Solutions Manual

Engineering Mechanics: Statics

2nd Edition

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Important Information about this Solutions Manual

We encourage you to occasionally visit

`http://www.mhhe.com/pgc2e`

to obtain the most up-to-date version of this solutions manual.

Contact the Authors

If you find any errors and/or have questions concerning a solution, please do not hesitate to contact the authors and editors via email at:

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`stat_solns@email.esm.psu.edu`

We welcome your input.

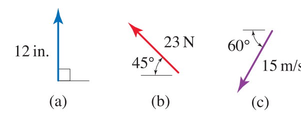
Accuracy of Numbers in Calculations

Throughout this solutions manual, we will generally assume that the data given for problems is accurate to 3 significant digits. When calculations are performed, all intermediate numerical results are reported to 4 significant digits. Final answers are usually reported with 3 or 4 significant digits. If you verify the calculations in this solutions manual using the rounded intermediate numerical results that are reported, you should obtain the final answers that are reported to 3 significant digits.

Chapter 2 Solutions

Problem 2.1

For each vector, write two expressions using polar vector representations, one using a positive value of θ and the other a negative value, where θ is measured counterclockwise from the right-hand horizontal direction.



Solution

Part (a)

$$\boxed{\vec{r} = 12\,\text{in.}\ @\ 90^\circ\measuredangle \quad \text{or} \quad \vec{r} = 12\,\text{in.}\ @\ -270^\circ\measuredangle.} \tag{1}$$

Part (b)

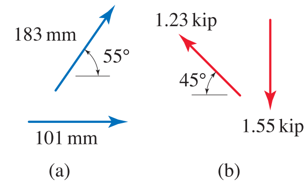
$$\boxed{\vec{F} = 23\,\text{N}\ @\ 135^\circ\measuredangle \quad \text{or} \quad \vec{F} = 23\,\text{N}\ @\ -225^\circ\measuredangle.} \tag{2}$$

Part (c)

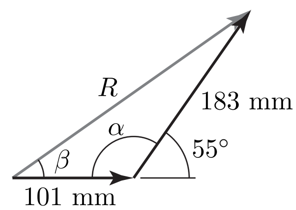
$$\boxed{\vec{v} = 15\,\text{m/s}\ @\ 240^\circ\measuredangle \quad \text{or} \quad \vec{v} = 15\,\text{m/s}\ @\ -120^\circ\measuredangle.} \tag{3}$$

Problem 2.2

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. Note that α is given by $\alpha = 180° - 55° = 125°$. Knowing this angle, the law of cosines may be used to determine R

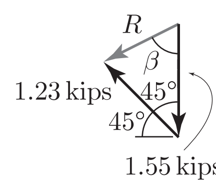
$$R = \sqrt{(101\,\text{mm})^2 + (183\,\text{mm})^2 - 2(101\,\text{mm})(183\,\text{mm})\cos 125°} = 254.7\,\text{mm}. \tag{1}$$

Next, the law of sines may be used to determine the angle β:

$$\frac{R}{\sin\alpha} = \frac{183\,\text{mm}}{\sin\beta} \quad\Rightarrow\quad \beta = \sin^{-1}\left(\frac{183\,\text{mm}}{254.7\,\text{mm}}\sin 125°\right) = 36.05°. \tag{2}$$

Using these results, we may report the vector $\vec{R}$ using polar vector representation as

$$\boxed{\vec{R} = 255\,\text{mm} \ @\ 36.0° \measuredangle.} \tag{3}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. The law of cosines may be used to determine R

$$R = \sqrt{(1.23\,\text{kip})^2 + (1.55\,\text{kip})^2 - 2(1.23\,\text{kip})(1.55\,\text{kip})\cos 45°} = 1.104\,\text{kip}. \tag{4}$$

Using the law of sines, we find that

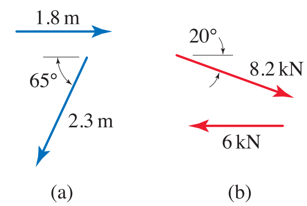
$$\frac{R}{\sin 45°} = \frac{1.23\,\text{kip}}{\sin\beta} \quad\Rightarrow\quad \beta = \sin^{-1}\left(\frac{1.23\,\text{kip}}{1.104\,\text{kip}}\sin 45°\right) = 51.97°. \tag{5}$$

The direction of $\vec{R}$ measured from the right-hand horizontal direction is $-90° - 51.97° = -142°$. Using these results, we may report $\vec{R}$ using polar vector representation as

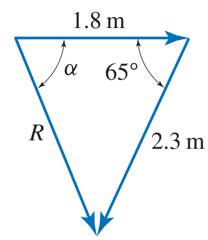
$$\boxed{\vec{R} = 1.10\,\text{kip} \ @\ -142° \measuredangle.} \tag{6}$$

Problem 2.3

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. The law of cosines may be used to determine R as

$$\begin{aligned} R &= \sqrt{(1.8\,\text{m})^2 + (2.3\,\text{m})^2 - 2(1.8\,\text{m})(2.3\,\text{m})\cos 65^\circ} \\ &= 2.243\,\text{m}. \end{aligned} \tag{1}$$

The law of sines may be used to determine the angle α as

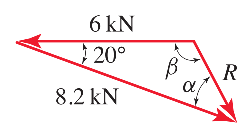
$$\frac{R}{\sin 65^\circ} = \frac{2.3\,\text{m}}{\sin\alpha} \quad \Rightarrow \quad \alpha = \sin^{-1}\frac{2.3\,\text{m}}{2.243\,\text{m}}\sin 65^\circ = 68.34^\circ. \tag{2}$$

Using polar vector representation, the resultant is

$$\boxed{\vec{R} = 2.243\,\text{m} \; @ \; -68.34^\circ \measuredangle.} \tag{3}$$

If desired, this resultant may be stated using a positive angle, where $360^\circ - 68.34^\circ = 291.7^\circ$, as

$$\boxed{\vec{R} = 2.243\,\text{m} \; @ \; 291.7^\circ \measuredangle.} \tag{4}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. The law of cosines may be used to determine R as

$$\begin{aligned} R &= \sqrt{(6\,\text{kN})^2 + (8.2\,\text{kN})^2 - 2(6\,\text{kN})(8.2\,\text{kN})\cos 20^\circ} \\ &= 3.282\,\text{kN}. \end{aligned} \tag{5}$$

Noting that β appears to be an obtuse angle (see the Common Pitfall margin note in the text), we will use the law of sines to determine α as

$$\frac{6\,\text{kN}}{\sin\alpha} = \frac{R}{\sin 20^\circ} \quad \Rightarrow \quad \alpha = \sin^{-1}\left(\frac{6\,\text{kN}}{3.282\,\text{kN}}\sin 20^\circ\right) = 38.70^\circ. \tag{6}$$

Angle β is obtained using

$$20^\circ + \alpha + \beta = 180^\circ, \tag{7}$$

$$\beta = 180^\circ - 20^\circ - 38.70^\circ = 121.3^\circ. \tag{8}$$

Using polar vector representation, the resultant is

$$\vec{R} = 3.282\,\text{kN @ } -(180^\circ - 121.3^\circ) \measuredangle \tag{9}$$

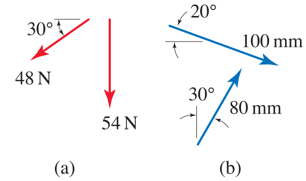
$$\boxed{\vec{R} = 3.282\,\text{kN @ } -58.70^\circ \measuredangle.} \tag{10}$$

If desired, the resultant may be stated using a positive angle, where $360^\circ - 58.70^\circ = 301.3^\circ$, as

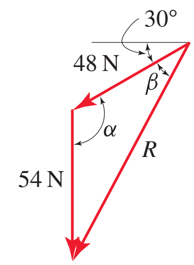
$$\boxed{\vec{R} = 3.282\,\text{kN @ } 301.3^\circ \measuredangle.} \tag{11}$$

Problem 2.4

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. Since the 54 N force is vertical, the angle α may be obtained by inspection as $\alpha = 90° + 30° = 120°$. The law of cosines may be used to determine R as

$$R = \sqrt{(48\,\text{N})^2 + (54\,\text{N})^2 - 2(48\,\text{N})(54\,\text{N})\cos 120°} = 88.39\,\text{N}. \tag{1}$$

The law of sines may be used to determine the angle β as

$$\frac{54\,\text{N}}{\sin\beta} = \frac{R}{\sin\alpha} \quad \Rightarrow \quad \beta = \sin^{-1}\frac{54\,\text{N}}{88.39\,\text{N}}\sin 120° = 31.95°. \tag{2}$$

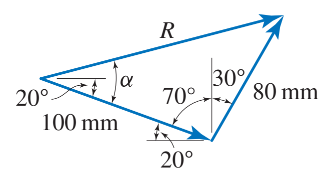
Using polar vector representation, the resultant is

$$\vec{R} = 88.39\,\text{N} \;@\; -(180° - 30° - \beta) \measuredangle \tag{3}$$

$$\boxed{\vec{R} = 88.39\,\text{N} \;@\; -118.1° \measuredangle.} \tag{4}$$

If desired, this resultant may be stated using a positive angle, where $360° - 118.1° = 241.9°$, as

$$\boxed{\vec{R} = 88.39\,\text{N} \;@\; 241.9° \measuredangle.} \tag{5}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. Given the 20° and 30° angles provided in the problem statement, we determine the angle opposite R to be $70° + 30° = 100°$. The law of cosines may be used to determine R as

$$R = \sqrt{(100\,\text{mm})^2 + (80\,\text{mm})^2 - 2(100\,\text{mm})(80\,\text{mm})\cos 100°} = 138.5\,\text{mm}. \tag{6}$$

The law of sines may be used to determine the angle α as

$$\frac{80\,\text{mm}}{\sin\alpha} = \frac{R}{\sin 100^\circ} \quad \Rightarrow \quad \alpha = \sin^{-1}\frac{80\,\text{mm}}{138.5\,\text{mm}}\sin 100^\circ = 34.67^\circ. \tag{7}$$

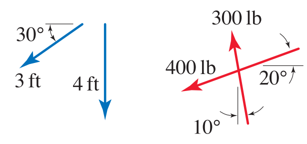
Using polar vector representation, the resultant is

$$\vec{R} = 138.5\,\text{mm} \;@\; 34.67^\circ - 20^\circ \measuredangle \tag{8}$$

$$\boxed{\vec{R} = 138.5\,\text{mm} \;@\; 14.67^\circ \measuredangle.} \tag{9}$$

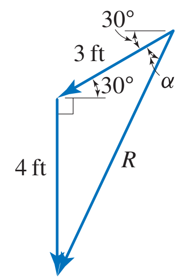
Problem 2.5

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



(a) (b)

Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain the resultant position vector $\vec{R}$. The law of cosines may be used to determine R as

$$\begin{aligned} R &= \sqrt{(3\,\text{ft})^2 + (4\,\text{ft})^2 - 2(3\,\text{ft})(4\,\text{ft})\cos 120°} \\ &= 6.083\,\text{ft}. \end{aligned} \tag{1}$$

The law of sines may be used to determine the angle α as

$$\frac{4\,\text{ft}}{\sin\alpha} = \frac{R}{\sin 120°} \quad\Rightarrow\quad \alpha = \sin^{-1}\frac{4\,\text{ft}}{6.083\,\text{ft}}\sin 120° = 34.72°. \tag{2}$$

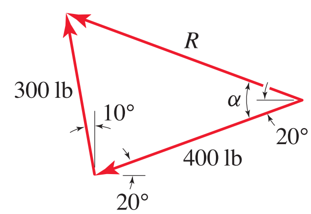
Using polar vector representation, the resultant is

$$\vec{R} = 6.083\,\text{ft} \;@\; -(180° - 30° - \alpha)\measuredangle \tag{3}$$

$$\boxed{\vec{R} = 6.083\,\text{ft} \;@\; -115.3°\measuredangle.} \tag{4}$$

If desired, this resultant may be stated using a positive angle, where $360° - 115.3° = 244.7°$, as

$$\boxed{\vec{R} = 6.083\,\text{ft} \;@\; 244.7°\measuredangle.} \tag{5}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain the resultant force vector $\vec{R}$. Given the 10° and 20° angles provided in the problem statement, we determine the angle opposite R to be $10° + 90° - 20° = 80°$. The law of cosines may be used to determine R as

$$\begin{aligned} R &= \sqrt{(300\,\text{lb})^2 + (400\,\text{lb})^2 - 2(300\,\text{lb})(400\,\text{lb})\cos 80°} \\ &= 456.4\,\text{lb}. \end{aligned} \tag{6}$$

The law of sines may be used to determine the angle α as

$$\frac{300\,\text{lb}}{\sin\alpha} = \frac{R}{\sin 80°} \quad\Rightarrow\quad \alpha = \sin^{-1}\frac{300\,\text{lb}}{456.4\,\text{lb}}\sin 80° = 40.34°. \tag{7}$$

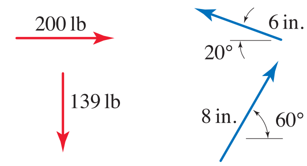
Using polar vector representation, the resultant is

$$\vec{R} = 456.4\,\text{lb} \;@\; 180° + 20° - \alpha\measuredangle \tag{8}$$

$$\boxed{\vec{R} = 456.4\,\text{lb} \;@\; 159.7°\measuredangle.} \tag{9}$$

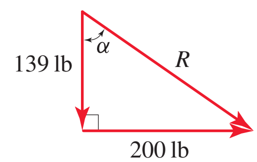
Problem 2.6

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



(a) (b)

Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. Since the two forces being added are perpendicular, basic trigonometry may be used to obtain R and α as

$$R = \sqrt{(139\,\text{lb})^2 + (200\,\text{lb})^2} = 243.6\,\text{lb}, \tag{1}$$

$$\alpha = \tan^{-1}\frac{200\,\text{lb}}{139\,\text{lb}} = 55.20°. \tag{2}$$

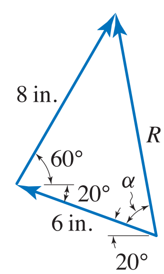
Using polar vector representation, the resultant is

$$\vec{R} = 243.6\,\text{lb} \; @ \; -(90° - 55.20°) \measuredangle \tag{3}$$

$$\boxed{\vec{R} = 243.6\,\text{lb} \; @ \; -34.80° \measuredangle.} \tag{4}$$

If desired, this resultant may be stated using a positive angle, where $360° - 34.80° = 325.2°$, as

$$\boxed{\vec{R} = 243.6\,\text{lb} \; @ \; 325.2° \measuredangle.} \tag{5}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. The law of cosines may be used to determine R as

$$\begin{aligned} R &= \sqrt{(6\,\text{in.})^2 + (8\,\text{in.})^2 - 2(6\,\text{in.})(8\,\text{in.})\cos 80°} \\ &= 9.129\,\text{in.} \end{aligned} \tag{6}$$

The law of sines may be used to determine the angle α as

$$\frac{8\,\text{in.}}{\sin\alpha} = \frac{R}{\sin 80°} \quad \Rightarrow \quad \alpha = \sin^{-1}\frac{8\,\text{in.}}{9.129\,\text{in.}}\sin 80° = 59.66°. \tag{7}$$

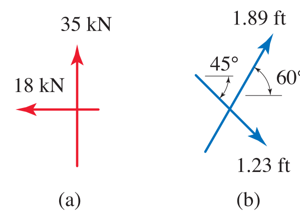
Using polar vector representation, the resultant is

$$\vec{R} = 9.129\,\text{in.} \; @ \; 180° - 20° - \alpha \measuredangle \tag{8}$$

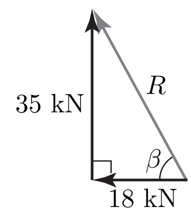
$$\boxed{\vec{R} = 9.129\,\text{in.} \; @ \; 100.3° \measuredangle.} \tag{9}$$

Problem 2.7

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution

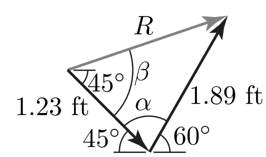


Part (a) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. Using this vector polygon, we determine R and β as

$$R = \sqrt{(35\,\text{kN})^2 + (18\,\text{kN})^2} = 39.36\,\text{kN}, \quad \beta = \tan^{-1}\left(\frac{35\,\text{kN}}{18\,\text{kN}}\right) = 62.78°. \qquad (1)$$

The direction for $\vec{R}$ measured from the right-hand horizontal direction is $180° - 62.78° = 117.2°$. Therefore, the polar vector representation for $\vec{R}$ is

$$\boxed{\vec{R} = 39.4\,\text{kN} \text{ @ } 117°\measuredangle.} \qquad (2)$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. We observe from this vector polygon that $\alpha = 180° - 60° - 45° = 75°$. Using the law of cosines

$$R = \sqrt{(1.23\,\text{ft})^2 + (1.89\,\text{ft})^2 - 2(1.23\,\text{ft})(1.89\,\text{ft})\cos 75°} = 1.970\,\text{ft}. \qquad (3)$$

Next, use the law of sines to find β, such that

$$\frac{1.89\,\text{ft}}{\sin\beta} = \frac{R}{\sin\alpha} \quad \Rightarrow \quad \beta = \sin^{-1}\left(\frac{1.89\,\text{ft}}{1.970\,\text{ft}}\sin 75°\right) = 67.91°. \qquad (4)$$

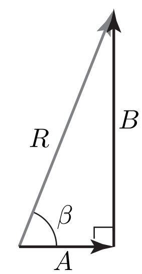
The direction of $\vec{R}$ measured from the right-hand horizontal direction is $67.91° - 45° = 22.91°$. Therefore, the polar vector representation of $\vec{R}$ is

$$\boxed{\vec{R} = 1.97\,\text{ft} \text{ @ } 22.9°\measuredangle.} \qquad (5)$$

Problem 2.8

> Let $\vec{A} = 2\,\mathrm{m}$ @ $0°\measuredangle$ and $\vec{B} = 6\,\mathrm{m}$ @ $90°\measuredangle$. Sketch the vector polygons and evaluate $\vec{R}$ for the following, reporting your answer using polar vector representation.
> (a) $\vec{R} = \vec{A} + \vec{B}$,
>
> (b) $\vec{R} = 2\vec{A} - \vec{B}$,
>
> (c) $\vec{R} = |\vec{A}|\,\vec{B} + |\vec{B}|\,\vec{A}$,
>
> (d) $\vec{R} = \dfrac{\vec{A}}{|\vec{A}|} + \dfrac{\vec{B}}{|\vec{B}|}$.

Solution



Part (a) The vector polygon is shown to the right. The magnitude R of vector $\vec{R}$ is given by

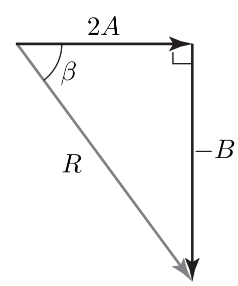
$$R = \sqrt{A^2 + B^2} = \sqrt{(2\,\mathrm{m})^2 + (6\,\mathrm{m})^2} = 6.325\,\mathrm{m}. \tag{1}$$

Referring to the figure again, we find β in the following manner:

$$R\cos\beta = A \quad\Rightarrow\quad \beta = \cos^{-1}\left(\frac{2\,\mathrm{m}}{6.325\,\mathrm{m}}\right) = 71.57°. \tag{2}$$

The polar vector representation of $\vec{R}$ is

$$\boxed{\vec{R} = 6.32\,\mathrm{m} \text{ @ } 71.6°\measuredangle.} \tag{3}$$

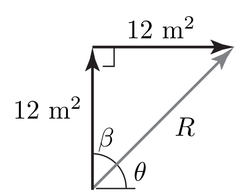


Part (b) Referring to the vector polygon shown at the right, we determine the values for R and β as

$$R = \sqrt{(2\cdot 2\,\mathrm{m})^2 + (6\,\mathrm{m})^2} = 7.211\,\mathrm{m}, \quad \beta = \sin^{-1}\left(\frac{6\,\mathrm{m}}{7.211\,\mathrm{m}}\right) = 56.31°. \tag{4}$$

The polar vector representation of $\vec{R}$ is

$$\boxed{\vec{R} = 7.21\,\mathrm{m} \text{ @ } -56.3°\measuredangle.} \tag{5}$$

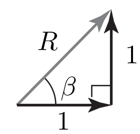


Part (c) Each vector $|\vec{A}|\vec{B}$ and $|\vec{B}|\vec{A}$ has a magnitude of $12\,\mathrm{m}^2$; since they are perpendicular to one another, it follows that $\beta = 45°$ and $\theta = 45°$. The magnitude of R is given by

$$R = \sqrt{(12\,\mathrm{m}^2)^2 + (12\,\mathrm{m}^2)^2} = 16.97\,\mathrm{m}^2. \tag{6}$$

The polar vector representation of $\vec{R}$ is

$$\boxed{\vec{R} = 17.0\,\mathrm{m}^2 \text{ @ } 45.0°\measuredangle.} \tag{7}$$



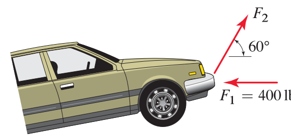
Part (d) Each vector $\vec{A}/|\vec{A}|$ and $\vec{B}/|\vec{B}|$ has a magnitude of one; since they are perpendicular to one another, it follows that $\beta = 45°$. The magnitude and polar vector representation of R are

$$R = \sqrt{(1)^2 + (1)^2} = 1.414. \tag{8}$$

$$\boxed{\vec{R} = 1.41 \text{ @ } 45.0°\measuredangle.} \tag{9}$$

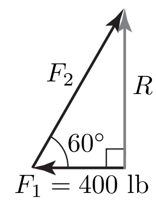
Problem 2.9

A tow truck applies forces $\vec{F}_1$ and $\vec{F}_2$ to the bumper of an automobile where $\vec{F}_1$ is horizontal. Determine the magnitude of $\vec{F}_2$ that will provide a vertical resultant force, and determine the magnitude of this resultant.



Solution

The resultant force is defined as $\vec{R} = \vec{F}_1 + \vec{F}_2$, and this resultant is to be vertical. The force polygon is shown at the right. Since F_1 is given,



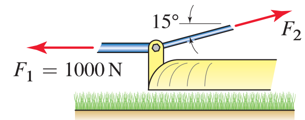
$$\boxed{F_2 \cos 60^\circ = 400\,\text{lb} \quad \Rightarrow \quad F_2 = \frac{400\,\text{lb}}{\cos 60^\circ} = 800\,\text{lb}.} \tag{1}$$

It then follows that R is given by

$$\boxed{R = F_2 \sin 60^\circ = (800\,\text{lb}) \sin 60^\circ = 693\,\text{lb}.} \tag{2}$$

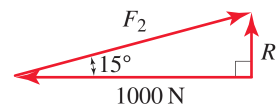
Problem 2.10

One of the support brackets for the lawn mowing deck of a garden tractor is shown where $\vec{F}_1$ is horizontal. Determine the magnitude of $\vec{F}_2$ so that the resultant of these two forces is vertical, and determine the magnitude of this resultant.



Solution

The vector polygon corresponding to the addition of $\vec{F}_1$ and $\vec{F}_2$ is shown at the right, where, as given in the problem statement, $\vec{R}$ is vertical. Thus,

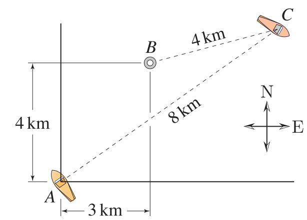


$$\boxed{F_2 \cos 15^\circ = 1000\,\mathrm{N} \quad \Rightarrow \quad F_2 = 1035\,\mathrm{N}} \tag{1}$$

$$\boxed{R = F_2 \sin 15^\circ = (1035\,\mathrm{N}) \sin 15^\circ = 267.9\,\mathrm{N}} \tag{2}$$

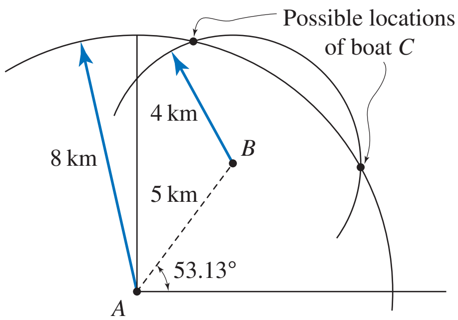
Problem 2.11

A buoy at point B is located 3 km east and 4 km north of boat A. Boat C is located 4 km from the buoy and 8 km from boat A. Determine the possible position vectors that give the position from boat A to boat C, $\vec{r}_{AC}$. State your answers using polar vector representation.



Solution

The locations of boat A and buoy B are shown. To determine the possible locations of boat C, we draw a circle with 8 km radius with center at A, and we draw a circle with 6 km radius with center at B; the intersections of these two circles are possible locations of boat C.



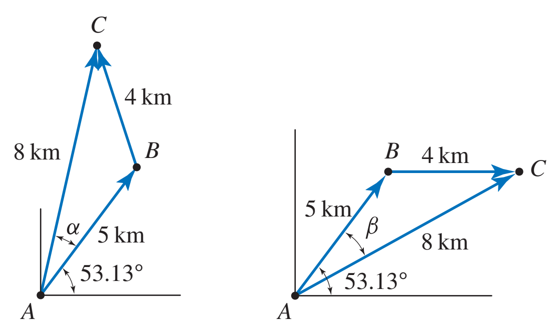
$$\tan^{-1}\frac{4\text{ km}}{3\text{ km}} = 53.13°,$$

$$r_{AB} = \sqrt{(3\text{ km})^2 + (4\text{ km})^2} = 5\text{ km}.$$

The two vector polygons corresponding to

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} \tag{1}$$

are shown below



For the vector polygon shown at the left, the law of cosines provides

$$(4\text{ km})^2 = (5\text{ km})^2 + (8\text{ km})^2 - 2(5\text{ km})(8\text{ km})\cos\alpha, \tag{2}$$

$$\alpha = \cos^{-1}\frac{(4\text{ km})^2 - (5\text{ km})^2 - (8\text{ km})^2}{-2(5\text{ km})(8\text{ km})} = 24.15°. \tag{3}$$

Hence, one of the possible position vectors from boat A to boat C is

$$\boxed{\begin{aligned}\vec{r}_{AC} &= 8\text{ km @ } \alpha + 53.13° \measuredangle \\ &= 8\text{ km @ } 77.28° \measuredangle\end{aligned}} \tag{4, 5}$$

For the vector polygon shown at the right, the law of cosines provides

$$(4\,\text{km})^2 = (5\,\text{km})^2 + (8\,\text{km})^2 - 2(5\,\text{km})(8\,\text{km})\cos\beta, \tag{6}$$

$$\beta = \cos^{-1}\frac{(4\,\text{km})^2 - (5\,\text{km})^2 - (8\,\text{km})^2}{-2(5\,\text{km})(8\,\text{km})} = 24.15°. \tag{7}$$

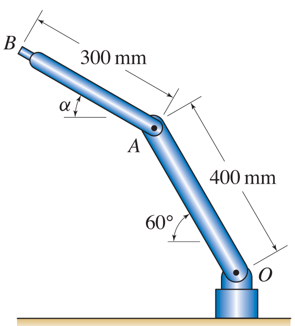
Hence, the other possible position vector from boat A to boat C is

$$\boxed{\begin{aligned}\vec{r}_{AC} &= 8\,\text{km} \;@\; 53.13° - \beta \measuredangle \quad (8)\\ &= 8\,\text{km} \;@\; 28.98° \measuredangle. \quad (9)\end{aligned}}$$

Remark: Equations (2) and (6) are identical, and hence $\alpha = \beta = 24.15°$. In fact, Eq. (2) has multiple solutions, two of which are $\alpha = \pm 24.15°$. Using this result, we could have arrived with both answers to this problem, namely Eqs. (5) and (9).

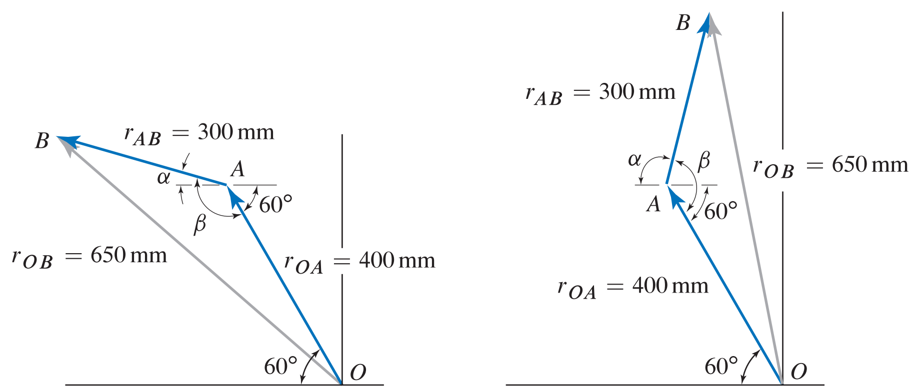
Problem 2.12

Arm OA of a robot is positioned as shown. Determine the value for angle α of arm AB so that the distance from point O to the actuator at B is 650 mm.



Solution

The two vector polygons shown below illustrate the addition $\vec{r}_{OB} = \vec{r}_{OA} + \vec{r}_{AB}$. These vector polygons show the two possible positions of arm AB such that the distance between points O and B is 650 mm.



First vector polygon: Applying the law of cosines, we obtain

$$650\,\text{mm} = \sqrt{(300\,\text{mm})^2 + (400\,\text{mm})^2 - 2(300\,\text{mm})(400\,\text{mm})\cos\beta}. \tag{1}$$

By squaring both sides and solving for β, we find that

$$\beta = \cos^{-1}\left[\frac{(650\,\text{mm})^2 - (300\,\text{mm})^2 - (400\,\text{mm})^2}{-2(300\,\text{mm})(400\,\text{mm})}\right] \tag{2}$$

$$= \cos^{-1}(-23/32) \tag{3}$$

$$= 136.0^\circ \tag{4}$$

To determine α, observe that

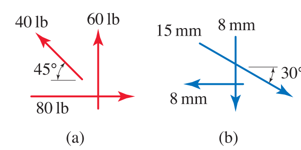
$$\boxed{180^\circ = \beta - \alpha + 60^\circ \quad \Rightarrow \quad \alpha = \beta + 60^\circ - 180^\circ = 16.0^\circ.} \tag{5}$$

Second vector polygon: Using the second vector polygon, Eq. (1) is still valid, which again provides $\beta = 136.0^\circ$. Thus,

$$\boxed{180^\circ = \alpha + \beta - 60^\circ \quad \Rightarrow \quad \alpha = 180^\circ - \beta + 60^\circ - = 104^\circ.} \tag{6}$$

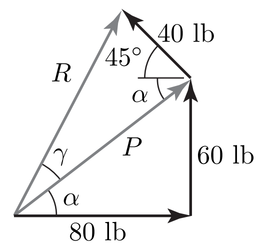
Problem 2.13

Add the three vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution

Part (a) The vector polygon shown at the right corresponds to the addition of the three force vectors to obtain a resultant force vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is given by



$$P = \sqrt{(60\,\text{lb})^2 + (80\,\text{lb})^2} = 100\,\text{lb}. \tag{1}$$

The angle α is found by

$$\tan\alpha = \frac{60\,\text{lb}}{80\,\text{lb}} \quad \Rightarrow \quad \alpha = \tan^{-1}\left(\frac{60\,\text{lb}}{80\,\text{lb}}\right) = 36.87°. \tag{2}$$

Next, use the law of cosines to find R

$$R = \sqrt{P^2 + (40\,\text{lb})^2 - 2P(40\,\text{lb})\cos(45° + \alpha)} = 102.3\,\text{lb}. \tag{3}$$

Use the law of sines to find γ

$$\frac{R}{\sin(45° + \alpha)} = \frac{40\,\text{lb}}{\sin\gamma} \quad \Rightarrow \quad \gamma = \sin^{-1}\left(\frac{40\,\text{lb}\sin\beta}{R}\right) = 22.77°. \tag{4}$$

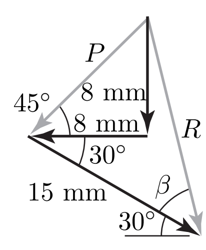
In polar vector representation, the direction of $\vec{R}$ measured from the right-hand horizontal direction is given by the sum of α and γ, such that

$$\boxed{\vec{R} = 102\,\text{lb} \ @\ 59.6°\measuredangle.} \tag{5}$$

Part (b) The vector polygon shown at the right corresponds to the addition of the three position vectors to obtain a resultant position vector $\vec{R}$. By inspection, the angle of $\vec{P}$ is 45°, while P is given given by

$$P = \sqrt{(8\,\text{mm})^2 + (8\,\text{mm})^2} = 11.31\,\text{mm}. \tag{6}$$

The law of cosines is used to find R



$$R = \sqrt{P^2 + (15\,\text{mm})^2 - 2P(15\,\text{mm})\cos(45° + 30°)} = 16.28\,\text{mm}. \tag{7}$$

The law of sines is used to determine the angle β as

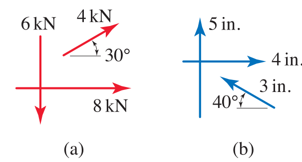
$$\frac{P}{\sin\beta} = \frac{R}{\sin(45° + 30°)} \quad \Rightarrow \quad \beta = \sin^{-1}\left[\frac{11.31\,\text{mm}\sin(75°)}{16.28\,\text{mm}}\right] = 42.15°. \tag{8}$$

The direction of $\vec{R}$ measured from the right-hand horizontal direction is given by $-\beta - 30° = -72.15°$, and the polar vector representation of $\vec{R}$ is

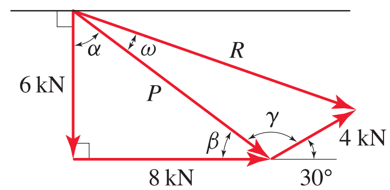
$$\boxed{\vec{R} = 16.3\,\text{mm} \ @\ {-72.2}°\measuredangle.} \tag{9}$$

Problem 2.14

Add the three vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the three force vectors to obtain a resultant force vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is

$$P = \sqrt{(6\,\text{kN})^2 + (8\,\text{kN})^2} = 10\,\text{kN}. \tag{1}$$

The angle α is found by

$$\tan\alpha = \frac{8\,\text{kN}}{6\,\text{kN}} \quad \Rightarrow \quad \alpha = \tan^{-1}\frac{8\,\text{kN}}{6\,\text{kN}} = 53.13^\circ, \tag{2}$$

and then, noting that $\alpha + \beta + 90^\circ = 180^\circ$,

$$\beta = 180^\circ - \alpha - 90^\circ = 36.87^\circ. \tag{3}$$

Considering the triangle formed by the 4 kN force, P, and R, the angle γ is obtained from $\beta + \gamma + 30^\circ = 180^\circ$ as

$$\gamma = 180^\circ - \beta - 30^\circ = 113.1^\circ. \tag{4}$$

Using the law of cosines

$$R = \sqrt{P^2 + (4\,\text{kN})^2 - 2P(4\,\text{kN})\cos\gamma} = 12.14\,\text{kN}. \tag{5}$$

Using the law of sines, the angle ω is obtained as follows

$$\frac{4\,\text{kN}}{\sin\omega} = \frac{R}{\sin\gamma}, \tag{6}$$

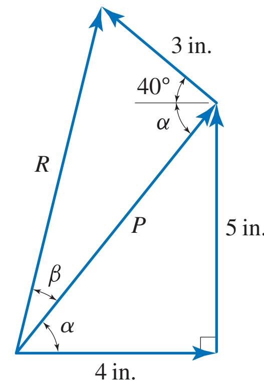
$$\omega = \sin^{-1}\frac{4\,\text{kN}}{12.14\,\text{kN}}\sin 113.1^\circ = 17.64^\circ. \tag{7}$$

Using polar vector representation, the resultant force vector is

$$\boxed{\begin{aligned} \vec{R} &= 12.14\,\text{kN} \;@\; -(90^\circ - \alpha - \omega)\measuredangle & (8)\\ &= 12.14\,\text{kN} \;@\; -19.23^\circ\measuredangle. & (9) \end{aligned}}$$

If desired, this resultant may be stated using a positive angle, where $360^\circ - 19.23^\circ = 340.8^\circ$, as

$$\boxed{\vec{R} = 12.14\,\text{kN} \;@\; 340.8^\circ\measuredangle.} \tag{10}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the three position vectors to obtain a resultant position vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is

$$P = \sqrt{(4\,\text{in.})^2 + (5\,\text{in.})^2} = 6.403\,\text{in.} \tag{11}$$

The angle α is found by

$$\tan\alpha = \frac{5\,\text{in.}}{4\,\text{in.}} \quad \Rightarrow \quad \alpha = \tan^{-1}\frac{5\,\text{in.}}{4\,\text{in.}} = 51.34°. \tag{12}$$

Considering the triangle formed by the 3 in. position vector, P, and R, the law of cosines may be used to obtain

$$R = \sqrt{(3\,\text{in.})^2 + P^2 - 2(3\,\text{in.})P\cos(40° + \alpha)} = 7.134\,\text{in.}, \tag{13}$$

and the law of sines may be used to determine the angle β as

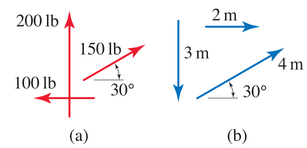
$$\frac{3\,\text{in.}}{\sin\beta} = \frac{R}{\sin(40° + \alpha)} \quad \Rightarrow \quad \beta = \sin^{-1}\frac{3\,\text{in.}}{7.134\,\text{in.}}\sin(40° + 51.34°) = 24.86°. \tag{14}$$

Using polar vector representation, the resultant position vector is

$$\boxed{\begin{aligned} \vec{R} &= 7.134\,\text{in.}\ @\ \alpha + \beta \measuredangle \quad (15)\\ &= 7.134\,\text{in.}\ @\ 76.20° \measuredangle. \quad (16) \end{aligned}}$$

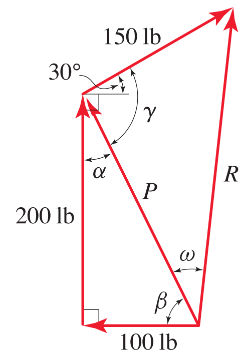
Problem 2.15

Add the three vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution

Part (a) The vector polygon shown at the right corresponds to the addition of the three force vectors to obtain a resultant force vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is



$$P = \sqrt{(100\,\text{lb})^2 + (200\,\text{lb})^2} = 223.6\,\text{lb}. \tag{1}$$

The angle α is found by

$$\tan\alpha = \frac{100\,\text{lb}}{200\,\text{lb}} \quad\Rightarrow\quad \alpha = \tan^{-1}\frac{100\,\text{lb}}{200\,\text{lb}} = 26.57°, \tag{2}$$

and then, noting that $\alpha + \beta + 90° = 180°$,

$$\beta = 180° - \alpha - 90° = 63.43°. \tag{3}$$

Considering the triangle formed by the 150 lb force, P, and R, the angle γ is obtained from $\gamma + \alpha = 90° + 30°$ as

$$\gamma = 90° + 30° - \alpha = 93.43°. \tag{4}$$

Using the law of cosines

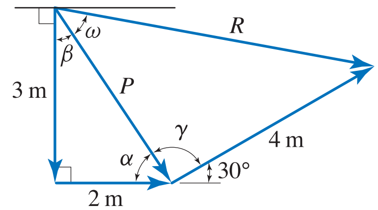
$$R = \sqrt{(150\,\text{lb})^2 + P^2 - 2(150\,\text{lb})P\cos\gamma} = 276.6\,\text{lb}. \tag{5}$$

Using the law of sines, the angle ω is obtained from

$$\frac{150\,\text{lb}}{\sin\omega} = \frac{R}{\sin\gamma} \quad\Rightarrow\quad \omega = \sin^{-1}\frac{150\,\text{lb}}{276.6\,\text{lb}}\sin 93.43° = 32.77°. \tag{6}$$

Using polar vector representation, the resultant force is

$$\boxed{\begin{aligned}\vec{R} &= 276.6\,\text{lb} \ @\ 180° - \beta - \omega \measuredangle \qquad (7)\\ &= 276.6\,\text{lb} \ @\ 83.79° \measuredangle. \qquad (8)\end{aligned}}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the three position vectors to obtain a resultant position vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is

$$P = \sqrt{(2\,\mathrm{m})^2 + (3\,\mathrm{m})^2} = 3.606\,\mathrm{m}. \tag{9}$$

The angle α is found by

$$\tan\alpha = \frac{3\,\mathrm{m}}{2\,\mathrm{m}} \quad\Rightarrow\quad \alpha = \tan^{-1}\frac{3\,\mathrm{m}}{2\,\mathrm{m}} = 56.31°, \tag{10}$$

and then, noting that $\alpha + \beta + 90° = 180°$,

$$\beta = 180° - \alpha - 90° = 33.69°. \tag{11}$$

Considering the triangle formed by the 4 m position vector, P, and R, the angle γ is obtained from $\alpha + \gamma + 30° = 180°$ as

$$\gamma = 180° - \alpha - 30° = 93.69°. \tag{12}$$

Using the law of cosines

$$R = \sqrt{(4\,\mathrm{m})^2 + P^2 - 2(4\,\mathrm{m})P\,\cos\gamma} = 5.555\,\mathrm{m}. \tag{13}$$

Using the law of sines,

$$\frac{4\,\mathrm{m}}{\sin\omega} = \frac{R}{\sin\gamma} \quad\Rightarrow\quad \omega = \sin^{-1}\frac{4\,\mathrm{m}}{5.555\,\mathrm{m}}\sin 93.69° = 45.94°. \tag{14}$$

Using polar vector representation, the resultant position vector is

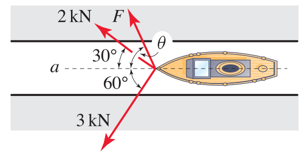
$$\boxed{\begin{aligned} \vec{R} &= 5.555\,\mathrm{m} \;@\; -(90° - \beta - \omega)\measuredangle && (15)\\ &= 5.555\,\mathrm{m} \;@\; -10.37°\measuredangle. && (16)\end{aligned}}$$

If desired, the resultant may be stated using a positive angle, where $360° - 10.37° = 349.6°$ as

$$\boxed{\vec{R} = 5.555\,\mathrm{m} \;@\; 349.6°\measuredangle.} \tag{17}$$

Problem 2.16

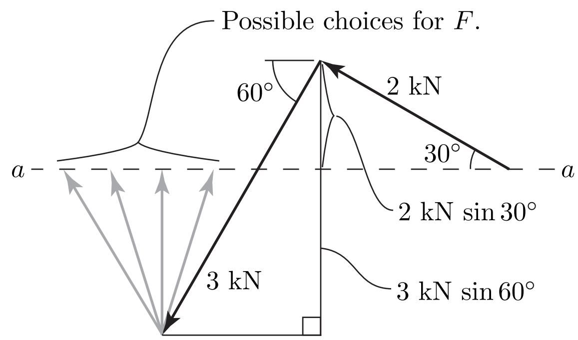
A ship is towed through a narrow channel by applying forces to three ropes attached to its bow. Determine the magnitude and orientation θ of the force $\vec{F}$ so that the resultant force is in the direction of line a and the magnitude of $\vec{F}$ is as small as possible.



Solution

The force polygon shown at the right corresponds to the addition of the forces applied by the three ropes to the ship. In sketching the force polygon, the known force vectors are sketched first (i.e., the 2 kN and 3 kN forces). There are many possible choices of $\vec{F}$ such that the resultant force will be parallel to line a. The smallest value of F occurs when $\vec{F}$ is perpendicular to line a; i.e., when

$$\boxed{\theta = 90^\circ.} \tag{1}$$



The magnitude of $\vec{F}$ is then found by using the force polygon to write

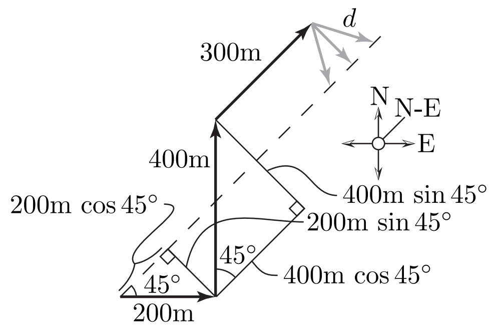
$$\boxed{F = (3\,\text{kN})\sin 60^\circ - (2\,\text{kN})\sin 30^\circ = 1.60\,\text{kN}.} \tag{2}$$

Problem 2.17

A surveyor needs to plant a marker directly northeast from where she is standing. Because of obstacles, she walks a route in the horizontal plane consisting of 200 m east, followed by 400 m north, followed by 300 m northeast. From this position, she would like to take the shortest-distance route back to the line that is directly northeast of her starting position. What direction should she travel and how far, and what will be her final distance from her starting point?

Solution

The vector polygon shown at the right corresponds to the addition of the four position vectors corresponding to the path walked by the surveyor. The first three position vectors take the surveyor to the point at which she begins to travel back to the line that is directly north-east of her starting position (this direction is shown as a dashed line in the vector polygon). The path she takes to reach this line has distance d, and several possibilities are shown. By examining the vector polygon, the smallest value of d results when she travels directly south-east, in which case d is given by



$$d = (400\,\text{m})\sin 45^\circ - (200\,\text{m})\sin 45^\circ = 141\,\text{m}. \tag{1}$$

To summarize,

$$\boxed{\text{The surveyor should walk 141 m in the S-E direction.}} \tag{2}$$

The distance R from her starting point to her final position is given by

$$\boxed{R = (200\,\text{m})\cos 45^\circ + (400\,\text{m})\cos 45^\circ + 300\,\text{m} = 724\,\text{m}.} \tag{3}$$

Problem 2.18

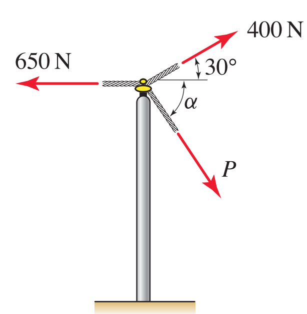
A utility pole supports a bundle of wires that apply the 400 and 650 N forces shown, and a guy wire applies the force $\vec{P}$.

(a) If $P = 0$, determine the resultant force applied by the wires to the pole and report your result using polar vector representation.

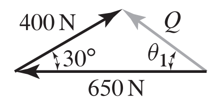
(b) Repeat Part (a) if $P = 500\,\mathrm{N}$ and $\alpha = 60°$.

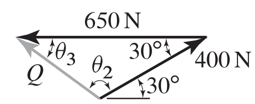
(c) With $\alpha = 60°$, what value of P will produce a resultant force that is vertical?

(d) If the resultant force is to be vertical and P is to be as small as possible, determine the value α should have and the corresponding value of P.



Solution





Part (a) Either of the force polygons shown at the right may be used to determine the resultant force Q. Regardless of which force polygon is used, the law of cosines provides

$$Q = \sqrt{(400\,\mathrm{N})^2 + (650\,\mathrm{N})^2 - 2(400\,\mathrm{N})(650\,\mathrm{N})\cos 30°} = 363.5\,\mathrm{N} \tag{1}$$

Using the *first* force polygon shown, the law of sines is used to determine the angle θ_1 as

$$\frac{400\,\mathrm{N}}{\sin\theta_1} = \frac{Q}{\sin 30°} \quad \Rightarrow \quad \theta_1 = \sin^{-1}\left(\frac{(400\,\mathrm{N})\sin 30°}{363.5\,\mathrm{N}}\right) = 33.38°. \tag{2}$$

The orientation of $\vec{Q}$ to be used for its polar vector representation is $180° - \theta_1 = 180° - 33.38° = 146.6°$, and hence the vector representation of $\vec{Q}$ is

$$\boxed{\vec{Q} = 364\,\mathrm{N}\ @\ 147°\measuredangle.} \tag{3}$$

Alternatively, the *second* force polygon could be used. As discussed above, Eq. (1) still applies, and $Q = 363.3\,\mathrm{N}$. Because angle θ_2 appears to be obtuse, we will avoid using the law of sines to determine its value (see the discussion in the text regarding the pitfall when using the law of sines to determine an obtuse angle). Using the law of sines to determine angle θ_3 provides

$$\frac{400\,\mathrm{N}}{\sin\theta_3} = \frac{Q}{\sin 30°} \quad \Rightarrow \quad \theta_3 = \sin^{-1}\left(\frac{(400\,\mathrm{N})\sin 30°}{363.5\,\mathrm{N}}\right) = 33.38°. \tag{4}$$

Once θ_3 is known, angle θ_2 is easily found as

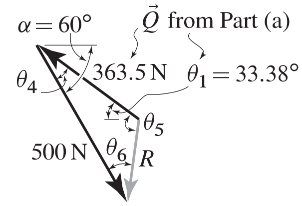
$$\theta_2 = 180° - \theta_3 - 30° = 180° - 33.38° - 30° = 116.6°. \tag{5}$$

The orientation of $\vec{Q}$ to be used for its polar vector representation is $30° + \theta_2 = 30° + 116.6° = 146.6°$, and hence the vector representation of $\vec{Q}$ is

$$\boxed{\vec{Q} = 364\,\mathrm{N}\ @\ 147°\measuredangle.} \tag{6}$$

As expected, the same result for $\vec{Q}$ is obtained regardless of which force polygon was used.

Part (b) Our strategy will be to add the force vector $\vec{P}$ to the result for $\vec{Q}$ obtained in Part (a). Thus, the force polygon is shown at the right, where Q from Eq. (1) and θ_1 from Eq. (2) are used, such that



$$\theta_4 = 60° - \theta_1 = 60° - 33.38° = 26.62°. \tag{7}$$

The law of cosines may be used to find R:

$$R = \sqrt{(500\,\mathrm{N})^2 + (363.5\,\mathrm{N})^2 - 2(500\,\mathrm{N})(363.3\,\mathrm{N})\cos\theta_4} = 239.1\,\mathrm{N}. \tag{8}$$

Since θ_5 is obtuse, we will avoid using the law of sines to determine it, and instead will use the law of sines to determine θ_6, as follows

$$\frac{R}{\sin\theta_4} = \frac{363.5\,\mathrm{N}}{\sin\theta_6} \quad \Rightarrow \quad \theta_6 = \sin^{-1}\left(\frac{(363.5\,\mathrm{N})\sin\theta_4}{R}\right) = 42.95°. \tag{9}$$

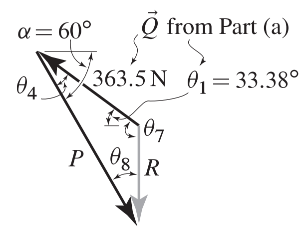
The angle θ_5 is given by

$$\theta_5 = 180° - \theta_4 - \theta_6 = 110.4°. \tag{10}$$

The orientation of $\vec{R}$ relative to the right-hand horizontal direction is the sum of the orientation of $\vec{Q}$ obtained in Part (a), namely 146.6°, plus θ_5. Thus

$$\boxed{\vec{R} = 239\,\mathrm{N} \ @ \ 257° \measuredangle.} \tag{11}$$

Part (c) The force polygon is shown at the right, where angle $\theta_4 = 26.62°$ was determined in Eq. (7). For the resultant force $\vec{R}$ to be vertical, $\theta_7 = 90° + \theta_1 = 90° + 33.38° = 123.4°$ Thus



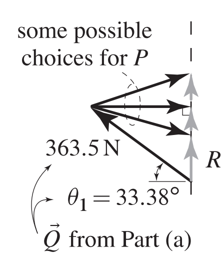
$$\theta_8 = 180° - \theta_4 - \theta_7 = 30°. \tag{12}$$

The law of sines is used to determine P as

$$\frac{363.5\,\mathrm{N}}{\sin\theta_8} = \frac{P}{\sin\theta_7} \tag{13}$$

$$\boxed{\Rightarrow \quad P = (363.5\,\mathrm{N})\frac{\sin 123.4°}{\sin 30°} = 607\,\mathrm{N}.} \tag{14}$$

Part (d) Using the results for $\vec{Q}$ from Part (a), and if the resultant force is to be vertical, then the force polygon is as shown at the right; three possible choices (among many possibilities) for P along with the corresponding resultant force are shown. The smallest value of P occurs when $\vec{P}$ is perpendicular to $\vec{R}$, hence



$$\boxed{\alpha = 0°.} \tag{15}$$

For this value of α,

$$\boxed{P = (363.5\,\mathrm{N})\cos 33.38° = 304\,\mathrm{N}.} \tag{16}$$

Problem 2.19

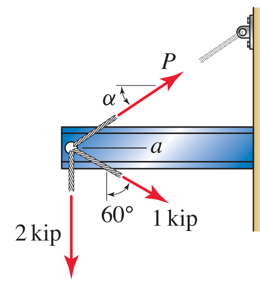
The end of a cantilever I beam supports forces from three cables.

(a) If $P = 0$, determine the resultant force applied by the two cables to the I beam and report your result using polar vector representation.

(b) Repeat Part (a) if $P = 1.5\,\text{kip}$ and $\alpha = 30°$.

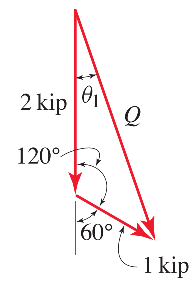
(c) With $\alpha = 30°$, what value of P will produce a resultant force that is horizontal?

(d) If the resultant force is to be horizontal and P is to be as small as possible, determine the value α should have and the corresponding value of P.



Solution

Part (a) The force polygon shown at the right may be used to determine the resultant force Q, Noting that the angle opposite Q is $180° - 60° = 120°$, the law of cosines may be used to obtain



$$Q = \sqrt{(1\,\text{kip})^2 + (2\,\text{kip})^2 - 2(1\,\text{kip})(2\,\text{kip})\cos 120°} = 2.646\,\text{kip}. \tag{1}$$

Using the law of sines, the angle θ_1, is obtained as follows

$$\frac{1\,\text{kip}}{\sin\theta_1} = \frac{Q}{\sin 120°} \quad\Rightarrow\quad \theta_1 = \sin^{-1}\left(\frac{1\,\text{kip}}{2.646\,\text{kip}}\sin 120°\right) = 19.11°. \tag{2}$$

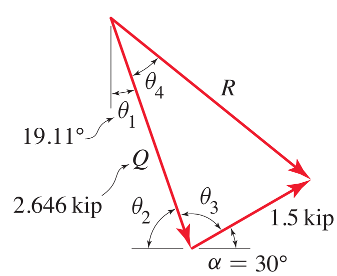
Using polar vector representation, the resultant force is

$$\boxed{\begin{aligned}\vec{Q} &= 2.646\,\text{kip} \ @ \ -(90° - \theta_1)\measuredangle \quad (3)\\ &= 2.646\,\text{kip} \ @ \ -70.89°\measuredangle. \quad (4)\end{aligned}}$$

If desired, the resultant force may be stated using a positive angle, where $360° - 70.89° = 289.1°$, as

$$\boxed{\vec{Q} = 2.646\,\text{kip} \ @ \ 289.1°\measuredangle.} \tag{5}$$

Part (b) Our strategy will be to add the force vector $\vec{P}$ to the result for $\vec{Q}$ obtained in Part (a). Thus, the force polygon is shown at the right where Q from Eq. (1) and θ_1 from Eq. (2) are used, and θ_2 is obtained from $\theta_1 + \theta_2 + 90° = 180°$ which provides



$$\theta_2 = 180° - \theta_1 - 90° = 70.89°. \tag{6}$$

The angle opposite force R is obtained by using $\theta_2 + \theta_3 + 30° = 180°$, which provides

$$\theta_3 = 180° - \theta_2 - 30° = 79.11°. \tag{7}$$

Using the law of cosines, the resultant force is

$$\begin{aligned} R &= \sqrt{(1.5\,\text{kip})^2 + (2.646\,\text{kip})^2 - 2(1.5\,\text{kip})(2.646\,\text{kip})\cos\theta_3} \\ &= 2.784\,\text{kip}. \end{aligned} \tag{8}$$

Using the law of sines, angle θ_4 may be determined

$$\frac{1.5\,\text{kip}}{\sin\theta_4} = \frac{R}{\sin\theta_3} \quad\Rightarrow\quad \theta_4 = \sin^{-1}\left(\frac{1.5\,\text{kip}}{2.784\,\text{kip}}\sin 79.11^\circ\right) = 31.95^\circ. \tag{9}$$

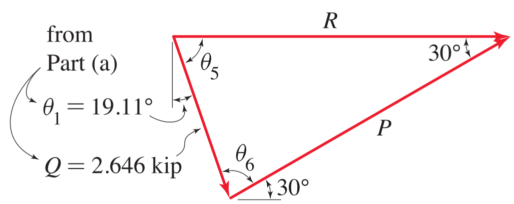
Using polar vector representation, the resultant force is

$$\boxed{\begin{aligned} \vec{R} &= 2.784\,\text{kip} \ @\ -(90^\circ - \theta_1 - \theta_4)\measuredangle \qquad (10)\\ &= 2.784\,\text{kip} \ @\ -38.95^\circ\measuredangle. \qquad (11) \end{aligned}}$$

If desired, the resultant may be stated using a positive angle, where $360^\circ - 38.95^\circ = 321.1^\circ$, as

$$\boxed{\vec{R} = 2.784\,\text{kip} \ @\ 321.1^\circ\measuredangle.} \tag{12}$$

Part (c) The force polygon is shown below



For the resultant force R to be horizontal, using $\theta_1 + \theta_5 = 90^\circ$, we obtain

$$\theta_5 = 90^\circ - \theta_1 = 70.89^\circ, \tag{13}$$

and noting that $\theta_5 + \theta_6 + 30^\circ = 180^\circ$, we obtain

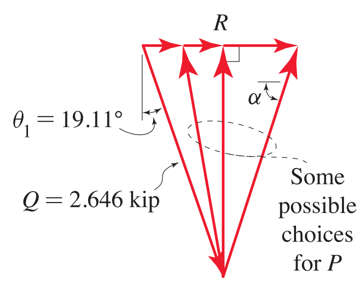
$$\theta_6 = 180^\circ - \theta_5 - 30^\circ = 79.11^\circ. \tag{14}$$

Using the law of sines, with $Q = 2.646\,\text{kip}$ from Part (a),

$$\frac{R}{\sin\theta_6} = \frac{Q}{\sin 30^\circ}, \tag{15}$$

$$\boxed{R = 2.646\,\text{kip}\,\frac{\sin 79.11^\circ}{\sin 30^\circ} = 5.196\,\text{kip}.} \tag{16}$$

Part (d) Using the results for Q from Part (a), and if the resultant force is to be horizontal, then the force polygon is shown at the right; three possible choices (among many possibilities) for P along with the corresponding resultant force R are shown. The smallest value of P occurs when $\vec{P}$ is perpendicular to $\vec{R}$, hence



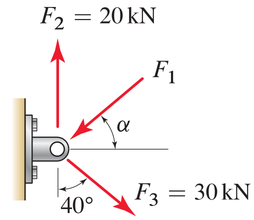
$$\boxed{\alpha = 90^\circ,} \tag{17}$$

and

$$\boxed{P = (2.646\,\text{kip})\cos 19.11^\circ = 2.500\,\text{kip}.} \tag{18}$$

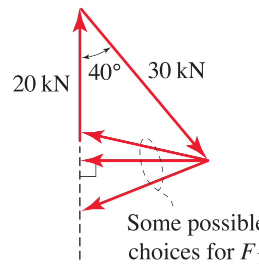
Problem 2.20

Determine the smallest force F_1 such that the resultant of the three forces F_1, F_2, and F_3 is vertical, and the angle α at which F_1 should be applied.



Solution

The force polygon, including various choices for $\vec{F}_1$, is shown at the right. The smallest value of F_1 occurs when the vector $\vec{F}_1$ is horizontal, hence



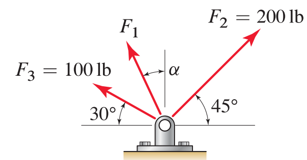
$$\boxed{\alpha = 0°,} \tag{1}$$

and the force is

$$\boxed{F_1 = (30\,\text{kN})\,\sin 40° = 19.28\,\text{kN}.} \tag{2}$$

Problem 2.21

Determine the smallest force F_1 such that the resultant of the three forces F_1, F_2, and F_3 is vertical, and the angle α at which F_1 should be applied.



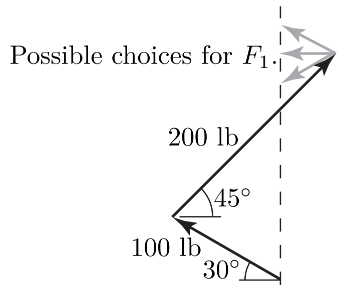
Solution

The force polygon, including various choices for $\vec{F}_1$, is shown to the right. The smallest value of F_1 occurs when the vector $\vec{F}_1$ is horizontal, i.e., when

$$\boxed{\alpha = 90°.} \tag{1}$$

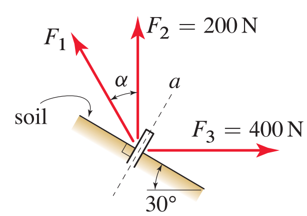
The value of F_1 is given by

$$\boxed{F_1 = (200\,\text{lb}) \cos 45° - (100\,\text{lb}) \cos 30° = 54.8\,\text{lb}.} \tag{2}$$

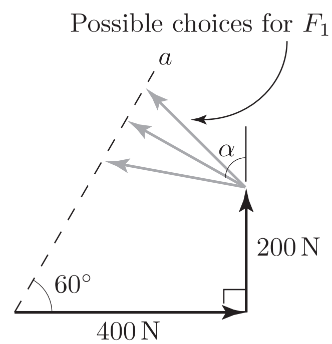


Problem 2.22

Forces F_1, F_2, and F_3 are applied to a soil nail to pull it out of a slope. If F_2 and F_3 are vertical and horizontal, respectively, with the magnitudes shown, determine the magnitude of the smallest force F_1 that can be applied and the angle α so that the resultant force applied to the nail is directed along the axis of the nail (direction a).



Solution



We begin by adding the two known forces, $\vec{F}_2$ and $\vec{F}_3$, as shown in the force polygon to the right. There are an infinite number of choices for $\vec{F}_1$, but we desire the one with the smallest magnitude. By examining the force polygon, F_1 is smallest when its direction is perpendicular to line a, i.e., when

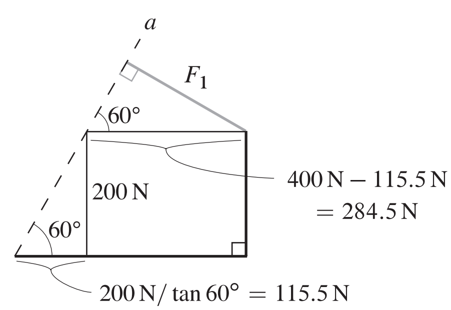
$$\boxed{\alpha = 60°.} \tag{1}$$

To determine the value of F_1, consider the sketch shown at the right. Noting that the hypotenuse of the upper triangle is given by

$$400\,\text{N} - \frac{200\,\text{N}}{\tan 60°} = 400\,\text{N} - 115.5\,\text{N} = 284.5\,\text{N}, \tag{2}$$

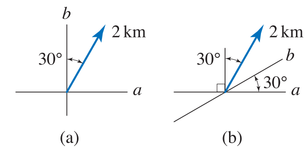
it follows that

$$\boxed{F_1 = (284.5\,\text{N})\sin 60° = 246\,\text{N}.} \tag{3}$$



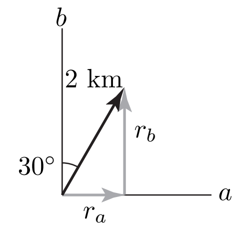
Problem 2.23

Determine the magnitudes of vectors $\vec{r}_a$ and $\vec{r}_b$ in the a and b directions, respectively, such that their sum is the 2 km position vector shown.



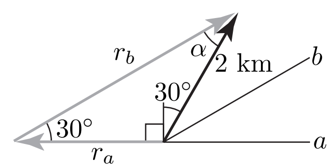
Solution

Part (a) Because the directions a and b of the two component vectors to be determined are orthogonal, determination of the magnitudes of the component vectors will be straightforward. The magnitudes r_a and r_b of vectors $\vec{r}_a$ and $\vec{r}_b$ are determined using



$$\boxed{\begin{aligned} r_a &= (2\,\text{km})\sin 30^\circ = 1.00\,\text{km}, \\ r_b &= (2\,\text{km})\cos 30^\circ = 1.73\,\text{km}. \end{aligned}} \qquad \begin{aligned} (1) \\ (2) \end{aligned}$$

Part (b) Because the directions a and b of the two component vectors to be determined are not orthogonal, determination of the magnitudes of the component vectors will be slightly more work than for Part (a). Observe that the angle $\alpha = 180^\circ - 30^\circ - 120^\circ = 30^\circ$. The magnitudes r_a and r_b of vectors $\vec{r}_a$ and $\vec{r}_b$ are determined using the law of sines to obtain

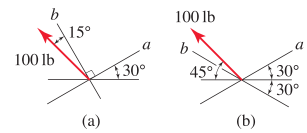


$$\boxed{\frac{2\,\text{km}}{\sin 30^\circ} = \frac{r_b}{\sin 120^\circ} = \frac{r_a}{\sin 30^\circ} \quad \Rightarrow \quad r_a = -2.00\,\text{km}, \quad \text{and} \quad r_b = 3.46\,\text{km},} \qquad (3)$$

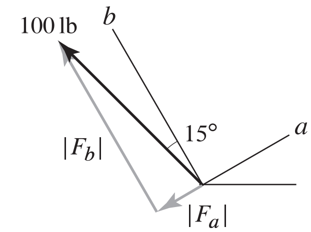
where the negative sign is inserted for r_a since it acts in the negative a direction.

Problem 2.24

Determine the magnitudes of vectors $\vec{F}_a$ and $\vec{F}_b$ in the a and b directions, respectively, such that their sum is the 100 lb force vector shown.



Solution



Part (a) Let F_a and F_b be the components (scalars) of force vectors $\vec{F}_a$ and $\vec{F}_b$, respectively. These components are determined using

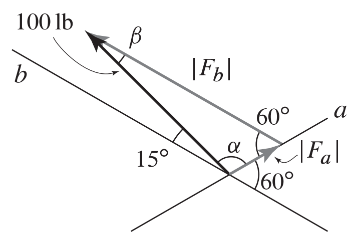
$$\boxed{\begin{aligned} F_a &= (-100\,\text{lb}) \sin 15° = -25.9\,\text{lb}, &\quad (1)\\ F_b &= (100\,\text{lb}) \cos 15° = 96.6\,\text{lb}, &\quad (2)\end{aligned}}$$

where F_a is negative since it acts in the negative a direction. Hence, the magnitudes of vectors $\vec{F}_a$ and $\vec{F}_b$ are

$$\boxed{\begin{aligned} |\vec{F}_a| &= 25.9\,\text{lb}, &\quad (3)\\ |\vec{F}_b| &= 96.6\,\text{lb}. &\quad (4)\end{aligned}}$$

Part (b) It is necessary to determine α and β, by noting that

$$\alpha = 180° - 15° - 60° = 105°, \quad \beta = 180° - \alpha - 60° = 15°. \quad (5)$$



The law of sines may then be used to find the components F_a and F_b (scalars) of vectors $\vec{F}_a$ and $\vec{F}_b$ as

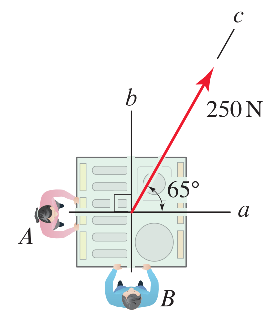
$$\boxed{\frac{100\,\text{lb}}{\sin 60°} = \frac{F_b}{\sin\alpha} = \frac{F_a}{\sin\beta} \quad \Rightarrow \quad F_a = 29.9\,\text{lb}, \quad F_b = 112\,\text{lb}.} \quad (6)$$

Hence, the magnitudes of vectors $\vec{F}_a$ and $\vec{F}_b$ are

$$\boxed{\begin{aligned} |\vec{F}_a| &= 29.9\,\text{lb}, &\quad (7)\\ |\vec{F}_b| &= 112\,\text{lb}. &\quad (8)\end{aligned}}$$

Problem 2.25

The child's play structure from Examples 2.2 and 2.3 on pp. 38 and 39 is shown again. The woman at A applies a force in the a direction and the man at B applies a force in the b direction, with the goal of producing a resultant force of 250 N in the c direction. Determine the forces the two people must apply, expressing the results as vectors.

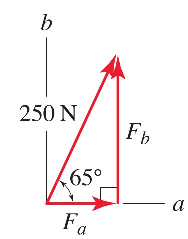


Solution

Let $\vec{F}$ denote the 250 N force vector acting in the c direction. Our objective is to determine the force vectors $\vec{F}_a$ acting in the a direction and $\vec{F}_b$ acting in the b direction such that

$$\vec{F} = \vec{F}_a + \vec{F}_b. \tag{1}$$

The force polygon corresponding to this addition is shown at the right. Since $\vec{F}_a$ and $\vec{F}_b$ are perpendicular, basic trigonometry provides



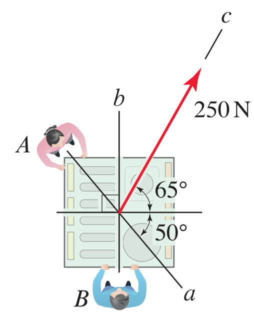
$$|F_a| = (250\,\mathrm{N})\cos 65^\circ = 105.7\,\mathrm{N} \tag{2}$$
$$|F_b| = (250\,\mathrm{N})\sin 65^\circ = 226.6\,\mathrm{N} \tag{3}$$

Using polar vector representation, the forces are

$$\boxed{\begin{aligned}\vec{F}_a &= 105.7\,\mathrm{N} \;@\; 0^\circ\measuredangle, \text{ and} \qquad (4)\\ \vec{F}_b &= 226.6\,\mathrm{N} \;@\; 90^\circ\measuredangle. \qquad (5)\end{aligned}}$$

Problem 2.26

The child's play structure from Examples 2.2 and 2.3 on pp. 38 and 39 is shown again. The woman at A applies a force in the a direction and the man at B applies a force in the b direction, with the goal of producing a resultant force of 250 N in the c direction. Determine the forces the two people must apply, expressing the results as vectors.

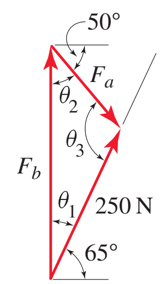


Solution

Let $\vec{F}$ denote the 250 N force vector acting in the c direction. Our objective is to determine the force vectors $\vec{F}_a$ acting in the a direction and $\vec{F}_b$ acting in the b direction such that

$$\vec{F} = \vec{F}_a + \vec{F}_b. \tag{1}$$

The force polygon corresponding to this addition is shown at the right. Since $\vec{F}_a$ and $\vec{F}_b$ are not perpendicular, the laws of sines and cosines must be used. The angles θ_1, θ_2, and θ_3 are easily determined as



$$\theta_1 = 90° - 65° = 25°, \tag{2}$$
$$\theta_2 = 90° - 50° = 40°, \tag{3}$$
$$\theta_3 = 180° - \theta_1 - \theta_2 = 115°. \tag{4}$$

The law of sines provides

$$\frac{250\,\text{N}}{\sin\theta_2} = \frac{|F_a|}{\sin\theta_1} = \frac{|F_b|}{\sin\theta_3}, \tag{5}$$

which yields

$$|F_a| = 250\,\text{N}\,\frac{\sin 25°}{\sin 40°} = 164.4\,\text{N}, \tag{6}$$

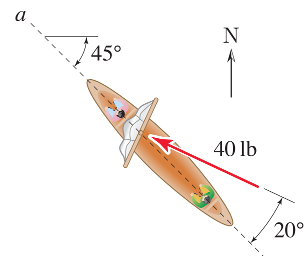
$$|F_b| = 250\,\text{N}\,\frac{\sin 115°}{\sin 40°} = 352.5\,\text{N}. \tag{7}$$

Using polar vector representation, the forces are

$$\boxed{\begin{aligned}\vec{F}_a &= 164.4\,\text{N} @ -50° \measuredangle, \text{ and} \\ \vec{F}_b &= 352.5\,\text{N} @ 90° \measuredangle.\end{aligned}} \tag{8, 9}$$

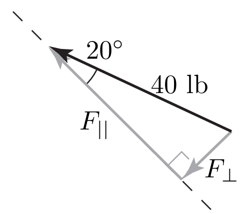
Problem 2.27

While canoes are normally propelled by paddle, if there is a favorable wind from the stern, adventurous users will sometimes employ a small sail. If a canoe is sailing north-west and the wind applies a 40 lb force perpendicular to the sail in the direction shown, determine the components of the wind force parallel and perpendicular to the keel of the canoe (direction a).



Solution

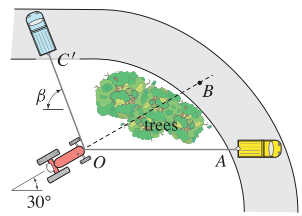
Let the force perpendicular to the keel be denoted by $F_\perp$ and the force parallel to the keel be denoted by $F_{||}$. The sketch shown at the right illustrates the addition of these two forces to yield the 40 lb force applied to the sail. Thus,



$$\boxed{\begin{aligned} F_\perp &= (40\,\text{lb})\sin 20^\circ = 13.7\,\text{lb}, \\ F_{||} &= (40\,\text{lb})\cos 20^\circ = 37.6\,\text{lb}. \end{aligned}} \qquad \begin{aligned} (1) \\ (2) \end{aligned}$$

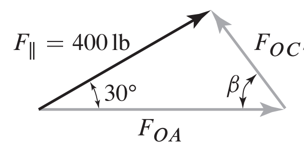
Problem 2.28

Repeat Part (b) of Example 2.5, using the optimization methods of calculus. *Hint:* Redraw the force polygon of Fig. 3 and rewrite Eq. (1) on p. 41 with the 45° angle shown there replaced by β, where β is defined in Fig. P2.28. Rearrange this equation to obtain an expression for $F_{OC'}$ as a function of β, and then determine the value of β that makes $dF_{OC'}/d\beta = 0$. While the approach described here is straightforward to carry out "by hand," you might also consider using symbolic algebra software such as Mathematica or Maple.



Solution

A relationship for $F_{OC'}$ in terms of $F_{||}$ and β is needed, and this may be obtained using the force polygon shown at the right with the law of sines



$$\frac{400\,\mathrm{lb}}{\sin\beta} = \frac{F_{OC'}}{\sin 30°} \quad \Rightarrow \quad F_{OC'} = (400\,\mathrm{lb})\frac{\sin 30°}{\sin\beta}. \tag{1}$$

To determine the minimum value of $F_{OC'}$ as a function of β, we make $F_{OC'}$ stationary by setting its derivative with respect to β equal to zero; i.e.,

$$\frac{dF_{OC'}}{d\beta} = 0 = (400\,\mathrm{lb})(\sin 30°)\frac{d}{d\beta}\left(\frac{1}{\sin\beta}\right) = (400\,\mathrm{lb})(\sin 30°)\frac{\cos\beta}{\sin\beta}\frac{1}{\sin\beta}. \tag{2}$$

Satisfaction of Eq. (2) requires $\cos\beta = 0$, which gives $\beta = 90°$. From Eq. (1) we obtain

$$\boxed{\beta = 90° \quad \Rightarrow \quad F_{OC'} = (400\,\mathrm{lb})\frac{\sin 30°}{\sin 90°} = 200\,\mathrm{lb}.} \tag{3}$$

Problem 2.29

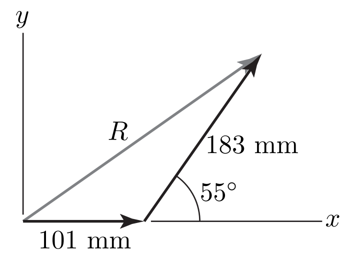
For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.2 on p. 43.

Solution

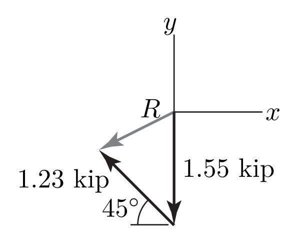
Part (a) The 101 mm and 183 mm position vectors are shown to the right with an xy Cartesian coordinate system, and the sum of these vectors is given by

$$\boxed{\begin{aligned}\vec{R} &= \left[101\,\hat{\imath} + 183(\cos 55°\,\hat{\imath} + \sin 55°\,\hat{\jmath})\right] \text{ mm} \\ &= (206\,\hat{\imath} + 150\,\hat{\jmath}) \text{ mm}.\end{aligned}} \tag{1}$$



Part (b) The 1.23 kip and 1.55 kip force vectors are shown to the right with an xy Cartesian coordinate system, and the sum of these vectors is given by

$$\boxed{\begin{aligned}\vec{R} &= \left[-1.55\,\hat{\jmath} + 1.23(-\cos 45°\,\hat{\imath} + \sin 45°\,\hat{\jmath})\right] \text{ kip}, \\ &= (-0.870\,\hat{\imath} - 0.680\,\hat{\jmath}) \text{ kip}.\end{aligned}} \tag{2}$$



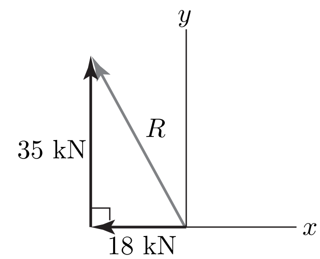
Problem 2.30

For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.3 on p. 43.

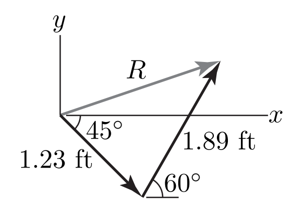
Solution

Part (a) The 18 kN and 35 kN force vectors are shown to the right with an xy Cartesian coordinate system, and the sum of these vectors is given by



$$\boxed{\vec{R} = (-18\,\hat{\imath} + 35\,\hat{\jmath})\ \text{kN}.} \tag{1}$$

The 1.23 ft and 1.89 ft position vectors are shown at the right with an xy Cartesian coordinate system, and the sum of these vectors is given by



$$\boxed{\begin{aligned}\vec{R} &= 1.23(\cos 45°\,\hat{\imath} - \sin 45°\,\hat{\jmath})\ \text{ft} + 1.89(\cos 60°\,\hat{\imath} + \sin 60°\,\hat{\jmath})\ \text{ft} && (2)\\ &= (1.81\,\hat{\imath} + 0.767\,\hat{\jmath})\ \text{ft}. && (3)\end{aligned}}$$

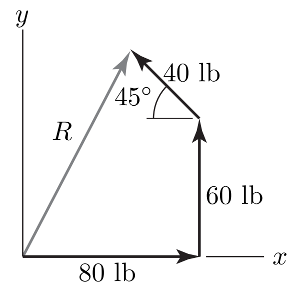
Problem 2.31

For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.13 on p. 45.

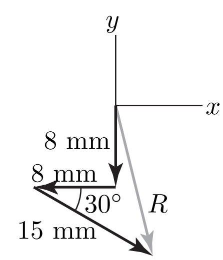
Solution

Part (a) The 40 lb, 60 lb, and 80 lb force vectors are shown at the right with an xy Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = \left[80\,\hat{\imath} + 60\,\hat{\jmath} + 40(-\cos 45°\,\hat{\imath} + \sin 45°\,\hat{\jmath})\right] \text{ lb} \tag{1}$$
$$= (51.7\,\hat{\imath} + 88.3\,\hat{\jmath}) \text{ lb.} \tag{2}$$

Part (b) The 8 mm, 8 mm, and 15 mm position vectors are shown to the right with an xy Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = \left[-8\,\hat{\imath} - 8\,\hat{\jmath} + 15(\cos 30°\,\hat{\imath} - \sin 30°\,\hat{\jmath})\right] \text{ mm} \tag{3}$$
$$= (4.99\,\hat{\imath} - 15.5\,\hat{\jmath}) \text{ mm.} \tag{4}$$

Problem 2.32

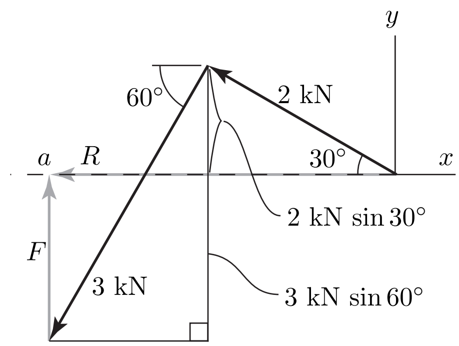
For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.16 on p. 45.

Solution

A force polygon shown at the right is constructed by selecting an xy Cartesian coordinate system, and then sketching the known force vectors (the 2 kN and 3 kN forces), followed by sketching the unknown force such that the resultant lies in the negative x direction (the a line in the problem statement). Based on this force polygon, $\vec{F}$ must act in the y direction, and its magnitude is given by

$$F = (3\,\text{kN})\sin 60° - (2\,\text{kN})\sin 30° = 1.598\,\text{kN}. \qquad (1)$$



Therefore, using Cartesian representation, $\vec{F}$ may be written as

$$\boxed{\vec{F} = 1.60\,\hat{\jmath}\,\text{kN}.} \qquad (2)$$

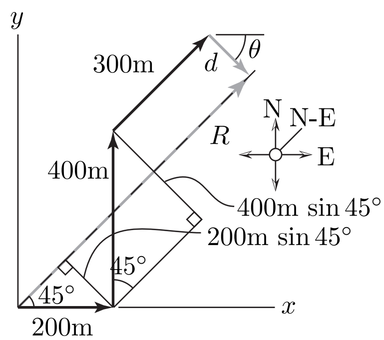
Problem 2.33

For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.17 on p. 45.

Solution

We sketch a vector polygon along with an xy Cartesian coordinate system. The vector $\vec{d}$ should be perpendicular to the dashed line shown. As such, $\theta = 45°$ and the magnitude d of vector $\vec{d}$ is given by



$$d = (400\,\mathrm{m})\sin 45° - (200\,\mathrm{m})\sin 45° = 141.4\,\mathrm{m}, \tag{1}$$

hence

$$\boxed{\vec{d} = (141.4\,\mathrm{m})(\cos 45°\,\hat{\imath} - \sin 45°\,\hat{\jmath}) = (100\,\hat{\imath} - 100\,\hat{\jmath})\ \mathrm{m}.} \tag{2}$$

where $\hat{\imath}$ and $\hat{\jmath}$ correspond to east and north, respectively.

The resultant vector $\vec{R}$ is given by the sum of the four vectors

$$\vec{R} = \left[200\,\hat{\imath} + 400\,\hat{\jmath} + 300(\cos 45°\,\hat{\imath} + \sin 45°\,\hat{\jmath}) + 100\,\hat{\imath} - 100\,\hat{\jmath}\ \mathrm{m} = 512.1(\hat{\imath} + \hat{\jmath})\right]\ \mathrm{m}. \tag{3}$$

The magnitude of the above expression is the final distance from her starting point to ending point, thus

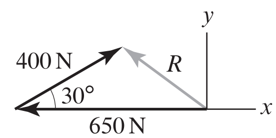
$$\boxed{R = \sqrt{(512.1\,\mathrm{m})^2 + (512.1\,\mathrm{m})^2} = 724\,\mathrm{m}.} \tag{4}$$

Problem 2.34

For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.18 on p. 45.

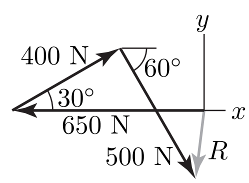
Solution



Part (a) Using the force polygon to the right, the resultant force $\vec{R}$ may be written as

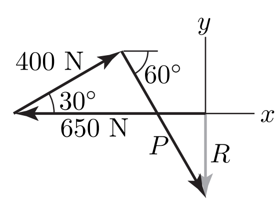
$$\boxed{\vec{R} = \left[-650\,\hat{\imath} + 400(\cos 30°\,\hat{\imath} + \sin 30°\,\hat{\jmath})\right]\,\text{N} = (-304\,\hat{\imath} + 200\,\hat{\jmath})\ \text{N}.} \tag{1}$$

Remark: In the solution to Prob. 2.18, the resultant force for Part (a) was called $\vec{Q}$.



Part (b) Using the force polygon to the right, the resultant force $\vec{R}$ may be written as

$$\boxed{\begin{aligned}\vec{R} &= \left[-650\,\hat{\imath} + 400(\cos 30°\,\hat{\imath} + \sin 30°\,\hat{\jmath}) + 500(\cos 60°\,\hat{\imath} - \sin 60°\,\hat{\jmath})\right]\,\text{N}\\ &= (-53.6\,\hat{\imath} - 233\,\hat{\jmath})\ \text{N}.\end{aligned}} \tag{2}$$



Part (c) The resultant force vector $\vec{R}$ is required to be vertical. Thus, we sketch the force polygon shown at the right. Using this force polygon, the x component of $\vec{P}$ is

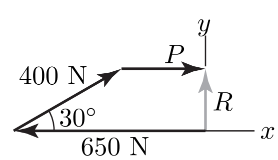
$$P_x = 650\,\text{N} - (400\,\text{N})\cos 30° = 303.6\,\text{N}, \tag{3}$$

and the y component is found by

$$\tan 60° = |P_y/P_x| \quad \Rightarrow \quad P_y = (-303.6\,\text{N})\tan 60° = -525.8\,\text{N}. \tag{4}$$

where the negative sign is inserted because the vertical component of $\vec{P}$ acts in the negative y direction. Thus it follows that

$$\boxed{P = \sqrt{P_x^2 + P_y^2} = 607\,\text{N}.} \tag{5}$$



Part (d) The 400 N and 650 N force vectors are shown in the force polygon at the right along with a vertical resultant force R. The smallest value of P occurs when this vector's direction is perpendicular to the resultant. Thus, it follows that

$$\boxed{P = 650\,\text{N} - (400\,\text{N})\cos 30° = 304\,\text{N} \quad \text{and} \quad \alpha = 0°.} \tag{6}$$

Problem 2.35

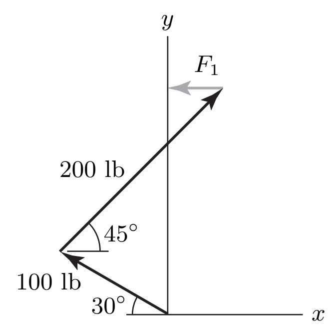
For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.21 on p. 46.

Solution

The force polygon shown at the right includes the 100 lb and 200 lb force vectors, along with the smallest possible force F_1 such that the resultant of the three vectors is vertical. Using this force polygon

$$\boxed{F_1 = (200\,\text{lb})\cos 45^\circ - (100\,\text{lb})\cos 30^\circ = 54.8\,\text{lb} \quad \text{and} \quad \alpha = 90^\circ.} \tag{1}$$



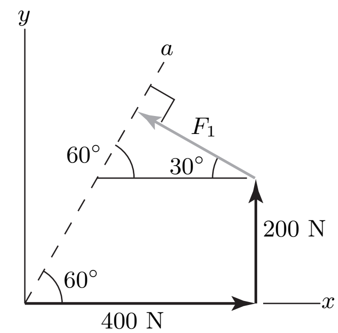
Problem 2.36

For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.22 on p. 46.

Solution

The dashed line in the figure to the right represents the direction along which the resultant force vector $\vec{R}$ is required to act. The horizontal and vertical components of $\vec{R}$ are given by



$$R_x = 400\,\text{N} - F_1 \cos 30°, \quad \text{and} \quad R_y = 200\,\text{N} + F_1 \sin 30°. \tag{1}$$

It follows that

$$\boxed{\tan 60° = \frac{R_y}{R_x} = \frac{200\,\text{N} + F_1 \sin 30°}{400\,\text{N} - F_1 \cos 30°} \quad \Rightarrow \quad F_1 = 246\,\text{N}.} \tag{2}$$

Note that α equals 60° since F_1 is perpendicular to line a. Using Cartesian representation, $\vec{F}_1$ is given by

$$\boxed{\vec{F}_1 = (246\,\text{N})(-\cos 30°\,\hat{\imath} + \sin 30°\,\hat{\jmath}) = (-213\,\hat{\imath} + 123\,\hat{\jmath})\,\text{N}.} \tag{3}$$

Problem 2.37

Let $\vec{A} = (150\,\hat{\imath} - 200\,\hat{\jmath})$ lb and $\vec{B} = (200\,\hat{\imath} + 480\,\hat{\jmath})$ lb. Evaluate the following, and for Parts (a) and (b) state the magnitude of $\vec{R}$.

(a) $\vec{R} = \vec{A} + \vec{B}$.

(b) $\vec{R} = 2\vec{A} - (1/2)\vec{B}$.

(c) Find a scalar s such that $\vec{R} = s\vec{A} + \vec{B}$ has an x component only.

(d) Determine a dimensionless unit vector in the direction $\vec{B} - \vec{A}$.

Solution

Part (a)

$$\vec{R} = (150\,\hat{\imath} - 200\,\hat{\jmath})\text{ lb} + (200\,\hat{\imath} + 480\,\hat{\jmath})\text{ lb} = (150 + 200)\hat{\imath}\text{ lb} + (-200 + 480)\,\hat{\jmath}\text{ lb}, \tag{1}$$

which simplifies to

$$\boxed{\vec{R} = (350\,\hat{\imath} + 280\,\hat{\jmath})\text{ lb.}} \tag{2}$$

The magnitude R is given by

$$\boxed{R = \sqrt{(350)^2 + (280)^2}\text{ lb} = 448\text{ lb.}} \tag{3}$$

Part (b)

$$\vec{R} = (300\,\hat{\imath} - 400\,\hat{\jmath})\text{ lb} - (100\,\hat{\imath} + 240\,\hat{\jmath})\text{ lb} = (300 - 100)\hat{\imath}\text{ lb} + (-400 - 240)\,\hat{\jmath}\text{ lb}, \tag{4}$$

which simplifies to

$$\boxed{\vec{R} = (200\,\hat{\imath} - 640\,\hat{\jmath})\text{ lb.}} \tag{5}$$

The magnitude R is given by

$$\boxed{R = \sqrt{(200)^2 + (-640)^2}\text{ lb} = 671\text{ lb.}} \tag{6}$$

Part (c)

$$\vec{R} = s(150\,\hat{\imath} - 200\,\hat{\jmath})\text{ lb} + (200\,\hat{\imath} + 480\,\hat{\jmath})\text{ lb} = (150s + 200)\hat{\imath}\text{ lb} + (-200s + 480)\,\hat{\jmath}\text{ lb}, \tag{7}$$

where, according to the problem statement,

$$R_y = 0 \quad \Rightarrow \quad -200s + 480 = 0 \quad \Rightarrow \quad \boxed{s = 480/200 = 2.40.} \tag{8}$$

Applying this value of s to Eq. (8) yields

$$\boxed{\vec{R} = [(150)(2.40) + 200]\,\hat{\imath}\text{ lb} = 560\,\hat{\imath}\text{ lb}, \quad R = \sqrt{(560)^2 + 0^2}\text{ lb} = 560\text{ lb.}} \tag{9}$$

Part (d)

$$\vec{R} = [(200\,\hat{\imath} + 480\,\hat{\jmath}) - (150\,\hat{\imath} - 200\,\hat{\jmath})]\ \text{lb} = (50\,\hat{\imath} + 680\,\hat{\jmath})\ \text{lb} \tag{10}$$

$$R = \sqrt{(50)^2 + (680)^2}\text{lb} = 681.8\,\text{lb}. \tag{11}$$

The unit vector in the direction of $\vec{R}$ is $\hat{R}$, and it is given by

$$\boxed{\hat{R} = \frac{\vec{R}}{R} = \frac{(50\,\hat{\imath} + 680\,\hat{\jmath})\ \text{lb}}{681.8\,\text{lb}} = 0.0733\,\hat{\imath} + 0.997\,\hat{\jmath}.} \tag{12}$$

Problem 2.38

Let $\vec{A} = (-6\,\hat{\imath} + 8\,\hat{\jmath})\,\text{kN}$ and $\vec{B} = (-9\,\hat{\imath} - 12\,\hat{\jmath})\,\text{kN}$. Evaluate the following, and for Parts (a) and (b) state the magnitude of $\vec{R}$.

(a) $\vec{R} = \vec{A} + \vec{B}$.

(b) $\vec{R} = -2\vec{A} + \vec{B}$.

(c) Find a scalar s such that $\vec{R} = -\vec{A} + s\vec{B}$ has a y component only.

(d) Determine a dimensionless unit vector in the direction $\vec{A} - \vec{B}$.

Solution

Part (a)

$$\begin{aligned} \vec{R} &= \vec{A} + \vec{B} && (1)\\ &= (-6\,\hat{\imath} + 8\,\hat{\jmath})\,\text{kN} + (-9\,\hat{\imath} - 12\,\hat{\jmath})\,\text{kN} && (2)\\ &= (-6 - 9)\,\hat{\imath}\,\text{kN} + (8 - 12)\,\hat{\jmath}\,\text{kN} && (3)\\ &= \boxed{(-15\,\hat{\imath} - 4\,\hat{\jmath})\,\text{kN}.} && (4) \end{aligned}$$

The magnitude of $\vec{R}$ is

$$\boxed{R = \sqrt{(-15\,\text{kN})^2 + (-4\,\text{kN})^2} = 15.52\,\text{kN}.} \qquad (5)$$

Part (b)

$$\begin{aligned} \vec{R} &= -2\vec{A} + \vec{B} && (6)\\ &= -2\,(-6\,\hat{\imath} + 8\,\hat{\jmath})\,\text{kN} + (-9\,\hat{\imath} - 12\,\hat{\jmath})\,\text{kN} && (7)\\ &= (12 - 9)\,\hat{\imath}\,\text{kN} + (-16 - 12)\,\hat{\jmath}\,\text{kN} && (8)\\ &= \boxed{(3\,\hat{\imath} - 28\,\hat{\jmath})\,\text{kN}.} && (9) \end{aligned}$$

The magnitude of $\vec{R}$ is

$$\boxed{R = \sqrt{(3\,\text{kN})^2 + (-28\,\text{kN})^2} = 28.16\,\text{kN}.} \qquad (10)$$

Part (c)

$$\begin{aligned} \vec{R} &= -\vec{A} + s\vec{B} && (11)\\ &= -(-6\,\hat{\imath} + 8\,\hat{\jmath})\,\text{kN} + s\,(-9\,\hat{\imath} - 12\,\hat{\jmath})\,\text{kN} && (12)\\ &= (6 - 9s)\,\hat{\imath}\,\text{kN} + (-8 - 12\,s)\,\hat{\jmath}\,\text{kN}. && (13) \end{aligned}$$

For Eq. (13) to have a y component only, the x component must be zero. Hence

$$R_x = 0 \quad \Rightarrow \quad (6 - 9s)\,\text{kN} = 0 \quad \Rightarrow \quad \boxed{s = \frac{6}{9} = 0.6667.} \qquad (14)$$

Part (d)

$$\vec{R} = \vec{A} - \vec{B} \tag{15}$$
$$= (-6\,\hat{\imath} + 8\,\hat{\jmath})\,\text{kN} - (-9\,\hat{\imath} - 12\,\hat{\jmath})\,\text{kN} \tag{16}$$
$$= (3\,\hat{\imath} + 20\,\hat{\jmath})\,\text{kN}, \tag{17}$$
$$R = \sqrt{(3\,\text{kN})^2 + (20\,\text{kN})^2} = 20.22\,\text{kN}. \tag{18}$$

The unit vector in the direction of $\vec{R}$ is $\hat{R}$, and it is given by

$$\boxed{\hat{R} = \frac{\vec{R}}{R} = \frac{(3\,\hat{\imath} + 20\,\hat{\jmath})\,\text{kN}}{20.22\,\text{kN}} = 0.1483\,\hat{\imath} + 0.9889\,\hat{\jmath}.} \tag{19}$$

Problem 2.39

Let $\vec{A} = (150\,\hat{\imath} + 200\,\hat{\jmath})$ mm, $\vec{B} = (300\,\hat{\imath} - 450\,\hat{\jmath})$ mm, and $\vec{C} = (-100\,\hat{\imath} - 250\,\hat{\jmath})$ mm. Evaluate the following, and for Parts (a) and (b) state the magnitude of $\vec{R}$.

(a) $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

(b) $\vec{R} = 3\vec{A} - 2\vec{B} + \vec{C}$.

(c) Find scalars r and s, if possible, such that $\vec{R} = r\vec{A} + s\vec{B} + \vec{C}$ has zero x and y components.

(d) Determine a dimensionless unit vector in the direction $\vec{A} + \vec{B} + \vec{C}$.

Solution

Part (a)

$$
\begin{aligned}
\vec{R} &= \vec{A} + \vec{B} + \vec{C} && (1)\\
&= (150\,\hat{\imath} + 200\,\hat{\jmath})\text{ mm} + (300\,\hat{\imath} - 450\,\hat{\jmath})\text{ mm} + (-100\,\hat{\imath} - 250\,\hat{\jmath})\text{ mm} && (2)\\
&= (150 + 300 - 100)\,\hat{\imath}\text{ mm} + (200 - 450 - 250)\,\hat{\jmath}\text{ mm} && (3)\\
&= \boxed{(350\,\hat{\imath} - 500\,\hat{\jmath})\text{ mm.}} && (4)
\end{aligned}
$$

The magnitude of $\vec{R}$ is

$$\boxed{R = \sqrt{(350\text{ mm})^2 + (-500\text{ mm})^2} = 610.3\text{ mm.}} \quad (5)$$

Part (b)

$$
\begin{aligned}
\vec{R} &= 3\vec{A} - 2\vec{B} + \vec{C} && (6)\\
&= 3(150\,\hat{\imath} + 200\,\hat{\jmath})\text{ mm} - 2(300\,\hat{\imath} - 450\,\hat{\jmath})\text{ mm} + (-100\,\hat{\imath} - 250\,\hat{\jmath})\text{ mm} && (7)\\
&= (450 - 600 - 100)\,\hat{\imath}\text{ mm} + (600 + 900 - 250)\,\hat{\jmath}\text{ mm} && (8)\\
&= \boxed{(-250\,\hat{\imath} + 1250\,\hat{\jmath})\text{ mm.}} && (9)
\end{aligned}
$$

The magnitude of $\vec{R}$ is

$$\boxed{R = \sqrt{(-250\text{ mm})^2 + (1250\text{ mm})^2} = 1275\text{ mm.}} \quad (10)$$

Part (c)

$$
\begin{aligned}
\vec{R} &= r\vec{A} + s\vec{B} + \vec{C} && (11)\\
&= r(150\,\hat{\imath} + 200\,\hat{\jmath})\text{ mm} + s(300\,\hat{\imath} - 450\,\hat{\jmath})\text{ mm} + (-100\,\hat{\imath} - 250\,\hat{\jmath})\text{ mm} && (12)\\
&= (150\,r + 300\,s - 100)\,\hat{\imath}\text{ mm} + (200\,r - 450\,s - 250)\,\hat{\jmath}\text{ mm.} && (13)
\end{aligned}
$$

For Eq. (13) to have zero x and y components requires

$$150\,r + 300\,s\ - 100 = 0, \tag{14}$$

$$200\,r - 450\,s - 250 = 0. \tag{15}$$

We solve Eq. (14) for r to obtain

$$r = \frac{1}{150}\,(-300\,s + 100), \tag{16}$$

and then we substitute this result into Eq. (15) to obtain

$$200\left[\frac{1}{150}(-300\,s + 100)\right] - 450\,s - 250 = 0. \tag{17}$$

Solving the above equation for s provides

$$\boxed{s = -0.1373\,,} \tag{18}$$

and substituting this result into Eq. (16) provides

$$\boxed{r = 0.9412\,.} \tag{19}$$

To check the accuracy of our solution, we substitute Eqs. (18) and (19) into Eqs. (14) and (15) to verify that they are satisfied.

Part (d) From Part (a),

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = (350\,\hat{\imath} - 500\,\hat{\jmath})\,\text{mm}. \tag{20}$$

The unit vector in the direction of $\vec{R}$ is $\hat{R}$, and it is given by

$$\boxed{\hat{R} = \frac{\vec{R}}{R} = \frac{(350\,\hat{\imath}\ - 500\,\hat{\jmath})\,\text{mm}}{610.3\,\text{mm}} = 0.5735\,\hat{\imath} - 0.8192\,\hat{\jmath}.} \tag{21}$$

Problem 2.40

A rope connecting points A and B supports the force F shown in the figure. Write expressions using Cartesian vector representation for the following:

(a) $\vec{r}_{AB}$: the position vector from A to B.

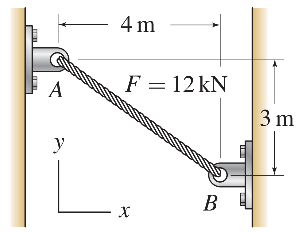
(b) $\vec{r}_{BA}$: the position vector from B to A.

(c) $\hat{u}_{AB}$: the unit vector in the direction from A to B.

(d) $\hat{u}_{BA}$: the unit vector in the direction from B to A.

(e) $\vec{F}_{AB}$: the force vector the rope applies to A.

(f) $\vec{F}_{BA}$: the force vector the rope applies to B.



Solution

Part (a)

$$\boxed{\vec{r}_{AB} = (4\,\hat{\imath} - 3\,\hat{\jmath})\,\text{m}.} \tag{1}$$

Part (b)

$$\boxed{\vec{r}_{BA} = (-4\,\hat{\imath} + 3\,\hat{\jmath})\,\text{m}.} \tag{2}$$

Part (c) Using the results of Part (a), we evaluate

$$|\vec{r}_{AB}| = \sqrt{(4\,\text{m})^2 + (-3\,\text{m})^2} = 5\,\text{m}, \tag{3}$$

and hence,

$$\boxed{\hat{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{4}{5}\,\hat{\imath} - \frac{3}{5}\,\hat{\jmath}.} \tag{4}$$

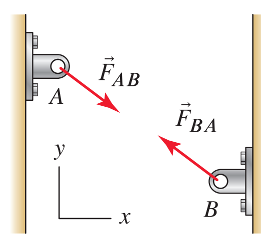
Part (d)

$$\boxed{\hat{u}_{BA} = -\hat{u}_{AB} = -\frac{4}{5}\,\hat{\imath} + \frac{3}{5}\,\hat{\jmath}.} \tag{5}$$

Part (e)

$$\vec{F}_{AB} = (12\,\text{kN})\,\hat{u}_{AB} = (12\,\text{kN})\left(\frac{4}{5}\,\hat{\imath} - \frac{3}{5}\,\hat{\jmath}\right), \tag{6}$$

$$\boxed{\vec{F}_{AB} = (9.6\,\hat{\imath} - 7.2\,\hat{\jmath})\,\text{kN}.} \tag{7}$$



Part (f) The sketch at the right shows how the forces F_{AB} and F_{BA} are related; namely $F_{BA} = -F_{AB}$. Thus

$$\boxed{\vec{F}_{BA} = -\vec{F}_{AB} = (-9.6\,\hat{\imath} + 7.2\,\hat{\jmath})\,\text{kN}.} \tag{8}$$

Problem 2.41

A rope connecting points A and B supports the force F shown in the figure. Write expressions using Cartesian vector representation for the following:

(a) $\vec{r}_{AB}$: the position vector from A to B.

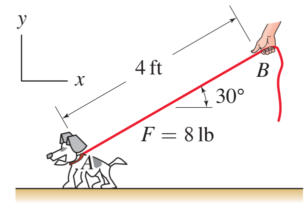
(b) $\vec{r}_{BA}$: the position vector from B to A.

(c) $\hat{u}_{AB}$: the unit vector in the direction from A to B.

(d) $\hat{u}_{BA}$: the unit vector in the direction from B to A.

(e) $\vec{F}_{AB}$: the force vector the rope applies to A.

(f) $\vec{F}_{BA}$: the force vector the rope applies to B.



Solution

Part (a)

$$\boxed{\vec{r}_{AB} = (4\,\text{ft})\left(\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath}\right) = (3.46\,\hat{\imath} + 2.00\,\hat{\jmath})\ \text{ft}.} \tag{1}$$

Part (b)

$$\boxed{\vec{r}_{BA} = (4\,\text{ft})\left(-\cos 30^\circ\,\hat{\imath} - \sin 30^\circ\,\hat{\jmath}\right) = (-3.46\,\hat{\imath} - 2.00\,\hat{\jmath})\ \text{ft}.} \tag{2}$$

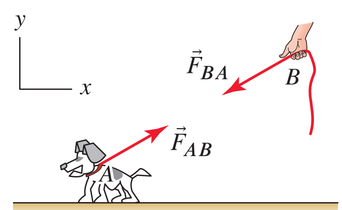
Part (c)

$$\boxed{\hat{u}_{AB} = \cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath} = 0.866\,\hat{\imath} + 0.500\,\hat{\jmath}.} \tag{3}$$

Part (d)

$$\boxed{\hat{u}_{BA} = -\hat{u}_{AB} = -\cos 30^\circ\,\hat{\imath} - \sin 30^\circ\,\hat{\jmath} = -0.866\,\hat{\imath} - 0.500\,\hat{\jmath}.} \tag{4}$$

Part (e)



$$\vec{F}_{AB} = (8\,\text{lb})\,\hat{u}_{AB} = (8\,\text{lb})\left(\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath}\right), \tag{5}$$

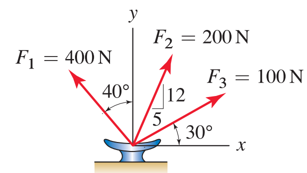
$$\boxed{\vec{F}_{AB} = (6.93\,\hat{\imath} + 4.00\,\hat{\jmath})\,\text{lb}.} \tag{6}$$

Part (f) The sketch at the right shows how the forces F_{AB} and F_{BA} are related; namely $F_{BA} = -F_{AB}$. Thus

$$\boxed{\vec{F}_{BA} = -\vec{F}_{AB} = (-6.93\,\hat{\imath} - 4.00\,\hat{\jmath})\,\text{kN}.} \tag{7}$$

Problem 2.42

A cleat on a boat is used to support forces from three ropes as shown. Determine the resultant force vector $\vec{R}$, using Cartesian representation, and determine the magnitude R. Also express $\vec{R}$ in polar vector representation.



Solution

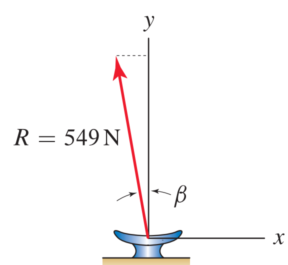
In Cartesian vector representation, $\vec{R}$ is expressed as

$$\boxed{\begin{aligned}\vec{R} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 && (1)\\ &= (400\,\mathrm{N})(-\sin 40°\,\hat{\imath} + \cos 40°\,\hat{\jmath}) + (200\,\mathrm{N})\left(\tfrac{5}{13}\hat{\imath} + \tfrac{12}{13}\hat{\jmath}\right) + (100\,\mathrm{N})(\cos 30°\,\hat{\imath} + \sin 30°\,\hat{\jmath}) && (2)\\ &= (-93.6\,\hat{\imath} + 541\,\hat{\jmath})\,\mathrm{N}. && (3)\end{aligned}}$$

The magnitude R of the resultant force is given by

$$\boxed{R = \sqrt{(-93.59)^2 + (541.0)^2}\,\mathrm{N} = 549\,\mathrm{N}.} \tag{4}$$

The resultant force is shown in its proper orientation in the figure to the right. The orientation β of the resultant, measured from the vertical, is



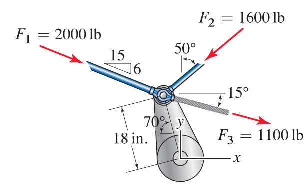
$$\beta = \tan^{-1}\left(\frac{541.0\,\mathrm{N}}{93.59\,\mathrm{N}}\right) = 9.814°. \tag{5}$$

The orientation θ of the resultant, measured from the right-hand horizontal direction, is $\theta = 90° + 9.814° = 99.81°$; therefore, the resultant force using polar vector representation is

$$\boxed{\vec{R} = 549\,\mathrm{N} \ @\ 99.8°\measuredangle.} \tag{6}$$

Problem 2.43

A control arm in a machine supports the three forces shown. Determine the resultant force vector $\vec{R}$, using Cartesian representation, and determine the magnitude R. Also express $\vec{R}$ in polar vector representation.



Solution

The resultant force is

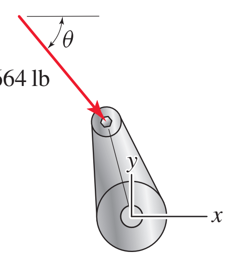
$$\begin{aligned}\vec{R} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \qquad (1)\\ &= 2000\,\text{lb}\,\frac{15\,\hat{\imath} - 6\,\hat{\jmath}}{\sqrt{(15)^2 + (-6)^2}}\\ &\quad + 1600\,\text{lb}\,(-\sin 50^\circ\ \hat{\imath} - \cos 50^\circ\,\hat{\jmath})\\ &\quad + 1100\,\text{lb}\,(\cos\ 15^\circ\,\hat{\imath} - \sin 15^\circ\ \hat{\jmath}) \qquad (2)\\ &= \boxed{(1694\,\hat{\imath} - 2056\,\hat{\jmath})\ \text{lb}.} \qquad (3)\end{aligned}$$

The magnitude of the resultant force is

$$\boxed{R = \sqrt{(1694\,\text{lb})^2 + (-2056\,\text{lb})^2} = 2664\,\text{lb}.} \qquad (4)$$

The resultant force is shown in its proper orientation to the right. The orientation θ of the resultant from the horizontal direction is

$$\theta = \tan^{-1}\left(\frac{2056\,\text{lb}}{1694\,\text{lb}}\right) = 50.52^\circ. \qquad (5)$$



Using polar vector representation, the resultant force is

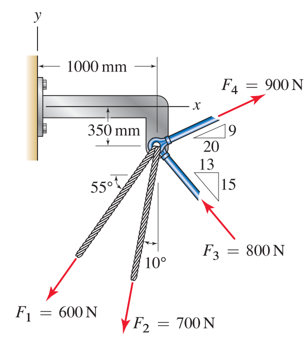
$$\boxed{\vec{R} = 2664\,\text{lb} \ @\ -50.52^\circ \measuredangle.} \qquad (6)$$

If desired, the above resultant force may be stated using a positive angle, where $360^\circ - 50.52^\circ = 309.5^\circ$, as

$$\boxed{\vec{R} = 2664\,\text{lb} \ @\ 309.5^\circ \measuredangle.} \qquad (7)$$

Problem 2.44

A cantilevered bracket supports forces from two bars and two cables, as shown. Determine the resultant force vector $\vec{R}$, using Cartesian representation, and determine the magnitude R. Also express $\vec{R}$ in polar vector representation.



Solution

The resultant force is

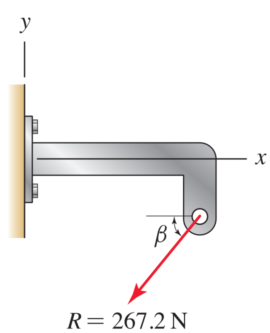
$$\begin{aligned}\vec{R} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 && (1)\\
&= 600\,\text{N}\,(-\cos 55^\circ\,\hat{\imath} - \sin 55^\circ\,\hat{\jmath})\\
&\quad + 700\,\text{N}(-\sin 10^\circ\,\hat{\imath} - \cos 10^\circ\,\hat{\jmath})\\
&\quad + 800\,\text{N}\left(\frac{-13\,\hat{\imath} + 15\,\hat{\jmath}}{\sqrt{(-13)^2+(15)^2}}\right)\\
&\quad + 900\,\text{N}\left(\frac{20\,\hat{\imath} + 9\,\hat{\jmath}}{\sqrt{(20)^2+(9)^2}}\right) && (2)\\
&= \boxed{(-168.9\,\hat{\imath} - 207.0\,\hat{\jmath})\,\text{N}.} && (3)\end{aligned}$$

The magnitude of the resultant force is

$$\boxed{R = \sqrt{(-168.9\,\text{N})^2 + (-207.0\,\text{N})^2} = 267.2\,\text{N}.} \quad (4)$$

The resultant force is shown in its proper orientation to the right. The orientation β of the resultant from the horizontal direction shown is

$$\beta = \tan^{-1}\left(\frac{207.0\,\text{N}}{168.9\,\text{N}}\right) = 50.78^\circ. \quad (5)$$



The orientation θ of the resultant measured from the right hand horizontal direction is $\theta = 180^\circ + 50.78^\circ = 230.8^\circ$. Therefore, the resultant force using polar vector representation is

$$\boxed{\vec{R} = 267.2\,\text{N} @ 230.8^\circ \measuredangle.} \quad (6)$$

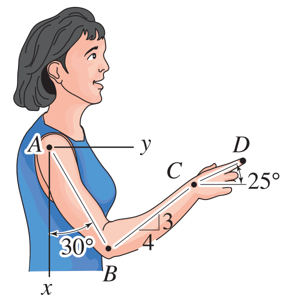
Problem 2.45

A model of a person's arm is used for ergonomics studies. Distances are $r_{AB} = 35\,\text{cm}$, $r_{BC} = 28\,\text{cm}$, and $r_{CD} = 19\,\text{cm}$.

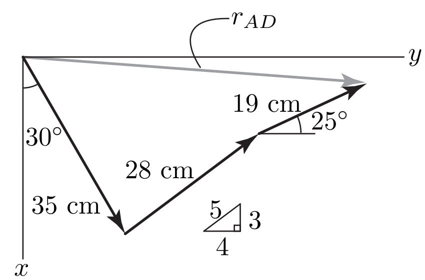
(a) Determine the position vector $\vec{r}_{AD}$ and its magnitude r_{AD}.

(b) Express $\vec{r}_{AD}$ using polar vector representation, measuring orientation positive counterclockwise from the right-hand horizontal direction.

(c) Determine a unit vector $\hat{u}_{AD}$ in the direction from A to D.



Solution



Part (a) The addition $\vec{r}_{AD} = \vec{r}_{AB} + \vec{r}_{BC} + \vec{r}_{CD}$ gives the vector polygon shown at the right. Thus,

$$\begin{aligned}\vec{r}_{AD} &= (35\,\text{cm})(\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath}) + (28\,\text{cm})(-\tfrac{3}{5}\,\hat{\imath} + \tfrac{4}{5}\,\hat{\jmath}) &(1)\\ &\quad + (19\,\text{cm})(-\sin 25^\circ\,\hat{\imath} + \cos 25^\circ\,\hat{\jmath}) \\ &= (5.481\,\hat{\imath} + 57.12\,\hat{\jmath})\ \text{cm}, &(2)\end{aligned}$$

and the magnitude of this vector is given by

$$r_{AD} = \sqrt{(5.481)^2 + (57.12)^2}\,\text{cm} = 57.38\,\text{cm}. \qquad (3)$$

In summary,

$$\boxed{\vec{r}_{AD} = (5.48\,\hat{\imath} + 57.1\,\hat{\jmath})\ \text{cm}, \quad r_{AD} = 57.4\,\text{cm}.} \qquad (4)$$

Part (b) The orientation of $\vec{r}_{AD}$ measured counterclockwise from the right-hand horizontal direction is

$$\theta = -\tan^{-1}\left(\frac{5.481\,\text{cm}}{57.12\,\text{cm}}\right) = -5.481^\circ. \qquad (5)$$

Using polar vector representation, the resultant vector is

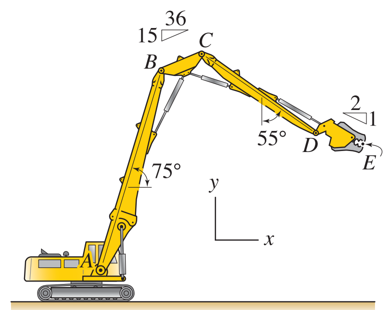
$$\boxed{\vec{r}_{AD} = 57.4\,\text{cm} \ @\ -5.48^\circ \measuredangle} \qquad (6)$$

Part (c) To determine the unit vector $\hat{u}_{AD}$, we use the results from Eqs. (2) and (3) to write

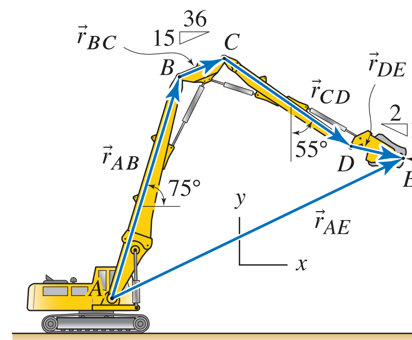
$$\boxed{\hat{u}_{AD} = \frac{\vec{r}_{AD}}{r_{AD}} = \frac{(5.481\,\hat{\imath} + 57.12\,\hat{\jmath})\,\text{cm}}{57.38\,\text{cm}} = 0.0955\,\hat{\imath} + 0.995\,\hat{\jmath}.} \qquad (7)$$

Problem 2.46

A Caterpillar Ultra High Demolition machine is shown. The distances between points A and B is 12.5 m, points B and C is 2.8 m, C and D is 7 m, and D and E is 2.5 m. Determine the position vectors $\vec{r}_{AB}$, $\vec{r}_{BC}$, $\vec{r}_{CD}$, and $\vec{r}_{DE}$, where $\vec{r}_{AB}$ is the position vector from point A to point B, and so on. Add these vectors to determine the position vector $\vec{r}_{AE}$.



Solution



$$\vec{r}_{AB} = 12.5\,\text{m}\,(\cos 75^\circ\,\hat{\imath} + \sin 75^\circ\,\hat{\jmath}) \tag{1}$$

$$= \boxed{(3.235\,\hat{\imath} + 12.07\,\hat{\jmath})\,\text{m},} \tag{2}$$

$$\vec{r}_{BC} = 2.8\,\text{m}\left(\frac{36\,\hat{\imath} + 15\,\hat{\jmath}}{\sqrt{(36)^2 + (15)^2}}\right) \tag{3}$$

$$= \boxed{(2.585\,\hat{\imath} + 1.077\,\hat{\jmath})\,\text{m},} \tag{4}$$

$$\vec{r}_{CD} = 7\,\text{m}\,(\sin 55^\circ\,\hat{\imath} - \cos 55^\circ\,\hat{\jmath}) \tag{5}$$

$$= \boxed{(5.734\,\hat{\imath} - 4.015\,\hat{\jmath})\,\text{m},} \tag{6}$$

$$\vec{r}_{DE} = 2.5\,\text{m}\left(\frac{2\,\hat{\imath} - \hat{\jmath}}{\sqrt{(2)^2 + (-1)^2}}\right) \tag{7}$$

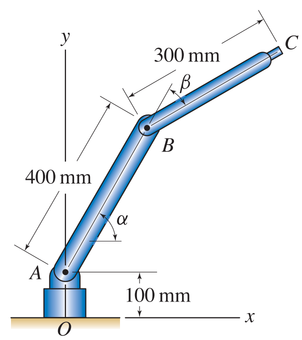
$$= \boxed{(2.236\,\hat{\imath} - 1.118\,\hat{\jmath})\,\text{m},} \tag{8}$$

$$\vec{r}_{AE} = \vec{r}_{AB} + \vec{r}_{BC} + \vec{r}_{CD} + \vec{r}_{DE} \tag{9}$$

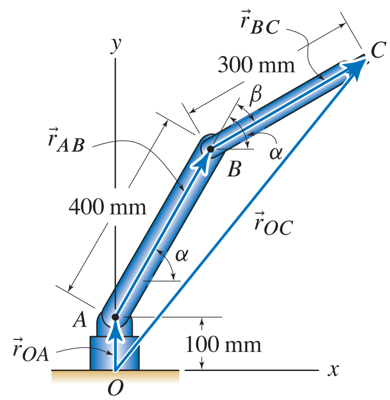
$$= \boxed{(13.79\,\hat{\imath} + 8.018\,\hat{\jmath})\,\text{m}.} \tag{10}$$

Problem 2.47

The actuator at point C of the robotic arm is positioned by specifying angles α and β, where α is measured positive counterclockwise from the positive x axis and β is measured positive clockwise from the AB direction to the BC direction. Determine the position vector from point O to point C in terms of angles α and β.



Solution



$$\vec{r}_{OA} = 100\,\text{mm}\,\hat{\jmath}, \tag{1}$$

$$\vec{r}_{AB} = 400\,\text{mm}\,(\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}), \tag{2}$$

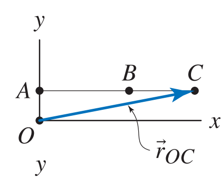
$$\vec{r}_{BC} = 300\,\text{mm}\,[\cos(\alpha-\beta)\,\hat{\imath} + \sin(\alpha-\beta)\,\hat{\jmath}], \tag{3}$$

$$\boxed{\begin{aligned}\vec{r}_{OC} &= \vec{r}_{OA} + \vec{r}_{AB} + \vec{r}_{BC} \qquad (4)\\ &= [400\,\text{mm}\,\cos\alpha + 300\,\text{mm}\,\cos(\alpha-\beta)]\,\hat{\imath}\\ &\quad + [100\,\text{mm} + 400\,\text{mm}\,\sin\alpha + 300\,\text{mm}\,\sin(\alpha-\beta)]\,\hat{\jmath}. \qquad (5)\end{aligned}}$$

Some simple checks may be performed to help verify the accuracy of $\vec{r}_{OC}$, as follows

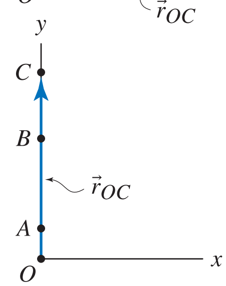
- If $\alpha = 0$ and $\beta = 0$, then

$$\vec{r}_{OC} = (700\,\hat{\imath} + 100\,\hat{\jmath})\,\text{mm}. \tag{6}$$



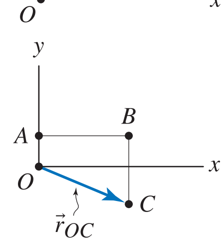
- If $\alpha = 90°$ and $\beta = 0°$, then

$$\vec{r}_{OC} = (0\,\hat{\imath} + 800\,\hat{\jmath})\,\text{mm}. \tag{7}$$



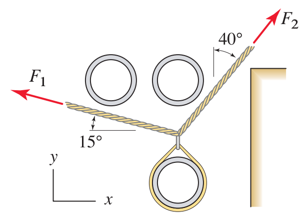
- If $\alpha = 0°$ and $\beta = 90°$, then

$$\vec{r}_{OC} = (400\,\hat{\imath} - 200\,\hat{\jmath})\,\text{mm}. \tag{8}$$



Problem 2.48

Two ropes are used to lift a pipe in a congested region. Determine the ratio F_2/F_1 so that the resultant of $\vec{F}_1$ and $\vec{F}_2$ is vertical.



Solution

If the resultant force $\vec{R}$ is given by

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = F_1(-\cos 15^\circ\,\hat{\imath} + \sin 15^\circ\,\hat{\jmath}) + F_2(\sin 40^\circ\,\hat{\imath} + \cos 40^\circ\,\hat{\jmath}) \quad (1)$$

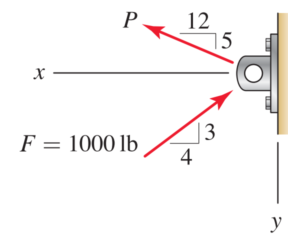
$$= (-F_1\cos 15^\circ + F_2\sin 40^\circ)\,\hat{\imath} + (F_1\sin 15^\circ + F_2\cos 40^\circ)\,\hat{\jmath}. \quad (2)$$

The problem statement asks for the x component of the resultant to be zero, thus

$$\boxed{0 = -F_1\cos 15^\circ + F_2\sin 40^\circ \quad \Rightarrow \quad \frac{F_2}{F_1} = \frac{\cos 15^\circ}{\sin 40^\circ} = 1.50.} \quad (3)$$

Problem 2.49

A welded steel tab is subjected to forces F and P. Determine the largest value P may have if $F = 1000\,\text{lb}$ and the magnitude of the resultant force cannot exceed 1500 lb.



Solution

The resultant $\vec{R}$ of the two forces applied to the tab is

$$\vec{R} = \vec{F} + \vec{P} = 1000\,\text{lb}(-\tfrac{4}{5}\,\hat{\imath} - \tfrac{3}{5}\,\hat{\jmath}) + P(\tfrac{12}{13}\,\hat{\imath} - \tfrac{5}{13}\,\hat{\jmath}) \tag{1}$$

$$= (-800\,\text{lb} + P\tfrac{12}{13})\,\hat{\imath} + (-600\,\text{lb} - P\tfrac{5}{13})\,\hat{\jmath} = R_x\,\hat{\imath} + R_y\,\hat{\jmath}. \tag{2}$$

Given that the resultant is $R = 1500\,\text{lb}$, and that $R^2 = R_x^2 + R_y^2$, we may write

$$(1500\,\text{lb})^2 = (-800\,\text{lb} + P\tfrac{12}{13})^2 + (-600\,\text{lb} - P\tfrac{5}{13})^2 \tag{3}$$

$$= (-800\,\text{lb})^2 + 2(-800\,\text{lb})(P\tfrac{12}{13}) + P^2(\tfrac{12}{13})^2 \tag{4}$$

$$+ (-600\,\text{lb})^2 + 2(-600\,\text{lb})(-P\tfrac{5}{13}) + P^2(\tfrac{5}{13})^2,$$

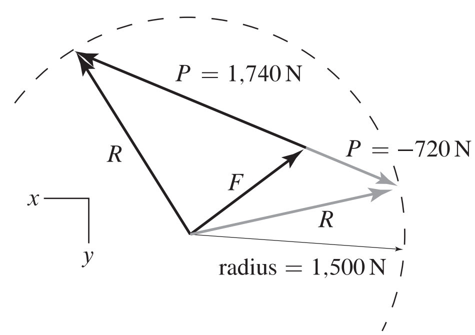
which may be simplified to yield

$$P^2 - \frac{1.32 \times 10^4\,\text{lb}}{13}P - 1.25\times 10^6\,\text{lb}^2 = 0. \tag{5}$$

Equation (5) is a quadratic equation, and it may be solved to obtain two solutions for P as follows

$$\boxed{P = \tfrac{1}{2}\left[\frac{1.32 \times 10^4}{13} \pm \sqrt{(-1.32 \times 10^4/13)^2 - 4(1)(-1.25 \times 10^6)}\right]\,\text{lb} = -720\,\text{lb},\ 1740\,\text{lb}.} \tag{6}$$

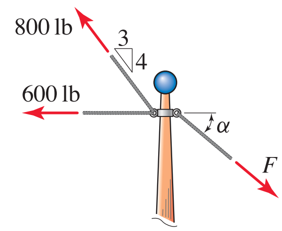
The results of Eq. (6) are graphically shown in the two force polygons at the right. The dashed circle is drawn with a radius of 1500 lb, and the resultant force vector must lie on this circle.



Remark: The force polygons shown at the right offer an alternative solution to this problem. For example, with $F = 1000\,\text{lb}$ and $R = 1500\,\text{lb}$, and knowing the directions for forces F and P, the laws of sines and cosines may be used to determine the two values of P obtained in Eq. (6).

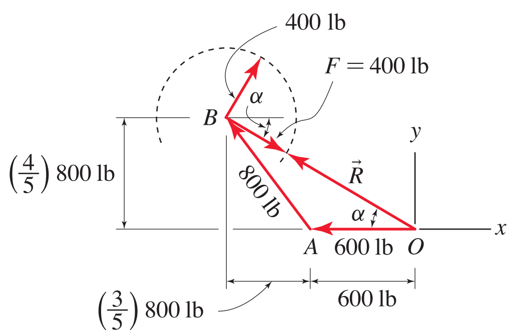
Problem 2.50

The mast of a ship supports forces from three cables as shown. If $F = 400$ lb, determine the value of α that minimizes the magnitude of the resultant of the three forces. Also, determine the magnitude of that resultant.



Solution

The force polygon corresponding to the addition of the three forces is shown at the right. It is constructed by first sketching the 600 lb and 800 lb forces, followed by force $\vec{F}$ which has 400 lb value but has direction to be determined. By examining the force polygon, it is seen that the resultant force $\vec{R}$ of smallest magnitude is obtained when $\vec{F}$ has a direction from point B to point O. Thus



$$\alpha = \tan^{-1}\frac{\left(\frac{4}{5}\right)800\,\text{lb}}{600\,\text{lb} + \left(\frac{3}{5}\right)800\,\text{lb}} = \boxed{30.65°.} \tag{1}$$

The resultant force vector is given by

$$\vec{R} = -600\,\text{lb}\,\hat{\imath} + 800\,\text{lb}\left(-\frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}\right) + 400\,\text{lb}\,(\cos 30.65°\,\hat{\imath} - \sin 30.65°\,\hat{\jmath}) \tag{2}$$

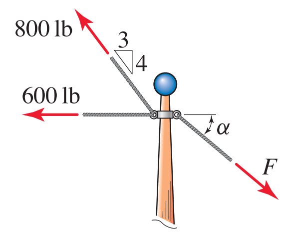
$$= \boxed{(-735.9\,\hat{\imath} + 436.1\,\hat{\jmath})\,\text{lb}.} \tag{3}$$

The magnitude of the resultant force is

$$\boxed{R = \sqrt{(-735.9\,\text{lb})^2 + (436.1\,\text{lb})^2} = 855.4\,\text{lb}.} \tag{4}$$

Problem 2.51

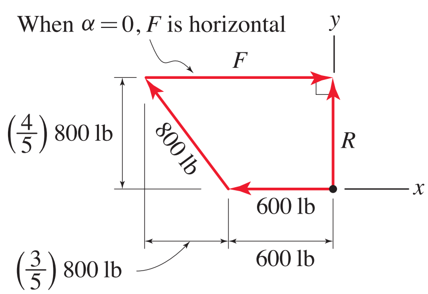
The mast of a ship supports forces from three cables as shown. If $\alpha = 0°$, determine the value of F that will make the magnitude of the resultant of the three forces smallest.



Solution

The force polygon shown at the right is sketched by first drawing the 600 lb and 800 lb forces. Force $\vec{F}$ is then drawn with orientation $\alpha = 0°$, although its size is to be determined. Adding the three force vectors provides the resultant force vector as

$$\vec{R} = -600\,\hat{\imath}\,\text{lb} + 800\,\text{lb}\left(\frac{-3}{5}\,\hat{\imath} + \frac{4}{5}\,\hat{\jmath}\right) + F\hat{\imath}. \tag{1}$$



By examining the force polygon, the resultant force $\vec{R}$ has the smallest magnitude when its direction is perpendicular to $\vec{F}$. For this situation, the x component of Eq. (1) is zero, which corresponds to

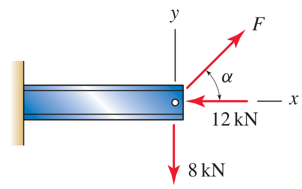
$$R_x = -600\,\text{lb} + 800\,\text{lb}\left(\frac{-3}{5}\right) + F = 0. \tag{2}$$

Thus,

$$\boxed{F = 600\,\text{lb} + 800\,\text{lb}\left(\frac{3}{5}\right) = 1080\,\text{lb}.} \tag{3}$$

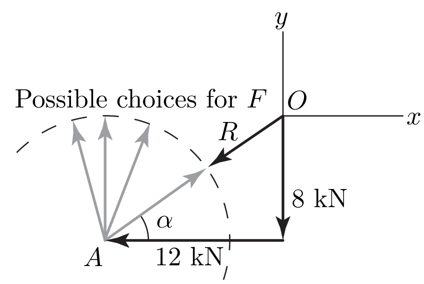
Problem 2.52

A short cantilever beam is subjected to three forces. If $F = 8\,\mathrm{kN}$, determine the value of α that minimizes the magnitude of the resultant of the three forces. Also, determine the magnitude of that resultant.



Solution

The force polygon corresponding to the addition of the three forces is shown at the right. It is constructed by first sketching the 8 kN and 12 kN forces, followed by force $\vec{F}$ which has 8 kN value, but has direction that is to be determined. By examining the force polygon, it is seen that the resultant force $\vec{R}$ of smallest magnitude is obtained when $\vec{F}$ has a direction from point A to O; in other words, when



$$\boxed{\alpha = \tan^{-1}\left(\frac{8\,\mathrm{kN}}{12\,\mathrm{kN}}\right) = 33.7°.} \tag{1}$$

The resultant force vector $\vec{R}$ is given by

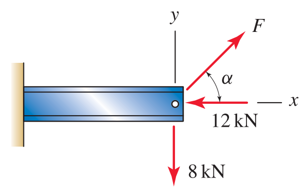
$$\vec{R} = (-8\,\mathrm{kN}\,\hat{\jmath}) - (12\,\mathrm{kN}\,\hat{\imath}) + 8\,\mathrm{kN}(\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}) = (-5.344\,\hat{\imath} - 3.562\,\hat{\jmath})\,\mathrm{kN}, \tag{2}$$

so that the magnitude R is then

$$\boxed{R = \sqrt{(-5.344)^2 + (-3.562)^2}\,\mathrm{kN} = 6.42\,\mathrm{kN}.} \tag{3}$$

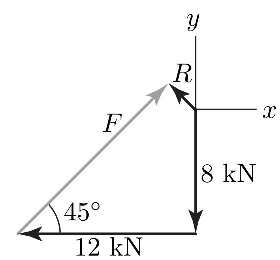
Problem 2.53

A short cantilever beam is subjected to three forces. If $\alpha = 45°$, determine the value of F that will make the magnitude of the resultant of the three forces smallest.



Solution

The force polygon shown at the right is sketched by first drawing the 8 kN and 12 kN forces. Force $\vec{F}$ is then drawn with orientation $\alpha = 45°$, although its size is to be determined. Adding the three force vectors provides the resultant force $\vec{R}$ as



$$\vec{R} = -(8\,\text{kN})\,\hat{\jmath} - (12\,\text{kN})\,\hat{\imath} + F(\cos 45°\,\hat{\imath} + \sin 45°\,\hat{\jmath}) \tag{1}$$

$$= (F\cos 45° - 12\,\text{kN})\,\hat{\imath} + (F\sin 45° - 8\,\text{kN})\,\hat{\imath}. \tag{2}$$

By examining the force polygon, the resultant force $\vec{R}$ has the smallest magnitude when its direction is perpendicular to $\vec{F}$, thus

$$\vec{R} = R(-\sin 45°\,\hat{\imath} + \cos 45°\,\hat{\jmath}). \tag{3}$$

Equating the x and y terms in Eqs. (2) and (3) gives the following expressions

$$R(-\sin 45°) = F\cos 45° - 12\,\text{kN} \quad\Rightarrow\quad R(-\sqrt{2}/2) = F\sqrt{2}/2 - 12\,\text{kN}, \tag{4}$$

$$R(\cos 45°) = F\sin 45° - 8\,\text{kN} \quad\Rightarrow\quad R(\sqrt{2}/2) = F\sqrt{2}/2 - 8\,\text{kN}. \tag{5}$$

The above equations can be solved for F by simply adding them to obtain

$$\boxed{0 = \sqrt{2}F - 20\,\text{kN} \quad\Rightarrow\quad F = \frac{20\,\text{kN}}{\sqrt{2}} = 14.1\,\text{kN}.} \tag{6}$$

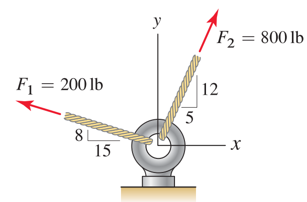
Problem 2.54

An eyebolt is loaded by forces F_1 and F_2. If the eyebolt has a maximum working load of 1200 lb, determine if the working load multipliers given in Fig. 1 of Example 2.6 are met for the following loading scenarios:

(a) Only F_1 is applied.

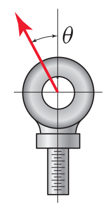
(b) Only F_2 is applied.

(c) Both F_1 and F_2 are applied simultaneously.



Solution

For convenience, the table of working load multipliers from Fig. 1 of Example 2.6 is repeated in the figure at the right.



θ	working load multipliers
$0° \le \theta < 15°$	100%
$15° \le \theta < 30°$	60%
$30° \le \theta < 45°$	33%
$45° \le \theta \le 90°$	20%

Part (a) When only F_1 is applied, the net force is oriented at an angle α_1 from the horizontal, where

$$\alpha_1 = \tan^{-1}(8/15) = 28.07°. \tag{1}$$

The orientation of F_1 from the vertical (i.e., axis of eyebolt) is therefore

$$\theta_1 = 90° - \alpha_1 = 61.93°. \tag{2}$$

Consulting the table of load multipliers, the working load multiplier for $\theta_1 = 61.93°$ is 20%. The maximum force the eyebolt may be subjected to is $(1200\,\text{lb})(0.20) = 240\,\text{lb}$; since this value is larger than $F_1 = 200\,\text{lb}$, the eyebolt is safe when only F_1 is applied. Hence,

$$\boxed{\text{This design is safe when only } F_1 \text{ is applied.}}$$

Part (b) When only F_2 is applied, the net force is oriented at an angle α_2 from the horizontal, where

$$\alpha_2 = \tan^{-1}(12/5) = 67.38°. \tag{3}$$

The orientation of F_2 from the vertical (i.e., axis of eyebolt) is therefore

$$\theta_2 = 90° - \alpha_2 = 22.62°. \tag{4}$$

Consulting the table of load multipliers, the working load multiplier for $\theta_2 = 22.62°$ is 60%. The maximum force the eyebolt may be subjected to is $(1200\,\text{lb})(0.60) = 720\,\text{lb}$; since this value is smaller than $F_2 = 800\,\text{lb}$, the eyebolt is not safe when only F_2 is applied. Hence,

$$\boxed{\text{This design is not safe when only } F_2 \text{ is applied.}}$$

Part (c) The resultant force is

$$\vec{R} = \vec{F}_1 + \vec{F}_2 \tag{5}$$
$$= 200\,\mathrm{lb}\left(\frac{-15\,\hat{\imath} + 8\,\hat{\jmath}}{17}\right) + 800\,\mathrm{lb}\left(\frac{5\,\hat{\imath} + 12\,\hat{\jmath}}{13}\right) \tag{6}$$
$$= (131.2\,\hat{\imath} + 832.6\,\hat{\jmath})\,\mathrm{lb}. \tag{7}$$

The magnitude of $\vec{R}$ and its orientation from the horizontal are

$$R = \sqrt{(131.2\,\mathrm{lb})^2 + (832.6\,\mathrm{lb})^2} = 842.9\,\mathrm{lb}, \tag{8}$$
$$\alpha_3 = \tan^{-1}(832.6\,\mathrm{lb}/131.2\,\mathrm{lb}) = 81.04°. \tag{9}$$

The orientation of $\vec{R}$ from the vertical (i.e., axis of eyebolt) is therefore

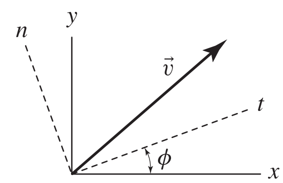
$$\theta_3 = 90° - \alpha_3 = 8.957°. \tag{10}$$

Consulting the table of load multipliers, the working load multiplier for $\theta_3 = 8.957°$ is 100%. The maximum force the eyebolt may be subjected to is $(1200\,\mathrm{lb})(1) = 1200\,\mathrm{lb}$; since this value is larger than $R = 842.9\,\mathrm{lb}$, the eyebolt is safe when F_1 and F_2 are applied simultaneously. Hence,

$$\boxed{\text{This design is safe when both } F_1 \text{ and } F_2 \text{ are applied simultaneously.}}$$

Problem 2.55

An important and useful property of vectors is they may be easily *transformed* from one Cartesian coordinate system to another. That is, if the x and y components of a vector are known, the t and n components can be found (or vice versa) by applying the formulas



$$\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath} = v_t\,\hat{t} + v_n\,\hat{n}, \tag{1}$$

$$\text{where} \quad v_t = v_x\cos\phi + v_y\sin\phi, \tag{2}$$

$$v_n = -v_x\sin\phi + v_y\cos\phi, \tag{3}$$

$$\text{or} \quad v_x = v_t\cos\phi - v_n\sin\phi, \tag{4}$$

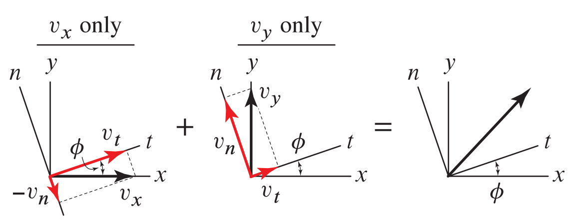
$$v_y = v_t\sin\phi + v_n\cos\phi. \tag{5}$$

In these equations, $\hat{t}$ and $\hat{n}$ are unit vectors in the t and n directions, respectively; ϕ is measured positive counterclockwise from the positive x direction to the positive t direction; and the y and n directions must be oriented 90° counterclockwise from the positive x and t directions, respectively.

(a) Derive the above transformation that gives v_t and v_n in terms of v_x and v_y. *Hint:* First consider a vector $\vec{v}_x$ that acts in the x direction, and resolve this into components in t and n directions. Then consider a vector $\vec{v}_y$ that acts in the y direction, and resolve this into components in t and n directions. Vectorially adding these results yields the transformation.

(b) For the eyebolt and post of Example 2.7, the x and y components of the resultant force are given by Eq. (4) of Example 2.6. Use these x and y components with the preceeding transformation equations to obtain the t and n components of the resultant force, and verify these are the same as those in Eq. (4) of Example 2.7.

Solution



Part (a) The suggestions made in Part (a) leads to the resolution of vectors shown in the figure at the right. Using the sketch for v_x only, v_x is resolved into the components v_t and v_n as follows

$$v_t = v_x\cos\phi, \quad v_n = -v_x\sin\phi. \tag{6}$$

Using the sketch for v_y only, v_y is resolved into the components v_t and v_n as follows

$$v_t = v_y\sin\phi, \quad v_n = v_y\cos\phi. \tag{7}$$

By observing that $\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath} = v_t\hat{t} + v_n\hat{n}$, we find that

$$\boxed{v_t = v_x\cos\phi + v_y\sin\phi, \quad v_n = -v_x\sin\phi + v_y\cos\phi,} \tag{8}$$

which are identical to Eqs. (2) and (3).

Part (b) The resultant force in terms of x and y components was found to be

$$R_x = -809.9\,\mathrm{lb}, \quad R_y = 373.9\,\mathrm{lb}. \tag{9}$$

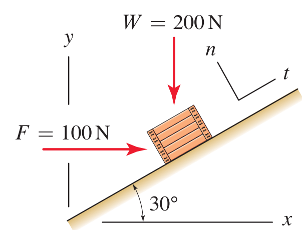
Since the t axis is rotated 40° counterclockwise from the x axis, $\phi = 40°$, and Eqs. (2) and (3) provide

$$\boxed{\begin{aligned} R_t &= (-809.9\,\mathrm{lb})\cos 40° + (373.9\,\mathrm{lb})\sin 40° = -380.0\,\mathrm{lb}, \\ R_n &= (-809.9\,\mathrm{lb})\sin 40° + (373.9\,\mathrm{lb})\cos 40° = 807.0\,\mathrm{lb}, \end{aligned}} \quad \begin{aligned} (10) \\ (11) \end{aligned}$$

with agree with Eq. (5) from Example 2.7 on page 52.

Problem 2.56

A box weighing 200 N rests on an inclined surface. A worker applies a horizontal force F to help position the box. Determine the x and y components of the resultant of forces W and F. Also determine the t and n components of the resultant force vector. Comment on why the t and n components might be useful to know.



Solution

The resultant $\vec{R}$ of the forces $\vec{F}$ and $\vec{W}$ applied to the box is

$$\vec{R} = \vec{F} + \vec{W} = 100\,\text{N}\,\hat{\imath} - 200\,\text{N}\,\hat{\jmath}, \tag{1}$$

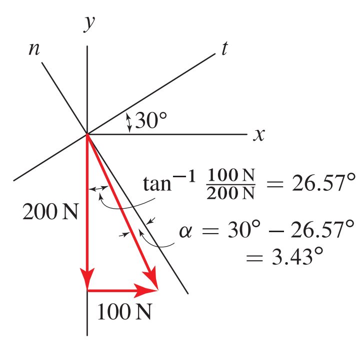
and the components of this resultant force are

$$\boxed{R_x = 100\,\text{N}, \quad R_y = -200\,\text{N}.} \tag{2}$$

The magnitude of $\vec{R}$ is

$$R = \sqrt{(100\,\text{N})^2 + -200\,\text{N})^2} = 223.6\,\text{N}. \tag{3}$$

Using the sketch at the right, we first determine the 26.57° angle, which then allows the angle $\alpha = 3.43°$ to be determined. Then, knowing that the resultant force is 223.6 N, the t and n components of the resultant force may be determined as



$$\boxed{\begin{aligned} R_t &= -R\sin\alpha = (-223.6\,\text{N})\sin 3.43° = -13.40\,\text{N}, \\ R_n &= -R\cos\alpha = (-223.6\,\text{N})\cos 3.43° = -223.2\,\text{N}. \end{aligned}} \quad \begin{aligned} (4) \\ (5) \end{aligned}$$

The t and n components are needed to determine if the box will slide using Coulomb's law of friction.

Alternate solution The t and n components of the resultant force may also be determined using vector transformation concepts. Using Eqs. (2) and (3) described in Problem 2.55, with $\phi = 30°$, we find that

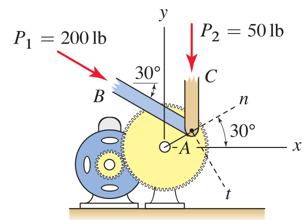
$$\boxed{\begin{aligned} R_t &= (100\,\text{N})\cos 30° + (-200\,\text{N})\sin 30° = -13.40\,\text{N}, \\ R_n &= -(100\,\text{N})\sin 30° + (-200\,\text{N})\cos 30° = -223.2\,\text{N}. \end{aligned}} \quad \begin{aligned} (6) \\ (7) \end{aligned}$$

Problem 2.57

A motor-driven gear is used to produce forces P_1 and P_2 in members AB and AC of a machine. Member AC is parallel to the y axis.

(a) Determine the x and y components of the resultant force vector at A due to forces P_1 and P_2.

(b) Determine the t and n components of the resultant force vector. Comment on why the t and n components might be useful to know.



Solution

Part (a) The resultant force vector is given by $\vec{R} = \vec{P}_1 + \vec{P}_2$. Thus

$$\vec{R} = (200\,\text{lb})(\cos 30°\,\hat{\imath} - \sin 30°\,\hat{\jmath}) + 50\,\text{lb}\,(-\hat{\jmath}) = (173.2\,\hat{\imath} - 150.0\,\hat{\jmath})\,\text{lb}, \tag{1}$$

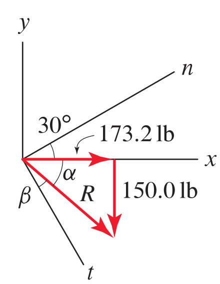
the components of the resultant force are

$$\boxed{R_x = 173\,\text{lb}, \quad R_y = -150\,\text{lb},} \tag{2}$$

and the magnitude of $\vec{R}$ is

$$R = \sqrt{(173.2)^2 + (-150.0)^2}\,\text{lb} = 229.1\,\text{lb}. \tag{3}$$

Part (b) Using the figure to the right, the angles α and β are given by



$$\alpha = \tan^{-1}(150.0\,\text{lb}/173.2\,\text{lb}) = 40.89°, \;\; \beta = 90° - 30° - \alpha = 19.11°. \tag{4}$$

The t and n components of $\vec{R}$ may be found using angle β as

$$\boxed{\begin{aligned} R_t &= R\cos\beta = (229.1\,\text{lb})\cos 19.11° = 217\,\text{lb}, \quad (5)\\ R_n &= R\sin\beta = (229.1\,\text{lb})\sin 19.11° = 75.0\,\text{lb}. \quad (6)\end{aligned}}$$

Alternate solution R_t and R_n may be determined using vector transformation concepts. Using Eqs. (2) and (3) described in Prob. 2.55, with $\phi = -60°$, we obtain

$$R_t = (173.2\,\text{lb})\cos(-60°) + (-150\,\text{lb})\sin(-60°) = 217\,\text{lb}, \tag{7}$$

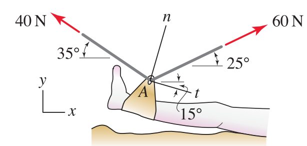
$$R_n = -(173.2\,\text{lb})\sin(-60°) + (-150\,\text{lb})\cos(-60°) = 75.0\,\text{lb}. \tag{8}$$

Problem 2.58

Traction is a medical procedure used to treat muscle and skeletal disorders by strategically applying one or more forces to a person's body for a specific length of time.

(a) Determine the x and y components of the resultant force vector at A due to the 40 N and 60 N forces.

(b) Determine the t and n components of the resultant force vector.



Solution

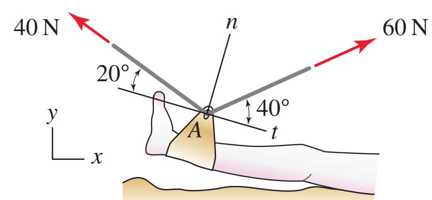
Part (a)

$$\vec{R} = 40\,\text{N}(-\cos 35°\,\hat{\imath} + \sin 35°\,\hat{\jmath}) + 60\,\text{N}\,(\cos 25°\,\hat{\imath} + \sin 25°\,\hat{\jmath}) \tag{1}$$
$$= (21.61\,\hat{\imath} + 48.30\,\hat{\jmath})\,\text{N}. \tag{2}$$

Thus, the x and y components are

$$\boxed{R_x = 21.61\,\text{N} \quad \text{and} \quad R_y = 48.30\,\text{N}.} \tag{3}$$

Part (b) The figure to the right has angles defined with respect to the t direction. Repeating the solution procedure from Part (a) provides



$$\vec{R} = 40\,\text{N}\,(-\cos 20°\,\hat{t} + \sin 20°\,\hat{n}) + 60\,\text{N}\,(\cos 40°\,\hat{t} + \sin 40°\,\hat{n}) \tag{4}$$
$$= (8.375\,\hat{t} + 52.25\,\hat{n})\,\text{N}. \tag{5}$$

Thus, the t and n components of the resultant force are

$$\boxed{R_t = 8.375\,\text{N} \quad \text{and} \quad R_n = 52.25\,\text{N}.} \tag{6}$$

As a partial check of solution accuracy, we may verify that Eqs. (3) and (5) have the same magnitude.

Alternate Solution R_t and R_n may be determined using vector transformation concepts. Using Eqs. (2) and (3) described in Prob. 2.55, with $\phi = -15°$, we obtain

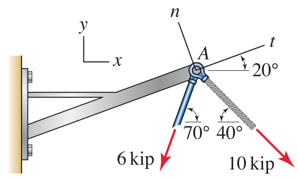
$$R_t = (21.61\,\text{N})\cos(-15°) + (48.30\,\text{N})\sin(-15°) = 8.375\,\text{N}, \tag{7}$$
$$R_n = -(21.61\,\text{N})\sin(-15°) + (48.30\,\text{N})\cos(-15°) = 52.25\,\text{N}. \tag{8}$$

Problem 2.59

A structure supports forces from a bar and cable as shown.

(a) Determine the x and y components of the resultant force vector at A due to the 6 kip and 10 kip forces.

(b) Determine the t and n components of the resultant force vector.



Solution

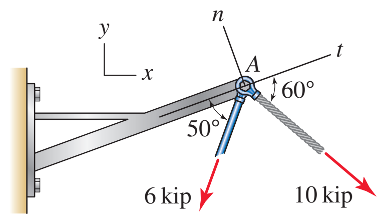
Part (a)

$$\begin{aligned}\vec{R} &= 6\,\text{kip}\,(-\cos 70°\,\hat{\imath} - \sin 70°\,\hat{\jmath}) \\ &\quad + 10\,\text{kip}\,(\cos 40°\,\hat{\imath} - \sin 40°\,\hat{\jmath}) && (1)\\ &= (5.608\,\hat{\imath} - 12.07\,\hat{\jmath})\,\text{kip}. && (2)\end{aligned}$$

Thus, the x and y components are

$$\boxed{R_x = 5.608\,\text{kip} \quad \text{and} \quad R_y = -12.07\,\text{kip}.} \tag{3}$$

Part (b) The figure to the right has angles defined with respect to the t direction. Repeating the solution procedure from Part (a) provides



$$\begin{aligned}\vec{R} &= 6\,\text{kip}\,(-\cos 50°\,\hat{t} - \sin 50°\,\hat{n}) \\ &\quad + 10\,\text{kip}\,(\cos 60°\,\hat{t} - \sin 60°\,\hat{n}) && (4)\\ &= (1.143\hat{t} - 13.26\hat{n})\,\text{kip}. && (5)\end{aligned}$$

Thus, the t and n components of the resultant force are

$$\boxed{R_t = 1.143\,\text{kip} \quad \text{and} \quad R_n = -13.26\,\text{kip}.} \tag{6}$$

As a partial check of solution accuracy, we may verify that Eqs. (3) and (5) have the same magnitude.

Alternate solution R_t and R_n may be determined using vector transformation concepts. Using Eqs. (2) and (3) described in Prob. 2.55, with $\phi = 20°$, we obtain

$$R_t = (5.608\,\text{kip})\cos 20° + (-12.07\,\text{kip})\sin 20° = 1.143\,\text{kip}, \tag{7}$$

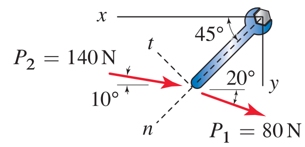
$$R_n = -(5.608\,\text{kip})\sin 20° + (-12.07\,\text{kip})\cos 20° = -13.26\,\text{kip}. \tag{8}$$

Problem 2.60

Two people apply forces P_1 and P_2 to the handle of a wrench as shown.

(a) Determine the x and y components of the resultant force vector applied to the handle of the wrench.

(b) Determine the t and n components of the resultant force vector. Comment on why the t and n components might be useful to know.



Solution

Part (a) The resultant force is given by $\vec{R} = \vec{P}_1 + \vec{P}_2$, where

$$\vec{R} = (80\,\mathrm{N})(-\cos 20°\,\hat{\imath} + \sin 20°\,\hat{\jmath}) + (140\,\mathrm{N})(-\cos 10°\,\hat{\imath} + \sin 10°\,\hat{\jmath}) = (-213.0\,\hat{\imath} + 51.67\,\hat{\jmath})\,\mathrm{N}, \quad (1)$$

so that the components of the resultant force are

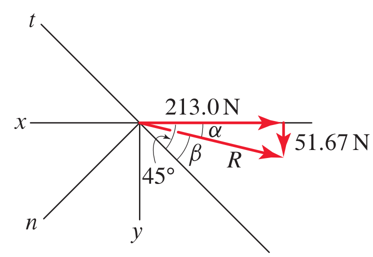
$$\boxed{R_x = -213\,\mathrm{N}, \quad R_y = 51.7\,\mathrm{N}.} \quad (2)$$

The magnitude of $\vec{R}$ is

$$R = \sqrt{(-213.0)^2 + (51.67)^2}\,\mathrm{N} = 219.2\,\mathrm{N}. \quad (3)$$

Part (b) Using the figure to the right, the angles α and β are

$$\alpha = \tan^{-1}(51.67\,\mathrm{N}/213.0\,\mathrm{N}) = 13.63°, \quad \beta = 45° - \alpha = 31.37°. \quad (4)$$



The t and n components of $\vec{R}$ may be found using β as

$$\boxed{\begin{aligned} R_t &= -R\cos\beta = -(219.2\,\mathrm{N})\cos 31.37° = -187\,\mathrm{N}, \\ R_n &= -R\sin\beta = -(219.2\,\mathrm{N})\sin 31.37° = -114\,\mathrm{N}. \end{aligned}} \quad \begin{aligned}(5)\\(6)\end{aligned}$$

Alternate solution R_t and R_n may be determined using vector transformation concepts. Using Eqs. (2) and (3) described in Problem 2.55, with $\phi = 315°$, or with $\phi = -45°$, we obtain

$$R_t = (-213.0\,\mathrm{N})\cos(-45°) + (51.67\,\mathrm{N})\sin(-45°) = -187\,\mathrm{N}, \quad (7)$$

$$R_n = -(-213.0\,\mathrm{N})\sin(-45°) + (51.67\,\mathrm{N})\cos(-45°) = -114\,\mathrm{N}. \quad (8)$$

Problem 2.61

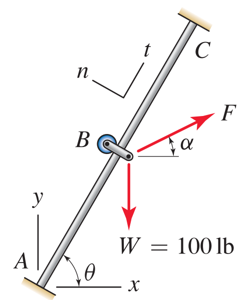
Bar AC is straight and has 106 in. length, B is a pulley that supports forces W and F, W is vertical, and the t direction is parallel to bar AC.

(a) If $F = 150\,\mathrm{lb}$ and $\alpha = 30°$, determine the coordinates of point C so that the t component of the resultant of F and W is zero.

(b) If $F = 150\,\mathrm{lb}$ and C is located at $(56, 90)$ in., determine angle α so that the t component of the resultant of F and W is zero.

(c) If C is located at $(56, 90)$ in. and $\alpha = 30°$, determine F so that the t component of the resultant of F and W is zero.

Hint: For each of these questions, first find the x and y components of the resultant force and then use the transformation given in Prob. 2.55 to obtain the t component.



Solution

Part (a) The resultant of $\vec{F}$ and $\vec{W}$ is given by

$$\vec{R} = (-100\,\mathrm{lb})\ \hat{\jmath} + (150\,\mathrm{lb})(\cos 30°\,\hat{\imath} + \sin 30°\,\hat{\jmath}) = (129.9\,\hat{\imath} - 25.00\,\hat{\jmath})\,\mathrm{lb}. \tag{1}$$

Using vector transformation (Eq. (2) in Prob. 2.55 with $\phi = \theta$), the component of the resultant force in the t direction is

$$R_t = R_x \cos\theta + R_y \sin\theta. \tag{2}$$

Substituting the values of R_x and R_y from Eq. (1) into Eq. (2), and noting that, according to the problem statement, $R_t = 0$, we obtain

$$R_t = (129.9\,\mathrm{lb})\cos\theta - (25.00\,\mathrm{lb})\sin\theta = 0. \tag{3}$$

Dividing Eq. (3) by $\sin\theta$ and simplifying, we obtain

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{129.9\,\mathrm{lb}}{25.00\,\mathrm{lb}} \quad\Rightarrow\quad \theta = \tan^{-1}\left(\frac{129.9\,\mathrm{lb}}{25.00\,\mathrm{lb}}\right) = 79.11°. \tag{4}$$

Since the length of the bar AC is 106 in., it follows that the coordinates of C are

$$\boxed{x_C = (106\,\mathrm{in.})\cos\theta = 20.0\,\mathrm{in.}, \quad y_C = (106\,\mathrm{in.})\sin\theta = 104\,\mathrm{in.}} \tag{5}$$

Part (b) With point C located at $(56, 90)$ in., the orientation θ of the bar is

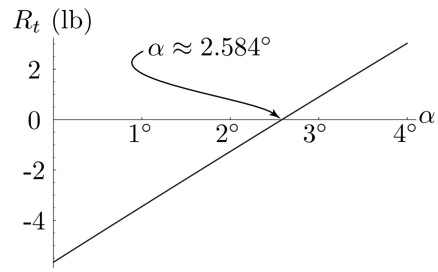
$$\theta = \tan^{-1}(90/56) = 58.11°. \tag{6}$$

The resultant of $\vec{F}$ and $\vec{W}$ is given by

$$\begin{aligned}\vec{R} &= (-100\,\mathrm{lb})\,\hat{\jmath} + F(\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}) &(7)\\ &= (150\,\mathrm{lb})\cos\alpha\,\hat{\imath} + \big[(150\,\mathrm{lb})\sin\alpha - 100\,\mathrm{lb}\big]\,\hat{\jmath} &(8)\\ &= R_x\,\hat{\imath} + R_y\,\hat{\jmath}. &(9)\end{aligned}$$

Using vector transformation (Eq. (2) in Prob. 2.55 with $\phi = \theta = 58.11°$), the component of the resultant force in the t direction, which is zero according to the problem statement, is

$$R_t = (150\,\text{lb})(\cos\alpha)(\cos 58.11°) + (150\,\text{lb}\sin\alpha - 100\,\text{lb})(\sin 58.11°) = 0, \tag{10}$$



Equation (10) has one unknown, namely α, and the equation may be solved using computer algebra software (e.g., Mathematica or Maple), by trial and error, or by simply plotting R_t versus α, as shown at the right. In this plot, the location where $R_t = 0$ corresponds to

$$\boxed{\alpha = 2.58°.} \tag{11}$$

Part (c) With point C located at $(56, 90)$ in., the orientation θ of the bar is given by Eq. (6), namely $\theta = 58.11°$. The resultant of $\vec{F}$ and $\vec{W}$ is given by

$$\vec{R} = (-100\,\text{lb})\ \hat{\jmath} + F(\cos 30°\,\hat{\imath} + \sin 30°\ \hat{\jmath}) \tag{12}$$
$$= F\cos 30°\,\hat{\imath} + (F\sin 30° - 100\,\text{lb})\ \hat{\jmath} \tag{13}$$
$$= R_x\,\hat{\imath} + R_y\ \hat{\jmath}. \tag{14}$$

Using vector transformation (Eq. (2) in Prob. 2.55 with $\phi = \theta = 58.11°$), the component of the resultant force in the t direction, which is zero according to the problem statement, is

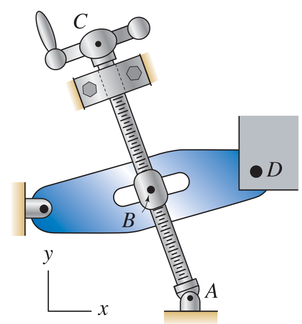
$$R_t = F(\cos 30°)(\cos 58.11°) + (F\sin 30° - 100\,\text{lb})(\sin 58.11°) = 0. \tag{15}$$

Solving Eq. (15) for F, we find that

$$\boxed{F\big[(\cos 30°)(\cos 58.11°) + (\sin 30°)(\sin 58.11°)\big] = (100\,\text{lb})(\sin 58.11°) \quad\Rightarrow\quad F = 96.3\,\text{lb}} \tag{16}$$

Problem 2.62

Screw AC is used to position point D of a machine. Points A and C have coordinates (185, 0) mm and (125, 144) mm, respectively, and are fixed in space by the bearings that support the screw. If point B is 52 mm from point A, determine the position vector $\vec{r}_{AB}$ and the coordinates of point B.



Solution

Begin by determining the vector $\vec{r}_{AC}$ (where point C is the head of the vector and point A is the tail):

$$\vec{r}_{AC} = ((125 - 185)\,\hat{\imath} + (144 - 0)\,\hat{\jmath})\text{ mm} = (-60\,\hat{\imath} + 144\,\hat{\jmath})\text{ mm}. \tag{1}$$

The magnitude of $\vec{r}_{AC}$ is given by

$$r_{AC} = \sqrt{(-60\text{ mm})^2 + (144\text{ mm})^2} = 156\text{ mm}. \tag{2}$$

The unit vector in the AB direction is given by $\vec{r}_{AC}/r_{AC}$, and hence the position vector $\vec{r}_{AB}$ is

$$\boxed{\vec{r}_{AB} = 52\text{ mm}\frac{\vec{r}_{AC}}{r_{AC}} = 52\text{ mm}\frac{-60\text{ mm}\,\hat{\imath} + 144\text{ mm}\,\hat{\jmath}}{156\text{ mm}} = (-20\,\hat{\imath} + 48\,\hat{\jmath})\text{ mm}.} \tag{3}$$

To determine the coordinates of point B, we write an expression for the position vector $\vec{r}_{AB}$ by taking the differences of the coordinates of the head and tail. With points B and A having coordinates (x_B, y_B) and (185.0 mm, 0), respectively, we obtain

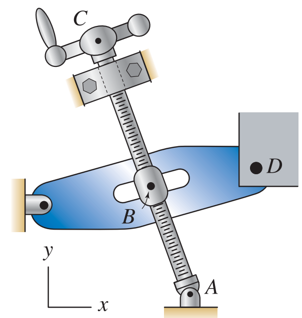
$$\vec{r}_{AB} = (x_B - 185\text{ mm})\,\hat{\imath} + (y_B - 0)\,\hat{\jmath}. \tag{4}$$

For Eqs. (3) and (4) to agree requires that

$$\boxed{-20\text{ mm} = x_B - 185\text{ mm} \quad\Rightarrow\quad x_B = 165\text{ mm}, \qquad 48\text{ mm} = y_B - 0\text{ mm} \quad\Rightarrow\quad y_B = 48\text{ mm}.} \tag{5}$$

Problem 2.63

Repeat Prob. 2.62 if point B is 39 mm from point C.



Solution

Begin by determining the vector $\vec{r}_{CA}$ (where point A is the head of the vector and point C is the tail):

$$\vec{r}_{CA} = (185\,\text{mm} - 125\,\text{mm})\,\hat{\imath} + (0 - 144\,\text{mm})\,\hat{\jmath} = (60\,\hat{\imath} - 144\,\hat{\jmath})\,\text{mm}. \tag{1}$$

The magnitude of $\vec{r}_{CA}$ is given by

$$r_{CA} = \sqrt{(60\,\text{mm})^2 + (-144\,\text{mm})^2} = 156\,\text{mm}. \tag{2}$$

The unit vector in the CB direction is given by $\vec{r}_{CA}/r_{CA}$, and hence the position vector $\vec{r}_{CB}$ is

$$\boxed{\vec{r}_{CB} = 39\,\text{mm}\frac{\vec{r}_{CA}}{r_{CA}} = 39\,\text{mm}\frac{(60\,\hat{\imath} - 144\,\hat{\jmath})\,\text{mm}}{156\,\text{mm}} = (15\,\hat{\imath} - 36\,\hat{\jmath})\,\text{mm}.} \tag{3}$$

To determine the coordinates of point B, we write an expression for the position vector $\vec{r}_{CB}$ by taking the differences of the coordinates of the head and tail. With points B and C having coordinates (x_B, y_B) and (125, 144) mm, respectively, we obtain

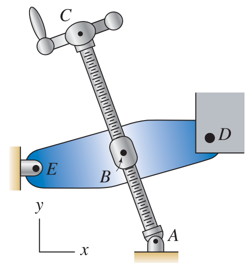
$$\vec{r}_{CB} = (x_B - 125\,\text{mm})\,\hat{\imath} + (y_B - 144\,\text{mm})\,\hat{\jmath}. \tag{4}$$

For Eqs. (3) and (4) to agree requires that

$$\boxed{15\,\text{mm} = x_B - 125\,\text{mm} \quad\Rightarrow\quad x_B = 140\,\text{mm}, \quad -36\,\text{mm} = y_B - 144\,\text{mm} \quad\Rightarrow\quad y_B = 108\,\text{mm}.} \tag{5}$$

Problem 2.64

Screw AC is used to position point D of a machine. Point A has coordinates $(185, 0)$ mm and is fixed in space by a bearing that supports the screw. The screw nut at point B is supported by lever ED and at the instant shown has coordinates $(160, 60)$ mm. If the length of the screw from A to C is 130 mm, determine the position vector $\vec{r}_{AC}$ and the coordinates of point C.



Solution

Begin by determining the vector $\vec{r}_{AB}$ (where point B is the head of the vector and point A is the tail):

$$\vec{r}_{AB} = (160\,\text{mm} - 185\,\text{mm})\,\hat{\imath} + (60\,\text{mm} - 0)\,\hat{\jmath} = (-25\,\hat{\imath} + 60\,\hat{\jmath})\,\text{mm}. \tag{1}$$

The magnitude of $\vec{r}_{AB}$ is given by

$$r_{AB} = \sqrt{(-25\,\text{mm})^2 + (60\,\text{mm})^2} = 65\,\text{mm}. \tag{2}$$

The unit vector in the AC direction is given by $\vec{r}_{AB}/r_{AB}$, and hence the position vector $\vec{r}_{AC}$ is

$$\boxed{\vec{r}_{AC} = 130\,\text{mm}\frac{\vec{r}_{AB}}{r_{AB}} = 130\,\text{mm}\frac{(-25\,\hat{\imath} + 60\,\hat{\jmath})\,\text{mm}}{65\,\text{mm}} = (-50\,\hat{\imath} + 120\,\hat{\jmath})\,\text{mm}.} \tag{3}$$

To determine the coordinates of point C, we write an expression for the position vector $\vec{r}_{AC}$ by taking the differences of the coordinates of the head and tail. With points C and A having coordinates (x_C, y_C) and $(185, 0)$ mm, respectively, we obtain

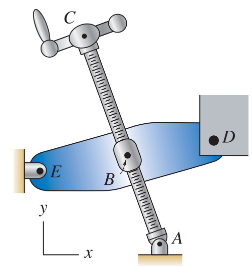
$$\vec{r}_{AC} = (x_C - 185\,\text{mm})\,\hat{\imath} + (y_C - 0)\,\hat{\jmath}. \tag{4}$$

For Eqs. (3) and (4) to agree requires that

$$\boxed{-50\,\text{mm} = x_C - 185\,\text{mm} \quad\Rightarrow\quad x_C = 135\,\text{mm}, \quad 120\,\text{mm} = y_C - 0\,\text{mm} \quad\Rightarrow\quad y_C = 120\,\text{mm}.}$$

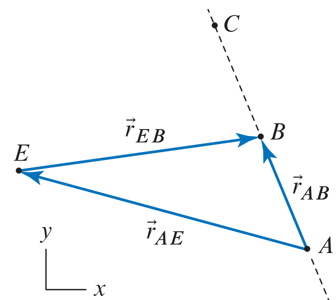
Problem 2.65

In Prob. 2.64 if point B is equidistant from points A and C and if $\vec{r}_{EA} = (185\,\hat{\imath} - 50\,\hat{\jmath})$ mm and $\vec{r}_{EB} = (155\,\hat{\imath} + 22\,\hat{\jmath})$ mm, determine the position vector $\vec{r}_{AC}$.



Solution

The vector polygon shown at the right corresponds to the vector expression $\vec{r}_{AB} = \vec{r}_{AE} + \vec{r}_{EB}$. Thus



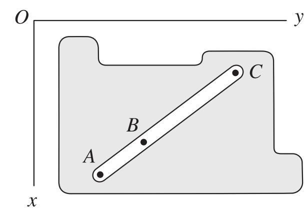
$$\vec{r}_{AB} = \vec{r}_{AE} + \vec{r}_{EB} \tag{1}$$
$$= -\vec{r}_{EA} + \vec{r}_{EB} \tag{2}$$
$$= -(185\,\hat{\imath} - 50\,\hat{\jmath})\text{ mm} + (155\,\hat{\imath} + 22\,\hat{\jmath})\text{ mm} \tag{3}$$
$$= (-30\,\hat{\imath} + 72\,\hat{\jmath})\text{ mm.} \tag{4}$$

Since point B is equidistant from points A and C, it follows that

$$\boxed{\vec{r}_{AC} = 2\,\vec{r}_{AB} = (-60\,\hat{\imath} + 144\,\hat{\jmath})\text{ mm.}}$$

Problem 2.66

A computer numerical control (CNC) milling machine is used to cut a slot in the component shown. The cutting tool starts at point A and advances to point B where it pauses while the depth of cut is increased, and then the tool continues on to point C. If the coordinates of points A and C are (95, 56) mm and (17, 160) mm, respectively, and B is located 45 mm from A, determine the position vectors $\vec{r}_{OA}$, $\vec{r}_{OB}$, and $\vec{r}_{OC}$ and the coordinates of B.



Solution

With the coordinates provided in the problem statement, the following three position vectors may immediately be written:

$$\boxed{\vec{r}_{OA} = (95\,\hat{\imath} + 56\,\hat{\jmath})\text{ mm},} \quad \vec{r}_{OB} = x_B\,\hat{\imath} + y_B\,\hat{\jmath}, \quad \boxed{\vec{r}_{OC} = (17\,\hat{\imath} + 160\,\hat{\jmath})\text{ mm}.} \tag{1}$$

Using these vectors, we may determine $\vec{r}_{AC}$ using

$$\begin{aligned} \vec{r}_{AC} &= \vec{r}_{AO} + \vec{r}_{OC} & (2)\\ &= -\vec{r}_{OA} + \vec{r}_{OC} & (3)\\ &= -(95\,\hat{\imath} + 56\,\hat{\jmath})\text{ mm} + (17\,\hat{\imath} + 160\,\hat{\jmath})\text{ mm} & (4)\\ &= (-78\,\hat{\imath} + 104\,\hat{\jmath})\text{ mm}. & (5) \end{aligned}$$

The magnitude of $\vec{r}_{AB}$ is given in the problem statement to be 45 mm, and thus we may write

$$\vec{r}_{AB} = 45\text{ mm}\frac{\vec{r}_{AC}}{r_{AC}} = 45\text{ mm}\frac{(-78\,\hat{\imath} + 104\,\hat{\jmath})\text{ mm}}{\sqrt{(-78)^2 + (104)^2}\text{ mm}} = (-27\,\hat{\imath} + 36\,\hat{\jmath})\text{ mm}. \tag{6}$$

An expression for $\vec{r}_{OB}$ may be written as

$$\begin{aligned} \vec{r}_{OB} &= \vec{r}_{OA} + \vec{r}_{AB} & (7)\\ &= (95\,\hat{\imath} + 56\,\hat{\jmath})\text{ mm} + (-27\,\hat{\imath} + 36\,\hat{\jmath})\text{ mm} & (8)\\ &= (68\,\hat{\imath} + 92\,\hat{\jmath})\text{ mm}. & (9) \end{aligned}$$

For Eq. (9) to agree with the expression for $\vec{r}_{OB}$ in Eq. (1) requires that

$$\boxed{x_B = 68\text{ mm} \quad \text{and} \quad y_B = 92\text{ mm},} \tag{10}$$

and with these coordinates, $\vec{r}_{OB}$ in Eq. (1) becomes

$$\boxed{\vec{r}_{OB} = (68\,\hat{\imath} + 92\,\hat{\jmath})\text{ mm}.} \tag{11}$$

Problem 2.67

A vector $\vec{r}$ has the two direction angles given below. Determine the possible values for the third direction angle, and describe the differences in the orientation of $\vec{r}$. Provide a sketch showing these different vectors.
$\theta_x = 36°$ and $\theta_y = 72°$.

Solution

The direction angle θ_z may be determined using

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1, \tag{1}$$

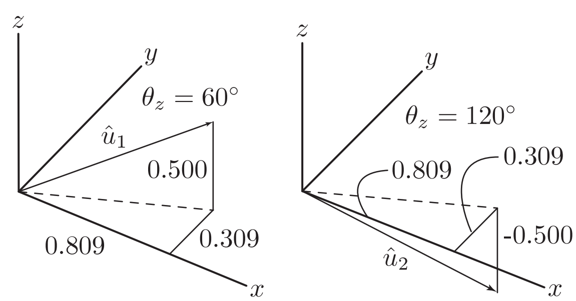
$$\begin{aligned}\theta_z &= \cos^{-1} \pm\sqrt{1-\cos^2\theta_x - \cos^2\theta_y} && (2)\\ &= \cos^{-1} \pm\sqrt{1-\cos^2 36° - \cos^2 72°} && (3)\\ &= \cos^{-1} \pm 0.5\,. && (4)\end{aligned}$$

Although there are an infinite number of solutions to the above equation for θ_z, there are only two solutions in the interval $0° \le \theta_z \le 180°$, and these are

$$\boxed{\theta_z = 60°, 120°.} \tag{5}$$

To show the vectors associated with these two solutions, consider the following two unit vectors for θ_x and θ_y given in the problem statement, and $\theta_z = 60°$ and $120°$:

$$\begin{aligned}\hat{u}_1 &= \cos(36°)\,\hat{\imath} + \cos(72°)\,\hat{\jmath} + \cos(60°)\,\hat{k} && (6)\\ &= 0.8090\,\hat{\imath} + 0.3090\,\hat{\jmath} + 0.5000\,\hat{k}, && (7)\\ \hat{u}_2 &= \cos(36°)\,\hat{\imath} + \cos(72°)\,\hat{\jmath} + \cos(120°)\,\hat{k} && (8)\\ &= 0.8090\,\hat{\imath} + 0.3090\,\hat{\jmath} - 0.5000\,\hat{k}. && (9)\end{aligned}$$



These two vectors are shown at the right, where the different θ_z values change the sign of the z-component.

Problem 2.68

A vector $\vec{r}$ has the two direction angles given below. Determine the possible values for the third direction angle, and describe the differences in the orientation of $\vec{r}$. Provide a sketch showing these different vectors.
$\theta_y = 60°$ and $\theta_z = 108°$.

Solution

The direction angle θ_x may be determined using

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1, \tag{1}$$

$$\theta_x = \cos^{-1} \pm\sqrt{1 - \cos^2\theta_y - \cos^2\theta_z} \tag{2}$$
$$= \cos^{-1} \pm\sqrt{1 - \cos^2 60° - \cos^2 108°} \tag{3}$$
$$= \cos^{-1} \pm 0.8090\,. \tag{4}$$

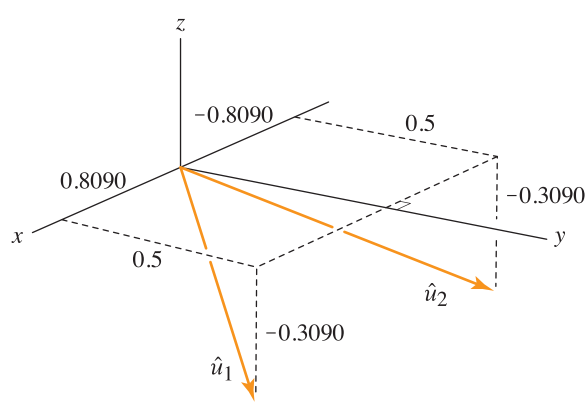
Although there are an infinite number of solutions to the above equation, the only two solutions in the interval $0° \leq \theta_x \leq 180°$ are

$$\boxed{\theta_x = 36°,\ 144°.} \tag{5}$$

To show the vectors associated with these two solutions, consider the following two unit vectors for θ_y and θ_z as given in the problem statement, and $\theta_x = 36°$ and $144°$:

$$\hat{u}_1 = \cos 36°\,\hat{\imath} + \cos 60°\,\hat{\jmath} + \cos 108°\,\hat{k} \tag{6}$$
$$= \quad 0.8090\,\hat{\imath} + 0.5\,\hat{\jmath} - 0.3090\,\hat{k}, \tag{7}$$
$$\hat{u}_2 = \cos 144°\,\hat{\imath} + \cos 60°\,\hat{\jmath} + \cos 108°\,\hat{k} \tag{8}$$
$$= -0.8090\,\hat{\imath} + 0.5\,\hat{\jmath} - 0.3090\,\hat{k}. \tag{9}$$

These two vectors are shown below, where the different θ_x values change the sign of the x component.



Problem 2.69

A vector $\vec{r}$ has the two direction angles given below. Determine the possible values for the third direction angle, and describe the differences in the orientation of $\vec{r}$. Provide a sketch showing these different vectors.
$\theta_x = 150°$ and $\theta_z = 100°$.

Solution

The direction angle θ_y may be determined using

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1, \tag{1}$$

$$\theta_y = \cos^{-1} \pm\sqrt{1-\cos^2\theta_x - \cos^2\theta_y} \tag{2}$$
$$= \cos^{-1} \pm\sqrt{1-\cos^2 150° - \cos^2 100°} \tag{3}$$
$$= \cos^{-1} \pm 0.4689\,. \tag{4}$$

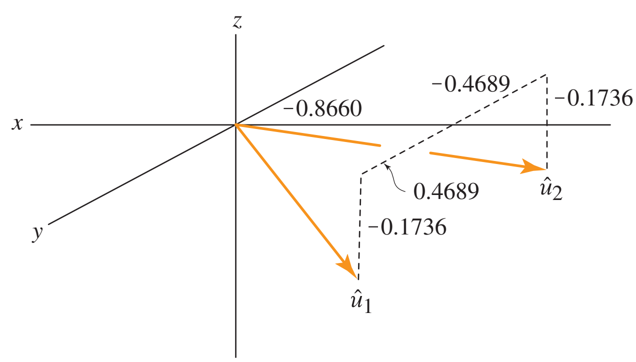
Although there are an infinite number of solutions to the above equation, the only two solutions in the interval $0° \le \theta_y \le 180°$ are

$$\boxed{\theta_y = 62.04°,\ 118.0°.} \tag{5}$$

To show the vectors associated with these two solutions, consider the following two unit vectors for θ_x and θ_z as given in the problem statement, and $\theta_y = 62.04°$ and $118.0°$:

$$\hat{u}_1 = \cos 150°\,\hat{\imath} + \cos 62.04°\,\hat{\jmath} + \cos 100°\,\hat{k} \tag{6}$$
$$= -0.8660\,\hat{\imath} + 0.4689\,\hat{\jmath} - 0.1736\,\hat{k}, \tag{7}$$
$$\hat{u}_2 = \cos 150°\,\hat{\imath} + \cos 118.0°\,\hat{\jmath} + \cos 100°\,\hat{k} \tag{8}$$
$$= -0.8660\,\hat{\imath} - 0.4689\,\hat{\jmath} - 0.1736\,\hat{k}. \tag{9}$$

These two vectors are shown below, where the different θ_y values change the sign of the y component.



Problem 2.70

Consider a vector $\vec{n}$ that makes equal angles with the x, y, and z axes (i.e., $\theta_x = \theta_y = \theta_z$). Determine the value of these angles and the value of the direction cosines. *Note:* In the study of stresses in mechanics of materials, the direction $\vec{n}$ plays an important role and is the normal direction to a special surface called the *octahedral plane*.

Solution

The direction angles all have the same value. Using the equation

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1, \tag{1}$$

with $\theta = \theta_x = \theta_y = \theta_z$, we obtain

$$3\cos^2\theta = 1. \tag{2}$$

Solving this provides the direction cosines as

$$\boxed{\cos\theta = \pm\sqrt{\frac{1}{3}} = \pm\,0.5774,} \tag{3}$$

and the direction angles are

$$\theta = \cos^{-1} \pm\sqrt{\frac{1}{3}} \tag{4}$$

$$= \boxed{54.74^\circ,\ 125.3^\circ.} \tag{5}$$

Problem 2.71

A person of dubious technical ability tells you that two of the direction angles for a particular vector are 30° and 40°. If these direction angles are feasible, then determine the possible values for the remaining direction angle. If these direction angles are not feasible, explain why.

Solution

Let $\theta_x = 30°$ and $\theta_y = 40°$. The remaining direction angle, θ_z, must satisfy the equation

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1 \tag{1}$$

$$\cos^2 30° + \cos^2 40 + \cos^2\theta_z = 1, \tag{2}$$

which may be solved for

$$\cos\theta_z = \pm\sqrt{1 - \cos^2 30° - \cos^2 40°} \tag{3}$$

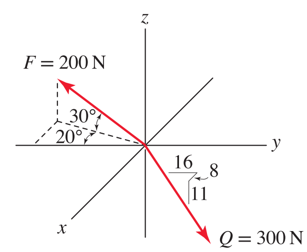
$$= \pm\sqrt{-0.3368}\,. \tag{4}$$

Since the direction cosine, $\cos\theta_z$, must be a real number, we conclude that

$$\boxed{\text{30° and 40° are not both direction angles.}} \tag{5}$$

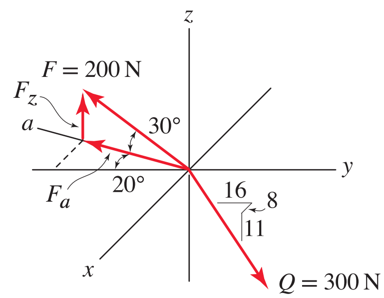
Problem 2.72

Write an expression for each force using Cartesian representation, and evaluate the resultant force vector. Sketch the resultant force in the xyz coordinate system.



Solution

Using the sketch shown, the projections of $\vec{F}$ in the z and a directions are



$$F_z = 200\,\text{N} \sin 30° = 100\,\text{N}, \tag{1}$$
$$F_a = 200\,\text{N} \cos 30° = 173.2\,\text{N}. \tag{2}$$

The component F_a may then be resolved into x and y components as follows

$$F_x = -F_a \sin 20° = -(173.2\,\text{N}) \sin 20° = -59.24\,\text{N}, \tag{3}$$
$$F_y = -F_a \cos 20° = -(173.2\,\text{N}) \cos 20° = -162.8\,\text{N}, \tag{4}$$

so we may write

$$\boxed{\vec{F} = (-59.24\,\hat{\imath} - 162.8\,\hat{\jmath} + 100\,\hat{k})\,\text{N}.} \tag{5}$$

Using the orientation geometry for $\vec{Q}$ provided in the problem statement, we may write

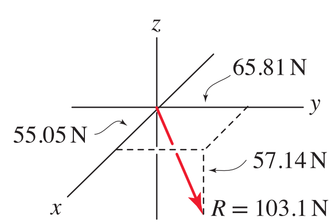
$$\vec{Q} = 300\,\text{N}\left(\frac{8\,\hat{\imath} + 16\,\hat{\jmath} - 11\,\hat{k}}{21}\right) \tag{6}$$
$$= (114.3\,\hat{\imath} + 228.6\,\hat{\jmath} - 157.1\,\hat{k})\,\text{N}. \tag{7}$$

The resultant force vector $\vec{R}$ is

$$\begin{aligned}\vec{R} &= \vec{F} + \vec{Q}\\ &= (-59.24 + 114.3)\,\hat{\imath}\,\text{N}\\ &\quad + (-162.8 + 228.6)\,\hat{\jmath}\,\text{N}\\ &\quad + (100 - 157.1)\,\hat{k}\,\text{N} && (8)\\ &= (55.05\,\hat{\imath} + 65.81\,\hat{\jmath} - 57.14\,\hat{k})\,\text{N}. && (9)\end{aligned}$$

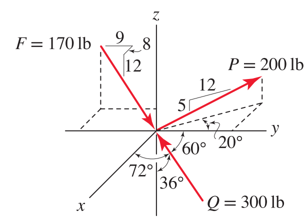
The magnitude of R is

$$R = \sqrt{(55.05\,\text{N})^2 + (65.81\,\text{N})^2 + (-57.14\,\text{N})^2} \tag{10}$$
$$= 103.1\,\text{N}. \tag{11}$$



Problem 2.73

Write an expression for each force using Cartesian representation, and evaluate the resultant force vector. Sketch the resultant force in the xyz coordinate system.



Solution

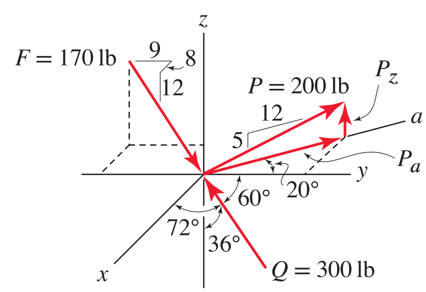
Using the orientation geometry for $\vec{F}$ provided in the problem statement, we may write

$$\boxed{\begin{aligned}\vec{F} &= 170\,\mathrm{lb}\left(\frac{8\,\hat{\imath} + 9\,\hat{\jmath} - 12\,\hat{k}}{17}\right) &(1)\\ &= (80\,\hat{\imath} + 90\,\hat{\jmath} - 120\,\hat{k})\,\mathrm{lb}. &(2)\end{aligned}}$$

Using the sketch shown, the projections of $\vec{P}$ in the z and a directions are

$$P_z = 200\,\mathrm{lb}\,\frac{5}{13} = 76.92\,\mathrm{lb}, \tag{3}$$

$$P_a = 200\,\mathrm{lb}\,\frac{12}{13} = 184.6\,\mathrm{lb}. \tag{4}$$



The component P_a may then be resolved into x and y components as follows

$$P_x = -P_a \sin 20° = -(184.6\,\mathrm{lb})\sin 20° = -63.14\,\mathrm{lb}, \tag{5}$$

$$P_y = P_a \cos 20° = 184.6\,\mathrm{lb}\cos 20° = 173.5\,\mathrm{lb}. \tag{6}$$

Thus

$$\boxed{\vec{P} = (-63.14\,\hat{\imath} + 173.5\,\hat{\jmath} + 76.92\,\hat{k})\,\mathrm{lb}.} \tag{7}$$

The 72° and 60° angles for $\vec{Q}$ are measured from the positive x and y axes, respectively, to the negative direction of the vector $\vec{Q}$. Hence the direction angles are

$$\theta_x = 180° - 72° = 108°, \tag{8}$$

$$\theta_y = 180° - 60° = 120°. \tag{9}$$

The 36° angle is measured from the negative z direction to the negative direction of the vector so that in the end,

$$\theta_z = 36°. \tag{10}$$

Thus

$$\boxed{\begin{aligned}\vec{Q} &= 300\,\text{lb}\,(\cos 108^\circ\,\hat{\imath} + \cos 120^\circ\,\hat{\jmath} + \cos 36^\circ\,\hat{k}) \quad (11)\\ &= (-92.71\,\hat{\imath} - 150\,\hat{\jmath} + 242.7\,\hat{k})\,\text{lb}. \quad (12)\end{aligned}}$$

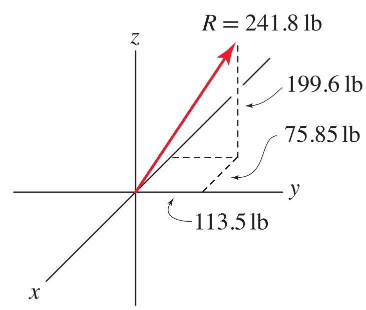
The reasoning behind these direction angles is perhaps subtle, so some useful checks are to verify that Eq. (12) has 300 lb magnitude, and that Eq. (12) has components in the correct directions; that is, we expect the x, y, and z components to be negative, negative, and positive, respectively.

The resultant force vector $\vec{R}$ is

$$\boxed{\begin{aligned}\vec{R} &= \vec{F} + \vec{P} + \vec{Q} \quad (13)\\ &= (80 - 63.14 - 92.71)\,\hat{\imath}\,\text{lb} +\\ &\quad\ (90 + 173.5 - 150)\,\hat{\jmath}\,\text{lb} +\\ &\quad\ (-120 + 76.92 + 242.7)\,\hat{k}\,\text{lb} \quad (14)\\ &= (-75.85\,\hat{\imath} + 113.5\,\hat{\jmath} + 199.6\,\hat{k})\,\text{lb}. \quad (15)\end{aligned}}$$

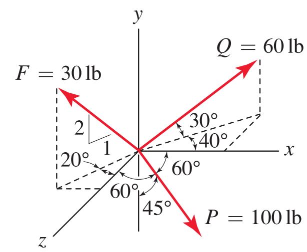
The magnitude of $\vec{R}$ is

$$R = \sqrt{(-75.85\,\text{lb})^2 + (113.5\,\text{lb})^2 + (199.6\,\text{lb})^2} = 241.8\,\text{lb}. \qquad (16)$$

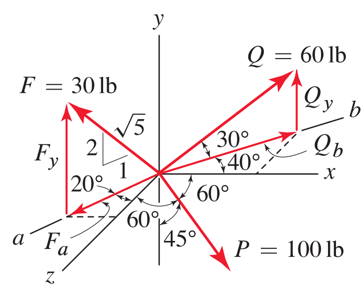


Problem 2.74

Write an expression for each force using Cartesian representation, and evaluate the resultant force vector. Sketch the resultant force in the xyz coordinate system.



Solution



Using the sketch shown, the projections of $\vec{F}$ in the y and a directions are

$$F_y = (30\,\text{lb})\,(2/\sqrt{5}) = 26.83\,\text{lb}, \quad F_a = (30\,\text{lb})\,(1/\sqrt{5}) = 13.42\,\text{lb}. \tag{1}$$

The component F_a may then be resolved into x and z components as follows

$$F_x = -F_a \sin 20° = (-13.42\,\text{lb}) \sin 20° = -4.589\,\text{lb}, \tag{2}$$

$$F_z = F_a \cos 20° = (13.42\,\text{lb}) \cos 20° = 12.61\,\text{lb}, \tag{3}$$

so we may write

$$\boxed{\vec{F} = (-4.59\,\hat{\imath} + 26.8\,\hat{\jmath} + 12.6\,\hat{k})\,\text{lb}.} \tag{4}$$

Using the sketch above, the projections of $\vec{Q}$ in the y and b directions are

$$Q_y = (60\,\text{lb}) \sin 30° = 30.00\,\text{lb}, \quad Q_a = (60\,\text{lb}) \cos 30° = 51.96\,\text{lb}. \tag{5}$$

The component Q_b may then be resolved into x and z components as follows

$$Q_x = Q_b \cos 40° = (51.96\,\text{lb}) \cos 40° = 39.80\,\text{lb}, \tag{6}$$

$$Q_z = -Q_b \cos 40° = (-51.96\,\text{lb}) \sin 40° = -33.40\,\text{lb}, \tag{7}$$

so we may write

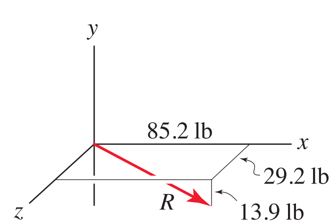
$$\boxed{\vec{Q} = (39.8\,\hat{\imath} + 30.0\,\hat{\jmath} - 33.4\,\hat{k})\,\text{lb}.} \tag{8}$$

Both of the 60° angles associated with $\vec{P}$ are measure from positive x and z axes, hence they are direction angles and $\theta_x = \theta_z = 60°$. Since the 45° angle is measured from the negative y axis, it is not a direction angle, although θ_y is easily determined by $\theta_y = 180° - 45° = 135°$. Thus

$$\boxed{\vec{P} = (100\,\text{lb})(\cos 60°\,\hat{\imath} + \cos 135°\,\hat{\jmath} + \cos 60°\,\hat{k}) = (50.0\,\hat{\imath} - 70.7\,\hat{\jmath} + 50.0\,\hat{k})\,\text{lb}.} \tag{9}$$

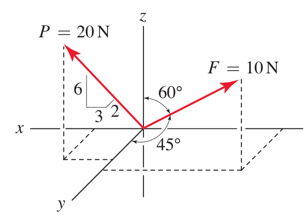
The resultant vector $\vec{R}$ is given by

$$\begin{aligned}
\vec{R} &= \vec{F} + \vec{Q} + \vec{P} && (10)\\
&= (-4.589 + 39.80 + 50.00)\,\hat{\imath}\,\text{lb}\\
&\quad + (26.83 + 30.00 - 70.71)\,\hat{\jmath}\,\text{lb} && (11)\\
&\quad + (12.61 - 33.40 + 50.00)\,\hat{k}\,\text{lb}\\
&= (85.2\,\hat{\imath} - 13.9\,\hat{\jmath} + 29.2\,\hat{k})\,\text{lb}. && (12)
\end{aligned}$$



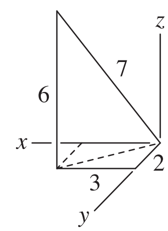
Problem 2.75

Write an expression for each force using Cartesian representation, and evaluate the resultant force vector. Sketch the resultant force in the xyz coordinate system.



Solution

The orientation of $\vec{P}$ is given by the dimensionless vector $\vec{r} = 3\,\hat{\imath} + 2\,\hat{\jmath} + 6\,\hat{k}$, which has magnitude $|\vec{r}| = \sqrt{3^2 + 2^2 + 6^2} = 7$. Thus, the components of $\vec{P}$ are



$$\vec{P} = 20\,\mathrm{N}\frac{\vec{r}}{|\vec{r}|} = 20\,\mathrm{N}\left(\frac{3}{7}\,\hat{\imath} + \frac{2}{7}\,\hat{\jmath} + \frac{6}{7}\,\hat{k}\right). \tag{1}$$

The 45° and 60° angles associated with $\vec{F}$ are both measured from the positive y and z axes, hence they are direction angles and $\theta_y = 45°$ and $\theta_z = 60°$. To determine θ_x, we use the following relation

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1, \tag{2}$$

$$\cos^2\theta_x = 1 - \cos^2 45° - \cos^2 60°, \tag{3}$$

$$\theta_x = \cos^{-1}\left(\pm\sqrt{1 - \cos^2\theta_y - \cos^2\theta_z}\right) = \cos^{-1}(\pm 0.5). \tag{4}$$

The two solutions to the above equation, for $0° \le \theta_x \le 180°$, are $\theta_x = 60°$ and $120°$. By observing the figure given in the problem statement, where the x component of $\vec{F}$ is clearly negative, it is apparent that $\theta_x > 90°$. Therefore, $\vec{F}$ is

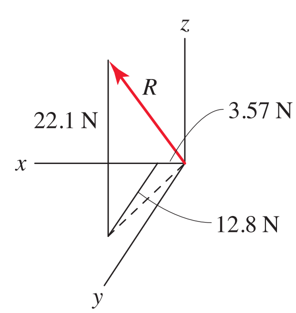
$$\boxed{\vec{F} = (10\,\mathrm{N})(\cos 120°\,\hat{\imath} + \cos 45°\,\hat{\jmath} + \cos 60°\,\hat{k}) = (-5.00\,\hat{\imath} + 7.07\,\hat{\jmath} + 5.00\,\hat{k})\,\mathrm{N}.} \tag{5}$$

The resultant vector $\vec{R}$ is given by

$$\vec{R} = \vec{F} + \vec{P} \tag{6}$$

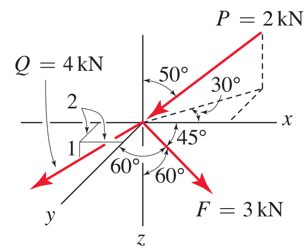
$$= (-5 + 8.571)\,\hat{\imath}\,\mathrm{N} + (7.071 + 5.714)\,\hat{\jmath}\,\mathrm{N} + (5 + 17.14)\,\hat{k}\,\mathrm{N} \tag{7}$$

$$= (3.57\,\hat{\imath} + 12.8\,\hat{\jmath} + 22.1\,\hat{k})\,\mathrm{N}. \tag{8}$$



Problem 2.76

Write an expression for each force using Cartesian representation, and evaluate the resultant force vector. Sketch the resultant force in the xyz coordinate system.



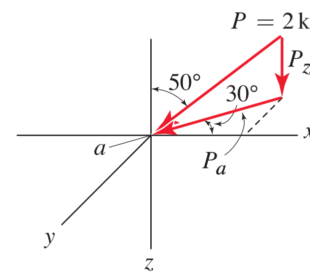
Solution

The orientation of $\vec{Q}$ is given by the dimensionless vector $\vec{r} = -2\,\hat{\imath} + 2\,\hat{\jmath} + 1\,\hat{k}$, which has magnitude $|\vec{r}| = \sqrt{(-2)^2 + 2^2 + 1^2} = 3$. Thus, the components of $\vec{Q}$ are

$$\vec{Q} = 4\,\text{kN}\frac{\vec{r}}{|\vec{r}|} = 4\,\text{kN}\left(\frac{-2}{3}\,\hat{\imath} + \frac{2}{3}\,\hat{\jmath} + \frac{1}{3}\,\hat{k}\right), \tag{1}$$

$$\boxed{\vec{Q} = \left(-2.667\,\hat{\imath} + 2.667\,\hat{\jmath} + 1.333\,\hat{k}\right)\ \text{kN}.} \tag{2}$$

Using the sketch to the right, the projections of $\vec{P}$ in the z and a directions are



$$P_z = (2\,\text{kN})\cos 50^\circ = 1.286\,\text{kN}, \tag{3}$$
$$P_a = (2\,\text{kN})\sin 50^\circ = 1.532\,\text{kN} \tag{4}$$

The component P_a may then be resolved into x and y components as follows

$$P_x = -P_a\cos 30^\circ = (-1.532\,\text{kN})\cos 30^\circ = -1.327\,\text{kN}, \tag{5}$$
$$P_y = P_a\sin 30^\circ = (1.532\,\text{kN})\sin 30^\circ = 0.7660\,\text{kN}, \tag{6}$$

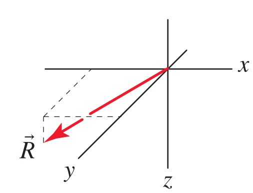
such that

$$\boxed{\vec{P} = \left(-1.327\,\hat{\imath} + 0.7660\,\hat{\jmath} + 1.286\,\hat{k}\right)\ \text{kN}.} \tag{7}$$

The angles reported for $\vec{F}$ in the problem statement are measured from the positive coordinate directions, so they are direction angles, and we may immediately write

$$\boxed{\vec{F} = (3\,\text{kN})(\cos 45^\circ\,\hat{\imath} + \cos 60^\circ\,\hat{\jmath} + \cos 60^\circ\,\hat{k}) = (2.121\,\hat{\imath} + 1.500\,\hat{\jmath} + 1.500\,\hat{k})\,\text{kN}.} \tag{8}$$

The resultant vector $\vec{R}$ is given by

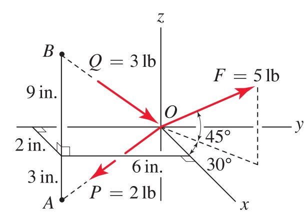


$$\vec{R} = \vec{F} + \vec{P} + \vec{Q} \tag{9}$$
$$\begin{aligned} &= (2.121 - 1.327 - 2.667)\,\hat{\imath}\,\text{kN} \\ &\quad + (1.500 + 0.7660 + 2.667)\,\hat{\jmath}\,\text{kN} \\ &\quad + (1.500 + 1.286 + 1.333)\,\hat{k}\,\text{kN}. \end{aligned} \tag{10}$$

$$\boxed{\vec{R} = (-1.87\,\hat{\imath} + 4.93\,\hat{\jmath} + 4.12\,\hat{k})\,\text{kN}.} \tag{11}$$

Problem 2.77

Write an expression for each force using Cartesian representation, and evaluate the resultant force vector. Sketch the resultant force in the xyz coordinate system.



Solution

The vectors $\vec{r}_{OA}$ and $\vec{r}_{BO}$ are given by

$$\vec{r}_{OA} = (2\,\hat{\imath} - 6\,\hat{\jmath} - 3\,\hat{k})\text{ in.},\quad \vec{r}_{BO} = (-2\,\hat{\imath} + 6\,\hat{\jmath} - 9\,\hat{k})\text{ in.}, \tag{1}$$

and the unit vectors in the directions of these position vectors are

$$\hat{u}_{OA} = \frac{(2\,\hat{\imath} - 6\,\hat{\jmath} - 3\,\hat{k})\text{ in.}}{\sqrt{2^2 + (-6)^2 + (-3)^2}\text{ in.}} = 0.2857\,\hat{\imath} - 0.8571\,\hat{\jmath} - 0.4286\,\hat{k}, \tag{2}$$

$$\hat{u}_{BO} = \frac{(-2\,\hat{\imath} + 6\,\hat{\jmath} - 9\,\hat{k})\text{ in.}}{\sqrt{(-2)^2 + 6^2 + (-9)^2}\text{ in.}} = -0.1818\,\hat{\imath} + 0.5455\,\hat{\jmath} - 0.8182\,\hat{k}. \tag{3}$$

Therefore, the vectors $\vec{P}$ and $\vec{Q}$ are given by

$$\boxed{\vec{P} = (2\text{ lb})\,\hat{u}_{OA} = (0.571\,\hat{\imath} - 1.71\,\hat{\jmath} - 0.857\,\hat{k})\text{ lb},} \tag{4}$$

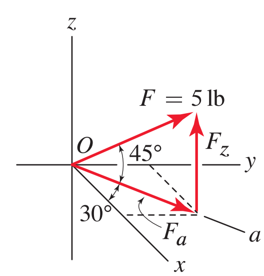
$$\boxed{\vec{Q} = (3\text{ lb})\,\hat{u}_{BO} = (-0.545\,\hat{\imath} + 1.64\,\hat{\jmath} - 2.45\,\hat{k})\text{ lb}.} \tag{5}$$

Using the sketch to the right, the components of $\vec{F}$ in the z and a directions are

$$F_z = (5\text{ lb})\sin 45^\circ = 3.536\text{ lb}\quad F_a = (5\text{ lb})\cos 45^\circ = 3.536\text{ lb}. \tag{6}$$

The component F_a may then be resolved into x and y components as follows

$$F_x = (3.536\text{ lb})\cos 30^\circ = 3.062\text{ lb},\quad F_y = (3.536\text{ lb})\sin 30^\circ = 1.768\text{ lb}. \tag{7}$$

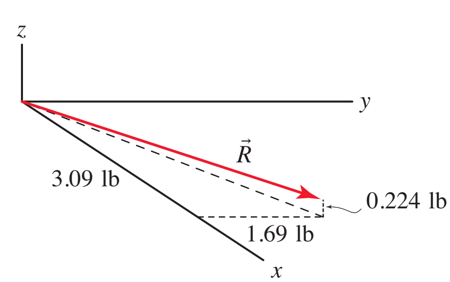


The Cartesian representation for $\vec{F}$ is

$$\boxed{\vec{F} = (3.06\,\hat{\imath} + 1.77\,\hat{\jmath} + 3.54\,\hat{k})\text{ lb}.} \tag{8}$$

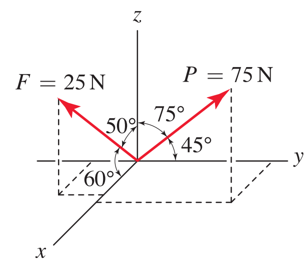
The resultant vector $\vec{R}$ is given by

$$\begin{aligned}
\vec{R} &= \vec{F} + \vec{P} + \vec{Q} && (9)\\
&= (3.062 + 0.5714 - 0.5455)\,\hat{\imath}\text{ lb}\\
&\quad + (1.768 - 1.714 + 1.636)\,\hat{\jmath}\text{ lb} && (10)\\
&\quad + (3.536 - 0.8572 - 2.455)\,\hat{k}\text{ lb}\\
&= (3.09\,\hat{\imath} + 1.69\,\hat{\jmath} + 0.224\,\hat{k})\text{ lb}. && (11)
\end{aligned}$$



Problem 2.78

For each vector, find the direction angles and write an expression for the vector using Cartesian representation. Evaluate the sum of the two vectors, and report the direction angles for the resultant. Also, sketch the resultant in the xyz coordinate system.



Solution

For force $\vec{F}$, the direction angles $\theta_x = 60°$ and $\theta_z = 50°$ are obtained from the figure in the problem statement, and the remaining direction angle is obtained from

$$\theta_y = \cos^{-1}(\pm\sqrt{1-\cos^2\theta_x - \cos^2\theta_z}) = \cos^{-1}(\pm 0.5804) = 54.52°, 125.5°. \tag{1}$$

By examining the orientation of $\vec{F}$ in the figure for the problem statement, the y component of $\vec{F}$ is clearly negative, and hence the correct y direction angle is $\theta_y = 125.5°$. Thus, for force $\vec{F}$,

$$\theta_x = 60°, \theta_y = 125.5°, \theta_z = 50°; \quad \vec{F} = (25\,\text{N})(\cos 60°\,\hat{\imath} + \cos 125.5°\,\hat{\jmath} + \cos 50°\,\hat{k}) \tag{2}$$
$$= (12.5\,\hat{\imath} - 14.5\,\hat{\jmath} + 16.1\,\hat{k})\,\text{N}. \tag{3}$$

For force $\vec{P}$, the direction angles $\theta_y = 45°$ and $\theta_z = 75°$ are obtained from the figure in the problem statement, and the remaining direction angle is obtained from

$$\theta_x = \cos^{-1}(\pm\sqrt{1-\cos^2\theta_y - \cos^2\theta_z}) = \cos^{-1}(\pm 0.6580) = 48.84°, 131.2°. \tag{4}$$

By examining the orientation of $\vec{P}$ in the figure for the problem statement, the x component of $\vec{P}$ is clearly positive, and hence the correct x direction angle is $\theta_x = 48.84°$. Thus, for force $\vec{P}$,

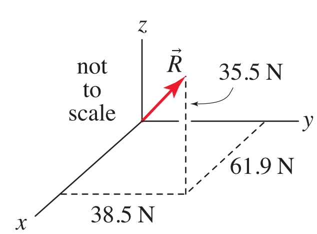
$$\theta_x = 48.84°, \theta_y = 45°, \theta_z = 75°; \quad \vec{P} = (75\,\text{N})(\cos 48.84°\,\hat{\imath} + \cos 45°\,\hat{\jmath} + \cos 75°\,\hat{k}) \tag{5}$$
$$= (49.4\,\hat{\imath} + 53.0\,\hat{\jmath} + 19.4\,\hat{k})\,\text{N}. \tag{6}$$

The sum of these vectors is given by

$$\vec{R} = \vec{F} + \vec{P} = (61.9\,\hat{\imath} + 38.5\,\hat{\jmath} + 35.5\,\hat{k})\,\text{N}, \tag{7}$$

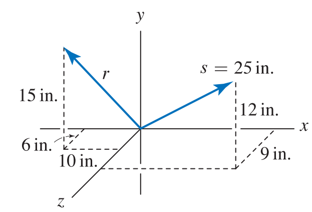
the magnitude is $R = \sqrt{(61.85\,\text{N})^2 + (38.52\,\text{N})^2 + (35.48\,\text{N})^2} = 81.05\,\text{N}$, and the direction angles are:

$$\cos\theta_x = R_x/R = 61.85\,\text{N}/81.05\,\text{N} = 0.7632 \tag{8}$$
$$\Rightarrow \theta_x = \cos^{-1}(0.7632) = 40.3°, \tag{9}$$
$$\cos\theta_y = R_y/R = 38.52\,\text{N}/81.05\,\text{N} = 0.4753 \tag{10}$$
$$\Rightarrow \theta_y = \cos^{-1}(0.4753) = 61.6°, \tag{11}$$
$$\cos\theta_z = R_z/R = 35.48\,\text{N}/81.05\,\text{N} = 0.4378 \tag{12}$$
$$\Rightarrow \theta_z = \cos^{-1}(0.4378) = 64.0°. \tag{13}$$



Problem 2.79

For each vector, find the direction angles and write an expression for the vector using Cartesian representation. Evaluate the sum of the two vectors, and report the direction angles for the resultant. Also, sketch the resultant in the xyz coordinate system.



Solution

From the figure in the problem statement, position vectors $\vec{r}$ and $\vec{s}$ may be immediately written as

$$\boxed{\vec{r} = (-10\,\hat{\imath} + 15\,\hat{\jmath} + 6\,\hat{k})\text{ in.}} \quad \text{and} \quad \vec{s} = (s_x\,\hat{\imath} + 12\,\hat{\jmath} + 9\,\hat{k})\text{ in.}, \tag{1}$$

where s_x denotes the x component of $\vec{s}$. Knowing that the magnitude of $\vec{s}$ is $s = 25$ in., the value of s_x may be determined using

$$s^2 = (25\text{ in.})^2 = s_x^2 + (12\text{ in.})^2 + (9\text{ in.})^2 \;\Rightarrow\; x = \pm 20\text{ in..} \tag{2}$$

The positive solution for x is selected so that $\vec{s}$ has the orientation shown in the problem statement. Hence, the vector representation of $\vec{s}$ is

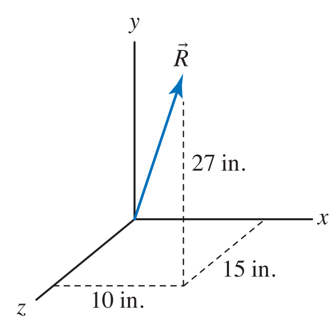
$$\boxed{\vec{s} = (20\,\hat{\imath} + 12\,\hat{\jmath} + 9\,\hat{k})\text{ in.}} \tag{3}$$

The resultant of $\vec{r}$ and $\vec{s}$ is given by

$$\boxed{\vec{R} = \vec{s} + \vec{r} = \big[(20-10)\,\hat{\imath} + (12+15)\,\hat{\jmath} + (9+6)\,\hat{k}\big]\text{ in.} = \big(10\,\hat{\imath} + 27\,\hat{\jmath} + 15\,\hat{k}\big)\text{ in..}} \tag{4}$$

The magnitude of $\vec{R}$ is given by $R = \sqrt{10^2 + 27^2 + 15^2}$ in. $= 32.47$ in., and the direction angles for $\vec{R}$ are obtained using

$$\boxed{\begin{aligned}
\cos\theta_x &= R_x/R = 10\text{ in.}/32.47\text{ in.} = 0.3080 && (5)\\
&\Rightarrow \theta_x = \cos^{-1}(0.3080) = 72.1^\circ, && (6)\\
\cos\theta_y &= R_y/R = 27\text{ in.}/32.47\text{ in.} = 0.8317 && (7)\\
&\Rightarrow \theta_y = \cos^{-1}(0.8317) = 33.7^\circ, && (8)\\
\cos\theta_z &= R_z/R = 15\text{ in.}/32.47\text{ in.} = 0.4620 && (9)\\
&\Rightarrow \theta_z = \cos^{-1}(0.4620) = 62.5^\circ. && (10)
\end{aligned}}$$



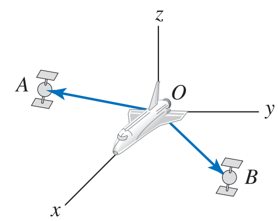
Problem 2.80

The Space Shuttle uses radar to determine the magnitudes and direction cosines of position vectors to satellites A and B as

for $\vec{r}_{OA}$: $|\vec{r}_{OA}| = 2\,\text{km},\ \cos\theta_x = 0.768,\ \cos\theta_y = -0.384,\ \cos\theta_z = 0.512,$

for $\vec{r}_{OB}$: $|\vec{r}_{OB}| = 4\,\text{km},\ \cos\theta_x = 0.743,\ \cos\theta_y = 0.557,\ \cos\theta_z = -0.371.$

Determine the distance between the satellites.



Solution

Begin by writing expressions for the position vectors $\vec{r}_{OA}$ and $\vec{r}_{OB}$ as

$$\vec{r}_{OA} = (2\,\text{km})(0.768\,\hat{\imath} - 0.384\,\hat{\jmath} + 0.512\,\hat{k}) = (1.536\,\hat{\imath} - 0.7680\,\hat{\jmath} + 1.024\,\hat{k})\,\text{km}, \tag{1}$$

$$\vec{r}_{OB} = (4\,\text{km})(0.743\,\hat{\imath} + 0.557\,\hat{\jmath} - 0.371\,\hat{k}) = (2.972\,\hat{\imath} + 2.228\,\hat{\jmath} - 1.484\,\hat{k})\,\text{km}. \tag{2}$$

The position vector from A to B is given by $\vec{r}_{AB}$, and it may be expressed as

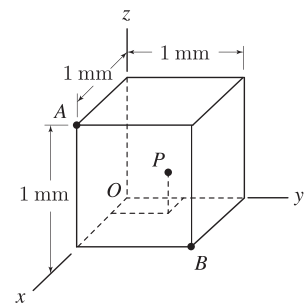
$$\begin{aligned}\vec{r}_{AB} &= \vec{r}_{AO} + \vec{r}_{OB} && (3)\\ &= -\vec{r}_{OA} + \vec{r}_{OB} && (4)\\ &= \big((-1.536 + 2.972)\,\hat{\imath} + (0.7680 + 2.228)\,\hat{\jmath} + (-1.024 - 1.484)\,\hat{k}\big)\,\text{km} && (5)\\ &= (1.436\,\hat{\imath} + 2.996\,\hat{\jmath} - 2.508\,\hat{k})\,\text{km}. && (6)\end{aligned}$$

The magnitude of $\vec{r}_{AB}$ is the distance between the satellites. Thus

$$\boxed{r_{AB} = \sqrt{(1.436\,\text{km})^2 + (2.996\,\text{km})^2 + (-2.508\,\text{km})^2} = 4.16\,\text{km}.} \tag{7}$$

Problem 2.81

A cube of material with 1 mm edge lengths is examined by a scanning electron microscope, and a small inclusion (i.e., a cavity) is found at point P. It is determined that the direction cosines for a vector from points A to P are $\cos\theta_x = -0.485$, and $\cos\theta_y = 0.485$, and $\cos\theta_z = -0.728$; and the direction cosines for a vector from points B to P are $\cos\theta_x = -0.667$, $\cos\theta_y = -0.667$, and $\cos\theta_z = 0.333$. Determine the coordinates of point P.



Solution

The distance between points A and P and points B and P is r_{AP} and r_{BP}, respectively. If $\vec{r}_{BA}$ is the position vector from point B to point A, given by

$$\vec{r}_{BA} = (-1\,\hat{\jmath} + 1\,\hat{k})\,\text{mm}, \tag{1}$$

then we may write

$$\vec{r}_{BA} + \vec{r}_{AP} = \vec{r}_{BP} \tag{2}$$

$$(-1\,\hat{\jmath} + 1\,\hat{k})\,\text{mm} + r_{AP}(-0.485\,\hat{\imath} + 0.485\,\hat{\jmath} - 0.728\,\hat{k}) = r_{BP}(-0.667\,\hat{\imath} - 0.667\,\hat{\jmath} + 0.333\,\hat{k}). \tag{3}$$

Equation (3) may be rewritten as

$$\begin{aligned}(-0.485\,r_{AP} + 0.667\,r_{BP})\,\hat{\imath} + (-1\,\text{mm} + 0.485\,r_{AP} + 0.667\,r_{BP})\,\hat{\jmath}&\\ + (1\,\text{mm} - 0.728\,r_{AP} - 0.333\,r_{BP})\,\hat{k} = \vec{0}.&\end{aligned} \tag{4}$$

For Eq. (4) to be satisfied, each term multiplying $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ must be zero. Furthermore, Eq. (4) contains only two unknowns, thus we use the equations corresponding to the $\hat{\imath}$ and $\hat{\jmath}$ terms to solve for r_{AP} and r_{BP} as follows

$$-0.485\,r_{AP} + 0.667\,r_{BP} = 0 \quad\Rightarrow\quad r_{AP} = 1.375\,r_{BP}, \tag{5}$$

$$-1\,\text{mm} + 0.485(1.375\,r_{BP}) + 0.667\,r_{BP} = 0 \quad\Rightarrow\quad r_{BP} = 0.7496\,\text{mm}. \tag{6}$$

Note that the above solutions for r_{AP} and r_{BP} also satisfy the equation corresponding to the $\hat{k}$ term.

To determine the coordinates of point P, we consider the vector $\vec{r}_{OP}$, which may be written as

$$\vec{r}_{OP} = \vec{r}_{OB} + \vec{r}_{BP}. \tag{7}$$

In Eq. (7), $\vec{r}_{OP} = x_P\,\hat{\imath} + y_P\,\hat{\jmath} + z_P\,\hat{k}$, the vector $\vec{r}_{BP}$ is known from the direction cosines given in the problem statement and Eq. (6), and $\vec{r}_{OB} = (1\,\hat{\imath} + 1\,\hat{\jmath})\,\text{mm}$. Thus, Eq. (7) becomes

$$x_P\,\hat{\imath} + y_P\,\hat{\jmath} + z_P\,\hat{k} = (1\,\hat{\imath} + 1\,\hat{\jmath})\,\text{mm} + (0.7496\,\text{mm})(-0.667\,\hat{\imath} - 0.667\,\hat{\jmath} + 0.333\,\hat{k})\,\text{mm} \tag{8}$$

$$= \big(0.500\,\hat{\imath} + 0.500\,\hat{\jmath} + 0.250\,\hat{k}\big). \tag{9}$$

Therefore, the coordinates of point P are

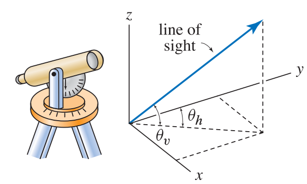
$$\boxed{(0.500, 0.500, 0.250)\,\text{mm}.} \tag{10}$$

Problem 2.82

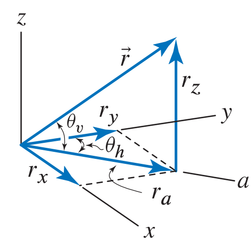
A theodolite is an instrument that measures horizontal and vertical angular orientations of a line of sight. The line of sight may be established optically, or by laser, and many forms of theodolite are used in surveying, construction, astronomy, and manufacturing. A simple optical theodolite is shown. After the instrument is aligned so that its base is in a horizontal plane and a desired reference direction is selected (such as perhaps north in the case of a surveying instrument), the telescopic sight is used to establish a line of sight, and then horizontal and vertical angles θ_h and θ_v are measured.

(a) If θ_h and θ_v are known, derive an expression that gives the direction angles θ_x, θ_y, and θ_z for the line of sight.

(b) If $\theta_h = 30°$ and $\theta_v = 60°$, use your answer to Part (a) to determine θ_x, θ_y, and θ_z.



Solution



Part (a) Let $\vec{r}$ be a vector along the line of sight of the theodolite, and let the magnitude of this vector be r. Using angle θ_v, this vector may be resolved into z direction and a direction components, as shown in the figure to the right, as

$$r_z = r \sin\theta_v \quad \text{and} \quad r_a = r \cos\theta_v. \tag{1}$$

The component r_a may then be resolved into x and y components as

$$r_x = r_a \sin\theta_h \quad \text{and} \quad r_y = r_a \cos\theta_h. \tag{2}$$

Combining Eqs. (1) and (2) allows an expression for $\vec{r}$ to be written as

$$\vec{r} = r\big(\sin\theta_h \cos\theta_v\,\hat{\imath} + \cos\theta_h \cos\theta_v\,\hat{\jmath} + \sin\theta_v\,\hat{k}\big). \tag{3}$$

In terms of direction angles θ_x, θ_y, and θ_z, the vector $\vec{r}$ may alternatively be written as

$$\vec{r} = \cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + \cos\theta_z\,\hat{k}. \tag{4}$$

For Eqs. (3) and (4) to agree, the direction angles are

$$\cos\theta_x = \sin\theta_h \cos\theta_v \quad \Rightarrow \quad \theta_x = \pm\cos^{-1}(\sin\theta_h \cos\theta_v), \tag{5}$$

$$\cos\theta_y = \cos\theta_h \cos\theta_v \quad \Rightarrow \quad \theta_y = \pm\cos^{-1}(\cos\theta_h \cos\theta_v), \tag{6}$$

$$\cos\theta_z = \sin\theta_v \quad \Rightarrow \quad \theta_z = \pm\cos^{-1}(\sin\theta_v) = \pm(90° - \theta_v). \tag{7}$$

Part (b) With $\theta_h = 30°$ and $\theta_v = 60°$, Eqs. (5)–(7) provide the direction angles for the line of sight as

$$\theta_x = \cos^{-1}(\sin 30° \cos 60°) = 75.5°, \tag{8}$$

$$\theta_y = \cos^{-1}(\cos 30° \cos 60°) = 64.3°, \tag{9}$$

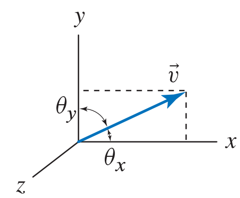
$$\theta_z = 90° - 60° = 30°. \tag{10}$$

Problem 2.83

> With direction angles, general practice is to use values of θ_x, θ_y, and θ_z between 0° and 180°, and this is sufficient to characterize the orientation of any vector in three dimensions. When direction angles are measured in this way, the sum of any two direction angles is always 90° or greater. Offer an argument that shows this statement is true.

Solution

We are asked to show that the quantities $\theta_x + \theta_y$, $\theta_x + \theta_z$, and $\theta_y + \theta_z$ are all equal to or greater than 90°. Begin by considering a vector $\vec{v}$ that lies in the xy plane, in the quadrant shown at the right. For this vector, $\theta_z = 90°$, and it necessarily follows that $\theta_x + \theta_y$ must then equal 90°, i.e.,



$$\theta_z = 90° \quad \Rightarrow \quad \theta_x + \theta_y = 90°. \tag{1}$$

Note that if the vector lies in any other quadrant of the xy plane, then Eq. (1) becomes $\theta_x + \theta_y > 90°$; hence the quadrant shown gives rise to the most conservative case for our purposes. Now, if we add a z component to this vector while keeping the x and y components fixed, it follows that $\theta_x + \theta_y > 90°$. Therefore, for any vector constructed by following this approach,

$$\theta_x + \theta_y \geq 90°. \tag{2}$$

Note that as we add a z component in the manner described, θ_z will decrease. Therefore, we cannot yet say anything about the quantities $\theta_x + \theta_z$ or $\theta_y + \theta_z$.

Now, repeat the above argument starting with a vector having only y and z components, and then add the x component. This leads to the conclusion that for any vector constructed in this manner, its direction angles satisfy

$$\theta_y + \theta_z \geq 90°. \tag{3}$$

Repeating this same line of reasoning once more, starting with a vector having x and z components, and then add the y component. This leads to the conclusion that for any vector constructed in this manner, its direction angles satisfy

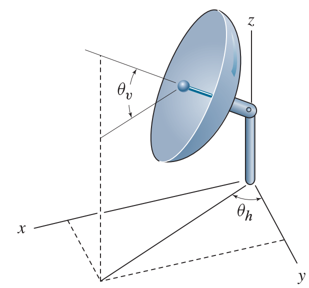
$$\theta_x + \theta_z \geq 90°. \tag{4}$$

Since any vector may be constructed by following any of these approaches, it is true that

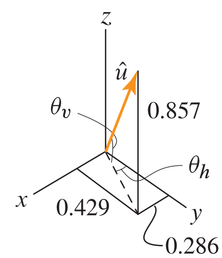
$$\boxed{\theta_x + \theta_y \geq 90°, \quad \theta_y + \theta_z \geq 90°, \quad \text{and} \quad \theta_x + \theta_z \geq 90°.} \tag{5}$$

Problem 2.84

If the direction cosines for the satellite antenna shown are to be $\cos\theta_x = 0.286$, $\cos\theta_y = 0.429$, and $\cos\theta_z = 0.857$, determine the values of angles θ_h and θ_v.



Solution



Let $\hat{u}$ be a unit vector that points in the direction of the antenna. Using the direction cosines given in the problem statement, this vector may be written as

$$\hat{u} = 0.286\,\hat{\imath} + 0.429\,\hat{\jmath} + 0.857\,\hat{k}. \tag{1}$$

The figure to the right shows the components of $\hat{u}$. By inspection of this figure, observe that

$$\tan\theta_h = \frac{x \text{ component of } \hat{u}}{y \text{ component of } \hat{u}} = \frac{\cos\theta_x}{\cos\theta_y} = \frac{0.286}{0.429}, \tag{2}$$

$$\sin\theta_v = \frac{z \text{ component of } \hat{u}}{1} = \frac{\cos\theta_z}{1} = \frac{0.857}{1}. \tag{3}$$

Hence,

$$\boxed{\theta_h = \tan^{-1}\frac{0.286}{0.429} = 33.7°, \qquad \theta_v = \sin^{-1}\frac{0.857}{1} = 59.0°.} \tag{4}$$

Remark These answers may be checked using the results of Prob. 2.82. Before carrying out this check, we must first verify that θ_h and θ_v are defined with respect to the x, y, and z axis in the same way in this problem and in Prob. 2.82, and indeed this is the case. Then, substituting the values of θ_x, θ_y, θ_z, θ_h, and θ_v into the relations from Prob. 2.82 shows that all of the following expressions are satisfied

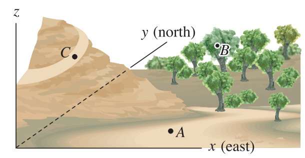
$$\cos\theta_x = \sin\theta_h\cos\theta_v = 0.2859, \tag{5}$$

$$\cos\theta_y = \cos\theta_h\cos\theta_v = 0.4288, \tag{6}$$

$$\cos\theta_z = \sin\theta_v = 0.8570. \tag{7}$$

Problem 2.85

Using a theodolite with a laser rangefinder, a researcher at point A locates a pair of rare birds nesting at point B and determines the direction angles $\theta_x = 53°$, $\theta_y = 38°$, and $\theta_z = 81°$ and distance 242 m for a position vector from point A to B. For subsequent observations, the researcher would like to use position C, and thus while at point A, she also measures the direction angles $\theta_x = 142°$, $\theta_y = 63°$, and $\theta_z = 65°$ and distance 309 m for a position vector from point A to C. For observation from point C, determine the direction angles and distance to the nest at B.



Solution

Our objective is to determine the position vector $\vec{r}_{CB}$, and we may accomplish this by using vector addition as follows

$$\begin{aligned}
\vec{r}_{CB} &= \vec{r}_{CA} + \vec{r}_{AB} && (1)\\
&= -\vec{r}_{AC} + \vec{r}_{AB} && (2)\\
&= -(309\,\mathrm{m})(\cos 142°\,\hat{\imath} + \cos 63°\,\hat{\jmath} + \cos 65°\,\hat{k})\\
&\quad +(242\,\mathrm{m})(\cos 53°\,\hat{\imath} + \cos 38°\,\hat{\jmath} + \cos 81°\,\hat{k}) && (3)\\
&= (389.1\,\hat{\imath} + 50.42\,\hat{\jmath} - 92.73\,\hat{k})\,\mathrm{m}. && (4)
\end{aligned}$$

The magnitude of $\vec{r}_{CB}$, and hence the distance from point C to the nest at point B, is

$$\boxed{r_{CB} = \sqrt{(389.1\,\mathrm{m})^2 + (50.42\,\mathrm{m})^2 + (-92.73\,\mathrm{m})^2} = 403.2\,\mathrm{m}.} \tag{5}$$

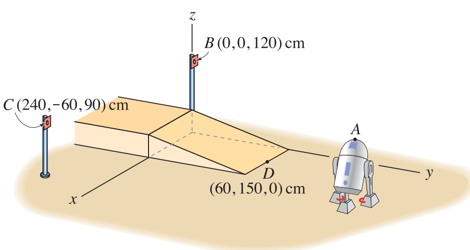
The direction angles for $\vec{r}_{CB}$ are

$$\boxed{\begin{aligned}
\cos\theta_x &= \frac{(r_{CB})_x}{r_{CB}} = \frac{389.1\,\mathrm{m}}{403.2\,\mathrm{m}} = 0.9651 \quad\Rightarrow\quad \theta_x = \cos^{-1}(0.9651) = 15.18°, && (6)\\
\cos\theta_y &= \frac{(r_{CB})_y}{r_{CB}} = \frac{50.42\,\mathrm{m}}{403.2\,\mathrm{m}} = 0.1250 \quad\Rightarrow\quad \theta_y = \cos^{-1}(0.1250) = 82.82°, && (7)\\
\cos\theta_z &= \frac{(r_{CB})_z}{r_{CB}} = \frac{-92.73\,\mathrm{m}}{403.2\,\mathrm{m}} = -0.2300 \quad\Rightarrow\quad \theta_z = \cos^{-1}(-0.2300) = 103.3°. && (8)
\end{aligned}}$$

Note that the direction angles in the problem statement are accurate to only two digits, hence the direction angles given in Eqs. (6)–(8) should be interpreted as being of similar accuracy.

Problem 2.86

A robot maneuvers itself by using a laser guidance system where the positions of reflective targets, such as at points B and C, are known. By scanning the room with a laser that emanates from point A, and sensing the reflections from targets B and C, the robot may compute the direction cosines of position vectors as given below. The locations of other points, such as point D at the base of the ramp, are also known, but such locations do not have reflective targets. Determine the position vector from points A to D so that the robot may move from its current position to the base of the ramp at point D.



for $\vec{r}_{AB}$: $\cos\theta_x = -0.4212$, $\cos\theta_y = -0.9025$, $\cos\theta_z = 0.0902$,
for $\vec{r}_{AC}$: $\cos\theta_x = 0.2676$, $\cos\theta_y = -0.9635$, $\cos\theta_z = 0$.

Solution

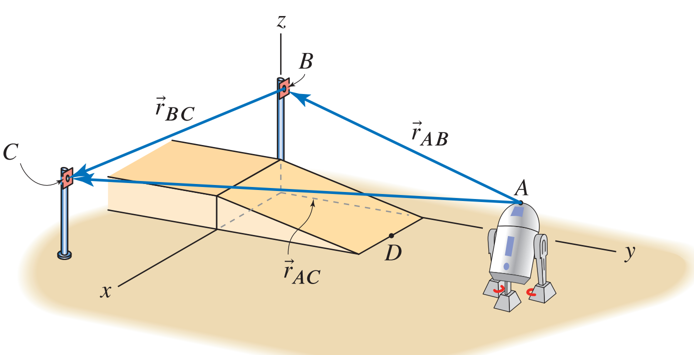
With the direction cosines provided in the problem statement, the following position vectors may be written

$$\vec{r}_{AB} = r_{AB}(-0.4212\,\hat{\imath} - 0.9025\,\hat{\jmath} + 0.0902\,\hat{k}), \tag{1}$$
$$\vec{r}_{AC} = r_{AC}(0.2676\,\hat{\imath} - 0.9635\,\hat{\jmath}). \tag{2}$$

With the coordinates of points B and C provided in the problem statement, the following position vector may be written

$$\vec{r}_{BC} = \left[(240-0)\,\hat{\imath} + (-60-0)\,\hat{\jmath} + (90-120)\,\hat{k}\right]\text{cm} \tag{3}$$
$$= (240\,\hat{\imath} - 60\,\hat{\jmath} - 30\,\hat{k})\,\text{cm}. \tag{4}$$

Using vector addition, as shown below, we may write



$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} \tag{5}$$

$$r_{AC}(0.2676\,\hat{\imath} - 0.9635\,\hat{\jmath}) = r_{AB}(-0.4212\,\hat{\imath} - 0.9025\,\hat{\jmath} + 0.0902\,\hat{k}) + (240\,\hat{\imath} - 60\,\hat{\jmath} - 30\,\hat{k})\,\text{cm}. \tag{6}$$

Equating the x, y, and z components of the above vector expression provides

$$r_{AC}(0.2676) = r_{AB}(-0.4212) + 240\,\text{cm}, \tag{7}$$
$$r_{AC}(-0.9635) = r_{AB}(-0.9025) - 60\,\text{cm}, \tag{8}$$
$$0 = r_{AB}(0.0902) - 30\,\text{cm}. \tag{9}$$

Equation (9) may be immediately solved to obtain

$$r_{AB} = \frac{30\,\text{cm}}{0.0902} = 332.6\,\text{cm}. \tag{10}$$

Then, either of Eqs. (7) or (8) may be used to determine r_{AC}, as follows. If Eq. (7) is used, then

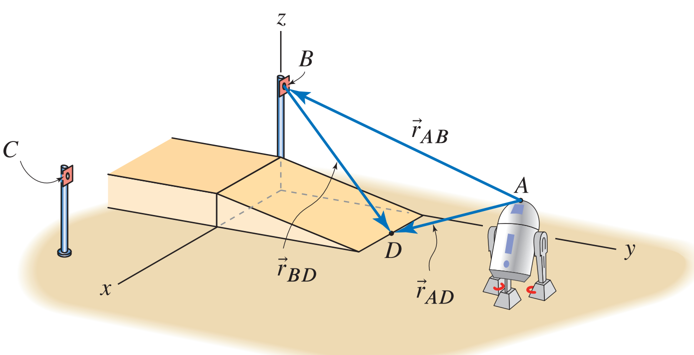
$$r_{AC} = \frac{r_{AB}(-0.4212) + 240\,\text{cm}}{0.2676} = 373.4\,\text{cm}, \tag{11}$$

and if Eq. (8) is used instead, then

$$r_{AC} = \frac{r_{AB}(-0.9025) - 60\,\text{cm}}{-0.9635} = 373.8\,\text{cm}. \tag{12}$$

The differences in the results in Eqs. (11) and (12) is small and is due to the small error in using four digits for the direction cosines in the problem statement. In the remainder of this solution, we will not need $\vec{r}_{AC}$, although if this were to be used, then it would probably be best to use an average of Eqs. (11) and (12).

Using vector addition, as shown below, we may write



$$\vec{r}_{AD} = \vec{r}_{AB} + \vec{r}_{BD}. \tag{13}$$

With the coordinates of points B and D provided in the problem statement, we have

$$\vec{r}_{BD} = \left[(60-0)\hat{\imath} + (150-0)\hat{\jmath} + (0 - 120\hat{k})\right]\text{cm} \tag{14}$$
$$= (60\hat{\imath} + 150\hat{\jmath} - 120\hat{k})\,\text{cm}. \tag{15}$$

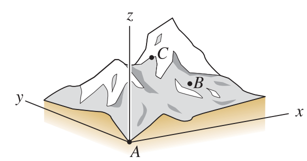
Hence,

$$\vec{r}_{AD} = 332.6\,\text{cm}(-0.4212\hat{\imath} - 0.9025\hat{\jmath} + 0.0902\hat{k}) + (60\hat{\imath} + 150\hat{\jmath} - 120\hat{k})\,\text{cm} \tag{16}$$

$$\boxed{\vec{r}_{AD} = (-80.09\hat{\imath} - 150.2\hat{\jmath} - 90.00\hat{k})\text{cm}.} \tag{17}$$

Problem 2.87

A mountain climbing team establishes base camps at points A and B, and the next camp is to be at point C. Camps A and B have elevations of 11,500 ft and 16,000 ft above sea level, respectively. With an optical telescope, the direction angles for position vectors from A to B, and B to C, and A to C are determined as follows:



$$\begin{aligned}
&\text{for } \vec{r}_{AB}: \quad \theta_x = 34.3^\circ, \quad \theta_y = 63.2^\circ, \quad \theta_z = 70.2^\circ,\\
&\text{for } \vec{r}_{BC}: \quad \theta_x = 104.7^\circ, \quad \theta_y = 27.3^\circ, \quad \theta_z = 67.6^\circ,\\
&\text{for } \vec{r}_{AC}: \quad \theta_x = 59.1^\circ, \quad \theta_y = 42.0^\circ, \quad \theta_z = 64.6^\circ.
\end{aligned}$$

Determine the elevation above sea level of camp C.

Solution

With the direction angles provided in the problem statement, the following position vectors may be written

$$\vec{r}_{AB} = r_{AB}(\cos 34.3^\circ \hat{\imath} + \cos 63.2^\circ \hat{\jmath} + \cos 70.2^\circ \hat{k}), \tag{1}$$

$$\vec{r}_{BC} = r_{BC}(\cos 104.7^\circ \hat{\imath} + \cos 27.3^\circ \hat{\jmath} + \cos 67.6^\circ \hat{k}), \tag{2}$$

$$\vec{r}_{AC} = r_{AC}(\cos 59.1^\circ \hat{\imath} + \cos 42.0^\circ \hat{\jmath} + \cos 64.6^\circ \hat{k}). \tag{3}$$

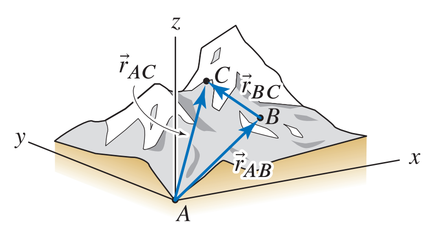
Using the elevations of points A and B given in the problem statement, the difference in elevation between points B and A is 16,000 ft − 11,500 ft = 4,500 ft. Hence, the z component of $\vec{r}_{AB}$ in Eq. (1) must be 4,500 ft, which provides

$$r_{AB} \cos 70.2^\circ = 4{,}500\,\text{ft} \Rightarrow r_{AB} = 13{,}285\,\text{ft}. \tag{4}$$

Using vector addition, as shown at the right, we may write

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} \tag{5}$$

$$= 13{,}285\,\text{ft}\,(\cos 34.3^\circ\,\hat{\imath} + \cos 63.2\,\hat{\jmath} + \cos 70.2\,\hat{k}) + r_{BC}(\cos 104.7^\circ\,\hat{\imath} + \cos 27.3^\circ\,\hat{\jmath} + \cos 67.6^\circ\,\hat{k}). \tag{6}$$



Equations (3) and (6) are equal, and we equate the x, y, and z components of these to obtain the following equations

$$r_{AC} \cos 59.1^\circ = (13{,}285\,\text{ft}) \cos 34.3^\circ + r_{BC} \cos 104.7^\circ, \tag{7}$$

$$r_{AC} \cos 42.0^\circ = (13{,}285\,\text{ft}) \cos 63.2^\circ + r_{BC} \cos 27.3^\circ, \tag{8}$$

$$r_{AC} \cos 64.6^\circ = (13{,}285\,\text{ft}) \cos 70.2^\circ + r_{BC} \cos 67.6^\circ. \tag{9}$$

As explained in the Closer Look discussion at the end of Example 2.11 on p. 73, only two of Eqs. (7)–(9) are independent, and any two of the three may be used to determine the unknowns r_{AC} and r_{BC}, with the following results:

$$\text{Using Eqs. (7) and (8)} \quad \Rightarrow \quad r_{AC} = 17{,}478\,\text{ft}, \quad \text{and} \quad r_{BC} = 7{,}876\,\text{ft}, \tag{10}$$

$$\text{Using Eqs. (8) and (9)} \quad \Rightarrow \quad r_{AC} = 17{,}519\,\text{ft}, \quad \text{and} \quad r_{BC} = 7{,}910\,\text{ft}, \tag{11}$$

$$\text{Using Eqs. (7) and (9)} \quad \Rightarrow \quad r_{AC} = 17{,}482\,\text{ft}, \quad \text{and} \quad r_{BC} = 7{,}869\,\text{ft}. \tag{12}$$

The small differences in these results are due to the small inaccuracy in the direction angles given in the problem statement. Thus, we will use the approx. values

$$r_{AC} = 17{,}500\,\text{ft} \quad \text{and} \quad r_{BC} = 7{,}890\,\text{ft}. \tag{13}$$

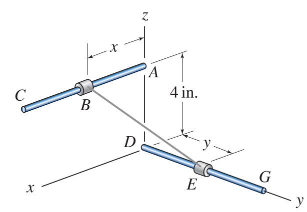
The elevation of point C relative to point A is given by the z component of $\vec{r}_{AC}$. Hence, the elevation above sea level of point C is approximately

$$h = 11{,}500\,\text{ft} + 17{,}500\,\text{ft}\,\cos 64.6° \tag{14}$$

$$= \boxed{19{,}000\,\text{ft.}} \tag{15}$$

Problem 2.88

Bars AC and DG are straight and parallel to the x and y axes, respectively. BE is a cable whose tensile force is 100 lb. For the dimensions given, determine expressions for the force the cable exerts on B and the force the cable exerts on E using Cartesian vector representation. $x = 4$ in. and $y = 7$ in.



Solution

Begin by writing an expression for the vector $\vec{r}_{BE}$, which is the position vector pointing from B to E

$$\vec{r}_{BE} = (-4\,\hat{\imath} + 7\,\hat{\jmath} - 4\,\hat{k})\,\text{in.}, \qquad |\vec{r}_{BE}| = \sqrt{(-4)^2 + (7)^2 + (-4)^2}\,\text{in.} = 9\,\text{in.} \tag{1}$$

Since the force in cable BE is tensile, the force that this cable applies to the collar at B is directed from B toward E, hence

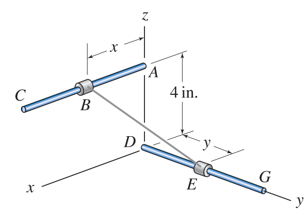
$$\boxed{\begin{aligned}\vec{F}_{BE} &= (100\,\text{lb})\frac{\vec{r}_{BE}}{|\vec{r}_{BE}|} = (100\,\text{lb})\frac{-4\,\hat{\imath} + 7\,\hat{\jmath} - 4\,\hat{k}}{9}\\ &= (-44.4\,\hat{\imath} + 77.8\,\hat{\jmath} - 44.4\,\hat{k})\,\text{lb.}\end{aligned}} \tag{2}$$

The force that cable BE applies to the collar at E is directed from E toward B, hence

$$\boxed{\begin{aligned}\vec{F}_{EB} &= -\vec{F}_{BE}\\ &= (44.4\,\hat{\imath} - 77.8\,\hat{\jmath} + 44.4\,\hat{k})\,\text{lb.}\end{aligned}} \tag{3}$$

Problem 2.89

Bars AC and DG are straight and parallel to the x and y axes, respectively. BE is a cable whose tensile force is 100 lb. For the dimensions given, determine expressions for the force the cable exerts on B and the force the cable exerts on E using Cartesian vector representation. $x = 6$ in. and $y = 12$ in.



Solution

Begin by writing an expression for the vector $\vec{r}_{BE}$, which is the position vector pointing from B to E

$$\vec{r}_{BE} = (-6\,\hat{\imath} + 12\,\hat{\jmath} - 4\,\hat{k})\text{ in.}, \qquad |\vec{r}_{BE}| = \sqrt{(-6)^2 + (12)^2 + (-4)^2}\text{ in.} = 14\text{ in.} \tag{1}$$

Since the force in cable BE is tensile, the force that this cable applies to the collar at B is directed from B toward E, hence

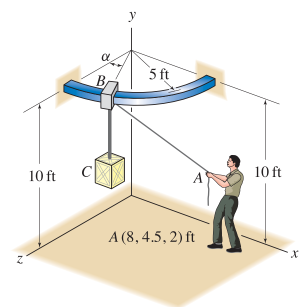
$$\boxed{\begin{aligned}\vec{F}_{BE} &= (100\text{ lb})\frac{\vec{r}_{BE}}{|\vec{r}_{BE}|} = (100\text{ lb})\frac{-6\,\hat{\imath} + 12\,\hat{\jmath} - 4\,\hat{k}}{14}\\ &= (-42.9\,\hat{\imath} + 85.7\,\hat{\jmath} - 28.6\,\hat{k})\text{ lb.}\end{aligned}} \tag{2}$$

The force that cable BE applies to the collar at E is directed from E toward B, hence

$$\boxed{\begin{aligned}\vec{F}_{EB} &= -\vec{F}_{BE}\\ &= (42.9\,\hat{\imath} - 85.7\,\hat{\jmath} + 28.6\,\hat{k})\text{ lb.}\end{aligned}} \tag{3}$$

Problem 2.90

A crane consists of a quarter-circular bar that lies in a plane parallel to the xz plane with a low-friction collar at point B that may slide on the bar. The forces supported by cables AB and BC are 30 lb and 1500 lb, respectively, and the coordinates of point A are given in the figure. For the value of angle α given below, determine expressions for the forces the two cables apply to the collar at point B. $\alpha = 0°$.



Solution

The coordinates of point A are given in the problem statement, and with $\alpha = 0°$, the coordinates of point B are (0, 10, 5) ft. Hence, the position vector from point B to point A is

$$\vec{r}_{BA} = (8\,\hat{\imath} - 5.5\,\hat{\jmath} - 3\,\hat{k})\text{ ft.} \tag{1}$$

The force that cable AB applies to the collar at point B is

$$\vec{F}_{BA} = 30\text{ lb}\,\frac{\vec{r}_{BA}}{r_{BA}} = 30\text{ lb}\,\frac{8\,\hat{\imath} - 5.5\,\hat{\jmath} - 3\,\hat{k}}{10.16} \tag{2}$$

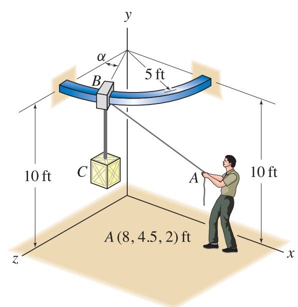
$$= \boxed{(23.62\,\hat{\imath} - 16.24\,\hat{\jmath} - 8.857\,\hat{k})\text{ lb.}} \tag{3}$$

By inspection, the force that cable BC applies to the collar at point B is

$$\boxed{\vec{F}_{BC} = -1500\text{ lb}\,\hat{\jmath}.} \tag{4}$$

Problem 2.91

A crane consists of a quarter-circular bar that lies in a plane parallel to the xz plane with a low-friction collar at point B that may slide on the bar. The forces supported by cables AB and BC are 30 lb and 1500 lb, respectively, and the coordinates of point A are given in the figure. For the value of angle α given below, determine expressions for the forces the two cables apply to the collar at point B. $\alpha = 30°$.



Solution

The coordinates of point A are given in the problem statement, and with $\alpha = 30°$, the coordinates of point B are $(5\,\text{ft} \sin 30°, 10\,\text{ft}, 5\,\text{ft} \cos 30°)$. Hence, the position vector from point B to point A is

$$\vec{r}_{BA} = (5.5\,\hat{\imath} - 5.5\,\hat{\jmath} - 2.330\,\hat{k})\,\text{ft}. \tag{1}$$

The force that cable AB applies to the collar at point B is

$$\vec{F}_{BA} = 30\,\text{lb}\,\frac{\vec{r}_{BA}}{r_{BA}} = 30\,\text{lb}\,\frac{5.5\,\hat{\imath} - 5.5\,\hat{\jmath} - 2.330\,\hat{k}}{8.120} \tag{2}$$

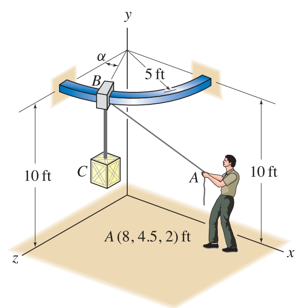
$$= \boxed{(20.32\,\hat{\imath} - 20.32\,\hat{\jmath} - 8.609\,\hat{k})\,\text{lb}.} \tag{3}$$

By inspection, the force that cable BC applies to the collar at point B is

$$\boxed{\vec{F}_{BC} = -1500\,\text{lb}\,\hat{\jmath}.} \tag{4}$$

Problem 2.92

A crane consists of a quarter-circular bar that lies in a plane parallel to the xz plane with a low-friction collar at point B that may slide on the bar. The forces supported by cables AB and BC are 30 lb and 1500 lb, respectively, and the coordinates of point A are given in the figure. For the value of angle α given below, determine expressions for the forces the two cables apply to the collar at point B. $\alpha = 60°$.



Solution

The coordinates of point A are given in the problem statement, and with $\alpha = 60°$, the coordinates of point B are (5 ft sin 60°, 10 ft, 5 ft cos 60°). Hence, the position vector from point B to point A is

$$\vec{r}_{BA} = (3.670\,\hat{\imath} - 5.5\,\hat{\jmath} - 0.5\,\hat{k})\ \text{ft}. \tag{1}$$

The force that cable AB applies to the collar at point B is

$$\vec{F}_{BA} = 30\,\text{lb}\,\frac{\vec{r}_{BA}}{r_{BA}} = 30\,\text{lb}\,\frac{3.670\,\hat{\imath} - 5.5\,\hat{\jmath} - 0.5\,\hat{k}}{6.631} \tag{2}$$

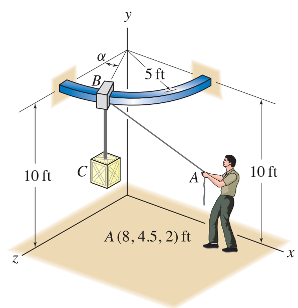
$$= \boxed{(16.60\,\hat{\imath} - 24.88\,\hat{\jmath} - 2.262\,\hat{k})\ \text{lb}.} \tag{3}$$

By inspection, the force that cable BC applies to the collar at point B is

$$\boxed{\vec{F}_{BC} = -1500\,\text{lb}\,\hat{\jmath}.} \tag{4}$$

Problem 2.93

A crane consists of a quarter-circular bar that lies in a plane parallel to the xz plane with a low-friction collar at point B that may slide on the bar. The forces supported by cables AB and BC are 30 lb and 1500 lb, respectively, and the coordinates of point A are given in the figure. For the value of angle α given below, determine expressions for the forces the two cables apply to the collar at point B. General values of α where $0° \le \alpha \le 90°$.



Solution

The coordinates of point A are given in the problem statement and the coordinates of point B are (5 ft sin α, 10 ft, 5 ft cos α). Hence, the position vector from point B to point A is

$$\vec{r}_{BA} = \left[(8 - 5\sin\alpha)\,\hat{\imath} - 5.5\,\hat{\jmath} + (2 - 5\cos\alpha)\,\hat{k}\right] \text{ft}, \tag{1}$$

and the magnitude of $\vec{r}_{BA}$ is

$$r_{BA} = \sqrt{(8 - 5\sin\alpha)^2 + (-5.5)^2 + (2 - 5\cos\alpha)^2} \text{ ft}. \tag{2}$$

The force that cable AB applies to the collar at point B is

$$\vec{F}_{BA} = 30\,\text{lb}\,\frac{\vec{r}_{BA}}{r_{BA}} \tag{3}$$

$$= \boxed{30\,\text{lb}\,\frac{(8 - 5\sin\alpha)\,\hat{\imath} - 5.5\,\hat{\jmath} + (2 - 5\cos\alpha)\,\hat{k}}{\sqrt{(8 - 5\sin\alpha)^2 + (-5.5)^2 + (2 - 5\cos\alpha)^2}}}\,. \tag{4}$$

By inspection, the force that cable BC applies to the collar at point B is

$$\boxed{\vec{F}_{BC} = -1500\,\text{lb}\,\hat{\jmath}.} \tag{5}$$

Remark: To help verify the accuracy of our result for $\vec{F}_{BA}$, Eq. (4) may be used to verify the answers for the previous three problems. For example:

If $\alpha = 0°$, Eq. (4) gives

$$\vec{F}_{BA} = (23.62\,\hat{\imath} - 16.24\,\hat{\jmath} - 8.857\,\hat{k})\,\text{lb}, \tag{6}$$

which agrees with the answer to Prob. 2.90.

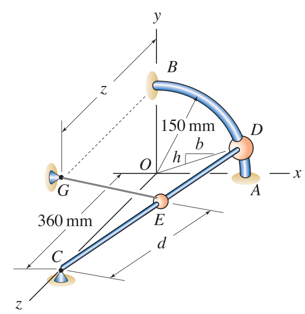
If $\alpha = 60°$, Eq. (4) gives

$$\vec{F}_{BA} = (16.60\,\hat{\imath} - 24.88\,\hat{\jmath} - 2.262\,\hat{k})\,\text{lb}, \tag{7}$$

which agrees with the answer to Prob. 2.92.

Problem 2.94

The structure consists of a quarter-circular rod AB with radius 150 mm that is fixed in the xy plane. CD is a straight rod where D may be positioned at different locations on the circular rod. GE is an elastic cord whose support at G lies in the yz plane, and the bead at E may have different positions d. For the values of b, h, d, and z provided, determine the coordinates of E, and if the elastic cord supports a tensile force of 100 N, write a vector expression using Cartesian vector representation for the force $\vec{F}_{EG}$ the cord exerts on bead E. $b = 4$, $h = 3$, $d = 260$ mm, and $z = 240$ mm.



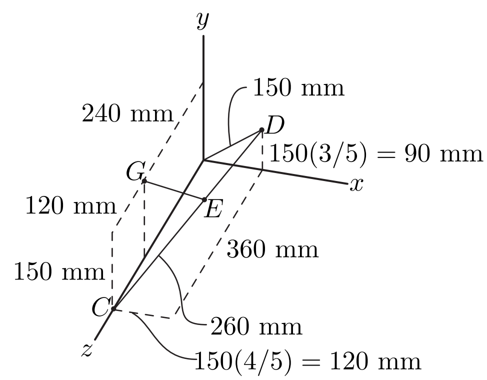
Solution

Let $\vec{r}_{CD}$ be the position vector from point C to point D. Using the figure to the right, this vector, its magnitude, and a unit vector in this direction, are given by

$$\vec{r}_{CD} = (120\,\hat{\imath} + 90\,\hat{\jmath} - 360\,\hat{k})\text{ mm}, \tag{1}$$

$$r_{CD} = \sqrt{120^2 + 90^2 + (-360)^2}\text{ mm} = 390\text{ mm}, \tag{2}$$

$$\hat{u}_{CD} = \frac{(120\,\hat{\imath} + 90\,\hat{\jmath} - 360\,\hat{k})\text{ mm}}{390\text{ mm}}$$
$$= 0.3077\,\hat{\imath} + 0.2308\,\hat{\jmath} - 0.9231\,\hat{k}, \tag{3}$$



The position vector from C to E is

$$\vec{r}_{CE} = (260\text{ mm})\,\hat{u}_{CD} = (260\text{ mm})(0.3077\,\hat{\imath} + 0.2308\,\hat{\jmath} - 0.9231\,\hat{k}),$$
$$= (80\,\hat{\imath} + 60\,\hat{\jmath} - 240\,\hat{k})\text{ mm}. \tag{4}$$

The coordinates of point E are easily obtained once vector $\vec{r}_{OE}$ has been determined, which can be accomplished using

$$\vec{r}_{OE} = \vec{r}_{OC} + \vec{r}_{CE} = (360\,\hat{k})\text{ mm} + (80\,\hat{\imath} + 60\,\hat{\jmath} - 240\,\hat{k})\text{ mm} \tag{5}$$
$$= (80\,\hat{\imath} + 60\,\hat{\jmath} + 120\,\hat{k})\text{ mm}. \tag{6}$$

Hence, the coordinates of point E are

$$\boxed{(80, 60, 120)\text{ mm.}} \tag{7}$$

Using the geometry provided in the problem statement, the vector pointing from C to G is given by $\vec{r}_{CG} = (150\,\hat{\jmath} - 90\,\hat{k})$ mm, and the vector $\vec{r}_{EG}$ can be obtained using

$$\vec{r}_{EG} = \vec{r}_{EC} + \vec{r}_{CG} = -\vec{r}_{CE} + \vec{r}_{CG} \tag{8}$$
$$= -(80\,\hat{\imath} + 60\,\hat{\jmath} - 240\,\hat{k})\text{ mm} + (150\,\hat{\jmath} - 120\,\hat{k})\text{ mm} \tag{9}$$
$$= (-80\,\hat{\imath} + 90\,\hat{\jmath} + 120\,\hat{k})\text{ mm}. \tag{10}$$

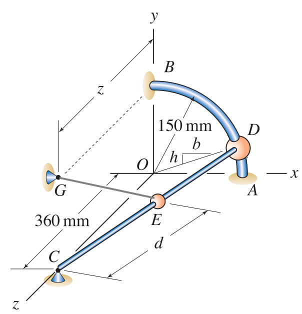
Letting r_{EG} denote the magnitude of $\vec{r}_{EG}$, the force $\vec{F}_{EG}$ is

$$\vec{F}_{EG} = (100\,\mathrm{N})\frac{\vec{r}_{EG}}{r_{EG}} = (100\,\mathrm{N})\frac{(-80\,\hat{\imath} + 90\,\hat{\jmath} + 120\,\hat{k})\,\mathrm{mm}}{\sqrt{(-80)^2 + 90^2 + 120^2}\,\mathrm{mm}}, \tag{11}$$

$$\boxed{\vec{F}_{EG} = (-47.1\,\hat{\imath} + 52.9\,\hat{\jmath} + 70.6\,\hat{k})\,\mathrm{N}.} \tag{12}$$

Problem 2.95

The structure consists of a quarter-circular rod AB with radius 150 mm that is fixed in the xy plane. CD is a straight rod where D may be positioned at different locations on the circular rod. GE is an elastic cord whose support at G lies in the yz plane, and the bead at E may have different positions d. For the values of b, h, d, and z provided, determine the coordinates of E, and if the elastic cord supports a tensile force of 100 N, write a vector expression using Cartesian vector representation for the force $\vec{F}_{EG}$ the cord exerts on bead E. $b = 3$, $h = 4$, $d = 195$ mm, and $z = 270$ mm.



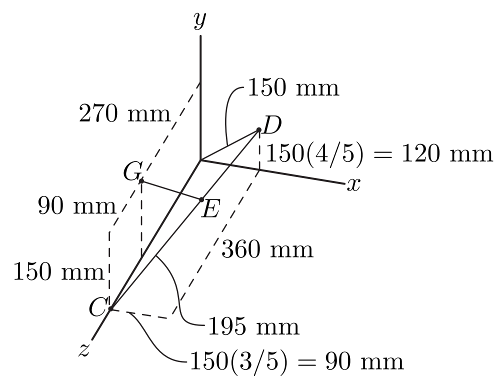
Solution

Let $\vec{r}_{CD}$ be the position vector from point C to point D. Using the figure to the right, this vector, its magnitude, and a unit vector in this direction, are given by

$$\vec{r}_{CD} = (90\,\hat{\imath} + 120\,\hat{\jmath} - 360\,\hat{k})\text{ mm}, \tag{1}$$

$$r_{CD} = \sqrt{90^2 + 120^2 + (-360)^2}\text{ mm} = 390\text{ mm}, \tag{2}$$

$$\hat{u}_{CD} = \frac{(90\,\hat{\imath} + 120\,\hat{\jmath} - 360\,\hat{k})\text{ mm}}{390\text{ mm}}$$
$$= 0.2308\,\hat{\imath} + 0.3077\,\hat{\jmath} - 0.9231\,\hat{k}, \tag{3}$$



The position vector from C to E is

$$\vec{r}_{CE} = (195\text{ mm})\,\hat{u}_{CD} = (195\text{ mm})(0.2308\,\hat{\imath} + 0.3077\,\hat{\jmath} - 0.9231\,\hat{k})$$
$$= (45\,\hat{\imath} + 60\,\hat{\jmath} - 180\,\hat{k})\text{ mm}. \tag{4}$$

The coordinates of point E are easily obtained once vector $\vec{r}_{OE}$ has been determined, which can be accomplished using

$$\vec{r}_{OE} = \vec{r}_{OC} + \vec{r}_{CE} = (360\,\hat{k})\text{ mm} + (45\,\hat{\imath} + 60\,\hat{\jmath} - 180\,\hat{k})\text{ mm} \tag{5}$$
$$= (45\,\hat{\imath} + 60\,\hat{\jmath} + 180\,\hat{k})\text{ mm}. \tag{6}$$

Hence, the coordinates of point E are

$$\boxed{(45, 60, 180)\text{ mm.}} \tag{7}$$

Using the geometry provided in the problem statement, the vector pointing from C to G is given by $\vec{r}_{CG} = (150\,\hat{\jmath} - 90\,\hat{k})$ mm, and the vector $\vec{r}_{EG}$ can be obtained using

$$\vec{r}_{EG} = \vec{r}_{EC} + \vec{r}_{CG} = -\vec{r}_{CE} + \vec{r}_{CG} \tag{8}$$
$$= -(45\,\hat{\imath} + 60\,\hat{\jmath} - 180\,\hat{k})\text{ mm} + (150\,\hat{\jmath} - 90\,\hat{k})\text{ mm} \tag{9}$$
$$= (-45\,\hat{\imath} + 90\,\hat{\jmath} + 90\,\hat{k})\text{ mm}. \tag{10}$$

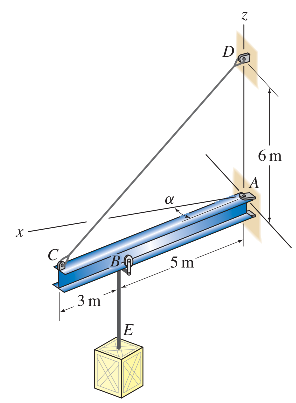
Letting r_{EG} denote the magnitude of $\vec{r}_{EG}$, the force $\vec{F}_{EG}$ is

$$\vec{F}_{EG} = (100\,\mathrm{N})\frac{\vec{r}_{EG}}{r_{EG}} = (100\,\mathrm{N})\frac{(-45\,\hat{\imath} + 90\,\hat{\jmath} + 90\,\hat{k})\,\mathrm{mm}}{\sqrt{(-45)^2 + 90^2 + 90^2}\,\mathrm{mm}}, \tag{11}$$

$$\boxed{\vec{F}_{EG} = (-33.3\,\hat{\imath} + 66.7\,\hat{\jmath} + 66.7\,\hat{k})\,\mathrm{N}.} \tag{12}$$

Problem 2.96

A wall-mounted jib crane consists of an I beam that is supported by a pin at point A and a cable at point C, where A and C lie in the xy plane. A crate at E is supported by a cable that is attached to a trolley at point B where the trolley may move along the length of the I beam. The forces supported by cables CD and BE are 3 kN and 5 kN, respectively. For the value of angle α given below, determine expressions for the forces the two cables apply to the I beam. $\alpha = 0°$.



Solution

The coordinates of point D are (0, 0, 6) m, and with $\alpha = 0°$, the coordinates of point C are (8, 0, 0) m. Hence, the position vector from point C to point D is

$$\vec{r}_{CD} = (-8\,\hat{\imath} + 6\,\hat{k})\,\text{m}. \tag{1}$$

The force that cable CD applies to the I beam is

$$\vec{F}_{CD} = 3\,\text{kN}\,\frac{\vec{r}_{CD}}{r_{CD}} = 3\,\text{kN}\,\frac{-8\,\hat{\imath} + 6\,\hat{k}}{10} \tag{2}$$

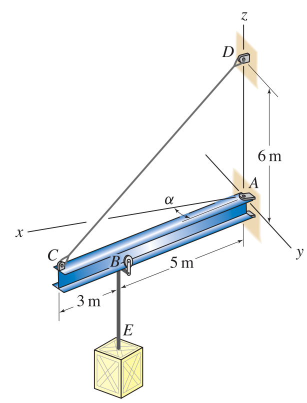
$$= \boxed{(-2.4\,\hat{\imath} + 1.8\,\hat{k})\,\text{kN}.} \tag{3}$$

By inspection, the force that cable BE applies to the I beam is

$$\boxed{\vec{F}_{BE} = -5\,\text{kN}\,\hat{k}.} \tag{4}$$

Problem 2.97

A wall-mounted jib crane consists of an I beam that is supported by a pin at point A and a cable at point C, where A and C lie in the xy plane. A crate at E is supported by a cable that is attached to a trolley at point B where the trolley may move along the length of the I beam. The forces supported by cables CD and BE are 3 kN and 5 kN, respectively. For the value of angle α given below, determine expressions for the forces the two cables apply to the I beam. $\alpha = 40°$.



Solution

The coordinates of point D are (0, 0, 6) m, and with $\alpha = 40°$, the coordinates of point C are (8 m cos 40°, 8 m sin 40°, 0). Hence, the position vector from point C to point D is

$$\vec{r}_{CD} = (-6.128\,\hat{\imath} - 5.142\,\hat{\jmath} + 6\,\hat{k})\text{ m.} \tag{1}$$

The force that cable CD applies to the I beam is

$$\vec{F}_{CD} = 3\text{ kN}\,\frac{\vec{r}_{CD}}{r_{CD}} = 3\text{ kN}\,\frac{-6.128\,\hat{\imath} - 5.142\,\hat{\jmath} + 6\,\hat{k}}{10.00} \tag{2}$$

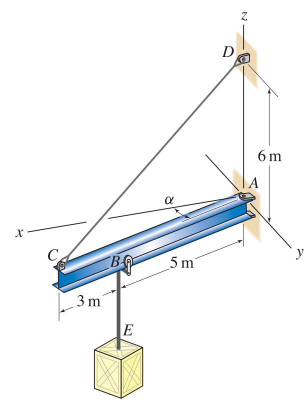
$$= \boxed{(-1.839\,\hat{\imath} - 1.543\,\hat{\jmath} + 1.800\,\hat{k})\text{ kN.}} \tag{3}$$

By inspection, the force that cable BE applies to the I beam is

$$\boxed{\vec{F}_{BE} = -5\text{ kN}\,\hat{k}.} \tag{4}$$

Problem 2.98

A wall-mounted jib crane consists of an I beam that is supported by a pin at point A and a cable at point C, where A and C lie in the xy plane. A crate at E is supported by a cable that is attached to a trolley at point B where the trolley may move along the length of the I beam. The forces supported by cables CD and BE are 3 kN and 5 kN, respectively. For the value of angle α given below, determine expressions for the forces the two cables apply to the I beam. $\alpha = 70°$.



Solution

The coordinates of point D are (0, 0, 6) m, and with $\alpha = 70°$, the coordinates of point C are (8 m cos 70°, 8 m sin 70°, 0). Hence, the position vector from point C to point D is

$$\vec{r}_{CD} = (-2.736\,\hat{\imath} - 7.518\,\hat{\jmath} + 6\,\hat{k})\,\text{m}. \tag{1}$$

The force that cable CD applies to the I beam is

$$\vec{F}_{CD} = 3\,\text{kN}\,\frac{\vec{r}_{CD}}{r_{CD}} = 3\,\text{kN}\,\frac{-2.736\,\hat{\imath} - 7.518\,\hat{\jmath} + 6\,\hat{k}}{10.00} \tag{2}$$

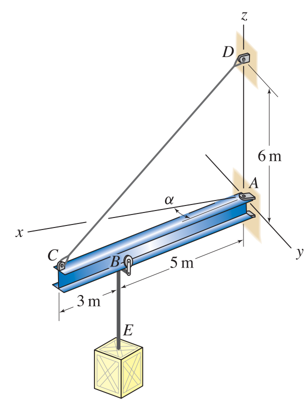
$$= \boxed{(-0.8208\,\hat{\imath} - 2.255\,\hat{\jmath} + 1.800\,\hat{k})\,\text{kN}} \tag{3}$$

By inspection, the force that cable BE applies to the I beam is

$$\boxed{\vec{F}_{BE} = -5\,\text{kN}\,\hat{k}.} \tag{4}$$

Problem 2.99

A wall-mounted jib crane consists of an I beam that is supported by a pin at point A and a cable at point C, where A and C lie in the xy plane. A crate at E is supported by a cable that is attached to a trolley at point B where the trolley may move along the length of the I beam. The forces supported by cables CD and BE are 3 kN and 5 kN, respectively. For the value of angle α given below, determine expressions for the forces the two cables apply to the I beam. General values of α where $-90° \le \alpha \le 90°$.



Solution

The coordinates of point D are (0, 0, 6) m and the coordinates of point C are ($8\,\text{m}\cos\alpha$, $8\,\text{m}\sin\alpha$, 0). Hence, the position vector from point C to point D is

$$\vec{r}_{CD} = (-8\cos\alpha\,\hat{\imath} - 8\sin\alpha\,\hat{\jmath} + 6\,\hat{k})\,\text{m}, \tag{1}$$

and the magnitude of $\vec{r}_{CD}$ is

$$r_{CD} = \sqrt{(-8\cos\alpha)^2 + (-8\sin\alpha)^2 + (6)^2}\,\text{m} \tag{2}$$

$$= \sqrt{64(\cos^2\alpha + \sin^2\alpha) + 36}\,\text{m}. \tag{3}$$

Since $\cos^2\alpha + \sin^2\alpha = 1$ for any value of α, the magnitude of $\vec{r}_{CD}$, which is the same as the length of cable CD, is always

$$r_{CD} = 10\,\text{m}. \tag{4}$$

The force cable CD applies to the I beam is

$$\vec{F}_{CD} = 3\,\text{kN}\,\frac{\vec{r}_{CD}}{r_{CD}} = 3\,\text{kN}\,\frac{-8\cos\alpha\,\hat{\imath} - 8\sin\alpha\,\hat{\jmath} + 6\,\hat{k}}{10} \tag{5}$$

$$= \boxed{(-2.4\cos\alpha\,\hat{\imath} - 2.4\sin\alpha\,\hat{\jmath} + 1.8\,\hat{k})\,\text{kN}.} \tag{6}$$

By inspection, the force that cable BE applies to the I beam is

$$\boxed{\vec{F}_{BE} = -5\,\text{kN}\,\hat{k}.} \tag{7}$$

Remark: To help verify the accuracy of our result for $\vec{F}_{CD}$, Eq. (6) may be used to verify the answers for the previous three problems. For example:

If $\alpha = 0°$, Eq. (6) gives

$$\vec{F}_{CD} = (-2.4\,\hat{\imath} + 1.8\,\hat{k})\,\text{kN}, \tag{8}$$

which agrees with the answer to Prob. 2.96.

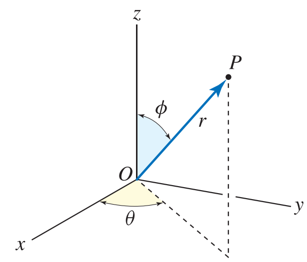
If $\alpha = 70°$, Eq. (6) gives

$$\vec{F}_{CD} = (-0.8208\,\hat{\imath} - 2.255\,\hat{\jmath} + 1.8\,\hat{k})\,\text{kN}, \tag{9}$$

which agrees with the answer to Prob. 2.98.

Problem 2.100

A coordinate system that is often used for problems in mechanics is the *spherical coordinate system*, as shown. With this coordinate system, the location of a point P in three dimensions is specified using a radial distance r where $r \geq 0$, an angle θ (sometimes called the azimuthal angle) where $0° \leq \theta \leq 360°$, and an angle ϕ (sometimes called the polar angle) where $0° \leq \phi \leq 180°$. If values for r, θ, and ϕ are known, determine the direction angles for the position vector from the origin of the coordinate system to point P.



Solution

By examining the figure provided in the problem statement, note that ϕ is measured from the positive z direction to the direction of vector $\vec{r}_{OP}$; hence ϕ is a direction angle and thus

$$\boxed{\theta_z = \phi.} \tag{1}$$

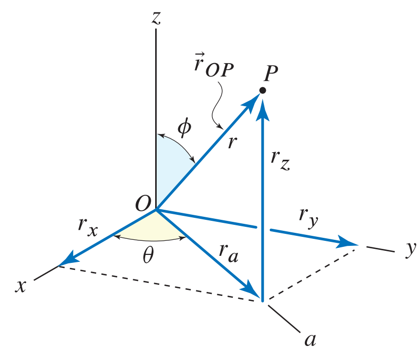
Using the sketch shown at the right, we determine the projection of $\vec{r}_{OP}$ in the a direction to be

$$r_a = r \sin\phi. \tag{2}$$

The projections of r_a in the x and y directions are then

$$r_x = r_a \cos\theta = r\sin\phi\cos\theta, \tag{3}$$

$$r_y = r_a \sin\theta = r\sin\phi\sin\theta. \tag{4}$$



The x and y direction angles are thus

$$\theta_x = \cos^{-1}\frac{r_x}{r} = \cos^{-1}\frac{r\sin\phi\cos\theta}{r} \tag{5}$$

$$= \boxed{\cos^{-1}(\sin\phi\cos\theta),} \tag{6}$$

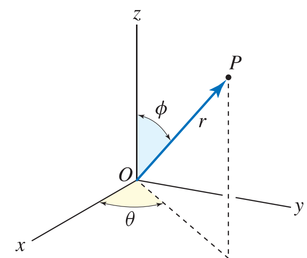
$$\theta_y = \cos^{-1}\frac{r_y}{r} = \cos^{-1}\frac{r\sin\phi\sin\theta}{r} \tag{7}$$

$$= \boxed{\cos^{-1}(\sin\phi\sin\theta).} \tag{8}$$

To help verify the accuracy of these results, you may wish to verify that $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$ for any values of ϕ and θ.

Problem 2.101

For the spherical coordinate system described in Prob. 2.100, if the direction angles for the position vector from the origin of the coordinate system to point P are known, determine the values for r, θ, and ϕ. *Hint:* Several approaches may be used to solve this problem, but a straightforward solution is to first solve Prob. 2.100, and then use those results to solve this problem.



Solution

Following the hint given in the problem statement, we first solve Prob. 2.100 to obtain the results

$$\theta_z = \phi, \tag{1}$$
$$\theta_x = \cos^{-1}(\sin\phi\cos\theta), \tag{2}$$
$$\theta_y = \cos^{-1}(\sin\phi\sin\theta). \tag{3}$$

From Eq. (1),

$$\boxed{\phi = \theta_z.} \tag{4}$$

Taking the cosine of both sides of Eq. (2), and combining with Eq. (4), provides

$$\cos\theta_x = \sin\phi\cos\theta \tag{5}$$
$$= \sin\theta_z\cos\theta. \tag{6}$$

Solving Eq. (6) for θ provides

$$\boxed{\theta = \cos^{-1}\frac{\cos\theta_x}{\sin\theta_z}.} \tag{7}$$

As an alternative to Eqs. (5) – (7), we could take the cosine of both sides of Eq. (3) to obtain

$$\boxed{\theta = \sin^{-1}\frac{\cos\theta_y}{\sin\theta_z}.} \tag{8}$$

Equations (7) and (8) yield the same result for θ, and either may be used. Also:

$\boxed{\text{It is not possible to determine } r \text{ if only the direction angles } \theta_x, \theta_y, \text{ and } \theta_z \text{ are known.}}$

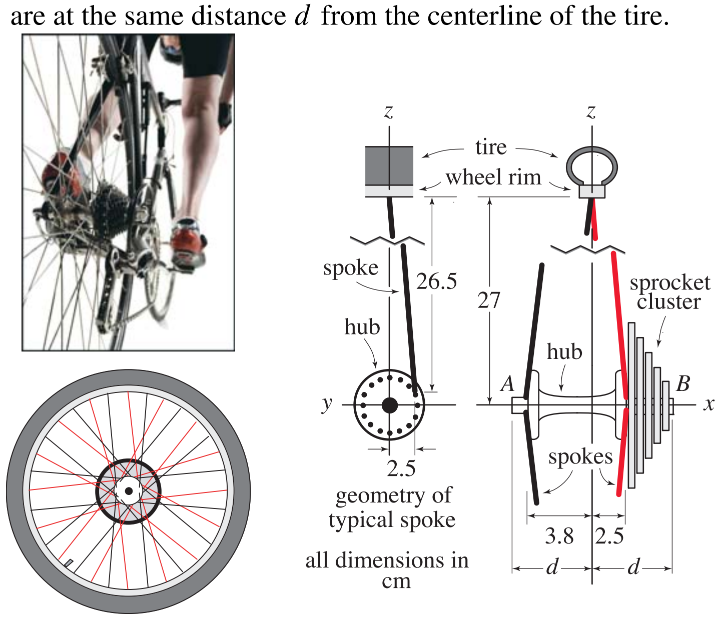
Problem 2.102

The rear wheel of a multispeed bicycle is shown. The wheel has 32 spokes, with one-half being on either side (spokes on sides A and B are shown in the figure in black and red, respectively). For the tire to be properly centered on the frame of the bicycle, points A and B of the hub must be positioned at the same distance d from the centerline of the tire. To make room for the sprocket cluster, bicycle manufacturers give spokes on side B of the wheel a different orientation than spokes on side A. For the following questions, assume the tire is in the process of being manufactured, so that all spokes on side A have the same force F_A and all spokes on side B have the same force F_B.

(a) Determine the ratio of spoke forces F_B/F_A so that the resultant force in the x direction applied to the hub by all 32 spokes is zero. *Hint:* Although each spoke has a different orientation, all spokes on side A have the same length, and similarly all spokes on side B have the same length. Furthermore, all spokes on side A have the same x component of force, and all spokes on side B have the same x component of force.

(b) On which side of the wheel are the spokes most severely loaded?

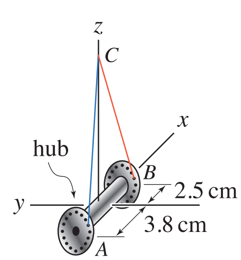
(c) Briefly describe a new design in which spokes on both sides of the wheel are equally loaded and points A and B are at the same distance d from the centerline of the tire.



side A of wheel

Solution

Part (a) Consider the two spokes shown in the figure at the right. Spoke AC is on side A of the wheel, and spoke BC is on side B of the wheel, where point C is the location where the two spokes attach to the rim of the wheel. Based on the geometry provided in the problem statement, position vectors from points A to C, and from points B to C may be written as



$$\vec{r}_{AC} = \left(3.8\,\hat{\imath} + 2.5\,\hat{\jmath} + 26.5\,\hat{k}\right)\text{ cm}, \quad \vec{r}_{BC} = \left(-2.5\,\hat{\imath} + 2.5\,\hat{\jmath} + 26.5\,\hat{k}\right)\text{ cm}. \quad (1)$$

Unit vectors in the directions of $\vec{r}_{AC}$ and $\vec{r}_{BC}$ are

$$\hat{u}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \frac{(3.8\,\hat{\imath} + 2.5\,\hat{\jmath} + 26.5\,\hat{k})\text{ cm}}{\sqrt{(3.8)^2 + (2.5)^2 + (26.5)^2}\text{ cm}} = 0.1413\,\hat{\imath} + 0.09298\,\hat{\jmath} + 0.9856\,\hat{k}, \tag{2}$$

$$\hat{u}_{BC} = \frac{\vec{r}_{BC}}{|\vec{r}_{BC}|} = \frac{(-2.5\,\hat{\imath} + 2.5\,\hat{\jmath} + 26.5\,\hat{k})\text{ cm}}{\sqrt{(-2.5)^2 + (2.5)^2 + (26.5)^2}\text{ cm}} = -0.09351\,\hat{\imath} + 0.09351\,\hat{\jmath} + 0.9912\,\hat{k}. \tag{3}$$

Let $\vec{F}_A$ be the force supported by spoke AC, and $\vec{F}_B$ be the force supported by spoke BC. Then,

$$\vec{F}_A = F_A\,\hat{u}_{AC} = F_A(0.1413\,\hat{\imath} + 0.09298\,\hat{\jmath} + 0.9856\,\hat{k}), \tag{4}$$

$$\vec{F}_B = F_B\,\hat{u}_{BC} = F_B(-0.09351\,\hat{\imath} + 0.09351\,\hat{\jmath} + 0.9912\,\hat{k}). \tag{5}$$

The net force applied by all of the spokes to the hub is obtained by adding the force vectors for all 32 spokes. Although each spoke will have different components of force in the y and z directions, depending upon each spoke's location around the wheel, all of the spokes on side A will have the same x component of force, and similarly, all of the spokes on side B will have the same x component of force. Since each side has the same number of spokes, we can consider the sum of forces $\vec{F}_A + \vec{F}_B$, with the requirement that the x component of this be zero. Thus, we add the x components of Eqs. (4) and (5) to obtain

$$F_A(0.1413) + F_B(-0.09351) = 0. \tag{6}$$

From Eq. (6), the ratio F_B/F_A is given by

$$\boxed{F_B/F_A = 0.1413/0.09351 = 1.51.} \tag{7}$$

Part (b) Based on the ratio of Eq. (7),

> the spokes on side B of the wheel are more severely loaded,
> with approximately 50% higher force than the spokes on side A.

Part (c) Solution not provided.

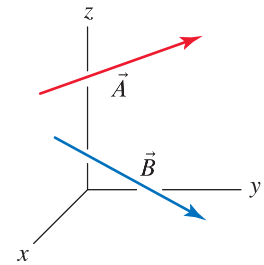
Photo credit: © Erik Isakson/ Rubberball Productions/Getty RF.

Problem 2.103

(a) Determine the angle between vectors $\vec{A}$ and $\vec{B}$.

(b) Determine the components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.

(c) Determine the vector components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.



$\vec{A} = (-\hat{\imath} + 8\hat{\jmath} + 4\hat{k})\,\text{N}$
$\vec{B} = (\hat{\imath} + 18\hat{\jmath} - 6\hat{k})\,\text{mm}$

Solution

Part (a) Begin by computing the magnitudes of the vectors $\vec{A}$ and $\vec{B}$

$$A = \sqrt{(-1)^2 + (8)^2 + (4)^2}\,\text{N} = 9\,\text{N}, \quad B = \sqrt{(1)^2 + (18)^2 + (-6)^2}\,\text{mm} = 19\,\text{mm}. \tag{1}$$

The angle θ between these vectors is given by

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left[\frac{(-1\,\text{N})(1\,\text{mm}) + (8\,\text{N})(18\,\text{mm}) + (4\,\text{N})(-6\,\text{mm})}{(9\,\text{N})(19\,\text{mm})}\right] \tag{2}$$

$$= \cos^{-1}\left(\frac{119}{171}\right) = \boxed{45.9^\circ.} \tag{3}$$

Part (b) The parallel component of $\vec{A}$ is given by $A_{\parallel}$ and the perpendicular component is given by $A_{\perp}$, such that

$$\boxed{A_{\parallel} = \vec{A}\cdot\frac{\vec{B}}{B} = \frac{(-1\,\text{N})(1\,\text{mm}) + (8\,\text{N})(18\,\text{mm}) + (4\,\text{N})(-6\,\text{mm})}{19\,\text{mm}} = 6.26\,\text{N},} \tag{4}$$

$$\boxed{A_{\perp} = \sqrt{A^2 - A_{\parallel}^2} = \sqrt{(9\,\text{N})^2 - (6.263\,\text{N})^2} = 6.46\,\text{N}.} \tag{5}$$

Part (c) The vector component of $\vec{A}$ parallel to $\vec{B}$ is given by

$$\boxed{\vec{A}_{\parallel} = A_{\parallel}\cdot\frac{\vec{B}}{B} = (6.263\,\text{N})\left(\frac{(\hat{\imath} + 18\,\hat{\jmath} - 6\,\hat{k})\,\text{mm}}{19\,\text{mm}}\right) = (0.330\,\hat{\imath} + 5.93\,\hat{\jmath} - 1.98\,\hat{k})\,\text{N}.} \tag{6}$$

The perpendicular vector component is then

$$\boxed{\begin{aligned}\vec{A}_{\perp} = \vec{A} - \vec{A}_{\parallel} &= (-\hat{\imath} + 8\,\hat{\jmath} + 4\,\hat{k})\,\text{N} - (0.3296\,\hat{\imath} + 5.934\,\hat{\jmath} - 1.978\,\hat{k})\,\text{N},\\ &= (-1.33\,\hat{\imath} + 2.07\,\hat{\jmath} + 5.98\,\hat{k})\,\text{N}.\end{aligned}} \tag{7}$$

As a partial check of the accuracy of our results, we evaluate the magnitude of Eq. (7) to obtain

$$A_{\perp} = \sqrt{(-1.330\,\text{N})^2 + (2.066\,\text{N})^2 + (5.978\,\text{N})^2} = 6.46\,\text{N}, \tag{8}$$

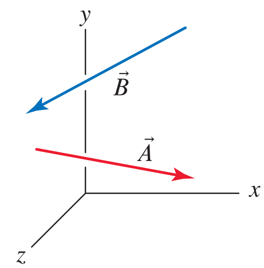
which agrees with the value found in Eq. (5).

Problem 2.104

(a) Determine the angle between vectors $\vec{A}$ and $\vec{B}$.

(b) Determine the components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.

(c) Determine the vector components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.



$\vec{A} = (6\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k})\,\text{lb}$
$\vec{B} = (-14\,\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k})\,\text{in.}$

Solution

Part (a) Begin by computing the magnitudes of the vectors $\vec{A}$ and $\vec{B}$

$$A = \sqrt{(6)^2 + (-2)^2 + (3)^2}\,\text{lb} = 7\,\text{lb}, \quad B = \sqrt{(-14)^2 + (-2)^2 + (5)^2}\,\text{in.} = 15\,\text{in.} \tag{1}$$

The angle θ between these vectors is given by

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left[\frac{(6\,\text{lb})(-14\,\text{in.}) + (-2\,\text{lb})(-2\,\text{in.}) + (3\,\text{lb})(5\,\text{in.})}{(7\,\text{lb})(15\,\text{in.})}\right] \tag{2}$$

$$= \cos^{-1}\left(\frac{-65}{105}\right) = \boxed{128^\circ.} \tag{3}$$

Part (b) The parallel component of $\vec{A}$ is given by $A_\parallel$ and the perpendicular component is given by $A_\perp$, such that

$$A_\parallel = \vec{A}\cdot\frac{\vec{B}}{B} = \frac{(6\,\text{lb})(-14\,\text{in.}) + (-2\,\text{lb})(-2\,\text{in.}) + (3\,\text{lb})(5\,\text{in.})}{15\,\text{in.}} = -4.33\,\text{lb}, \tag{4}$$

$$A_\perp = \sqrt{A^2 - A_\parallel^2} = \sqrt{(7\,\text{lb})^2 - (-4.333\,\text{lb})^2} = 5.50\,\text{lb}. \tag{5}$$

Part (c) The vector component of $\vec{A}$ parallel to $\vec{B}$ is given by

$$\vec{A}_\parallel = A_\parallel\cdot\frac{\vec{B}}{B} = (-4.333\,\text{lb})\left(\frac{(-14\,\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k})\,\text{in.}}{15\,\text{in.}}\right) = (4.04\,\hat{\imath} + 0.578\,\hat{\jmath} - 1.44\,\hat{k})\,\text{lb}. \tag{6}$$

The perpendicular vector component is then

$$\vec{A}_\perp = \vec{A} - \vec{A}_\parallel = (6\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k})\,\text{lb} - \vec{A}_\parallel = (1.96\,\hat{\imath} - 2.58\,\hat{\jmath} + 4.44\,\hat{k})\,\text{lb}. \tag{7}$$

As a partial check of the accuracy of our results, we evaluate the magnitude of Eq. (7) to obtain

$$A_\perp = \sqrt{(1.956\,\text{lb})^2 + (-2.578\,\text{lb})^2 + (4.444\,\text{lb})^2} = 5.50\,\text{lb}, \tag{8}$$

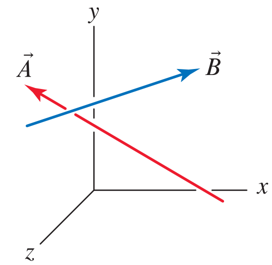
which agrees with the value found in Eq. (5).

Problem 2.105

(a) Determine the angle between vectors $\vec{A}$ and $\vec{B}$.

(b) Determine the components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.

(c) Determine the vector components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.



$\vec{A} = (-12\,\hat{\imath} + 16\,\hat{\jmath} - 15\,\hat{k})$ lb
$\vec{B} = (18\,\hat{\imath} + 6\,\hat{\jmath} - 13\,\hat{k})$ in.

Solution

Part (a) The magnitudes of vectors $\vec{A}$ and $\vec{B}$ are

$$A = \sqrt{(-12)^2 + (16)^2 + (-15)^2}\ \text{lb} = 25\ \text{lb}, \quad B = \sqrt{(18)^2 + (6)^2 + (-13)^2}\ \text{in.} = 23\ \text{in.} \tag{1}$$

The angle θ between these vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left[\frac{(-12\ \text{lb})(18\ \text{in.}) + (16\ \text{lb})(6\ \text{in.}) + (-15\ \text{lb})(-13\ \text{in.})}{(25\ \text{lb})(23\ \text{in.})}\right] \tag{2}$$

$$= \cos^{-1}\left(\frac{75}{575}\right) = \boxed{82.51^\circ.} \tag{3}$$

Part (b) The parallel component of $\vec{A}$ is given by $A_\parallel$ and the perpendicular component is given by $A_\perp$, where

$$\boxed{A_\parallel = \vec{A}\cdot\frac{\vec{B}}{B} = \frac{(-12\ \text{lb})(18\ \text{in.}) + (16\ \text{lb})(6\ \text{in.}) + (-15\ \text{lb})(-13\ \text{in.})}{23\ \text{in.}} = 3.261\ \text{lb},} \tag{4}$$

$$\boxed{A_\perp = \sqrt{A^2 - A_\parallel^2} = \sqrt{(25\ \text{lb})^2 - (3.261\ \text{lb})^2} = 24.79\ \text{lb}.} \tag{5}$$

Part (c) The vector component of $\vec{A}$ parallel to $\vec{B}$ is

$$\boxed{\vec{A}_\parallel = A_\parallel\frac{\vec{B}}{B} = 3.261\ \text{lb}\ \frac{(18\,\hat{\imath} + 6\,\hat{\jmath} - 13\,\hat{k})\ \text{in.}}{23\ \text{in.}} = (2.552\,\hat{\imath} + 0.8507\,\hat{\jmath} - 1.843\,\hat{k})\ \text{lb}.} \tag{6}$$

The perpendicular vector component is

$$\vec{A}_\perp = \vec{A} - \vec{A}_\parallel = (-12\,\hat{\imath} + 16\,\hat{\jmath} - 15\,\hat{k})\ \text{lb} - (2.552\,\hat{\imath} + 0.8507\,\hat{\jmath} - 1.843\,\hat{k})\ \text{lb} \tag{7}$$

$$= (-14.55\,\hat{\imath} + 15.15\,\hat{\jmath} - 13.16\,\hat{k})\ \text{lb}. \tag{8}$$

As a partial check of accuracy of our results, we evaluate the magnitude of Eq. (8) to obtain

$$A_\perp = \sqrt{(-14.55\ \text{lb})^2 + (15.15\ \text{lb})^2 + (-13.16\ \text{lb})^2} = 24.79\ \text{lb}, \tag{9}$$

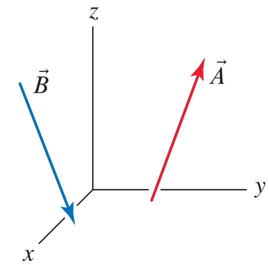
which agrees with the value found in Eq. (5).

Problem 2.106

(a) Determine the angle between vectors $\vec{A}$ and $\vec{B}$.

(b) Determine the components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.

(c) Determine the vector components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.



$\vec{A} = (4\,\hat{\imath} + 3\,\hat{\jmath} + 12\,\hat{k})\,\text{N}$
$\vec{B} = (-2\,\hat{\imath} + 5\,\hat{\jmath} - 14\,\hat{k})\,\text{m}$

Solution

Part (a) The magnitude of vectors $\vec{A}$ and $\vec{B}$ are

$$A = \sqrt{(4)^2 + (3)^2 + (12)^2}\,\text{N} = 13\,\text{N}, \quad B = \sqrt{(-2)^2 + (5)^2 + (-14)^2}\ \text{m} = 15\ \text{m}. \tag{1}$$

The angle θ between these vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left[\frac{(4\ \text{N})(-2\ \text{m}) + (3\ \text{N})(5\ \text{m}) + (12\ \text{N})(-14\ \text{m})}{(13\ \text{N})(15\ \text{m})}\right] \tag{2}$$

$$= \cos^{-1}\frac{-161}{195} = \boxed{145.7^\circ.} \tag{3}$$

Part (b) The parallel component of $\vec{A}$ is given by $A_\parallel$ and the perpendicular component is given by $A_\perp$, where

$$\boxed{A_\parallel = \vec{A}\cdot\frac{\vec{B}}{B} = \frac{(4\ \text{N})(-2\ \text{m}) + (3\ \text{N})(5\ \text{m}) + (12\ \text{N})(-14\ \text{m})}{15\ \text{m}} = -10.73\ \text{N},} \tag{4}$$

$$\boxed{A_\perp = \sqrt{A^2 - A_\parallel^2} = \sqrt{(13\ \text{N})^2 - (-10.73\ \text{N})^2} = 7.335\ \text{N}.} \tag{5}$$

Part (c) The vector component of $\vec{A}$ parallel to $\vec{B}$ is

$$\boxed{\vec{A}_\parallel = A_\parallel\frac{\vec{B}}{B} = -10.73\ \text{N}\,\frac{(-2\,\hat{\imath} + 5\,\hat{\jmath} - 14\,\hat{k})\ \text{m}}{15\ \text{m}} = (1.431\,\hat{\imath} - 3.578\,\hat{\jmath} + 10.02\,\hat{k})\,\text{N}.} \tag{6}$$

The perpendicular vector component is

$$\boxed{\vec{A}_\perp = \vec{A} - \vec{A}_\parallel = (4\,\hat{\imath} + 3\,\hat{\jmath} + 12\,\hat{k})\,\text{N} - (1.431\,\hat{\imath} - 3.578\,\hat{\jmath} + 10.02\,\hat{k})\,\text{N}} \tag{7}$$

$$\boxed{= (2.569\,\hat{\imath} + 6.578\,\hat{\jmath} + 1.982\,\hat{k})\,\text{N}.} \tag{8}$$

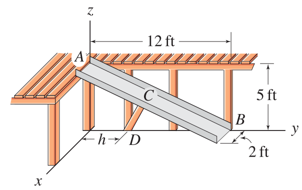
As a partial check of accuracy of our results, we evaluate the magnitude of Eq. (8) to obtain

$$A_\perp = \sqrt{(2.569\ \text{N})^2 + (6.578\ \text{N})^2 + (1.982\ \text{N})^2} = 7.335\ \text{N}, \tag{9}$$

which agrees with the value found in Eq. (5).

Problem 2.107

A slide on a child's play structure is to be supported in part by strut CD (railings are omitted from the sketch for clarity). End C of the strut is to be positioned along the outside edge of the slide, halfway between ends A and B. End D of the strut is to be positioned on the y axis so that the angle $\angle ACD$ between the slide and the strut is a right angle. Determine the distance h that point D should be positioned.



Solution

The problem statement asks that we determine h such that the angle between $\vec{r}_{CD}$ and $\vec{r}_{CA}$ is 90°, i.e.,

$$\frac{\vec{r}_{CD}\cdot\vec{r}_{CA}}{r_{CD}\,r_{CA}} = \cos\theta = \cos 90^\circ = 0 \quad\Rightarrow\quad \vec{r}_{CD}\cdot\vec{r}_{CA} = 0. \tag{1}$$

To determine $\vec{r}_{CD}$, the coordinates of point D are $(0, h, 0)$ and the coordinates of point C are $(2, 6, 2.5)$ ft, hence

$$\vec{r}_{CD} = -2\,\text{ft}\,\hat{\imath} + (h - 6\,\text{ft})\,\hat{\jmath} - 2.5\,\text{ft}\,\hat{k}. \tag{2}$$

Point A is located at the origin of the coordinate system, and the coordinates of point C are cited above, so $\vec{r}_{CA}$ is given by

$$\vec{r}_{CA} = (-2\,\hat{\imath} - 6\,\hat{\jmath} + 2.5\,\hat{k})\,\text{ft}. \tag{3}$$

Substituting these vectors to Eq. (1) leads to

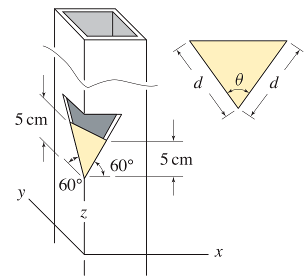
$$(-2\,\text{ft})(-2\,\text{ft}) + (h - 6\,\text{ft})(-6\,\text{ft}) + (-2.5\,\text{ft})(2.5\,\text{ft}) = 0, \tag{4}$$

which may be solved to obtain

$$\boxed{h = \frac{-4\,\text{ft}^2 + 6.25\,\text{ft}^2}{-6\,\text{ft}} + 6\,\text{ft} = 5.63\,\text{ft}.} \tag{5}$$

Problem 2.108

A whistle is made of a square tube with a notch cut in its edge, into which a baffle is brazed. Determine the dimensions d and θ for the baffle.



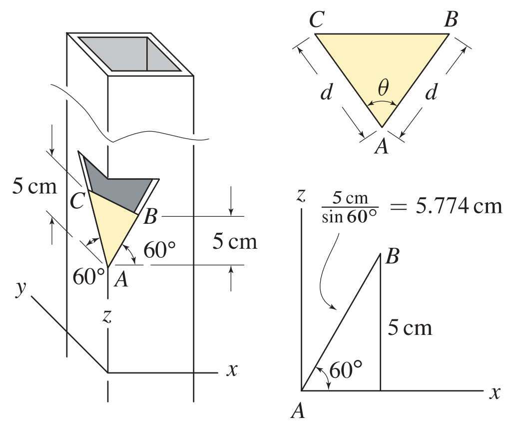
Solution

Let points A, B, and C be located at the three corners of the triangular baffle, as shown. Position vectors may then be written as

$$\vec{r}_{AB} = (5.774\,\text{cm})(\cos 60^\circ\,\hat{\imath} + \sin 60^\circ\,\hat{k}), \qquad (1)$$

$$\vec{r}_{AC} = (5.774\,\text{cm})(\cos 60^\circ\,\hat{\jmath} + \sin 60^\circ\,\hat{k}). \qquad (2)$$

$$\qquad (3)$$



From the geometry shown at the right, the dimension d of the baffle is

$$\boxed{d = 5.77\,\text{cm}.} \qquad (4)$$

The angle θ for the baffle, which is the angle between vectors $\vec{r})AB$ and r_{AC}, is given by

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{AB}\cdot\vec{r}_{AC}}{r_{AB}\,r_{AC}}\right) \qquad (5)$$

$$= \cos^{-1}\left[\frac{\big((5.774\,\text{cm})(\cos 60^\circ\,\hat{\imath} + \sin 60^\circ\,\hat{k})\big)\cdot\big((5.774\,\text{cm})(\cos 60^\circ\,\hat{\jmath} + \sin 60^\circ\,\hat{k})\big)}{(5.774\,\text{cm})(5.774\,\text{cm})}\right] \qquad (6)$$

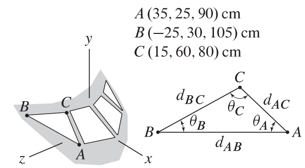
$$= \cos^{-1}\big[(\cos 60^\circ)(0) + (0)(\cos 60^\circ) + (\sin 60^\circ)(\sin 60^\circ)\big] = \cos^{-1}(0.75), \qquad (7)$$

such that

$$\boxed{\theta = 41.4^\circ.} \qquad (8)$$

Problem 2.109

A flat, triangular-shaped window for the cockpit of an airplane is to have the corner coordinates shown. Specify the angles θ_A, θ_B, and θ_C and dimensions d_{AB}, d_{BC}, and d_{AC} for the window.



Solution

Begin by writing the following vectors:

$$\vec{r}_{AB} = \left[(-25-35)\,\hat{\imath} + (30-25)\,\hat{\jmath} + (105-90)\,\hat{k}\right]\text{cm} = (-60\,\hat{\imath} + 5\,\hat{\jmath} + 15\,\hat{k})\,\text{cm}, \tag{1}$$

$$\vec{r}_{AC} = \left[(15-35)\,\hat{\imath} + (60-25)\,\hat{\jmath} + (80-90)\,\hat{k}\right]\text{cm} = (-20\,\hat{\imath} + 35\,\hat{\jmath} - 10\,\hat{k})\,\text{cm}, \tag{2}$$

$$\vec{r}_{BC} = \left[(15-(-25))\,\hat{\imath} + (60-30)\,\hat{\jmath} + (80-105)\,\hat{k}\right]\text{cm} = (40\,\hat{\imath} + 30\,\hat{\jmath} - 25\,\hat{k})\,\text{cm}. \tag{3}$$

The magnitudes of these vectors are

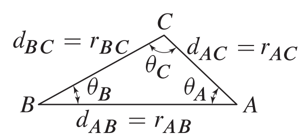
$$r_{AB} = r_{BA} = \sqrt{(-60)^2 + 5^2 + 15^2}\,\text{cm} = 62.05\,\text{cm}, \tag{4}$$

$$r_{AC} = r_{CA} = \sqrt{(-20)^2 + 35^2 + (-10)^2}\,\text{cm} = 41.53\,\text{cm}, \tag{5}$$

$$r_{BC} = r_{CB} = \sqrt{40^2 + 30^2 + (-25)^2}\,\text{cm} = 55.90\,\text{cm}. \tag{6}$$

The dimensions d_{AB}, d_{BC}, and d_{AC} corresponding to these magnitudes are shown at the right, and are given by

$$\boxed{d_{AB} = 62.05\,\text{cm}, \quad d_{BC} = 55.90\,\text{cm}, \quad d_{AC} = 41.53\,\text{cm}.} \tag{7}$$



Before determining the angles, consider the following dot products (where $\vec{r}_{BA} = -\vec{r}_{AB}$, etc.),

$$\vec{r}_{AB}\cdot\vec{r}_{AC} = (-60\,\hat{\imath} + 5\,\hat{\jmath} + 15\,\hat{k})\,\text{cm}\cdot(-20\,\hat{\imath} + 35\,\hat{\jmath} - 10\,\hat{k})\,\text{cm} = 1225\,\text{cm}^2, \tag{8}$$

$$\vec{r}_{BC}\cdot\vec{r}_{BA} = (40\,\hat{\imath} + 30\,\hat{\jmath} - 25\,\hat{k})\,\text{cm}\cdot(60\,\hat{\imath} - 5\,\hat{\jmath} - 15\,\hat{k})\,\text{cm} = 2625\,\text{cm}^2, \tag{9}$$

$$\vec{r}_{CB}\cdot\vec{r}_{CA} = (-40\,\hat{\imath} - 30\,\hat{\jmath} + 25\,\hat{k})\,\text{cm}\cdot(20\,\hat{\imath} - 35\,\hat{\jmath} + 10\,\hat{k})\,\text{cm} = 500\,\text{cm}^2. \tag{10}$$

The angles are then found using

$$\boxed{\theta_A = \cos^{-1}\left(\frac{\vec{r}_{AB}\cdot\vec{r}_{AC}}{r_{AB}\,r_{AC}}\right) = \cos^{-1}\left[\frac{1225\,\text{cm}^2}{(62.05\,\text{cm})(41.53\,\text{cm})}\right] = 61.6^\circ,} \tag{11}$$

$$\boxed{\theta_B = \cos^{-1}\left(\frac{\vec{r}_{BC}\cdot\vec{r}_{BA}}{r_{BC}\,r_{BA}}\right) = \cos^{-1}\left[\frac{2625\,\text{cm}^2}{(55.90\,\text{cm})(62.05\,\text{cm})}\right] = 40.8^\circ,} \tag{12}$$

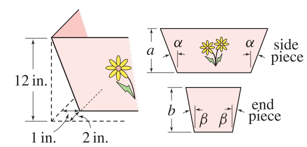
$$\boxed{\theta_C = \cos^{-1}\left(\frac{\vec{r}_{CB}\cdot\vec{r}_{CA}}{r_{CB}\,r_{CA}}\right) = \cos^{-1}\left[\frac{500\,\text{cm}^2}{(55.90\,\text{cm})(41.53\,\text{cm})}\right] = 77.6^\circ.} \tag{13}$$

As a partial check of accuracy, we may add the above angles to verify that their sum equals 180°, i.e.,

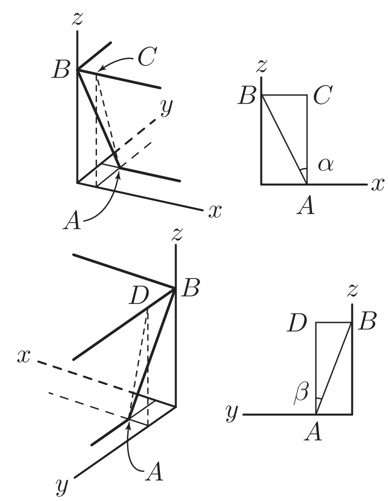
$$\theta_A + \theta_B + \theta_C = 61.62^\circ + 40.82^\circ + 77.56^\circ = 180^\circ. \tag{14}$$

Problem 2.110

The corner of an infant's bassinet is shown. Determine angles α and β and dimensions a and b of the side and end pieces so the corners of the bassinet will properly meet when assembled.



Solution



The top two figures to the right are used to find the dimension a and the angle α. Based on these figures, we obtain

$$\vec{r}_{AB} = (-\hat{\imath} - 2\,\hat{\jmath} + 12\,\hat{k})\text{ in.,} \tag{1}$$

$$r_{AB} = \sqrt{(-1)^2 + (-2)^2 + (12)^2}\text{ in.} = 12.21\text{ in.,} \tag{2}$$

$$\vec{r}_{AC} = (-2\,\hat{\jmath} + 12\,\hat{k})\text{ in.,} \tag{3}$$

$$r_{AC} = \sqrt{(-2)^2 + (12)^2}\text{ in.} = 12.17\text{ in.} \tag{4}$$

The bottom two figures to the right are used to find $\vec{r}_{AD}$ and r_{AD}, such that

$$\vec{r}_{AD} = \left(-\hat{\imath} + 12\,\hat{k}\right)\text{ in.,} \tag{5}$$

$$r_{AD} = \sqrt{(-1)^2 + (12)^2}\text{ in.} = 12.04\text{ in.} \tag{6}$$

The angles α and β are given by

$$\boxed{\alpha = \cos^{-1}\left(\frac{\vec{r}_{AC}\cdot\vec{r}_{AB}}{r_{AC}\,r_{AB}}\right) = \cos^{-1}\left(\frac{0 + (-2)(-2) + (12)(12)}{(12.17)(12.21)}\right) = 4.70^\circ,} \tag{7}$$

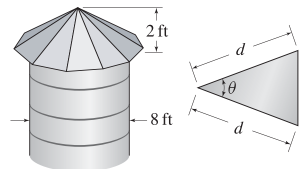
$$\boxed{\beta = \cos^{-1}\left(\frac{\vec{r}_{AD}\cdot\vec{r}_{AB}}{r_{AD}\,r_{AB}}\right) = \cos^{-1}\left(\frac{(-1)(-1) + 0 + (12)(12)}{(12.04)(12.21)}\right) = 9.43^\circ.} \tag{8}$$

The dimensions a and b are given by the magnitudes r_{AC} and r_{AD}, such that

$$\boxed{a = r_{AC} = 12.2\text{ in.,}\quad b = r_{AD} = 12.0\text{ in.}} \tag{9}$$

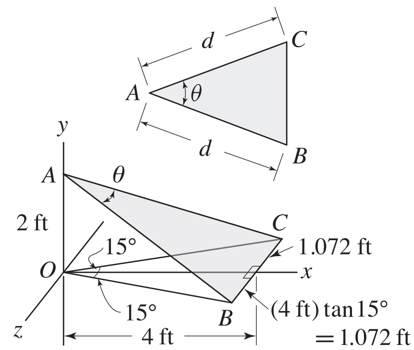
Problem 2.111

The roof of an 8 ft diameter grain silo is to be made using 12 identical triangular panels. Determine the value of angle θ needed and the smallest value of d that can be used.



Solution

The figure to the right shows a single triangular panel of the roof. Since there are 12 such panels, the projected angle of each panel onto the xz plane is $360°/12 = 30°$. With the x axis taken to be through the middle of the panel, the angle to each side of the horizontal projection is $30°/2 = 15°$, as shown. To determine the smallest value d may have, we use the radius of the silo (4 ft) as the length in the x direction from the origin of the coordinate system to the edge of the panel, as shown. Our strategy is to write expressions for vectors $\vec{r}_{AB}$ and $\vec{r}_{AC}$, and then use the dot product between these to determine θ. These vectors, and their magnitudes, are given by



$$\vec{r}_{AB} = (4\,\hat{\imath} - 2\,\hat{\jmath} + 1.072\,\hat{k})\text{ ft}, \qquad r_{AB} = \sqrt{(4)^2 + (-2)^2 + (1.072)^2}\text{ ft} = 4.599\text{ ft}, \tag{1}$$

$$\vec{r}_{AC} = (4\,\hat{\imath} - 2\,\hat{\jmath} - 1.072\,\hat{k})\text{ ft}, \qquad r_{AB} = \sqrt{(4)^2 + (-2)^2 + (-1.072)^2}\text{ ft} = 4.599\text{ ft}. \tag{2}$$

The angle θ is found using

$$\boxed{\theta = \cos^{-1}\left(\frac{\vec{r}_{AB}\cdot\vec{r}_{AC}}{r_{AB}\,r_{AC}}\right) = \cos^{-1}\left[\frac{(4)(4) + (-2)(-2) + (1.072)(-1.072)}{(4.590)(4.599)}\right] = 27.0°.} \tag{3}$$

The distance d is given by either magnitude r_{AB} or r_{AC}, i.e.,

$$\boxed{d = 4.60\text{ ft.}} \tag{4}$$

Problem 2.112

For the description and figure indicated below, determine the components of the cord force in directions parallel and perpendicular to rod CD. If released from rest, will the cord force tend to make bead E slide toward C or D?

Use the description and figure for Prob. 2.94 on p 82.

Solution

From the solution to Problem 2.94, we know that

$$\vec{r}_{CD} = (120\,\hat{\imath} + 90\,\hat{\jmath} - 360\,\hat{k})\,\text{mm}, \qquad r_{CD} = 390\,\text{mm}, \tag{1}$$

$$\vec{F}_{EG} = (-47.06\,\hat{\imath} + 52.94\,\hat{\jmath} + 70.59\,\hat{k})\,\text{N}, \qquad F_{EG} = 100\,\text{N}. \tag{2}$$

The component of $\vec{F}_{EG}$ parallel to rod CD is given by

$$\boxed{F_\parallel = \vec{F}_{EG} \cdot \frac{\vec{r}_{CD}}{r_{CD}} = (-47.06\,\hat{\imath} + 52.94\,\hat{\jmath} + 70.59\,\hat{k})\,\text{N} \cdot \frac{(120\,\hat{\imath} + 90\,\hat{\jmath} - 360\,\hat{k})}{390} = -67.4\,\text{N}.} \tag{3}$$

Using this result, the perpendicular component is then

$$\boxed{F_{EG}^2 = F_\perp^2 + F_\parallel^2 \quad \Rightarrow \quad F_\perp = \sqrt{F_{EG}^2 - F_\parallel^2} = \sqrt{(100\,\text{N})^2 - (-67.42\,\text{N})^2} = 73.9\,\text{N}.} \tag{4}$$

The vector form of the parallel component, $\vec{F}_\parallel$, is

$$\boxed{\vec{F}_\parallel = F_\parallel \frac{\vec{r}_{CD}}{r_{CD}} = (-67.42\,\text{N}) \frac{(120\,\hat{\imath} + 90\,\hat{\jmath} - 360\,\hat{k})\,\text{mm}}{390\,\text{mm}} = (-20.7\,\hat{\imath} - 15.6\,\hat{\jmath} + 62.2\,\hat{k})\,\text{N}.} \tag{5}$$

The perpendicular component $\vec{F}_\perp = \vec{F}_{EG} - \vec{F}_\parallel$ is then

$$\begin{aligned} \vec{F}_\perp &= (-47.06\,\hat{\imath} + 52.94\,\hat{\jmath} + 70.59\,\hat{k})\,\text{N} - (-20.75\,\hat{\imath} - 15.56\,\hat{\jmath} + 62.23\,\hat{k})\,\text{N}, \\ &\boxed{= (-26.3\,\hat{\imath} + 68.5\,\hat{\jmath} + 8.35\,\hat{k})\,\text{N}.} \end{aligned} \tag{6}$$

As a partial check of the accuracy of our answers, we evaluate the magnitudes of $\vec{F}_\parallel$ and $\vec{F}_\perp$ in Eqs. (5) and (6) to obtain

$$F_\parallel = \sqrt{(-20.75\,\text{N})^2 + (-15.56\,\text{N})^2 + (62.23\,\text{N})^2} = 67.4\,\text{N}, \tag{7}$$

$$F_\perp = \sqrt{(-26.31\,\text{N})^2 + (68.50\,\text{N})^2 + (8.354\,\text{N})^2} = 73.9\,\text{N}, \tag{8}$$

and these magnitudes agree with the absolute values of Eqs. (3) and (4).

Since $F_\parallel$ is negative in Eq. (3), the bead will slide in a direction opposite to $\vec{r}_{CD}$, so that $\boxed{\text{the bead slides toward } C.}$

Problem 2.113

For the description and figure indicated below, determine the components of the cord force in directions parallel and perpendicular to rod CD. If released from rest, will the cord force tend to make bead E slide toward C or D?

Use the description and figure for Prob. 2.95 on p 82.

Solution

From the solution to Problem 2.95, we know that

$$\vec{r}_{CD} = (90\,\hat{\imath} + 120\,\hat{\jmath} - 360\,\hat{k})\,\text{mm}, \qquad r_{CD} = 390\,\text{mm}, \tag{1}$$

$$\vec{F}_{EG} = (-33.33\,\hat{\imath} + 66.67\,\hat{\jmath} + 66.67\,\hat{k})\,\text{N}, \qquad F_{EG} = 100\,\text{N}. \tag{2}$$

The component of $\vec{F}_{EG}$ parallel to rod CD is given by

$$\boxed{F_{\parallel} = \vec{F}_{EG} \cdot \frac{\vec{r}_{CD}}{r_{CD}} = (-33.33\,\hat{\imath} + 66.67\,\hat{\jmath} + 66.67\,\hat{k})\,\text{N} \cdot \frac{(90\,\hat{\imath} + 120\,\hat{\jmath} - 360\,\hat{k})\,\text{mm}}{390\,\text{mm}} = -48.7\,\text{N}.} \tag{3}$$

Using this result, the perpendicular component is then

$$\boxed{F_{EG}^2 = F_{\perp}^2 + F_{\parallel}^2 \quad \Rightarrow \quad F_{\perp} = \sqrt{F_{EG}^2 - F_{\parallel}^2} = \sqrt{(100\,\text{N})^2 - (-48.72\,\text{N})^2} = 87.3\,\text{N}.} \tag{4}$$

The vector form of the parallel component, $\vec{F}_{\parallel}$, is

$$\boxed{\vec{F}_{\parallel} = F_{\parallel}\frac{\vec{r}_{CD}}{r_{CD}} = (-48.72\,\text{N})\frac{(90\,\hat{\imath} + 120\,\hat{\jmath} - 360\,\hat{k})\,\text{mm}}{390\,\text{mm}} = (-11.2\,\hat{\imath} - 15.0\,\hat{\jmath} + 45.0\,\hat{k})\,\text{N}.} \tag{5}$$

The perpendicular component $\vec{F}_{\perp} = \vec{F}_{EG} - \vec{F}_{\parallel}$ is then

$$\boxed{\begin{aligned}\vec{F}_{\perp} &= (-33.33\,\hat{\imath} + 66.67\,\hat{\jmath} + 66.67\,\hat{k})\,\text{N} - (-11.24\,\hat{\imath} - 14.99\,\hat{\jmath} + 44.97\,\hat{k})\,\text{N},\\ &= (-22.1\,\hat{\imath} + 81.7\,\hat{\jmath} + 21.7\,\hat{k})\,\text{N}.\end{aligned}} \tag{6}$$

As a partial check of the accuracy of our answers, we evaluate the magnitudes of $\vec{F}_{\parallel}$ and $\vec{F}_{\perp}$ in Eqs. (5) and (6) to obtain

$$F_{\parallel} = \sqrt{(-11.24\,\text{N})^2 + (-14.99\,\text{N})^2 + (44.97\,\text{N})^2} = 48.7\,\text{N}, \tag{7}$$

$$F_{\perp} = \sqrt{(-22.09\,\text{N})^2 + (81.66\,\text{N})^2 + (21.70\,\text{N})^2} = 87.3\,\text{N}. \tag{8}$$

and these magnitudes agree with the absolute values of Eqs. (3) and (4).

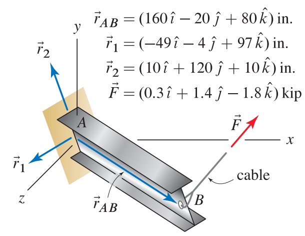
Since $F_{\parallel}$ is negative in Eq. (3), the bead will slide in a direction opposite to $\vec{r}_{CD}$, so that

$$\boxed{\text{the bead slides toward } C.}$$

Problem 2.114

A cantilever I beam has a cable at end B that supports a force $\vec{F}$, and $\vec{r}_{AB}$ is the position vector from end A of the beam to end B. Position vectors $\vec{r}_1$ and $\vec{r}_2$ are parallel to the flanges and web of the I beam, respectively. For determination of the internal forces in the beam (discussed in Chapter 8), and for mechanics of materials analysis, it is necessary to know the components of the force in the axial direction of the beam (AB) and in directions parallel to the web and flanges.

Using the dot product, show that $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_{AB}$ are orthogonal to one another.



Solution

Recall that two vectors are orthogonal if the dot product between them is zero. Thus,

$$\boxed{\vec{r}_{AB} \cdot \vec{r}_1 = \big[(160\,\text{in.})(-49\,\text{in.}) + (-20\,\text{in.})(-4\,\text{in.}) + (80\,\text{in.})(97\,\text{in.})\big] = 0,} \tag{1}$$

hence $\vec{r}_{AB}$ and $\vec{r}_1$ are orthogonal.

$$\boxed{\vec{r}_{AB} \cdot \vec{r}_2 = \big[(160\,\text{in.})(10\,\text{in.}) + (-20\,\text{in.})(120\,\text{in.}) + (80\,\text{in.})(10\,\text{in.})\big] = 0,} \tag{2}$$

hence $\vec{r}_{AB}$ and $\vec{r}_2$ are orthogonal.

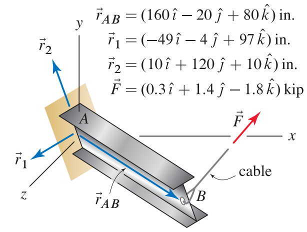
$$\boxed{\vec{r}_1 \cdot \vec{r}_2 = \big[(-49\,\text{in.})(10\,\text{in.}) + (-4\,\text{in.})(120\,\text{in.}) + (97\,\text{in.})(10\,\text{in.})\big] = 0,} \tag{3}$$

hence $\vec{r}_1$ and $\vec{r}_2$ are orthogonal.

Problem 2.115

A cantilever I beam has a cable at end B that supports a force $\vec{F}$, and $\vec{r}_{AB}$ is the position vector from end A of the beam to end B. Position vectors $\vec{r}_1$ and $\vec{r}_2$ are parallel to the flanges and web of the I beam, respectively. For determination of the internal forces in the beam (discussed in Chapter 8), and for mechanics of materials analysis, it is necessary to know the components of the force in the axial direction of the beam (AB) and in directions parallel to the web and flanges.

Determine the scalar and vector components of $\vec{F}$ in direction $\vec{r}_{AB}$.



Solution

Let F_{AB} be the component of $\vec{F}$ in the direction of $\vec{r}_{AB}$. Noting that the magnitude of $\vec{r}_{AB}$ is $r_{AB} = 180$ in., then

$$\boxed{\begin{aligned} F_{AB} &= \vec{F}\cdot\frac{\vec{r}_{AB}}{r_{AB}} = \big[(0.3\,\text{kip})(160\,\text{in.}) + (1.4\,\text{kip})(-20\,\text{in.}) + (-1.8\,\text{kip})(80\,\text{in.})\big]\frac{1}{180\,\text{in.}} \\ &= \frac{-124}{180}\,\text{kip} = -0.689\,\text{kip}. \end{aligned}} \tag{1}$$

The vector component of $\vec{F}$ in the direction of $\vec{r}_{AB}$ is

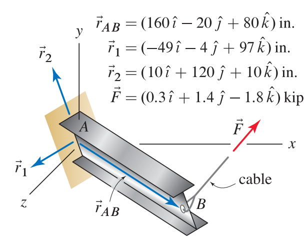
$$\boxed{\begin{aligned} \vec{F}_{AB} &= F_{AB}\frac{\vec{r}_{AB}}{r_{AB}} = \Big(\frac{-124}{180}\text{kip}\Big)(160\,\hat{\imath} - 2\,\hat{\jmath} + 80\,\hat{k})\frac{1}{180} \\ &= (-0.612\,\hat{\imath} + 0.0765\,\hat{\jmath} - 0.306\,\hat{k})\,\text{kip}. \end{aligned}} \tag{2}$$

As a partial check of accuracy, we evaluate the magnitude of Eq. (2) to verify that it agrees with the absolute value of Eq. (1).

Problem 2.116

A cantilever I beam has a cable at end B that supports a force $\vec{F}$, and $\vec{r}_{AB}$ is the position vector from end A of the beam to end B. Position vectors $\vec{r}_1$ and $\vec{r}_2$ are parallel to the flanges and web of the I beam, respectively. For determination of the internal forces in the beam (discussed in Chapter 8), and for mechanics of materials analysis, it is necessary to know the components of the force in the axial direction of the beam (AB) and in directions parallel to the web and flanges.

Determine the scalar and vector components of $\vec{F}$ in direction $\vec{r}_1$.



Solution

Let F_1 be the component of $\vec{F}$ in the direction of $\vec{r}_1$. Noting that the magnitude of $\vec{r}_1$ is $r_1 = 108.7$ in., then

$$\boxed{\begin{aligned} F_1 &= \vec{F}\cdot\frac{\vec{r}_1}{r_1} = \big[(0.3\,\text{kip})(-49\,\text{in.}) + (1.4\,\text{kip})(-4\,\text{in.}) + (-1.8\,\text{kip})(97\,\text{in.})\big]\frac{1}{108.7\,\text{in.}} \\ &= -1.792\,\text{kip}. \end{aligned}} \tag{1}$$

The vector component of $\vec{F}$ in the direction of $\vec{r}_1$ is

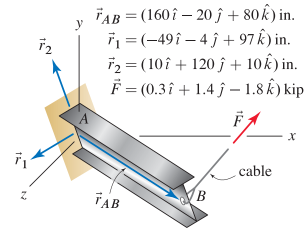
$$\boxed{\begin{aligned} \vec{F}_1 &= F_1\frac{\vec{r}_1}{r_1} = (-1.792\,\text{kip})(-49\,\hat{\imath} - 4\,\hat{\jmath} + 97\,\hat{k})\frac{1}{108.7} \\ &= (0.808\,\hat{\imath} + 0.0659\,\hat{\jmath} - 1.60\,\hat{k})\,\text{kip}. \end{aligned}} \tag{2}$$

As a partial check of accuracy, we evaluate the magnitude of Eq. (2) to verify that it agrees with the absolute value of Eq. (1).

Problem 2.117

A cantilever I beam has a cable at end B that supports a force $\vec{F}$, and $\vec{r}_{AB}$ is the position vector from end A of the beam to end B. Position vectors $\vec{r}_1$ and $\vec{r}_2$ are parallel to the flanges and web of the I beam, respectively. For determination of the internal forces in the beam (discussed in Chapter 8), and for mechanics of materials analysis, it is necessary to know the components of the force in the axial direction of the beam (AB) and in directions parallel to the web and flanges.

Determine the scalar and vector components of $\vec{F}$ in direction $\vec{r}_2$.



Solution

Let F_2 be the component of $\vec{F}$ in the direction of $\vec{r}_2$. Noting that the magnitude of $\vec{r}_2$ is $r_2 = 120.8$ in., then

$$\boxed{\begin{aligned} F_2 &= \vec{F}\cdot\frac{\vec{r}_2}{r_2} = \big[(0.3\,\text{kip})(10\,\text{in.}) + (1.4\,\text{kip})(120\,\text{in.}) + (-1.8\,\text{kip})(10\,\text{in.})\big]\frac{1}{120.8\,\text{in.}} \\ &= 1.266\,\text{kip}. \end{aligned}} \tag{1}$$

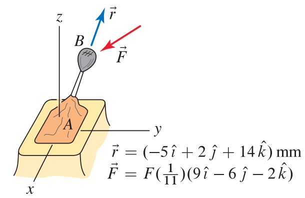
The vector component of $\vec{F}$ in the direction of $\vec{r}_2$ is

$$\boxed{\begin{aligned} \vec{F}_2 &= F_2\frac{\vec{r}_2}{r_2} = (1.266\,\text{kip})(10\,\hat{\imath} + 120\,\hat{\jmath} + 10\,\hat{k})\frac{1}{120.8} \\ &= (0.105\,\hat{\imath} + 1.26\,\hat{\jmath} + 0.105\,\hat{k})\,\text{kip}. \end{aligned}} \tag{2}$$

As a partial check of accuracy, we evaluate the magnitude of Eq. (2) to verify that it agrees with the absolute value of Eq. (1).

Problem 2.118

The gearshift lever AB for the transmission of a sports car has position vector $\vec{r}$ whose line of action passes through points A and B. The driver applies a force $\vec{F}$ to the knob of the lever to shift gears. If the component of $\vec{F}$ perpendicular to the gearshift lever must be 5 N to shift gears, determine the magnitude F of the force.



Solution

Our strategy will be to use the dot product to determine the component of $\vec{F}$ that is parallel to $\vec{r}$, namely $F_{\parallel}$. We will then use the Pythagorean theorem to determine the component of $\vec{F}$ that is perpendicular to $\vec{r}$, namely $F_{\perp}$. According to the problem statement, $F_{\perp} = 5\,\mathrm{N}$, which will allow us to solve for the value of F.

Noting that the magnitude of $\vec{r}$ is $r = 15\,\mathrm{mm}$, the component of $\vec{F}$ that is parallel to $\vec{r}$ is

$$F_{\parallel} = \vec{F} \cdot \frac{\vec{r}}{r} = F\left(\tfrac{1}{11}\right)(9\,\hat{\imath} - 6\,\hat{\jmath} - 3\,\hat{k}) \cdot \frac{-5\,\hat{\imath} + 2\,\hat{\jmath} + 14\,\hat{k}}{15} \tag{1}$$

$$= \frac{F}{165}\Big[(9)(-5) + (-6)(2) + (-2)(14)\Big] \tag{2}$$

$$= F\left(\frac{-85}{165}\right). \tag{3}$$

The parallel and perpendicular components of $\vec{F}$ are related by the Pythagorean theorem, $F^2 = F_{\parallel}^2 + F_{\perp}^2$. Rearranging this and noting that the perpendicular component $F_{\perp} = 5\,\mathrm{N}$, we obtain

$$F_{\perp}^2 = F^2 - F_{\parallel}^2 \tag{4}$$

$$(5\,\mathrm{N})^2 = F^2 - \left[F\left(\frac{-85}{165}\right)\right]^2 \tag{5}$$

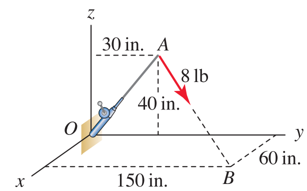
$$(5\,\mathrm{N})^2 = F^2(0.7346). \tag{6}$$

Solving Eq. (6) provides

$$\boxed{F = 5.83\,\mathrm{N}.} \tag{7}$$

Problem 2.119

A force of magnitude 8 lb is applied to line AB of a fishing rod OA. Determine the vector components of the 8 lb force in directions parallel and perpendicular to the rod.



Solution

The position vector describing the direction of fishing rod OA is $\vec{r}_{OA}$, and the magnitude of this vector is r_{OA}, as follows

$$\vec{r}_{OA} = (30\,\hat{\jmath} + 40\,\hat{k})\text{ in.}, \quad r_{OA} = \sqrt{(30)^2 + (40)^2}\text{ in.} = 50\text{ in.} \tag{1}$$

The force $\vec{F}_{AB}$ is expressed as

$$\vec{F}_{AB} = (8\text{ lb})\frac{\vec{r}_{AB}}{r_{AB}} = (8\text{ lb})\frac{(60\,\hat{\imath} + 120\,\hat{\jmath} - 40\,\hat{k})\text{ in.}}{\sqrt{(60)^2 + (120)^2 + (-40)^2}\text{ in.}} = (3.429\,\hat{\imath} + 6.857\,\hat{\jmath} - 2.286\,\hat{k})\text{ lb.} \tag{2}$$

The parallel component of the force is given by

$$F_{\parallel} = \vec{F}_{AB} \cdot \frac{\vec{r}_{OA}}{r_{OA}} = \frac{\big[(3.429)(0) + (6.857)(30) + (-2.286)(40)\big]\text{ in.}\cdot\text{lb}}{50\text{ in.}} = 2.286\text{ lb} \tag{3}$$

The perpendicular component is then

$$F_{\perp} = \sqrt{F_{AB}^2 - F_{\parallel}^2} = \sqrt{(8\text{ lb})^2 - (2.286\text{ lb})^2} = 7.667\text{ lb.} \tag{4}$$

The vector component of the force parallel to the rod is given by

$$\boxed{\vec{F}_{\parallel} = F_{\parallel}\frac{\vec{r}_{OA}}{r_{OA}} = (2.286\text{ lb})\frac{(30\,\hat{\jmath} + 40\,\hat{k})\text{ in.}}{50\text{ in.}} = (1.372\,\hat{\jmath} + 1.829\,\hat{k})\text{ lb.}} \tag{5}$$

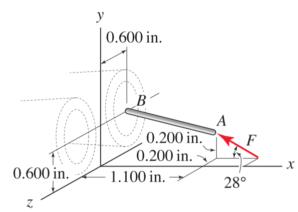
The vector component of the force perpendicular to the rod is determined using $\vec{F}_{AB} = \vec{F}_{\parallel} + \vec{F}_{\perp}$, which after rearrangement gives

$$\boxed{\begin{aligned}\vec{F}_{\perp} &= \vec{F}_{AB} - \vec{F}_{\parallel} = (3.429\,\hat{\imath} + 6.857\,\hat{\jmath} - 2.286\,\hat{k})\text{ lb} - (1.372\,\hat{\jmath} + 1.829\,\hat{k})\text{ lb}\\ &= (3.429\,\hat{\jmath} + 5.485\,\hat{\jmath} - 4.114\,\hat{k})\text{ lb}\end{aligned}} \tag{6}$$

As a partial check of the accuracy of our answers, we may evaluate the magnitudes of $\vec{F}_{\parallel}$ and $\vec{F}_{\perp}$ in Eqs. (5) and (6) to verify that they agree with Eqs. (3) and (4), respectively.

Problem 2.120

Rod AB is the firing pin for the right-hand barrel of a side-by-side shotgun. During firing, end A is struck by the hammer, which imparts a force F to the firing pin. The force F lies in a plane parallel to the xy plane, but the firing pin has a three-dimensional orientation as shown. Determine the components of F in directions parallel and perpendicular to the firing pin.



Solution

The coordinates of points A and B are $A\,(1.1, 0.2, -0.2)$ in., and $B\,(0, 0.6, -0.6)$ in.. Thus, the position vector from A to B is given by

$$\vec{r}_{AB} = \big((0-1.1)\,\hat{\imath} + (0.6-0.2)\,\hat{\jmath} + (-0.6-(-0.2))\,\hat{k}\big)\,\text{in.} = (-1.1\,\hat{\imath} + 0.4\,\hat{\jmath} - 0.4\,\hat{k})\,\text{in.}, \tag{1}$$

and the magnitude of this vector is

$$r_{AB} = \sqrt{(-1.1\,\text{in.})^2 + (0.4\,\text{in.})^2 + (-0.4\,\text{in.})^2} = 1.237\,\text{in.} \tag{2}$$

The vector expression for the force $\vec{F}$ applied to the firing pin is

$$\vec{F} = F(-\cos 28°\,\hat{\imath} + \sin 28°\,\hat{\jmath}). \tag{3}$$

The parallel component of the force is given by

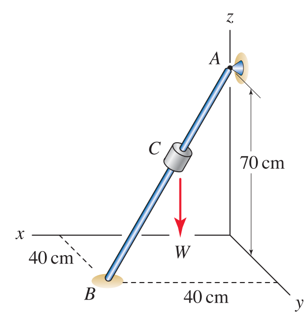
$$\boxed{F_{\parallel} = \vec{F}\cdot\frac{\vec{r}_{AB}}{r_{AB}} = F\,\frac{(-\cos 28°)(-1.1\,\text{in.}) + (\sin 28°)(0.4\,\text{in.}) + (0)(-0.4\,\text{in.})}{1.237\,\text{in.}} = (0.9370)\,F,} \tag{4}$$

while the perpendicular component is

$$\boxed{F_{\perp} = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{F^2 - \big[(0.9370)\,F\big]^2} = (0.349)\,F.} \tag{5}$$

Problem 2.121

The collar C is fixed to rod AB and supports a weight $W = 15\,\mathrm{N}$, acting in the negative z direction. Determine the vector components of the weight that are parallel and perpendicular to rod AB.



Solution

The vector expression for the weight is $\vec{W} = -15\,\hat{k}\,\mathrm{N}$. The position vector from A to B is denoted by $\vec{r}_{AB}$, such that

$$\vec{r}_{AB} = (40\,\hat{\imath} + 40\,\hat{\jmath} - 70\,\hat{k})\,\mathrm{cm}, \quad r_{AB} = \sqrt{40^2 + 40^2 + (-70)^2}\,\mathrm{cm} = 90\,\mathrm{cm}. \tag{1}$$

The scalar component of $\vec{W}$ parallel to $\vec{r}_{AB}$ is

$$W_\parallel = \vec{W} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{0 + 0 + (-15\,\mathrm{N})(-70\,\mathrm{cm})}{90\,\mathrm{cm}} = 11.67\,\mathrm{N}. \tag{2}$$

Although $W_\perp$ is not asked for in this problem, determining it will provide a check of accuracy for our results for $\vec{W}_\perp$. Thus, the perpendicular component is

$$W_\perp = \sqrt{W^2 - W_\parallel^2} = \sqrt{(-15\,\mathrm{N})^2 - (11.67\,\mathrm{N})^2} = 9.428\,\mathrm{N}. \tag{3}$$

The parallel component of $\vec{W}$ in vector form is then

$$\boxed{\vec{W}_\parallel = W_\parallel \frac{\vec{r}_{AB}}{r_{AB}} = (11.67\,\mathrm{N})\frac{(40\,\hat{\imath} + 40\,\hat{\jmath} - 70\,\hat{k})\,\mathrm{cm}}{90\,\mathrm{cm}} = (5.19\,\hat{\imath} + 5.19\,\hat{\jmath} - 9.07\,\hat{k})\,\mathrm{N}.} \tag{4}$$

The perpendicular vector component is

$$\boxed{\vec{W}_\perp = \vec{W} - \vec{W}_\parallel = (-15\,\mathrm{N})\,\hat{k} - (5.185\,\hat{\imath} + 5.185\,\hat{\jmath} - 9.04\,\hat{k})\,\mathrm{N} = (-5.19\,\hat{\imath} - 5.19\,\hat{\imath} - 5.93\,\hat{k})\,\mathrm{N}.} \tag{5}$$

As a partial check of the accuracy of our results, we use Eqs. (4) and (5) to evaluate the magnitudes $W_\parallel$ and $W_\perp$ as

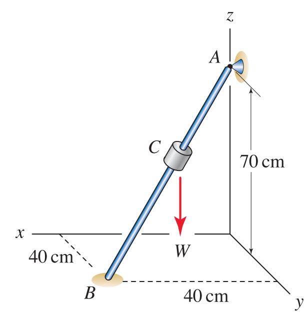
$$W_\parallel = \sqrt{(5.185)^2 + (5.185)^2 + (-9.074)^2}\,\mathrm{N} = 11.67\,\mathrm{N}, \tag{6}$$

$$W_\perp = \sqrt{(-5.185)^2 + (-5.185)^2 + (-5.926)^2}\,\mathrm{N} = 9.428\,\mathrm{N}, \tag{7}$$

which agree with the values reported in Eqs. (2) and (3).

Problem 2.122

The collar C is fixed to rod AB using a glued bond that allows a maximum force of 35 N parallel to the axis of the rod. The collar has weight W acting in the negative z direction. Determine the weight W of the collar that will cause the glued bond to break.



Solution

The vector expression for the weight is $\vec{W} = -W\,\hat{k}$. The position vector from A to B is denoted by $\vec{r}_{AB}$, such that

$$\vec{r}_{AB} = (40\,\hat{\imath} + 40\,\hat{\jmath} - 70\,\hat{k})\,\text{cm}, \quad r_{AB} = \sqrt{40^2 + 40^2 + (-70)^2}\,\text{cm} = 90\,\text{cm}. \tag{1}$$

Since the maximum force in the AB direction is 35 N, it follows that

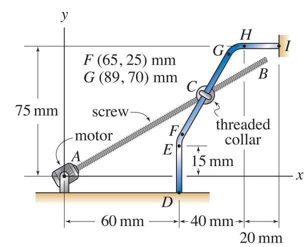
$$W_{\parallel} = 35\,\text{N} = \vec{W} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{0 + 0 + (-W)(-70\,\text{cm})}{90\,\text{cm}}. \tag{2}$$

Solving the above equation provides

$$\boxed{W = (90/70)35\,\text{N} = 45.0\,\text{N}.} \tag{3}$$

Problem 2.123

Components of a machine (not shown in the figure) are actuated by motion of a threaded collar C that slides on a fixed bar $DEFGHI$. Segment DE of the bar is straight and vertical, segment FG is straight where the coordinated of points F and G are given in the figure, and segment HI is straight and horizontal. The collar's motion is controlled by screw AB, which is turned by the motor at point A. If the force in the screw is 50 N compression, use the dot product to determine the component of the screw force in the direction of the bar for the positions of the collar given below. Also determine the component of the screw force perpendicular to the bar.



(a) Collar C is 2 mm below point E.

(b) Collar C is 1/3 the distance from point F to point G.

Solution

Part (a) When collar C is 2 mm below point E, then

$$\vec{r}_{AC} = (60\,\hat{\imath} + 13\,\hat{\jmath})\,\text{mm}, \tag{1}$$

$$\vec{F}_{AC} = 50\,\text{N}\,\frac{\vec{r}_{AC}}{r_{AC}} = 50\,\text{N}\,\frac{60\,\hat{\imath} + 13\,\hat{\jmath}}{\sqrt{(60)^2 + (13)^2}} = (48.87\,\hat{\imath} + 10.59\,\hat{\jmath})\,\text{N}. \tag{2}$$

The component of the screw force in the direction of the bar from point D to point E, which is the $+y$ direction, is simply the y component of Eq. (2), which provides

$$\boxed{F_{\parallel} = 10.59\ \text{N}.} \tag{3}$$

To obtain this result more formally, we take the dot product of Eq. (2) with a unit vector in the direction from D to E, namely $\hat{\jmath}$, to obtain

$$\boxed{F_{\parallel} = \vec{F}_{AC} \cdot \hat{\jmath} = (48.87\ \text{N})(0) + (10.59\ \text{N})(1) = 10.59\ \text{N}.} \tag{4}$$

The component of the screw force in the direction perpendicular to the bar is

$$\boxed{F_{\perp} = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{(50\ \text{N})^2 - (10.59\ \text{N})^2} = 48.87\ \text{N}.} \tag{5}$$

Part (b) When collar C is $1/3$ the distance from point F to point G, the coordinates of C are equal to the coordinates of F plus $1/3$ of the difference between the coordinates of points G and F. Hence

$$x_C = 65\ \text{mm} + \frac{1}{3}(89\ \text{mm} - 65\ \text{mm}) = 73\ \text{mm}, \tag{6}$$

$$y_C = 25\ \text{mm} + \frac{1}{3}(70\ \text{mm} - 25\ \text{mm}) = 40\ \text{mm}. \tag{7}$$

Thus

$$\vec{r}_{AC} = (73\,\hat{\imath} + 40\,\hat{\jmath})\,\text{mm}, \tag{8}$$

$$\vec{F}_{AC} = 50\text{ N}\,\frac{\vec{r}_{AC}}{r_{AC}} = 50\text{ N}\,\frac{73\,\hat{\imath} + 40\,\hat{\jmath}}{\sqrt{(73)^2 + (40)^2}} = (43.85\,\hat{\imath} + 24.03\,\hat{\jmath})\,\text{N}. \tag{9}$$

The position vector from F to G is

$$\vec{r}_{FG} = [(89 - 65)\,\hat{\imath} + (70 - 25)\,\hat{\jmath}]\,\text{mm} = (24\,\hat{\imath} + 45\,\hat{\jmath})\,\text{mm}. \tag{10}$$

The component of the screw force in the direction of the bar from point F to point G is

$$\boxed{F_{\parallel} = \vec{F}_{AC} \cdot \frac{\vec{r}_{FG}}{r_{FG}} = \frac{(43.85\text{ N})(24\text{ mm}) + (24.03\text{ N})(45\text{ mm})}{\sqrt{(24\text{ mm})^2 + (45\text{ mm})^2}} = 41.83\text{ N}.} \tag{11}$$

The component of the screw force in the direction perpendicular to the bar is

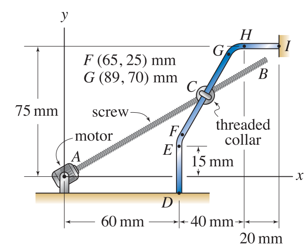
$$\boxed{F_{\perp} = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{(50\text{ N})^2 - (41.83\text{ N})^2} = 27.38\text{ N}.} \tag{12}$$

Problem 2.124

Components of a machine (not shown in the figure) are actuated by motion of a threaded collar C that slides on a fixed bar $DEFGHI$. Segment DE of the bar is straight and vertical, segment FG is straight where the coordinated of points F and G are given in the figure, and segment HI is straight and horizontal. The collar's motion is controlled by screw AB, which is turned by the motor at point A. If the force in the screw is 50 N compression, use the dot product to determine the component of the screw force in the direction of the bar for the positions of the collar given below. Also determine the component of the screw force perpendicular to the bar.

(a) Collar C is 2/3 the distance from point F to point G.

(b) Collar C is 1/2 the distance from point H to point I.



Solution

Part (a) When collar C is $2/3$ the distance from point F to point G, the coordinates of C are equal to the coordinates of F plus $2/3$ of the difference between the coordinates of points G and F. Hence

$$x_C = 65\text{ mm} + \frac{2}{3}(89\text{ mm} - 65\text{ mm}) = 81\text{ mm}, \tag{1}$$

$$y_C = 25\text{ mm} + \frac{2}{3}(70\text{ mm} - 25\text{ mm}) = 55\text{ mm}. \tag{2}$$

Thus,

$$\vec{r}_{AC} = (81\,\hat{\imath} + 55\,\hat{\jmath})\text{ mm}, \tag{3}$$

$$\vec{F}_{AC} = 50\text{ N}\,\frac{\vec{r}_{AC}}{r_{AC}} = 50\text{ N}\,\frac{81\,\hat{\imath} + 55\,\hat{\jmath}}{\sqrt{(81)^2 + (55)^2}} = (41.37\,\hat{\imath} + 28.09\,\hat{\jmath})\text{ N}. \tag{4}$$

The position vector from F to G is

$$\vec{r}_{FG} = \big[(89 - 65)\,\hat{\imath} + (70 - 25\,\hat{\jmath})\big]\text{ mm} = (24\,\hat{\imath} + 45\,\hat{\jmath})\text{ mm}. \tag{5}$$

The component of the screw force in the direction of the bar from point F to point G is

$$\boxed{F_{\parallel} = \vec{F}_{AC} \cdot \frac{\vec{r}_{FG}}{r_{FG}} = \frac{(41.37\text{ N})(24\text{ mm}) + (28.09\text{ N})(45\text{ mm})}{\sqrt{(24\text{ mm})^2 + (45\text{ mm})^2}} = 44.25\text{ N}.} \tag{6}$$

The component of the screw force in the direction perpendicular to the bar is

$$\boxed{F_{\perp} = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{(50\text{ N})^2 - (44.25\text{ N})^2} = 23.28\text{ N}.} \tag{7}$$

Part (b) When collar C is $1/2$ the distance from point H to point I, then

$$\vec{r}_{AC} = (110\,\hat{\imath} + 75\,\hat{\jmath})\,\text{mm}, \tag{8}$$

$$\vec{F}_{AC} = 50\text{ N}\,\frac{\vec{r}_{AC}}{r_{AC}} = 50\text{ N}\,\frac{110\,\hat{\imath} + 75\,\hat{\jmath}}{\sqrt{(110)^2 + (75)^2}} = (41.31\,\hat{\imath} + 28.17\,\hat{\jmath})\,\text{N}. \tag{9}$$

The component of the screw force in the direction of the bar from point H to point I, which is the $+x$ direction, is simply the x component of Eq. (9), which provides

$$\boxed{F_{\parallel} = 41.31\text{ N}.} \tag{10}$$

To obtain this result more formally, we take the dot product of Eq. (9) with a unit vector in the direction from H to I, namely $\hat{\imath}$, to obtain

$$\boxed{F_{\parallel} = \vec{F}_{AC} \cdot \hat{\imath} = (41.31\text{ N})(1) + (28.17\text{ N})(0) = 41.31\text{ N}.} \tag{11}$$

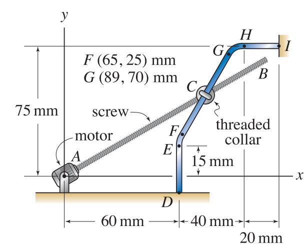
The component of the screw force in the direction perpendicular to the bar is

$$\boxed{F_{\perp} = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{(50\text{ N})^2 - (41.31\text{ N})^2} = 28.17\text{ N}.} \tag{12}$$

Problem 2.125

Components of a machine (not shown in the figure) are actuated by motion of a threaded collar C that slides on a fixed bar $DEFGHI$. Segment DE of the bar is straight and vertical, segment FG is straight where the coordinated of points F and G are given in the figure, and segment HI is straight and horizontal. The collar's motion is controlled by screw AB, which is turned by the motor at point A. If the force in the screw is 50 N compression, use the dot product to determine the component of the screw force in the direction of the bar for the positions of the collar given below. Also determine the component of the screw force perpendicular to the bar.

Let the collar be positioned on the straight segment of the bar between points F and G. Determine the point along this segment of the bar where the force from the screw in the direction of the bar is largest, and determine the value of this force. Also determine the component of the screw force perpendicular to the bar.



Solution

The component of the screw force in the direction of the bar from point F to point G will be greatest when the angle between $\vec{F}_{AC}$ (which is parallel to $\vec{r}_{AC}$) and $\vec{r}_{FG}$ is smallest. Thus, by inspection, this occurs when the collar C is positioned at point G. Thus,

$$\vec{r}_{AC} = \vec{r}_{AG} = (89\,\hat{\imath} + 70\,\hat{\jmath})\,\text{mm}, \tag{1}$$

$$\vec{F}_{AC} = 50\text{ N}\,\frac{\vec{r}_{AC}}{r_{AC}} = 50\text{ N}\,\frac{89\,\hat{\imath} + 70\,\hat{\jmath}}{\sqrt{(89)^2 + (70)^2}} = (39.30\,\hat{\imath} + 30.91\,\hat{\jmath})\text{ N}. \tag{2}$$

The position vector from point F to point G is

$$\vec{r}_{FG} = \big[(89 - 65)\,\hat{\imath} + (70 - 25\,\hat{\jmath})\big]\,\text{mm} = (24\,\hat{\imath} + 45\,\hat{\jmath})\,\text{mm}. \tag{3}$$

The component of the screw force in the direction from point F to point G is

$$\boxed{F_{\parallel} = \vec{F}_{AC} \cdot \frac{\vec{r}_{FG}}{r_{FG}} = \frac{(39.03\text{ N})(24\text{ mm}) + (30.91\text{ N})(45\text{ mm})}{\sqrt{(24\text{ mm})^2 + (45\text{ mm})^2}} = 45.77\text{ N}.} \tag{4}$$

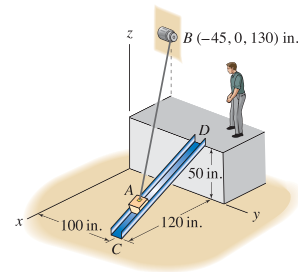
The component of the screw force perpendicular to the bar is

$$\boxed{F_{\perp} = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{(50\text{ N})^2 - (45.77\text{ N})^2} = 20.13\text{ N}.} \tag{5}$$

Problem 2.126

A winch at point B is used to slide a box up a chute CD in a warehouse. If the component of the force from cable AB in the direction of the chute must be 400 lb, determine the force in the cable for the position of the box given below. The chute is parallel to the xz plane. Neglect the size of the box and the winch.

The box is at the base of the chute, point C.



Solution

When the box A is at the base of the chute, point C, then

$$\vec{r}_{AB} = (-165\,\hat{\imath} - 100\,\hat{\jmath} + 130\,\hat{k})\text{ in.,} \tag{1}$$

$$\vec{F}_{AB} = F_{AB}\frac{\vec{r}_{AB}}{r_{AB}} = F_{AB}\frac{-165\,\hat{\imath} - 100\,\hat{\jmath} + 130\,\hat{k}}{232.6}. \tag{2}$$

The position vector from point C to D of the chute is

$$\vec{r}_{CD} = (-120\,\hat{\imath} + 50\,\hat{k})\text{ in.} \tag{3}$$

The component of $\vec{F}_{AB}$ in the direction of the chute must be 400 lb, hence

$$F_{\parallel} = \vec{F}_{AB}\cdot\frac{\vec{r}_{CD}}{r_{CD}} = 400\text{ lb} \tag{4}$$

$$F_{AB}\frac{-165\,\hat{\imath} - 100\,\hat{\jmath} + 130\,\hat{k}}{232.6}\cdot\frac{(-120\,\hat{\imath} + 50\,\hat{k})\text{ in.}}{130\text{ in.}} = 400\text{ lb} \tag{5}$$

$$F_{AB}\frac{(-165)(-120) + (-100)(0) + (130)(50)}{(232.6)(130)} = 400\text{ lb.} \tag{6}$$

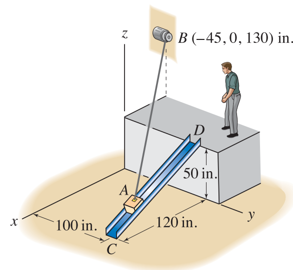
Thus, the force in the cable must be

$$\boxed{F_{AB} = 460.0\text{ lb.}} \tag{7}$$

Problem 2.127

A winch at point B is used to slide a box up a chute CD in a warehouse. If the component of the force from cable AB in the direction of the chute must be 400 lb, determine the force in the cable for the position of the box given below. The chute is parallel to the xz plane. Neglect the size of the box and the winch.

The box is 1/3 the distance up the chute from point C.



Solution

When box A is $1/3$ the distance up the chute from point C, then the coordinates of box A are

$$x_A = 120\text{ in.} - \frac{1}{3}(120\text{ in.}) = 80\text{ in.}, \tag{1}$$
$$y_A = 100\text{ in.}, \tag{2}$$
$$z_A = \frac{1}{3}(50\text{ in.}) = 16.67\text{ in.} \tag{3}$$

Then

$$\vec{r}_{AB} = \left[(-45-80)\,\hat{\imath} + (0-100)\,\hat{\jmath} + (130-16.67)\,\hat{k}\right]\text{ in.} \tag{4}$$
$$= (-125\,\hat{\imath} - 100\,\hat{\jmath} + 113.3\,\hat{k})\text{ in.}, \tag{5}$$
$$\vec{F}_{AB} = F_{AB}\frac{\vec{r}_{AB}}{r_{AB}} = F_{AB}\frac{-125\,\hat{\imath} - 100\,\hat{\jmath} + 113.3\,\hat{k}}{196.1}. \tag{6}$$

The position vector from point C to D of the chute is

$$\vec{r}_{CD} = (-120\,\hat{\imath} + 50\,\hat{k})\text{ in.} \tag{7}$$

The component of $\vec{F}_{AB}$ in the direction of the chute must be 400 lb, hence

$$F_{\parallel} = \vec{F}_{AB}\cdot\frac{\vec{r}_{CD}}{r_{CD}} = 400\text{ lb} \tag{8}$$

$$F_{AB}\frac{-125\,\hat{\imath} - 100\,\hat{\jmath} + 113.3\,\hat{k}}{196.1}\cdot\frac{(-120\,\hat{\imath} + 50\,\hat{k})\text{ in.}}{130\text{ in.}} = 400\text{ lb} \tag{9}$$
$$F_{AB}\frac{(-125)(-120) + (-100)(0) + (113.3)(50)}{(196.1)(130)} = 400\text{ lb.} \tag{10}$$

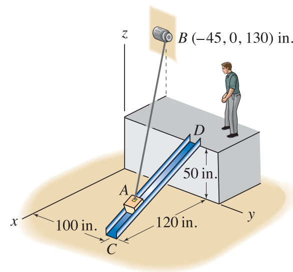
Thus, the force in the cable must be

$$\boxed{F_{AB} = 493.5\text{ lb.}} \tag{11}$$

Problem 2.128

A winch at point B is used to slide a box up a chute CD in a warehouse. If the component of the force from cable AB in the direction of the chute must be 400 lb, determine the force in the cable for the position of the box given below. The chute is parallel to the xz plane. Neglect the size of the box and the winch.

The box is 3/4 the distance up the chute from point C.



Solution

When box A is 3/4 the distance up the chute from point C, then the coordinates of the box A are

$$x_A = 120 \text{ in.} - \frac{3}{4}(120 \text{ in.}) = 30 \text{ in.}, \tag{1}$$

$$y_A = 100 \text{ in.}, \tag{2}$$

$$z_A = \frac{3}{4}(50 \text{ in.}) = 37.5 \text{ in.} \tag{3}$$

Then

$$\vec{r}_{AB} = [(-45-30)\,\hat{\imath} + (0-100)\,\hat{\jmath} + (130-37.5)\,\hat{k}] \text{ in.} \tag{4}$$

$$= (-75\,\hat{\imath} - 100\,\hat{\jmath} + 92.5\,\hat{k}) \text{ in.}, \tag{5}$$

$$\vec{F}_{AB} = F_{AB}\frac{\vec{r}_{AB}}{r_{AB}} = F_{AB}\frac{-75\,\hat{\imath} - 100\,\hat{\jmath} + 92.5\,\hat{k}}{155.5}. \tag{6}$$

The position vector from point C to D of the chute is

$$\vec{r}_{CD} = (-120\,\hat{\imath} + 50\,\hat{k}) \text{ in.} \tag{7}$$

The component of $\vec{F}_{AB}$ in the direction of the chute must be 400 lb, hence

$$F_{\parallel} = \vec{F}_{AB} \cdot \frac{\vec{r}_{CD}}{r_{CD}} = 400 \text{ lb} \tag{8}$$

$$F_{AB}\frac{-75\,\hat{\imath} - 100\,\hat{\jmath} + 92.5\,\hat{k}}{155.5} \cdot \frac{(-120\,\hat{\imath} + 50\,\hat{k}) \text{ in.}}{130 \text{ in.}} = 400 \text{ lb} \tag{9}$$

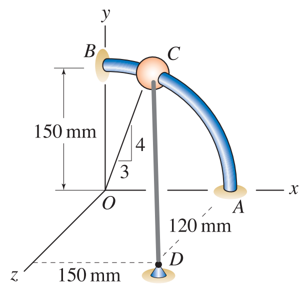
$$F_{AB}\frac{(-75)(-120) + (-100)(0) + (92.5)(50)}{(155.5)(130)} = 400 \text{ lb.} \tag{10}$$

Thus, the force in the cable must be

$$\boxed{F_{AB} = 593.5 \text{ lb.}} \tag{11}$$

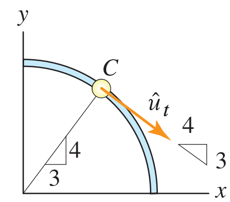
Problem 2.129

The structure consists of a quarter-circular rod AB with 150 mm radius that is fixed in the xy plane. An elastic cord supporting a force of 100 N is attached to a support at D and a bead at C. Determine the components of the cord force in directions tangent and normal to the curved rod AB at point C.



Solution

The unit vector $\hat{u}_t$ tangent to the ring at point C is shown at the right and is given by



$$\hat{u}_t = \frac{4\,\hat{\imath} - 3\,\hat{\jmath}}{\sqrt{4^2 + (-3)^2}} = \tfrac{4}{5}\,\hat{\imath} - \tfrac{3}{5}\,\hat{\jmath}. \tag{1}$$

The coordinates of point C are

$$\Big(\big(\tfrac{3}{5}\big)\,150\text{ mm},\,\big(\tfrac{4}{5}\big)150\text{ mm},\,0\Big),$$

and those of point D are

$$\big(150\text{ mm},\,0,\,120\text{ mm}\big).$$

Therefore,

$$\vec{r}_{CD} = \big[150\text{ mm} - \big(\tfrac{3}{5}\big)\,150\text{ mm}\big]\,\hat{\imath} + \big[0 - \big(\tfrac{4}{5}\big)\,150\text{ mm}\big]\,\hat{\jmath} + (120\text{ mm} - 0)\,\hat{k} = (60\,\hat{\imath} - 120\,\hat{\jmath} + 120\,\hat{k})\text{ mm}, \tag{2}$$

and

$$r_{CD} = \sqrt{(60\text{ mm})^2 + (-120\text{ mm})^2 + (120\text{ mm})^2} = 180\text{ mm}. \tag{3}$$

The force applied by the cord to the bead at C is given by

$$\vec{F}_{CD} = (100\text{ N})\frac{\vec{r}_{CD}}{r_{CD}} = (100\text{ N})\frac{(60\,\hat{\imath} - 120\,\hat{\jmath} + 120\,\hat{k})\text{ mm}}{180\text{ mm}} = (33.33\,\hat{\imath} - 66.67\,\hat{\jmath} + 66.67\,\hat{k})\text{ N}. \tag{4}$$

The component of $\vec{F}_{CD}$ in the tangent direction is

$$\boxed{F_{\parallel} = \vec{F}_{CD}\cdot\hat{u}_t = \big((33.33)(4/5) + (-66.67)(-3/5) + (66.67)(0)\big)\text{ N} = 66.7\text{ N},} \tag{5}$$

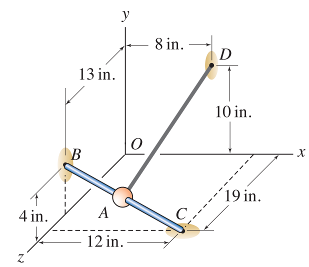
and the component of the cord force in the normal direction to the ring is

$$\boxed{F_{\perp} = \sqrt{F_{CD}^2 - F_{\parallel}^2} = \sqrt{(100\text{ N})^2 - (66.67\text{ N})^2} = 74.5\text{ N}.} \tag{6}$$

Problem 2.130

Bead A has negligible weight and slides without friction on rigid fixed bar BC. An elastic cord AD, which supports a 10 lb tensile force, is attached to the bead. At the instant shown, the bead has zero velocity. For the position of the bead given below, determine the vector components of the cord tension that act parallel and perpendicular to direction BC of the bar (your answers should be vectors). Due to the cord tension, will the bead slide toward point B or C?

Bead A is positioned halfway between points B and C.



Solution

$$\vec{r}_{BC} = (12\,\hat{\imath} - 4\,\hat{\jmath} + 6\,\hat{k})\text{ in.}, \qquad r_{BC} = 14\text{ in.} \tag{1}$$

When bead A is positioned half way between points B and C,

$$\vec{r}_{BA} = (6\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k})\text{ in.} \tag{2}$$

While we could write an expression for $\vec{r}_{AD}$ by taking the difference of the coordinates of D (head of vector) and A (tail of vector), we will use the following vector addition instead

$$\vec{r}_{AD} = \underbrace{\vec{r}_{AB}}_{-\vec{r}_{BA}} + \vec{r}_{BO} + \vec{r}_{OD} \tag{3}$$

$$= (-6\,\hat{\imath} + 2\,\hat{\jmath} - 3\,\hat{k})\text{ in.} + (0\,\hat{\imath} - 4\hat{\jmath} - 13\,\hat{k})\text{ in.} + (8\,\hat{\imath} + 10\,\hat{\jmath} + 0\,\hat{k})\text{ in.} \tag{4}$$

$$= (2\,\hat{\imath} + 8\,\hat{\jmath} - 16\,\hat{k})\text{ in.}, \tag{5}$$

$$r_{AD} = 18\text{ in.}, \tag{6}$$

$$\vec{T}_{AD} = 10\text{ lb}\,\frac{\vec{r}_{AD}}{r_{AD}} = 10\text{ lb}\,\frac{2\,\hat{\imath} + 8\,\hat{\jmath} - 16\,\hat{k}}{18}. \tag{7}$$

The component and vector component of $\vec{T}_{AD}$ parallel to rod BC is

$$T_\parallel = \vec{T}_{AD} \cdot \frac{\vec{r}_{BC}}{r_{BC}} = \frac{10\text{ lb}\left[(2)(12) + (8)(-4) + (-16)(6)\right]}{(18)(14)} = -4.127\text{ lb}, \tag{8}$$

$$\vec{T}_\parallel = T_\parallel \frac{\vec{r}_{BC}}{r_{BC}} = (-4.127\text{ lb})\frac{12\,\hat{\imath} - 4\,\hat{\jmath} + 6\,\hat{k}}{14} \tag{9}$$

$$= \boxed{(-3.537\,\hat{\imath} + 1.179\,\hat{\jmath} - 1.769\,\hat{k})\text{ lb.}} \tag{10}$$

The vector component of $\vec{T}_{AD}$ perpendicular to rod BC is

$$\boxed{\vec{T}_\perp = \vec{T}_{AD} - \vec{T}_\parallel = (4.648\,\hat{\imath} + 3.265\,\hat{\jmath} - 7.120\,\hat{k})\text{ lb.}} \tag{11}$$

As a partial check of accuracy, we evaluate the magnitude of Eq. (11) to obtain $T_\perp = 9.108$ lb, and then using $T_\parallel = -4.127$ lb, we evaluate $\sqrt{T_\parallel^2 + T_\perp^2}$ to verify that the result is 10 lb.

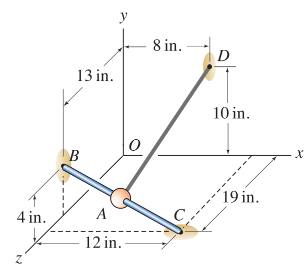
Sliding direction Since $T_\parallel$ is negative, the projection of $\vec{T}_{AD}$ in the BC direction is from C to B. Hence,

$\boxed{\text{The bead will tend to slide toward } B \text{ due to the cord force.}}$

Problem 2.131

Bead A has negligible weight and slides without friction on rigid fixed bar BC. An elastic cord AD, which supports a 10 lb tensile force, is attached to the bead. At the instant shown, the bead has zero velocity. For the position of the bead given below, determine the vector components of the cord tension that act parallel and perpendicular to direction BC of the bar (your answers should be vectors). Due to the cord tension, will the bead slide toward point B or C?

Bead A is 1/4 the distance from point B to point C.



Solution

$$\vec{r}_{BC} = (12\,\hat{\imath} - 4\,\hat{\jmath} + 6\,\hat{k})\text{ in.}, \qquad r_{BC} = 14\text{ in.} \tag{1}$$

When bead A is positioned $1/4$ the distance from point B to point C,

$$\vec{r}_{BA} = (3\,\hat{\imath} - \hat{\jmath} + 1.5\,\hat{k})\text{ in.} \tag{2}$$

While we could write an expression for $\vec{r}_{AD}$ by taking the difference of the coordinates of D (head of vector) and A (tail of vector), we will use the following vector addition instead

$$\vec{r}_{AD} = \underbrace{\vec{r}_{AB}}_{-\vec{r}_{BA}} + \vec{r}_{BO} + \vec{r}_{OD} \tag{3}$$

$$= (-3\,\hat{\imath} + \hat{\jmath} - 1.5\,\hat{k})\text{ in.} + (0\,\hat{\imath} - 4\,\hat{\jmath} - 13\,\hat{k})\text{ in.} + (8\,\hat{\imath} + 10\,\hat{\jmath} + 0\,\hat{k})\text{ in.} \tag{4}$$

$$= (5\,\hat{\imath} + 7\,\hat{\jmath} - 14.5\,\hat{k})\text{ in.}, \tag{5}$$

$$\vec{T}_{AD} = 10\text{ lb}\,\frac{\vec{r}_{AD}}{r_{AD}} = 10\text{ lb}\,\frac{5\,\hat{\imath} + 7\,\hat{\jmath} - 14.5\,\hat{k}}{16.86}. \tag{6}$$

The component and vector component of $\vec{T}_{AD}$ parallel to rod BC is

$$T_{\parallel} = \vec{T}_{AD}\cdot\frac{\vec{r}_{BC}}{r_{BC}} = \frac{10\text{ lb}\left[(5)(12) + (7)(-4) + (-14.5)(6)\right]}{(16.86)(14)} = -2.330\text{ lb}, \tag{7}$$

$$\vec{T}_{\parallel} = T_{\parallel}\frac{\vec{r}_{BC}}{r_{BC}} = (-2.330\text{ lb})\,\frac{12\,\hat{\imath} - 4\,\hat{\jmath} + 6\,\hat{k}}{14} \tag{8}$$

$$= \boxed{(-1.997\,\hat{\imath} + 0.6658\,\hat{\jmath} - 0.9986\,\hat{k})\text{ lb}.} \tag{9}$$

The vector component of $\vec{T}_{AD}$ perpendicular to rod BC is

$$\boxed{\vec{T}_{\perp} = \vec{T}_{AD} - \vec{T}_{\parallel} = (4.963\,\hat{\imath} + 3.486\,\hat{\jmath} - 7.602\,\hat{k})\text{ lb}.} \tag{10}$$

As a partial check of accuracy, we evaluate the magnitude of Eq. (10) to obtain $T_{\perp} = 9.725$ lb, and then using $T_{\parallel} = -2.330$ lb, we evaluate $\sqrt{T_{\parallel}^2 + T_{\perp}^2}$ to verify that the result is 10 lb.

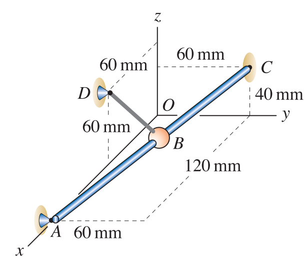
Sliding direction Since $T_{\parallel}$ is negative, the projection of $\vec{T}_{AD}$ in the BC direction is from C to B. Hence,

> The bead will tend to slide toward B due to the cord force.

Problem 2.132

Bead B has negligible weight and slides without friction on rigid fixed bar AC. An elastic cord BD, which supports a 60 N tensile force, is attached to the bead. At the instant shown, the bead has zero velocity. For the position of the bead given below, determine the components of the cord tension that act parallel and perpendicular to direction AC of the bar. Due to the cord tension, will the bead slide toward point A or C?

Bead B is positioned halfway between points A and C.



Solution

The position vector from A to C is

$$\vec{r}_{AC} = (-120\,\hat{\imath} + 60\,\hat{\jmath} + 40\,\hat{k})\,\text{mm}, \quad r_{AC} = \sqrt{(-120)^2 + 60^2 + 40^2}\,\text{mm} = 140\,\text{mm}. \tag{1}$$

Since B is at the midpoint of AC, it follows that the position vector from A to B is

$$\vec{r}_{AB} = \left(\frac{140\,\text{mm}}{2}\right)\frac{(-120\,\hat{\imath} + 60\,\hat{\jmath} + 40\,\hat{k})\,\text{mm}}{140\,\text{mm}} = (-60\,\hat{\imath} + 30\,\hat{\jmath} + 20\,\hat{k})\,\text{mm}. \tag{2}$$

The position vector from A to D is given by $\vec{r}_{AD} = (-60\,\hat{\imath} + 60\,\hat{k})$ mm. The position vector from D to B, $\vec{r}_{BD}$, may be determined using

$$\begin{aligned}\vec{r}_{BD} &= \vec{r}_{BA} + \vec{r}_{AD}\\ &= -\vec{r}_{AB} + \vec{r}_{AD}\\ &= -(-60\,\hat{\imath} + 30\,\hat{\jmath} + 20\,\hat{k})\,\text{mm} + (-60\,\hat{\imath} + 60\,\hat{k})\,\text{mm}\\ &= (-30\,\hat{\jmath} + 40\,\hat{k})\,\text{mm}.\end{aligned} \tag{3}$$

The vector expression for the 60 N tensile force the cord applies to bead B is

$$\vec{F}_{BD} = (60\,\text{N})\frac{(-30\,\hat{\jmath} + 40\,\hat{k})\,\text{mm}}{\sqrt{(-30)^2 + (40)^2}\,\text{mm}} = (-36\,\hat{\jmath} + 48\,\hat{k})\,\text{N}. \tag{4}$$

The component of the cord tension parallel to AC is given by

$$\boxed{F_\parallel = \vec{F}_{BD} \cdot \frac{\vec{r}_{AC}}{r_{AC}} = \frac{(-36\,\hat{\jmath} + 48\,\hat{k})\,\text{N} \cdot (-120\,\hat{\imath} + 60\,\hat{\jmath} + 40\,\hat{k})\,\text{mm}}{140\,\text{mm}} = -1.71\,\text{N}.} \tag{5}$$

The perpendicular component of the cord tension is

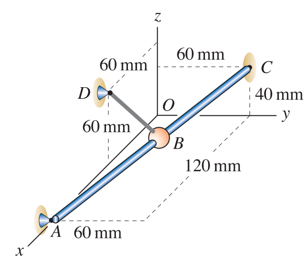
$$\boxed{F_\perp = \sqrt{F_{BD}^2 - F_\parallel^2} = \sqrt{(60\,\text{N})^2 - (-1.714\,\text{N})^2} = 59.98\,\text{N}.} \tag{6}$$

Since the value of $F_\parallel$ found in Eq. (5) is negative, the component of the cord force is actually in the direction opposite $\vec{r}_{AC}$, and hence is in the direction from C to A. Thus, $\boxed{\text{the bead slides toward } A.}$

Problem 2.133

Bead B has negligible weight and slides without friction on rigid fixed bar AC. An elastic cord BD, which supports a 60 N tensile force, is attached to the bead. At the instant shown, the bead has zero velocity. For the position of the bead given below, determine the components of the cord tension that act parallel and perpendicular to direction AC of the bar. Due to the cord tension, will the bead slide toward point A or C?

Bead B is 2/5 the distance from point A to point C.



Solution

The position vector from A to C is

$$\vec{r}_{AC} = (-120\,\hat{\imath} + 60\,\hat{\jmath} + 40\,\hat{k})\text{ mm}, \quad r_{AC} = \sqrt{(-120)^2 + (60)^2 + (40)^2}\text{ mm} = 140\text{ mm}. \tag{1}$$

Since B is $2/5$ the distance from point A to C, it follows that the position vector from A to bead B is

$$\vec{r}_{AB} = \frac{2}{5}(-120\,\hat{\imath} + 60\,\hat{\jmath} + 40\,\hat{k})\text{ mm} = (-48\,\hat{\imath} + 24\,\hat{\jmath} + 16\,\hat{k})\text{ mm}. \tag{2}$$

The position vector from A to D is $\vec{r}_{AD} = (-60\,\hat{\imath} + 60\,\hat{k})$ mm. The position vector from D to bead B is

$$\begin{aligned}\vec{r}_{BD} &= \vec{r}_{BA} + \vec{r}_{AD}\\ &= -\vec{r}_{AB} + \vec{r}_{AD}\\ &= -(-48\,\hat{\imath} + 24\,\hat{\jmath} + 16\,\hat{k})\text{ mm} + (-60\,\hat{\imath} + 0\,\hat{\jmath} + 60\,\hat{k})\text{ mm}\\ &= (-12\,\hat{\imath} - 24\,\hat{\jmath} + 44\,\hat{k})\text{ mm}.\end{aligned} \tag{3}$$

The vector expression for the 60 N tensile force the cord applies to bead B is

$$\vec{F}_{BD} = 60\text{ N}\,\frac{\vec{r}_{BD}}{r_{BD}} = 60\text{ N}\,\frac{-12\,\hat{\imath} - 24\,\hat{\jmath} + 44\,\hat{k}}{\sqrt{(-12)^2 + (-24)^2 + (44)^2}} = (-13.97\,\hat{\imath} - 27.94\,\hat{\jmath} + 51.23\,\hat{k})\text{ N}. \tag{4}$$

The component of the cord force parallel to AC is

$$\boxed{F_{\parallel} = \vec{F}_{BD}\cdot\frac{\vec{r}_{AC}}{r_{AC}} = \frac{(-13.97\text{ N})(-120\text{ mm}) + (-27.94\text{N})(60\text{ mm}) + (51.23\text{ N})(40\text{ mm})}{140\text{ mm}} = 14.64\text{ N}.} \tag{5}$$

The perpendicular component of the cord force is

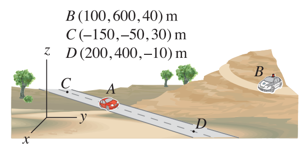
$$\boxed{F_{\perp} = \sqrt{F_{BD}^2 - F_{\parallel}^2} = \sqrt{(60\text{ N})^2 - (14.64\text{ N})^2} = 58.19\text{ N}.} \tag{6}$$

Since the value of $F_{\parallel}$ found in Eq. (5) is positive, the component of the cord force is in the direction from A to C. Thus, $\boxed{\text{the bead will slide toward } C.}$

Problem 2.134

A sports car at point A drives down a straight stretch of road CD. A police car at point B uses radar to measure the speed of the car, and obtains a reading of 80 km/h. For the position of the car given below, determine the speed of the car. Note that the 80 km/h speed measured by the radar gun is the rate of change of the distance between the car and the radar gun.

The car is 30% of the distance from point C to point D.



Solution

Let the actual speed of the car along the line from C to D be denoted by v, and then the vector representation of this (i.e., the velocity) is $\vec{v} = v(\vec{r}_{CD}/r_{CD})$. Using the coordinates of points C and D given in the problem statement,

$$\vec{r}_{CD} = \Big[\big(200-(-150)\big)\,\hat{\imath} + \big(400-(-50)\big)\,\hat{\jmath} + (-10-30)\,\hat{k}\Big]\,\text{m}$$

$$= (350\,\hat{\imath} + 450\,\hat{\jmath} - 40\,\hat{k})\,\text{m}, \tag{1}$$

$$\vec{v} = v\,\frac{\vec{r}_{CD}}{r_{CD}} = v\,\frac{350\,\hat{\imath} + 450\,\hat{\jmath} - 40\,\hat{k}}{571.5}. \tag{2}$$

When the car is positioned 30% of the distance from C to D, then the position vector from C to A is

$$\vec{r}_{CA} = (0.3)\,\vec{r}_{CD} = (105\,\hat{\imath} + 135\,\hat{\jmath} - 12\,\hat{k})\,\text{m}. \tag{3}$$

Let s be the speed measured by the radar gun. This speed is the component of the car's velocity $\vec{v}$ in the direction AB. Hence

$$s = \vec{v}\cdot\frac{\vec{r}_{AB}}{r_{AB}}. \tag{4}$$

The position vector $\vec{r}_{AB}$ may be determined using vector addition, as follows

$$\vec{r}_{AB} = \vec{r}_{AC} + \vec{r}_{CB} \tag{5}$$

$$= -\,\vec{r}_{CA} + \vec{r}_{CB} \tag{6}$$

$$= -(105\,\hat{\imath} + 135\,\hat{\jmath} - 12\,\hat{k})\,\text{m} + \Big[\big(100-(-150)\big)\,\hat{\imath} + \big(600-(-50)\big)\,\hat{\jmath} + (40-30)\,\hat{k}\Big]\,\text{m} \tag{7}$$

$$= -(105\,\hat{\imath} + 135\,\hat{\jmath} - 12\,\hat{k})\,\text{m} + (250\,\hat{\imath} + 650\,\hat{\jmath} + 10\,\hat{k})\,\text{m} \tag{8}$$

$$= (145\,\hat{\imath} + 515\,\hat{\jmath} + 22\,\hat{k})\,\text{m}, \tag{9}$$

$$r_{AB} = \sqrt{(145)^2 + (515)^2 + (22)^2}\,\text{m} = 535.5\,\text{m}. \tag{10}$$

Evaluating Eq. (4) provides

$$s = v\,\frac{350\,\hat{\imath} + 450\,\hat{\jmath} - 40\,\hat{k}}{571.5}\cdot\frac{145\,\hat{\imath} + 515\,\hat{\jmath} + 22\,\hat{k}}{535.5} \tag{11}$$

$$= v\,\frac{(350)(145) + (450)(515) + (-40)(22)}{(571.5)(535.5)}. \tag{12}$$

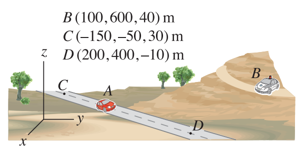
When the speed measured by the radar gun is $s = 80$ km/h, Eq. (12) may be solved to obtain the speed of the car as

$$\boxed{v = 86.93\ \text{km/h}.} \tag{13}$$

Problem 2.135

A sports car at point A drives down a straight stretch of road CD. A police car at point B uses radar to measure the speed of the car, and obtains a reading of 80 km/h. For the position of the car given below, determine the speed of the car. Note that the 80 km/h speed measured by the radar gun is the rate of change of the distance between the car and the radar gun.

The car is 60% of the distance from point C to point D.



Solution

Let the actual speed of the car along the line from C to D be denoted by v, and then the vector representation of this (i.e., the velocity) is $\vec{v} = v(\vec{r}_{CD}/r_{CD})$. Using the coordinates of points C and D given in the problem statement,

$$\vec{r}_{CD} = \Big[\big(200 - (-150)\big)\,\hat{\imath} + \big(400 - (-50)\big)\,\hat{\jmath} + (-10 - 30)\,\hat{k}\Big]\,\text{m}$$

$$= (350\,\hat{\imath} + 450\,\hat{\jmath} - 40\,\hat{k})\,\text{m}, \tag{1}$$

$$\vec{v} = v\,\frac{\vec{r}_{CD}}{r_{CD}} = v\,\frac{350\,\hat{\imath} + 450\,\hat{\jmath} - 40\,\hat{k}}{571.5}. \tag{2}$$

When the car is positioned 60% of the distance from C to D, then the position vector from C to A is

$$\vec{r}_{CA} = (0.6)\,\vec{r}_{CD} = (210\,\hat{\imath} + 270\,\hat{\jmath} - 24\,\hat{k})\,\text{m}. \tag{3}$$

Let s be the speed measured by the radar gun. This speed is the component of the car's velocity $\vec{v}$ in the direction AB. Hence

$$s = \vec{v}\cdot\frac{\vec{r}_{AB}}{r_{AB}}. \tag{4}$$

The position vector $\vec{r}_{AB}$ may be determined using vector addition, as follows

$$\vec{r}_{AB} = \vec{r}_{AC} + \vec{r}_{CB} \tag{5}$$

$$= -\vec{r}_{CA} + \vec{r}_{CB} \tag{6}$$

$$= -(210\,\hat{\imath} + 270\,\hat{\jmath} - 24\,\hat{k})\,\text{m} + \Big[\big(100 - (-150)\big)\,\hat{\imath} + \big(600 - (-50)\big)\,\hat{\jmath} + (40 - 30)\,\hat{k}\Big]\,\text{m} \tag{7}$$

$$= -(210\,\hat{\imath} + 270\,\hat{\jmath} - 24\,\hat{k})\,\text{m} + (250\,\hat{\imath} + 650\,\hat{\jmath} + 10\,\hat{k})\,\text{m} \tag{8}$$

$$= (40\,\hat{\imath} + 380\,\hat{\jmath} + 34\,\hat{k})\,\text{m}, \tag{9}$$

$$r_{AB} = \sqrt{(40)^2 + (380)^2 + (34)^2}\,\text{m} = 383.6\,\text{m}. \tag{10}$$

Evaluating Eq. (4) provides

$$s = v\,\frac{350\,\hat{\imath} + 450\,\hat{\jmath} - 40\,\hat{k}}{571.5}\cdot\frac{40\,\hat{\imath} + 380\,\hat{\jmath} + 34\,\hat{k}}{383.6} \tag{11}$$

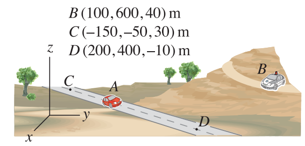
$$= v\,\frac{(350)(40) + (450)(380) + (-40)(34)}{(571.5)(383.6)}. \tag{12}$$

When the speed measured by the radar gun is $s = 80$ km/h, Eq. (12) may be solved to obtain the speed of the car as

$$\boxed{v = 95.50\,\text{km/h}.} \tag{13}$$

Problem 2.136

Let the road shown be straight between points C and D and beyond. Determine the shortest distance between the police car at point B and the road. Determine if the point where the road is closest to the police car falls within segment CD, or outside of this.



Solution

Our strategy will be to write expressions for $\vec{r}_{CB}$ and $\vec{r}_{CD}$. The component of $\vec{r}_{CB}$ that is perpendicular to $\vec{r}_{CD}$ is the shortest distance between the road and police car, while the component of $\vec{r}_{CB}$ that is parallel to $\vec{r}_{CD}$ will identify the point on the road where the road is closest to the police car. Using the coordinates provided in the problem statement, the following position vectors may be written

$$\vec{r}_{CB} = \Big[\big(100-(-150)\big)\,\hat{\imath} + \big(600-(-50)\big)\,\hat{\jmath} + (40-30)\,\hat{k}\Big]\,\text{m}$$

$$= (250\,\hat{\imath} + 650\,\hat{\jmath} + 10\,\hat{k})\,\text{m}, \tag{1}$$

$$r_{CB} = \sqrt{(250)^2 + (650)^2 + (10)^2}\,\text{m} = 696.5\ \text{m}, \tag{2}$$

$$\vec{r}_{CD} = \Big[\big(200-(-150)\big)\,\hat{\imath} + \big(400-(-50)\big)\,\hat{\jmath} + (-10-30)\,\hat{k}\Big]\,\text{m}$$

$$= (350\,\hat{\imath} + 450\,\hat{\jmath} - 40\,\hat{k})\,\text{m}, \tag{3}$$

$$r_{CD} = \sqrt{(350)^2 + (450)^2 + (-40)^2}\,\text{m} = 571.5\ \text{m}. \tag{4}$$

The component of $\vec{r}_{CB}$ that is parallel to $\vec{r}_{CD}$ is

$$r_{\parallel} = \vec{r}_{CB}\cdot\frac{\vec{r}_{CD}}{r_{CD}} = \frac{(250\ \text{m})(350\ \text{m}) + (650\ \text{m})(450\ \text{m}) + (10\ \text{m})(-40\ \text{m})}{571.5\ \text{m}}$$

$$= 664.2\ \text{m}. \tag{5}$$

Since the result in Eq. (5) is larger than the distance between points C and D (which is 571.5 m),

the point where the road is closest to the police car lies outside of segment *CD* of the road.

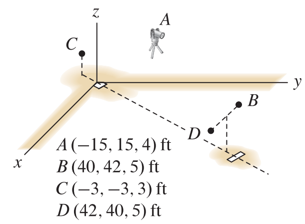
The component of $\vec{r}_{CB}$ that is perpendicular to $\vec{r}_{CD}$ is

$$r_{\perp} = \sqrt{r_{CB}^2 - r_{\parallel}^2} = \sqrt{(696.5\ \text{m})^2 - (664.2\ \text{m})^2} = 209.5\ \text{m}. \tag{6}$$

Hence, the shortest distance between the police car and the road is 209.5 m.

Problem 2.137

The manager of a baseball team plans to use a radar gun positioned at point A to measure the speed of pitches for a right-handed pitcher. If the person operating the radar gun measures a speed s when the baseball is one-third the distance from the release point at B to the catcher's glove at C, what is the actual speed of the pitch? Assume the pitch follows a straight-line path, and express your answer in terms of s. Note that the value s measured by the radar gun is the rate of change of the distance between point A and the ball.



Solution

Let the actual speed of the ball along the line from B to C be denoted by v, and then the vector representation of this (i.e., the velocity) is $\vec{v} = v(\vec{r}_{BC}/r_{BC})$. Consider the point at which the speed is measured by the radar gun (one third of the distance from B to C) to be denoted by point E. The value of speed measured by the radar gun is then the component of the speed in the AE direction

$$s = \vec{v} \cdot \frac{\vec{r}_{AE}}{r_{AE}}. \tag{1}$$

To evaluate Eq. (1), expressions for $\vec{r}_{BC}$ and $\vec{r}_{AE}$, and their magnitudes r_{BC} and r_{AE}, must be determined. Since the coordinates of points B and C are given, we may write

$$\vec{r}_{BC} = \left\{[(-3) - (40)]\,\hat{\imath} + [(-3) - (42)]\,\hat{\jmath} + [(3) - (5)]\,\hat{k}\right\}\text{ft} = (-43\,\hat{\imath} - 45\,\hat{\jmath} - 2\,\hat{k})\text{ ft}, \tag{2}$$

$$r_{BC} = \sqrt{(-43\text{ ft})^2 + (-45\text{ ft})^2 + (-2\text{ ft})^2} = 62.27\text{ ft}. \tag{3}$$

Since E is one-third of the way along $\vec{r}_{BC}$, it follows that

$$\begin{aligned}\vec{r}_{AE} &= \vec{r}_{AB} + \left(\tfrac{1}{3}\right)\vec{r}_{BC}\\ &= \left\{[(40) - (-15)]\,\hat{\imath} + [(42) - (15)]\,\hat{\jmath} + [(5) - (4)]\,\hat{k}\right\}\text{ft} + \tfrac{1}{3}(-43\,\hat{\imath} - 45\,\hat{\jmath} - 2\,\hat{k})\text{ ft}\\ &= (40.67\,\hat{\imath} + 12.00\,\hat{\jmath} + 0.3333\,\hat{k})\text{ ft},\end{aligned} \tag{4}$$

$$r_{AE} = \sqrt{(40.67\text{ ft})^2 + (12.00\text{ ft})^2 + (0.3333\text{ ft})^2} = 42.40\text{ ft}. \tag{5}$$

Using these relations and Eq. (1), we obtain

$$s = \vec{v} \cdot \frac{\vec{r}_{AE}}{r_{AE}} = v\frac{(-43\text{ ft})(40.67\text{ ft}) + (-45\text{ ft})(12.00\text{ ft}) + (-2\text{ ft})(0.3333\text{ ft})}{(62.27\text{ ft})(42.40\text{ ft})} = (-0.8670)\,v. \tag{6}$$

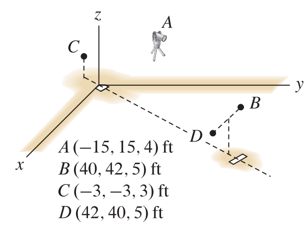
So, if the radar gun reads a speed of s, the actual speed of the pitch is

$$\boxed{v = \frac{s}{0.8670} = (1.153)s.} \tag{7}$$

Note that in reporting our answer in Eq. (7), we have disregarded the negative sign in Eq. (6) as being irrelevant.

Problem 2.138

Repeat Prob. 2.137 for a left-handed pitcher whose release point is D.



Solution

Let the actual speed of the ball along the line from D to C be denoted by v, and then the vector representation of this (i.e., the velocity) is $\vec{v} = v(\vec{r}_{DC}/r_{DC})$. Consider the point at which the speed is measured by the radar gun (one third of the distance from D to C) to be denoted by point F. The value of speed measured by the radar gun is then the component of the speed in the AF direction

$$s = \vec{v} \cdot \frac{\vec{r}_{AF}}{r_{AF}}. \tag{1}$$

To evaluate Eq. (1), expressions for $\vec{r}_{DC}$ and $\vec{r}_{AF}$, and their magnitudes r_{DC} and r_{AF}, must be determined. Since the coordinates of points D and C are given, we may write

$$\vec{r}_{DC} = \left\{[(-3) - (42)]\,\hat{\imath} + [(-3) - (40)]\,\hat{\jmath} + [(3) - (5)]\,\hat{k}\right\}\text{ft} = \left(-45\,\hat{\imath} - 43\,\hat{\jmath} - 2\,\hat{k}\right)\text{ft}, \tag{2}$$

$$r_{DC} = \sqrt{(-45\,\text{ft})^2 + (-43\,\text{ft})^2 + (-2\,\text{ft})^2} = 62.27\,\text{ft}. \tag{3}$$

Since F is one-third of the way along $\vec{r}_{DC}$, it follows that

$$\vec{r}_{AF} = \vec{r}_{AD} + \left(\tfrac{1}{3}\right)\vec{r}_{DC} \tag{4}$$

$$= \left\{[(42) - (-15)]\,\hat{\imath} + [(40) - (15)]\,\hat{\jmath} + [(5) - (4)]\,\hat{k}\right\}\text{ft} + \tfrac{1}{3}(-45\,\hat{\imath} - 43\,\hat{\jmath} - 2\,\hat{k})\,\text{ft} \tag{5}$$

$$= (42.00\,\hat{\imath} + 10.67\,\hat{\jmath} + 0.3333\,\hat{k})\,\text{ft}, \tag{6}$$

$$r_{BC} = \sqrt{(42.00\,\text{ft})^2 + (10.67\,\text{ft})^2 + (0.3333\,\text{ft})^2} = 43.33\,\text{ft}. \tag{7}$$

Using these relations and Eq. (1), we obtain

$$s = \vec{v} \cdot \frac{\vec{r}_{AF}}{r_{AF}} = v\,\frac{(-45\,\text{ft})(42.00\,\text{ft}) + (-43\,\text{ft})(10.67\,\text{ft}) + (-2\,\text{ft})(0.3333\,\text{ft})}{(62.27\,\text{ft})(43.33\,\text{ft})} = (-0.8706)\,v. \tag{8}$$

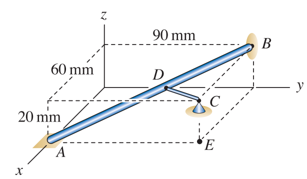
So, if the radar gun reads a speed of s, the actual speed of the pitch is

$$\boxed{v = \frac{s}{0.8706} = (1.149)s.} \tag{9}$$

Note that in reporting our answer in Eq. (9), we have disregarded the negative sign in Eq. (8) as being irrelevant.

Problem 2.139

Structural member AB is to be supported by a strut CD. Determine the smallest length CD may have, and specify where D must be located for a strut of this length to be used.



Solution

The strategy is to first find $\vec{r}_{AC}$ and then determine the components of $\vec{r}_{AC}$ parallel to AB and perpendicular to AB (these will be designated $r_\parallel$ and $r_\perp$, respectively).* The shortest possible length of the strut is $r_\perp$. Using the coordinates provided in the problem description, $\vec{r}_{AB}$ and its magnitude r_{AB} are

$$\vec{r}_{AB} = (-60\,\hat{\imath} + 90\,\hat{\jmath} + 20\,\hat{k})\,\text{mm}, \quad r_{AB} = \sqrt{(-60)^2 + 90^2 + 20^2} = 110\,\text{mm}. \tag{1}$$

Similarly, $\vec{r}_{AC}$ and r_{AC} are given by

$$\vec{r}_{AC} = (90\,\hat{\jmath} + 20\,\hat{k})\,\text{mm}, \quad r_{AC} = \sqrt{90^2 + 20^2} = 92.20\,\text{mm}. \tag{2}$$

The component of $\vec{r}_{AC}$ that is parallel to $\vec{r}_{AB}$ is

$$r_\parallel = \vec{r}_{AC} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{0 + (90\,\text{mm})(90\,\text{mm}) + (20\,\text{mm})(20\,\text{mm})}{110\,\text{mm}} = 77.27\,\text{mm}. \tag{3}$$

The smallest length of strut CD is given by $r_\perp$, which is found using

$$\boxed{r_\perp = \sqrt{r_{AC}^2 - r_\parallel^2} = \sqrt{(92.20\,\text{mm})^2 - (77.27\,\text{mm})^2} = 50.3\,\text{mm}.} \tag{4}$$

With the above results, the position vector from A to D may be written as

$$\vec{r}_{AD} = r_\parallel \frac{\vec{r}_{AB}}{r_{AB}} = (77.27\,\text{mm})\frac{(-60\,\hat{\imath} + 90\,\hat{\jmath} + 20\,\hat{k})\,\text{mm}}{110\,\text{mm}} = (-42.15\,\hat{\imath} + 63.22\,\hat{\jmath} + 14.05\,\hat{k})\,\text{mm}. \tag{5}$$

Alternatively, with the coordinates of point D being (x_D, y_D, z_D), by taking the differences between the coordinates of the head and tail of a vector, $\vec{r}_{AD}$ may be written as

$$\begin{aligned} \vec{r}_{AD} &= (x_D - 60\,\text{mm})\,\hat{\imath} + (y_D - 0)\,\hat{\jmath} + (z_D - 0)\,\hat{k}, \\ &= (x_D - 60\,\text{mm})\,\hat{\imath} + y_D\,\hat{\jmath} + z_D\,\hat{k}. \end{aligned} \tag{6}$$

For Eqs. (5) and (6) to agree requires

$$-42.15\,\text{mm} = x_D - 60\,\text{mm}, \quad \Rightarrow \quad x_D = 17.85\,\text{mm}, \tag{7}$$

$$63.22\,\text{mm} = y_D, \quad \Rightarrow \quad y_D = 63.22\,\text{mm}, \tag{8}$$

$$14.05\,\text{mm} = z_D, \quad \Rightarrow \quad z_D = 14.05\,\text{mm}. \tag{9}$$

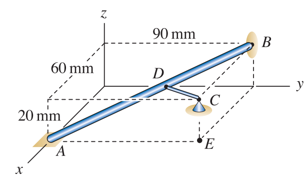
To summarize, the coordinates of point D are

$$\boxed{(17.9, 63.2, 14.0)\,\text{mm}.} \tag{10}$$

*Rather than $\vec{r}_{AC}$, we could begin with $\vec{r}_{BC}$. Note that the perpendicular component, $r_\perp$, of both of these vectors is the same.

Problem 2.140

Determine the smallest distance between member AB and point E.



Solution

The strategy is to first find $\vec{r}_{AE}$ and then determine the components of $\vec{r}_{AE}$ parallel to AB and perpendicular to AB (these will be designated $r_\parallel$ and $r_\perp$, respectively).* The shortest distance between point E and member AB is $r_\perp$. Using the coordinates provided in the problem description, $\vec{r}_{AB}$ and its magnitude r_{AB} are

$$\vec{r}_{AB} = (-60\,\hat{\imath} + 90\,\hat{\jmath} + 20\,\hat{k})\,\text{mm}, \quad r_{AB} = \sqrt{(-60)^2 + 90^2 + 20^2} = 110\,\text{mm}, \tag{1}$$

and the position vector from A to E is given by

$$\vec{r}_{AE} = 90\,\hat{\jmath}\,\text{mm}, \quad r_{AE} = 90\,\text{mm}. \tag{2}$$

The component of $\vec{r}_{AE}$ parallel to $\vec{r}_{AB}$ is

$$r_\parallel = \vec{r}_{AE} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{0 + (90\,\text{mm})(90\,\text{mm}) + 0}{110\,\text{mm}} = 73.64\,\text{mm}. \tag{3}$$

The perpendicular component, which is also the shortest distance between member AB and E, is

$$\boxed{r_\perp = \sqrt{r_{AE}^2 - r_\parallel^2} = \sqrt{(90\,\text{mm})^2 - (73.64\,\text{mm})^2} = 51.7\,\text{mm}.} \tag{4}$$

* Rather than $\vec{r}_{AE}$, we could begin with $\vec{r}_{BE}$. Note that the perpendicular component, $r_\perp$, of both of these vectors is the same.

Problem 2.141

In Example 2.13 on p. 76, determine the smallest distance between point D and the infinite line passing through points A and B. Is this distance the same as the smallest distance to rod AB? Explain.

Solution

The shortest distance between point D and the infinite line passing through points A and B is the component of $\vec{r}_{BD}$ that is perpendicular to line AB.* Using the coordinates of points A, B, and D given in Fig. 1 of Example 2.13, the following position vectors, and their magnitudes, may be written

$$\vec{r}_{BD} = (-300\,\hat{\jmath} + 280\,\hat{k})\text{ mm}, \qquad r_{BD} = \sqrt{(-300)^2 + (280)^2}\text{ mm} = 410.4\text{ mm}, \tag{1}$$

$$\vec{r}_{BA} = (-120\,\hat{\imath} - 240\,\hat{\jmath} + 240\,\hat{k})\text{ mm}, \quad r_{BA} = \sqrt{(-120)^2 + (-240)^2 + (240)^2}\text{ mm} = 360.0\text{ mm}. \tag{2}$$

The component of $\vec{r}_{BD}$ that is parallel to $\vec{r}_{BA}$ is given by

$$r_{\parallel} = \vec{r}_{BD} \cdot \frac{\vec{r}_{BA}}{r_{BA}} = \frac{0 + (-300\text{ mm})(-240\text{ mm}) + (280\text{ mm})(240\text{ mm})}{360.0\text{ mm}} = 386.7\text{ mm}. \tag{3}$$

Because the 386.7 mm result in Eq. (3) exceeds the 360.0 mm length of rod AB, the shortest distance between point D and the infinite line passing through points A and B *is not* the same as the shortest distance between point D and the finite-length line segment connecting points A and B.

The shortest distance between point D and the infinite line passing through points A and B is

$$\boxed{r_{\perp} = \sqrt{r_{BD}^2 - r_{\parallel}^2} = \sqrt{(410.4\text{ mm})^2 - (386.7\text{ mm})^2} = 137\text{ mm}.} \tag{4}$$

If attention is restricted to points on rod AB only, then the shortest distance between point D and the rod is the distance between points A and D. Hence,

$$\vec{r}_{AD} = (120\,\hat{\imath} - 60\,\hat{\jmath} + 40\,\hat{k})\text{ mm}, \quad r_{AD} = \sqrt{(120)^2 + (-60)^2 + (40)^2}\text{ mm} = 140\text{ mm}. \tag{5}$$

As expected, the value in Eq. (5) is greater than $r_{\perp}$ given in Eq. (4).

* Rather than using $\vec{r}_{BD}$, this solution could be developed using $\vec{r}_{AD}$ instead.

Problem 2.142

In Example 2.18 on p. 94, determine the smallest distance between point O and the infinite line passing through points A and B. Is this distance the same as the smallest distance to rod AB? Explain.

Solution

The shortest distance between point O and the infinite line passing through points A and B is the component of $\vec{r}_{AO}$ that is perpendicular to the line AB.* Using the coordinates of points A and B given in Fig. 1 of Example 2.18, and noting that point O is at the origin of the coordinate system, the following position vectors, and their magnitudes, may be written

$$\vec{r}_{AO} = (-12\,\hat{\imath} - 8\,\hat{\jmath})\,\text{in.}, \qquad r_{AO} = \sqrt{(-12)^2 + (-8)^2}\,\text{in.} = 14.42\,\text{in.}, \tag{1}$$

$$\vec{r}_{AB} = (-12\,\hat{\imath} - 4\,\hat{\jmath} + 18\,\hat{k})\,\text{in.}, \qquad r_{AB} = \sqrt{(-12)^2 + (-4)^2 + (18)^2}\,\text{in.} = 22.00\,\text{in.} \tag{2}$$

The component of $\vec{r}_{AO}$ that is parallel to $\vec{r}_{AB}$ is given by

$$r_{\parallel} = \vec{r}_{AO} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{(-12\,\text{in.})(-12\,\text{in.}) + (-8\,\text{in.})(-4\,\text{in.}) + 0}{22.00\,\text{in.}} = 8.000\,\text{in.} \tag{3}$$

Because the 8.000 in. result in Eq. (3) is less than the 22.00 in. length of rod AB, the shortest distance between point O and the infinite line passing through points A and B is indeed the same as the shortest distance between point O and the finite-length line segment connecting points A and B.

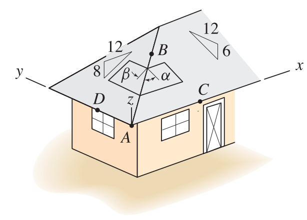
The shortest distance between point O and rod AB is

$$\boxed{r_{\perp} = \sqrt{r_{AO}^2 - r_{\parallel}^2} = \sqrt{(14.42\,\text{in.})^2 - (8.000\,\text{in.})^2} = 12.0\,\text{in.}} \tag{4}$$

* Rather than using $\vec{r}_{AO}$, this solution could be developed using $\vec{r}_{BO}$ instead.

Problem 2.143

A building's roof has "6 in 12" slope in the front and back and "8 in 12" slope on the sides. Determine the angles α and β that should be used for cutting sheets of plywood so they properly meet along edge AB of the roof. *Hint:* Write the position vector $\vec{r}_{AB}$ (where B is some point along the edge of the roof) two ways: $\vec{r}_{AB} = \vec{r}_{AC} + \vec{r}_{CB}$ and $\vec{r}_{AB} = \vec{r}_{AD} + \vec{r}_{DB}$. Then use the roof slopes to help write $\vec{r}_{CB}$ and $\vec{r}_{DB}$ such that the magnitudes of the two expressions for $\vec{r}_{AB}$ are the same.



Solution

Using the suggestion given in the problem statement, we will make use of the following relations

$$\vec{r}_{AB} = \vec{r}_{AC} + \vec{r}_{CB} = \vec{r}_{AD} + \vec{r}_{DB}. \tag{1}$$

Letting the x coordinate of point C be x_C, and the y coordinate of point D be y_D, Eq. (1) may be rewritten as

$$\vec{r}_{AB} = x_C\,\hat{\imath} + r_{CB}\frac{12\,\hat{\jmath} + 6\,\hat{k}}{\sqrt{12^2 + 6^2}} = y_D\,\hat{\jmath} + r_{DB}\frac{12\,\hat{\imath} + 8\,\hat{k}}{\sqrt{12^2 + 8^2}}, \tag{2}$$

where r_{CB} and r_{DB} represent the lengths from points C to B and from points D to B, respectively. Note that when writing Eq. (2), the x coordinates of points B and C are taken to be the same, and the y coordinates of points B and D are taken to be the same. Equating the x, y, and z components of Eq. (2) leads to the following relations

$$x_C = r_{DB}(12/\sqrt{208}), \quad y_D = r_{CB}(12/\sqrt{180}), \quad r_{CB}(6/\sqrt{180}) = r_{DB}(8/\sqrt{208}). \tag{3}$$

The third of the three relations in Eq. (3) provides $r_{DB} = \sqrt{13/20}\,r_{CB}$, and this may be substituted into the first two relations in Eq. (3) to write

$$x_C = (3/\sqrt{20})r_{CB}, \quad y_D = (2/\sqrt{5})r_{CB}. \tag{4}$$

Using Eq. (4) and $r_{DB} = \sqrt{13/20}\,r_{CB}$, Eqn. (2) becomes

$$\vec{r}_{AB} = r_{CB}\left(\frac{3}{\sqrt{20}}\,\hat{\imath} + \frac{12}{\sqrt{180}}\,\hat{\jmath} + \frac{6}{\sqrt{180}}\,\hat{k}\right) = \frac{1}{2\sqrt{5}}\,r_{CB}\left(3\,\hat{\imath} + 4\,\hat{\jmath} + 2\,\hat{k}\right). \tag{5}$$

The angle α shown in the problem statement is the angle between the vectors $\vec{r}_{BA}$ and $\vec{r}_{BC}$, where

$$\vec{r}_{BA} = -\vec{r}_{AB} = \frac{-1}{2\sqrt{5}}\,r_{CB}\left(3\,\hat{\imath} + 4\,\hat{\jmath} + 2\,\hat{k}\right), \tag{6}$$

$$r_{BA} = \frac{1}{2\sqrt{5}}\,r_{CB}\sqrt{(3)^2 + (4)^2 + (2)^2} = \sqrt{\frac{29}{20}}\,r_{CB}, \tag{7}$$

$$\vec{r}_{BC} = -r_{CB}\frac{12\,\hat{\jmath} + 6\,\hat{k}}{\sqrt{12^2 + 6^2}} = \frac{-1}{\sqrt{5}}\,r_{CB}(2\,\hat{\jmath} + \hat{k}). \tag{8}$$

Thus, the angle α is given by

$$\alpha = \cos^{-1}\left(\frac{\vec{r}_{BC}\cdot\vec{r}_{BA}}{r_{BC}\,r_{BA}}\right) = \cos^{-1}\left\{\frac{\left(\frac{-1}{\sqrt{5}}\,r_{CB}\right)\left(\frac{-1}{2\sqrt{5}}\,r_{CB}\right)}{r_{CB}\,\sqrt{\frac{29}{20}}\,r_{CB}}\Big[0 + (2)(4) + (1)(2)\Big]\right\} \tag{9}$$

$$= \boxed{33.9°.}$$

Similarly, to obtain the angle β, we will consider the vector $\vec{r}_{BD}$ and its magnitude

$$\vec{r}_{BD} = -r_{DB}\frac{12\,\hat{\imath} + 8\,\hat{k}}{\sqrt{12^2+8^2}} = \frac{-1}{\sqrt{20}}\,r_{CB}(3\,\hat{\imath} + 2\,\hat{k}), \tag{10}$$

$$r_{BD} = r_{DB} = \sqrt{\frac{13}{20}}\,r_{CB}. \tag{11}$$

The angle β is then given by

$$\alpha = \cos^{-1}\left(\frac{\vec{r}_{BD}\cdot\vec{r}_{BA}}{r_{BD}\,r_{BA}}\right) = \cos^{-1}\left\{\frac{\left(\frac{-1}{\sqrt{20}}\,r_{CB}\right)\left(\frac{-1}{2\sqrt{5}}\,r_{CB}\right)}{\sqrt{\frac{13}{20}}\,r_{CB}\;\sqrt{\frac{29}{20}}\,r_{CB}}\Big[(3)(3)+0+(2)(2)\Big]\right\} \tag{12}$$

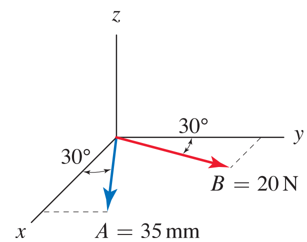
$$= \boxed{48.0^\circ.}$$

Problem 2.144

Vectors $\vec{A}$ and $\vec{B}$ lie in the xy plane.

(a) Use Eq. (2.48) on p. 101 to evaluate $\vec{A} \times \vec{B}$, expressing the resulting vector using Cartesian representation.

(b) Evaluate $\vec{A} \times \vec{B}$ by computing the determinant of a matrix, using either Method 1 or Method 2.



Solution

Part (a) Using Eq. (2.48), where by application of the right-hand rule $\hat{u} = \hat{k}$, we obtain

$$\boxed{\vec{A} \times \vec{B} = (|\vec{A}||\vec{B}|\sin\theta)\,\hat{u} = (35\,\text{mm})(20\,\text{N})\big(\sin 30°\big)\;\hat{k} = (350\,\text{N·mm})\;\hat{k}.} \tag{1}$$

Part (b) Vectors $\vec{A}$ and $\vec{B}$ are

$$\vec{A} = (35\,\text{mm})(\cos 30°\,\hat{\imath} + \sin 30°\,\hat{\jmath}), \qquad \vec{B} = (20\,\text{N})(\sin 30°\,\hat{\imath} + \cos 30°\,\hat{\jmath}). \tag{2}$$

Evaluating the cross product using the determinant provides

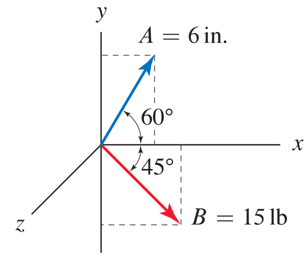
$$\begin{aligned}
\vec{A} \times \vec{B} &= (35\,\text{mm})(20\,\text{N}) \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos 30° & \sin 30° & 0 \\ \sin 30° & \cos 30° & 0 \end{vmatrix} \\
&= 700\,\text{N}\cdot\text{mm}\left\{\hat{\imath}\,(0-0) - \hat{\jmath}\,(0-0) + \hat{k}\,\big[(\cos 30°)(\cos 30°) - (\sin 30°)(\sin 30°)\big]\right\} \\
&= (700\,\text{N·mm})\,(0.7500 - 0.2500)\,\hat{k} \\
&= \boxed{(350\,\text{N·mm})\;\hat{k}.}
\end{aligned} \tag{3}$$

Problem 2.145

Vectors $\vec{A}$ and $\vec{B}$ lie in the xy plane.

(a) Use Eq. (2.48) on p. 101 to evaluate $\vec{A} \times \vec{B}$, expressing the resulting vector using Cartesian representation.

(b) Evaluate $\vec{A} \times \vec{B}$ by computing the determinant of a matrix, using either Method 1 or Method 2.



Solution

Part (a) Using Eq. (2.48) where by application of the right-hand rule $\hat{u} = -\hat{k}$, we obtain

$$\boxed{\vec{A} \times \vec{B} = (|\vec{A}||\vec{B}| \sin\theta)\,\hat{u} = (6\,\text{in.})(15\,\text{lb}) \sin 105°(-\hat{k}) = (-86.9\,\text{in.}\cdot\text{lb})\ \hat{k}.} \tag{1}$$

Part (b) Vectors $\vec{A}$ and $\vec{B}$ are

$$\vec{A} = (6\,\text{in.})(\cos 60°\,\hat{\imath} + \sin 60°\,\hat{\jmath}), \qquad \vec{B} = (15\,\text{lb})(\cos 45°\,\hat{\imath} - \sin 45°\,\hat{\jmath}). \tag{2}$$

Evaluating the cross product using the determinant provides

$$\begin{aligned}
\vec{A} \times \vec{B} &= (6\,\text{in.})(15\,\text{lb}) \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos 60° & \sin 60° & 0 \\ \cos 45° & -\sin 45° & 0 \end{vmatrix} \\
&= 90\,\text{in.}\cdot\text{lb} \left\{ \hat{\imath}\,(0-0) - \hat{\jmath}\,(0-0) + \hat{k}\,\left[(\cos 60°)(-\sin 45°) - (\sin 60°)(\cos 45°)\right] \right\} \qquad (3) \\
&= (90\,\text{in.}\cdot\text{lb})\,(-0.3536 - 0.6124)\hat{k} \\
&= \boxed{(-86.9\,\text{in.}\cdot\text{lb})\,\hat{k}.}
\end{aligned}$$

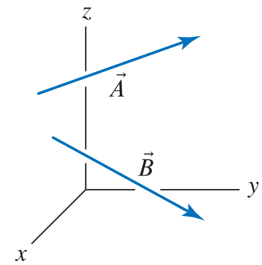
Problem 2.146

(a) Evaluate $\vec{A} \times \vec{B}$.

(b) Evaluate $\vec{B} \times \vec{A}$.

(c) Comment on any differences between the results of Parts (a) and (b).

(d) Use the dot product to show the result of Part (a) is orthogonal to vectors $\vec{A}$ and $\vec{B}$.



$\vec{A} = (-\hat{\imath} + 8\hat{\jmath} + 4\hat{k})\,\text{mm}$
$\vec{B} = (\hat{\imath} + 18\hat{\jmath} - 6\hat{k})\,\text{mm}$

Solution

Part (a) Evaluating the cross product $\vec{A} \times \vec{B}$ using the determinant provides

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 8 & 4 \\ 1 & 18 & -6 \end{vmatrix} \text{mm}^2 \tag{1}$$

$$= \left\{ \hat{\imath}\ [(8)(-6) - (4)(18)] - \hat{\jmath}\ [(-1)(-6) - (4)(1)] + \hat{k}\ [(-1)(18) - (8)(1)] \right\}\ \text{mm}^2 \tag{2}$$

$$= (-120\,\hat{\imath} - 2\,\hat{\jmath} - 26\,\hat{k})\,\text{mm}^2. \tag{3}$$

Part (b) Evaluating the cross product $\vec{B} \times \vec{A}$ using the determinant provides

$$\vec{B} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 18 & -6 \\ -1 & 8 & 4 \end{vmatrix} \text{mm}^2 \tag{4}$$

$$= \left\{ \hat{\imath}\ [(18)(4) - (-6)(8)] - \hat{\jmath}\ [(1)(4) - (-6)(-1)] + \hat{k}\ [(1)(8) - (18)(-1)] \right\}\ \text{mm}^2 \tag{5}$$

$$= (120\,\hat{\imath} + 2\,\hat{\jmath} + 26\,\hat{k})\,\text{mm}^2. \tag{6}$$

Part (c) Comparing the results of Parts (a) and (b), observe that $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are vectors with equal magnitude but opposite direction.

Part (d) Consider the vector $\vec{C} = \vec{A} \times \vec{B}$, where from Part (a), $\vec{C} = (-120\,\hat{\imath} - 2\,\hat{\jmath} - 26\,\hat{k})\,\text{mm}^2$. Noting that two vectors are orthogonal if the dot product between them is zero, we evaluate the following dot products

$$\vec{C} \cdot \vec{A} = (-120\,\text{mm}^2)(-1\,\text{mm}) + (-2\,\text{mm}^2)(8\,\text{mm}) + (-26\,\text{mm}^2)(4\,\text{mm}) = 0, \tag{7}$$

$$\vec{C} \cdot \vec{B} = (-120\,\text{mm}^2)(1\,\text{mm}) + (-2\,\text{mm}^2)(18\,\text{mm}) + (-26\,\text{mm}^2)(-6\,\text{mm}) = 0. \tag{8}$$

Thus,

$$\boxed{\vec{C} \text{ is orthogonal to both } \vec{A} \text{ and } \vec{B}.} \tag{9}$$

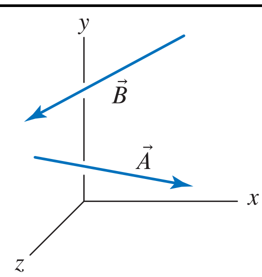
Problem 2.147

(a) Evaluate $\vec{A} \times \vec{B}$.

(b) Evaluate $\vec{B} \times \vec{A}$.

(c) Comment on any differences between the results of Parts (a) and (b).

(d) Use the dot product to show the result of Part (a) is orthogonal to vectors $\vec{A}$ and $\vec{B}$.



$\vec{A} = (6\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k})$ in.
$\vec{B} = (-14\,\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k})$ in.

Solution

Part (a) Evaluating the cross product $\vec{A} \times \vec{B}$ using the determinant provides

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & -2 & 3 \\ -14 & -2 & 5 \end{vmatrix} \text{in.}^2 \tag{1}$$

$$= \Big\{\hat{\imath}\ [(-2)(5) - (3)(-2)] - \hat{\jmath}\ [(6)(5) - (3)(-14)] + \hat{k}\ [(6)(-2) - (-2)(-14)]\Big\} \text{ in.}^2 \tag{2}$$

$$= (-4\,\hat{\imath} - 72\,\hat{\jmath} - 40\,\hat{k})\,\text{in.}^2. \tag{3}$$

Part (b) Evaluating the cross product $\vec{B} \times \vec{A}$ using the determinant provides

$$\vec{B} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -14 & -2 & 5 \\ 6 & -2 & 3 \end{vmatrix} \text{in.}^2 \tag{4}$$

$$= \Big\{\hat{\imath}\ [(-2)(3) - (5)(-2)] - \hat{\jmath}\ [(-14)(3) - (5)(6)] + \hat{k}\ [(-14)(-2) - (-2)(6)]\Big\} \text{ in.}^2 \tag{5}$$

$$= (4\,\hat{\imath} + 72\,\hat{\jmath} + 40\,\hat{k})\,\text{in.}^2. \tag{6}$$

Part (c) Comparing the results of Parts (a) and (b), observe that $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are vectors with equal magnitude but opposite direction.

Part (d) Consider the vector $\vec{C} = \vec{A} \times \vec{B}$, where from Part (a), $\vec{C} = (-4\,\hat{\imath} - 72\,\hat{\jmath} - 40\,\hat{k})\,\text{in.}^2$. Noting that two vectors are orthogonal if the dot product between them is zero, we evaluate the following dot products

$$\vec{C} \cdot \vec{A} = (-4\,\text{in.}^2)(6\,\text{in.}) + (-72\,\text{in.}^2)(-2\,\text{in.}) + (-40\,\text{in.}^2)(3\,\text{in.}) = 0, \tag{7}$$

$$\vec{C} \cdot \vec{B} = (-4\,\text{in.}^2)(-14\,\text{in.}) + (-72\,\text{in.}^2)(-2\,\text{in.}) + (-40\,\text{in.}^2)(5\,\text{in.}) = 0. \tag{8}$$

Thus,

$$\boxed{\vec{C} \text{ is orthogonal to both } \vec{A} \text{ and } \vec{B}.} \tag{9}$$

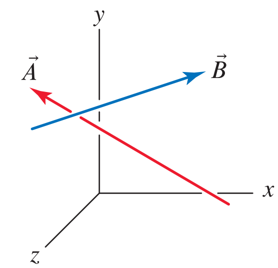
Problem 2.148

(a) Evaluate $\vec{A} \times \vec{B}$.

(b) Evaluate $\vec{B} \times \vec{A}$.

(c) Comment on any differences between the results of Parts (a) and (b).

(d) Use the dot product to show the result of Part (a) is orthogonal to vectors $\vec{A}$ and $\vec{B}$.



$\vec{A} = (-12\,\hat{\imath} + 16\,\hat{\jmath} - 15\,\hat{k})$ lb
$\vec{B} = (18\,\hat{\imath} + 6\,\hat{\jmath} - 13\,\hat{k})$ in.

Solution

Part (a) Evaluating the cross product $\vec{A} \times \vec{B}$ yields

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -12 & 16 & -15 \\ 18 & 6 & -13 \end{vmatrix} \text{in.·lb} \tag{1}$$

$$= \Big\{ \hat{\imath} \big[(16)(-13) - (-15)(6)\big] - \hat{\jmath} \big[(-12)(-13) - (-15)(18)\big] + \hat{k} \big[(-12)(6) - (16)(18)\big] \Big\} \text{ in.·lb} \tag{2}$$

$$= \boxed{(-118\,\hat{\imath} - 426\,\hat{\jmath} - 360\,\hat{k}) \text{ in.·lb.}} \tag{3}$$

Part (b) Evaluating the cross product $\vec{B} \times \vec{A}$ yields

$$\vec{B} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 18 & 6 & -13 \\ -12 & 16 & -15 \end{vmatrix} \text{in.·lb} \tag{4}$$

$$= \Big\{ \hat{\imath} \big[(6)(-15) - (-13)(16)\big] - \hat{\jmath} \big[(18)(-15) - (-13)(-12)\big] + \hat{k} \big[(18)(16) - (6)(-12)\big] \Big\} \text{ in.·lb} \tag{5}$$

$$= \boxed{(118\,\hat{\imath} + 426\,\hat{\jmath} + 360\,\hat{k}\,) \text{ in.·lb.}} \tag{6}$$

Part (c) Comparing the results of parts (a) and (b), observe that $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are vectors with equal magnitude but opposite direction.

Part (d) Consider the vector $\vec{C} = \vec{A} \times \vec{B}$, where from Part (a), $\vec{C} = (-118\,\hat{\imath} - 426\,\hat{\jmath} - 360\,\hat{k})$ in.·lb. Noting that two vectors are orthogonal if the dot product between them is zero, we evaluate the following dot products

$$\vec{C} \cdot \vec{A} = (-118 \text{ in.·lb})(-12 \text{ lb}) + (-426 \text{ in.·lb})(16 \text{ lb}) + (-360 \text{ in.·lb})(-15 \text{ lb}) = 0, \tag{7}$$

$$\vec{C} \cdot \vec{B} = (-118 \text{ in.·lb})(18 \text{ in}) + (-426 \text{ in.·lb})(6 \text{ in}) + (-360 \text{ in.·lb})(-13 \text{ in.}) = 0. \tag{8}$$

Thus,

$$\boxed{\text{The vector } \vec{C} = \vec{A} \times \vec{B} \text{ is orthogonal to both } \vec{A} \text{ and } \vec{B}.}$$

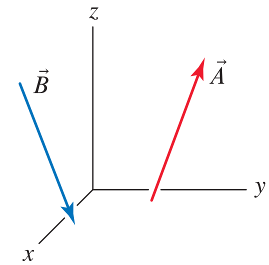
Problem 2.149

(a) Evaluate $\vec{A} \times \vec{B}$.

(b) Evaluate $\vec{B} \times \vec{A}$.

(c) Comment on any differences between the results of Parts (a) and (b).

(d) Use the dot product to show the result of Part (a) is orthogonal to vectors $\vec{A}$ and $\vec{B}$.



$\vec{A} = (4\,\hat{\imath} + 3\,\hat{\jmath} + 12\,\hat{k})\,\mathrm{N}$
$\vec{B} = (-2\,\hat{\imath} + 5\,\hat{\jmath} - 14\,\hat{k})\,\mathrm{m}$

Solution

Part (a) Evaluating the cross product $\vec{A} \times \vec{B}$ yields

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 3 & 12 \\ -2 & 5 & -14 \end{vmatrix} \mathrm{N{\cdot}m} \tag{1}$$

$$= \Big\{ \hat{\imath}\,\big[(3)(-14) - (12)(5)\big] - \hat{\jmath}\,\big[(4)(-14) - (12)(-2)\big] + \hat{k}\,\big[(4)(5) - (3)(-2)\big] \Big\}\ \mathrm{N{\cdot}m} \tag{2}$$

$$= \boxed{(-102\,\hat{\imath} + 32\,\hat{\jmath} + 26\,\hat{k})\ \mathrm{N{\cdot}m}.} \tag{3}$$

Part (b) Evaluating the cross product $\vec{B} \times \vec{A}$ yields

$$\vec{B} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 5 & -14 \\ 4 & 3 & 12 \end{vmatrix} \mathrm{N{\cdot}m} \tag{4}$$

$$= \Big\{ \hat{\imath}\,\big[(5)(12) - (-14)(3)\big] - \hat{\jmath}\big[(-2)(12) - (-14)(4)\big] + \hat{k}\,\big[(-2)(3) - (5)(4)\big] \Big\}\ \mathrm{N{\cdot}m}. \tag{5}$$

$$= \boxed{(102\,\hat{\imath} - 32\,\hat{\jmath} - 26\,\hat{k})\,\mathrm{N{\cdot}m}.} \tag{6}$$

Part (c) Comparing the results of Parts (a) and (b), observe that $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are vectors with equal magnitude but opposite direction.

Part (d) Consider the vector $\vec{C} = \vec{A} \times \vec{B}$, where from Part (a), $\vec{C} = (-102\,\hat{\imath} + 32\,\hat{\jmath} + 26\,\hat{k})\,\mathrm{N{\cdot}m}$. Noting that two vectors are orthogonal if the dot product between them is zero, we evaluate the following dot products

$$\vec{C} \cdot \vec{A} = (-102\ \mathrm{N{\cdot}m})(4\ \mathrm{N}) + (32\ \mathrm{N{\cdot}m})(3\ \mathrm{N}) + (26\ \mathrm{N{\cdot}m})(12\ \mathrm{N}) = 0, \tag{7}$$

$$\vec{C} \cdot \vec{B} = (-102\ \mathrm{N{\cdot}m})(-2\ \mathrm{m}) + (32\ \mathrm{N{\cdot}m})(5\ \mathrm{N}) + (26\ \mathrm{N{\cdot}m})(-14\ \mathrm{m}) = 0. \tag{8}$$

Thus,

$$\boxed{\text{The vector } \vec{C} = \vec{A} \times \vec{B} \text{ is orthogonal to both } \vec{A} \text{ and } \vec{B}.}$$

Problem 2.150

> Describe how the cross product operation can be used to determine (or "test") whether two vectors $\vec{A}$ and $\vec{B}$ are orthogonal. Is this test as easy to use as the test based on the dot product? Explain, perhaps using an example to support your remarks. **Note:** Concept problems are about *explanations,* not computations.

Solution

Consider the following relation:

$$\vec{C} = \vec{A} \times \vec{B} = AB \sin\theta\, \hat{u}. \tag{1}$$

If the vectors $\vec{A}$ and $\vec{B}$ are orthogonal, then $\theta = 90°$ and $\sin\theta = 1$. Therefore, the following test of orthogonality, which consists of three steps, may be used:

1) Compute the vector $\vec{C}$ using $\vec{C} = \vec{A} \times \vec{B}$.

2) Evaluate the magnitudes A, B, and C.

3) If $C = AB$, then the vectors are orthogonal; if $C \neq AB$, then the vectors are not orthogonal.

Using the dot product to test for orthogonality requires only two steps:

1) Compute the dot product $\vec{A} \cdot \vec{B}$.

2) If $\vec{A} \cdot \vec{B} = 0$, then the vectors are orthogonal; if $\vec{A} \cdot \vec{B} \neq 0$, then the vectors are not orthogonal.

Furthermore, the dot product is usually easier to evaluate than the cross product. Hence, the dot product test is easier to use compared to the cross product test described above.

Problem 2.151

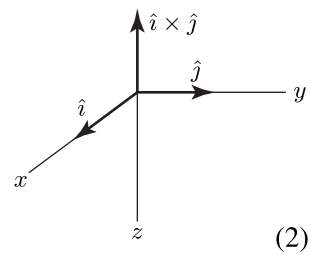
Imagine a left-hand coordinate system has inadvertently been used for a problem. That is, if the x and y directions have been selected first, the z direction has been taken in the wrong direction for a right-hand coordinate system. What consequences will this have for dot products and cross products? Perhaps use an example to support your remarks.

Solution

If a left-hand coordinate system is used, the dot product may still be employed and all remarks made in the book regarding its use and interpretation are still applicable. To see why, consider

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z. \tag{1}$$

If the z direction has been taken incorrectly for a right-hand system, the signs on both A_z and B_z are changed—if both are changed, there will be no net change in the resulting dot product. For the cross product, vector directions given by the determinant will be wrong. To see why, consider the left-hand coordinate system shown in the figure to the right. Using the basic definition of the cross product to evaluate $\hat{\imath} \times \hat{\jmath}$, we write



$$\hat{\imath} \times \hat{\jmath} = |\hat{\imath}|\ |\hat{\jmath}| \cos\theta\, \hat{u}, \tag{2}$$

where $\hat{u}$ is a unit vector given by the right-hand rule; according to this definition, $\hat{\imath} \times \hat{\jmath}$ is a vector that points upward (in the $-z$ direction) in the figure.

Evaluating $\hat{\imath} \times \hat{\jmath}$ using the determinant method provides

$$\hat{\imath} \times \hat{\jmath} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{\imath}\,(0) - \hat{\jmath}\,(0) + \hat{k}\,[(1)(1) - 0] = \hat{k}. \tag{3}$$

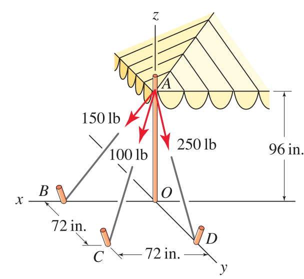
where $\hat{k}$, by definition, is a unit vector in the positive z direction. Therefore, for Eq.(3) to give proper directions, and hence agree with Eq. (2), a right-hand coordinate system must be used.

Problem 2.152

The corner of a tent is supported using three ropes having the forces shown. We wish to compute the sum of the cross products $\vec{M}_O = \vec{r}_{OA} \times \vec{F}_{AB} + \vec{r}_{OA} \times \vec{F}_{AC} + \vec{r}_{OA} \times \vec{F}_{AD}$ where $\vec{r}_{OA}$ is the position vector from points O to A, $\vec{F}_{AB}$ is the force directed from points A to B, and so on.

(a) Rather than compute three separate cross products to find $\vec{M}_O$, do the properties of the cross product permit $\vec{M}_O$ to be found using just one cross product? Explain.

(b) Determine $\vec{M}_O$.



Solution

Part (a) As specified in the problem statement, the vector $\vec{M}_O$ is given by

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F}_{AB} + \vec{r}_{OA} \times \vec{F}_{AC} + \vec{r}_{OA} \times \vec{F}_{AD}. \tag{1}$$

Rather than evaluate Eq. (1), the distributive property of the cross product may be used to rewrite Eq. (1) as

$$\boxed{\vec{M}_O = \vec{r}_{OA} \times (\vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD}).} \tag{2}$$

Because Eq. (2) requires the evaluation of only one cross product, it will probably be more convenient to use than Eq. (1) for determining $\vec{M}_O$.

Part (b) To carry out the evaluation for $\vec{M}_O$, either Eq. (1) or Eq. (2) may be used, and we will use Eq. (2). Expressions for the three force vectors are

$$\vec{F}_{AB} = 150\,\text{lb}\left(\frac{72\,\hat{\imath} - 96\,\hat{k}}{\sqrt{(72)^2 + (-96)^2}}\right) = (90.00\,\hat{\imath} - 120.0\,\hat{k})\,\text{lb}, \tag{3}$$

$$\vec{F}_{AC} = 100\,\text{lb}\left(\frac{72\,\hat{\imath} + 72\,\hat{\jmath} - 96\,\hat{k}}{\sqrt{(72)^2 + (72)^2 + (-96)^2}}\right) = (51.45\,\hat{\imath} + 51.45\,\hat{\jmath} - 68.60\,\hat{k})\,\text{lb}, \tag{4}$$

$$\vec{F}_{AD} = 250\,\text{lb}\left(\frac{72\,\hat{\jmath} - 96\,\hat{k}}{\sqrt{(72)^2 + (-96)^2}}\right) = (150.0\,\hat{\jmath} - 200.0\,\hat{k})\,\text{lb}, \tag{5}$$

such that

$$\vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} = (141.4\,\hat{\imath} + 201.4\,\hat{\jmath} - 388.6\,\hat{k})\,\text{lb}. \tag{6}$$

The position vector $\vec{r}_{OA} = 96\,\text{in.}\,\hat{k}$. Therefore, using Eq. (2), we obtain

$$\vec{M}_O = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 96 \\ 141.4 & 201.4 & -388.6 \end{vmatrix} \text{in.}\cdot\text{lb} \tag{7}$$

$$= \Big\{[0 - (96)(201.4)]\,\hat{\imath} - [0 - (96)(141.4)]\,\hat{\jmath} + [0 - 0]\,\hat{k}\Big\}\ \text{in.}\cdot\text{lb} \tag{8}$$

$$= (-19300\,\hat{\imath} + 13600\,\hat{\jmath})\,\text{in.}\cdot\text{lb} \tag{9}$$

Problem 2.153

For the triangular-shaped window of Prob. 2.109 on p. 95, use the cross product to determine the outward normal unit vector (i.e., pointing away from the origin) and the area of the window.

Solution

Our strategy will be to determine the outward normal vector $\vec{n}$ to the window by taking the cross product between two vectors that lie along the edges of the window. Among the several vectors that may be used, we will employ the following

$$\vec{r}_{AB} = \Big[(-25-35)\,\hat{\imath} + (30-25)\,\hat{\jmath} + (105-90)\,\hat{k}\Big]\,\text{cm} = (-60\,\hat{\imath} + 5\,\hat{\jmath} + 15\,\hat{k})\,\text{cm}, \tag{1}$$

$$\vec{r}_{AC} = \Big[(15-35)\,\hat{\imath} + (60-25)\,\hat{\jmath} + (80-90)\,\hat{k}\Big]\,\text{cm} = (-20\,\hat{\imath} + 35\,\hat{\jmath} - 10\,\hat{k})\,\text{cm}. \tag{2}$$

To obtain an outward normal vector (as opposed to an inward normal vector), we examine the figure in Prob. 2.109, and apply the right-hand rule, to determine that the vectors in the cross product must be taken in the order $\vec{r}_{AC}$ first and $\vec{r}_{AB}$ second, as follows

$$\vec{n} = \vec{r}_{AC} \times \vec{r}_{AB} \tag{3}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -20 & 35 & -10 \\ -60 & 5 & 15 \end{vmatrix}\,\text{cm}^2 \tag{4}$$

$$= \Big\{[(35)(15) - (-10)(5)]\,\hat{\imath} - [(-20)(15) - (-10)(-60)]\,\hat{\jmath} + [(-20)(5) - (35)(-60)]\,\hat{k}\Big\}\,\text{cm}^2 \tag{5}$$

$$= (575\,\hat{\imath} + 900\,\hat{\jmath} + 2000\,\hat{k})\,\text{cm}^2, \tag{6}$$

$$n = \sqrt{(575\,\text{cm})^2 + (900\,\text{cm})^2 + (2000\,\text{cm})^2} = 2267\,\text{cm}^2. \tag{7}$$

We may confirm that the result in Eq. (6) is indeed an outward normal vector by verifying that the components in Eq. (6) correspond to the outward direction from the window (i.e., away from the origin of the coordinate system). Using Eqs. (6) and (7), the outward normal unit vector is then given by

$$\boxed{\hat{u} = \frac{\vec{n}}{n} = \frac{(575\,\hat{\imath} + 900\,\hat{\jmath} + 2000\,\hat{k})\,\text{cm}^2}{2267\,\text{cm}^2} = 0.254\,\hat{\imath} + 0.397\,\hat{\jmath} + 0.882\,\hat{k}.} \tag{8}$$

The magnitude of the cross product $\vec{r}_{AC} \times \vec{r}_{AB}$ is the area of a parallelogram, which is twice the area of the triangular window ABC. Thus, using Eq. (7), we find that the area A_{ABC} of the window is

$$\boxed{A_{ABC} = \frac{2267\,\text{cm}^2}{2} = 1130\,\text{cm}^2.} \tag{9}$$

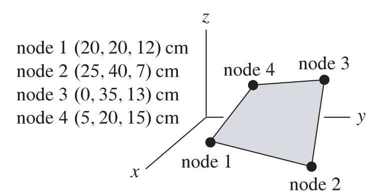
Problem 2.154

A flat quadrilateral plate finite element is shown (see Example 2.21 on p. 110 for a description of what a finite element is).

(a) Describe and perform a test, using two cross products that will verify if all four nodes (corners) lie in the same plane.

(b) Determine the unit outward normal direction to the surface (i.e., pointing away from the origin).

(c) Determine the surface area of the element.



Solution

Part (a) We will begin by computing two vectors that are normal to the surface: $\vec{n}_1 = \vec{r}_{12} \times \vec{r}_{14}$ and $\vec{n}_2 = \vec{r}_{34} \times \vec{r}_{32}$. If $\vec{n}_1/n_1$ equals $\vec{n}_2/n_2$, that is, if the unit normal vectors are identical, then the plane formed by points 1, 2, and 4 and the plane formed by points 2, 3, and 4 are the same and hence all four corners of the plate finite element will lie in the same plane. The four position vectors to be used are

$$\vec{r}_{12} = \Big[(25-20)\,\hat{\imath} + (40-20)\,\hat{\jmath} + (7-12)\,\hat{k}\Big]\text{ cm} = (5\,\hat{\imath} + 20\,\hat{\jmath} - 5\,\hat{k})\text{ cm}, \tag{1}$$

$$\vec{r}_{14} = \Big[(5-20)\,\hat{\imath} + (20-20)\,\hat{\jmath} + (15-12)\,\hat{k}\Big]\text{ cm} = (-15\,\hat{\imath} + 0\,\hat{\jmath} + 3\,\hat{k})\text{ cm}, \tag{2}$$

$$\vec{r}_{32} = \Big[(25-0)\,\hat{\imath} + (40-35)\,\hat{\jmath} + (7-13)\,\hat{k}\Big]\text{ cm} = (25\,\hat{\imath} + 5\,\hat{\jmath} - 6\,\hat{k})\text{ cm}, \tag{3}$$

$$\vec{r}_{34} = \Big[(5-0)\,\hat{\imath} + (20-35)\,\hat{\jmath} + (15-13)\,\hat{k}\Big]\text{ cm} = (5\,\hat{\imath} - 15\,\hat{\jmath} + 2\,\hat{k})\text{ cm}. \tag{4}$$

The normal direction vectors discussed above are given by

$$\vec{n}_1 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 20 & -5 \\ -15 & 0 & 3 \end{vmatrix}\text{ cm}^2 \tag{5}$$

$$= \Big\{[(20)(3) - (-5)(0)]\,\hat{\imath} - [(5)(3) - (-5)(-15)]\,\hat{\jmath} + [(5)(0) - (20)(-15)]\,\hat{k}\Big\}\text{ cm}^2 \tag{6}$$

$$= (60\,\hat{\imath} + 60\,\hat{\jmath} + 300\,\hat{k})\text{ cm}^2, \tag{7}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & -15 & 2 \\ 25 & 5 & -6 \end{vmatrix}\text{ cm}^2 \tag{8}$$

$$= \Big\{[(-15)(-6) - (2)(5)]\,\hat{\imath} - [(5)(-6) - (2)(25)]\,\hat{\jmath} + [(5)(5) - (-15)(25)]\,\hat{k}\Big\}\text{ cm}^2 \tag{9}$$

$$= (80\,\hat{\imath} + 80\,\hat{\jmath} + 400\,\hat{k})\text{ cm}^2, \tag{10}$$

where the magnitudes of the above vectors are

$$n_1 = \sqrt{(60)^2 + (60)^2 + (300)^2}\text{ cm}^2 = 311.8\text{ cm}^2, \quad n_2 = \sqrt{(80)^2 + (80)^2 + (400)^2}\text{ cm}^2 = 415.7\text{ cm}^2. \tag{11}$$

The unit vectors corresponding to $\vec{n}_1$ and $\vec{n}_2$ are

$$\hat{u}_1 = \frac{\vec{n}_1}{n_1} = \frac{(60\,\hat{\imath} + 60\,\hat{\jmath} + 300\,\hat{k})\text{ cm}^2}{311.8\text{ cm}^2} = 0.1925\,\hat{\imath} + 0.1925\,\hat{\jmath} + 0.9623\,\hat{k}, \tag{12}$$

$$\hat{u}_2 = \frac{\vec{n}_2}{n_2} = \frac{(80\,\hat{\imath} + 80\,\hat{\jmath} + 400\,\hat{k})\,\mathrm{cm}^2}{415.7\,\mathrm{cm}^2} = 0.1925\,\hat{\imath} + 0.1925\,\hat{\jmath} + 0.9623\,\hat{k}. \tag{13}$$

Comparing Eqs. (12) and (13), we see that $\hat{u}_1 = \hat{u}_2$, and therefore

$$\boxed{\text{all the nodes (1,2,3,4) lie in the same plane.}}$$

Part (b) The outward normal direction is $\hat{u} = \hat{u}_1 = \hat{u}_2$, hence

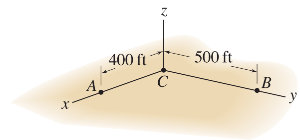
$$\boxed{\hat{u} = 0.192\,\hat{\imath} + 0.192\,\hat{\jmath} + 0.962\,\hat{k}.} \tag{14}$$

Part (c) The magnitude of $\vec{n}_1$, which is given by n_1 in Eq. (11), gives the area of the parallelogram with three of the corners given by points 1, 2, and 4. Similarly, the magnitude of $\vec{n}_2$, which is given by n_2 in Eq. (11), gives the area of the parallelogram with three of the corners given by points 2, 3, and 4. The area of the triangle 1-2-4 is given by $n_1/2$ and the area of the triangle 2-3-4 is given by $n_2/2$. Hence, the surface area of the quadrilateral plate element is given by

$$\boxed{A_{1234} = \frac{n_1}{2} + \frac{n_2}{2} = \frac{311.8\,\mathrm{cm}^2}{2} + \frac{415.7\,\mathrm{cm}^2}{2} = 364\,\mathrm{cm}^2.} \tag{15}$$

Problem 2.155

A contractor is required to prepare a soil surface so that it is planar and the normal direction to the surface is within 0.5° of vertical. If the elevations at points A and B, relative to the elevation at point C, are -3.3 ft and -4.2 ft, respectively, determine the angle between the soil surface's normal direction and the vertical, and if the surface is sufficiently level.



Solution

Our strategy will be to use the cross product to determine the normal direction to surface ABC, $\vec{n} = \vec{r}_{CA} \times \vec{r}_{CB}$. We will then take the dot product of $\vec{n}$ with $\hat{k}$ to determine the angle between $\vec{n}$ and $\hat{k}$.

$$\vec{r}_{CA} = (400\,\hat{\imath} - 3.3\,\hat{k})\text{ ft}, \tag{1}$$

$$\vec{r}_{CB} = (500\,\hat{\jmath} - 4.2\,\hat{k})\text{ ft}, \tag{2}$$

$$\vec{n} = \vec{r}_{CA} \times \vec{r}_{CB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 400 & 0 & -3.3 \\ 0 & 500 & -4.2 \end{vmatrix} \text{ft}^2 \tag{3}$$

$$= \Big\{ \hat{\imath}\,\big[(0)(-4.2) - (-3.3)(500)\big] - \hat{\jmath}\,\big[(400)(-4.2) - (-3.3)(0)\big] + \hat{k}\,\big[(400)(500) - (0)(0)\big]\Big\}\text{ ft}^2 \tag{4}$$

$$= (1650\,\hat{\imath} + 1680\,\hat{\jmath} + 200{,}000\,\hat{k})\text{ ft}^2, \tag{5}$$

$$n = |\vec{n}| = \sqrt{(1650)^2 + (1680)^2 + (200{,}000)^2}\text{ ft}^2 = 200{,}014\text{ ft}^2. \tag{6}$$

The angle θ between $\vec{n}$ and the vertical direction (z axis) can be obtained using the dot product of $\vec{n}$ with a vector in the z direction, namely $\hat{k}$. Thus,

$$\theta = \cos^{-1}\frac{\vec{n}\cdot\hat{k}}{n(1)} \tag{7}$$

$$= \cos^{-1}\frac{(1650\text{ ft}^2)(0) + (1680\text{ ft}^2)(0) + (200{,}000\text{ ft}^2)(1)}{(200{,}014\text{ ft}^2)(1)} \tag{8}$$

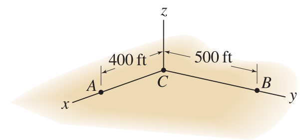
$$= \boxed{0.6746^\circ.} \tag{9}$$

Thus,

$$\boxed{\text{The soil surface is not sufficiently level.}}$$

Problem 2.156

Repeat Prob. 2.155 if the elevations at points A and B, relative to the elevation at point C, are 2.6 ft and −3.1 ft, respectively.



Solution

Our strategy will be to use the cross product to determine the normal direction to surface ABC, $\vec{n} = \vec{r}_{CA} \times \vec{r}_{CB}$. We will then take the dot product of $\vec{n}$ with $\hat{k}$ to determine the angle between $\vec{n}$ and $\hat{k}$.

$$\vec{r}_{CA} = (400\,\hat{\imath} + 2.6\,\hat{k})\text{ ft}, \tag{1}$$
$$\vec{r}_{CB} = (500\,j - 3.1\,\hat{k})\text{ ft}, \tag{2}$$
$$\vec{n} = \vec{r}_{CA} \times \vec{r}_{CB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 400 & 0 & 2.6 \\ 0 & 500 & -3.1 \end{vmatrix} \text{ft}^2 \tag{3}$$
$$= \Big\{ \hat{\imath}\,\big[(0)(-3.1) - (2.6)(500)\big] - \hat{\jmath}\,\big[(400)(-3.1) - (2.6)(0)\big] + \hat{k}\,\big[(400)(500) - (0)(0)\big]\Big\}\text{ ft}^2 \tag{4}$$
$$= (-1300\,\hat{\imath} + 1240\,\hat{\jmath} + 200{,}000\,\hat{k})\text{ ft}^2, \tag{5}$$
$$n = \left|\vec{n}\right| = \sqrt{(-1300)^2 + (1240)^2 + (200{,}000)^2}\text{ ft}^2 = 200{,}008\text{ ft.}^2 \tag{6}$$

The angle θ between $\vec{n}$ and the vertical direction (z axis) can be obtained using the dot product of $\vec{n}$ with a vector in the z direction, namely $\hat{k}$. Thus,

$$\theta = \cos^{-1}\frac{\vec{n}\cdot\hat{k}}{n(1)} \tag{7}$$
$$= \cos^{-1}\frac{(-800\text{ ft}^2)(0) + (1240\text{ ft}^2)(0) + (200{,}000\text{ ft}^2)(1)}{(200,008\text{ ft}^2)(1)} \tag{8}$$
$$= \boxed{0.5147^\circ.} \tag{9}$$

Thus,

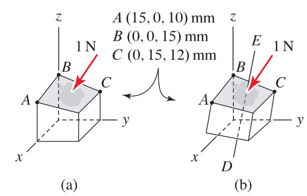
$$\boxed{\text{The soil surface is not sufficiently level.}}$$

Problem 2.157

An ergonomically designed key for a computer keyboard has an approximately flat surface defined by points A, B, and C and is subjected to a 1 N force in the direction normal to the key's surface.

(a) In Fig. P2.157(a), motion of the key is in the z direction. Determine the components of the force in directions normal and tangent to the key's motion. Comment on why it might be important to know these components.

(b) In Fig. P2.157(b), the switch mechanism is repositioned so that the motion of the key is in the direction of the line connecting points D and E, where this line has x, y, and z direction cosines of 0.123, 0.123, and 0.985, respectively. Determine the components of the force in directions normal and tangent to the key's motion. Comment on why this design might be more effective than that in Part (a).



Solution

Part (a) To determine the vector representation for the 1 N force, we must determine the direction of this force, which is normal to the surface of the key. Thus, we begin by determining the normal direction to the key, which is defined by the plane containing points A, B, and C; to accomplish this the following vectors will be used

$$\vec{r}_{BA} = \Big[(15-0)\,\hat{\imath} + (0-0)\,\hat{\jmath} + (10-15)\,\hat{k}\Big]\,\text{mm} = (15\,\hat{\imath} + 0\,\hat{\jmath} - 5\,\hat{k})\,\text{mm}, \tag{1}$$

$$\vec{r}_{BC} = \Big[(0-0)\,\hat{\imath} + (15-0)\,\hat{\jmath} + (12-15)\,\hat{k}\Big]\,\text{mm} = (0\,\hat{\imath} + 15\,\hat{\jmath} - 3\,\hat{k})\,\text{mm}. \tag{2}$$

A vector that is normal to the surface of the key is given by

$$\vec{n} = \vec{r}_{BA} \times \vec{r}_{BC} \tag{3}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 15 & 0 & -5 \\ 0 & 15 & -3 \end{vmatrix}\,\text{mm}^2 \tag{4}$$

$$= \Big\{[(0)(-3) - (-5)(15)]\,\hat{\imath} - [(15)(-3) - (-5)(0)]\,\hat{\jmath} + [(15)(15) - (0)(0)]\,\hat{k}\Big\}\,\text{mm}^2 \tag{5}$$

$$= (75\,\hat{\imath} + 45\,\hat{\jmath} + 225\,\hat{k})\,\text{mm}^2, \tag{6}$$

$$n = \sqrt{(75)^2 + (-45)^2 + (225)^2}\,\text{mm}^2 = 241.4\,\text{mm}^2, \tag{7}$$

where $\vec{n}$ points upward from the surface of the key (i.e., positive z component). The vector representation of the force is then

$$\vec{F} = (-1\,\text{N})\,\frac{\vec{n}}{n} = (-1\,\text{N})\frac{(75\,\hat{\imath} + 45\,\hat{\jmath} + 225\,\hat{k})\,\text{mm}^2}{241.4\,\text{mm}^2} = (-0.3107\,\hat{\imath} - 0.1864\,\hat{\jmath} - 0.9321\,\hat{k})\,\text{N}. \tag{8}$$

Due to the 1 N force, the motion of the key is in the $-z$ direction. Therefore, the component of the force in this direction (i.e., tangent to the direction of motion) is given by the negative of the z component of $\vec{F}$ in Eq. (8), (i.e., the dot product of $\vec{F}$ with $-\hat{k}$) which is

$$\boxed{F_{\parallel} = 0.932\,\text{N}.} \tag{9}$$

The component of force normal to this direction is obtained using the Pythagorean theorem

$$\boxed{F_\perp = \sqrt{(1\,\mathrm{N})^2 - (-0.9321\,\mathrm{N})^2} = 0.362\,\mathrm{N}.} \tag{10}$$

Both $F_\parallel$ and $F_\perp$ are important to know for purposes of designing the key. In particular, $F_\parallel$ is the portion of the 1 N force that causes motion of the key.

Part (b) The direction of the key's motion is $\vec{r}_{ED}$, where $\vec{r}_{ED} = -\vec{r}_{DE}$. Thus

$$\begin{aligned}\vec{r}_{ED} &= -\vec{r}_{DE} = -(0.123\,\hat{\imath} + 0.123\,\hat{\jmath} + 0.985\,\hat{k}) \\ &= -0.123\,\hat{\imath} - 0.123\,\hat{\jmath} - 0.985\,\hat{k}.\end{aligned} \tag{11}$$

With $\vec{F}$ given by Eq. (8), the component of the force tangent (parallel) to $\vec{r}_{ED}$ is

$$F_\parallel = \vec{F}\cdot\frac{\vec{r}_{ED}}{r_{ED}} = (-0.3107\,\hat{\imath} - 0.1864\,\hat{\jmath} - 0.9321\,\hat{k})\,\mathrm{N}\cdot(-0.123\,\hat{\imath} - 0.123\,\hat{\jmath} - 0.985\,\hat{k}) = 0.9792\,\mathrm{N}. \tag{12}$$

The component of force normal to this direction is obtained using the Pythagorean theorem

$$F_\perp = \sqrt{(1\,\mathrm{N})^2 - (0.9792\,\mathrm{N})^2} = 0.2028\,\mathrm{N}. \tag{13}$$

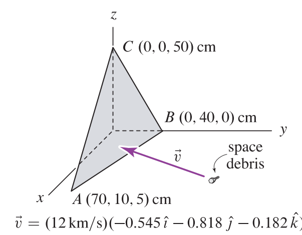
Thus, the components are given by

$$\boxed{F_\parallel = -0.979\,\mathrm{N}, \quad F_\perp = 0.203\,\mathrm{N}.} \tag{14}$$

This design may be more effective than that given in Part (a) for two reasons. First, the tangential component of the force in Part (b) is larger, meaning that a greater force is available to cause motion of the key. Second, the normal component of the force for Part (b) is approximately one-half of the normal component of the force found in Part (a). Note that a large value of the normal component of the force is probably not desirable since it does not help produce motion of the key and it may lead to wear of the key.

Problem 2.158

Impact of debris, both natural and artificial, is a significant hazard for spacecraft. Space Shuttle windows are routinely replaced because of damage due to impact with small objects, and recent flights have employed evasive maneuvers to avoid impact with larger objects, whose orbits NASA constantly monitors. For the triangular-shaped window and the relative velocity of approach $\vec{v}$ shown, determine the components and vector components of the velocity in directions normal and tangent to the window. Note that this information is needed before an analysis of damage due to impact can be performed.



Solution

Our strategy will be to determine the outward* normal vector $\vec{n}$ to the window by taking the cross product between two vectors that lie along the edges of the window. Among the several vectors that may be used, we will employ the following

$$\vec{r}_{AB} = \Big[(0-70)\,\hat{\imath} + (40-10)\,\hat{\jmath} + (0-5)\,\hat{k}\Big]\,\mathrm{cm} = (-70\,\hat{\imath} + 30\,\hat{\jmath} - 5\,\hat{k})\,\mathrm{cm}, \tag{1}$$

$$\vec{r}_{AC} = \Big[(0-70)\,\hat{\imath} + (0-10)\,\hat{\jmath} + (50-5)\,\hat{k}\Big]\,\mathrm{cm} = (-70\,\hat{\imath} - 10\,\hat{\jmath} + 45\,\hat{k})\,\mathrm{cm}. \tag{2}$$

To obtain an outward normal vector, we examine the figure in the problem statement, and apply the right-hand rule, to determine that the vectors in the cross product must be taken in the order $\vec{r}_{AB}$ first and $\vec{r}_{AC}$ second, as follows

$$\vec{n} = \vec{r}_{AB} \times \vec{r}_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -70 & 30 & -5 \\ -70 & -10 & 45 \end{vmatrix}\,\mathrm{cm}^2 \tag{3}$$

$$= \Big\{[(30)(45) - (-5)(-10)]\,\hat{\imath} - [(-70)(45) - (-5)(-70)]\,\hat{\jmath} + [(-70)(-10) - (30)(-70)]\,\hat{k}\Big\}\,\mathrm{cm}^2 \tag{4}$$

$$= (1300\,\hat{\imath} + 3500\,\hat{\jmath} + 2800\,\hat{k})\,\mathrm{cm}^2, \tag{5}$$

$$n = \sqrt{(1300)^2 + (3500)^2 + (2800)^2}\,\mathrm{cm}^2 = 4667\,\mathrm{cm}^2. \tag{6}$$

To determine the component of $\vec{v}$ normal to the surface, use the dot product with unit vector $\vec{n}/n$, as follows

$$v_n = \vec{v} \cdot \frac{\vec{n}}{n} = (12\,\mathrm{km/s})(-0.545\,\hat{\imath} - 0.818\,\hat{\jmath} - 0.182\,\hat{k}) \cdot \frac{(1300\,\hat{\imath} + 3500\,\hat{\jmath} + 2800\,\hat{k})\,\mathrm{cm}^2}{4667\,\mathrm{cm}^2} \tag{7}$$

$$= -10.49\,\mathrm{km/s}. \tag{8}$$

Noting that the magnitude of $\vec{v}$ is $12\,\mathrm{km/s}$, the component of $\vec{v}$ tangent to the surface is given by

$$v_t = \sqrt{v^2 - v_n^2} = \sqrt{(12\,\mathrm{km/s})^2 - (-10.49\,\mathrm{km/s})^2} = 5.821\,\mathrm{km/s}. \tag{9}$$

*The inward normal vector could be used just as well. If this is the case, v_n in Eq. (8) becomes $10.49\,\mathrm{km/s}$, while the results in Eqs. (9) and (11)–(13) remain the same.

The vector components of the velocity are then given by

$$\boxed{\begin{aligned} \vec{v}_n &= v_n \frac{\vec{n}}{n} = (-10.49\,\text{km/s}) \frac{(1300\,\hat{\imath} + 3500\,\hat{\jmath} + 2800\,\hat{k})\,\text{cm}^2}{4667\,\text{cm}^2}, && (10)\\ &= (-2.92\,\hat{\imath} - 7.87\,\hat{\jmath} - 6.30\,\hat{k})\,\text{km/s}, && (11)\\ \vec{v}_t &= \vec{v} - \vec{v}_n = (-3.62\,\hat{\imath} - 1.95\,\hat{\jmath} + 4.11\,\hat{k})\,\text{km/s}. && (12) \end{aligned}}$$

The magnitudes of the velocity components can be determined from Eqs. (11) and (12), or by taking the absolute values of the components found in Eqs. (8) and (9). Hence,

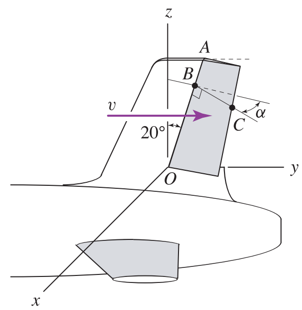
$$\boxed{|\vec{v}_n| = 10.5\,\text{km/s}, \quad |\vec{v}_t| = 5.82\,\text{km/s}.} \tag{13}$$

Problem 2.159

The velocity of air approaching the rudder of an aircraft has magnitude 900 ft/s in the y direction. The rudder rotates about line OA.

(a) The position vector from B to C has the x, y, and z direction cosines $\sin\alpha$, $(\cos\alpha)(\cos 20°)$, and $(-\cos\alpha)(\sin 20°)$, respectively. If the rudder is rotated so that $\alpha = 10°$, determine the components, and vector components, of the air velocity in directions normal and tangent to the surface of the rudder.

(b) Using the geometry shown in Fig. P2.159, verify the direction cosines stated in Part (a).



Solution

Part (a) Begin by writing $\vec{r}_{BC}$, for $\alpha = 10°$, as

$$\vec{r}_{BC} = \sin\alpha\,\hat{\imath} + \cos\alpha\cos 20°\,\hat{\jmath} - \cos\alpha\sin 20°\,\hat{k} = 0.1736\,\hat{\imath} + 0.9254\,\hat{\jmath} - 0.3368\,\hat{k}, \tag{1}$$

and $\vec{r}_{BO}$ as

$$\vec{r}_{BO} = -\sin 20°\,\hat{\jmath} - \cos 20°\,\hat{k} = -0.3420\,\hat{\jmath} - 0.9397\,\hat{k}. \tag{2}$$

A vector $\vec{n}$ that is normal to the rudder is given by

$$\vec{n} = \vec{r}_{BO} \times \vec{r}_{BC} \tag{3}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -0.3420 & -0.9397 \\ 0.1736 & 0.9254 & -0.3368 \end{vmatrix} \tag{4}$$

$$= [(-0.3420)(-0.3368) - (-0.9397)(0.9254)]\,\hat{\imath} - [(0)(-0.3368) - (-0.9397)(0.1736)]\,\hat{\jmath}$$
$$+ [(0)(0.9254) - (-0.3420)(0.1736)]\,\hat{k} \tag{5}$$

$$= 0.9848\,\hat{\imath} - 0.1632\,\hat{\jmath} + 0.05939\,\hat{k} \tag{6}$$

The normal vector $\vec{n}$ given in Eq. (6) is a unit vector, and henceforth will be called $\hat{n}$ in this solution. This can be seen by verifying that Eq. (6) has unit magnitude, or by noticing that since $\vec{r}_{BO}$ and $\vec{r}_{BC}$ are unit vectors *and* the angle between these vectors is 90°, their cross product is a unit vector. Based on the problem description, the velocity of air approaching the rudder is

$$\vec{v} = 900\,\text{ft/s}\,\hat{\jmath}. \tag{7}$$

The components of $\vec{v}$ in the normal and tangential directions to the rudder are

$$v_n = \vec{v}\cdot\hat{n} = (900\,\text{ft/s})(-0.1632) = -146.9\,\text{ft/s}, \tag{8}$$

$$v_t = \sqrt{v^2 - v_t^2} = \sqrt{(900\,\text{ft/s})^2 - (-146.9\,\text{ft/s})^2} = 887.9\,\text{ft/s}. \tag{9}$$

Thus, the vector representations of the normal and tangential components of the velocity are

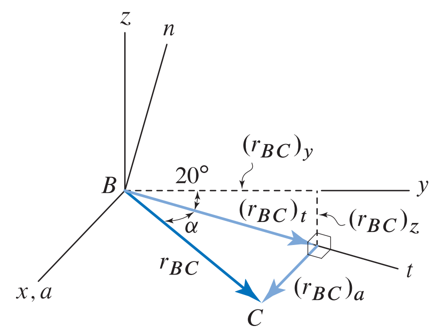
$$\vec{v}_n = v_n\hat{n} = (-146.9\,\text{ft/s})(0.9848\,\hat{\imath} - 0.1632\,\hat{\jmath} + 0.05939\,\hat{k}) \tag{10}$$

$$= (-145\,\hat{\imath} + 24.0\,\hat{\jmath} - 8.72\,\hat{k})\,\text{ft/s}, \tag{11}$$

$$\vec{v}_t = \vec{v} - \vec{v}_n = (145\,\hat{\imath} + 876\,\hat{\jmath} + 8.72\,\hat{k})\,\text{ft/s}. \tag{12}$$

The magnitudes of the velocity components can be determined from Eqs. (11) and (12), or by taking the absolute values of the components found in Eqs. (8) and (9). Hence,

$$\boxed{|\vec{v}_n| = 147\,\text{ft/s}, \quad |\vec{v}_t| = 888\,\text{ft/s}.} \tag{13}$$



Part (b) Consider the figure to the right, where a second coordinate system with axes a, t, and n has been included. (Note that the a and x axes coincide, and the t and n axes lie in the xy plane.) Begin by writing the components of $\vec{r}_{BC}$ in terms of the new coordinate axes, which results in

$$(r_{BC})_a = r_{BC}\sin\alpha,\ (r_{BC})_t = r_{BC}\cos\alpha,\ \text{and}\ (r_{BC})_n = 0. \tag{14}$$

Next, we write the components of the same vector $\vec{r}_{BC}$ in terms of the x, y, and z axes, which results in

$$(r_{BC})_x = (r_{BC})_a = r_{BC}\sin\alpha, \tag{15}$$
$$(r_{BC})_y = (r_{BC})_t\cos 20^\circ = r_{BC}\cos\alpha\ \cos 20^\circ, \tag{16}$$
$$(r_{BC})_z = -(r_{BC})_t\sin 20^\circ = -r_{BC}\cos\alpha\ \sin 20^\circ. \tag{17}$$

Based on the definition of direction cosines, the direction cosines for $\vec{r}_{BC}$ are

$$\cos\theta_x = \frac{(r_{BC})_x}{r_{BC}} = \sin\alpha, \tag{18}$$
$$\cos\theta_y = \frac{(r_{BC})_y}{r_{BC}} = \cos\alpha\cos 20^\circ, \tag{19}$$
$$\cos\theta_z = \frac{(r_{BC})_z}{r_{BC}} = -\cos\alpha\sin 20^\circ. \tag{20}$$

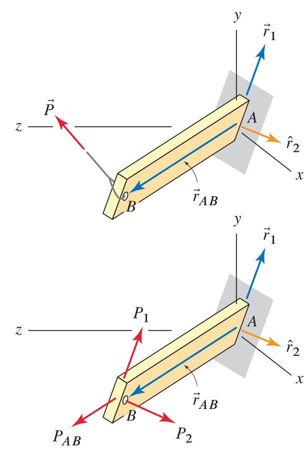
Problem 2.160

Beam AB has rectangular cross section where point A is at the origin of the coordinate system and point B has the coordinates $B\,(1.2, -0.3, 2.4)$ m. The vector $\vec{r}_1 = (1\,\hat{\imath} + 12\,\hat{\jmath} + 1\,\hat{k})$ m is perpendicular to the axis AB of the beam ($\vec{r}_{AB}$), and is parallel to the thin dimension of the cross section.

(a) Verify that $\vec{r}_1$ is indeed perpendicular to $\vec{r}_{AB}$.

(b) Determine the unit vector $\hat{r}_2$ that is perpendicular to both $\vec{r}_1$ and $\vec{r}_{AB}$ and has the orientation shown in the figure.

(c) The force $\vec{P}$ applied to point B of the beam has 1000 N magnitude and direction angles $\theta_x = 144°$, $\theta_y = 72°$, and $\theta_z = 60°$. Determine the components of the 1000 N force, namely P_{AB}, P_1, and P_2, in the directions $\vec{r}_{AB}$, $\vec{r}_1$, and $\hat{r}_2$, respectively, as shown in the figure. *Note:* For future work in mechanics of materials, it will be necessary to know these forces.



Solution

Part (a) We will use the dot product to verify that $\vec{r}_1$ and $\vec{r}_{AB}$ are orthogonal, as follows.

$$\vec{r}_{AB} = (1.2\,\hat{\imath} - 0.3\,\hat{\jmath} + 2.4\,\hat{k}\,)\text{ m}, \qquad r_{AB} = 2.7\text{ m}, \tag{1}$$

$$\vec{r}_1 \cdot \vec{r}_{AB} = (1\text{ m})(1.2\text{ m}) + (12\text{ m})(-0.3\text{ m}) + (1\text{ m})(2.4\text{ m}) = 0. \tag{2}$$

$$\boxed{\text{Since } \vec{r}_1 \cdot \vec{r}_{AB} = 0, \text{ the vectors } \vec{r}_1 \text{ and } \vec{r}_{AB} \text{ are perpendicular.}}$$

Part (b) We will use the cross product to determine a vector $\vec{r}_2$ that is orthogonal to both $\vec{r}_1$ and $\vec{r}_{AB}$ (note: for $\vec{r}_2$ to have the orientation shown in the problem statement, $\vec{r}_1$ must be taken as the first vector in the cross product and $\vec{r}_{AB}$ is the second vector).

$$\vec{r}_2 = \vec{r}_1 \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 12 & 1 \\ 1.2 & -0.3 & 2.4 \end{vmatrix} \text{m}^2 \tag{3}$$

$$= \Big\{ \hat{\imath}\,\big[(12)(2.4) - (1)(-0.3)\big] - \hat{\jmath}\,\big[(1)(2.4) - (1)(1.2)\big] \tag{4}$$

$$+\,\hat{k}\,\big[(1)(-0.3) - (12)(1.2)\big]\Big\}\text{ m}^2$$

$$= (29.1\,\hat{\imath} - 1.2\,\hat{\jmath} - 14.7\,\hat{k})\text{ m}^2. \tag{5}$$

The unit vector in the direction of $\vec{r}_2$ is

$$\hat{r}_2 = \frac{\vec{r}_2}{r_2} = \frac{(29.1\,\hat{\imath} - 1.2\,\hat{\jmath} - 14.7\,\hat{k})\text{ m}^2}{32.62\text{ m}^2} \tag{6}$$

$$= \boxed{0.8920\,\hat{\imath} - 0.03678\,\hat{\jmath} - 0.4506\,\hat{k}.} \tag{7}$$

Part (c) Using the information provided in the problem statement,

$$\vec{P} = 1000\,\text{N}\,(\cos 144°\;\hat{\imath} + \cos 72°\;\hat{\jmath} + \cos 60°\;\hat{k}) \tag{8}$$

$$= (-809.0\,\hat{\imath} + 309.0\,\hat{\jmath} + 500\,\hat{k})\ \text{N}. \tag{9}$$

The dot product is used to determine the components of $\vec{P}$ in the $\vec{r}_{AB}$, $\vec{r}_1$, and $\hat{r}_2$ directions, as follows

$$P_{AB} = \vec{P}\cdot\frac{\vec{r}_{AB}}{r_{AB}} = \frac{(-809.0\ \text{N})(1.2\ \text{m}) + (309.0\ \text{N})(-0.3\ \text{m}) + (500\ \text{N})(2.4\ \text{m})}{2.7\ \text{m}} \tag{10}$$

$$= \boxed{50.55\ \text{N},} \tag{11}$$

$$P_1 = \vec{P}\cdot\frac{\vec{r}_1}{r_1} = \frac{(-809.0\ \text{N})(1\ \text{m}) + (309.0\ \text{N})(12\ \text{m}) + (500\ \text{N})(1\ \text{m})}{12.08\ \text{m}} \tag{12}$$

$$= \boxed{281.3\ \text{N},} \tag{13}$$

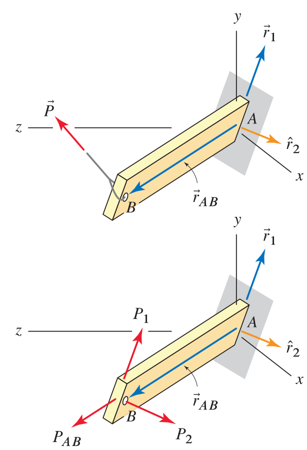
$$P_2 = \vec{P}\cdot\hat{r}_2 = (-809.0\ \text{N})(0.8920) + (309.0\ \text{N})(-0.03678) + (500\,\text{N})(-0.4506) \tag{14}$$

$$= \boxed{-958.3\ \text{N}.} \tag{15}$$

As a partial check of solution accuracy, we use the results in Eqs. (11), (13), and (15) to verify that $\sqrt{P_{AB}^2 + P_1^2 + P_2^2} = 1000$ N.

Problem 2.161

Repeat Prob. 2.160 if the coordinates of point B are $B\,(130, -60, 180)$ in., $\vec{r}_1 = (6\,\hat{\imath} + 31\,\hat{\jmath} + 6\,\hat{k})$ in., and the force $\vec{P}$ has 500 lb magnitude and direction angles $\theta_x = 108°$, $\theta_y = 60°$, and $\theta_z = 36°$.



Solution

Part (a) We will use the dot product to verify that $\vec{r}_1$ and $\vec{r}_{AB}$ are orthogonal, as follows.

$$\vec{r}_{AB} = (130\,\hat{\imath} - 60\,\hat{\jmath} + 180\,\hat{k})\text{ in.},\qquad r_{AB} = 230\text{ in.,} \tag{1}$$

$$\vec{r}_1 \cdot \vec{r}_{AB} = (6\text{ in.})(130\text{ in.}) + (31\text{ in.})(-60\text{ in.}) + (6\text{ in.})(180\text{ in.}) = 0. \tag{2}$$

$$\boxed{\text{Since } \vec{r}_1 \cdot \vec{r}_{AB} = 0, \text{ the vectors } \vec{r}_1 \text{ and } \vec{r}_{AB} \text{ are perpendicular.}}$$

Part (b) We will use the cross product to determine a vector $\vec{r}_2$ that is orthogonal to both $\vec{r}_1$ and $\vec{r}_{AB}$ (note: for $\vec{r}_2$ to have the orientation shown in the problem statement, $\vec{r}_1$ must be taken as the first vector in the cross product and $\vec{r}_{AB}$ is the second vector).

$$\vec{r}_2 = \vec{r}_1 \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & 31 & 6 \\ 130 & -60 & 180 \end{vmatrix}\text{ in.}^2 \tag{3}$$

$$= \Big\{ \hat{\imath}\,\big[(31)(180) - (6)(-60)\big] - \hat{\jmath}\,\big[(6)(180) - (6)(130)\big] + \hat{k}\,\big[(6)(-60) - (31)(130)\big]\Big\}\text{ in.}^2 \tag{4}$$

$$= (5940\,\hat{\imath} - 300\,\hat{\jmath} - 4390\,\hat{k})\text{ in.}^2 \tag{5}$$

The unit vector in the direction of $\vec{r}_2$ is

$$\hat{r}_2 = \frac{\vec{r}_2}{r_2} = \frac{(5940\,\hat{\imath} - 300\,\hat{\jmath} - 4390\,\hat{k})\text{ in.}^2}{7392\text{ in.}^2} \tag{6}$$

$$= \boxed{0.8035\,\hat{\imath} - 0.04058\,\hat{\jmath} - 0.5939\,\hat{k}.} \tag{7}$$

Part (c) Using the information provided in the problem statement,

$$\vec{P} = 500 \text{ lb} (\cos 108^\circ\ \hat{\imath} + \cos 60^\circ\ \hat{\jmath} + \cos 36^\circ\ \hat{k}) \tag{8}$$
$$= (-154.5\ \hat{\imath} + 250\ \hat{\jmath} + 404.5\ \hat{k}) \text{ lb}. \tag{9}$$

The dot product is used to determine the components of $\vec{P}$ in the $\vec{r}_{AB}$, $\vec{r}_1$, and $\hat{r}_2$ directions, as follows.

$$P_{AB} = \vec{P} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{(-154.5 \text{ lb})(130 \text{ in.}) + (250 \text{ lb})(-60 \text{ in.}) + (404.5 \text{ lb})(180 \text{ in.})}{230 \text{ in.}} \tag{10}$$
$$= \boxed{164.0 \text{ lb},} \tag{11}$$
$$P_1 = \vec{P} \cdot \frac{\vec{r}_1}{r_1} = \frac{(-154.5 \text{ lb})(6 \text{ in.}) + (250 \text{ lb})(31 \text{ in.}) + (404.5 \text{ lb})(6 \text{ in.})}{32.14 \text{ in.}} \tag{12}$$
$$= \boxed{287.8 \text{ lb},} \tag{13}$$
$$P_2 = \vec{P} \cdot \hat{r}_2 = (-154.5 \text{ lb})(0.8035) + (250 \text{ lb})(-0.04058) + (404.5 \text{ lb})(-0.5939) \tag{14}$$
$$= \boxed{-374.5 \text{ lb}.} \tag{15}$$

As a partial check of solution accuracy, we use the results in Eqs. (11), (13), and (15) to verify that $\sqrt{P_{AB}^2 + P_1^2 + P_2^2} = 500$ lb.

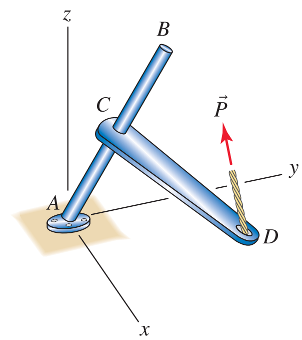
Problem 2.162

Structure $ABCD$ is rigid and is built-in at point A. Point A is at the origin of the coordinate system, points B and D have the coordinates $B\,(16, 24, 48)$ in. and $D\,(26, 52, -2)$ in., and point C is at the midpoint of the straight bar AB. Portion CD of the structure is perpendicular to bar AB.

(a) Verify that portion CD of the structure is indeed perpendicular to bar AB.

(b) Determine the unit vector $\hat{r}$ that is perpendicular to both AB and CD such that this vector has a positive x component.

(c) The force $\vec{P}$ applied to point D of the structure has 600 lb magnitude and direction angles $\theta_x = 108°$, $\theta_y = 36°$, and $\theta_z = 60°$. Determine the components of the 600 lb force, namely P_{AB}, P_{CD}, and P_r, in the directions AB, CD, and $\hat{r}$, respectively.



NOTE: The first printing of this book has an error in the last sentence of the statement for Part (c). The force referenced in the last sentence should be 600 lb (not 50 N). This error is corrected in the second and subsequent printings of the book.

Solution

Part (a) We will use the dot product to verify that the position vectors $\vec{r}_{AB}$ and $\vec{r}_{CD}$ are orthogonal, as follows, where the coordinates of point C are $C(8, 12, 24)$ in.

$$\vec{r}_{AB} = (16\,\hat{\imath} + 24\,\hat{\jmath} + 48\,\hat{k})\text{ in.},\qquad r_{AB} = 56\text{ in.},\tag{1}$$

$$\vec{r}_{CD} = \left[(26-8)\,\hat{\imath} + (52-12)\,\hat{\jmath} + (-2-24)\,\hat{k}\right]\text{ in.}\tag{2}$$

$$= (18\,\hat{\imath} + 40\hat{\jmath} - 26\,\hat{k})\text{ in.},\tag{3}$$

$$\vec{r}_{AB}\cdot\vec{r}_{CD} = (16\text{ in.})(18\text{ in.}) + (24\text{ in.})(40\text{ in.}) + (48\text{ in.})(-26\text{ in.}) = 0.\tag{4}$$

$$\boxed{\text{Since } \vec{r}_{AB}\cdot\vec{r}_{CD} = 0\text{, the vectors } \vec{r}_{AB} \text{ and } \vec{r}_{CD} \text{ are perpendicular.}}$$

Part (b) We will use the cross product to determine a vector $\vec{r}$ that is orthogonal to both $\vec{r}_{AB}$ and $\vec{r}_{CD}$ (note: for $\vec{r}$ to have a positive x component, we will take $\vec{r}_{CD}$ to be the first vector in the cross product and $\vec{r}_{AB}$ is the second vector).

$$\vec{r} = \vec{r}_{CD}\times\vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 18 & 40 & -26 \\ 16 & 24 & 48 \end{vmatrix}\text{ in.}^2\tag{5}$$

$$= \Big\{ \hat{\imath}\left[(40)(48) - (-26)(24)\right] - \hat{\jmath}\left[(18)(48) - (-26)(16)\right]$$

$$+ \hat{k}\left[(18)(24) - (40)(16)\right]\Big\}\text{ in.}^2\tag{6}$$

$$= (2544\,\hat{\imath} - 1280\hat{\jmath} - 208\,\hat{k})\text{ in.}^2\tag{7}$$

The unit vector in the direction of $\vec{r}$ is

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(2544\,\hat{\imath} - 1280\,\hat{\jmath} - 208\,\hat{k})\text{ in.}^2}{2855.5\text{ in.}^2}\tag{8}$$

$$= \boxed{0.8909\,\hat{\imath} - 0.4483\,\hat{\jmath} - 0.07284\,\hat{k}.} \tag{9}$$

Part (c) Using the information provided in the problem statement,

$$\vec{P} = 600\text{ lb}\,(\cos 108°\,\hat{\imath} + \cos 36°\,\hat{\jmath} + \cos 60°\,\hat{k}) \tag{10}$$

$$= (-185.4\,\hat{\imath} + 485.4\,\hat{\jmath} + 300\,\hat{k})\text{ lb}. \tag{11}$$

The dot product is used to determine the components of $\vec{P}$ in the $\vec{r}_{AB}$, $\vec{r}_{CD}$, and $\hat{r}$ directions, as follows.

$$P_{AB} = \vec{P}\cdot\frac{\vec{r}_{AB}}{r_{AB}} = \frac{(-185.4\text{ lb})(16\text{ in.}) + (485.4\text{ lb})(24\text{ in.}) + (300\text{ lb})(48\text{ in.})}{56\text{ in.}} \tag{12}$$

$$= \boxed{412.2\text{ lb},} \tag{13}$$

$$P_{CD} = \vec{P}\cdot\frac{\vec{r}_{CD}}{r_{CD}} = \frac{(-185.4\text{ lb})(18\text{ in.}) + (485.4\text{ lb})(40\text{ in.}) + (300\text{ lb})(-26\text{ in.})}{50.99\text{ in.}} \tag{14}$$

$$= \boxed{162.4\text{ lb},} \tag{15}$$

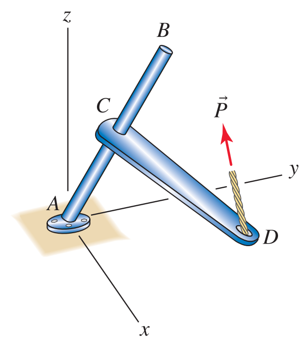
$$P_r = \vec{P}\cdot\hat{r} = (-185.4\text{ lb})(0.8909) + (485.4\text{ lb})(-0.4483) + (300\text{ lb})(-0.07284) \tag{16}$$

$$= \boxed{-404.6\text{ lb}.} \tag{17}$$

As a partial check of solution accuracy, we use the results in Eqs. (13), (15), and (17) to verify that $\sqrt{P_{AB}^2 + P_{CD}^2 + P_r^2} = 600$ lb.

Problem 2.163

Repeat Prob. 2.162 if the coordinates of points B and D are $B\,(20, 60, 90)$ mm and $D\,(40, 110, -15)$ mm, and the force $\vec{P}$ has 50 N magnitude and direction angles $\theta_x = 135°$, $\theta_y = 60°$, and $\theta_z = 60°$.



Solution

Part (a) We will use the dot product to verify that the position vectors $\vec{r}_{AB}$ and $\vec{r}_{CD}$ are orthogonal, as follows, where the coordinates of point C are $C(10,\ 30,\ 45)$ mm.

$$\vec{r}_{AB} = (20\,\hat{\imath} + 60\,\hat{\jmath} + 90\,\hat{k})\ \text{mm}, \qquad r_{AB} = 110\ \text{mm}, \tag{1}$$

$$\vec{r}_{CD} = \big[(40-10)\,\hat{\imath} + (110-30)\,\hat{\jmath} + (-15-45)\,\hat{k}\big]\ \text{mm} \tag{2}$$

$$= (30\,\hat{\imath} + 80\,\hat{\jmath} - 60\,\hat{k})\ \text{mm}, \tag{3}$$

$$\vec{r}_{AB} \cdot \vec{r}_{CD} = (20\ \text{mm})(30\ \text{mm}) + (60\ \text{mm})(80\ \text{mm}) + (90\ \text{mm})(-60\ \text{mm}) = 0. \tag{4}$$

Since $\vec{r}_{AB} \cdot \vec{r}_{CD} = 0$, the vectors $\vec{r}_{AB}$ and $\vec{r}_{CD}$ are perpendicular.

Part (b) We will use the cross product to determine a vector $\vec{r}$ that is orthogonal to both $\vec{r}_{AB}$ and $\vec{r}_{CD}$ (note: for $\vec{r}$ to have a positive x component, we will take $\vec{r}_{CD}$ to be the first vector in the cross product and $\vec{r}_{AB}$ is the second vector).

$$\vec{r} = \vec{r}_{CD} \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 30 & 80 & -60 \\ 20 & 60 & 90 \end{vmatrix}\ \text{mm}^2 \tag{5}$$

$$= \Big\{ \hat{\imath}\,\big[(80)(90) - (-60)(60)\big] - \hat{\jmath}\,\big[(30)(90) - (-60)(20)\big] + \hat{k}\,\big[(30)(60) - (80)(20)\big] \Big\}\ \text{mm}^2 \tag{6}$$

$$= (10{,}800\,\hat{\imath} - 3900\,\hat{\jmath} + 200\,\hat{k})\ \text{mm}^2. \tag{7}$$

The unit vector in the direction of $\vec{r}$ is

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(10{,}800\,\hat{\imath} - 3900\,\hat{\jmath} + 200\,\hat{k})\ \text{mm}^2}{11484\ \text{mm}^2} \tag{8}$$

$$= \boxed{0.9404\,\hat{\imath} - 0.3396\,\hat{\jmath} + 0.01742\,\hat{k}.} \tag{9}$$

Part (c) Using the information provided in the problem statement,

$$\vec{P} = 50\text{ N}\,(\cos 135^\circ\,\hat{\imath} + \cos 60^\circ\,\hat{\jmath} + \cos 60^\circ\,\hat{k}) \tag{10}$$

$$= (-35.36\,\hat{\imath} + 25\,\hat{\jmath} + 25\,\hat{k})\text{ N}. \tag{11}$$

The dot product is used to determine the components of $\vec{P}$ in the $\vec{r}_{AB}$, $\vec{r}_{CD}$, and $\hat{r}$ directions, as follows.

$$P_{AB} = \vec{P}\cdot\frac{\vec{r}_{AB}}{r_{AB}} = \frac{(-35.36\text{ N})(20\text{ mm}) + (25\text{ N})(60\text{ mm}) + (25\text{ N})(90\text{ mm})}{110\text{ mm}} \tag{12}$$

$$= \boxed{27.66\text{ N},} \tag{13}$$

$$P_{CD} = \vec{P}\cdot\frac{\vec{r}_{CD}}{r_{CD}} = \frac{(-35.36\text{ N})(30\text{ mm}) + (25\text{ N})(80\text{ mm}) + (25\text{ N})(-60\text{ mm})}{104.4\text{ mm}} \tag{14}$$

$$= \boxed{-5.370\text{ N},} \tag{15}$$

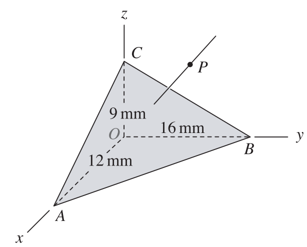
$$P_r = \vec{P}\cdot\hat{r} = (-35.36\text{ N})(0.9404) + (25\text{ N})(-0.3396) + (25\text{ N})(0.01742) \tag{16}$$

$$= \boxed{-41.30\text{ N}.} \tag{17}$$

As a partial check of solution accuracy, we use the results in Eqs. (13), (15), and (17) to verify that $\sqrt{P_{AB}^2 + P_{CD}^2 + P_r^2} = 50$ N.

Problem 2.164

Determine the smallest distance between point O and the infinite plane containing points A, B, and C.



Solution

To determine the smallest distance between point O and the infinite plane containing points A, B, and C, we will use a position vector between any convenient point on the plane and point O, or vice versa. We will use $\vec{r}_{AO}$, although $\vec{r}_{BO}$ and $\vec{r}_{CO}$ are equally good choices. Thus,

$$\vec{r}_{AO} = -12\,\text{mm}\,\hat{\imath}. \tag{1}$$

The normal direction to the plane is needed, and will be determined by taking the cross product between two vectors that lie in the plane, and among the many possible choices, we will use

$$\vec{r}_{AB} = (-12\,\hat{\imath} + 16\,\hat{\jmath})\,\text{mm}, \quad \text{and} \quad \vec{r}_{AC} = (-12\,\hat{\imath} + 9\,\hat{k})\,\text{mm}. \tag{2}$$

The normal direction $\vec{n}$ to the plane is

$$\vec{n} = \vec{r}_{AB} \times \vec{r}_{AC} \tag{3}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -12 & 16 & 0 \\ -12 & 0 & 9 \end{vmatrix} \text{mm}^2 = \Big\{[(16)(9) - 0]\;\hat{\imath} - [(-12)(9) - 0]\;\hat{\jmath} + [0 - (16)(-12)]\;\hat{k}\Big\}\;\text{mm}^2 \tag{4}$$

$$= (144.0\,\hat{\imath} + 108.0\,\hat{\jmath} + 192.0\,\hat{k})\,\text{mm}^2, \tag{5}$$

$$n = \sqrt{(144.0)^2 + (108.8)^2 + (192.0)^2}\,\text{mm}^2 = 263.2\,\text{mm}^2. \tag{6}$$

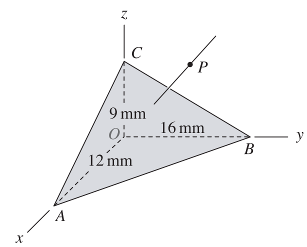
The dot product of $\vec{r}_{AO}$ with the unit vector normal to the plane is the smallest distance between point O and the plane. Hence,

$$d = \vec{r}_{AO} \cdot \frac{\vec{n}}{n} = (-12\,\text{mm})\,\hat{\imath} \cdot \frac{(144.0\,\hat{\imath} + 108.0\,\hat{\jmath} + 192.0\,\hat{k})\,\text{mm}^2}{263.2\,\text{mm}^2} = -6.566\,\text{mm}. \tag{7}$$

The negative sign in Eq. (7) is irrelevant, thus smallest distance between point O and the plane is $\boxed{6.57\,\text{mm.}}$

Problem 2.165

The vector from point O to point P has magnitude 40 mm and has equal direction angles with the x, y, and z axes. Determine the smallest distance from point P to the infinite plane containing points A, B, and C.



Solution

Direction angles must satisfy $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$, and since $\theta_x = \theta_y = \theta_z$, we may write

$$3\cos^2\theta = 1 \quad \Rightarrow \quad \cos\theta = \sqrt{1/3} = 0.5774 \tag{1}$$

where θ is the value each of the three direction angles. It follows that $\vec{r}_{OP}$ is given by

$$\vec{r}_{OP} = (40\,\text{mm})(0.5774\,\hat{\imath} + 0.5774\,\hat{\jmath} + 0.5774\,\hat{k}) = (23.09\,\hat{\imath} + 23.09\,\hat{\jmath} + 23.09\,\hat{k})\,\text{mm}. \tag{2}$$

To determine the smallest distance between point P and the plane containing points A, B, and C, we will need a position vector between some convenient point on the plane and point P. We will use $\vec{r}_{AP}$ ($\vec{r}_{BP}$ and $\vec{r}_{CP}$ are also good choices), which can be written as

$$\begin{aligned}\vec{r}_{AP} &= \vec{r}_{AO} + \vec{r}_{OP} = (-12\,\text{mm})\,\hat{\imath} + (23.09\,\hat{\imath} + 23.09\,\hat{\jmath} + 23.09\,\hat{k})\,\text{mm}\\ &= (11.09\,\hat{\imath} + 23.09\,\hat{\jmath} + 23.09\,\hat{k})\,\text{mm}.\end{aligned} \tag{3}$$

The normal direction $\vec{n}$ to the plane may be determined by taking the cross product between two vectors that lie in the plane. Among the many possible choices, we will use

$$\vec{r}_{AB} = (-12\,\hat{\imath} + 16\,\hat{\jmath})\,\text{mm} \quad \text{and} \quad \vec{r}_{AC} = (-12\,\hat{\imath} + 9\,\hat{k})\,\text{mm}, \tag{4}$$

so that the normal direction is

$$\vec{n} = \vec{r}_{AB} \times \vec{r}_{AC} \tag{5}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -12 & 16 & 0 \\ -12 & 0 & 9 \end{vmatrix} \text{mm}^2 = \left\{[(16)(9) - 0]\,\hat{\imath} - [(-12)(9) - 0]\,\hat{\jmath} + [0 - (16)(-12)]\,\hat{k}\right\}\,\text{mm}^2 \tag{6}$$

$$= (144.0\,\hat{\imath} + 108.0\,\hat{\jmath} + 192.0\,\hat{k})\,\text{mm}^2, \tag{7}$$

$$n = \sqrt{(144.0)^2 + (108.8)^2 + (192.0)^2}\,\text{mm}^2 = 263.2\,\text{mm}^2. \tag{8}$$

The smallest distance between the plane and point P is given by the perpendicular line from the plane to point P, and this may be determined using the following dot product

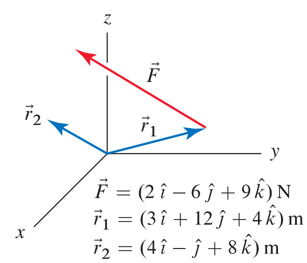
$$\begin{aligned} d &= \vec{r}_{AP} \cdot \frac{\vec{n}}{n}\\ &= (11.09\,\hat{\imath} + 23.09\,\hat{\jmath} + 23.09\,\hat{k})\,\text{mm} \cdot \frac{(144.0\,\hat{\imath} + 108.0\,\hat{\jmath} + 192.0\,\hat{k})\,\text{mm}^2}{263.2\,\text{mm}^2}\\ &= \boxed{32.4\,\text{mm}.}\end{aligned} \tag{9}$$

Problem 2.166

The product $\vec{r}_1 \times \vec{F}$ produces a vector $\vec{M}$. The product $\vec{M} \cdot \vec{r}_2/|\vec{r}_2|$ produces a scalar $M_\|$, which is the component of $\vec{M}$ in the direction of $\vec{r}_2$.

(a) Evaluate $M_\|$ by finding $\vec{M}$ first, followed by the dot product.

(b) Evaluate $M_\|$ using the scalar triple product.



Solution

Part (a) Begin by determining $\vec{M}$ using

$$\vec{M} = \vec{r}_1 \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 12 & 4 \\ 2 & -6 & 9 \end{vmatrix} \text{ N·m} \tag{1}$$

$$= \left\{[(12)(9) - (4)(-6)]\,\hat{\imath} - [(3)(9) - (4)(2)]\,\hat{\jmath} + [(3)(-6) - (12)(2)]\,\hat{k}\right\} \text{N·m} \tag{2}$$

$$= (132\,\hat{\imath} - 19\,\hat{\jmath} - 42\,\hat{k})\,\text{N·m}. \tag{3}$$

We then determine $M_\|$ using the dot product

$$\boxed{M_\| = \vec{M} \cdot \frac{\vec{r}_2}{r_2} = \frac{\big((132)(4) + (-19)(-1) + (-42)(8)\big)\,\text{N·m}}{\sqrt{(4)^2 + (-1)^2 + (8)^2}} = 23.4\,\text{N} \cdot \text{m}.} \tag{4}$$

Part (b) The scalar triple product provides

$$M_\| = (\vec{r}_1 \times \vec{F}) \cdot \left(\frac{\vec{r}_2}{r_2}\right) = \begin{vmatrix} 4/9 & -1/9 & 8/9 \\ 3 & 12 & 4 \\ 2 & -6 & 9 \end{vmatrix} \text{N·m} \tag{5}$$

$$= \left\{(4/9)\big[(12)(9) - (4)(-6)\big] - (-1/9)\big[(3)(9) - (4)(2)\big] + (8/9)\big[(3)(-6) - (12)(2)\big]\right\} \text{N·m} \tag{6}$$

$$= \boxed{23.4\,\text{N·m},} \tag{7}$$

which is the same result that was obtained in Part (a).

Problem 2.167

As described in connection with Fig. 2.34 on p. 105, the scalar triple product $(\vec{A} \times \vec{B}) \cdot \vec{C}$ provides the volume of the parallelepiped formed by $\vec{A}$, $\vec{B}$, and $\vec{C}$. Comment on how the results of the following triple products compare to the value provided by $(\vec{A} \times \vec{B}) \cdot \vec{C}$:

(a) $(\vec{A} \times \vec{C}) \cdot \vec{B}$.

(b) $(\vec{B} \times \vec{C}) \cdot \vec{A}$.

(c) $(\vec{C} \times \vec{B}) \cdot \vec{A}$.

(d) $(\vec{C} \times \vec{A}) \cdot \vec{B}$.

Note: Concept problems are about *explanations,* not computations.

Solution

The volume V of the parallelepiped is given by

$$V = (\vec{A} \times \vec{B}) \cdot \vec{C}. \tag{1}$$

Part (a)

$$\boxed{(\vec{A} \times \vec{C}) \cdot \vec{B} = -V.} \tag{2}$$

Part (b)

$$\boxed{(\vec{B} \times \vec{C}) \cdot \vec{A} = V.} \tag{3}$$

Part (c)

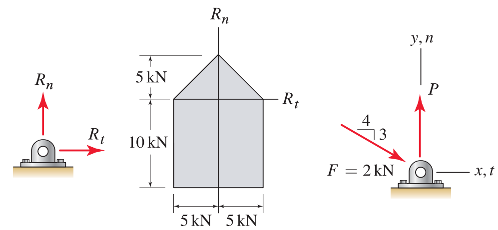
$$\boxed{(\vec{C} \times \vec{B}) \cdot \vec{A} = -V.} \tag{4}$$

Part (d)

$$\boxed{(\vec{C} \times \vec{A}) \cdot \vec{B} = V.} \tag{5}$$

Problem 2.168

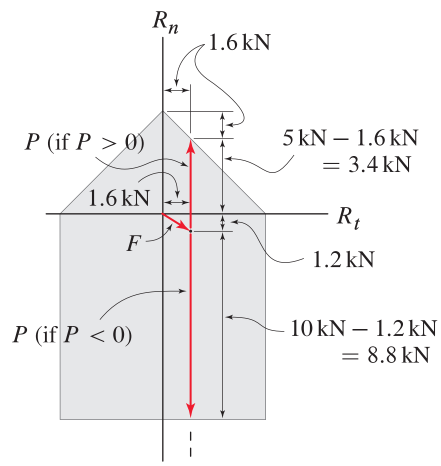
The manufacturer of a welded steel bracket specifies the working loads depicted in the figure of R_n versus R_t. Values of R_n and R_t that lie within the shaded region are allowable, while values that lie outside of the region are unsafe. Such a diagram is often called an *interaction diagram* because it characterizes the combined effect that multiple loads have on the strength of a component. For the loading and geometry shown, determine the range of values load P may have and still satisfy the manufacturer's combined loading criterion.



Solution

Begin by writing the resultant force $\vec{R}$ for any value of P

$$\begin{aligned}\vec{R} &= \vec{F} + \vec{P}\\ &= 2\,\mathrm{kN}(4/5)\,\hat{\imath} - 2\,\mathrm{kN}(3/5)\,\hat{\jmath} + P\,\hat{\jmath}\\ &= 1.6\,\mathrm{kN}\,\hat{\imath} + (P - 1.2\,\mathrm{kN})\,\hat{\jmath}. \qquad (1)\end{aligned}$$



In the figure at the right, the sum $\vec{F} + \vec{P}$ is shown. The force $\vec{F}$ has known magnitude and direction, and it is sketched first. Then, two possible vectors for force $\vec{P}$ are shown; one possibility corresponds to $P > 0$ and the other to $P < 0$. Using some geometry, the values of P that cause the resultant force to tocuh the surface of allowable working loads is determined, with the result

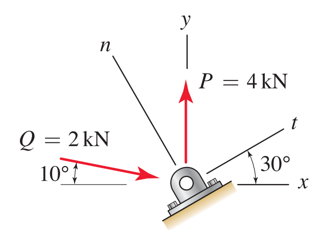
$$-(10 - 1.2)\,\mathrm{kN} \le P \le (5 - 1.6 + 1.2)\,\mathrm{kN}, \qquad (2)$$

or

$$\boxed{-8.8\,\mathrm{kN} \le P \le 4.6\,\mathrm{kN}.} \qquad (3)$$

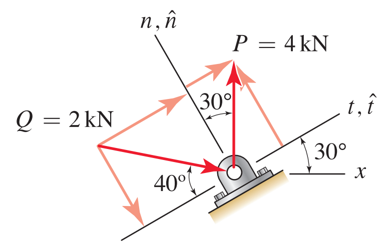
Problem 2.169

For the loading and geometry shown, use the interaction diagram of Prob. 2.168 to determine if the manufacturer's combined loading criterion is satisfied.



Solution

Let $\hat{t}$ and $\hat{n}$ be unit vectors in the t and n directions, respectively. Using the sketch to the right, forces $\vec{P}$ and $\vec{Q}$ may be written in terms of t and n components as



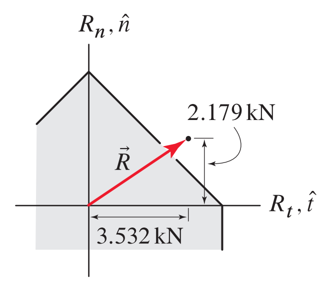
$$\vec{P} = (4\,\text{kN})(\sin 30°\,\hat{t} + \cos 30°\,\hat{n}), \tag{1}$$

$$\vec{Q} = (2\,\text{kN})(\cos 40°\,\hat{t} - \sin 40°\,\hat{n}). \tag{2}$$

The resultant force $\vec{R}$ is then given by

$$\begin{aligned}\vec{R} &= \vec{P} + \vec{Q} \\ &= \big[(4\,\text{kN})\sin 30° + (2\,\text{kN})\cos 40°\big]\,\hat{t} + \big[(4\,\text{kN})\cos 30° - (2\,\text{kN})\sin 40°\big]\,\hat{n} \\ &= (3.532\,\hat{t} + 2.179\,\hat{n})\,\text{kN}.\end{aligned} \tag{3}$$

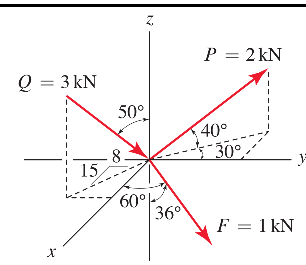
The components of Eq. (3) are plotted in the figure at the right with the interaction diagram from Prob. 2.168. Since the resultant force from Eq. (3) lies outside of the range of permissible values, $\boxed{\text{this loading is not safe.}}$



Problem 2.170

(a) Determine the resultant $\vec{R}$ of the three forces $\vec{F} + \vec{P} + \vec{Q}$.

(b) If an additional force $\vec{T}$ in the $\pm x$ direction is to be added, determine the magnitude it should have so that the magnitude of the resultant is as small as possible.



Solution

Part (a) For force $\vec{F}$, by examining the figure in the problem statement, $\theta_x = 60°$. The 36° angle is not a direction angle, but θ_z can be easily determined using

$$\theta_z = 180° - 36° = 144°. \tag{1}$$

The remaining direction angle θ_y may be determined by using $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$, which can be rearranged for

$$\cos\theta_y = \pm\sqrt{1 - \cos^2\theta_x - \cos^2\theta_z} = \pm 0.3090 \quad \Rightarrow \quad \cos\theta_y = 0.3090. \tag{2}$$

In Eq. (2), we have selected the postive solution since force $\vec{F}$ clearly has a positive y component. Thus, the vector expression for $\vec{F}$ is

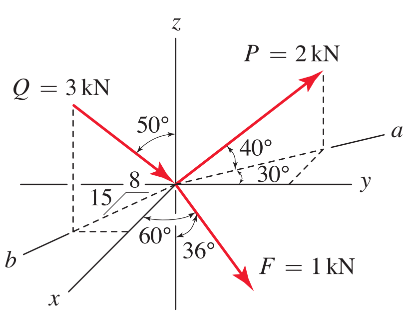
$$\begin{aligned}\vec{F} &= (1\,\text{kN})(\cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + \cos\theta_z\,\hat{k}) \\ &= (1\,\text{kN})(0.5000\,\hat{\imath} + 0.3090\,\hat{\jmath} - 0.8090\,\hat{k}) \\ &= (0.5000\,\hat{\imath} + 0.3090\,\hat{\jmath} - 0.8090\,\hat{k})\,\text{kN}.\end{aligned} \tag{3}$$

For the force $\vec{P}$, we begin by determining the components in the z and a directions, where the a direction is defined in the sketch at the right. Thus

$$P_z = (2\,\text{kN})\sin 40° \quad \text{and} \quad P_a = (2\,\text{kN})\cos 40°. \tag{4}$$

The x and y components are then found from P_a as follows

$$P_x = -P_a \sin 30° = -(2\,\text{kN})\cos 40° \sin 30°, \tag{5}$$
$$P_y = P_a \cos 30° = (2\,\text{kN})\cos 40° \cos 30°. \tag{6}$$



Thus, the vector expression for $\vec{P}$ is

$$\begin{aligned}\vec{P} &= -(2\,\text{kN})\cos 40° \sin 30°\,\hat{\imath} + (2\,\text{kN})\cos 40° \cos 30°\,\hat{\jmath} + (2\,\text{kN})\sin 40°\,\hat{k} \\ &= (-0.7660\,\hat{\imath} + 1.327\,\hat{\jmath} + 1.286\,\hat{k})\,\text{kN}.\end{aligned} \tag{7}$$

A partial check of accuracy, you should verify that the magnitude of Eq. (7) is 2 kN.

For the force $\vec{Q}$, we begin by determining the components in the z and b directions, where the b direction is defined in the sketch shown earlier. Thus

$$Q_z = -(3\,\text{kN})\cos 50° \quad \text{and} \quad Q_b = (3\,\text{kN})\sin 50°. \tag{8}$$

Noting that $\sqrt{15^2 + 8^2} = 17$, the x and y components are then found from Q_b as follows

$$Q_x = -Q_b(15/17) = -(3\,\text{kN})\sin 50°(15/17), \tag{9}$$
$$Q_y = Q_b(8/17) = (3\,\text{kN})\sin 50°(8/17). \tag{10}$$

Thus, the vector expression for $\vec{Q}$ is

$$\begin{aligned}\vec{Q} &= -(3\,\text{kN})\sin 50°(15/17)\,\hat{\imath} + (3\,\text{kN})\sin 50°(8/17)\,\hat{\jmath} - (3\,\text{kN})\cos 50°\,\hat{k} \\ &= (-2.028\,\hat{\imath} + 1.081\,\hat{\jmath} - 1.928\,\hat{k})\,\text{kN}.\end{aligned} \tag{11}$$

As a partial check of accuracy, you should verify that the magnitude of Eq. (11) is 3 kN.

The resultant force is found by adding Eqs. (3), (7), and (11) to obtain

$$\boxed{\vec{R} = \vec{F} + \vec{P} + \vec{Q} = (-2.29\,\hat{\imath} + 2.72\,\hat{\jmath} - 1.45\,\hat{k})\,\text{kN}.} \tag{12}$$

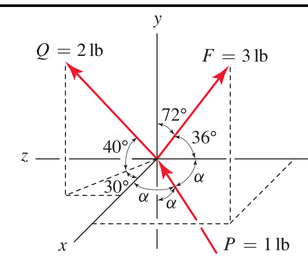
Part (b) To determine the force $\vec{T}$ that will make the resultant force $\vec{R}$ as small as possible, where $\vec{T}$ is required to be in the $\pm x$ direction, we choose a force that will render the x component of $\vec{R}$ equal to zero, hence

$$\boxed{\vec{T} = (2.29\,\hat{\imath})\,\text{kN}.} \tag{13}$$

Problem 2.171

(a) Determine the resultant $\vec{R}$ of the three forces $\vec{F} + \vec{P} + \vec{Q}$.

(b) If an additional force $\vec{T}$ in the $\pm x$ direction is to be added, determine the magnitude it should have so that the magnitude of the resultant is as small as possible.



Solution

Part (a) For force $\vec{F}$, by examining the figure in the problem statement, $\theta_y = 72°$. The 36° angle is not a direction angle, but θ_z can be easily determined using

$$\theta_z = 180° - 36° = 144°. \tag{1}$$

The remaining direction angle θ_x may be determined by using

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1, \tag{2}$$

which can be rearranged for

$$\cos\theta_x = \pm\sqrt{1 - \cos^2\theta_y - \cos^2\theta_z} = \pm 0.5000 \quad \Rightarrow \quad \cos\theta_x = 0.5000. \tag{3}$$

In Eq. (3), we have selected the postive solution since force $\vec{F}$ clearly has a positive x component. Thus, the vector expression for $\vec{F}$ is

$$\begin{aligned}\vec{F} &= (3\,\text{lb})(\cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + \cos\theta_z\,\hat{k})\\ &= (3\,\text{lb})(0.5000\,\hat{\imath} + 0.3090\,\hat{\jmath} - 0.8090\,\hat{k})\\ &= (1.5\,\hat{\imath} + 0.9271\,\hat{\jmath} - 2.427\,\hat{k})\,\text{lb}.\end{aligned} \tag{4}$$

For the force $\vec{P}$, we observe from the figure in the problem statement that the value of α must satisfy an expression similar to Eq. (2), which leads to *

$$3\cos^2\alpha = 1. \tag{5}$$

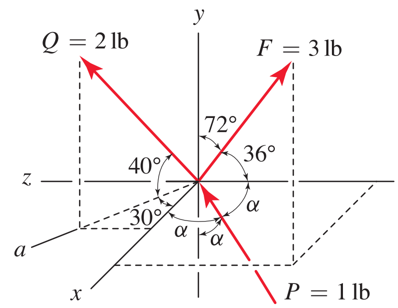
Solving Eq. (5) provides

$$\cos\alpha = \pm 0.5774 \quad \Rightarrow \quad \alpha = 54.74°, 125.3°. \tag{6}$$

By examining the figure in the problem statement, α is clearly an accute angle, so we select $\alpha = 54.74°$ in Eq. (6). By noting that the x, y, and z components of $\vec{P}$ are negative, positive, and positive, respectively, the vector expression for $\vec{P}$ is

$$\begin{aligned}\vec{P} &= (1\,\text{lb})(-0.5774\,\hat{\imath} + 0.5774\,\hat{\jmath} + 0.5774\,\hat{k})\\ &= (-0.5774\,\hat{\imath} + 0.5774\,\hat{\jmath} + 0.5774\,\hat{k})\,\text{lb}.\end{aligned} \tag{7}$$

*By carefully examining the figure in the problem statement, the direction angles for $\vec{P}$ are $\theta_x = 180° - \alpha$, $\theta_y = \alpha$, and $\theta_z = \alpha$. Substituting these direction angles in Eq.(2) and solving provides the solutions $\alpha = 54.74°, 125.3°$; the first of these, which corresponds to an acute angle, is the physically correct solution for this problem.



For the force $\vec{Q}$, we begin by determining the components in the y and a directions, where the a direction is defined in the sketch at the right. Thus

$$Q_y = (2\,\text{lb}) \sin 40°, \quad Q_a = (2\,\text{lb}) \cos 40°. \tag{8}$$

The x and z components are then found from Q_a as follows

$$Q_x = Q_a \cos 30° = (2\,\text{lb}) \cos 40° \cos 30°, \tag{9}$$

$$Q_z = Q_a \sin 30° = (2\,\text{lb}) \cos 40° \sin 30°. \tag{10}$$

Thus, the vector expression for $\vec{Q}$ is

$$\vec{Q} = (2\,\text{lb}) \cos 40° \cos 30°\, \hat{\imath} + (2\,\text{lb}) \sin 40°\, \hat{\jmath} + (2\,\text{lb}) \cos 40° \sin 30°\, \hat{k} \tag{11}$$

$$= (1.327\,\hat{\imath} + 1.286\,\hat{\jmath} + 0.7660\,\hat{k})\,\text{lb}. \tag{12}$$

As a partial check of accuracy, you should verify that the magnitude of Eq. (12) is 2 lb.

The resultant force is found by adding Eqs. (4), (7), and (12) to obtain

$$\boxed{\vec{R} = \vec{F} + \vec{P} + \vec{Q} = (2.25\,\hat{\imath} + 2.79\,\hat{\jmath} - 1.08\,\hat{k})\,\text{lb}.} \tag{13}$$

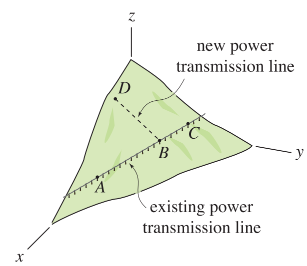
Part (b) To determine the force $\vec{T}$ that will make the resultant force $\vec{R}$ as small as possible, where $\vec{T}$ is required to be in the $\pm x$ direction, we choose a force that will render the x component of $\vec{R}$ equal to zero, hence

$$\boxed{\vec{T} = -2.25\,\hat{\imath}\,\text{lb}.} \tag{14}$$

Problem 2.172

An electrical power transmission line runs between points A, B, and C, as shown, and a new power transmission line between points B and D is to be constructed. Using the coordinates of points A, C, and D given below, determine the distance from point A to point B where the new power line should connect to the existing power line such that the length of the new power line is as small as possible. Also determine the length of the new power line. Assume that power lines ABC and BD are straight.

The coordinates of points A, C, and D are $A(6000, 1000, 500)$ ft, $C(700, 4500, 900)$ ft, and $D(2000, 400, 1400)$ ft.



Solution

Our strategy will be to begin by writing a position vector from A to D, namely $\vec{r}_{AD}$ (we could just as well begin with $\vec{r}_{CD}$). We will then resolve this vector into a component parallel to AB, and a component perpendicular to AB. The parallel component will provide the distance from point A to point B*, and the perpendicular component will provide the length of the new power line.

Using the coordinates of points A, C, and D provided in the problem statement,

$$\vec{r}_{AC} = \left[(700 - 6000)\,\hat{\imath} + (4500 - 1000)\,\hat{\jmath} + (900 - 500)\,\hat{k}\right] \text{ft} \tag{1}$$

$$= (-5300\,\hat{\imath} + 3500\,\hat{\jmath} + 400\,\hat{k})\ \text{ft}, \tag{2}$$

$$\vec{r}_{AD} = \left[(2000 - 6000)\,\hat{\imath} + (400 - 1000)\,\hat{\jmath} + (1400 - 500)\,\hat{k}\right] \text{ft} \tag{3}$$

$$= (-4000\,\hat{\imath} - 600\,\hat{\jmath} + 900\,\hat{k})\ \text{ft}. \tag{4}$$

The component of $\vec{r}_{AD}$ parallel to $\vec{r}_{AC}$ is

$$r_{||} = \vec{r}_{AD} \cdot \frac{\vec{r}_{AC}}{r_{AC}} = \frac{(-4000\text{ ft})(-5300\text{ ft}) + (-600\text{ ft})(3500\text{ ft}) + (900\text{ ft})(400\text{ ft})}{6364\text{ ft}} = 3058\text{ ft}. \tag{5}$$

The component of $\vec{r}_{AD}$ perpendicular to $\vec{r}_{AC}$ is

$$r_{\perp} = \sqrt{r_{AD}^2 - r_{||}^2} = \sqrt{(4144\text{ ft})^2 - (3058\text{ ft})^2} = 2796\text{ ft}. \tag{6}$$

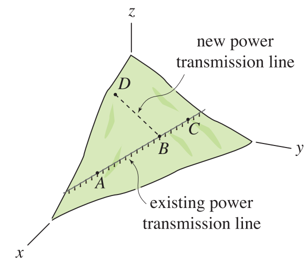
Thus, the distance from point A to point B is 3058 ft and the length of the new power line is 2796 ft.

*If $\vec{r}_{CD}$ is used instead of $\vec{r}_{AD}$, then the component of $\vec{r}_{CD}$ parallel to AB will provide the distance from point C to B.

Problem 2.173

An electrical power transmission line runs between points A, B, and C, as shown, and a new power transmission line between points B and D is to be constructed. Using the coordinates of points A, C, and D given below, determine the distance from point A to point B where the new power line should connect to the existing power line such that the length of the new power line is as small as possible. Also determine the length of the new power line. Assume that power lines ABC and BD are straight.

The coordinates of points A, C, and D are $A(3000, 600, 300)$ m, $C(600, 2200, 500)$ m, and $D(1000, 300, 800)$ m.



Solution

Our strategy will be to begin by writing a position vector from A to D, namely $\vec{r}_{AD}$ (we could just as well begin with $\vec{r}_{CD}$). We will then resolve this vector into a component parallel to AB, and a component perpendicular to AB. The parallel component will provide the distance from point A to point B*, and the perpendicular component will provide the length of the new power line.

Using the coordinates of points A, C, and D provided in the problem statement,

$$\vec{r}_{AC} = \left[(600 - 3000)\,\hat{\imath} + (2200 - 600)\,\hat{\jmath} + (500 - 300)\,\hat{k}\,\right] \text{m} \tag{1}$$

$$= (-2400\,\hat{\imath} + 1600\,\hat{\jmath} + 200\,\hat{k})\ \text{m}, \tag{2}$$

$$\vec{r}_{AD} = \left[(1000 - 3000)\,\hat{\imath} + (300 - 600)\,\hat{\jmath} + (800 - 300)\,\hat{k}\right] \text{m} \tag{3}$$

$$= (-2000\,\hat{\imath} - 300\,\hat{\jmath} + 500\,\hat{k})\ \text{m}. \tag{4}$$

The component of $\vec{r}_{AD}$ parallel to $\vec{r}_{AC}$ is

$$r_{||} = \vec{r}_{AD} \cdot \frac{\vec{r}_{AC}}{r_{AC}} = \frac{(-2000\ \text{m})(-2400\ \text{m}) + (-300\ \text{m})(1600\ \text{m}) + (500\ \text{m})(200\ \text{m})}{2891\ \text{m}} = 1529\ \text{m}. \tag{5}$$

The component of $\vec{r}_{AD}$ perpendicular to $\vec{r}_{AC}$ is

$$r_{\perp} = \sqrt{r_{AD}^2 - r_{||}^2} = \sqrt{(2083\ \text{m})^2 - (1529\ \text{m})^2} = 1415\ \text{m}. \tag{6}$$

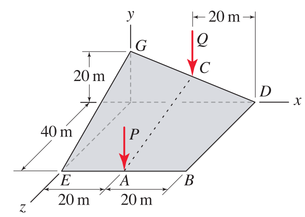
Thus, the distance from point A to point B is 1529 m and the length of the new power line is 1415 m.

*If $\vec{r}_{CD}$ is used instead of $\vec{r}_{AD}$, then the component of $\vec{r}_{CD}$ parallel to AB will provide the distance from point C to B.

Problem 2.174

An architect specifies the roof geometry shown for a building. Each of lines EAB, BD, DCG, EG, and AC are straight. Two forces of magnitude P and Q acting in the $-y$ direction are applied at the positions shown.

Determine the components of the force P in directions normal and parallel to the roof at point A. Express your answer in terms of P.



Solution

The following vectors are obtained from the figure in the problem statement:

$$\vec{P} = -P\,\hat{\jmath}, \quad \vec{r}_{AB} = (20\,\hat{\imath})\,\text{m}, \quad \text{and} \quad \vec{r}_{AC} = (10\,\hat{\jmath} - 40\,\hat{k})\,\text{m}. \tag{1}$$

The vector perpendicular to $\vec{r}_{AB}$ and $\vec{r}_{AC}$ is given by

$$\vec{n} = \vec{r}_{AB} \times \vec{r}_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 20 & 0 & 0 \\ 0 & 10 & -40 \end{vmatrix} \text{m}^2 \tag{2}$$

$$= \left\{(0-0)\,\hat{\imath} - [(20)(-40) - (0)(0)]\,\hat{\jmath} + [(20)(10) - (0)(0)]\,\hat{k}\right\}\text{m}^2 \tag{3}$$

$$= (800\,\hat{\jmath} + 200\,\hat{k})\,\text{m}^2, \tag{4}$$

$$n = \sqrt{(800\,\text{m}^2)^2 + (200\,\text{m}^2)^2} = 824.6\,\text{m}^2. \tag{5}$$

The component of $\vec{P}$ that is perpendicular to the roof is given by

$$\boxed{P_\perp = \vec{P} \cdot \frac{\vec{n}}{n} = \frac{0 + (-P)(800\,\text{m}^2) + 0}{824.6\,\text{m}^2} = (-0.970)\,P.} \tag{6}$$

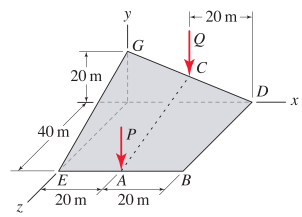
The component of $\vec{P}$ that is parallel to the roof is given by

$$\boxed{P_\parallel = \sqrt{P^2 - P_\perp^2} = \sqrt{P^2 - (-0.970P)^2} = (0.243)\,P.} \tag{7}$$

Problem 2.175

An architect specifies the roof geometry shown for a building. Each of lines EAB, BD, DCG, EG, and AC are straight. Two forces of magnitude P and Q acting in the $-y$ direction are applied at the positions shown.

Determine the components of the force Q in directions normal and parallel to the roof at point C. Express your answer in terms of Q.



Solution

The following vectors are obtained from the figure in the problem statement:

$$\vec{Q} = -Q\,\hat{\jmath}, \quad \vec{r}_{AC} = \left(10\,\hat{\jmath} - 40\,\hat{k}\right)\text{m}, \quad \text{and} \quad \vec{r}_{CD} = \left(20\,\hat{\imath} - 10\,\hat{\jmath}\right)\text{m}. \tag{1}$$

The vector perpendicular to $\vec{r}_{CD}$ and $\vec{r}_{AC}$ is given by

$$\vec{n} = \vec{r}_{CD} \times \vec{r}_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 20 & -10 & 0 \\ 0 & 10 & -40 \end{vmatrix}\ \text{m}^2 \tag{2}$$

$$= \left\{[(-10)(-40) - (0)(10)]\,\hat{\imath} - [(20)(-40) - (0)(0)]\,\hat{\jmath} + [(20)(10) - (10)(0)]\,\hat{k}\right\}\text{m}^2 \tag{3}$$

$$= (400\,\hat{\imath} + 800\,\hat{\jmath} + 200\,\hat{k})\,\text{m}^2, \tag{4}$$

$$n = \sqrt{(800\,\text{m}^2)^2 + (200\,\text{m}^2)^2} = 916.5\,\text{m}^2. \tag{5}$$

The component of $\vec{Q}$ that is perpendicular to the roof is given by

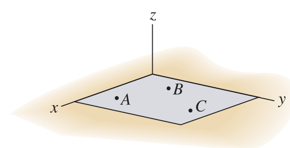
$$\boxed{Q_\perp = \vec{Q} \cdot \frac{\vec{n}}{n} = \frac{0 + (-Q)(800\,\text{m}^2) + 0}{916.5\,\text{m}^2} = (-0.873)\,Q.} \tag{6}$$

The component of $\vec{Q}$ that is parallel to the roof is given by

$$\boxed{Q_\parallel = \sqrt{Q^2 - Q_\perp^2} = \sqrt{Q^2 - (-0.873Q)^2} = (0.488)\,Q.} \tag{7}$$

Problem 2.176

A concrete surface for a parking lot is to be prepared such that it is planar and the normal direction to the surface is $2° \pm 0.5°$ of vertical so that rainwater will have adequate drainage. If the coordinates of points A, B, and C are A (300, 50, 2) ft, B (20, 150, −5) ft, and C (280, 450, −8) ft, determine the angle between the concrete surface's normal direction and the vertical, and if the concrete surface meets the drainage specification cited above.



Solution

Our strategy will be to use the cross product to determine the normal direction to surface ABC, $\vec{n} = \vec{r}_{AC} \times \vec{r}_{AB}$. We will then take the dot product of $\vec{n}$ with $\hat{k}$ to determine the angle between $\vec{n}$ and $\hat{k}$.

$$\vec{r}_{AC} = \left[(280-300)\,\hat{\imath} + (450-50)\,\hat{\jmath} + (-8-2)\,\hat{k}\,\right]\text{ ft} \tag{1}$$
$$= (-20\,\hat{\imath} + 400\,\hat{\jmath} - 10\,\hat{k})\text{ ft,} \tag{2}$$
$$\vec{r}_{AB} = \left[(20-300)\,\hat{\imath} + (150-50)\,\hat{\jmath} + (-5-2)\,\hat{k}\right]\text{ ft} \tag{3}$$
$$= (-280\,\hat{\imath} + 100\,\hat{\jmath} - 7\,\hat{k})\text{ ft,} \tag{4}$$
$$\vec{n} = \vec{r}_{AC} \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -20 & 400 & -10 \\ -280 & 100 & -7 \end{vmatrix}\text{ ft}^2 \tag{5}$$
$$= \Big\{\hat{\imath}\big[(400)(-7) - (-10)(100)\big] - \hat{\jmath}\big[(-20)(-7) - (-10)(-280)\big]$$
$$+ \hat{k}\big[(-20)(100) - (400)(-280)\big]\Big\}\text{ ft}^2 \tag{6}$$
$$= (-1800\,\hat{\imath} + 2660\,\hat{\jmath} + 110{,}000\,\hat{k})\text{ ft}^2, \tag{7}$$
$$n = |\vec{n}| = \sqrt{(1800)^2 + (2660)^2 + (110{,}000)^2}\text{ ft}^2 = 110{,}047\text{ ft}^2. \tag{8}$$

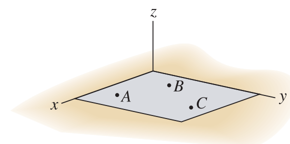
The angle θ between $\vec{n}$ and the vertical direction (z axis) can be obtained using the dot product of $\vec{n}$ with a vector in the z direction, namely $\hat{k}$. Thus,

$$\theta = \cos^{-1}\frac{\vec{n}\cdot\hat{k}}{n\;(1)} \tag{9}$$
$$= \cos^{-1}\frac{(-1800\text{ ft}^2)(0) + (2660\text{ ft}^2)(0) + (110{,}000\text{ ft}^2)(1)}{(110{,}047\text{ ft}^2)(1)} \tag{10}$$
$$= \boxed{1.672°.} \tag{11}$$

Since θ is between 1.5° and 2.5°, $\boxed{\text{the concrete surface meets the drainage specification.}}$

Problem 2.177

Repeat Prob. 2.176 if the coordinates of points A, B, and C are A (100, 20, 1) m, B (10, 50, −3) m, and C (110, 160, 2) m.



Solution

Our strategy will be to use the cross product to determine the normal direction to surface ABC, $\vec{n} = \vec{r}_{AC} \times \vec{r}_{AB}$. We will then take the dot product of $\vec{n}$ with $\hat{k}$ to determine the angle between $\vec{n}$ and $\hat{k}$.

$$\vec{r}_{AC} = \left[(110-100)\,\hat{\imath} + (160-20)\,\hat{\jmath} + (2-1)\,\hat{k}\right]\text{ m} \tag{1}$$

$$= (10\,\hat{\imath} + 140\,\hat{\jmath} + 1\,\hat{k})\text{ m}, \tag{2}$$

$$\vec{r}_{AB} = \left[(10-100)\,\hat{\imath} + (50-20)\,\hat{\jmath} + (-3-1)\,\hat{k}\right]\text{ m} \tag{3}$$

$$= (-90\,\hat{\imath} + 30\,\hat{\jmath} - 4\,\hat{k})\text{ m}, \tag{4}$$

$$\vec{n} = \vec{r}_{AC} \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 10 & 140 & 1 \\ -90 & 30 & -4 \end{vmatrix}\text{ m}^2 \tag{5}$$

$$= \Big\{\hat{\imath}\left[(140)(-4) - (1)(30)\right] - \hat{\jmath}\left[(10)(-4) - (1)(-90)\right] + \hat{k}\left[(10)(30) - (140)(-90)\right]\Big\}\text{ m}^2 \tag{6}$$

$$= (-590\,\hat{\imath} - 50\,\hat{\jmath} + 12{,}900\,\hat{k})\text{ m}^2, \tag{7}$$

$$n = |\vec{n}| = \sqrt{(-590)^2 + (-50)^2 + (12{,}900)^2}\text{ m}^2 = 12{,}914\text{ m}^2. \tag{8}$$

The angle θ between $\vec{n}$ and the vertical direction (z axis) can be obtained by using the dot product of $\vec{n}$ with a vector in the z direction, namely $\hat{k}$. Thus,

$$\theta = \cos^{-1}\frac{\vec{n}\cdot\hat{k}}{n\,(1)} \tag{9}$$

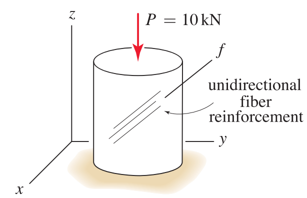
$$= \cos^{-1}\frac{(-590\text{ m}^2)(0) + (-50\text{ m}^2)(0) + (12{,}900\text{ m}^2)(1)}{(12{,}914\text{ m}^2)(1)} \tag{10}$$

$$= \boxed{2.628^\circ.} \tag{11}$$

Since θ is not between 1.5° and 2.5°, $\boxed{\text{the concrete surface does not meet the drainage specification.}}$

Problem 2.178

A specimen of composite material consisting of ceramic matrix and unidirectional ceramic fiber reinforcing is tested in a laboratory under compressive loading. If a 10 kN force is applied in the $-z$ direction, determine the components, and vector components, of this force in directions parallel and perpendicular to the fiber direction f, where this direction has direction angle $\theta_z = 40°$ and remaining direction angles that are equal (i.e., $\theta_x = \theta_y$).



Solution

Begin by determining the unit vector in the direction of the fiber, $\hat{f} = \cos\theta_x\,\hat{\imath} + \theta_y\,\hat{\jmath} + \theta_z\,\hat{k}$, where according to the problem statement, $\theta_x = \theta_y$ and $\theta_z = 40°$. Using Eq. (2.26),

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1, \tag{1}$$

we rearrange to obtain

$$\cos^2\theta_x + \cos^2\theta_y = 1 - \cos^2\theta_z \quad\Rightarrow\quad 2\cos^2\theta_x = 1 - \cos^2\theta_z, \tag{2}$$

which yields the solutions

$$\cos\theta_x = \pm\sqrt{\frac{1-\cos^2\theta_z}{2}} = \pm\sqrt{\frac{1-\cos^2(40°)}{2}} = \pm 0.4545. \tag{3}$$

Based on the figure in the problem statement, we select the positive solution in Eq. (3), namely $\theta_x = 0.4545$. The unit vector $\hat{f}$ is

$$\hat{f} = 0.4545\,\hat{\imath} + 0.4545\,\hat{\jmath} + 0.7660\,\hat{k}. \tag{4}$$

The force applied to the specimen is

$$\vec{P} = -10\,\hat{k}\ \text{kN}. \tag{5}$$

The component of $\vec{P}$ that is parallel to $\hat{f}$ is

$$\boxed{P_\parallel = \vec{P}\cdot\hat{f} = 0 + 0 + (-10\,\text{kN})(0.7660) = 7.66\,\text{kN}.} \tag{6}$$

The component of $\vec{P}$ that is perpendicular to $\hat{f}$ is

$$\boxed{P_\perp = \sqrt{P^2 - P_\parallel^2} = \sqrt{(10\,\text{kN})^2 - (7.660\,\text{kN})^2} = 6.43\,\text{kN}.} \tag{7}$$

To determine the vector component of $\vec{P}$ parallel to the the fiber direction $\hat{f}$, we simply multiply the component $P_\parallel$ by $\hat{f}$, noting that the force acts in the direction opposite to $\hat{f}$ (hence the negative sign):

$$\boxed{\begin{aligned}\vec{P}_\parallel &= P_\parallel\hat{f} = -(7.660\,\text{kN})(0.4545\,\hat{\imath} + 0.4545\,\hat{\jmath} + 0.7660\,\hat{k})\\ &= (-3.48\,\hat{\imath} - 3.48\,\hat{\jmath} - 5.87\,\hat{k})\ \text{kN}.\end{aligned}} \tag{8}$$

The vector component of $\vec{P}$ perpendicular to the the fiber direction is then

$$\boxed{\begin{aligned}\vec{P}_\perp &= \vec{P} - \vec{P}_\parallel = (-10\,\hat{k})\,\text{kN} - (-3.482\,\hat{\imath} - 3.482\,\hat{\jmath} - 5.868\,\hat{k})\,\text{kN} \\ &= (3.48\,\hat{\imath} + 3.48\,\hat{\jmath} - 4.13\,\hat{k})\,\text{kN}.\end{aligned}} \tag{9}$$

As a partial check of accuracy, we evaluate the magnitudes of Eqs. (8) and (9) to obtain

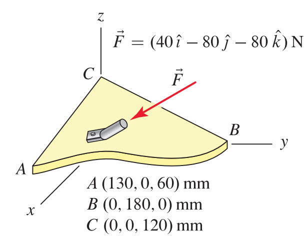
$$|\vec{P}_\parallel| = \sqrt{(-3.482)^2 + (-3.482)^2 + (-5.868)^2}\,\text{kN} = 7.660\,\text{kN}, \tag{10}$$

$$|\vec{P}_\perp| = \sqrt{(3.482)^2 + (3.482)^2 + (-4.132)^2}\,\text{kN} = 6.428\,\text{kN}, \tag{11}$$

and we observe that these values agree with the components found in Eqs. (6) and (7), respectively.

Problem 2.179

An automobile body panel is subjected to a force $\vec{F}$ from a stiffening strut. Assuming that region ABC of the panel is planar, determine the components, and vector components, of $\vec{F}$ in the directions normal and parallel to the panel.



Solution

The following vectors are obtained from the figure in the problem statement:

$$\vec{r}_{BC} = \left[(0-0)\,\hat{\imath} + (0-180)\,\hat{\jmath} + (120-0)\,\hat{k}\right]\text{mm} = (0\,\hat{\imath} - 180\,\hat{\jmath} + 120\,\hat{k}\text{ mm}, \tag{1}$$

$$\vec{r}_{BA} = \left[(130-0)\,\hat{\imath} + (0-180)\,\hat{\jmath} + (60-0)\,\hat{k}\right]\text{mm} = (130\,\hat{\imath} - 180\,\hat{\jmath} + 60\,\hat{k})\text{ mm}. \tag{2}$$

A vector $\vec{n}$ in the normal direction to the panel is given by

$$\vec{n} = \vec{r}_{BC} \times \vec{r}_{BA} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -180 & 120 \\ 130 & -180 & 60 \end{vmatrix} \text{mm}^2 \tag{3}$$

$$\begin{aligned} &= \{[(-180)(60) - (120)(-180)]\,\hat{\imath} - [(0)(60) - (120)(130)]\,\hat{\jmath} \\ &\quad + [(0)(-180) - (-180)(130)]\,\hat{k}\}\text{ mm}^2 \end{aligned} \tag{4}$$

$$= (108\times10^2\,\hat{\imath} + 156\times10^2\,\hat{\jmath} + 234\times10^2\,\hat{k})\text{ mm}^2, \tag{5}$$

$$n = \sqrt{(108\times10^2)^2 + (156\times10^2)^2 + (234\times10^2)^2}\text{ mm}^2 = 301.3\times10^2\text{ mm}^2. \tag{6}$$

The unit vector in the $\vec{n}$ direction is

$$\hat{n} = \frac{\vec{n}}{n} = \frac{\left(108\times10^2\,\hat{\imath} + 156\times10^2\,\hat{\jmath} + 234\times10^2\,\hat{k}\right)\text{ mm}^2}{301.3\times10^2\text{ mm}^2} = 0.3585\,\hat{\imath} + 0.5178\,\hat{\jmath} + 0.7767\,\hat{k}. \tag{7}$$

The component of $\vec{F}$ normal to the panel is found using the dot product as

$$\boxed{F_\perp = \vec{F}\cdot\hat{n} = (40\text{ N})(0.3585) + (-80\text{ N})(0.5178) + (-80\text{ N})(0.7767) = -89.2\text{ N}.} \tag{8}$$

The magnitude of the force $\vec{F}$ is

$$F = \sqrt{(40\text{ N})^2 + (-80\text{ N})^2 + (-80\text{ N})^2} = 120\text{ N}. \tag{9}$$

Hence, the component of the force tangent to the surface of the panel is

$$\boxed{F_\parallel = \sqrt{F^2 - F_\perp^2} = \sqrt{(120\text{ N})^2 - (-89.23\text{ N})^2} = 80.2\text{ N}.} \tag{10}$$

The vector component of $\vec{F}$ in the direction normal to the panel is given by

$$\boxed{\begin{aligned} \vec{F}_\perp &= F_\perp\hat{n} = (-89.23\text{ N})(0.3585\,\hat{\imath} + 0.5178\,\hat{\jmath} + 0.7767\,\hat{k}) \\ &= (-32.0\,\hat{\imath} - 46.2\,\hat{\jmath} - 69.3\,\hat{k})\text{ N}, \end{aligned}} \tag{11}$$

and the vector component of $\vec{F}$ in the direction tangent to the panel is given by

$$\boxed{\begin{aligned}\vec{F}_{\parallel} &= \vec{F} - \vec{F}_{\perp} = (40\,\hat{\imath} - 80\,\hat{\jmath} - 80\,\hat{k})\,\mathrm{N} - (-31.99\,\hat{\imath} - 46.20\,\hat{\jmath} - 69.31\,\hat{k})\,\mathrm{N} \\ &= (72.0\,\hat{\imath} - 33.8\,\hat{\jmath} - 10.7\,\hat{k})\,\mathrm{N}.\end{aligned}} \tag{12}$$

As a partial check of accuracy, we evaluate the magnitudes of Eqs. (11) and (12) to obtain

$$|\vec{F}_{\perp}| = \sqrt{(-31.99)^2 + (-46.20)^2 + (-69.31)^2}\,\mathrm{N} = 89.2\,\mathrm{N}, \tag{13}$$

$$|\vec{F}_{\parallel}| = \sqrt{(71.99)^2 + (-33.80)^2 + (-10.69)^2}\,\mathrm{N} = 80.2\,\mathrm{N}, \tag{14}$$

and we observe that these values agree with the absolute values of the components found in Eqs. (8) and (10), respectively.

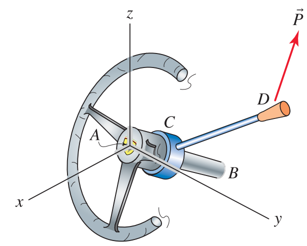
Problem 2.180

The steering wheel and gearshift lever of an automobile are shown. Points A, C, and B lie on the axis of the steering column, where point A is at the origin of the coordinate system, point B has the coordinates $B\,(-120, 0, -50)$ mm, and point C is 60 mm from point A. The gearshift lever from C to D has 240 mm length and the direction angles given below.

(a) Two of the direction angles for the position vector from point C to D are $\theta_y = 50°$ and $\theta_z = 60°$. Knowing that this position vector has a negative x component, determine θ_x.

(b) Determine the unit vector $\hat{r}$ that is perpendicular to directions AB and CD such that this vector has a positive z component.

(c) If the force applied to the gearshift lever is $\vec{P} = (-6\,\hat{\imath} - 4\,\hat{\jmath} + 12\,\hat{k})$ N, determine the component of $\vec{P}$, namely P_r, in the direction of $\hat{r}$.



Solution

Part (a) The y and z direction angles are given, and the remaining direction angle must satisfy

$$(\cos\theta_x)^2 + (\cos 50°)^2 + (\cos 60°)^2 = 1. \quad (1)$$

Solving for θ_x provides

$$\cos\theta_x = \pm\sqrt{1 - (\cos 50°)^2 - (\cos 60°)^2} \quad (2)$$

$$\theta_x = \cos^{-1} \pm\sqrt{1 - (\cos 50°)^2 - (\cos 60°)^2} \quad (3)$$

$$= 54.52°,\ 125.5°. \quad (4)$$

Since the position vector from C to D, namely $\vec{r}_{CD}$, has a negative x component, the correct direction angle is

$$\boxed{\theta_x = 125.5°.} \quad (5)$$

Part (b) Using the coordinates of points A and B given in the problem statement, and the direction angles from Part (a),

$$\vec{r}_{AB} = (-120\,\hat{\imath} - 0\,\hat{\jmath} - 50\,\hat{k})\text{ mm}, \quad (6)$$

$$\vec{r}_{CD} = (240\text{ mm})(\cos 125.5°\,\hat{\imath} + \cos 50°\,\hat{\jmath} + \cos 60°\,\hat{k}) \quad (7)$$

$$= (-139.3\,\hat{\imath} + 154.3\,\hat{\jmath} + 120\,\hat{k})\text{ mm}. \quad (8)$$

The vector $\vec{r}$ that is perpendicular to both $\vec{r}_{AB}$ and $\vec{r}_{CD}$, where we use the right hand rule to select the order of the vectors in the cross product so that the result will have a positive z component, is

$$\vec{r} = \vec{r}_{CD} \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -139.3 & 154.3 & 120 \\ -120 & 0 & -50 \end{vmatrix} \text{mm}^2 \tag{9}$$

$$= \Big\{ \hat{\imath} \left[(154.3)(-50) - (120)(0)\right] - \hat{\jmath} \left[(-139.3)(-50) - (120)(-120)\right]$$

$$+ \hat{k} \left[(-139.3)(0) - (154.3)(-120)\right]\Big\} \text{ mm}^2 \tag{10}$$

$$= (-7713\,\hat{\imath} - 21{,}364\,\hat{\jmath} + 18{,}512\,\hat{k}) \text{ mm}^2. \tag{11}$$

The unit vector in the direction of $\vec{r}$ is

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(-7713\,\hat{\imath} - 21{,}364\,\hat{\jmath} + 18{,}512\,\hat{k}) \text{ mm}^2}{29{,}303 \text{ mm}^2} \tag{12}$$

$$= \boxed{-0.2632\,\hat{\imath} - 0.7291\,\hat{\jmath} + 0.6318\,\hat{k}.} \tag{13}$$

Part (c)

$$P_r = \vec{P} \cdot \hat{r} = (-6 \text{ N})(-0.2632) + (-4 \text{ N})(-0.7291) + (12 \text{ N})(0.6318)$$

$$= \boxed{12.08 \text{ N.}} \tag{14}$$

Some checks of solution accuracy:

- Verify that $\theta_x = 125.5°$, $\theta_y = 50°$, and $\theta_z = 60°$ are indeed direction angles by checking that Eq. (1) is satisfied.
- Verify that Eq. (8) has 240 mm magnitude.
- Verify that $\vec{r}$ given by Eq. (11) is orthogonal to both $\vec{r}_{AB}$ and $\vec{r}_{CD}$ by checking that $\vec{r} \cdot \vec{r}_{AB} = 0$ and $\vec{r} \cdot \vec{r}_{CD} = 0$.
- Verify that $\hat{r}$ in Eq. (13) is a unit vector.

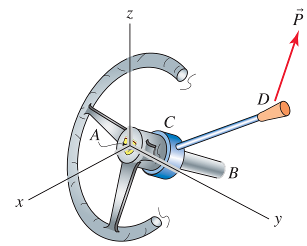
Problem 2.181

The steering wheel and gearshift lever of an automobile are shown. Points A, C, and B lie on the axis of the steering column, where point A is at the origin of the coordinate system, point B has the coordinates $B\,(-120, 0, -50)$ mm, and point C is 60 mm from point A. The gearshift lever from C to D has 240 mm length and the direction angles given below.

(a) Two of the direction angles for the position vector from point C to D are $\theta_y = 45°$ and $\theta_z = 70°$. Knowing that this position vector has a negative x component, determine θ_x.

(b) Determine the unit vector $\hat{r}$ that is perpendicular to directions AB and CD such that this vector has a positive z component.

(c) If the force applied to the gear shift lever is $\vec{P} = (-9\,\hat{\imath} - 6\,\hat{\jmath} + 18\,\hat{k})$ N, determine the component of $\vec{P}$, namely P_r, in the direction of $\hat{r}$.



Solution

Part (a) The y and z direction angles are given, and the remaining direction angle must satisfy

$$(\cos\theta_x)^2 + (\cos 45°)^2 + (\cos 70°)^2 = 1. \tag{1}$$

Solving for θ_x provides

$$\cos\theta_x = \pm\sqrt{1 - (\cos 45°)^2 - (\cos 70°)^2} \tag{2}$$

$$\theta_x = \cos^{-1} \pm\sqrt{1 - (\cos 45°)^2 - (\cos 70°)^2} \tag{3}$$

$$= 51.76°,\ 128.2°. \tag{4}$$

Since the position vector from C to D, namely $\vec{r}_{CD}$, has a negative x component, the correct direction angle is

$$\boxed{\theta_x = 128.2°.} \tag{5}$$

Part (b) Using the coordinates of points A and B given in the problem statement, and the direction angles from Part (a),

$$\vec{r}_{AB} = (-120\,\hat{\imath} - 0\,\hat{\jmath} - 50\,\hat{k})\ \text{mm}, \tag{6}$$

$$\vec{r}_{CD} = (240\ \text{mm})(\cos 128.2°\,\hat{\imath} + \cos 45°\,\hat{\jmath} + \cos 70°\,\hat{k}) \tag{7}$$

$$= (-148.5\,\hat{\imath} + 169.7\,\hat{\jmath} + 82.08\,\hat{k})\ \text{mm}. \tag{8}$$

The vector $\vec{r}$ that is perpendicular to both $\vec{r}_{AB}$ and $\vec{r}_{CD}$, where we use the right hand rule to select the order of the vectors in the cross product so that the result will have positive z component, is

$$\vec{r} = \vec{r}_{CD} \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -148.5 & 169.7 & 82.08 \\ -120 & 0 & -50 \end{vmatrix} \text{mm}^2 \tag{9}$$

$$= \Big\{ \hat{\imath} \big[(169.7)(-50) - (82.08)(0)\big] - \hat{\jmath} \big[(-148.5)(-50) - (82.08)(-120)\big]$$
$$+ \hat{k} \big[(-148.5)(0) - (169.7)(-120)\big]\Big\} \text{ mm}^2 \tag{10}$$

$$= (-8485\,\hat{\imath} - 17{,}277\,\hat{\jmath} + 20{,}365\,\hat{k}) \text{ mm}^2. \tag{11}$$

The unit vector in the direction of $\vec{r}$ is

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(-8485\,\hat{\imath} - 17{,}277\,\hat{\jmath} + 20{,}365\,\hat{k}) \text{ mm}^2}{28{,}022 \text{ mm}^2} \tag{12}$$

$$= \boxed{-0.3028\,\hat{\imath} - 0.6166\,\hat{\jmath} + 0.7268\,\hat{k}.} \tag{13}$$

Part (c)

$$P_r = \vec{P} \cdot \hat{r} = (-9 \text{ N})(-0.3028) + (-6 \text{ N})(-0.6166) + (18 \text{ N})(0.7268)$$
$$= \boxed{19.51 \text{ N.}} \tag{14}$$

Some checks of solution accuracy:

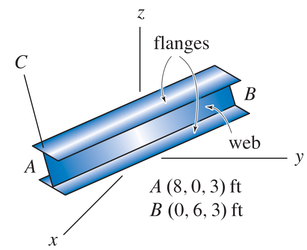
- Verify that $\theta_x = 128.2°$, $\theta_y = 45°$, and $\theta_z = 70°$ are indeed direction angles by checking that Eq. (1) is satisfied.
- Verify that Eq. (8) has 240 mm magnitude.
- Verify that $\vec{r}$ given by Eq. (11) is orthogonal to both $\vec{r}_{AB}$ and $\vec{r}_{CD}$ by checking that $\vec{r} \cdot \vec{r}_{AB} = 0$ and $\vec{r} \cdot \vec{r}_{CD} = 0$.
- Verify that $\hat{r}$ in Eq. (13) is a unit vector.

Problem 2.182

An I beam is positioned from points A to B. Because its strength and deformation properties for bending about an axis through the web of the cross section are different than those for bending about an axis parallel to the flanges, it is necessary to also characterize these directions. This can be accomplished by specifying just one of the direction angles for the direction of the web from A to C, which is perpendicular to line AB, plus the octant of the coordinate system in which line AC lies.

(a) If direction angle $\theta_z = 30°$ for line AC, determine the remaining direction angles for this line.

(b) Determine the unit vector in the direction perpendicular to the web of the beam (i.e., perpendicular to lines AB and AC).



Solution

Part (a) The two unknowns to be determined are the x and y direction angles for line AC, namely θ_x and θ_y. It will be necessary to write two equation so that these unknowns may be determined. Our strategy will be to write an expression requiring line AC to be perpendicular to line AB (we will use $\hat{r}_{AC} \cdot \vec{r}_{AB} = 0$), and we will write an expression requiring the squares of the direction cosines to sum to one. Based on the problem description, the following position vectors may be written

$$\begin{aligned} \hat{r}_{AC} &= \cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + \cos\theta_z\,\hat{k}, \\ &= \cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + \cos 30°\,\hat{k}, \\ &= \cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + 0.8660\,\hat{k}, && (1) \\ \vec{r}_{AB} &= (-8\,\hat{\imath} + 6\,\hat{\jmath} + 0\,\hat{k})\,\text{ft}, && (2) \\ r_{AB} &= 10\,\text{ft}. && (3) \end{aligned}$$

Note that $\hat{r}_{AC}$ is a unit vector, while $\vec{r}_{AB}$ is not. As described in the problem statement, vectors $\hat{r}_{AC}$ and $\vec{r}_{AB}$ are orthogonal, hence

$$\hat{r}_{AC} \cdot \vec{r}_{AB} = 0$$
$$\cos\theta_x(-8\,\text{ft}) + \cos\theta_y(6\,\text{ft}) + (0.8660)(0) = 0. \tag{4}$$

Equation (4) gives

$$\cos\theta_x = \tfrac{3}{4}\cos\theta_y. \tag{5}$$

The three direction angles must also satisfy

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2 30° = 1$$
$$\cos^2\theta_x + \cos^2\theta_y + (0.8660)^2 = 1. \tag{6}$$

Substituting Eq. (5) into Eq. (6) gives

$$\left(\tfrac{3}{4}\cos\theta_y\right)^2 + \cos^2\theta_y + (0.8660)^2 = 1 \tag{7}$$

$$\Rightarrow \quad \cos\theta_y = \pm\sqrt{\frac{1-(0.8660)^2}{(3/4)^2+1}} = \pm 0.4000 \tag{8}$$

$$\Rightarrow \quad \theta_y = 66.42°, 113.6°. \tag{9}$$

Referring to the figure in the problem statement, the position vector $\hat{r}_{AC}$ has negative y component, thus we select the negative solution for $\cos\theta_y$ in Eq. (8) and the corresponding value of θ_y in Eq. (9), i.e.,

$$\cos\theta_y = -0.4000 \quad \Rightarrow \quad \boxed{\theta_y = 114°.} \tag{10}$$

Substituting $\cos\theta_y = -0.4000$ into Eq. (5) provides

$$\cos\theta_x = \tfrac{3}{4}(-0.4000) = -0.3000 \quad \Rightarrow \quad \boxed{\theta_x = 107°.} \tag{11}$$

Part (b) The unit vector $\hat{r}_\perp$ that is perpendicular to AB and AC is found using the cross product as

$$\hat{r}_\perp = \frac{\vec{r}_{AB}}{r_{AB}} \times \hat{r}_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -8/10 & 6/10 & 0 \\ -0.3000 & -0.4000 & 0.8600 \end{vmatrix} \tag{12}$$

$$= [(0.6)(0.8600) - 0]\,\hat{\imath} - [(-0.8)(0.8600) - 0]\,\hat{\jmath} + [(-0.8)(-0.4) - (0.6)(-0.3)]\,\hat{k} \tag{13}$$

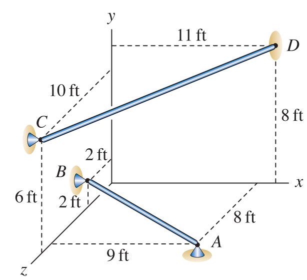
$$= \boxed{0.520\,\hat{\imath} + 0.693\,\hat{\jmath} + 0.500\,\hat{k}.} \tag{14}$$

Note that an acceptable answer is also obtained if the vectors in the cross product of Eq. (12) are interchanged, thus the following is also a correct answer

$$\boxed{\hat{r}_\perp = -0.520\,\hat{\imath} - 0.693\,\hat{\jmath} - 0.500\,\hat{k}.} \tag{15}$$

Problem 2.183

Determine the smallest distance between the infinite lines passing through bars AB and CD.



Solution

Our strategy will be to write a position vector from some convenient point on one bar to some convenient point on the other bar, and we will use $\vec{r}_{BC}$. We will then determine the portion (or component) of this vector that acts in the normal direction to vectors $\vec{r}_{CD}$ and $\vec{r}_{AB}$, and the result of this is the smallest distance between the two bars.

Based on the figure in the problem statement, the following position vectors may be written

$$\vec{r}_{BC} = (4\,\hat{\jmath} + 8\,\hat{k})\,\text{ft}, \tag{1}$$

$$\vec{r}_{AB} = (-9\,\hat{\imath} + 2\,\hat{\jmath} - 6\,\hat{k})\,\text{ft}, \tag{2}$$

$$\vec{r}_{CD} = (11\,\hat{\imath} + 2\,\hat{\jmath} - 10\,\hat{k})\,\text{ft}. \tag{3}$$

The direction perpendicular to the two bars is $\vec{n}$, computed using the cross product as

$$\vec{n} = \vec{r}_{CD} \times \vec{r}_{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 11 & 2 & -10 \\ -9 & 2 & -6 \end{vmatrix}\,\text{ft}^2 \tag{4}$$

$$= \Big\{[(2)(-6) - (-10)(2)]\,\hat{\imath} - [(11)(-6) - (-10)(-9)]\,\hat{\jmath} + [(11)(2) - (2)(-9)]\,\hat{k}\Big\}\,\text{ft}^2 \tag{5}$$

$$= (8\,\hat{\imath} + 156\,\hat{\jmath} + 40\,\hat{k})\,\text{ft}^2, \tag{6}$$

$$n = \sqrt{(8\,\text{ft}^2)^2 + (156\,\text{ft}^2)^2 + (40\,\text{ft}^2)^2} = 161.2\,\text{ft}^2. \tag{7}$$

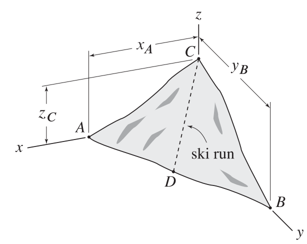
The component of $\vec{r}_{BC}$ in the direction $\vec{n}$ is the distance d between the bars. Hence

$$\boxed{d = \vec{r}_{BC} \cdot \frac{\vec{n}}{n} = \frac{(0\,\text{ft})(8) + (4\,\text{ft})(156) + (8\,\text{ft})(40)}{161.2} = 5.85\,\text{ft}.} \tag{8}$$

Problem 2.184

A portion of a downhill ski run between points C and D is to be constructed on a mountainside, as shown. Let the portion of the mountainside defined by points A, B, and C be idealized to be planar. For the values of x_A, y_B, and z_C given below, determine the distance from point A to point D where the ski run should intersect line AB so that the run will be as steep as possible. *Hint:* Consider a force acting on the slope in the $-z$ direction, such as perhaps a skier's weight – or better yet, a 1 lb weight – and follow the approach used in Example 2.20 on p. 108 to resolve this weight into normal and tangential components; the direction of the tangential component will give the direction of steepest descent.

$x_A = 1500$ ft, $y_B = 2000$ ft, and $z_C = 800$ ft.



Solution

Our strategy will be to first determine the normal direction to the slope, $\vec{n}$. We will then follow the hint given in the problem statement and consider a 1 lb weight resting on the slope. We will then determine the components and vector components of this weight in directions normal and tangent to the slope. Finally, we will use vector addition to determine the distance from point A to point D.

Using the coordinates given in the problem statement,

$$\vec{r}_{AB} = (-1500\,\hat{\imath} + 2000\,\hat{\jmath})\text{ ft}, \qquad r_{AB} = 2500\text{ ft}, \tag{1}$$

$$\vec{r}_{AC} = (-1500\,\hat{\imath} + 800\,\hat{k})\text{ ft}, \qquad r_{AC} = 1700\text{ ft}. \tag{2}$$

The normal direction to the slope is

$$\vec{n} = \vec{r}_{AB} \times \vec{r}_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1500 & 2000 & 0 \\ -1500 & 0 & 800 \end{vmatrix} \text{ft}^2 \tag{3}$$

$$= \Big\{ \hat{\imath}\,\big[(2000)(800) - (0)(0)\big] - \hat{\jmath}\,\big[(-1500)(800) - (0)(-1500)\big] + \hat{k}\,\big[(-1500)(0) - (2000)(-1500)\big] \Big\}\text{ ft}^2 \tag{4}$$

$$= (1.6 \times 10^6\,\hat{\imath} + 1.2 \times 10^6\,\hat{\jmath} + 3 \times 10^6\,\hat{k})\text{ ft}^2, \tag{5}$$

$$n = |\vec{n}| = 3.606 \times 10^6\text{ ft}^2. \tag{6}$$

The unit vector in the direction of $\vec{n}$ is

$$\hat{n} = \frac{\vec{n}}{n} = \frac{(1.6 \times 10^6\,\hat{\imath} + 1.2 \times 10^6\,\hat{\jmath} + 3 \times 10^6\,\hat{k})\text{ ft}^2}{3.606 \times 10^6\text{ ft}^2} \tag{7}$$

$$= 0.4438\,\hat{\imath} + 0.3328\,\hat{\jmath} + 0.8321\,\hat{k}. \tag{8}$$

The vector describing a 1 lb weight resting on the slope is

$$\vec{W} = -1\text{ lb}\,\hat{k}. \tag{9}$$

The component of this weight in the normal direction to the slope is

$$W_n = \vec{W} \cdot \hat{n} = (0)(0.4438) + (0)(0.3328) + (-1\text{ lb})(0.8321) = -0.8321\text{ lb} \tag{10}$$

The vector component of the weight in the normal direction is

$$\vec{W}_n = W_n\,\hat{n} = (-0.8321\text{ lb})(0.4438\,\hat{\imath} + 0.3328\,\hat{\jmath} + 0.8321\,\hat{k}) \tag{11}$$
$$= (-0.3692\,\hat{\imath} - 0.2769\,\hat{\jmath} - 0.6923\,\hat{k})\text{ lb}. \tag{12}$$

The vector component of the weight that is tangent to the slope is $\vec{W}_t$, and noting that $\vec{W} = \vec{W}_t + \vec{W}_n$, we have

$$\vec{W}_t = \vec{W} - \vec{W}_n \tag{13}$$
$$= -1\text{ lb}\,\hat{k} - (-0.3692\,\hat{\imath} - 0.2769\,\hat{\jmath} - 0.6923\,\hat{k})\text{ lb} \tag{14}$$
$$= (0.3692\,\hat{\imath} + 0.2769\,\hat{\jmath} - 0.3077\,\hat{k})\text{ lb}, \tag{15}$$
$$W_t = \left|\vec{W}_t\right| = \sqrt{(0.3692\text{ lb})^2 + (0.2769\text{ lb})^2 + (-0.3077\text{ lb})^2} = 0.5547\text{ lb}. \tag{16}$$

As a partial check of solution accuracy, we use the Pythagorean theorem $W = \sqrt{W_t^2 + W_n^2}$, where $W = 1$ lb and W_n is given by Eq. (10), to write

$$W_t = \sqrt{W^2 - W_n^2} = \sqrt{(1\text{ lb})^2 - (-0.8321\text{ lb})^2} = 0.5547\text{ lb}, \tag{17}$$

and we observe that this result agrees with Eq. (16).

The unit vector in the direction of $\vec{W}_t$ is

$$\hat{t} = \frac{\vec{W}_t}{W_t} = \frac{(0.3692\,\hat{\imath} + 0.2769\,\hat{\jmath} - 0.3077\,\hat{k})\text{ lb}}{0.5547\text{ lb}} \tag{18}$$
$$= 0.6656\,\hat{\imath} + 0.4992\,\hat{\jmath} - 0.5547\,\hat{k}. \tag{19}$$

The position vector from C to D is

$$\vec{r}_{CD} = r_{CD}\,\hat{t} = r_{CD}(0.6656\,\hat{\imath} + 0.4992\,\hat{\jmath} - 0.5547\,\hat{k}). \tag{20}$$

We then use vector addition to write

$$\vec{r}_{CD} = \vec{r}_{CA} + \vec{r}_{AD} = -\vec{r}_{AC} + r_{AD}\,\frac{\vec{r}_{AB}}{r_{AB}}, \tag{21}$$

which becomes

$$r_{CD}(0.6656\,\hat{\imath} + 0.4992\,\hat{\jmath} - 0.5547\,\hat{k}) = -(-1500\,\hat{\imath} + 800\,\hat{k})\text{ ft} + r_{AD}\,\frac{(-1500\,\hat{\imath} + 2000\,\hat{\jmath})\text{ ft}}{2500\text{ ft}}. \tag{22}$$

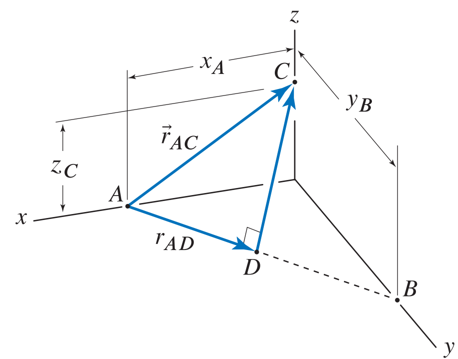
Equating terms that multiply $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ gives the following three scalar equations

$$r_{CD}(0.6656) = 1500\text{ ft} - r_{AD}\,\frac{1500}{2500}, \tag{23}$$
$$r_{CD}(0.4992) = r_{AD}\,\frac{2000}{2500}, \tag{24}$$
$$r_{CD}(-0.5547) = -800\text{ ft}. \tag{25}$$

Solving Eq. (25) for r_{CD} and then solving Eq. (24) for r_{AD} provides

$$r_{CD} = 1442\text{ ft} \quad\text{and}\quad r_{AD} = 900\text{ ft}. \tag{26}$$

Note that these solutions also satisfy Eq. (23). Thus, $\boxed{\text{the distance from point } A \text{ to point } D \text{ is 900 ft.}}$



Alternative solution Rather than using the hint given in the problem statement, a straightforward, although perhaps less intuitive, alternative solution can developed by noting that the run will be steepest when lines AB and CD are perpendicular, as shown at the right. With the dimensions given in the problem statement, the following position vectors may be written.

$$\vec{r}_{AC} = (-1500\,\hat{\imath} + 800\,\hat{k})\text{ ft}, \tag{27}$$

$$\vec{r}_{AB} = (-1500\,\hat{\imath} + 2000\,\hat{\jmath})\text{ ft}, \quad r_{AB} = 2500\text{ ft}. \tag{28}$$

The distance r_{AD} is then obtained using

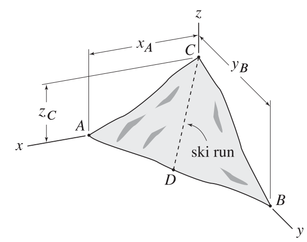
$$r_{AD} = \vec{r}_{AC} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{(-1500)(-1500) + (0)(2000) + (800)(0)}{2500} = 900\text{ ft}. \tag{29}$$

Thus, $\boxed{\text{the distance from point } A \text{ to point } D \text{ is 900 ft.}}$

Problem 2.185

A portion of a downhill ski run between points C and D is to be constructed on a mountainside, as shown. Let the portion of the mountainside defined by points A, B, and C be idealized to be planar. For the values of x_A, y_B, and z_C given below, determine the distance from point A to point D where the ski run should intersect line AB so that the run will be as steep as possible. *Hint:* Consider a force acting on the slope in the $-z$ direction, such as perhaps a skier's weight – or better yet, a 1 lb weight – and follow the approach used in Example 2.20 on p. 108 to resolve this weight into normal and tangential components; the direction of the tangential component will give the direction of steepest descent.

$x_A = 1200\,\text{ft}$, $y_B = 1600\,\text{ft}$, and $z_C = 900\,\text{ft}$.



Solution

Our strategy will be to first determine the normal direction to the slope, $\vec{n}$. We will then follow the hint given in the problem statement and consider a 1 lb weight resting on the slope. We will then determine the components and vector components of this weight in directions normal and tangent to the slope. Finally, we will use vector addition to determine the distance from point A to point D.

Using the coordinates given in the problem statement,

$$\vec{r}_{AB} = (-1200\,\hat{\imath} + 1600\,\hat{\jmath})\ \text{ft}, \qquad r_{AB} = 2000\ \text{ft}, \tag{1}$$

$$\vec{r}_{AC} = (-1200\,\hat{\imath} + 900\,\hat{k})\ \text{ft}, \qquad r_{AC} = 1500\ \text{ft}. \tag{2}$$

The normal direction to the slope is

$$\vec{n} = \vec{r}_{AB} \times \vec{r}_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1200 & 1600 & 0 \\ -1200 & 0 & 900 \end{vmatrix}\ \text{ft}^2 \tag{3}$$

$$= \Big\{ \hat{\imath}\,\big[(1600)(900) - (0)(0)\big] - \hat{\jmath}\,\big[(-1200)(900) - (0)(-1200)\big] + \hat{k}\,\big[(-1200)(0) - (1600)(-1200)\big]\Big\}\ \text{ft}^2 \tag{4}$$

$$= (1.44 \times 10^6\,\hat{\imath} + 1.08 \times 10^6\,\hat{\jmath} + 1.92 \times 10^6\,\hat{k})\ \text{ft}^2, \tag{5}$$

$$n = |\vec{n}| = 2.632 \times 10^6\ \text{ft}^2. \tag{6}$$

The unit vector in the direction of $\vec{n}$ is

$$\hat{n} = \frac{\vec{n}}{n} = \frac{(1.44 \times 10^6\,\hat{\imath} + 1.08 \times 10^6\,\hat{\jmath} + 1.92 \times 10^6\,\hat{k})\ \text{ft}^2}{2.632 \times 10^6\ \text{ft}^2} \tag{7}$$

$$= 0.5472\,\hat{\imath} + 0.4104\,\hat{\jmath} + 0.7295\,\hat{k}. \tag{8}$$

The vector describing a 1 lb weight resting on the slope is

$$\vec{W} = -1\ \text{lb}\,\hat{k}. \tag{9}$$

The component of this weight in the normal direction to the slope is

$$W_n = \vec{W} \cdot \hat{n} = (0)(0.5472) + (0)(0.4104) + (-1\ \text{lb})(0.7295) = -0.7295\ \text{lb}. \tag{10}$$

The vector component of the weight in the normal direction is

$$\vec{W}_n = W_n\,\hat{n} = (-0.7295\text{ lb})(0.5472\,\hat{\imath} + 0.4104\,\hat{\jmath} + 0.7295\,\hat{k}) \tag{11}$$
$$= (-0.3992\,\hat{\imath} - 0.2994\,\hat{\jmath} - 0.5322\,\hat{k})\text{ lb}. \tag{12}$$

The vector component of the weight that is tangent to the slope is $\vec{W}_t$, and noting that $\vec{W} = \vec{W}_t + \vec{W}_n$, we have

$$\vec{W}_t = \vec{W} - \vec{W}_n \tag{13}$$
$$= -1\text{ lb}\,\hat{k} - (-0.3992\,\hat{\imath} - 0.2994\,\hat{\jmath} - 0.5332\,\hat{k})\text{ lb} \tag{14}$$
$$= (0.3992\,\hat{\imath} + 0.2994\,\hat{\jmath} - 0.4678\,\hat{k})\text{ lb}, \tag{15}$$
$$W_t = \left|\vec{W}_t\right| = \sqrt{(0.3992\text{ lb})^2 + (0.2994\text{ lb})^2 + (-0.4678\text{ lb})^2} = 0.6839\text{ lb}. \tag{16}$$

As a partial check of solution accuracy, we use the Pythagorean theorem $W = \sqrt{W_t^2 + W_n^2}$, where $W = 1$ lb and W_n is given by Eq. (10), to write

$$W_t = \sqrt{W^2 - W_n^2} = \sqrt{(1\text{ lb})^2 - (-0.7295\text{ lb})^2} = 0.6839\text{ lb}, \tag{17}$$

and we observe that this result agrees with Eq. (16).

The unit vector in the direction of $\vec{W}_t$ is

$$\hat{t} = \frac{\vec{W}_t}{W_t} = \frac{(0.3992\,\hat{\imath} + 0.2994\,\hat{\jmath} - 0.4678\,\hat{k})\text{ lb}}{0.6839\text{ lb}} \tag{18}$$
$$= 0.5836\,\hat{\imath} + 0.4377\,\hat{\jmath} - 0.6839\,\hat{k}. \tag{19}$$

The position vector from C to D is

$$\vec{r}_{CD} = r_{CD}\,\hat{t} = r_{CD}(0.5836\,\hat{\imath} + 0.4377\,\hat{\jmath} - 0.6839\,\hat{k}). \tag{20}$$

We then use vector addition to write

$$\vec{r}_{CD} = \vec{r}_{CA} + \vec{r}_{AD} = -\vec{r}_{AC} + r_{AD}\,\frac{\vec{r}_{AB}}{r_{AB}}, \tag{21}$$

which becomes

$$r_{CD}(0.5836\,\hat{\imath} + 0.4377\,\hat{\jmath} - 0.6839\,\hat{k}) = -(-1200\,\hat{\imath} + 900\,\hat{k})\text{ ft} + r_{AD}\,\frac{(-1200\,\hat{\imath} + 1600\,\hat{\jmath})\text{ ft}}{2000\text{ ft}}. \tag{22}$$

Equating terms that multiply $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ gives the following three scalar equations

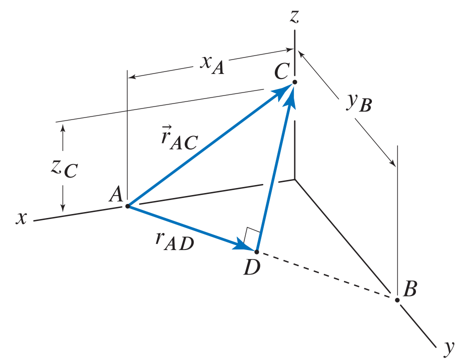
$$r_{CD}(0.5836) = 1200\text{ ft} - r_{AD}\frac{1200}{2000}, \tag{23}$$
$$r_{CD}(0.4377) = r_{AD}\frac{1600}{2000}, \tag{24}$$
$$r_{CD}(-0.6839) = -900\text{ ft}. \tag{25}$$

Solving Eq. (25) for r_{CD} and then solving Eq. (24) for r_{AD} provides

$$r_{CD} = 1316\text{ ft} \quad \text{and} \quad r_{AD} = 720\text{ ft}. \tag{26}$$

Note that these solutions also satisfy Eq. (23). Thus, $\boxed{\text{the distance from point } A \text{ to point } D \text{ is 720 ft.}}$

Alternative solution Rather than using the hint given in the problem statement, a straightforward, although perhaps less intuitive, alternative solution can developed by noting that the run will be steepest when lines AB and CD are perpendicular, as shown at the right. With the dimensions given in the problem statement, the following position vectors may be written.



$$\vec{r}_{AC} = (-1200\,\hat{\imath} + 900\,\hat{k})\,\text{ft}, \tag{27}$$

$$\vec{r}_{AB} = (-1200\,\hat{\imath} + 1600\,\hat{\jmath})\,\text{ft}, \quad r_{AB} = 2000\,\text{ft}. \tag{28}$$

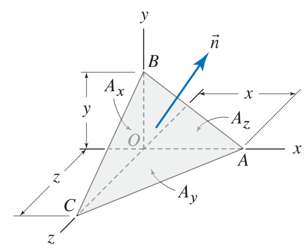
The distance r_{AD} is then obtained using

$$r_{AD} = \vec{r}_{AC} \cdot \frac{\vec{r}_{AB}}{r_{AB}} = \frac{(-1200)(-1200) + (0)(1600) + (900)(0)}{2000} = 720\,\text{ft}. \tag{29}$$

Thus, $\boxed{\text{the distance from point } A \text{ to point } D \text{ is 720 ft.}}$

Problem 2.186

The tetrahedron shown arises in advanced mechanics, and it is necessary to relate the areas of the four surfaces. Show that the surface areas are related by $A_x = A\cos\theta_x$, $A_y = A\cos\theta_y$, and $A_z = A\cos\theta_z$ where A is the area of surface ABC, and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for normal direction $\vec{n}$. *Hint:* Find $\vec{n}$ by taking the cross product of vectors along edges AB, AC, and/or BC, and note the magnitude of this vector is $2A$. Then, by inspection, write expressions for A_x, A_y, and A_z (e.g., $A_x = yz/2$ and so on).



Solution

Using the suggestion in the problem statement, begin by writing the vectors

$$\vec{r}_{AB} = -x\,\hat{\imath} + y\,\hat{\jmath} + 0\,\hat{k} \quad \text{and} \quad \vec{r}_{AC} = -x\,\hat{\imath} + 0\,\hat{\jmath} + z\,\hat{k}. \tag{1}$$

Next, the vector $\vec{n}$ that is normal to surface ABC is found using

$$\vec{n} = \vec{r}_{AB} \times \vec{r}_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -x & y & 0 \\ -x & 0 & z \end{vmatrix} \tag{2}$$

$$= [(y)(z) - (0)(0)]\,\hat{\imath} - [(-x)(z) - (0)(-x)]\,\hat{\jmath} + [(-x)(0) - (y)(-x)]\,\hat{k} \tag{3}$$

$$= yz\,\hat{\imath} + xz\,\hat{\jmath} + xy\,\hat{k}, \tag{4}$$

$$n = \sqrt{(yz)^2 + (xz)^2 + (xy)^2} = 2A, \tag{5}$$

where A is the area of surface ABC. From the figure in the problem statement, note that $A_x = yz/2$, $A_y = xz/2$, and $A_z = xy/2$. Therefore, using Eqs. (4) and (5), the unit vector $\hat{n}$ in the direction normal to surface ABC may be written as

$$\hat{n} = \frac{\vec{n}}{n} = \frac{2A_x\,\hat{\imath} + 2A_y\,\hat{\jmath} + 2A_z\,\hat{k}}{2A}. \tag{6}$$

Alternatively, $\hat{n}$ may be written as

$$\hat{n} = \cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + \cos\theta_z\,\hat{k}, \tag{7}$$

where θ_x, θ_y, and θ_z are the direction angles for the normal vector to surface ABC. For Eqs. (6) and (7) to agree requires that the terms multiplying $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ each be equal. Hence

$$\boxed{\begin{aligned} \frac{2A_x}{2A} &= \cos\theta_x \quad \Rightarrow \quad A_x = A\cos\theta_x, & (8)\\ \frac{2A_y}{2A} &= \cos\theta_y \quad \Rightarrow \quad A_y = A\cos\theta_y, & (9)\\ \frac{2A_z}{2A} &= \cos\theta_z \quad \Rightarrow \quad A_z = A\cos\theta_z. & (10) \end{aligned}}$$