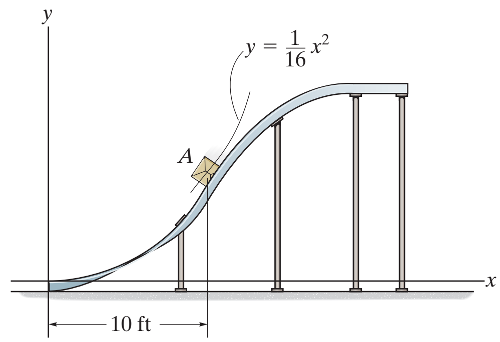
$$a_t = \dot{v} = 4t\,|_{t=3.1554\text{ s}} = 12.62\ \text{ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.91^2}{40} = 9.91\ \text{ft/s}^2$$

$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{12.62^2 + 9.91^2} = 16.0\ \text{ft/s}^2$ **Ans.**

12–145.

If the speed of the crate at A is 15 ft/s, which is increasing at a rate $\dot{v} = 3\ \text{ft/s}^2$, determine the magnitude of the acceleration of the crate at this instant.



SOLUTION

Radius of Curvature:

$$y = \frac{1}{16}x^2$$

$$\frac{dy}{dx} = \frac{1}{8}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{8}$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{8}x\right)^2\right]^{3/2}}{\left|\frac{1}{8}\right|}\right|_{x=10\text{ ft}} = 32.82\text{ ft}$$

Acceleration:

$$a_t = \dot{v} = 3\text{ft/s}^2$$

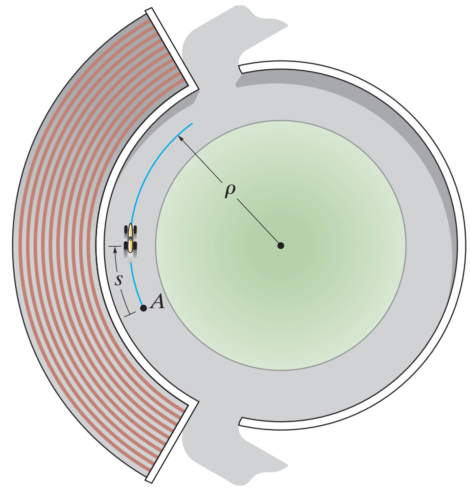
$$a_n = \frac{v^2}{\rho} = \frac{15^2}{32.82} = 6.856\text{ ft/s}^2$$

The magnitude of the crate's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 6.856^2} = 7.48\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–146.

The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



SOLUTION

$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^{v} v\,dv$$

$$\left.\frac{0.4s^2}{2}\right|_0^s = \left.\frac{v^2}{2}\right|_{15}^{v}$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$\left.1\text{n}\left(s + \sqrt{s^2 + 562.5}\right)\right|_0^s = 0.632\,456t$$

$$1\text{n}\left(s + \sqrt{s^2 + 562.5}\right) - 3.166\,196 = 0.632\,456t$$

At $s = 20$ m,

$t = 1.21$ s **Ans.**

12–147.

A boy sits on a merry-go-round so that he is always located at $r = 8$ ft from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at $2\ \text{ft/s}^2$. Determine the time needed for his acceleration to become $4\ \text{ft/s}^2$.

SOLUTION

$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = 2$$

$$v = v_0 + a_c t$$

$$v = 0 + 2t$$

$$a_n = \frac{v^2}{\rho} = \frac{(2t)^2}{8}$$

$$4 = \sqrt{(2)^2 + \left(\frac{(2t)^2}{8}\right)^2}$$

$$16 = 4 + \frac{16\,t^4}{64}$$

$$t = 2.63\ \text{s}$$ **Ans.**

***12–148.**

A particle travels along the path $y = a + bx + cx^2$, where a, b, c are constants. If the speed of the particle is constant, $v = v_0$, determine the x and y components of velocity and the normal component of acceleration when $x = 0$.

SOLUTION

$$y = a + bx + cx^2$$

$$\dot{y} = b\dot{x} + 2\,c\,x\,\dot{x}$$

$$\ddot{y} = b\,\ddot{x} + 2c\,(\dot{x})^2 + 2c\,x\ddot{x}$$

When $x = 0$, $\qquad \dot{y} = b\,\dot{x}$

$$v_0^2 + \dot{x}^2 + b^2\,\dot{x}^2$$

$$v_x = \dot{x} = \frac{v_0}{\sqrt{1 + b^2}}$$ **Ans.**

$$v_y = \frac{v_0\,b}{\sqrt{1 + b^2}}$$ **Ans.**

$$a_n = \frac{v_0^2}{\rho}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\frac{dy}{dx} = b + 2c\,x$$

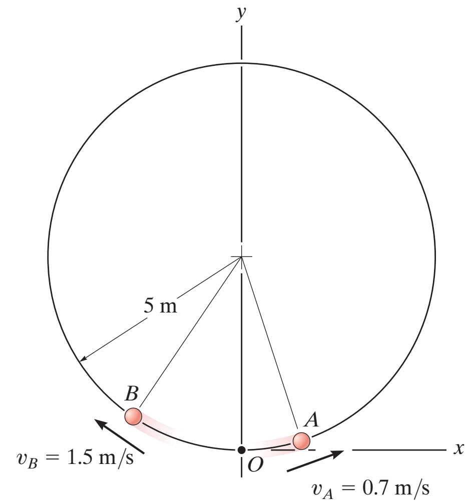
$$\frac{d^2y}{dx^2} = 2\,c$$

At $x = 0$, $\qquad \rho = \dfrac{(1 + b^2)^{3/2}}{2\,c}$

$$a_n = \frac{2\,c\,v_0^2}{(1 + b^2)^{3/2}}$$ **Ans.**

12–149.

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine in $t = 2$ s, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.



SOLUTION

(a) $s_A = 0.7(2) = 1.40$ m **Ans.**

$s_B = 1.5(2) = 3$ m **Ans.**

(b) $\theta_A = \dfrac{1.40}{5} = 0.280 \text{ rad.} = 16.04°$

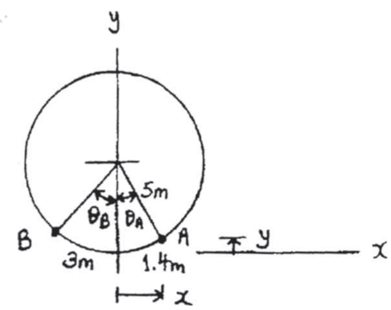
$\theta_B = \dfrac{3}{5} = 0.600 \text{ rad.} = 34.38°$

For A

$x = 5 \sin 16.04° = 1.382 = 1.38$ m

$y = 5(1 - \cos 16.04°) = 0.1947 = 0.195$ m

$\mathbf{r}_A = \{1.38\mathbf{i} + 0.195\mathbf{j}\}$ m **Ans.**



For B

$x = -5 \sin 34.38° = -2.823 = -2.82$ m

$y = 5(1 - \cos 34.38°) = 0.8734 = 0.873$ m

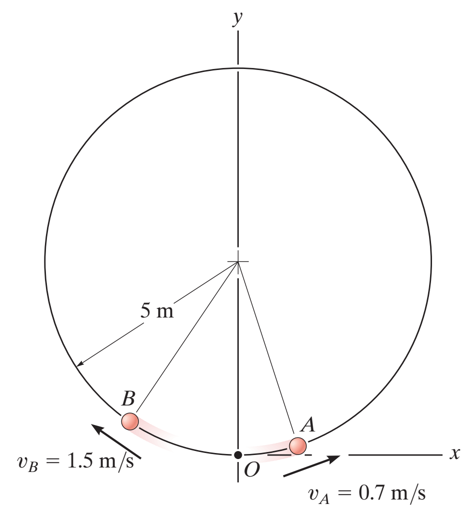
$\mathbf{r}_B = \{-2.82\mathbf{i} + 0.873\mathbf{j}\}$ m **Ans.**

(c) $\Delta\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = \{-4.20\mathbf{i} + 0.678\mathbf{j}\}$ m

$\Delta r = \sqrt{(-4.20)^2 + (0.678)^2} = 4.26$ m **Ans.**

12–150.

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7\ \text{m/s}$ and $v_B = 1.5\ \text{m/s}$, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



SOLUTION

$s_t = 2\pi(5) = 31.4159\ \text{m}$

$s_A = 0.7\,t$

$s_B = 1.5\,t$

Require

$s_A + s_B = 31.4159$

$0.7\,t + 1.5\,t = 31.4159$

$t = 14.28\ \text{s} = 14.3\ \text{s}$ **Ans.**

$a_B = \dfrac{v_B^2}{\rho} = \dfrac{(1.5)^2}{5} = 0.45\ \text{m/s}^2$ **Ans.**

12–151.

The position of a particle traveling along a curved path is $s = (3t^3 - 4t^2 + 4)$ m, where t is in seconds. When $t = 2$ s, the particle is at a position on the path where the radius of curvature is 25 m. Determine the magnitude of the particle's acceleration at this instant.

SOLUTION

Velocity:

$$v = \frac{d}{dt}\left(3t^3 - 4t^2 + 4\right) = \left(9t^2 - 8t\right)\text{m/s}$$

When $t = 2$ s,

$$v|_{t=2\text{ s}} = 9\left(2^2\right) - 8(2) = 20 \text{ m/s}$$

Acceleration:

$$a_t = \frac{dv}{ds} = \frac{d}{dt}\left(9t^2 - 8t\right) = \left(18t - 8\right)\text{m/s}^2$$

$$a_t|_{t=2\text{ s}} = 18(2) - 8 = 28 \text{ m/s}^2$$

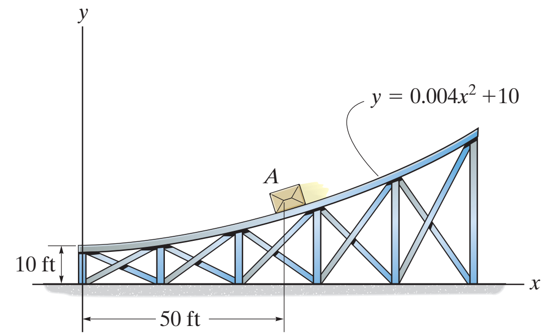
$$a_n = \frac{\left(v|_{t=2\text{s}}\right)^2}{\rho} = \frac{20^2}{25} = 16 \text{ m/s}^2$$

Thus,

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{28^2 + 16^2} = 32.2 \text{ m/s}^2$$ **Ans.**

***12–152.**

If the speed of the box at point A on the track is 30 ft/s which is increasing at the rate of $\dot{v} = 5\ \text{ft/s}^2$, determine the magnitude of the acceleration of the box at this instant.



SOLUTION

Radius of Curvature:

$$y = 0.004x^2 + 10$$

$$\frac{dy}{dx} = 0.008x$$

$$\frac{d^2y}{dx^2} = 0.008$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + (0.008x)^2\right]^{3/2}}{|0.008|}\right|_{x=50\ \text{ft}} = 156.17\ \text{ft}$$

Acceleration:

$$a_n = \frac{v^2}{\rho} = \frac{30^2}{156.17} = 5.763\ \text{ft/s}^2$$

$$a_t = \dot{v} = 5\ \text{ft/s}^2$$

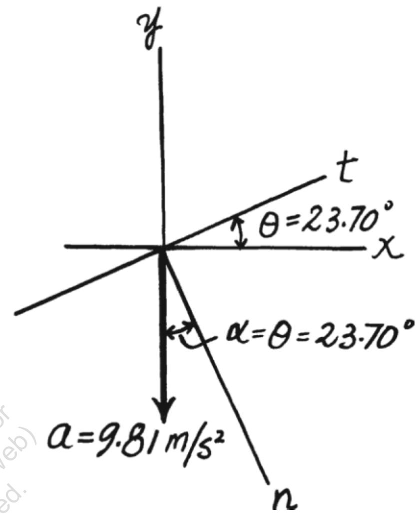
The magnitude of the box's acceleration at A is therefore

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{5^2 + 5.763^2} = 7.63\ \text{ft/s}^2$$ **Ans.**

■ **12–153.**

A go-cart moves along a circular track of radius 100 ft such that its speed for a short period of time, $0 \le t \le 4$ s, is $v = 60(1 - e^{-t^2})$ ft/s. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s? Use Simpson's rule with $n = 50$ to evaluate the integral.

SOLUTION



$v = 60(1 - e^{-t^2})$

$a_t = \dfrac{dv}{dt} = 60(-e^{-t^2})(-2t) = 120\, t\, e^{-t^2}$

$a_t|_{t=2} = 120(2)e^{-4} = 4.3958$

$v|_{t=2} = 60(1 - e^{-4}) = 58.9011$

$a_n = \dfrac{(58.9011)^2}{100} = 34.693$

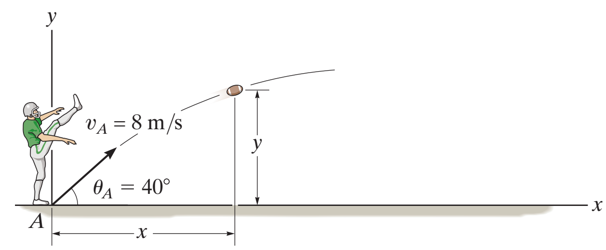
$a = \sqrt{(4.3958)^2 + (34.693)^2} = 35.0 \text{ m/s}^2$ **Ans.**

$\int_0^s ds = \int_0^2 60(1 - e^{-t^2})\, dt$

$s = 67.1$ ft **Ans.**

■ **12–154.**

The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25$ s.

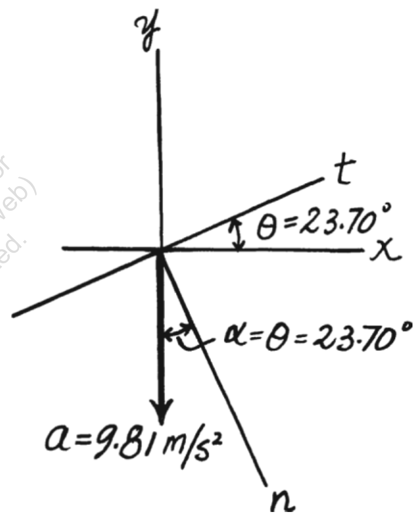


SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8 \cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad \textbf{(1)}$$



Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8 \sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)\left(t^2\right) \qquad \textbf{(2)}$$

Eliminate t from Eqs (1) and (2),we have

$$y = \{0.8391x - 0.1306x^2\}\text{ m} = \{0.839x - 0.131x^2\}\text{ m} \qquad \textbf{Ans.}$$

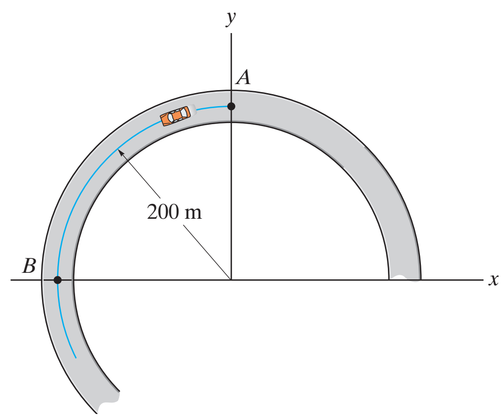
Acceleration: When $t = 0.25$ s, from Eq. (1), $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\dfrac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\dfrac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1} 0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81\text{ m/s}^2$ and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = -a \sin \alpha = -9.81 \sin 23.70° = -3.94\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = a \cos \alpha = 9.81 \cos 23.70° = 8.98\text{ m/s}^2 \qquad \textbf{Ans.}$$

12–155.

The race car travels around the circular track with a speed of 16 m/s. When it reaches point A it increases its speed at $a_t = (\frac{4}{3}v^{1/4})\ \text{m/s}^2$, where v is in m/s. Determine the magnitudes of the velocity and acceleration of the car when it reaches point B. Also, how much time is required for it to travel from A to B?



SOLUTION

$$a_t = \frac{4}{3}v^{\frac{1}{4}}$$

$$dv = a_t\ dt$$

$$dv = \frac{4}{3}v^{\frac{1}{4}}\ dt$$

$$\int_{16}^{v} 0.75\frac{dv}{v^{\frac{1}{4}}} = \int_0^t dt$$

$$v^{\frac{3}{4}}\Big|_{16}^{v} = t$$

$$v^{\frac{3}{4}} - 8 = t$$

$$v = (t + 8)^{\frac{4}{3}}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (t + 8)^{\frac{4}{3}}\,dt$$

$$s = \frac{3}{7}(t + 8)^{\frac{7}{3}}\Big|_0^t$$

$$s = \frac{3}{7}(t + 8)^{\frac{7}{3}} - 54.86$$

For $s = \frac{\pi}{2}(200) = 100\pi = \frac{3}{7}(t + 8)^{\frac{7}{3}} - 54.86$

$t = 10.108\ \text{s} = 10.1\ \text{s}$ **Ans.**

$v = (10.108 + 8)^{\frac{4}{3}} = 47.551 = 47.6\ \text{m/s}$ **Ans.**

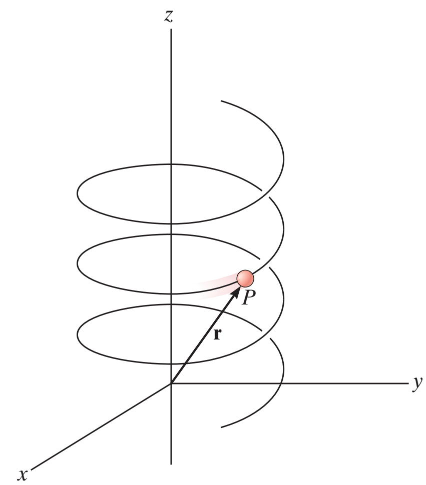
$$a_t = \frac{4}{3}(47.551)^{\frac{1}{4}} = 3.501\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(47.551)^2}{200} = 11.305\ \text{m/s}^2$$

$a = \sqrt{(3.501)^2 + (11.305)^2} = 11.8\ \text{m/s}^2$ **Ans.**

***12–156.**

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$$

When $t = 8$ s,

$$\mathbf{v}_P = -0.2\sin(0.8\text{ rad})\mathbf{i} + 0.15\cos(0.8\text{ rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = -0.02\cos(0.8\text{ rad})\mathbf{i} - 0.015\sin(0.8\text{ rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$$

Since the binormal vector is perpendicular to the plane containing the n–t axis, and $\mathbf{a}_p$ and $\mathbf{v}_p$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14\,347 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$$

$$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans.**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans.**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans.**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_p \times \mathbf{v}_p$.

For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans.**

12–157.

The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

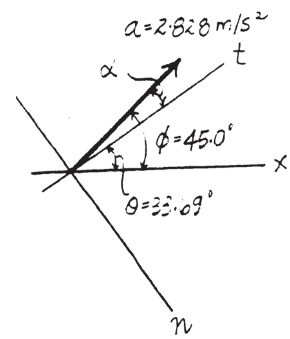
SOLUTION

Velocity: Here, $\mathbf{r} = \left\{\left(2t + t^2\right)\mathbf{i} + t^2\mathbf{j}\right\}$ m. To determine the velocity **v**, apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\,\mathbf{i} + 2t\mathbf{j}\}\text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2}$ $= 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \quad \text{and} \quad v_t = 7.21\text{ m/s} \qquad \textbf{Ans.}$$



The velocity **v** makes an angle $\theta = \tan^{-1}\frac{4}{6} = 33.69°$ with the x axis.

Acceleration: To determine the acceleration **a**, apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\}\text{ m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828\text{ m/s}^2$$

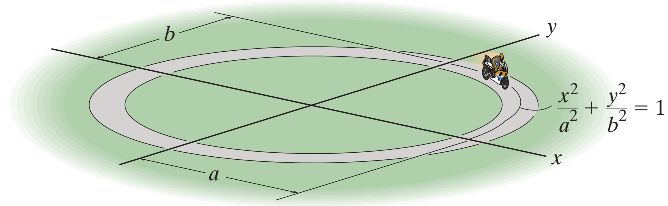
The acceleration **a** makes an angle $\phi = \tan^{-1}\frac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a\sin\alpha = 2.828\sin 11.31° = 0.555\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_t = a\cos\alpha = 2.828\cos 11.31° = 2.77\text{ m/s}^2 \qquad \textbf{Ans.}$$

12–158.

The motorcycle travels along the elliptical track at a constant speed v. Determine the greatest magnitude of the acceleration if $a > b$.



SOLUTION

Acceleration: Differentiating twice the expression $y = \frac{b}{a}\sqrt{a^2 - x^2}$, we have

$$\frac{dy}{dx} = -\frac{bx}{a\sqrt{a^2 - x^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{ab}{(a^2 - x^2)^{3/2}}$$

The radius of curvature of the path is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{bx}{a\sqrt{a^2 - x^2}}\right)^2\right]^{3/2}}{\left|-\frac{ab}{(a^2 - x^2)^{3/2}}\right|} = \frac{\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}}{\frac{ab}{(a^2 - x^2)^{3/2}}} \qquad \textbf{(1)}$$

To have the maximum normal acceleration, the radius of curvature of the path must be a minimum. By observation, this happens when $y = 0$ and $x = a$. When $x \to a$, $\frac{b^2x^2}{a^2(a^2 - x^2)} >> 1$. Then, $\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} \to \left[\frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} = \frac{b^3x^3}{a^3(a^2 - x^2)^{3/2}}$.

Substituting this value into Eq. [1] yields $\rho = \frac{b^2}{a^4}x^3$. At $x = a$,

$$\rho = \frac{b^2}{a^4}\left(a^3\right) = \frac{b^2}{a}$$

To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_{\max} = \frac{v^2}{\rho} = \frac{v^2}{b^2/a} = \frac{a}{b^2}v^2$$

Since the motorcycle is traveling at a constant speed, $a_t = 0$. Thus,

$$a_{\max} = (a_n)_{\max} = \frac{a}{b^2}v^2 \qquad \textbf{Ans.}$$

12–159.

A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

SOLUTION

When $\theta = \frac{\pi}{6}$ rad, $\quad \frac{\pi}{6} = \cos 2t \quad t = 0.5099$ s

$$\dot{\theta} = \frac{d\theta}{dt} = -2\sin 2t\bigg|_{t=0.5099\text{ s}} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4\cos 2t\bigg|_{t=0.5099\text{ s}} = -2.0944 \text{ rad/s}^2$$

$r = 4 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2$$ **Ans.**

***12–160.**

A particle travels around a limaçon, defined by the equation $r = b - a\cos\theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

SOLUTION

$$r = b - a\cos\theta$$

$$\dot{r} = a\sin\theta\dot{\theta}$$

$$\dot{r} = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$

$$v_r = \dot{r} = a\sin\theta\dot{\theta}$$ **Ans.**

$$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta}$$ **Ans.**

$$a_r = \ddot{r} - r\dot{\theta}^2 = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta} - (b - a\cos\theta)\dot{\theta}^2$$

$$= (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\,\ddot{\theta}$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} = (b - a\cos\theta)\ddot{\theta} + 2\left(a\sin\theta\dot{\theta}\right)\dot{\theta}$$

$$= (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta$$ **Ans.**

12–161.

If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

When $t = 2$ s,

$r = 4(1+\sin t) = 7.637$

$\dot{r} = 4\cos t = -1.66459$

$\ddot{r} = -4\sin t = -3.6372$

$\theta = 2\,e^{-t}$

$\dot{\theta} = -2\,e^{-t} = -0.27067$

$\ddot{\theta} = 2\,e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66 \text{ m/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97 \text{ m/s}^2$ **Ans.**

12–162.

An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h^2. If the propeller has a diameter of 6 ft and is rotating at an angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

SOLUTION

$$v_{Pl} = \left(\frac{200\text{ mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 293.3\text{ ft/s}$$

$$a_{Pl} = \left(\frac{3\text{ mi}}{\text{h}^2}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = 0.001\,22\ \text{ft/s}^2$$

$$v_{Pr} = 120(3) = 360\text{ ft/s}$$

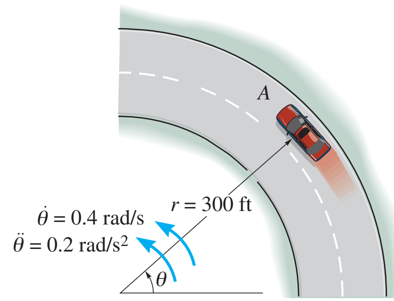
$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464\text{ ft/s} \qquad \textbf{Ans.}$$

$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43\,200\text{ ft/s}^2$$

$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.001\,22)^2 + (43\,200)^2} = 43.2(10^3)\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–163.

A car is traveling along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s^2. Determine the magnitudes of the car's velocity and acceleration at this instant.



SOLUTION

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300\left(0.4^2\right) = -48.0 \text{ ft/s}^2$$

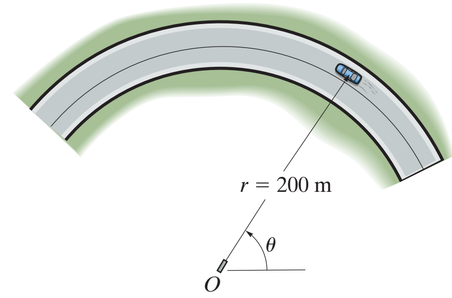
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

*12–164.

A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1$ rad/s and angular acceleration of $\ddot{\theta} = 0.025$ rad/s^2, at the instant $\theta = 45°$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200$ m. Determine the magnitudes of velocity and acceleration of the car at this instant.



SOLUTION

Time Derivatives: Since r is constant,

$$\dot{r} = \ddot{r} = 0$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 200(0.1) = 20 \text{ m/s}$$

Thus, the magnitude of the car's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{0^2 + 20^2} = 20 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 200(0.1^2) = -2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(0.025) + 0 = 5 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-2)^2 + 5^2} = 5.39 \text{ m/s}^2$$ **Ans.**

12–165.

If a particle moves along a path such that $r = (2\cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$r = 2\cos t \quad \dot{r} = -2\sin t \qquad \ddot{r} = -2\cos t$

$\theta = \dfrac{t}{2} \qquad \dot{\theta} = \dfrac{1}{2} \qquad \ddot{\theta} = 0$

$v_r = \dot{r} = -2\sin t$ **Ans.**

$v_\theta = r\dot{\theta} = (2\cos t)\left(\dfrac{1}{2}\right) = \cos t$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = -2\cos t - (2\cos t)\left(\dfrac{1}{2}\right)^2 = -\dfrac{5}{2}\cos t$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cos t(0) + 2(-2\sin t)\left(\dfrac{1}{2}\right) = -2\sin t$ **Ans.**

12–166.

If a particle's position is described by the polar coordinates $r = (2 \sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and transverse components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$\theta = 4\,t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2 \sin 2\theta = 1.9787$

$\dot{r} = 4 \cos 2\theta\, \dot{\theta} = -2.3280$

$\ddot{r} = -8 \sin 2\theta (\dot{\theta})^2 + 8 \cos 2\theta\, \ddot{\theta} = -126.638$

$v_r = \dot{r} = -2.33 \text{ m/s}$ **Ans.**

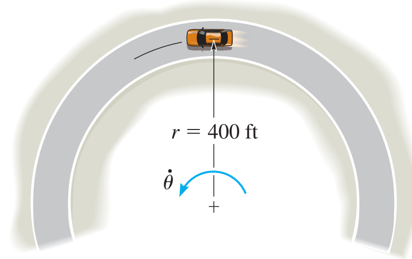
$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6 \text{ m/s}^2$ **Ans.**

12–167.

The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s^2. Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.



SOLUTION

$r = 400 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

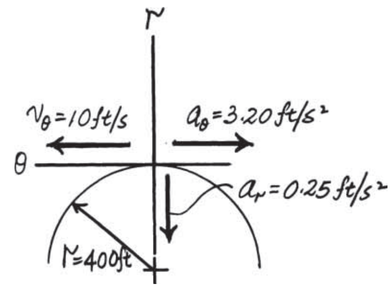
$\dot{\theta} = 0.025 \qquad \theta = -0.008$

$v_r = \dot{r} = 0$ **Ans.**

$v_\theta = r\dot{\theta} = 400(0.025) = 10$ ft/s **Ans.**

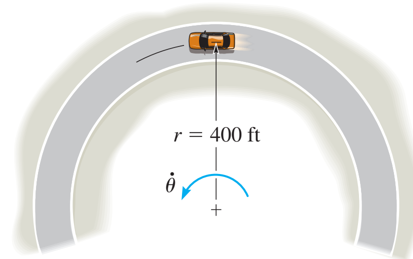
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25$ ft/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(-0.008) + 0 = -3.20$ ft/s^2 **Ans.**



***12–168.**

The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.



SOLUTION

$r = 400 \text{ ft} \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 400\left(\dot{\theta}\right)$

$v = \sqrt{(0)^2 + \left(400\,\dot{\theta}\right)^2} = 30$

$\dot{\theta} = 0.075 \text{ rad/s}$ **Ans.**

$\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(0) + 2(0)(0.075) = 0$

$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25 \text{ ft/s}^2$ **Ans.**

12–169.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

$$\ddot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\,\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\mathbf{u}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3r\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans.}$$

12–170.

A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine are in radians, and t is in seconds. Determine the acceleration of the particle at $\theta = 30°$. The particle starts from rest at $\theta = 0°$.

SOLUTION

$r = 6 \text{ in.}, \qquad \dot{r} = 0, \qquad \ddot{r} = 0$

$\theta = \sin 3t$

$\dot{\theta} = 3 \cos 3t$

$\ddot{\theta} = -9 \sin 3t$

At $\theta = 30°$,

$\frac{30°}{180°}\pi = \sin 3t$

$t = 10.525 \text{ s}$

Thus,

$\dot{\theta} = 2.5559 \text{ rad/s}$

$\ddot{\theta} = -4.7124 \text{ rad/s}^2$

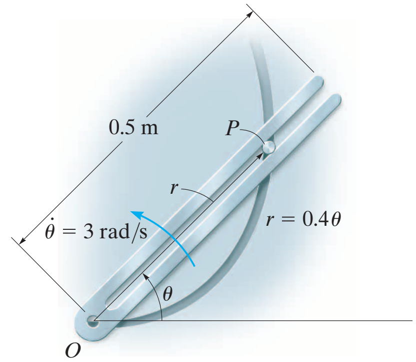
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$

$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3 \text{ in./s}^2$ **Ans.**

12–171.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$$\dot{r} = 0.4\,\dot{\theta}$$

$$\ddot{r} = 0.4\,\ddot{\theta}$$

At $\theta = \dfrac{\pi}{3}, \quad r = 0.4189$

$$\dot{r} = 0.4(3) = 1.20$$

$$\ddot{r} = 0.4(0) = 0$$

$v = \dot{r} = 1.20$ m/s **Ans.**

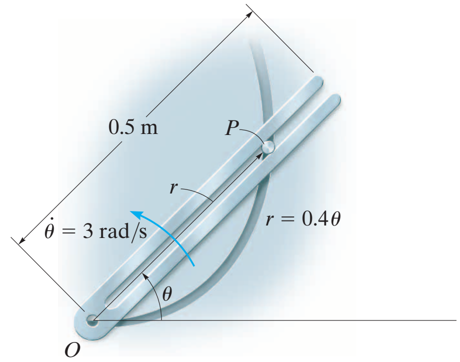
$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans.**

12–172.

Solve Prob. 12–171 if the slotted link has an angular acceleration $\ddot{\theta} = 8 \text{ rad/s}^2$ when $\dot{\theta} = 3 \text{ rad/s}$ at $\theta = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3 \text{ rad/s} \qquad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\theta = \dfrac{\pi}{3}$

$\dot{\theta} = 3$

$\ddot{\theta} = 8$

$r = 0.4189$

$\dot{r} = 1.20$

$\ddot{r} = 0.4(8) = 3.20$

$v_r = \dot{r} = 1.20 \text{ m/s}$ **Ans.**

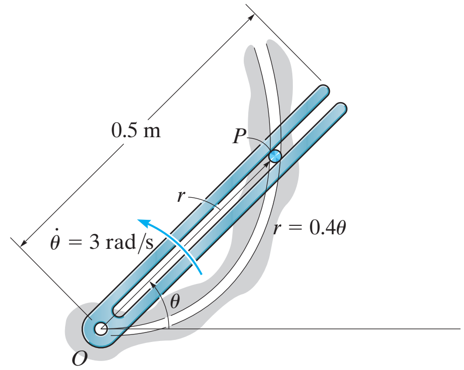
$v_\theta = r\,\dot{\theta} = 0.4189(3) = 1.26 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.20 - 0.4189(3)^2 = -0.570 \text{ m/s}^2$ **Ans.**

$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0.4189(8) + 2(1.20)(3) = 10.6 \text{ m/s}^2$ **Ans.**

12–173.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



SOLUTION

$r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\dot{\theta} = 3$

$\ddot{\theta} = 0$

At $r = 0.5$ m,

$$\theta = \frac{0.5}{0.4} = 1.25 \text{ rad}$$

$\dot{r} = 1.20$

$\ddot{r} = 0$

$v_r = \dot{r} = 1.20$ m/s **Ans.**

$v_\theta = r\,\dot{\theta} = 0.5(3) = 1.50$ m/s **Ans.**

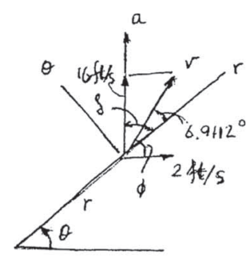
$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50\ \text{m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20\ \text{m/s}^2$ **Ans.**

12–174.

A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION



$\mathbf{r} = 2t\mathbf{i} + 4\,t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 8\,t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$

$\mathbf{a} = 8\mathbf{j}$

$\theta = \tan^{-1}\left(\frac{16}{4}\right) = 75.964°$

$v = \sqrt{(2)^2 + (16)^2} = 16.1245 \text{ ft/s}$

$\phi = \tan^{-1}\left(\frac{16}{2}\right) = 82.875°$

$a = 8 \text{ ft/s}^2$

$\phi - \theta = 6.9112°$

$v_r = 16.1245 \cos 6.9112° = 16.0 \text{ ft/s}$ **Ans.**

$v_\theta = 16.1245 \sin 6.9112° = 1.94 \text{ ft/s}$ **Ans.**

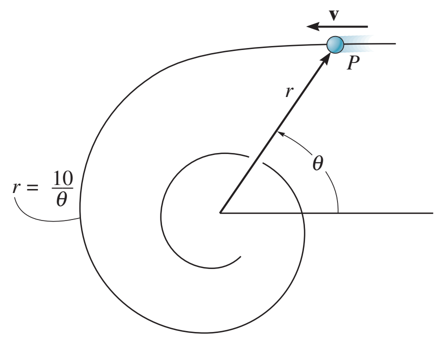
$\delta = 90° - \theta = 14.036°$

$a_r = 8 \cos 14.036° = 7.76 \text{ ft/s}^2$ **Ans.**

$a_\theta = 8 \sin 14.036° = 1.94 \text{ ft/s}^2$ **Ans.**

12–175.

A particle P moves along the spiral path $r = (10/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 20$ ft/s, determine the magnitudes v_r and v_θ as functions of θ and evaluate each at $\theta = 1$ rad.



SOLUTION

$$r = \frac{10}{\theta}$$

$$\dot{r} = -\left(\frac{10}{\theta^2}\right)\dot{\theta}$$

Since $v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\dot{\theta}^2 + \left(\frac{10^2}{\theta^2}\right)\dot{\theta}^2$$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)(1 + \theta^2)\dot{\theta}^2$$

Thus, $\dot{\theta} = \dfrac{2\theta^2}{\sqrt{1 + \theta^2}}$

$$v_r = \dot{r} = -\left(\frac{10}{\theta^2}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = -\frac{20}{\sqrt{1 + \theta^2}}$$ **Ans.**

$$v_\theta = r\dot{\theta} = \left(\frac{10}{\theta}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = \frac{20\theta}{\sqrt{1 + \theta^2}}$$ **Ans.**

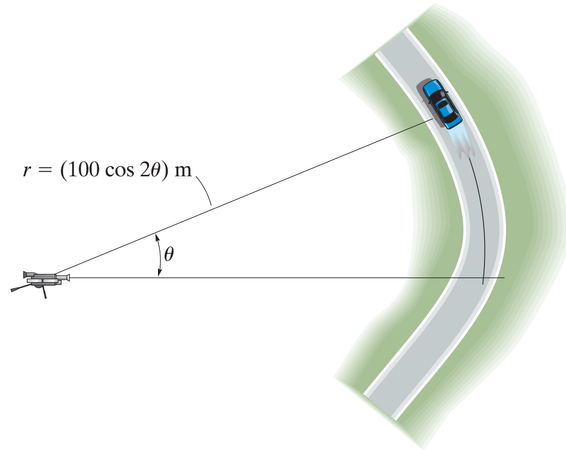
When $\theta = 1$ rad

$$v_r = \left(-\frac{20}{\sqrt{2}}\right) = -14.1 \text{ ft/s}$$ **Ans.**

$$v_\theta = \left(\frac{20}{\sqrt{2}}\right) = 14.1 \text{ ft/s}$$ **Ans.**

***12–176.**

The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15°$.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\dot{\theta}) \text{ m/s}$$

At $\theta = 15°$,

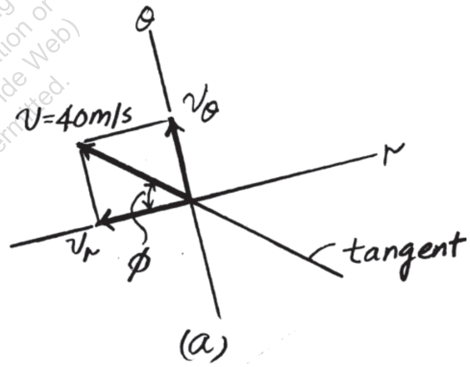
$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

Velocity: Referring to Fig. *a*, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta} \quad \textbf{(1)}$$



and

$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta} \quad \textbf{(2)}$$

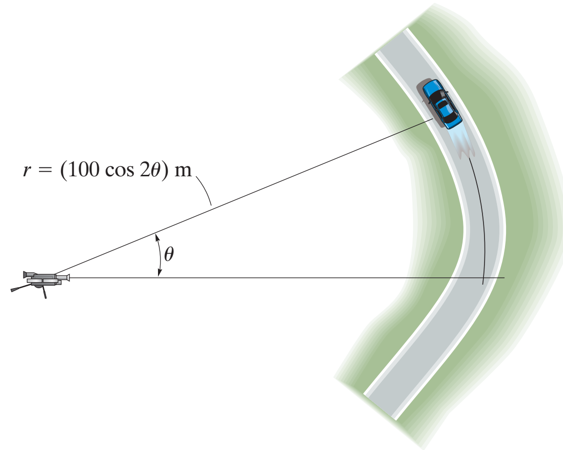
Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.3024 \text{ rad/s} = 0.302 \text{ rad/s} \quad \textbf{Ans.}$$

12–177.

When $\theta = 15°$, the car has a speed of 50 m/s which is increasing at 6 m/s^2. Determine the angular velocity of the camera tracking the car at this instant.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\, \dot{\theta})\ \text{m/s}$$

$$\ddot{r} = -200\left[(\sin\ 2\theta)\, \ddot{\theta} + 2\,(\cos 2\theta)\, \dot{\theta}^2\right]\ \text{m/s}^2$$

At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60\ \text{m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30° \dot{\theta} = -100\dot{\theta}\ \text{m/s}$$

$$\dot{r}\big|_{\theta=15°} = -200\left[\sin 30° \ddot{\theta} + 2 \cos 30° \dot{\theta}^2\right] = \left(-100\ddot{\theta} - 346.41\dot{\theta}^2\right)\ \text{m/s}^2$$

Velocity: Referring to Fig. *a*, $v_r = -50 \cos \phi$ and $v_\theta = 50 \sin \phi$. Thus,

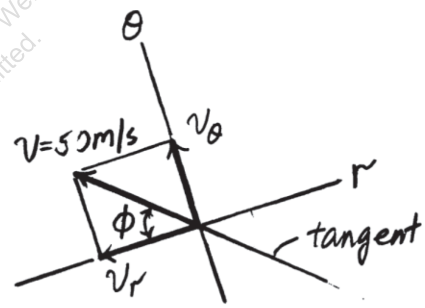
$$v_r = \dot{r}$$

$$-50 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$50 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$



(a)

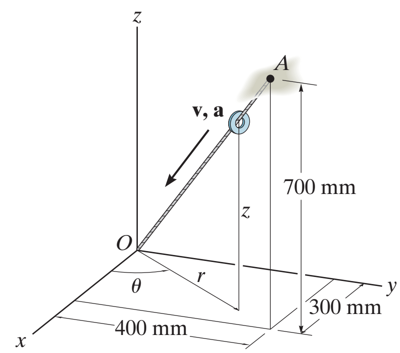
Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.378\ \text{rad/s} \qquad \textbf{Ans.}$$

12–178.

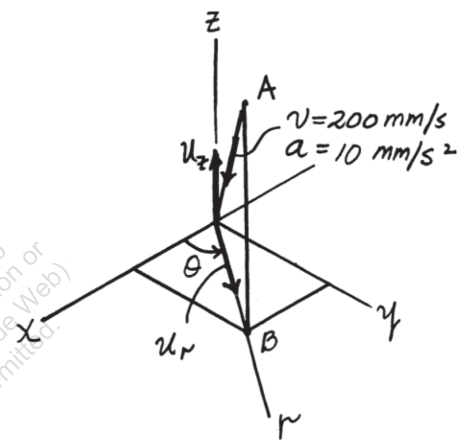
The small washer slides down the cord OA. When it is at the midpoint, its speed is 200 mm/s and its acceleration is 10 mm/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



SOLUTION

$$OA = \sqrt{(400)^2 + (300)^2 + (700)^2} = 860.23 \text{ mm}$$

$$OB = \sqrt{(400)^2 + (300)^2} = 500 \text{ mm}$$



$$v_r = (200)\left(\frac{500}{860.23}\right) = 116 \text{ mm/s}$$

$$v_\theta = 0$$

$$v_z = (200)\left(\frac{700}{860.23}\right) = 163 \text{ mm/s}$$

Thus, $\mathbf{v} = \{-116\mathbf{u}_r - 163\mathbf{u}_z\}$ mm/s **Ans.**

$$a_r = 10\left(\frac{500}{860.23}\right) = 5.81$$

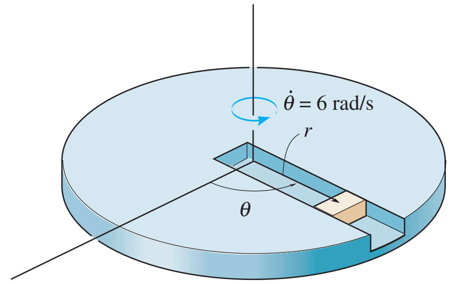
$$a_\theta = 0$$

$$a_z = 10\left(\frac{700}{860.23}\right) = 8.14$$

Thus, $\mathbf{a} = \{-5.81\mathbf{u}_r - 8.14\mathbf{u}_z\}$ mm/s^2 **Ans.**

12–179.

A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



SOLUTION

$\dot{r} = 4t|_{t=1} = 4 \qquad \ddot{r} = 4$

$\dot{\theta} = 6 \qquad \ddot{\theta} = 0$

$$\int_0^1 dr = \int_0^1 4t\,dt$$

$r = 2t^2\Big]_0^1 = 2$ m

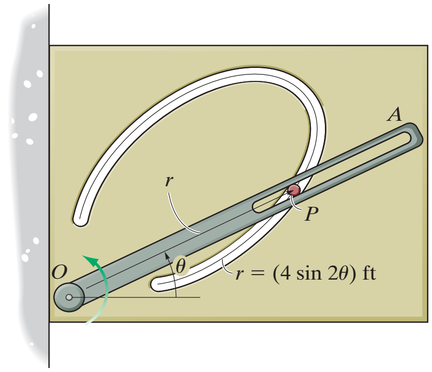
$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{(4)^2 + [2(6)]^2} = 12.6$ m/s **Ans.**

$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\,\dot{r}\dot{\theta})^2} = \sqrt{[4 - 2(6)^2]^2 + [0 + 2(4)(6)]^2}$ **Ans.**

$= 83.2$ m/s^2

***12–180.**

Pin P is constrained to move along the curve defined by the lemniscate $r = (4 \sin 2\theta)$ ft. If the slotted arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of peg P when $\theta = 60°$.



SOLUTION

Time Derivatives:

$$r = 4\sin 2\theta$$

$$\dot{r} = (8(\cos 2\theta)\dot{\theta})\ \text{ft/s} \qquad \dot{\theta} = 1.5\ \text{rad/s}$$

$$\ddot{r} = 8[(\cos 2\theta)\ddot{\theta} - 2\sin 2\theta(\dot{\theta})^2]\ \text{ft/s}^2 \qquad \ddot{\theta} = 0$$

When $\theta = 60°$,

$$r|_{\theta=60°} = 4\sin 120° = 3.464\ \text{ft}$$

$$\dot{r}|_{\theta=60°} = 8\cos 120°(1.5) = -6\ \text{ft/s}$$

$$\ddot{r}|_{\theta=60°} = 8[0 - 2\sin 120°(1.5^2)] = -31.18\ \text{ft/s}^2$$

Velocity:

$$v_r = \dot{r} = -6\ \text{ft/s} \qquad v_\theta = r\dot{\theta} = 3.464(1.5) = 5.196\ \text{ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-6)^2 + 5.196^2} = 7.94\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -31.18 - 3.464(1.5^2) = -38.97\ \text{ft/s}^2$$

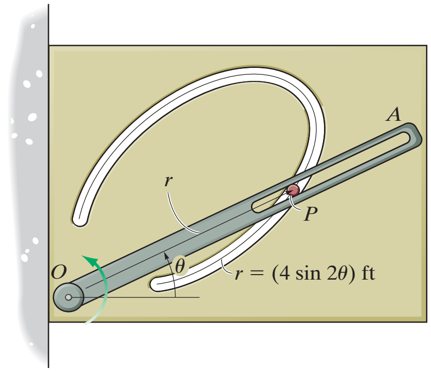
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-6)(1.5) = -18\ \text{ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-38.97)^2 + (-18)^2} = 42.9\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–181.

Pin P is constrained to move along the curve defined by the lemniscate $r = (4 \sin 2\theta)$ ft. If the angular position of the slotted arm OA is defined by $\theta = (3t^{3/2})$ rad, determine the magnitudes of the velocity and acceleration of the pin P when $\theta = 60°$.



SOLUTION

Time Derivatives:

$$r = 4 \sin 2\theta$$

$$\dot{r} = (8(\cos 2\theta)\dot{\theta}) \text{ ft/s}$$

$$\ddot{r} = 8[(\cos 2\theta)\ddot{\theta} - 2(\sin 2\theta)\dot{\theta}^2] \text{ ft/s}^2$$

When $\theta = 60° = \dfrac{\pi}{3}$ rad,

$$\frac{\pi}{3} = 3t^{3/2} \qquad t = 0.4958 \text{ s}$$

Thus, the angular velocity and angular acceleration of arm OA when $\theta = \dfrac{\pi}{3}$ rad ($t = 0.4958$ s) are

$$\dot{\theta} = \frac{9}{2}t^{1/2}\Big|_{t=0.4958\text{ s}} = 3.168 \text{ rad/s}$$

$$\ddot{\theta} = \frac{9}{4}t^{1/2}\Big|_{t=0.4958\text{ s}} = 3.196 \text{ rad/s}^2$$

Thus,

$$r|_{\theta=60°} = 4 \sin 120° = 3.464 \text{ ft}$$

$$\dot{r}|_{\theta=60°} = 8 \cos 120°(3.168) = -12.67 \text{ ft/s}$$

$$\ddot{r}|_{\theta=60°} = 8[\cos 120°(3.196) - 2 \sin 120°(3.168^2)] = -151.89 \text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = -12.67 \text{ ft/s} \qquad v_\theta = r\dot{\theta} = 3.464(3.168) = 10.98 \text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-12.67)^2 + 10.98^2} = 16.8 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -151.89 - 3.464(3.168^2) = -186.67 \text{ ft/s}^2$$

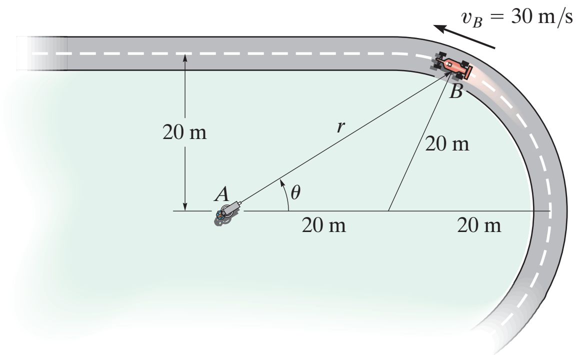
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.464(3.196) + 2(-12.67)(3.168) = -69.24 \text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-186.67)^2 + (-69.24)^2} = 199 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–182.

A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.



SOLUTION

$$r = 2(20)\cos\theta = 40\cos\theta$$

$$\dot{r} = -(40\sin\theta)\,\dot{\theta}$$

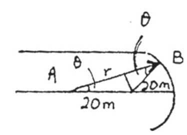
$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\,\dot{\theta}\,\mathbf{u}_\theta$$

$$v = \sqrt{(\dot{r})^2 + (r\,\dot{\theta})^2}$$

$$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$$

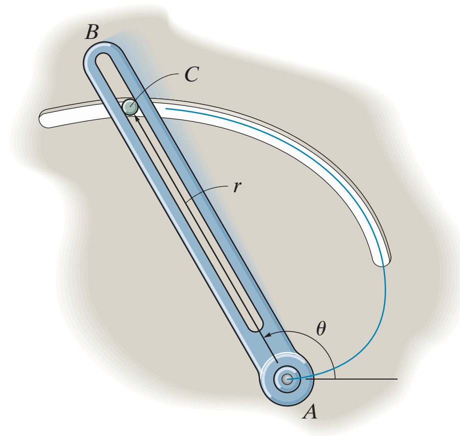
$$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$$

$$\dot{\theta} = \frac{30}{40} = 0.75\text{ rad/s}$$ **Ans.**



12–183.

The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \qquad \dot{r} = a\dot{\theta} \qquad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = a\dot{\theta}$$ **Ans.**

$$v_\theta = r\dot{\theta} = a\theta\dot{\theta}$$ **Ans.**

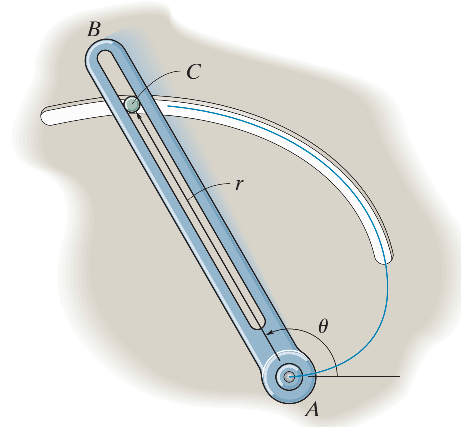
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2$$ **Ans.**

***12–184.**

The slotted arm AB drives pin C through the spiral groove described by the equation $r = (1.5\,\theta)$ ft, where θ is in radians. If the arm starts from rest when $\theta = 60°$ and is driven at an angular velocity of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin C when $t = 1$ s.



SOLUTION

Time Derivatives: Here, $\dot{\theta} = 4t$ and $\ddot{\theta} = 4\text{ rad/s}^2$.

$$r = 1.5\theta \qquad \dot{r} = 1.5\dot{\theta} = 1.5(4t) = 6t \qquad \ddot{r} = 1.5\ddot{\theta} = 1.5(4) = 6\text{ ft/s}^2$$

Velocity: Integrate the angular rate, $\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$, we have $\theta = \frac{1}{3}(6t^2 + \pi)$ rad.

Then, $r = \left\{\frac{1}{2}(6t^2 + \pi)\right\}$ ft. At $t = 1$ s, $r = \frac{1}{2}\left[6(1^2) + \pi\right] = 4.571$ ft, $\dot{r} = 6(1) = 6.00$ ft/s.

and $\dot{\theta} = 4(1) = 4$ rad/s. Applying Eq. 12–25, we have

$$v_r = \dot{r} = 6.00\text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4.571\,(4) = 18.3\text{ ft/s} \qquad \textbf{Ans.}$$

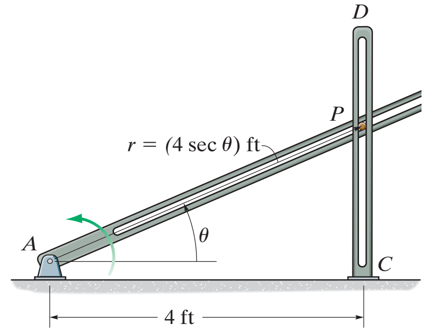
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.571\left(4^2\right) = -67.1\text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.571(4) + 2(6)\,(4) = 66.3\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–185.

If the slotted arm AB rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB.



SOLUTION

Time Derivatives:

$$r = 4\sec\theta$$

$$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})\text{ ft/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta(\sec^2\theta)\dot{\theta} + \tan\theta\sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 0$$

$$= 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}^2(\sec 3\theta + \tan^2\theta\sec\theta)]\text{ ft/s}^2$$

When $\theta = 30°$,

$$r|_{\theta=30°} = 4\sec 30° = 4.619\text{ ft}$$

$$\dot{r}|_{\theta=30°} = (4\sec 30°\tan 30°)(2) = 5.333\text{ ft/s}$$

$$\ddot{r}|_{\theta=30°} = 4[0 + 2^2(\sec^3 30° + \tan^2 30°\sec 30°)] = 30.79\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\text{ ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 30.79 - 4.619(2^2) = 12.32\text{ ft/s}^2$$

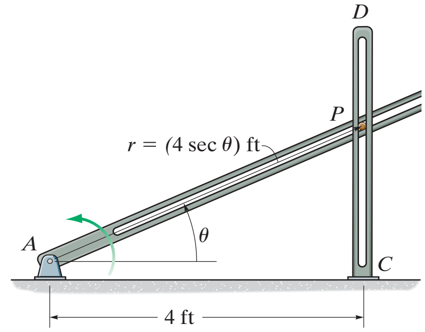
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(5.333)(2) = 21.23\text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{12.32^2 + 21.23^2} = 24.6\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–186.

The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB. When $\theta = 30^\circ$, the angular velocity and angular acceleration of arm AB are $\dot{\theta} = 2\ \text{rad/s}$ and $\ddot{\theta} = 3\ \text{rad/s}^2$, respectively. Determine the magnitudes of the velocity and acceleration of the peg P at this instant.



SOLUTION

Time Derivatives:

$$r = 4\sec\theta$$

$$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})\ \text{ft/s} \qquad \dot{\theta} = 2\ \text{rad/s}$$

$$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta\sec^2\theta\dot{\theta} + \tan\theta\sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 3\ \text{rad/s}^2$$

$$= 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}^2(\sec^3\theta^\circ + \tan^2\theta^\circ\sec\theta^\circ)]\ \text{ft/s}^2$$

When $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = 4\sec 30^\circ = 4.619\ \text{ft}$$

$$\dot{r}|_{\theta=30^\circ} = (4\sec 30^\circ\tan 30^\circ)(2) = 5.333\ \text{ft/s}$$

$$\ddot{r}|_{\theta=30^\circ} = 4[(\sec 30^\circ\tan 30^\circ)(3) + 2^2(\sec^3 30^\circ + \tan^2 30^\circ\sec 30^\circ)] = 38.79\ \text{ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\ \text{ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\ \text{ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.79 - 4.619(2^2) = 20.32\ \text{ft/s}^2$$

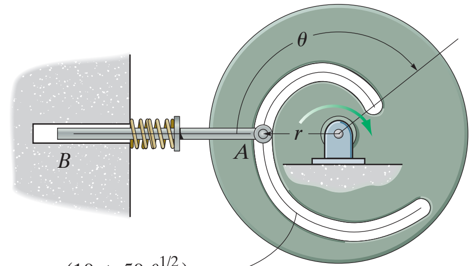
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.619(3) + 2(5.333)(2) = 35.19\ \text{ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{20.32^2 + 35.19^2} = 40.6\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–187.

If the circular plate rotates clockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB when $\theta = 2/3\pi$ rad.



SOLUTION

Time Derivaties:

$$r = \left(10 + 50\theta^{1/2}\right)\text{mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\ \text{mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\dot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\ \text{mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\ \text{mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[0 - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = -9.279\ \text{mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

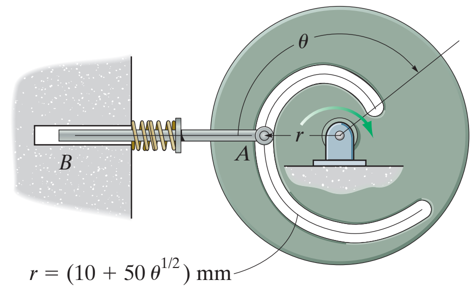
$$v_r = \dot{r} = 25.9\ \text{mm/s}$$ **Ans.**

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -9.279 - 82.36\left(1.5^2\right) = -195\ \text{mm/s}^2$$ **Ans.**

***12–188.**

When $\theta = 2/3\pi$ rad, the angular velocity and angular acceleration of the circular plate are $\dot{\theta} = 1.5$ rad/s and $\ddot{\theta} = 3$ rad/s^2, respectively. Determine the magnitudes of the velocity and acceleration of the rod AB at this instant.



SOLUTION

Time Derivatives:

$$r = (10 + 50\theta^{1/2})\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{ mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[\left(\frac{2\pi}{3}\right)^{-1/2}(3) - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = 42.55\text{ mm/s}^2$$

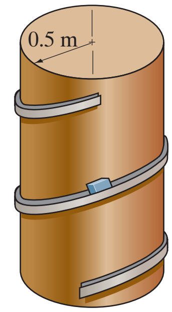
For the rod,

$$v = \dot{r} = 25.9\text{ mm/s}$$ **Ans.**

$$a = \ddot{r} = 42.5\text{ mm/s}^2$$ **Ans.**

12–189.

The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



SOLUTION

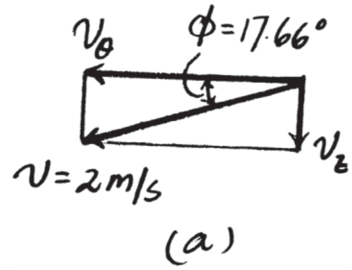
Velocity: The inclination angle of the ramp is $\phi = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\left[\dfrac{1}{2\pi(0.5)}\right] = 17.66°$.

Thus, from Fig. a, $v_\theta = 2\cos 17.66° = 1.906$ m/s and $v_z = 2\sin 17.66° = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$



Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

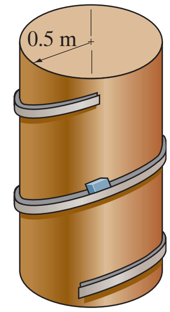
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$

Since $\mathbf{v}_z$ is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2} + a_z^{\,2}} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–190.

The box slides down the helical ramp which is defined by $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



SOLUTION

Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right) \text{ rad/s} \qquad \ddot{\theta} = (3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \qquad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}\big|_{t=2.325\text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}\big|_{t=2.325\text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}\big|_{t=2.325\text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}\big|_{t=2.325\text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2} + v_z^{\,2}} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

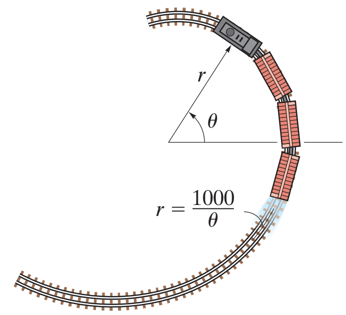
$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2} + a_z^{\,2}} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–191.

For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



SOLUTION

$$r = \frac{1000}{\theta}$$

$$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$$

Since

$$v^2 = (\dot{r})^2 + (r\,\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$$

Thus,

$$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$$

At $\theta = \dfrac{9\pi}{4}$

$$\dot{\theta} = 0.140$$

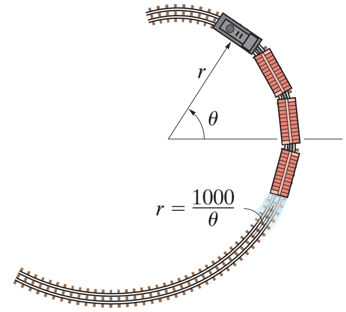
$$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$$

$v_r = \dot{r} = -2.80$ m/s **Ans.**

$v_\theta = r\dot{\theta} = \dfrac{1000}{(9\pi/4)}(0.140) = 19.8$ m/s **Ans.**

12–192.

For a *short distance* the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



SOLUTION

$\dot{\theta} = 0.2$

$\ddot{\theta} = 0$

$r = \dfrac{1000}{\theta}$

$\dot{r} = -1000(\theta^{-2})\,\dot{\theta}$

$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\,\ddot{\theta}$

When $\theta = \dfrac{9\pi}{4}$

$r = 141.477$

$\dot{r} = -4.002812$

$\ddot{r} = 0.226513$

$v_r = \dot{r} = -4.00$ m/s **Ans.**

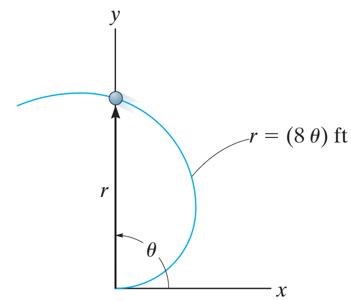
$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60$ m/s^2 **Ans.**

12–193.

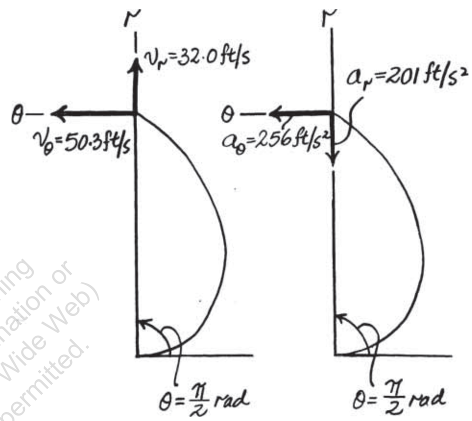
A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \qquad \ddot{r} = 8\ddot{\theta} = 0$$



Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s} \qquad \textbf{Ans.}$$

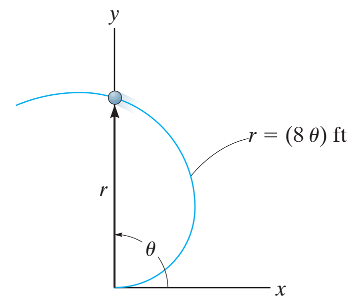
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi\left(4^2\right) = -201 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–194.

Solve Prob. 12–193 if the particle has an angular acceleration $\ddot{\theta} = 5\ \text{rad/s}^2$ when $\dot{\theta} = 4\ \text{rad/s}$ at $\theta = \pi\ 2$ rad.



SOLUTION

Time Derivatives: Here,

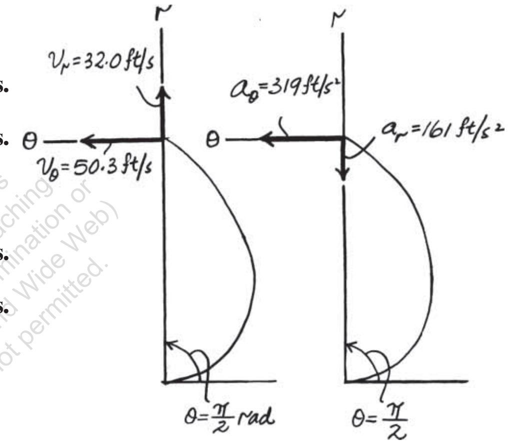
$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi\ \text{ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0\ \text{ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40\ \text{ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0\ \text{ft/s}$$ **Ans.**

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3\ \text{ft/s}$$ **Ans.**



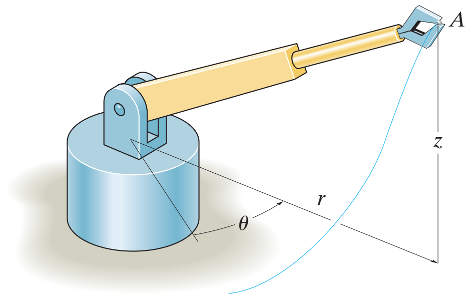
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi\left(4^2\right) = -161\ \text{ft/s}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319\ \text{ft/s}^2$$ **Ans.**

12–195.

The arm of the robot has a length of $r = 3$ ft grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t \qquad r = 3 \qquad z = 3 \sin 2t$

$\dot{\theta} = 0.5 \qquad \dot{r} = 0 \qquad \dot{z} = 6 \cos 2t$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = -12 \sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95 \text{ ft/s}$ **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

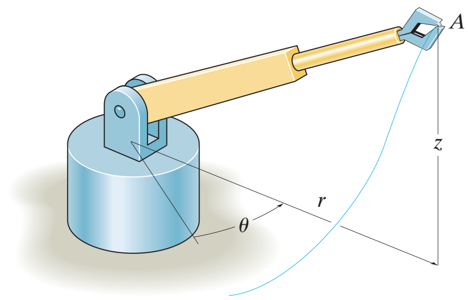
$a_\theta = 0 + 0 = 0$

$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44 \text{ ft/s}^2$ **Ans.**

***12–196.**

For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t$ rad $\qquad r = 3$ ft $\qquad z = 4\,t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\qquad \dot{r} = 1.5$ ft/s $\qquad \dot{z} = 8\,t$ ft/s

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \qquad r = 3 \qquad z = 36$

$\dot{\theta} = 0.5 \qquad \dot{r} = 1.5 \qquad \dot{z} = 24$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

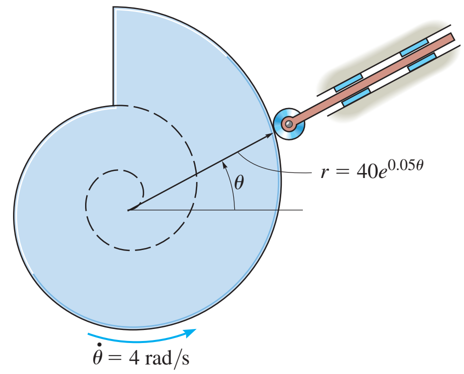
$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans.**

12–197.

The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



SOLUTION

$r = 40e^{0.05\,\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.0\,5\,\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\,\theta}\ddot{\theta}$

$\theta = \dfrac{\pi}{6}$

$\dot{\theta} = -4$

$\ddot{\theta} = 0$

$r = 40e^{\,0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{\,0.05\left(\frac{\pi}{6}\right)}\,(-4) = -8.2122$

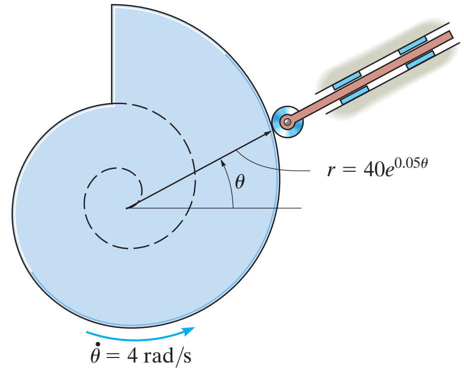
$\ddot{r} = 0.1e^{\,0.05\left(\frac{\pi}{6}\right)}\,(-4)^2 + 0 = 1.64244$

$v = \dot{r} = -8.2122 = 8.21$ mm/s **Ans.**

$a = \ddot{r} - r\dot{\theta}^2 = 1.642\,44 - 41.0610(-4)^2 = -665.33 = -665$ mm/s^2 **Ans.**

12–198.

Solve Prob. 12–197, if the cam has an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$ when its angular velocity is $\dot{\theta} = 4\ \text{rad/s}$ at $\dot{\theta} = 30°$.



SOLUTION

$r = 40e^{0.05\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.05\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\theta}\ddot{\theta}$

$\theta = \frac{\pi}{6}$

$\dot{\theta} = -4$

$\ddot{\theta} = -2$

$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\left(\left(\frac{\pi}{6}\right)\right)}(-4) = -8.2122$

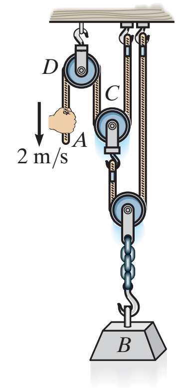
$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 2e^{0.05\left(\frac{\pi}{6}\right)}(-2) = -2.4637$

$v = \dot{r} = 8.2122 = 8.21\ \text{mm/s}$

$a = \ddot{r} - r\dot{\theta}^2 = -2.4637 - 41.0610(-4)^2 = -659\ \text{mm/s}^2$

12–199.

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



SOLUTION

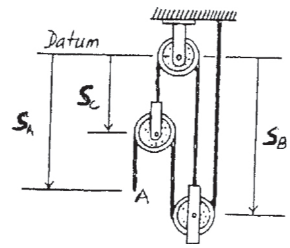
Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B, and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad \textbf{(1)}$$

$$s_B + s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields

$$s_A + 4s_B = l_1 = 2l_2$$



Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{(3)}$$

Since $v_A = 2$ m/s, from Eq. (3)

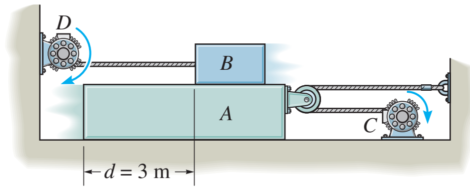
$(+\downarrow)$

$$2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

***12–200.**

The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

For A:

$s_A + (s_A - s_C) = l$

$2v_A = v_C$

$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \quad \rightarrow$

$v_A = 0.5t^3 \quad \rightarrow$

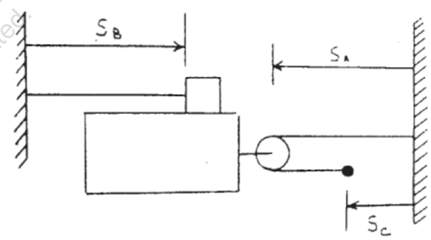
$s_A = 0.125\,t^4 \quad \rightarrow$

For B:

$a_B = 5\ \text{m/s}^2 \quad \leftarrow$

$v_B = 5t \quad \leftarrow$

$s_B = 2.5t^2 \quad \leftarrow$



Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2 \qquad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$t = 1.0656 = 1.07$ s **Ans.**

$v_A = 0.5(1.0656)^3 = 0.6050$

$v_B = 5(1.0656) = 5.3281\ \text{m/s}$

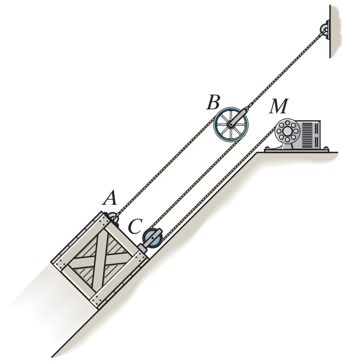
$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\,\mathbf{i}$

$v_{A/B} = 5.93\ \text{m/s} \quad \rightarrow$ **Ans.**

12–201.

The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with a constant speed of 4 ft/s.

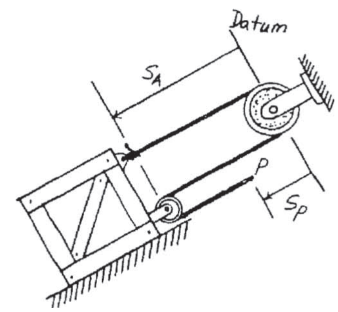


SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley B. The position of point P and crate A with respect to datum are s_P and s_A, respectively.

$$2s_A + (s_A - s_P) = l$$

$$3s_A - s_P = 0$$



Time Derivative: Taking the time derivative of the above equation yields

$$3v_A - v_P = 0 \qquad \textbf{(1)}$$

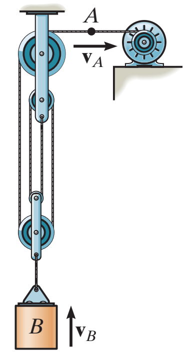
Since $v_A = 4$ ft/s, from Eq. [1]

$(+\ )$ $$3(4) - v_P = 0$$

$$v_P = 12 \text{ ft/s} \qquad \textbf{Ans.}$$

12–202.

Determine the time needed for the load at B to attain a speed of 8 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.2 m/s^2.



SOLUTION

$$4s_B + s_A = l$$

$$4v_B = -v_A$$

$$4a_B = -a_A$$

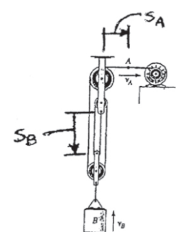
$$4a_B = -0.2$$

$$a_B = -0.05 \text{ m/s}^2$$

$$(+\downarrow) \quad v_B = (v_B)_0 + a_B t$$

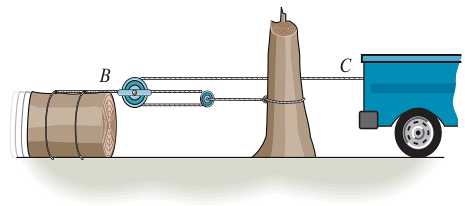
$$-8 = 0 - (0.05)(t)$$

$$t = 160 \text{ s} \qquad \textbf{Ans.}$$

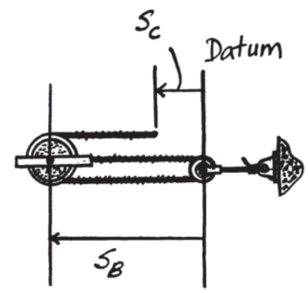


12–203.

Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



SOLUTION



$2s_B + (s_B - s_C) = l$

$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

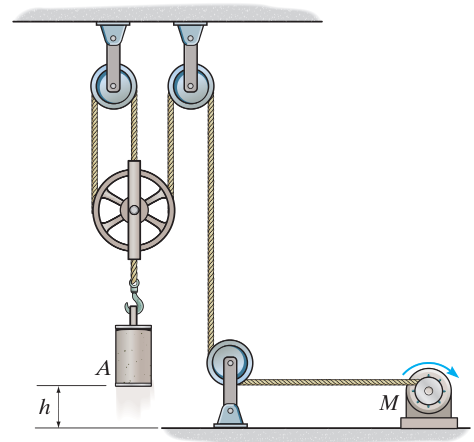
Since $\Delta s_C = -4$, then

$3\Delta s_B = -4$

$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$ **Ans.**

***12–204.**

Determine the speed of cylinder A, if the rope is drawn towards the motor M at a constant rate of 10 m/s.



SOLUTION

Position Coordinates: By referring to Fig. *a*, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

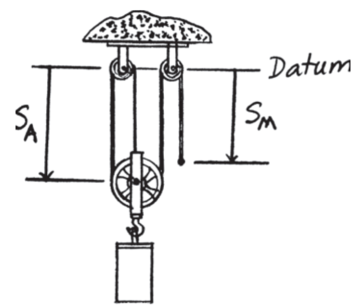
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = 10$ m/s. Thus,

$$3v_A + 10 = 0$$

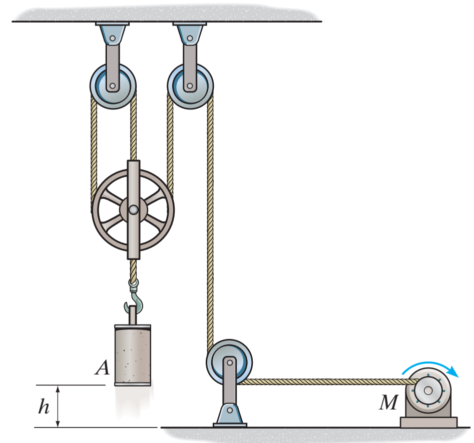
$$v_A = -3.33 \text{ m/s} = 3.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

12–205.

If the rope is drawn toward the motor M at a speed of $v_M = (5t^{3/2})$ m/s, where t is in seconds, determine the speed of cylinder A when $t = 1$ s.



SOLUTION

Position Coordinates: By referring to Fig. a, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

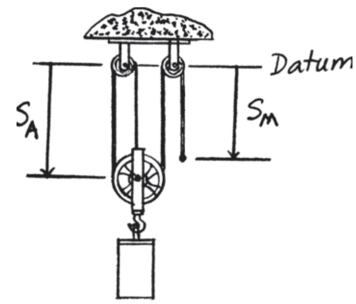
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = \left(5t^{3/2}\right)$ m/s. Thus,

$$3v_A + 5t^{3/2} = 0$$

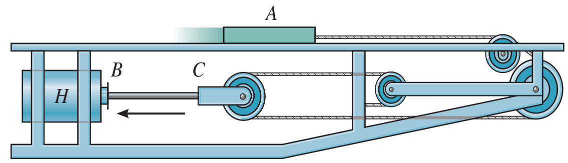
$$v_A = \left(-\frac{5}{3}t^{3/2}\right)\text{m/s} = \left(\frac{5}{3}t^{3/2}\right)\text{m/s}\bigg|_{t=1\text{ s}} = 1.67\text{ m/s} \qquad \textbf{Ans.}$$



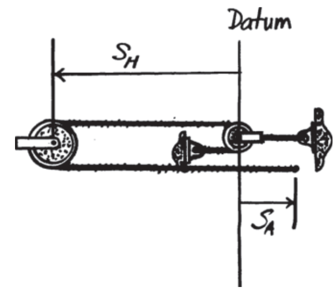
(a)

12–206.

If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A.



SOLUTION



$2s_H + s_A = l$

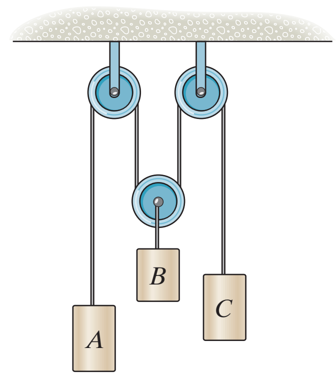
$2v_H = -v_A$

$2(2) = -v_A$

$v_A = -4\text{ ft/s} = 4\text{ ft/s} \leftarrow$ **Ans.**

12–207.

If block A is moving downward with a speed of 4 ft/s while C is moving up at 2 ft/s, determine the speed of block B.



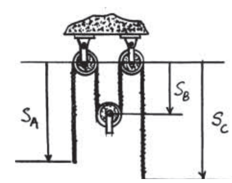
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

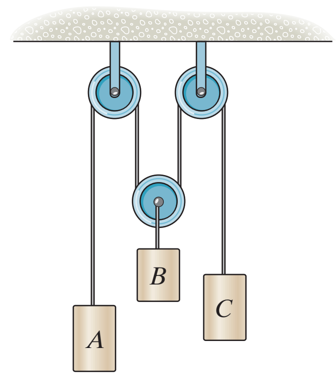
$4 + 2v_B - 2 = 0$

$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow$ **Ans.**



***12–208.**

If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the speed of block B.



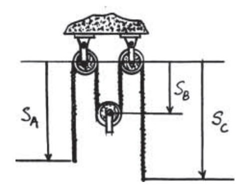
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

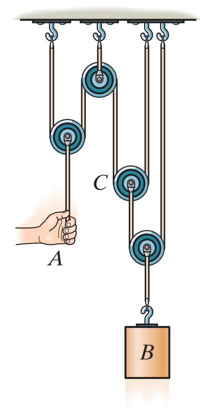
$6 + 2v_B + 18 = 0$

$v_B = -12 \text{ ft/s} = 12 \text{ ft/s} \uparrow$ **Ans.**



12–209.

Determine the displacement of the block B if A is pulled down 4 ft.



SOLUTION

$2s_A + 2s_C = l_1$

$\Delta s_A = -\Delta s_C$

$s_B - s_C + s_B = l_2$

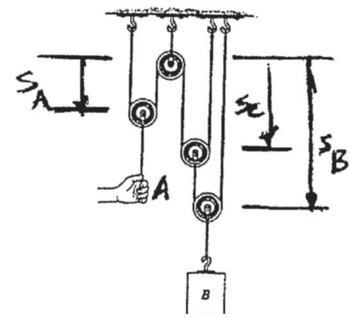
$2\,\Delta s_B = \Delta s_C$

Thus,

$2\,\Delta s_B = -\Delta s_A$

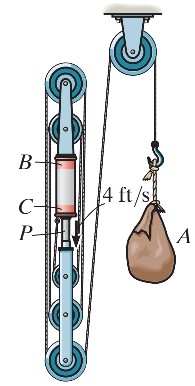
$2\,\Delta s_B = -4$

$\Delta s_B = -2\text{ ft} = 2\text{ ft}\uparrow$ **Ans.**



12–210.

The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger *P* is pushed downward with a speed of 4 ft/s, determine the speed of the load at *A*.



SOLUTION

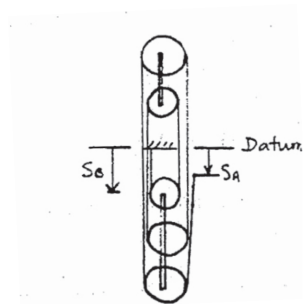
$5\,s_B + (s_B - s_A) = l$

$6\,s_B - s_A = l$

$6\,v_B - v_A = 0$

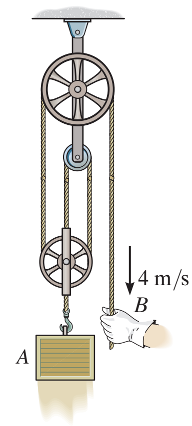
$6(4) = v_A$

$v_A = 24\text{ ft/s}$ **Ans.**



12–211.

Determine the speed of block A if the end of the rope is pulled down with a speed of 4 m/s.



SOLUTION

Position Coordinates: By referring to Fig. *a*, the length of the cord written in terms of the position coordinates s_A and s_B is

$$s_B + s_A + 2(s_A - a) = l$$

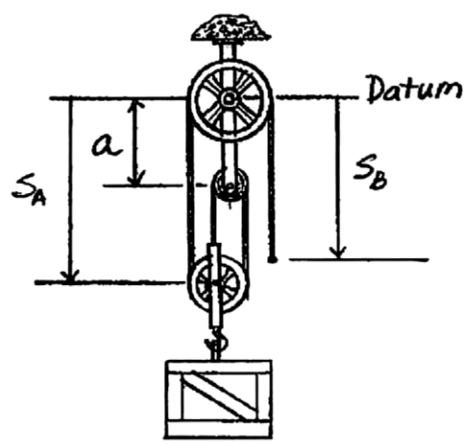
$$s_B + 3s_A = l + 2a$$

Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 3v_A = 0$$

Here, $v_B = 4$ m/s. Thus,

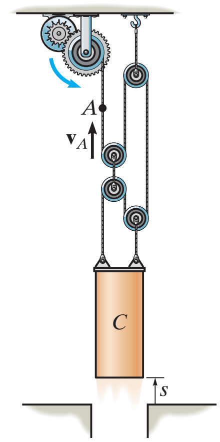
$$4 + 3v_A = 0 \qquad v_A = -133 \text{ m/s} = 1.33 \text{ m/s} \uparrow$$ **Ans.**



(a)

***12–212.**

The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the cylinder.



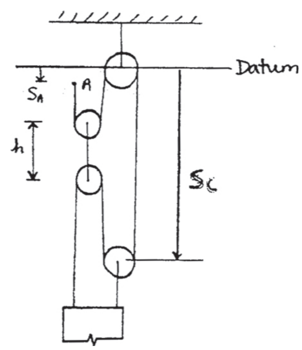
SOLUTION

$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

$$l = 3s_C - 2h - s_A$$

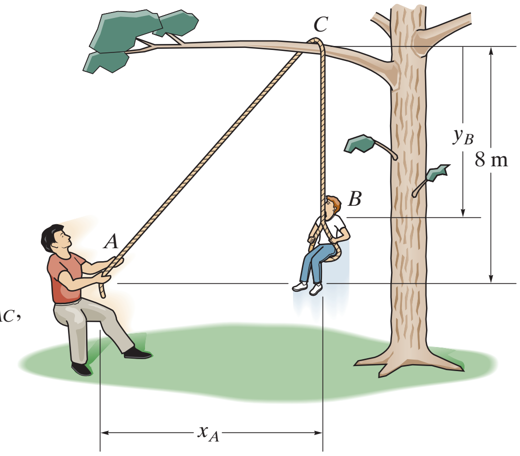
$$0 = 3v_C - v_A$$

$$v_C = \frac{v_A}{3} = \frac{-2}{3} = -0.667 \text{ m/s} = 0.667 \text{ m/s} \uparrow$$ **Ans.**



12–213.

The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s. Determine the speed at which the boy is being lifted at the instant $x_A = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) and realizing that $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

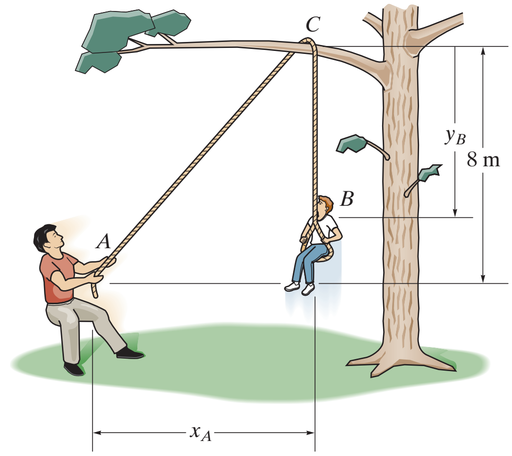
At the instant $x_A = 4$ m, from Eq. [2]

$$v_B = -\frac{4}{\sqrt{4^2 + 64}}(1.5) = -0.671 \text{ m/s} = 0.671 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

12–214.

The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) Where $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

At the instant $y_B = 4$ m, from Eq. (1), $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

$$v_A^2 = (v_0)_A^2 + 2(a_c)_A\left[s_A - (s_0)_A\right]$$

$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$

$$v_A = 1.891\ \text{m/s}$$

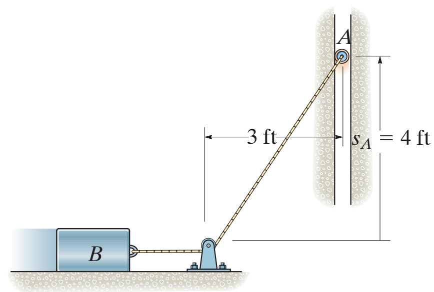
Substitute the above results into Eq. (2) yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41\ \text{m/s} = 1.41\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

12–215.

The roller at A is moving upward with a velocity of $v_A = 3$ ft/s and has an acceleration of $a_A = 4\text{ ft/s}^2$ when $s_A = 4$ ft. Determine the velocity and acceleration of block B at this instant.



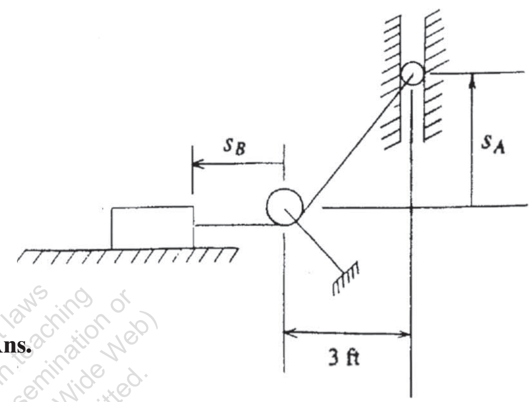
SOLUTION

$$s_B + \sqrt{(s_A)^2 + 3^2} = l$$

$$\dot{s}_B + \frac{1}{2}\left[(s_A)^2 + 3^2\right]^{-\frac{1}{2}}(2s_A)\,\dot{s}_A = 0$$

$$\dot{s}_B + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\dot{s}_A\right) = 0$$

$$\ddot{s}_B - \left[(s_A)^2 + 9\right]^{-\frac{3}{2}}\left(s_A^2\,\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$



At $s_A = 4$ ft, $\dot{s}_A = 3$ ft/s, $\ddot{s}_A = 4\text{ ft/s}^2$

$$\dot{s}_B + \left(\frac{1}{5}\right)(4)(3) = 0$$

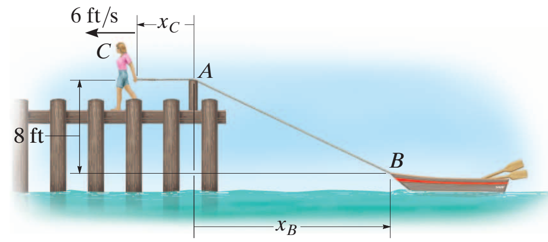
$v_B = -2.4\text{ ft/s} = 2.40\text{ ft/s} \rightarrow$ **Ans.**

$$\ddot{s}_B - \left(\frac{1}{5}\right)^3(4)^2(3)^2 + \left(\frac{1}{5}\right)(3)^2 + \left(\frac{1}{5}\right)(4)(4) = 0$$

$a_B = -3.85\text{ ft/s}^2 = 3.85\text{ ft/s}^2 \rightarrow$ **Ans.**

***12–216.**

The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



SOLUTION

The length l of cord is

$$\sqrt{(8)^2 + x_B^2} + x_C = l$$

Taking the time derivative:

$$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2}\, 2\, x_B \dot{x}_B + \dot{x}_C = 0 \qquad \textbf{(1)}$$

$$\dot{x}_C = 6 \text{ ft/s}$$

When $AB = 50$ ft,

$$x_B = \sqrt{(50)^2 - (8)^2} = 49.356 \text{ ft}$$

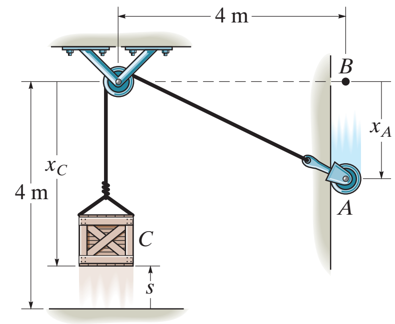
From Eq. (1)

$$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2}\, 2(49.356)(\dot{x}_B) + 6 = 0$$

$$\dot{x}_B = -6.0783 = 6.08 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

12–217.

The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2$ m/s along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

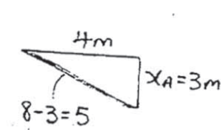


SOLUTION

$$x_C + \sqrt{x_A^2 + (4)^2} = l$$

$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$



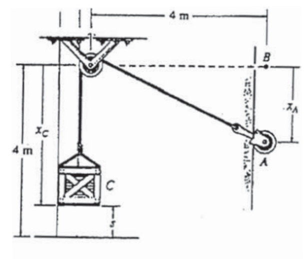
$l = 8$ m, and when $s = 1$ m,

$x_C = 3$ m

$x_A = 3$ m

$v_A = \dot{x}_A = 2$ m/s

$a_A = \ddot{x}_A = 0$



Thus,

$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$

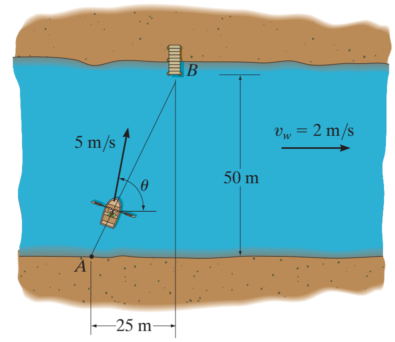
$v_C = -1.2 \text{ m/s} = 1.2 \text{ m/s} \uparrow$ **Ans.**

$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$

$a_C = -0.512 \text{ m/s}^2 = 0.512 \text{ m/s}^2 \uparrow$ **Ans.**

12–218.

The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle θ he must direct the boat so that it travels from A to B.



SOLUTION

Solution I

Vector Analysis: Here, the velocity $\mathbf{v}_b$ of the boat is directed from A to B. Thus, $\phi = \tan^{-1}\left(\frac{50}{25}\right) = 63.43°$. The magnitude of the boat's velocity relative to the flowing river is $v_{b/w} = 5$ m/s. Expressing $\mathbf{v}_b$, $\mathbf{v}_w$, and $\mathbf{v}_{b/w}$ in Cartesian vector form, we have $\mathbf{v}_b = v_b \cos 63.43\mathbf{i} + v_b \sin 63.43\mathbf{j} = 0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j}$, $\mathbf{v}_w = [2\mathbf{i}]$ m/s, and $\mathbf{v}_{b/w} = 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_{b/w}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = 2\mathbf{i} + 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = (2 + 5\cos\theta)\mathbf{i} + 5\sin\theta\mathbf{j}$$

Equating the **i** and **j** components, we have

$$0.4472v_b = 2 + 5\cos\theta \qquad \textbf{(1)}$$

$$0.8944v_b = 5\sin\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_b = 5.56 \text{ m/s} \qquad \theta = 84.4° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. a and applying the law of cosines,

$$5^2 = 2^2 + v_b^{\,2} - 2(2)(v_b)\cos 63.43°$$

$$v_b^{\,2} - 1.789v_b - 21 = 0$$

$$v_b = \frac{-(-1.789) \pm \sqrt{(-1.789)^2 - 4(1)(-21)}}{2(1)}$$

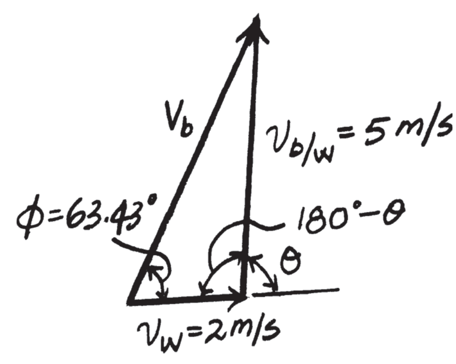
Choosing the positive root,

$$v_b = 5.563 \text{ m/s} = 5.56 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of v_b and applying the law of sines,

$$\frac{\sin(180° - \theta)}{5.563} = \frac{\sin 63.43°}{5}$$

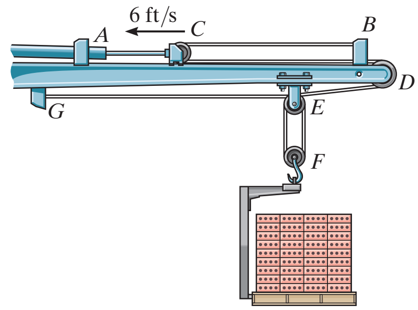
$$\theta = 84.4° \qquad \textbf{Ans.}$$



(a)

12–219.

Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load 2 ft. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.



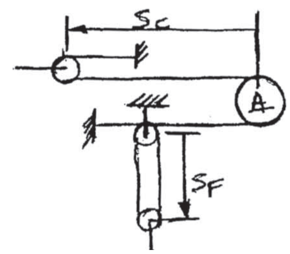
SOLUTION

$2\,s_C + 2\,s_F = l$

$2\,\Delta s_C = -2\,\Delta s_F$

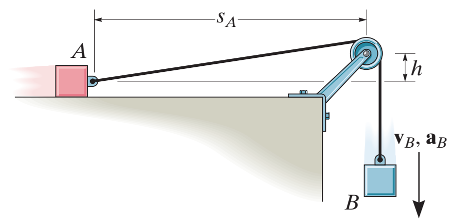
$\Delta s_C = -\Delta s_F$

$\Delta s_C = -(-2\text{ ft}) = 2\text{ ft}$ **Ans.**



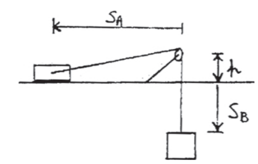
12–220.

If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_B^2 + h^2}$$



$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}2s_A\,\dot{s}_A$$

$$v_A = \dot{s}_A = \frac{-\,\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

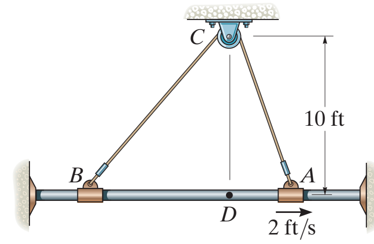
$$v_A = -v_B\,(1 + \left(\frac{h}{s_A}\right)^2)^{1/2}$$ **Ans.**

$$a_A = \dot{v}_A = -\,\dot{v}_B(1 + \left(\frac{h}{s_A}\right)^2)^{1/2} - v_B\left(\frac{1}{2}\right)(1 + \left(\frac{h}{s_A}\right)^2)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B(1+\left(\frac{h}{s_B}\right)^2)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\,(1+\left(\frac{h}{s_A}\right)^2)^{-1/2}$$ **Ans.**

12–221.

Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is 24 ft to the left of D. If A moves at a constant speed of 2 ft/s to the right, determine the speed of B when A is 4 ft to the right of D.



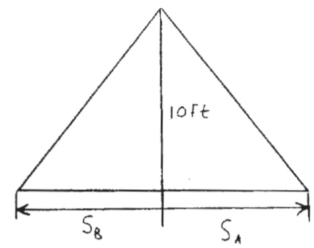
SOLUTION

$$l = \sqrt{(24)^2 + (10)^2} + 10 = 36 \text{ ft}$$

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + s_A^2} = 36$$

$$\frac{1}{2}(100 + s_B^2)^{-\frac{1}{2}}(2s_B\dot{s}_B) + \frac{1}{2}(100 + s_A^2)^{-\frac{1}{2}}(2s_A\dot{s}_A) = 0$$

$$\dot{s}_B = -\left(\frac{s_A\dot{s}_A}{s_B}\right)\left(\frac{100 + s_B^2}{100 + s_A^2}\right)^{\frac{1}{2}}$$



At $s_A = 4$,

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + (4)^2} = 36$$

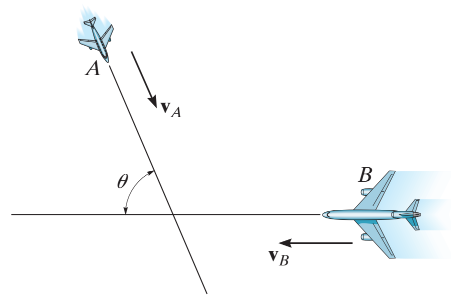
$$s_B = 23.163 \text{ ft}$$

Thus,

$$\dot{s}_B = -\left(\frac{4(2)}{23.163}\right)\left(\frac{100 + (23.163)^2}{100 + 4^2}\right)^{\frac{1}{2}} = -0.809 \text{ ft/s} = 0.809 \text{ ft/s} \rightarrow \quad \textbf{Ans.}$$

12–222.

Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 600$ km/h and $v_B = 500$ km/h such that the angle between their straight-line courses is $\theta = 75°$, determine the velocity of plane B with respect to plane A.



SOLUTION

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[500 \leftarrow] = [600 \overset{75°}{\measuredangle}\theta] + v_{B/A}$$

$(\overset{+}{\leftarrow})$ $\quad 500 = -600 \cos 75° + (v_{B/A})_x$

$$(v_{B/A})_x = 655.29 \leftarrow$$

$(+\uparrow)$ $\quad 0 = -600 \sin 75° + (v_{B/A})_y$

$$(v_{B/A})_y = 579.56 \uparrow$$

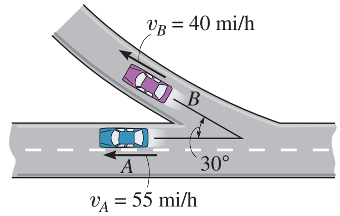
$$(v_{B/A}) = \sqrt{(655.29)^2 + (579.56)^2}$$

$$v_{B/A} = 875 \text{ km/h} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{579.56}{655.29}\right) = 41.5° \qquad \textbf{Ans.}$$

12–223.

At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by $1200\ \text{mi/h}^2$, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



SOLUTION

$$v_B = -40 \cos 30°\mathbf{i} + 40 \sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}\ \text{mi/h}$$

$$v_A = \{-55\mathbf{i}\}\ \text{mi/h}$$

$$v_{B/A} = \nu_B - \nu_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}\ \text{mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5\ \text{mi/h}$$ **Ans.**

$$\theta = \tan^{-1}\frac{20}{20.36} = 44.5°\ \measuredangle$$ **Ans.**

$$(a_B)_n = \frac{v_A^2}{\rho} = \frac{40^2}{0.5} = 3200\ \text{mi/h}^2 \qquad (a_B)_t = 1200\ \text{mi/h}^2$$

$$\mathbf{a}_B = (3200 \cos 60° - 1200 \cos 30°)\mathbf{i} + (3200 \sin 60° + 1200 \sin 30°)\mathbf{j}$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\ \ \text{mi/h}^2$$

$$\mathbf{a}_A = 0$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

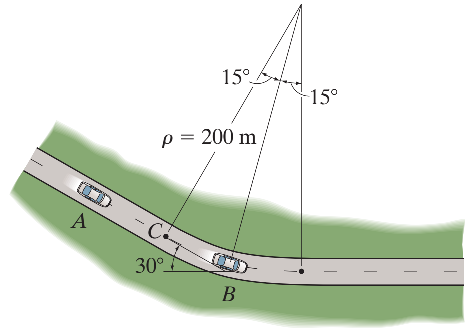
$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\ \ \text{mi/h}^2$$

$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418\ \text{mi/h}^2$$ **Ans.**

$$\theta = \tan^{-1}\frac{3371.28}{560.77} = 80.6°\ \measuredangle$$ **Ans.**

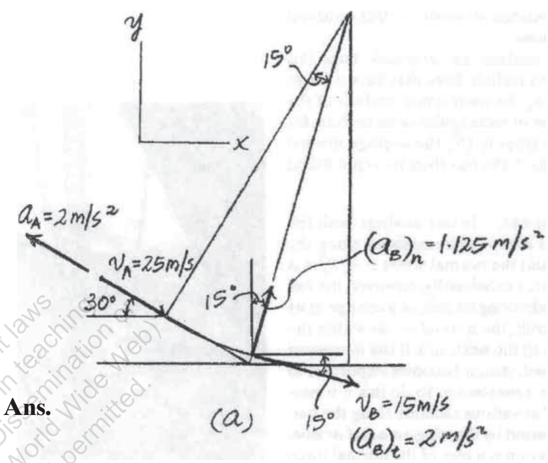
12–224.

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.



SOLUTION

Velocity: Referring to Fig. a, the velocity of cars A and B expressed in Cartesian vector form are



$$\mathbf{v}_A = [25 \cos 30°\,\mathbf{i} - 25 \sin 30°\,\mathbf{j}]\text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_B = [15 \cos 15°\,\mathbf{i} - 15 \sin 15°\,\mathbf{j}]\text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

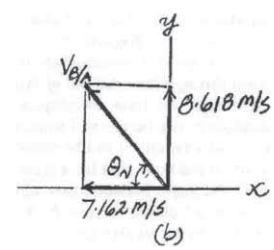
$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2\text{ m/s}$$ **Ans.**

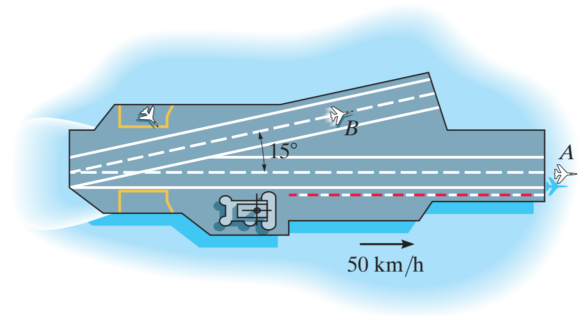
The direction angle θ_v of $\mathbf{v}_{B/A}$ measured down from the negative x axis, Fig. b is

$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3°$$ **Ans.**



12–225.

An aircraft carrier is traveling forward with a velocity of 50 km/h. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed of 200 km/h, measured from still water. If the plane at B is traveling along the runway of the carrier at 175 km/h in the direction shown, determine the velocity of A with respect to B.



SOLUTION

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\mathbf{v}_B = 50\mathbf{i} + 175\cos 15°\mathbf{i} + 175\sin 15°\mathbf{j} = 219.04\mathbf{i} + 45.293\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

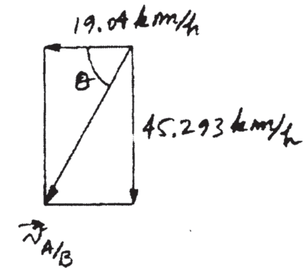
$$200\mathbf{i} = 219.04\mathbf{i} + 45.293\mathbf{j} + (v_{A/B})_x\mathbf{i} + (v_{A/B})_y\mathbf{j}$$

$$200 = 219.04 + (v_{A/B})_x$$

$$0 = 45.293 + (v_{A/B})_y$$

$$(v_{A/B})_x = -19.04$$

$$(v_{A/B})_y = -45.293$$



$$v_{A/B} = \sqrt{(-19.04)^2 + (-45.293)^2} = 49.1 \text{ km/h}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{45.293}{19.04}\right) = 67.2° \measuredangle$$ **Ans.**

12–226.

A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is directed toward the east. If the car's speed is 80 km/h, the instrument indicates that the wind is directed toward the north-east. Determine the speed and direction of the wind.

SOLUTION

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_1\,\mathbf{i}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 50\mathbf{j} + (v_{w/c})_1\,\mathbf{i}$$

$$\mathbf{v}_w = (v_{w/c})_1\mathbf{i} + 50\mathbf{j} \qquad \textbf{(1)}$$

For the second case, $v_C = [80\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_2\cos 45°\mathbf{i} + (v_{W/C})_2\sin 45°\,\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 80\mathbf{j} + (v_{w/c})_2\cos 45°\mathbf{i} + (v_{w/c})_2\sin 45°\,\mathbf{j}$$

$$\mathbf{v}_w = (v_{w/c})_2\cos 45°\,\mathbf{i} + \left[80 + (v_{w/c})_2\sin 45°\right]\mathbf{j} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) and then the **i** and **j** components,

$$(v_{w/c})_1 = (v_{w/c})_2\cos 45° \qquad \textbf{(3)}$$

$$50 = 80 + (v_{w/c})_2\sin 45° \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43\text{ km/h} \qquad (v_{w/c})_1 = -30\text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}]\text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_W$ is

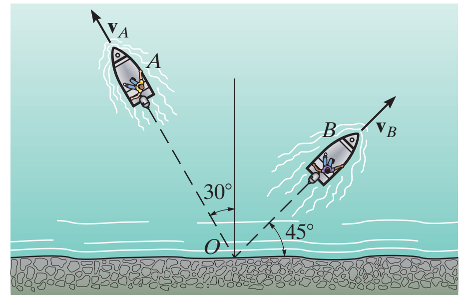
$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3\text{ km/h} \qquad \textbf{Ans.}$$

and the directional angle θ that $\mathbf{v}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{50}{30}\right) = 59.0° \; ⦪ \qquad \textbf{Ans.}$$

12–227.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20\text{ ft/s}$ and $v_B = 15\text{ ft/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



SOLUTION

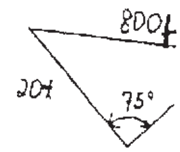
$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-20 \sin 30°\mathbf{i} + 20 \cos 30°\mathbf{j} = 15 \cos 45°\mathbf{i} + 15 \sin 45°\mathbf{j} + \mathbf{v}_{A/B}$

$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} + 6.714\mathbf{j}\}\text{ ft/s}$

$v_{A/B} = \sqrt{(-20.61)^2 + (+6.714)^2} = 21.7\text{ ft/s}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0°$ **Ans.**



$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t)\cos 75°$

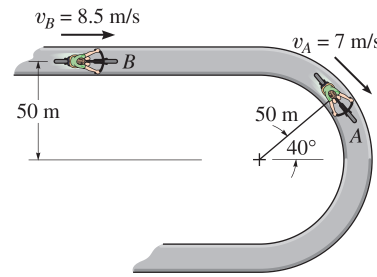
$t = 36.9\text{ s}$ **Ans.**

Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9\text{ s}$$ **Ans.**

***12–228.**

At the instant shown, the bicyclist at A is traveling at 7 m/s around the curve on the race track while increasing his speed at 0.5 m/s^2. The bicyclist at B is traveling at 8.5 m/s along the straight-a-way and increasing his speed at 0.7 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = v_B + v_{A/B}$$

$$[7 \searrow_{40°}] = [8.5 \rightarrow] + [(v_{A/B})_x \rightarrow] + [(v_{A/B})_y \downarrow]$$

$$(\overset{+}{\rightarrow}) \qquad 7 \sin 40° = 8.5 + (v_{A/B})_x$$

$$(+\downarrow) \qquad 7 \cos 40° = (v_{A/B})_y$$

Thus,

$$(v_{A/B})_x = 4.00 \text{ m/s} \leftarrow$$

$$(v_{A/B})_y = 5.36 \text{ m/s} \downarrow$$

$$(v_{A/B}) = \sqrt{(4.00)^2 + (5.36)^2}$$

$$v_{A/B} = 6.69 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{5.36}{4.00}\right) = 53.3° \; \swarrow \qquad \textbf{Ans.}$$

$$(a_A)_n = \frac{7^2}{50} = 0.980 \text{ m/s}^2$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$[0.980] \overset{\theta}{\swarrow}_{40°} + [0.5] \searrow_{40°} = [0.7 \rightarrow] + [(a_{A/B})_x \rightarrow] + [(a_{A/B})_y \downarrow]$$

$$(+\rightarrow) \qquad -0.980 \cos 40° + 0.5 \sin 40° = 0.7 + (a_{A/B})_x$$

$$(a_{A/B})_x = 1.129 \text{ m/s}^2 \leftarrow$$

$$(+\downarrow) \qquad 0.980 \sin 40° + 0.5 \cos 40° = (a_{A/B})_y$$

$$(a_{A/B})_y = 1.013 \text{ m/s}^2 \downarrow$$

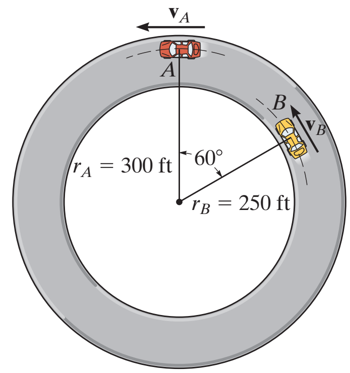
$$(a_{A/B}) = \sqrt{(1.129)^2 + (1.013)^2}$$

$$a_{A/B} = 1.52 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{1.013}{1.129}\right) = 41.9° \; \swarrow \qquad \textbf{Ans.}$$

12–229.

Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



SOLUTION

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-90\mathbf{i} = -105 \sin 30° \mathbf{i} + 105 \cos 30° \mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4 \text{ ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{90.93}{37.5}\right) = 67.6°$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$-15\mathbf{i} - \frac{(90)^2}{300}\mathbf{j} = 25 \cos 60° \mathbf{i} - 25 \sin 60° \mathbf{j} - 44.1 \sin 60° \mathbf{i} - 44.1 \cos 60° \mathbf{j} + \mathbf{a}_{A/B}$$

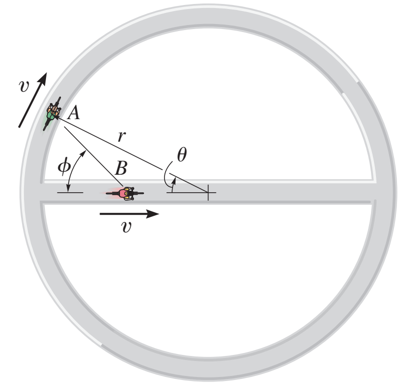
$$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\} \text{ ft/s}^2$$

$$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8 \text{ ft/s}^2$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{16.70}{10.69}\right) = 57.4°$$ **Ans.**

12–230.

The two cyclists A and B travel at the same constant speed v. Determine the speed of A with respect to B if A travels along the circular track, while B travels along the diameter of the circle.



SOLUTION

$$\mathbf{v}_A = v \sin\theta\mathbf{i} + v\cos\theta\mathbf{j} \qquad \mathbf{v}_B = v\mathbf{i}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (v\sin\theta\mathbf{i} + v\cos\theta\mathbf{j}) - v\mathbf{i}$$

$$= (v\sin\theta - v)\mathbf{i} + v\cos\theta\mathbf{j}$$

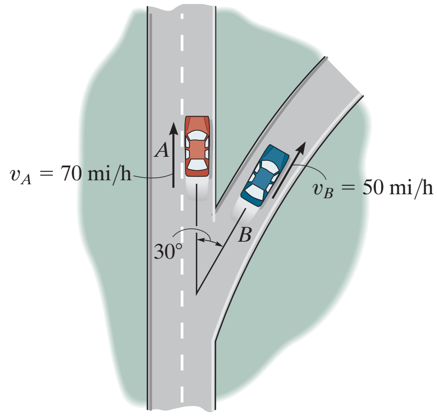
$$v_{A/B} = \sqrt{(v\sin\theta - v)^2 + (v\cos\theta)^2}$$

$$= \sqrt{2v^2 - 2v^2\sin\theta}$$

$$= v\sqrt{2(1 - \sin\theta)} \qquad \textbf{Ans.}$$

12–231.

At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is increasing its speed by 1100 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



SOLUTION

Relative Velocity:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$50 \sin 30°\mathbf{i} + 50 \cos 30°\mathbf{j} = 70\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{25.0\mathbf{i} - 26.70\mathbf{j}\} \text{ mi/h}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{25.0^2 + (-26.70)^2} = 36.6 \text{ mi/h} \qquad \textbf{Ans.}$$

The direction of the relative velocity is the same as the direction of that for relative acceleration. Thus

$$\theta = \tan^{-1}\frac{26.70}{25.0} = 46.9° \searrow \qquad \textbf{Ans.}$$

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43 \text{ mi/h}^2$. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(1100 \sin 30° + 3571.43 \cos 30°)\mathbf{i} + (1100 \cos 30° - 3571.43 \sin 30°)\mathbf{j} = 0 + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{3642.95\mathbf{i} - 833.09\mathbf{j}\} \text{ mi/h}^2$$

Thus, the magnitude of the relative velocity $\mathbf{a}_{B/A}$ is

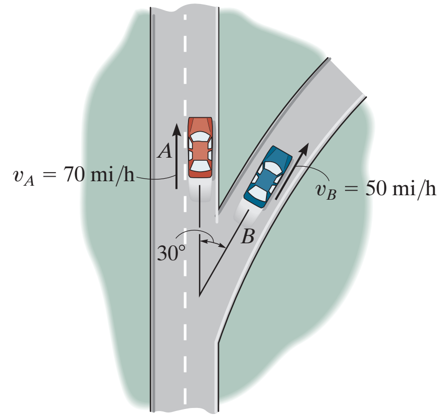
$$a_{B/A} = \sqrt{3642.95^2 + (-833.09)^2} = 3737 \text{ mi/h}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\phi = \tan^{-1}\frac{833.09}{3642.95} = 12.9° \searrow \qquad \textbf{Ans.}$$

12–232.

At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is decreasing its speed at 1400 mi/h^2 while A is increasing its speed at 800 mi/h^2, determine the acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



SOLUTION

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43\ \text{mi/h}^2$. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(3571.43 \cos 30° - 1400 \sin 30°)\mathbf{i} + (-1400 \cos 30° - 3571.43 \sin 30°)\mathbf{j} = 800\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{2392.95\mathbf{i} - 3798.15\mathbf{j}\}\ \text{mi/h}^2$$

Thus, the magnitude of the relative acc. $\mathbf{a}_{B/A}$ is

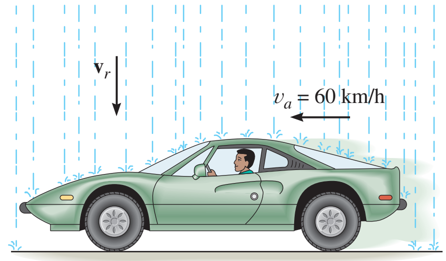
$$a_{B/A} = \sqrt{2392.95^2 + (-3798.15)^2} = 4489\ \text{mi/h}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\phi = \tan^{-1}\frac{3798.15}{2392.95} = 57.8° \qquad \textbf{Ans.}$$

12–233.

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant)velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



SOLUTION

$v_r = v_a + v_{r/a}$

$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$

$(\overset{+}{\rightarrow})$ $\quad 0 = -60 + v_{r/a}\cos 30°$

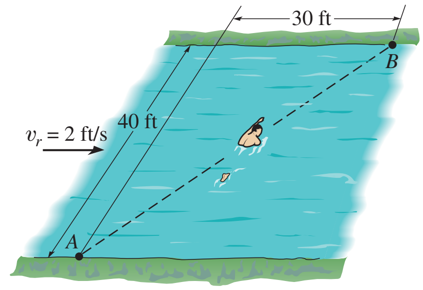
$(+\uparrow)$ $\quad -v_r = 0 - v_{r/a}\sin 30°$

$v_{r/a} = 69.3$ km/h

$v_r = 34.6$ km/h **Ans.**

12–234.

A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself toward point B to reach this point. Why?

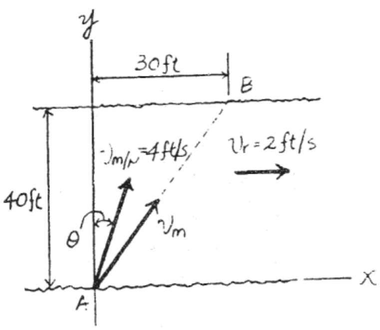


SOLUTION

Relative Velocity:

$$v_m = v_r + v_{m/r}$$

$$\frac{3}{5}v_m\mathbf{i} + \frac{4}{5}v_m\mathbf{j} = 2\mathbf{i} + 4\sin\theta\mathbf{i} + 4\cos\theta\mathbf{j}$$



Equating the i and j components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \qquad \textbf{(1)}$$

$$\frac{4}{5}v_m = 4\cos\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta = 13.29^\circ$$

$$v_m = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \qquad \textbf{Ans.}$$

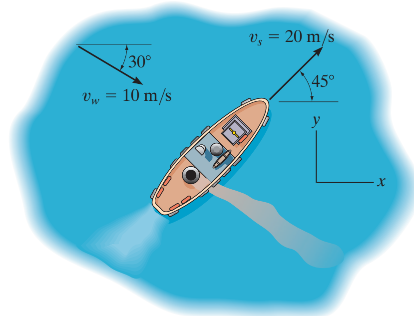
Thus, the time t required by the boat to travel from points A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \qquad \textbf{Ans.}$$

In order for the man to reached point B, the man has to direct himself at an angle $\theta = 13.3^\circ$ with y axis.

12–235.

The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION

Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20\cos 45°\,\mathbf{i} + 20\sin 45°\,\mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10\cos 30°\,\mathbf{i} - 10\sin 30°\,\mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s}$$ **Ans.**

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

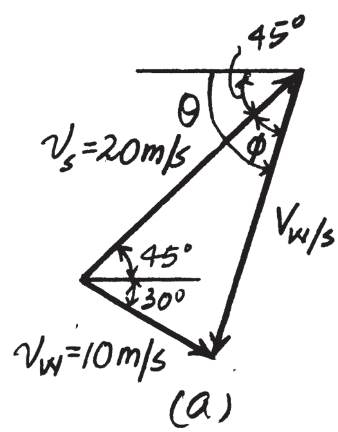
$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0° \;\measuredangle$$ **Ans.**

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. *a*,

$$v_{w/s} = \sqrt{20^2 + 10^2 - 2(20)(10)\cos 75°}$$

$$= 19.91 \text{ m/s} = 19.9 \text{ m/s}$$ **Ans.**



Using the result of $v_{w/s}$ and applying the law of sines,

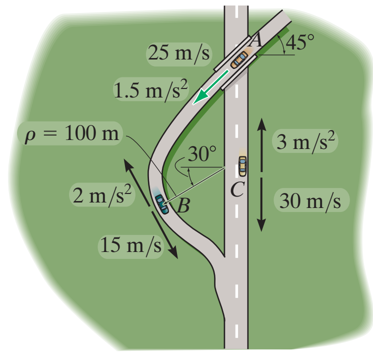
$$\frac{\sin\phi}{10} = \frac{\sin 75°}{19.91} \qquad \phi = 29.02°$$

Thus,

$$\theta = 45° + \phi = 74.0° \;\measuredangle$$ **Ans.**

***12–236.**

Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



SOLUTION

Velocity: The velocity of cars A and C expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45°\mathbf{i} - 25\sin 45°\mathbf{j}]\text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9° \qquad \textbf{Ans.}$$

Acceleration: The acceleration of cars A and C expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45°\mathbf{i} - 1.5\sin 45°\mathbf{j}]\text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

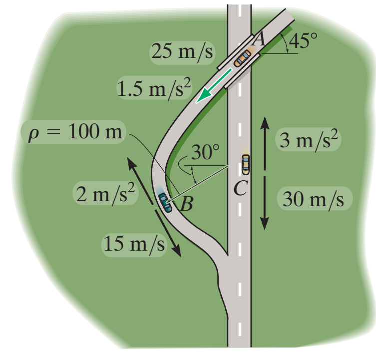
$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4° \qquad \textbf{Ans.}$$

12–237.

Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car B relative to car C.



SOLUTION

Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15\cos 60^\circ\,\mathbf{i} - 15\sin 60^\circ\,\mathbf{j}]\text{ m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{B/C} = \sqrt{7.5^2 + 17.01^2} = 18.6\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2^\circ \ \measuredangle \qquad \textbf{Ans.}$$

Acceleration: The normal component of car B's acceleration is $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{100} = 2.25\text{ m/s}^2$. Thus, the tangential and normal components of car B's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2\cos 60^\circ\,\mathbf{i} + 2\sin 60^\circ\mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}]\text{ m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25\cos 30^\circ\,\mathbf{i} + 2.25\sin 30^\circ\,\mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

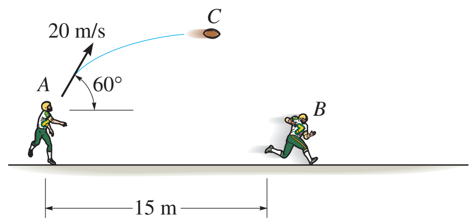
$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57^\circ \ \triangleleft \qquad \textbf{Ans.}$$

12–238.

At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



SOLUTION

Ball:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$s_C = 0 + 20 \cos 60^\circ\, t$

$(+\uparrow)\qquad v = v_0 + a_c t$

$-20 \sin 60^\circ = 20 \sin 60^\circ - 9.81\, t$

$t = 3.53\ s$

$s_C = 35.31\ \text{m}$

Player B:

$(\overset{+}{\rightarrow})\ s_B = s_0 + \nu_B\, t$

Require,

$35.31 = 15 + v_B(3.53)$

$v_B = 5.75\ \text{m/s}$ **Ans.**

At the time of the catch

$(v_C)_x = 20 \cos 60^\circ = 10\ \text{m/s} \rightarrow$

$(v_C)_y = 20 \sin 60^\circ = 17.32\ \text{m/s} \downarrow$

$v_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$10\mathbf{i} - 17.32\mathbf{j} = 5.751\mathbf{i} + (v_{C/B})_x\mathbf{i} + (v_{C/B})_y\mathbf{j}$

$(\overset{+}{\rightarrow})\qquad 10 = 5.75 + (v_{C/B})_x$

$(+\uparrow)\qquad -17.32 = (v_{C/B})_y$

$(v_{C/B})_x = 4.25\ \text{m/s} \rightarrow$

$(v_{C/B})_y = 17.32\ \text{m/s} \downarrow$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8\ \text{m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{17.32}{4.25}\right) = 76.2^\circ$ **Ans.**

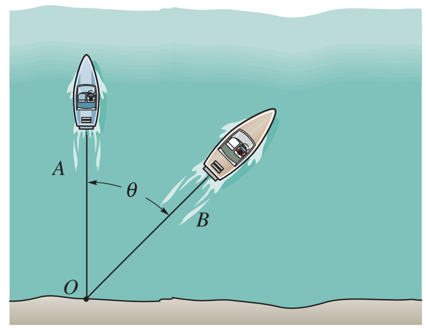
$a_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

$-9.81\,\mathbf{j} = 0 + \mathbf{a}_{C/B}$

$a_{C/B} = 9.81\ \text{m/s}^2 \downarrow$ **Ans.**

12–239.

Both boats A and B leave the shore at O at the same time. If A travels at v_A and B travels at v_B, write a general expression to determine the velocity of A with respect to B.



SOLUTION

Relative Velocity:

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_A\mathbf{j} = v_B \sin\theta\mathbf{i} + v_B \cos\theta\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = -v_B \sin\theta\mathbf{i} + (v_A - v_B \cos\theta)\mathbf{j}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-v_B \sin\theta)^2 + (v_A - v_B\cos\theta)^2}$$

$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta} \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{v_A - v_B\cos\theta}{v_B \sin\theta}\right) \qquad \textbf{Ans.}$$