Instructor's Manual

Essential Mathematics for Economic Analysis

Fifth edition

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Preface

This Instructor's Manual accompanies *Essential Mathematics for Economic Analysis*, 5th Edition. Its main purpose is to provide instructors with a collection of problems that might be used for tutorials and exams. It supplements the problems in MyMathLab.

Most of the problems are taken from previous exams and problem sets at the Department of Economics, University of Oslo, and at Stanford University. We have endeavoured to select problems of varying difficulty, including some problems that might challenge even the best students. The number in parentheses after each problem indicates the appropriate section of the text that should be covered before attempting the (whole) problem.

For each chapter we offer some comments on the text. Sometimes we explain why certain topics are included and others are excluded. There are also occasional hints based on our experience of teaching the material. In some cases, we also comment on alternative approaches, sometimes with mild criticism of other ways of dealing with the material that we believe to be less suitable.

Chapters 2 and 3 in the main text review elementary algebra. This manual includes a Test 1 (page 213), designed for the students themselves to see if they need to review particular sections of Chapters 2 and 3. Many students using our text will probably have some background in calculus. The accompanying Test 2 (page 216) is designed to give information to both the students and the instructors about what students actually know about single variable calculus, and about what needs to be studied more closely, perhaps in Chapters 6–9 of the text.

For instructors who are unwilling to spend more than 5–10 minutes for a test of essentials, we have made Test 0 (page 211). Based on our experience, some instructors might be in for a shock if this test is given to students who have been away from mathematics for some time, even if they have an acceptable mathematical background.

As with the main text, we would like to acknowledge the extensive help from Cristina Maria Igreja in converting the original plain TEX files for this manual into LATEX.

Oslo and Coventry, April 2016

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CHAPTER 1

Essentials of Logic and Set Theory

Section 1.2 offers a very brief introduction to some key concepts in logic, and Section 1.3 attempts to give ambitious students a short discussion of proofs. Set theory, treated in Section 1.1, is in our opinion, not crucial for economics students, except when the need for it arises in their statistics courses.

Problem 1-01 (1.1)

In a group of 100 students, 25 study economics, 30 study political science, and 5 study both subjects. How many students study neither economics nor political science?

Problem 1-02 (1.1)

Given the sets $A = \{2, 3, 4, 5\}$, $B = \{1, 2, 3, 4, 7\}$, and $C = \{1, 3, 6, 7\}$, which of the following statements are true?

(a) $2 \in A \cap B$ (b) $(A \cup B) \cap C = \{1, 3, 7\}$ (c) $(A \setminus B) \cap C = \{2\}$ (d) $A \cap C \subseteq B$

Problem 1-03 (1.1)

Let A, B, and C be three sets. Which of the following statements are true? (Use Venn diagrams.)

(a) $A \cap B = A \cap C$ and $A \neq \emptyset \Rightarrow B = C$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $(A \setminus B) \setminus C = A \setminus (B \cup C)$

(d) $A \subseteq B \Rightarrow A \cup (B \setminus A) = B$

Problem 1-04 (1.2)

Which of the following statements are true?

(a) $x^3 + y^3 = 0 \Leftrightarrow x = y = 0$

(b) $x^2(1 + x) > 0 \Leftrightarrow x > -1$ and $x \neq 0$

(c) $x = \sqrt{16} \Rightarrow x^2 = 16$

(d) $x = 3$ and $y = 5 \Rightarrow 2x + 4y = 26$

Problem 1-05 (1.2)

Which of the following implications can be reversed?

(a) $x = 3 \Rightarrow x^3 = 27$

(b) $x = 0 \Rightarrow x(x^4 + 1) = 0$

(c) $x \geq 3 \Rightarrow (x + 2)^2(x - 3) \geq 0$

(d) $x = 3 \Rightarrow \sqrt{1 + x} = 5 - x$

Problem 1-06 (1.2)

Consider the statement: "A matrix can have an inverse only if its determinant is not 0." Which of the following statements express the same? (You do not need to know the meaning of the concepts.)

(a) A sufficient condition for a matrix to have an inverse is that its determinant is not 0.

(b) A matrix with determinant equal to 0 has no inverse.

(c) A necessary condition for a matrix to have an inverse is that its determinant is not 0.

Problem 1-07 (1.3)

Prove that $\sqrt{2}+3$ is irrational.

Problem 1-08 (Harder problem.) (1.3)

Let a and b be positive rational numbers. Prove that if $\sqrt{a}+\sqrt{b}$ is rational, so is $\sqrt{a}$.

Problem 1-09 (1.4)

Prove by induction that for all natural numbers $n \geq 3$,

$$2n+1<2^n \qquad (*)$$

Problem 1-10 (1.4)

Prove by induction that the following equations hold for all natural numbers n.

(a) $1\cdot 2+2\cdot 3+\cdots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$

(b) $1^3+2^3+\cdots+n^3=\frac{1}{4}n^2(n+1)^2$

Problem 1-11

Let n be a positive integer and consider the expression $s_n = n^2 - n + 41$. Verify that s_n is a prime number (and so has no factor except 1 and itself) for n = 1, 2, 3, 4, and 5. With some effort, one can prove that s_n is a prime number for n = 6, 7, …, 40 as well. Is s_n a prime for all n? (This problem was first suggested by the Swiss mathematician L. Euler.)

CHAPTER 2

Algebra

The main purpose of this chapter in the text is to help those students who need to review elementary algebra. (Those who never learned it will need more intensive help than a text of this kind can provide.)

We strongly advise instructors to test the elementary algebra level of the students at the outset of the course, using Test 1 on page 213, or at least Test 0 on page 211. Reports we have received suggest that instructors who are not used to giving such tests, sometimes have been shocked by the results when they do, and have had to adjust the start of their course accordingly. But we do feel that reviewing elementary algebra should primarily be left to the individual students. That's why the text supplies a rather extensive review with many problems.

We recommend illustrating power rules (also with negative exponents) with compound interest calculations (as in Section 2.2 in the text), which are needed by economics students anyway.

We often encounter students who have a purely memory based, mechanistic approach to the algebraic rules reviewed in Section 2.3. A surprisingly large number of students seem unaware of how algebraic rules can be illustrated in the way we do in Figure 2.3.1.

We find the sign diagrams introduced in Section 2.6 to be useful devices for seeing when certain products or quotients are positive, and when they are negative. Alternative ways of solving such problems can be used, of course.

Economists sometimes need to consider lengthy sums and it is useful to have a convenient notation. In the text, a general introduction to the summation notation for finite sums is given in Sections 2.8–2.11. (Infinite sums are studied in Section 10.4.) In fact, the summation notation is a topic that often causes difficulties to the untutored. Simple examples illustrate the general notation. It is important to understand that the index of summation is a "dummy variable," and what to do with indices that are not indices of summation. The binomial theorem is discussed in Section 2.10.

Problem 2-01 (2.1)

Classify the following numbers as integers, rationals or irrationals:

(a) -33 (b) 1.23 (c) $-3/5$ (d) 0.090909 … (e) 1.313113111311113 …

Problem 2-02 (2.1)

If $x = 0.090909$ …, then $100x = 9.090909$ … and $100x - x = 9$, so $99x = 9$ and thus $x = 1/11$. Try to find a fraction representing $x = 0.151515$ ….

Problem 2-03 (2.3)

Write the following in terms of algebraic expressions:

(a) Half of a number x increased by 3.

(b) The quotient of a and the difference between b and 10.

(c) One third of the sum of n and three sevenths of p.

(d) Four times a number x reduced by the same number results in five times the number plus 1.

(e) One tenth of a number a increased by the product of 10 and b.

Problem 2-04 (2.2)

Solve each of the following equations for x:

(a) $5^2 \cdot 5^x = 5^7$ (b) $10^x = 1$ (c) $10^x \div 10^5 = 10^{-2}$

(d) $(25)^2 = 5^x$ (e) $2^{10} - 2^2 \cdot 2^x = 0$ (f) $(x + 3)^2 = x^2 + 3^2$

Problem 2-05 (2.2)

(a) $(1^{-2} + 2^{-2} + 3^{-2})^{-1} =$ (b) If $\left(1+\dfrac{1}{n}\right)\left(1-\dfrac{1}{m}\right)=1$, then $m = \ldots$?

Problem 2-06 (2.2)

Which of the following equalities are correct?

(a) $3^5 = 5^3$ (b) $(5^2)^3 = 5^{2^3}$ (c) $(3^3)^4 = (3^4)^3$

(d) $(5 + 7)^2 = 5^2 + 7^2$ (e) $\dfrac{2x+4}{2} = x + 4$ (f) $2(x - y) = x \cdot 2 - y \cdot 2$

Problem 2-07 (2.2)

(a) An amount 40,000 earns interest at 2.5% per year. What will this amount have grown to after 10 years?

(b) How much should you have deposited 8 years ago in order to have 30,000 today, if the interest rate has been 6% every year?

Problem 2-08 (2.3)

If $xy = B$ and $\frac{1}{x^2} + \frac{1}{y^2} = A$, then $\left(\frac{1}{x} + \frac{1}{y}\right)^2 =$

Problem 2-09 (2.4)

Simplify:

(a) $\frac{5a-3}{25a^2-9}$ (b) $\frac{4x^2yz}{2xy+2xyz}$ (c) $\frac{t^4-16}{(t-2)(t^2+4)}$

Problem 2-10 (2.4)

Simplify: $\frac{\frac{100}{p}}{\left(1+\frac{p}{100}\right)^{-1}} - 1$

Problem 2-11 (2.5)

Simplify:

(a) $\frac{556^2-555^2}{1111}$ (b) $\frac{125^{-2/3}}{5^{-3}}$ (c) $\left(\frac{2}{3^2}-\frac{1}{6}\right)^{-1}$ (d) $\frac{x^{\alpha/2}y^{-\beta/3}z^{\gamma}}{(x^{-\alpha}y^{8\beta/3}z^{2\gamma})^{-1/2}}$

Problem 2-12 (2.5)

Simplify:

(a) $\frac{896 \cdot 897 - 897}{895}$ (b) $\frac{1}{1-\frac{1}{2}} + \frac{1}{1-\frac{1}{4}} + \frac{1}{1+\frac{1}{2}}$ (c) $\frac{(p^{\alpha}q^{-\beta/2})^2}{(p^{-2\alpha/3}q^{4\beta/3})^{-3/2}}$

Problem 2-13 (2.5)

Simplify:

(a) $\frac{9986 \cdot 9987 - 9987}{9985}$ (b) $\left(\frac{1}{r}\right)^{-3} \div r^2$

(c) $125^{-2/3}$

(d) $\left(\dfrac{\sqrt{c}}{\sqrt{c}-\sqrt{d}}-\dfrac{d}{\sqrt{c}\sqrt{c}-\sqrt{c}\sqrt{d}}\right)\dfrac{5c\sqrt{c}}{\sqrt{c}+\sqrt{d}}$

Problem 2-14 (2.5)

Simplify:

(a) $3(\sqrt{a})^3-2a\sqrt{a}-(a^{1/4})^2/a^{-1}$

(b) $\dfrac{x^{2\beta}(x^2y^2)^{\gamma}}{x^{\beta+2\gamma}}$

(c) $\sqrt[3]{-64x^6}$

(d) $\left(\sqrt{x^{-1}\sqrt{1+x}}\right)^{-4}$

Problem 2-15 (2.5)

Simplify:

(a) $\sqrt{25^2-15^2}$

(b) $\dfrac{(-2a)^3a^{-2/3}}{-32(2a)^{-2}a^{1/3}}$

(c) $\sqrt[3]{\dfrac{5^{pq-q}\cdot 5^{2p}}{5^{pq-p}\cdot 5^{2q}}}$

(d) $\dfrac{1}{2}[(P+Q+R)^2-P^2-Q^2-R^2]$

Problem 2-16 (2.5)

Simplify:

(a) $2^{10}(32)^{-9/5}$

(b) $\sqrt{5^2+12^2}-10$

(c) $\dfrac{(a^{3c})^{-1}a^{3c}}{a^{-5c}(a^{2c})^2}$

(d) $\dfrac{1}{1+x}+\dfrac{1}{1-x^2}+\dfrac{1}{1-x}$

Problem 2-17 (2.5)

Simplify:

(a) $4^0-(0.4)^{-1}+\frac{3}{2}+4\cdot 4^{-1}$

(b) $64\cdot 32^{-3/5}$

(c) $\dfrac{8}{x^2-4x}+\dfrac{2}{x}-\dfrac{2}{x-4}$

Problem 2-18 (2.5)

The surface area S and the volume V of a sphere of radius r are $S = 4\pi r^2$ and $V=\frac{4}{3}\pi r^3$, respectively.

(a) Express S in terms of V by eliminating r.

(b) A sphere of capacity 100 m^3 is to have its outside surface painted. One litre of paint covers 5 m^2. How many litres of paint are needed?

Problem 2-19 (2.6)

Use sign diagrams to find for what values of x each of the following inequalities holds:

(a) $-\frac{1}{2}(x-5) \le 2x-1$ (b) $(3-x)(x+2) > 0$ (c) $\dfrac{(x+2)(2-x)}{x(x+4)} \le 0$

Problem 2-20 (2.6)

Which of the following inequalities are satisfied for all p in (0, 1)?

(a) $p > \sqrt{p}$ (b) $\dfrac{1}{p} > \sqrt{p}$ (c) $p^3 > p^2$ (d) $p > \dfrac{1}{p}$

Problem 2-21 (2.7)

By using the definition of absolute value, solve the inequality in (a) and write the expressions in (b) and (c) without using the absolute value sign. (You will get different expressions over different intervals.)

(a) $|3-x| < 6$ (b) $|x+1| + |x+4|$ (c) $|x+1| - |x+4|$

Problem 2-22 (2.7)

(a) For what values of x is $|x-1| < |x+1|$?

(b) For what values of x is $|x+a| < |x+1|$?

(c) For what values of x is $\left|\dfrac{x-1}{x+1}\right| > \dfrac{x-1}{x+1}$?

Problem 2-23 (2.8)

Find the sums:

(a) $\sum_{k=1}^{5}(5+k)$ (b) $\sum_{i=1}^{4}(5+3^i)$ (c) $\sum_{k=0}^{5}(1-x)^k y^{5-k}$ (d) $\sum_{i=0}^{4}(i^2-1)$

Problem 2-24 (2.8)

Express using summation notation:

(a) $3 + 6 + 9 + 12 + 18 + 21$

(b) $x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5$

(c) $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36}$

(d) $1 + \frac{t}{3} + \frac{t^2}{5} + \frac{t^3}{7} + \cdots + \frac{t^{12}}{25}$

Problem 2-25 (2.8)

Find the sums:

(a) $\sum_{i=1}^{100} 6$ (b) $\sum_{k=0}^{3} (k+1)^{k-1}$ (c) $\sum_{j=0}^{4} j \cdot 2^{j+1}$ (d) $\sum_{k=1}^{3} (-1)^k k^k$

Problem 2-26 (2.8)

Fill in the blank spaces:

(a) $\sum_{j=1}^{9} (2j-3) = \sum_{k=\square}^{\square} (2k+1)$

(b) $\sum_{k=1}^{5} (k^k - 1) = \sum_{j=-2}^{\square} \boxed{}$

Problem 2-27 (2.8)

(a) Express the sum $a^5 + (1-b)a^4 + (1-b)^2a^3 + (1-b)^3a^2 + (1-b)^4a + (1-b)^5$ by means of summation notation.

(b) How many terms are there in the sum $\sum_{k=10}^{60} k^k$?

(c) Write with a summation sign: $\frac{1}{9}(x-1) + \frac{1}{27}(x-1)^2 + \frac{1}{81}(x-1)^3 + \frac{1}{243}(x-1)^4$

Problem 2-28 (2.9)

Which of the following summations are wrong?

(a) $\sum_{j=1}^{3} j^2 = 14$ (b) $\sum_{i=1}^{4} (-1)^i 3^{i-1} = 20$ (c) $\sum_{k=0}^{3} (k+1)^{k-1} = 22$ (d) $\sum_{i=1}^{10} 6 = 60$

Problem 2-29 (2.9)

Find the following sums:

(a) $\sum_{j=1}^{200} j$ (b) $\sum_{i=1}^{7} i^2$ (c) $\sum_{k=1}^{10} k^3$ (d) $\sum_{i=1}^{n} (k^3 - \frac{1}{2}n^2 k)$

Problem 2-30 (2.9)

Which of the following equalities are correct?

(a) $\sum_{j=1}^{3} a_j = \sum_{s=5}^{7} a_{s-4}$ (b) $\sum_{i=1}^{3} 4a_{i+1,j} = 4\sum_{i=1}^{3} a_{i+1,j}$

(c) $\sum_{j=1}^{3} (-1)^j a^{j-1} = -\sum_{k=0}^{2} (-1)^k a^k$ (d) $\sum_{n=1}^{4} (a_n^3 + b_n^3) = \sum_{n=1}^{4} (a_n + b_n)^3$

Problem 2-31 (2.10)

Using the binomial formula, find the coefficient of a^8 in $(a + 2)^{10}$.

Problem 2-32 (2.10)

Using Newton's binomial formula, expand

(a) $(2x + y)^4$ (b) $(1 - x)^6$

CHAPTER 3

Solving Equations

Mathematics for economic analysis often involves solving equations, which is the topic of this chapter. In particular, it is important to train the students in handling equations with parameters, which arise in so many economic applications. (We often see that students who are used to equations involving x and y have problems when the variables are Y, C, etc.) The examples and problems in Section 3.4 deal with some types of nonlinear equation that frequently occur in optimization problems. These can be postponed until the techniques for them are needed.

Problem 3-01 (3.1)

Solve the equations:

(a) $\frac{1}{5}x - 3 = 2x + 6$ (b) $\frac{x}{x-2} + 3 = \frac{-2}{2-x}$ (c) $Y = 0.4\,(Y - (300 + 0.5Y)) + 200$

Problem 3-02 (3.1)

In a sports league where no drawn games are possible, a team had 10 more wins than twice its losses. It played a total of 52 matches. How many did it lose?

Problem 3-03 (3.1)

A school has 300 students, of whom 144 are boys. In the first year, 45% are boys, while 50% of the other years are boys. What is the number of first year students?

Problem 3-04 (3.1)

A swimming pool can be filled by any one of three different hosepipes in 20, 30, and 60 minutes, respectively. How long will it take to fill the pool if all three hosepipes are used at the same time, without reducing the water pressure?

Problem 3-05 (3.2)

Solve the following equations:

(a) $\frac{a^{-2} - b^{-2}}{a^{-1} - b^{-1}} = \frac{x}{a} + \frac{x}{b}$ for x $(a \neq \pm b)$ (b) $Y = I + a(Y - (c + dY))$ for Y

Problem 3-06 (3.2)

Consider the macro model:

(i) $Y = C + \bar{I} + G$, (ii) $C = a + b(Y - T)$, (iii) $T = d + tY$

where the parameters b and t lie in the interval (0, 1), Y is the gross domestic product (GDP), C is consumption, $\bar{I}$ is total investment, T denotes taxes, and G is government expenditure.

(a) Express Y in terms of $\bar{I}$, G, and the parameters.

(b) What happens to Y as t increases?

Problem 3-07 (3.2)

Consider the macro model:

$$Y = C + I^* + G^* + X^* - M, \quad C = aY + b, \quad M = mY + M^*$$

Express Y in terms of the variables marked with an asterisk, as well as the constants a, b, and c.

Problem 3-08 (3.2)

Solve $\frac{1}{p} + \frac{1}{q} = \frac{1}{T}$ for q.

Problem 3-09 (3.3)

Solve the equations:

(a) $\frac{x^2 - 3x - 10}{\sqrt{x - 5}} = 0$ (b) $\sqrt{x^2 - 4x + 3} = 6 - 2x$

Problem 3-10 (3.3)

For what values of x is $|x^2 - 2x - 3| > 5$? (*Hint:* $|a| > 5$ if either $a > 5$ or $a < -5$.)

Problem 3-11 (3.4)

Solve the following equations (or systems of equations):

(a) $x(16 - x^2) = 0$ (b) $5^{2p-1} = 125^{-p}$ (c) $\begin{array}{c} P + 2Q = 1 \\ P^2 + Q^2 = 2 \end{array}$ (d) $\begin{array}{c} 9x - xy^2 = 0 \\ 3x^2 - 2y = 0 \end{array}$