3-1

$$\sigma = \frac{P}{A} = \frac{10\times10^3}{\pi/4\, d^2} \le 100$$

$$d \ge 11.28 \text{ mm}$$

$$\epsilon = \frac{\sigma}{E} = \frac{\frac{10\times10^3}{\pi d^2/4}}{45\times10^3} \le 0.1\%$$

$$d \ge 16.82 \text{ mm (governs)}$$

magnesium alloy elastic modulus:
45 GPa for tension or compression.

3-2

(a) $A_{req} = \frac{5\times10^3}{150} = 33.33 \text{ mm}^2$

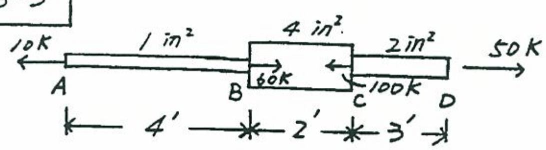
$A_{req} = \frac{PL}{\Delta E} = \frac{5\times10^3\times10\times10^3}{4\times210\times10^3} = 59.52 \text{ mm}^2$ (governs)

$\longrightarrow$ stiffness control.

(b) $P = K\Delta$

$\Rightarrow K = \frac{P}{\Delta} = \frac{5}{4} = 1.25$ KN/mm

3-3



(a) $\Sigma F = 0$, $P_2 = 60K$

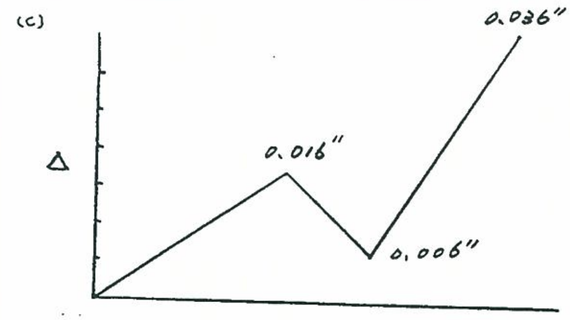
(b) $\Delta = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD}$

$= \frac{10(4\times12)}{1(30\times10^3)} + \frac{(-50)(2\times12)}{4(30\times10^3)} + \frac{50(3\times12)}{2(30\times10^3)}$

$= 0.016 - 0.010 + 0.03$

$= 0.036''$

(c)



(d) $f_{eff.} = \Sigma \frac{L_i}{(EA)_i}$

$= \frac{1}{30\times10^3}\left(\frac{4\times12}{1} + \frac{2\times12}{4} + \frac{3\times12}{2}\right)$

$= 2.4\times10^{-3}$ in/k

$K_{eff.} = \frac{1}{2.4\times10^{-3}} = 416.67$ K/in

3-4

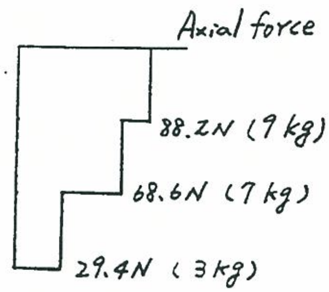
$$\Delta = \frac{PL}{AE} = \frac{mgL}{AE}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{AE}{mL}}$$

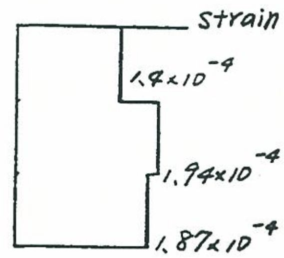
$$= \frac{1}{2\pi}\sqrt{\frac{\frac{\pi}{4}\cdot 20^2\cdot 180\times10^3}{2\times0.4}} = 1.3 \text{ KHz}$$

3-5

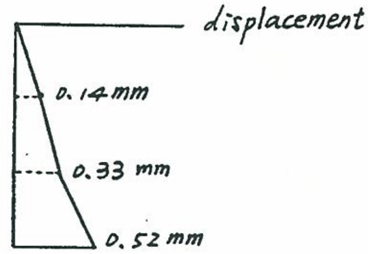
(a)



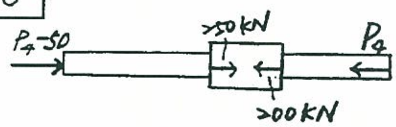
(b)



(c)



3-6



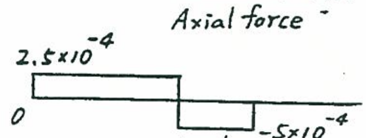
$$\Delta = \Sigma \left(\frac{PL}{EA}\right)_i$$

$$= -\frac{(P_4-50)\times 2000}{1000\times E} - \frac{(P_4+200)\times 1000}{2000\times E} - \frac{P_4\times 1500}{1000\times E}$$

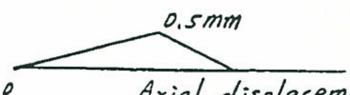
$$= 0$$

$$\rightarrow \quad P_4 = 0$$

Axial force

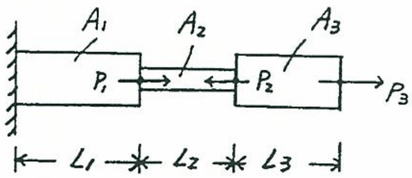


Axial strain



Axial displacement

3-7

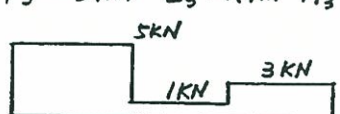


$P_1 = 4$ kN $\quad L_1 = 0.6\,m \quad A_1 = 2\times10^{-4}\,m^2$

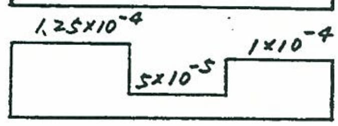
$P_2 = 2$ kN $\quad L_2 = 0.5\,m \quad A_2 = 1\times10^{-4}\,m^2$

$P_3 = 3$ kN $\quad L_3 = 0.4\,m \quad A_3 = 1.5\times10^{-4}\,m^2$

Axial force



Axial strain

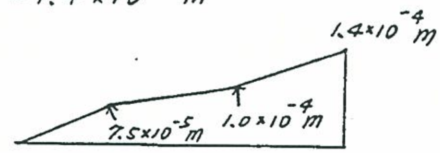


$$\Delta = \frac{PL}{AE} = \epsilon L$$

$$\Delta_1 = 1.25\times10^{-4}(0.6) = 7.5\times10^{-5}\ m$$

$$\Delta_2 = \Delta_1 + \Delta_2 = (7.5\times10^{-5}) + (5\times10^{-5})(0.5)$$
$$= 1.0\times10^{-4}\ m$$

$$\Delta_3 = \Delta_1 + \Delta_2 + \Delta_3$$
$$= 1.0\times10^{-4} + (1\times10^{-4})(0.4)$$
$$= 1.4\times10^{-4}\ m$$



Axial displacement

$$K = \frac{P}{\Delta} \quad \text{(use unit Force)}$$

$$\Delta = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

$$= \frac{1}{200\times10^6}\left(\frac{0.6}{2\times10^{-4}} + \frac{0.5}{1\times10^{-4}} + \frac{0.4}{1.5\times10^{-4}}\right)$$

$$= 5.33\times10^{-5}$$

$$\Rightarrow K = \frac{P}{\Delta} = \frac{1}{5.33\times10^{-5}} = 1.875\times10^4\ kN/m$$

3-8

(a) $\sigma_{AL} = E\epsilon = 70\times10^3\times873\times10^{-6}$

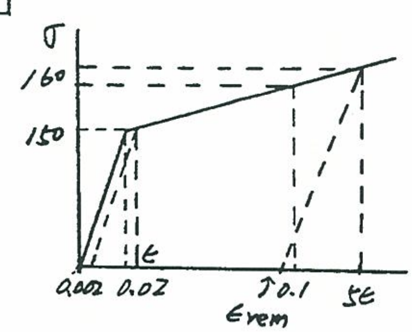
$= 61.1\ MPa = \sigma_{st}$

$$P = \frac{\pi}{4}d^2\sigma = \frac{\pi}{4}\times50^2\times61.1 = 120\ kN$$

(b) $\epsilon_{st} = \dfrac{\sigma_{st}}{E_{st}} = \dfrac{61.1}{200\times10^3} = 305.5\times10^{-6}$

$$\Delta = [(305.5\times1500) + (873\times500)]\times10^{-6}$$
$$= 0.89\ mm$$

3-9

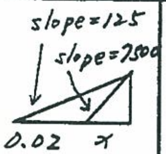


$$125(0.02+x) = 7500\times x$$

$x = 0.00034$

$\epsilon = 0.0034 + 0.22 = 0.2234$

$5\epsilon = 0.1117$



$\sigma = 150 + (0.1117 - 0.02) \times 125 = 161.4625$ MPa

$F = 161.4625 \times 0.2 = 32.29$ kN

$\epsilon_{rem} = 0.1117 - \dfrac{161.4625}{7500} = 0.09$

$\Delta_{rem} = 0.09 \times 600 = 54$ mm.

3-10

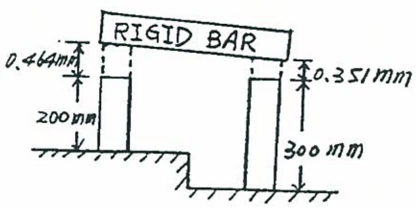
(a) $\Delta_{AL} = \alpha L \Delta T = 23.2 \times 10^{-6} \times 200 \times 100$

$= 0.464$ mm

$\Delta_{st} = \alpha_{st} L_{st} \Delta T = 11.7 \times 10^{-6} \times 300 \times 100$

$= 0.351$ mm

Inclination $\simeq \dfrac{0.464 - 0.351}{400} = 2.83 \times 10^{-4}$ rad



(b) $\Delta = \dfrac{PL}{EA} = \dfrac{\sigma L}{E} \Rightarrow \sigma = \dfrac{\Delta E}{L}$

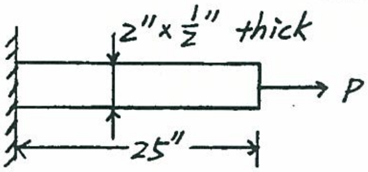
$\sigma_{AL} = \dfrac{\Delta_{AL} E_{AL}}{L} = \dfrac{0.464 \times 75 \times 10^3}{200}$

$= 174$ MPa

$\sigma_{st} = \dfrac{\Delta_{st} E_{st}}{L_{st}} = \dfrac{0.351 \times 200 \times 10^3}{300}$

$= 234$ MPa

3-11



$\Delta_x = 0.5 \times 10^{-3}$ in

$\epsilon_x = \dfrac{-0.3 \times 10^{-3}}{2} = -0.15 \times 10^{-3}$

$\nu = \dfrac{0.15 \times 10^{-3}}{\epsilon_a} = 0.25$

$\epsilon_a = 6 \times 10^{-4}$

$\sigma = E\epsilon_a = (30 \times 10^3) \times 6 \times 10^{-4}$

$= 18$ ksi

$P = 18 \times 2 \times \frac{1}{2} = 18$ k

$\Delta = 25 \times 6 \times 10^{-4} = 0.015$ in

3-12

$\epsilon_x = \dfrac{-16 \times 10^{-3}}{180} = -8.9 \times 10^{-5}$

$\Rightarrow 8.89 \times 10^{-5} = -\nu \dfrac{\sigma_y}{E}$

$\sigma_y = \dfrac{8.89 \times 10^{-5} \times 2 \times 10^5}{0.25} = 71.2$ N/mm²

$P_{a\text{-}a} = \sigma_y A = 71.2 \times 180 \times 10 = 12.8 \times 10^4$ N

$2g \times 400 = P_{a\text{-}a}$

$g = \dfrac{12.8 \times 10^4}{800} = 160$ N/mm

$P_x = 2gx = 2 \times 160x = 320x$ N

$\Delta = \int \dfrac{P_x dx}{AE} = \dfrac{320}{180 \times 10 \times 2 \times 10^5} \times \dfrac{x^2}{2} \Big|_0^{2000}$

$= 1.8$ mm

3-13

(a) $F = \int_0^L f\,dy = \int_0^L ky^2 dy = \dfrac{kL^3}{3}$

$k = \dfrac{3F}{L^3}$

$dF = f\,dy = ky^2 dy$

$P = \int_0^y dP = \int_0^y ky^2 dy = \dfrac{ky^3}{3} = \dfrac{y^3}{L^3} F$

$\Delta = \int_0^L \dfrac{P}{AE} dy = \int_0^L \dfrac{F}{AE} \dfrac{y^3}{L^3} dy = \dfrac{FL}{4AE}$

(b) $\Delta = \dfrac{400 \times 10^3 (10 \times 10^3)}{4(64000)(10^4)}$

$= 1.56$ mm

3-14

(a) $\Delta_{co} = \dfrac{PL}{AE} = \dfrac{\frac{2}{3} \times 2000 \times 60}{0.1 \times 20 \times 10^6} = 0.04''$

$\Delta_{AL} = \dfrac{PL}{AE} = \dfrac{\frac{1}{3} \times 2000 \times 100}{0.2 \times 10 \times 10^6} = 0.033''$

$\Delta_W = \Delta_{co} - \frac{1}{3}(\Delta_{co} - \Delta_{AL})$

$= 0.04 - \frac{1}{3}(0.04 - 0.03\dot{3})$

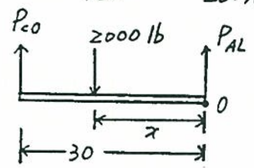
$= 0.038''$

(b) $\Delta_{co} = \Delta_{AL}$

$\Rightarrow \left(\dfrac{PL}{AE}\right)_{AL} = \left(\dfrac{PL}{AE}\right)_{co}$

$P_{AL} = \dfrac{E_{AL} A_{AL}}{L_{AL}} \times \dfrac{L_{co}}{E_{co} A_{co}} P_{co}$

$= \dfrac{10 \times 10^6 \times 0.2}{100} \times \dfrac{60}{20 \times 10^6 \times 0.1} P_{co} = 0.6 P_{co}$



$P_{co} + P_{AL} = 2000$

$1.6\, P_{co} = 2000$

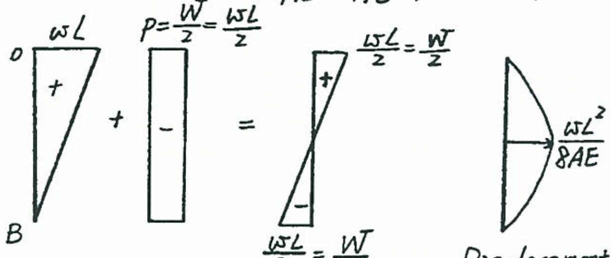
$P_{co} = 1250$ lb

$\Sigma M_o = 0 \Rightarrow 1250 \times 30 = 2000 \times x$

$x = 18.75$ inch from the right end

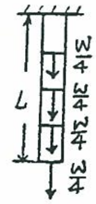
3-15

$\Delta_B = 0 \quad \dfrac{WL}{2AE} = \dfrac{PL}{AE}, \quad P = \dfrac{W}{2}$



3-16

(a) subdivide L into 4 segments



$\Delta_4 = \displaystyle\sum_{i=1}^{4} \frac{P_i L_i}{EA}$

$= \dfrac{1}{EA} \cdot \dfrac{L}{4}\left(W + \frac{3}{4}W + \frac{2}{4}W + \frac{1}{4}W\right)$

$= \dfrac{1}{EA} \times \dfrac{L}{4} \times \dfrac{10}{4} W = \dfrac{5WL}{8EA}$

(b) subdivide L into 10 segments

$\Delta_{10} = \displaystyle\sum_{i=1}^{10} \frac{P_i L_i}{EA}$

$= \dfrac{1}{EA} \times \dfrac{L}{10} \times \dfrac{1}{10}(W + 2W + 3W + \cdots + 10W)$

$= \dfrac{L}{EA} \times \dfrac{55}{100} W = \dfrac{11WL}{20EA}$

(c) subdivide L into 20 segments

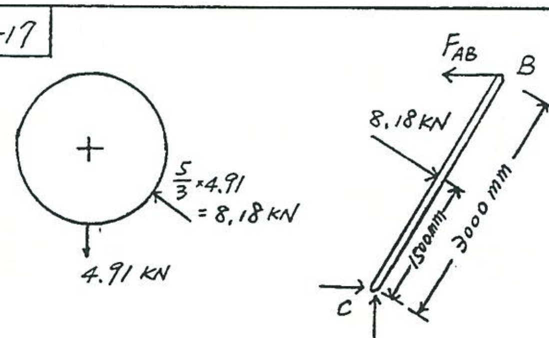
$\Delta_{20} = \displaystyle\sum_{i=1}^{20} \frac{P_i L_i}{EA}$

$= \dfrac{1}{EA} \times \dfrac{L}{20} \times \dfrac{1}{20}(W + 2W + \cdots + 20W)$

$= \dfrac{L}{EA} \times \dfrac{210W}{400} = \dfrac{21WL}{40EA}$

$\therefore \Delta_4 > \Delta_{10} > \Delta_{20} \simeq \Delta = \dfrac{WL}{2EA}$

3-17



$\Sigma M_c = 0$

$8.18 \times 1500 = F_{AB} \times 2400$

$F_{AB} = 5.11$ kN

$\Delta = \dfrac{F_{AB} L}{AE} = \dfrac{5.11 \times 10^3 (1800)}{5(200 \times 10^3)}$

$= 9.20$ mm

3-18

vertical bar force from C: $16 \times \frac{3}{4}$

vertical bar force from D: $16 \times \frac{4}{3}$

$$\Delta = \frac{PL}{AE} = \frac{1 \times 10^3}{100 \times 200} \times 16 \times \left(\frac{3}{4} + \frac{4}{3}\right) = 1.67 \text{ mm}$$

3-19

$$\Delta = \frac{PL}{EA} = \frac{\sigma L}{E} = \frac{15 \times 20}{30 \times 10^3}$$

$= 0.01''$ (elongation of each rod)

$\Delta_B = \Delta = 0.01$ in

$\Delta_D = 0$

$\Delta_E = \Delta = 0.01$ in

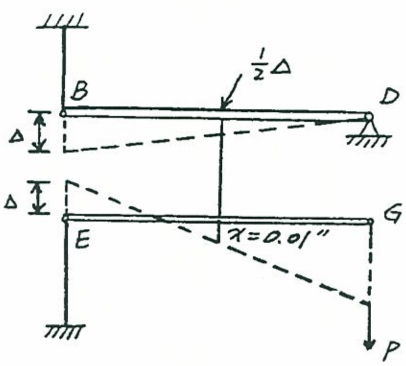
$$\Delta_{CF} = \frac{15 \times 10}{30 \times 10^3} = 0.005 \text{ in}$$

$x - \frac{1}{2}\Delta = 0.005$

$x = 0.005 + 0.005 = 0.01$

$$\frac{\Delta_G - \Delta}{2} = 0.001$$

$\Delta_G = 0.02 + 0.01 = 0.03$ in



3-20

$$\Delta = \frac{wL^2}{2AE}$$

$$L^2 = \frac{2AE\Delta}{w} = \frac{2 \times 1 \times 10^6 \times 0.25}{1.17 \times 12}$$

$= 356\,000 \text{ ft}^2$

$\therefore L = 596.76$ ft

3-21

$$\epsilon^n = E^{-1}\sigma, \quad \epsilon = \left(\frac{\sigma}{E}\right)^{\frac{1}{n}} = \left(\frac{P_x}{A_x E}\right)^{\frac{1}{n}}$$

$$\Delta = \int_0^L \epsilon\, dx = \int_0^L \left(\frac{wx}{AE}\right)^{\frac{1}{n}} dx$$

$$= \left(\frac{w}{AE}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right)\left[x^{\frac{n+1}{n}}\right]\Big|_0^L$$

$$\Delta = \left(\frac{w}{AE}\right)^{\frac{1}{n}} (L)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) L$$

$$= \left(\frac{wL}{AE}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) L$$

3-22

(a) $\sigma_1 = 5/1 = 5$ ksi

$\sigma_2 = 5/0.5 = 10$ ksi

$$\Delta = \left[\frac{5}{16000} + \left(\frac{5}{165}\right)^3\right] \times 50 + \left[\frac{10}{16000} + \left(\frac{10}{165}\right)^3\right] \times 100$$

$= 0.102''$

(b) $$d\epsilon = \frac{1}{16000} d\sigma + \frac{3}{165^2} \sigma^2 d\sigma$$

$$E = \frac{d\sigma}{d\epsilon}\Big|_{\sigma=0} = 16000 \text{ ksi}$$

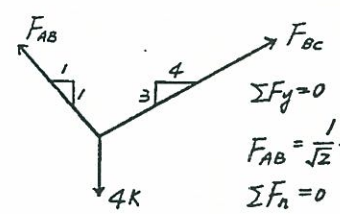
$$\Delta_E = \frac{5}{E} \times 50 + \frac{10}{E} \times 100$$

$$= \frac{5}{16000} \times 50 + \frac{10}{16000} \times 100 = 0.078''$$

$\Delta_{residual} = \Delta - \Delta_E$

$= 0.102 - 0.078 = 0.024''$

3-23



$\Sigma F_y = 0$

$F_{AB} \frac{1}{\sqrt{2}} + F_{BC} \frac{3}{5} = 20k$

$\Sigma F_n = 0$

$F_{AB}\left(\frac{1}{\sqrt{2}}\right) = F_{BC}\left(\frac{4}{5}\right)$

Solve: $\Rightarrow$ $F_{BC} = 14.3$ K

$F_{AB} = 16.15$ K

wire AB

$$\sigma_{0 \to 80}\ ; \quad \Delta = \frac{PL}{AE} = \frac{80 \times 130 (900 \times \sqrt{2})}{130 (80 \times 10^3)} = 1.273 \text{ mm}$$

$\sigma_{80\to}$; $\Delta = \dfrac{PL}{AE} = \dfrac{(14300-10400)(900\sqrt{2})}{130\,(40\times10^3)}$

$= 0.955$ mm

$\Delta_{Total} = 1.27 + 0.96 = 2.23$ mm

Wire BC

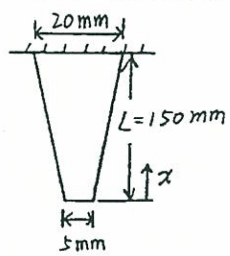
$\sigma_{0\to80}$ $\Delta = \dfrac{PL}{AE} = \dfrac{80\times100\,(3.05\times10^3)}{100\,(80\times10^3)} = 3.05$ mm

$80\to$ $\Delta = \dfrac{PL}{AE} = \dfrac{(16150-8000)(3.05\times10^3)}{100\,(40\times10^3)}$

$= 6.21$ mm

$\Delta_{Total} = 9.26$ mm

3-24



$A_x = 4\times[5 + \frac{(20-5)}{150}x] = 4(5 + \frac{1}{10}x)$

$P_x = \gamma g \times \frac{1}{2}\times 4\,(10 + \frac{1}{10}x)\,x$

$= \gamma g\,(20 + \frac{1}{5}x)\,x$

$\Delta = \int_0^{150} \frac{P_x}{A_x E}\,dx = \int_0^{150} \frac{\gamma g (20 + \frac{1}{5}x)x}{4(5+\frac{1}{10}x)E}\,dx$

$= \int_0^{150} \frac{\gamma g (100 + x)x}{(100+2x)E}\,dx$

$= 7637.5\,\frac{\gamma g}{E}$

3-25

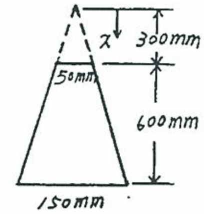
$\Delta_A = \int_0^{L_A} \frac{P}{2E}\,dx = \frac{PL_A}{2E}$

$\Delta_B = \int_0^{L_B} \frac{P}{(1+\frac{2x}{L_B})}\,dx = \frac{PL_B}{2E}\ln 3$

$\Delta_A = \Delta_B$, $\frac{PL_A}{2E} = \frac{PL_B}{2E}\ln 3$

$\frac{L_A}{L_B} = \ln 3 = 1.10$

3-26



$d = \frac{x}{12}$

$A = \pi d^2 = \pi (\frac{x}{12})^2$

$P = [\frac{\pi}{3}(\frac{x}{12})^2 x - \frac{\pi}{3}\times 25^2 \times 300]\,\gamma$

$= \frac{\pi\gamma}{3}(\frac{x^3}{144} - 187500)$

$\Delta = \int_{300}^{900} \frac{P}{AE}\,dx = \int_{300}^{900} \frac{\pi\gamma}{3}\,\frac{(\frac{x^3}{144} - 187500)}{\pi(\frac{x}{12})^2 E}\,dx$

$= \frac{\gamma}{3E}\int_{300}^{900} \frac{x^3 - 27\times10^6}{x^2}\,dx$

$= \frac{\gamma}{3E}[\frac{1}{2}x^2 + 27\times10^6\,\frac{1}{x}]\Big|_{300}^{900}$

$= \frac{\gamma}{3E}(36\times10^4 - 6\times10^4) = 10^5\,\frac{\gamma}{E}$

3-27

$P = \int_r^L (\frac{\gamma A\,dr}{g})\,\omega^2 r = \frac{\gamma A \omega^2}{2g}(L^2 - r^2)$

$\Delta = \int_0^L \frac{2P\,dr}{AE} = 2\int_0^L \frac{\gamma A \omega^2}{2g}(L^2 - r^2)\frac{dr}{AE}$

$= \frac{\gamma\omega^2}{gE}[L^2 r - \frac{r^3}{3}]\Big|_0^L = \frac{2\gamma\omega^2 L^3}{3gE}$

3-28

$F = \int_a^x -kx'\,dx' = -\frac{1}{2}kx^2\Big|_a^x$

$= -\frac{k}{2}(x^2 - a^2)$

$\Delta = \int_a^{2a} \frac{F\,dx}{AE} = \int_a^{2a} \frac{-k}{2EA}(x^2 - a^2)\,dx$

$= \frac{-k}{2AE}(\frac{x^3}{3} - a^2 x)\Big|_a^{2a}$

$= -\frac{k}{2AE}[\frac{1}{3}(8a^3 - a^3) - a^3] = \frac{-2ka^3}{3EA}$

$P = \int_a^{2a} kx\,dx = \frac{kx^2}{2}\Big|_a^{2a} = \frac{k}{2}(4a^2 - a^2)$

$= \frac{3ka^2}{2} \Rightarrow k = \frac{2P}{3a^2}$

$\therefore \Delta = -\frac{2a^3}{3EA}\times\frac{2P}{3a^2} = -\frac{4Pa}{9EA}$

3-29

$$\begin{cases} F_{AB}\sin 26.6^\circ = F_{BC}\sin 45^\circ \\ F_{AB}\cos 26.6^\circ + F_{BC}\cos 45^\circ = 3 \end{cases} \Longrightarrow \begin{array}{l} F_{AB} = 2.24 \text{ kips} \\ F_{BC} = 1.42 \text{ kips} \end{array}$$

$$\Delta_{AB} = \frac{2.24 \times 6.71}{0.25 \times 0.5 \times 10.6 \times 10^3} = 0.0113 \text{ in}$$

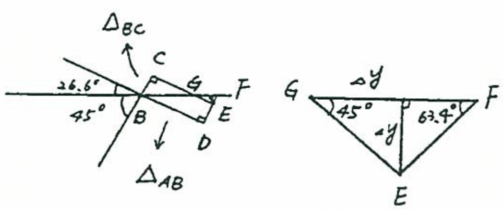
$$\Delta_{BC} = \frac{1.42 \times 8.49}{0.25 \times 0.875 \times 10.6 \times 10^3} = 5.20 \times 10^{-3} \text{ in}$$

$$GF = \frac{\Delta AB}{\cos 26.6^\circ} - \frac{\Delta_{BC}}{\cos 45^\circ} = 5.28 \times 10^{-3} \text{ in}$$

$$GF = \Delta y + \Delta y / \tan 63.4^\circ = 5.28 \times 10^{-3} \text{ in}$$

$$\therefore \Delta y = 3.52 \times 10^{-3} \text{ in}$$

$$\Delta x = \frac{\Delta_{BC}}{\cos 45^\circ} + \Delta y = 0.0109 \text{ in}$$



3-30

$$\Sigma F = 0, \quad \frac{1}{\sqrt{2}} F_{AB} + \frac{\sqrt{3}}{2} F_{BC} = 10 \qquad F_{AB} = 5.177 \text{ k}$$

$$\frac{1}{\sqrt{2}} F_{AB} = \frac{1}{2} F_{BC} \qquad \Longrightarrow \qquad F_{BC} = 7.321 \text{ k}$$

$$\Delta_{AB} = \left(\frac{PL}{AE}\right)_{AB} = \frac{5.177 \times 100\sqrt{2}}{10^4} = 0.07321''$$

$$\Delta_{BC} = \left(\frac{PL}{AE}\right)_{BC} = \frac{7.321 \times 100 \times \frac{2}{\sqrt{3}}}{10^4} = 0.08454''$$

$$EF = \sqrt{2} \times \Delta_{AB} - \frac{2}{\sqrt{3}} \times \Delta_{AB}$$

$$= \sqrt{2} \times 0.07321 - \frac{2}{\sqrt{3}} \times 0.08454$$

$$= 0.00592$$

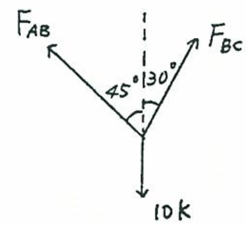
$$EF = \frac{\Delta x}{\sqrt{3}} + \Delta x = 0.00592$$

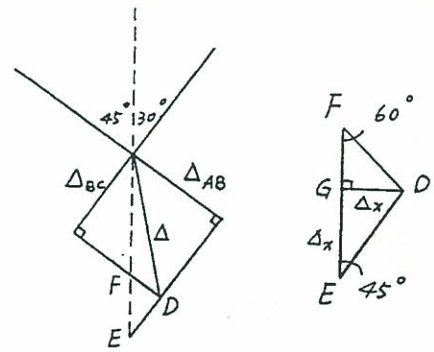
$$\Delta x = 0.00375 \text{ in}$$

$$\Delta y = \sqrt{2} \times \Delta_{AB} - \Delta x$$

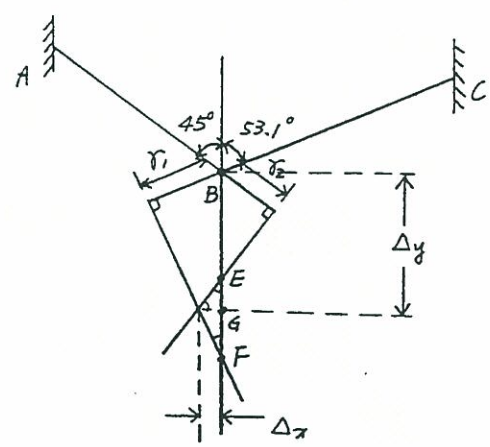
$$= \sqrt{2} \times 0.007321 - 0.00375$$

$$= 0.09978 \text{ in}$$





3-31



From Prob. 3-23

$$\gamma_1 = \frac{(14.3 \times 10^3) \times (3.05 \times 10^3)}{100 \times 80 \times 10^3} = 5.45 \text{ mm}$$

$$\gamma_2 = \frac{(16.15 \times 10^3) \times (0.9\sqrt{2} \times 10^3)}{130 \times 80 \times 10^3} = 1.98 \text{ mm}$$

$$BF = BE + \Delta_x \tan 45° + \frac{\Delta_x}{\tan 36.9°}$$

where $BF = \frac{\gamma_1}{\cos 53.1°}$, $BE = \frac{\gamma_2}{\cos 45°}$

$$\Delta_x = \frac{\frac{\gamma_1}{\cos 53.1°} - \frac{\gamma_2}{\cos 45°}}{\tan 45° + \frac{1}{\tan 36.9°}} = 2.69\ mm$$

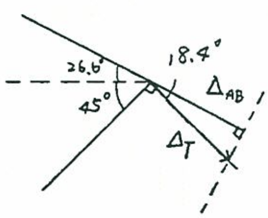
$$\Delta_y = \Delta_x + BE = 5.49\ mm$$

3-32

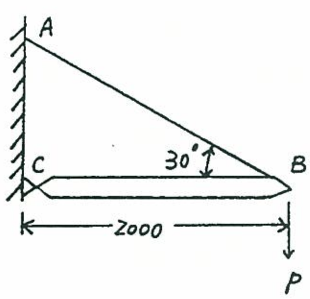
$\Delta_{AB} = 8.656 \times 10^{-3}$ in

$\Delta_{BC} = 0$

$$\Delta_T = \frac{\Delta_{AB}}{\cos 18.4°} = \frac{8.656 \times 10^{-3}}{\cos 18.4°} = 9.122 \times 10^{-3}\ in$$



3-33



(a) crane stiffness

$P = 1$ (unit force)

$P_{AB} \times \frac{1}{2} = 1$

$\Rightarrow P_{AB} = 2$ (tensile)

$P_{BC} = P_{AB} \times \frac{\sqrt{3}}{2} = 1.732$ (compressive)

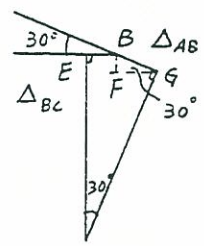
$$\Delta_{AB} = \frac{PL}{AE} = \frac{2(2000 \times \frac{2}{\sqrt{3}})}{3 \times 10^{-4}(200 \times 10^9)} = 7.70 \times 10^{-5}\ mm$$

$$\Delta_{BC} = \frac{1.732 \times 2000}{(3.2 \times 10^{-4})(200 \times 10^9)} = 5.41 \times 10^{-5}\ mm$$

$$EG = \Delta_{BC} + \frac{\sqrt{3}}{2}\Delta_{AB} = 1.21 \times 10^{-4}\ mm$$

$$x = EG \times \sqrt{3} + \frac{1}{2}\Delta_{AB} = 2.48 \times 10^{-4}\ mm$$

$$K = \frac{P}{\Delta} = \frac{1}{2.48 \times 10^{-7}} = 4.0 \times 10^6\ N/m$$



(b) Deflection from 16 kN

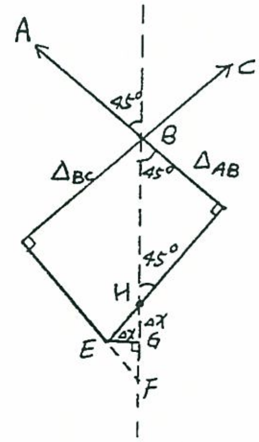
$$\Delta = \frac{P}{K} = \frac{16 \times 10^3}{4.0 \times 10^6} = 4.0 \times 10^{-3}\ m = 4.0\ mm$$

3-34

$$\Delta = \frac{mg}{K}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.0 \times 10^6}{2000}} = 7.12\ Hz$$

3-35



$\alpha = 13.0 \times 10^{-6}$ per °F

$\Delta_{AB} = 100 \times 13 \times 10^{-6} \times 900\sqrt{2}$

$= 1.655$ mm

$\Delta_{BC} = 100 \times 13 \times 10^{-6} \times 3050$

$= 3.965$ mm

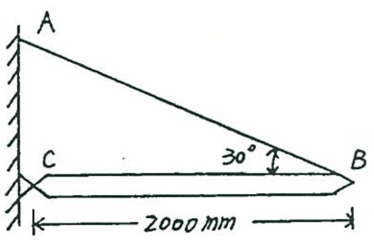
$HF = \frac{5}{3}\Delta_{BC} - \sqrt{2}\Delta_{AB} = 4.268$ mm

$HF = \Delta x + \Delta x \times \frac{4}{3} = 4.268$ mm

$\Delta x = 1.829$ mm

$\Delta y = \sqrt{2}\Delta_{AB} + \Delta x = 4.170$ mm

3-36



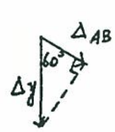
$\Delta_{BC} = 0$

$\Delta_{AB} = 11.7 \times 10^{-6}\,(200 \times \frac{2}{\sqrt{3}})(80°)$

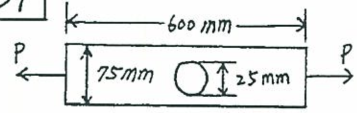
$= 2.162$ mm

$\Delta x = \Delta_{BC} = 0$

$\Delta y = \frac{\Delta_{AB}}{\cos 60°} = 4.324$ mm



3-37



from graph on fig. 3-11

$K = 2.18$

$$P = \frac{\sigma A}{K} = \frac{0.22(75-25)\times 6}{2.18} = 30.28\ KN$$

3-38

from graph on Fig 3-11,

$K = 1.75$

$$\frac{\sigma}{\sigma_{max}} = \frac{\frac{P}{A}}{K\frac{P}{A}} = \frac{1}{K} = \frac{1}{1.75} = 0.57$$

3-39

$\frac{r_1}{d} = \frac{8}{40} = 0.20, \quad K_1 = 1.63$

$\Rightarrow P_1 = \frac{40 \times 10 \times 60}{1.63} = 14.7\ kN$

$\frac{r_2}{d} = \frac{12}{60} = 0.20, \quad K_2 = 2.30$

$\Rightarrow P_2 = \frac{(60-24)\times 10 \times 60}{2.30} = 9.4\ KN$

3-40

small bar $\frac{r}{d} = \frac{6}{20} = 0.3, \quad k_1 = 1.53$

bend $\frac{r}{d} = \frac{20}{35} = 0.57, \quad k_2 = 1.37$

pin hole $\frac{r}{d} = \frac{10}{50} = 0.2, \quad k_3 = 2.3$

(a)

$P_1 = \frac{80}{1.53} \times 20\,t_1 = 12000$

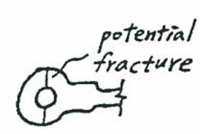
$\Rightarrow t_1 = 11.5$ mm

$P_2 = \frac{80}{1.37} \times 35\,t_2 = 12000$

$\Rightarrow t_2 = 5.9$ mm

$P_3 = \frac{80}{2.3} \times 30\,t_3 = 12000$

$\Rightarrow t_3 = 11.5$ mm



(b) The potential fracture might occur in the pin hole section.

3-41

Number of cycles : 10×10^6

$\Rightarrow$ stress amplitude 185 MPa

$$\frac{r_1}{d}=\frac{2}{10}=0.2 \quad \Rightarrow \quad K_1=1.63$$

$$\Rightarrow \sigma_{1max}=1.63\times185=301.55\ MPa$$

or $301.55\times10\ mm^2=3015.5\ N$
$=3.0155\ kN$

$$\frac{r_2}{d}=\frac{4}{20}=0.2 \quad \Rightarrow \quad K_2=2.30$$

$$\Rightarrow \sigma_{2max}=2.30\times185=425.5\ MPa$$

or $425.5\times(20-8)\times1=5106\ N$
$=5.106\ kN$

3-42

(a) from graph on Fig. 3-11,

$$\frac{r}{D}=\frac{10}{60}=\frac{1}{6}, \qquad K=2.35$$

$$\sigma_{max}=K\frac{P}{A}=2.35\times\frac{300}{60\times10}=1.175\ GPa$$

(b) $$\Delta=\sum\frac{PL}{AE}=\frac{300\times120}{200\times60\times10}+\frac{300\times120}{200\times40\times10}$$
$=0.75\ mm$

(c) $$\Delta=0.02\times120+\frac{350\times120}{200\times60\times10}=2.75\ mm$$

(d) $$\Delta_{res}=\Delta-\Delta_E$$
$$=2.75-\left(\frac{350\times120}{200\times60\times10}+\frac{350\times120}{200\times40\times10}\right)$$
$$=2.75-0.875=1.875\ mm$$

3-43

for aluminum,

$$U_{r,\,elastic}=\frac{44^2}{2\times10.6}=91.32\ psi$$

$$U_{r,\,hyper}=\frac{60^2}{2\times10.6}=169.81\ psi$$

for magnesium, $$U_{r,elastic}=\frac{22^2}{2\times6.5}=37.23\ psi$$

$$U_{r,hyper}=\frac{40^2}{2\times6.5}=123.08\ psi$$

for steel, $$U_{r,\,elastic}=\frac{36^2}{2\times30}=21.6\ psi$$

$$U_{r,\,hyper}=\frac{65^2}{2\times30}=325\ psi$$

3-44

for bar 12,

$$u\simeq\frac{18.33\times0.5}{\frac{\pi}{4}\times0.505^2\times0.5}=91.51\ ksi$$

for bar 15,

$$u\simeq\frac{18.33\times1}{\frac{\pi}{4}\times0.505\times3.5}=26.15\ ksi$$

3-45 from example 3-4

$\sigma_{AB}=17.8\ ksi$, $P_{AB}=2.23\ kips$

$L_{AB}=6.71''$

$\sigma_{BC}=12.9\ ksi$, $P_{BC}=2.83\ kips$

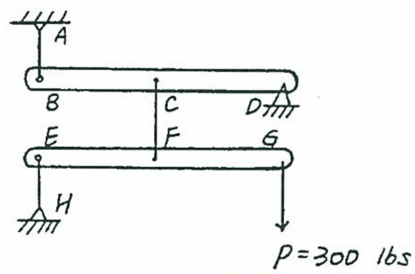
$L_{BC}=8.49''$

$$\frac{P\Delta}{2}=\sum\frac{\sigma^2}{2E}AL=\sum\frac{\sigma PL}{2E}$$

$$\Delta=\frac{1}{3\times10.6\times10^3}[17.8\times2.23\times6.71+12.9\times2.83\times8.49]$$

$=0.018''$

3-46



Forces: $\Sigma M_F = 0 \Rightarrow F_{EH} = 300\ lbs$

$\Sigma M_E = 0 \Rightarrow F_{CF} = 600\ lbs$

$A_{CF} = \frac{F_{CF}}{\sigma} = \frac{600}{15\times10^3} = 0.04\ in^2$

$\Sigma M_D = 0 \Rightarrow F_{AB} = 300\ lbs$

$A_{AB} = \frac{F_{AB}}{\sigma} = \frac{300}{15\times10^3} = 0.02\ in^2$

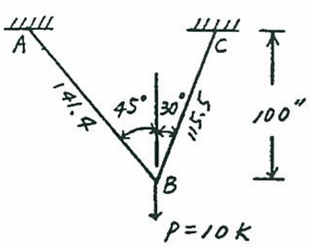
$$U = \frac{1}{2}\left[\left(\frac{P^2L}{AE}\right)_{AB} + \left(\frac{P^2L}{AE}\right)_{CF} + \left(\frac{P^2L}{AE}\right)_{EH}\right]$$

$$= \frac{1}{2}P\Delta$$

$$P\Delta = \left[\frac{0.3^2\times20}{30\times10^3\times0.02} + \frac{0.3^2\times10}{30\times10^3\times0.02} + \frac{0.6^2\times20}{30\times10^3\times0.04}\right]$$

$\therefore \Delta = 0.035\ in$

3-47



$\Sigma F_y = 0 \Rightarrow F_{AB}\cos45° + F_{CB}\cos30° = 10K$

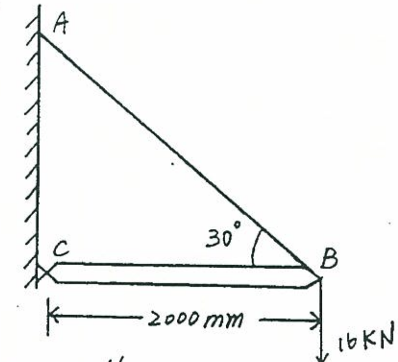
$\Sigma F_x = 0 \Rightarrow F_{AB}\sin45° + F_{CB}\sin30° = 0$

solve, $F_{AB} = 5.177$, $F_{CB} = 7.231$

$$u = \frac{1}{2}\frac{(5.177)^2\,141.4}{10^4} + \frac{1}{2}\frac{(7.321)^2\,115.5}{10^4} = \frac{10}{2}\Delta$$

$\Rightarrow \Delta = 0.0998\ in$

3-48



$P_{AB} = \frac{16}{\sin30°} = 32\ KN$

$P_{BC} = 27.712\ KN$

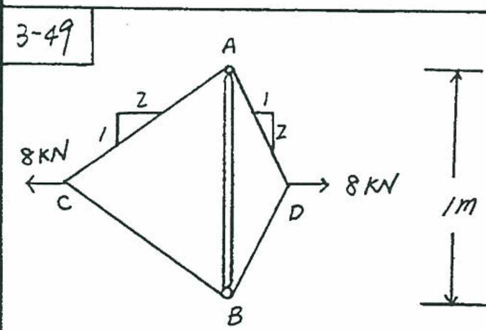
$$\frac{1}{2}P\Delta = \frac{1}{2}\frac{P_{AB}^2 L_{AB}}{A_{AB}E} + \frac{1}{2}\frac{P_{BC}^2 L_{BC}}{A_{BC}E}$$

$$\Rightarrow \frac{(32\times10^3)^2(2.31)}{(300\times10^{-6})(200\times10^9)} + \frac{(27.712\times10^3)^2(2.0)}{(320\times10^{-6})(200\times10^9)}$$

$$= 16\times10^3\,\Delta$$

$\Rightarrow \Delta = 0.004m = 4mm$

3-49



$T_{AD} = T_{DB} = 8.94\ KN$

$T_{CB} = T_{CA} = 4.47\ KN$

$T_{AB} = 8KN + 2KN = 10\ KN$

$$\frac{1}{2}\times2\times\frac{(4.47\times1000)^2\,1.118}{20\times200\times1000} + \frac{1}{2}\times2\,\frac{(8.94\times1000)^2\times0.559}{40\times200\times1000} + \frac{1}{2}\,\frac{(10\times1000)^2\times1}{10\times200\times1000} = \frac{1}{2}\times8.0\Delta$$

$\Delta = 3.42\ mm$

3-50

static force $= mg = 1.5 \times 9.81$
$= 14.715$

Bar 'A' $\Rightarrow \Delta_{ST} = \frac{PL}{AE} = \frac{14.7(2000)}{5^2 \times \pi (200 \times 10^3)}$

$= 0.0019$ mm

$$\sigma_{max} = \frac{14.7}{5^2 \pi}\left(1 + \sqrt{1 + \frac{2 \times 1000}{0.0019}}\right) = 192.3 \text{ MPa}$$

Bar 'B' $\Rightarrow \Delta_{ST} = \frac{PL}{AE} = \frac{14.7 \times 2000}{(\frac{15}{2})^2 \pi (200 \times 10^3)}$

$= 0.0008$ mm

$$\sigma_{max} = \frac{14.7}{\frac{15^2}{4}\pi}\left(1 + \sqrt{1 + \frac{2000}{0.0008}}\right) = 131.6 \text{ MPa}$$

Bar 'C' $\Rightarrow \Delta_{ST} = \frac{14.7(1000)}{78.5(200 \times 10^3)} + \frac{14.7(1000)}{\frac{15^2}{4}\pi (200 \times 10^3)}$

$= 1.35 \times 10^{-3}$ mm

for lower rod

$$\sigma_{max} = \frac{14.7}{78.5}\left(1 + \sqrt{1 + \frac{2000}{1.35 \times 10^{-3}}}\right) = 228.1 \text{ MPa}$$

for upper rod

$$\sigma_{max} = \frac{78.5}{176.7} \times 228.1 = 101.3 \text{ MPa}$$

3-51

$\Delta = 200$ mm

$u = \frac{1}{2} k \Delta^2 = \frac{1}{2} m v^2$

$$\Rightarrow k = \frac{mv^2}{\Delta^2} = \frac{(1 \text{ kg}) 3^2}{(0.02 m)^2}$$

$= 45000$ N/m

$= 45$ kN/m