3 MOTION IN TWO AND THREE DIMENSIONS

EXERCISES

Section 3.1 Vectors

13. **INTERPRET** This problem involves finding the magnitude and direction of a vector in two dimensions.

DEVELOP In two dimensions, a vector can be written as $\Delta\vec{r} = \Delta r_x\hat{i} + \Delta r_y\hat{j}$, where Δr_x and Δr_y are the *x*- and *y*-components of the displacement, respectively, and $\hat{i}$ and $\hat{j}$ are unit vectors in the *x*- and *y*-directions, respectively. For this problem, Δr_x = 250 m, Δr_y = 160 m, $\hat{i}$ indicates east, and $\hat{j}$ indicates north. The magnitude of $\Delta\vec{r}$ is $\Delta r = \sqrt{(\Delta r_x)^2 + (\Delta r_y)^2}$, and the angle $\Delta\vec{r}$ makes with the +*x* axis is

$$\theta = \text{atan}\left(\frac{\Delta r_y}{\Delta r_x}\right)$$

EVALUATE The magnitude of the vector is therefore

$$|\Delta\vec{r}| = \sqrt{(250\text{ m})^2 + (160\text{ m})^2} = 296.81\text{ m} \approx 297\text{ m}$$

to two significant figures. The direction of the vector is

$$\theta = \text{atan}\left(\frac{160\text{ m}}{250\text{ m}}\right) = 32.61° \approx 33°$$

with respect to the positive-*x* axis (and to two significant figures).

ASSESS With the coordinate system we have chosen, this vector lies in the second quadrant.

14. **INTERPRET** This problem involves finding the distance of a semicircular arc given its radius, then finding the magnitude the resulting displacement if one travels along this arc.

DEVELOP The length of an arc is the radius multiplied by the angle in radians, or $s = r\theta$ (see Figure 1.2). For a semicircular arc, $\theta = \pi$ radians.

EVALUATE **(a)** The length of the semicircle is $s = \pi r = \pi(15.8\text{ cm}) = 49.637\text{ cm}$ **(b)** The magnitude of the displacement vector, from the start of the semicircle to its end, is just the diameter of the circle, which is twice the radius. Therefore, $d = 2r = 2(15.8\text{ cm}) = 31.6\text{ cm}$.

ASSESS The displacement may also be found by subtracting the final position vector $\vec{r}_2$ from the initial position vector $\vec{r}_1$. This gives $|\vec{r}_2 - \vec{r}_1| = 15.8\text{ cm} + 15.8\text{ cm} = 31.6\text{ cm}$.

15. **INTERPRET** This problem involves the addition of two displacement vectors in two dimensions and finding the magnitude and direction of the resultant vector.

DEVELOP Using Equation 3.1, we see that in two dimensions, a vector $\vec{A}$ can be written in unit vector notation as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} = A\left[\cos(\theta_A)\hat{i} + \sin(\theta_A)\hat{j}\right]$$

where $A = \sqrt{A_x^2 + A_y^2}$ and $\theta_A = \text{atan}(A_y/A_x)$. Similarly, we express a second vector $\vec{B}$ as $\vec{B} = B_x\hat{i} + B_y\hat{j} = B\left[\cos(\theta_B)\hat{i} + \sin(\theta_B)\hat{j}\right]$. The resultant vector $\vec{C}$ is

$$\vec{C}=\vec{A}+\vec{B}=\left(A_x+B_x\right)\hat{i}+\left(A_y+B_y\right)\hat{j}=\left[A\cos\left(\theta_A\right)+B\cos\left(\theta_B\right)\right]\hat{i}+\left[A\sin\left(\theta_A\right)+B\sin\left(\theta_B\right)\right]\hat{j}=C_x\hat{i}+C_y\hat{j}$$

EVALUATE From the problem statement, $A=360$ km and $\theta_A=135°$ (see figure below). The first segment of travel can thus be written as

$$\vec{A}=A\left(\cos\theta_A\hat{i}+\sin\theta_A\hat{j}\right)=(360\text{ km})\left[\cos(135°)\hat{i}+\sin(135°)\hat{j}\right]=(-254.6\text{ km})\hat{i}+(254.6\text{ km})\hat{j}$$

Similarly, the second segment of the travel can be expressed as (with $B=400$ km and $\theta_B=90°$)

$$\vec{B}=B\left[\cos\left(\theta_B\right)\hat{i}+\sin\left(\theta_B\right)\hat{j}\right]=(400\text{ km})\hat{j}$$

Thus, the resultant displacement vector is

$$\vec{C}=\vec{A}+\vec{B}=C_x\hat{i}+C_y\hat{j}=(-254.6\text{ km})\hat{i}+(254.6\text{ km}+400\text{ km})\hat{j}=(-254.6\text{ km})\hat{i}+(654.6\text{ km})\hat{j}$$

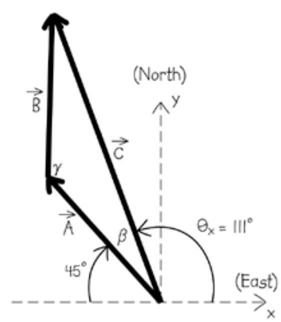
The magnitude of $\vec{C}$ is

$$C=\sqrt{C_x^2+C_y^2}=\sqrt{(-254.6\text{ km})^2+(654.6\text{ km})^2}=702.4\text{ km}\approx 700\text{ km}$$

to two significant figures. Its direction is

$$\theta=\tan^{-1}\left(\frac{C_y}{C_x}\right)=\text{atan}\left(\frac{654.6\text{ km}}{-254.6\text{ km}}\right)=-68.75°,\text{ or }111°$$

We choose the latter solution (110° to two significant figures) because the vector (with $C_x<0$ and $C_y>0$) lies in the second quadrant.



ASSESS As depicted in the figure, the resultant displacement vector $\vec{C}$ lies in the second quadrant. The direction of $\vec{C}$ can be specified as $111°$ CCW from the *x*-axis (east), or $45°+23.7°=68.7$ °N of W.

16. **INTERPRET** This problem involves finding the magnitude and direction of a vector that results from the addition of two given vectors.

DEVELOP Choose a coordinate system where $\hat{i}$ points to the right and $\hat{j}$ points up. In this case the two vectors can be written as $\vec{A}=(4.0\text{ m})\hat{i}$ and $\vec{B}=(3.0\text{ m})\hat{j}$, so A_i = 4.0 m, A_j = 0.0 m, B_i = 0.0 m, and B_j = 3.0 m. The vector equation $\vec{A}+\vec{B}+\vec{C}=0$ actually is two equations, one for the $\hat{i}$ components and one for the $\hat{j}$ components. These two equations are

$$A_i+B_i+C_i=0$$
$$A_j+B_j+C_j=0$$

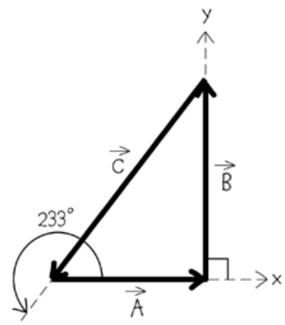
Because we are given the components of $\vec{A}$ and $\vec{B}$, we can solve for the two components of $\vec{C}$.

EVALUATE Inserting the given quantities into the above equation, we find the components of $\vec{C}$ are

$$A_i+B_i+C_i=4.0\text{ m}+0.0\text{ m}+C_i=0$$
$$C_i=-4.0\text{ m}$$
$$A_j+B_j+C_j=0.0\text{ m}+3.0\text{ m}+C_i=0$$
$$C_j=-3.0\text{ m}$$

so the resulting vector is $\vec{C} = (-4.0\text{ m})\hat{i} + (-3.0\text{ m})\hat{j}$. From Equations 3.1, we find its magnitude and direction to be $C = \sqrt{(-4.0\text{ m})^2 + (-3.0\text{ m})^2} = 5.0\text{ m}$ and θ = atan(−3/4) = 143° with respect to the $\hat{i}$ direction (to two significant figures).

ASSESS To sum vectors graphically, simply put them head-to-tail, as shown in the figure below. Notice that the vector $\vec{C}$ terminates where the vector $\vec{A}$ begins, which is the graphical way to say that the sum $\vec{A} + \vec{B} + \vec{C} = 0$.



17. **INTERPRET** We are asked to express a displacement with the unit vectors $\hat{i}$ and $\hat{j}$.

DEVELOP We're given the magnitude of the vector, $A = 120$ km, and the angle, $\theta = 29°$. The *x* and *y* components are given in Equation 3.2:

$$A_x = A\cos\theta = (120\text{ km})\cos 29° = 105\text{ km}$$
$$A_y = A\sin\theta = (120\text{ km})\sin 29° = 58\text{ km}$$

Since the vector points above the *x*-axis and to the right of the *y*-axis, both the *x* and *y* components should be positive.

EVALUATE In terms of unit vectors, the displacement is

$$\vec{A} = A_x\hat{i} + A_y\hat{j} = 105\hat{i} + 58\hat{j}\text{ km}$$

ASSESS We can verify that indeed this is right by checking that $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = A_y / A_x$ from Equation 3.1.

18. **INTERPRET** This problem involves finding the magnitude and direction of a given vector.

DEVELOP Choose a coordinate system where $\hat{i}$ corresponds to the positive-*x* axis. Use Equations 3.1 to find the magnitude and direction of the given vector $\vec{A}$, whose components are A_i = 33 m and A_j = 14 m.

EVALUATE The magnitude of $\vec{A}$ is $A = \sqrt{(33\text{ m})^2 + (14\text{ m})^2} = 35.84\text{ m} = 35.9\text{ m}$ to two significant figures. The direction of $\vec{A}$ with respect to the *x* axis is θ = atan(13/34) = 22.98° or 23° to two significant figures. We choose the first solution because we know that this vector lies in the first quadrant because both of its components are positive.

ASSESS Because $A_j = 14\text{ m} < A_i = 33\text{ m}$ (i.e., the *x* component of $\vec{A}$ is greater than the *y* component), we expect the angle with respect to the *x*-axis to be less than 45°. Our result indeed confirms this.

19. **INTERPRET** This problem involves finding the magnitude and direction of a given vector.

DEVELOP Choose a coordinate system where $\hat{i}$ corresponds to the positive-*x* axis. Use Equations 3.1 to find the magnitude and direction of the given vector $\vec{A}$, whose components are A_i = 1 and A_j = 1.

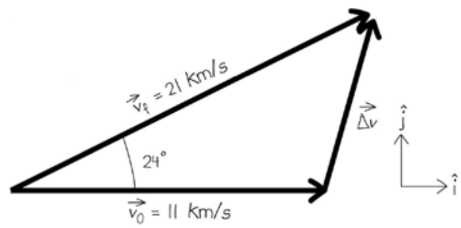
EVALUATE The magnitude of $\vec{A}$ is $A = \sqrt{1^2 + 1^2} = \sqrt{2}$. The direction of $\vec{A}$ with respect to the *x* axis is θ= atan(1/1) = 45°.

ASSESS This vector lies in the first quadrant because both components are positive.

Section 3.2 Velocity and Acceleration Vectors

20. **INTERPRET** You are asked to report on the acceleration produced by the rocket firing. This can be determined from the velocity data.

DEVELOP You can calculate the average acceleration, using $\bar{\vec{a}} = \Delta\vec{v} / \Delta t$ (Equation 3.5). The change in velocity here is the difference in velocity before and after the rocket fires, $\Delta\vec{v} = \vec{v}_f - \vec{v}_0$, so it will be helpful to put the velocities in coordinate form. The figure below should help.



In the figure, we have chosen the direction of the unit vectors, $\hat{i}$ and $\hat{j}$. We use that to break the vectors into components. From there, we find the change in velocity and divide this change by the time to obtain the average acceleration.

EVALUATE According to our figure, the velocities in component form are

$$\vec{v}_0 = 11\hat{i} \text{ km/s}$$
$$\vec{v}_f = 21\cos 24°\hat{i} + 21\sin 24°\hat{j} = 19.18\hat{i} + 8.54\hat{j} \text{ km/s}$$

The change in velocity is then

$$\Delta\vec{v} = (19.18 - 11)\hat{i} + (8.54 - 0)\hat{j} \text{ km/s} = 7.18\hat{i} + 8.54\hat{j} \text{ km/s}$$

The average acceleration is this change in velocity divided by the time of 10 min, or 600 s.

$$\bar{\vec{a}} = \frac{7.18\hat{i} + 8.54\hat{j} \text{ km/s}}{600 \text{ s}} = 11.96\hat{i} + 14.23\hat{j} \text{ m/s}^2$$

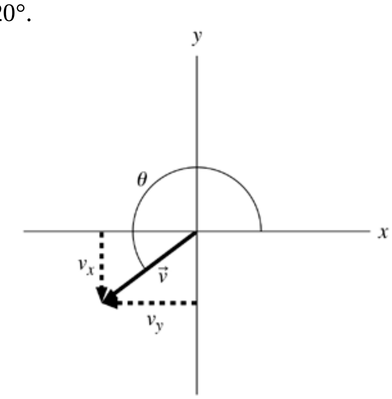
and its magnitude is $|\bar{\vec{a}}| = \sqrt{(11.96 \text{ m/s}^2)^2 + (14.23 \text{ m/s}^2)^2} = 18.59 \text{ m/s}^2$.

You can report these numbers, but perhaps the most relevant information is that the asteroid has been effectively accelerated in the $\hat{j}$- direction, steering it away from a collision with Earth presumably.

ASSESS It's hard to estimate whether this acceleration is reasonable or not without knowing the mass of the asteroid. Large rocket engines accelerate the space shuttle at about 30 m/s^2, though, so it's probably about right.

21. **INTERPRET** This problem asks us to find the components of a vector given its magnitude and direction.

DEVELOP Draw a diagram of the vector (see figure below). Use Equations 3.2, with the magnitude of the vector being v = 18 m/s and the angle θ = 220°.



EVALUATE The x component of the vector v is $v_x = v\cos(\theta)$ = (18 m/s)cos(220°) = −14 m/s and the y component is $v_y = v\sin(\theta) = (18 \text{ m/s})\sin(220°) = -12 \text{ m/s}$.

ASSESS Note that both components are negative, as expected from the plot.

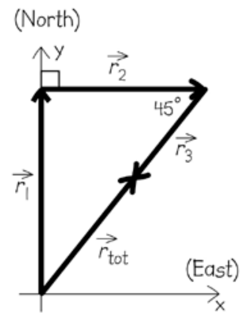
22. **INTERPRET** We interpret this as a problem involving the addition of three displacements in two dimensions and finding the magnitude and direction of the resultant vector. The key concepts here are displacement and average velocity.

DEVELOP The trip can be divided into three segments, each of which can be described by the displacement vector in Equation 3.3: $\Delta\vec{r} = \vec{v}\Delta t$. We will be adding these vectors together to get the total displacement $\Delta\vec{r}_{\text{tot}}$. The average velocity for the full trip is just:

$$\bar{\vec{v}} = \frac{\Delta\vec{r}_{\text{tot}}}{\Delta t_{\text{tot}}}$$

EVALUATE **(a)** To get started, let's find the magnitude of the displacement (or distance) and the duration of the trip for each segment. The car starts off going north at $v_1 = 38$ mi/h for $\Delta t_1 = 10$ min , which corresponds to a distance of $\Delta r_1 = (38\text{ mi/h})(10\text{ min}) = 6.33\text{ mi}$. It then turns east and drives $\Delta r_2 = 5.0\text{ mi}$ at a speed of $v_2 = 70\text{ mi/h}$, which implies a duration of $\Delta t_2 = (5.0\text{ mi})/(70\text{ mi/h}) = 4.28\text{min}$. Lastly, it turns southwest and moves at $v_3 = 40\text{ mi/h}$ for $\Delta t_3 = 6.0\text{ min}$, which corresponds to a distance of $\Delta r_3 = (40\text{ mi/h})(6.0\text{ min}) = 4.0\text{ mi}$.

With the above distances, we are ready to write the displacement vectors in component form. It will help to sketch a picture, as shown below.



Note that we chose a coordinate system with x axis east, y axis north, and origin at the starting point. The vectors are therefore:

$$\Delta\vec{r}_1 = 6.33\hat{j}\text{ mi}; \qquad \Delta\vec{r}_2 = 5.0\hat{i}\text{ mi};$$
$$\Delta\vec{r}_3 = 4.0\cos 225°\hat{i} + 4.0\sin 225°\hat{j}\text{ mi} = -2.83\hat{i} - 2.83\hat{j}\text{ mi}$$

Note that we have used $225°$ to represent the direction southwest. The total displacement is the sum of these vectors

$$\Delta\vec{r}_{\text{tot}} = (5.0 - 2.83)\hat{i} + (6.67 - 2.83)\hat{j}\text{ mi} = 2.17\hat{i} + 3.5\hat{j}\text{ mi} \approx 2.2\hat{i} + 3.5\hat{j}\text{ mi}$$

(b) The average velocity is

$$\bar{\vec{v}} = \frac{2.2\hat{i} + 3.5\hat{j}\text{ mi}}{(10\text{ min} + 5.0\text{ min} + 6.0\text{ min})}\left(\frac{60\text{min}}{1\text{ h}}\right) = 6.29\hat{i} + 10\hat{j}\text{ mi/h}$$

ASSESS We expect both $\Delta\vec{r}_{\text{tot}}$ and $\bar{\vec{v}}$ to be in the first quadrant since their components are all positive. Instead of unit-vector notation, we can specify these vectors by their magnitudes:

$$\Delta\vec{r}_{\text{tot}} = \sqrt{(2.2\text{ mi})^2 + (3.5\text{ mi})^2} = 4.13\text{ mi}$$

and $\bar{\vec{v}} = 15\text{ mi/h}$, as well as their common direction:

$$\theta = \tan^{-1}\left[(3.5\text{ mi})/(2.2\text{ mi})\right] = 57.8°\ \text{N of E.}$$

23. **INTERPRET** We're asked to find the acceleration given the velocity as a function of time.

DEVELOP Equation 3.6 relates the instantaneous velocity to the instantaneous acceleration: $\vec{a} = d\vec{v}/dt$.

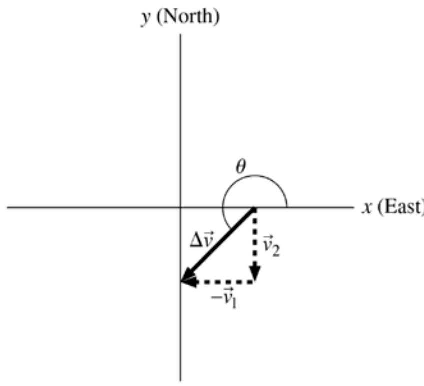
EVALUATE Taking the derivate of the velocity expression gives

$$\vec{a} = \frac{d}{dt}\left[ct^3\hat{i} + d\hat{j}\right] = 3ct^2\hat{i}$$

ASSESS The object has a constant velocity in the *y*-direction, so there is no acceleration in that direction.

24. **INTERPRET** This problem involves calculating an average acceleration vector given its initial and final velocity.

DEVELOP Draw a diagram of the situation (see figure below) to display graphically the difference $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, where v_1 is the initial velocity and v_2 is the final velocity. From Equation 3.5, we know that the average acceleration is in the same direction as the change in velocity, $\bar{\vec{a}}\Delta t = \Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.

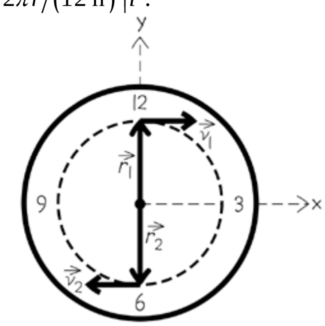


EVALUATE The angle made by $\Delta\vec{v}$ is $\theta = \text{atan}\left(\dfrac{v_2}{v_1}\right) = \text{atan}(-1) = 225°$, where we have used $|\vec{v}_1| = |\vec{v}_2|$, which we know because the initial and final speed is the same. Thus, the average acceleration is oriented at 225° counter-clockwise from the x axis.

ASSESS The direction may also be reported as 135° clockwise from the x axis.

25. **INTERPRET** This problem involves calculating the average velocity and average acceleration given the initial and final positions of an object.

DEVELOP Draw a diagram of the situation (see figure below) to display graphically the difference $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$, where r_1 is the initial position of the tip of the clock hand and r_2 is its final position. Use Equation 3.3 $\bar{\vec{v}} = \Delta\vec{r}/\Delta t$ to find the average velocity and Equation 3.5 $\bar{\vec{a}} = \Delta\vec{v}/\Delta t$ to find the average acceleration. The magnitude of $\vec{r}$ is 2.6 cm, so $\vec{r}_1 = (2.6\text{ cm})\hat{j}$ and $\vec{r}_2 = (-2.6\text{ cm})\hat{j}$. For part **(b)**, note that the tip of the clock moves at a constant speed, which is $v = 2\pi r/(12\text{ h})$. The direction of the velocity vector at any moment is perpendicular to the clock hand, so $\vec{v}_1 = [2\pi r/(12\text{ h})]\hat{i}$ and $\vec{v}_2 = [-2\pi r/(12\text{ h})]\hat{i}$.



EVALUATE **(a)** The change in position is $\Delta r = \vec{r}_2 - \vec{r}_1 = (-2.6\text{ cm})\hat{j} + (-2.6\text{ cm})\hat{j} = (-5.2\text{ cm})\hat{j}$, so the average velocity is $\bar{\vec{v}} = \Delta\vec{r}/\Delta t = (-5.2\text{ cm})\hat{j}/(6\text{ h}) = (-0.86\text{ cm/h})\hat{j} = (-2.4\times10^{-7}\text{ m/s})\hat{j}$. **(b)** From Equation 3.5, the average acceleration is

$$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{-4\pi(2.6\text{ cm})/(12\text{ h})}{6\text{ h}}\hat{i} = (-0.45\text{ cm/h}^2)\hat{i} = (-3.5\times10^{-10}\text{ m/s}^2)\hat{i}$$

ASSESS Notice that the average acceleration is perpendicular to the average velocity, as expected for circular motion (see Figure 3.9).

26. **INTERPRET** This problem involves finding the angle between the initial velocity and the acceleration, given the magnitude of each and of the final velocity.

DEVELOP From Equation 3.5, we know that $\bar{\vec{a}} = \Delta\vec{v}/\Delta t = (\vec{v}_2 - \vec{v}_1)/\Delta t$, which we can rearrange to give $\bar{\vec{a}}\Delta t = \vec{v}_2 - \vec{v}_1$. Let the initial velocity define the x axis [so $\vec{v}_1 = (v_1)\hat{i}$]and equate the x and y components of the vector equation $\bar{\vec{a}}\Delta t = \vec{v}_2 - \vec{v}_1$. This gives the following two equations:

$$v_1 + \bar{a}\Delta t\cos(\alpha) = v_2\cos(\theta)$$
$$\bar{a}\Delta t\sin(\alpha) = v_2\sin(\theta)$$

where the first is for the x-component and the second is for the y-component, and α and θ are the angles between the positive x axis and the acceleration and final velocity, respectively. The two unknowns are the angles, which we can solve for using these two equations.

EVALUATE Upon inspection of the above system of two equations, we see that $\alpha = 0$, $\theta = 0$ is a solution, because this gives $(2.4\text{ m/s}) + (1.5\text{ m/s}^2)(3.0\text{ s}) = 6.9\text{ m/s}$ for the first equation and 0 m/s = 0 m/s for the second equation. Thus, the all three vectors are parallel and the angle α between the skater's initial velocity and her acceleration is 0°.

ASSESS If the angle α between the acceleration and the initial velocity were greater than zero, the final velocity would be less than that found.

27. **INTERPRET** This problem asks us to find the final velocity given the initial velocity, acceleration, and the time interval for the acceleration.

DEVELOP Use Equation 3.5, $\bar{\vec{a}} = \Delta\vec{v}/\Delta t = (\vec{v}_2 - \vec{v}_1)/\Delta t$ to find the final velocity.

EVALUATE Solving Equation 3.5 for $\vec{v}_2$ and inserting the given quantities gives

$$\vec{v}_2 = \vec{v}_1 + \bar{\vec{a}}\Delta t = (1.1\text{ m/s})\hat{i} + (0.52\text{ m/s}^2)(5.2\text{ s})\hat{j} = (1.1\text{ m/s})\hat{i} + (2.7\text{ m/s})\hat{j}$$

The magnitude of the final velocity is $v_2 = \sqrt{(1.1\text{ m/s})^2 + (2.72\text{ m/s})^2} = 2.9\text{ m/s}$, and its angle with respect to the positive x axis is $\theta = \text{atan}(2.7\text{ m/s}/1.1\text{ m/s}) = 67.8°$.

ASSESS Given the components of each vector, we simply added the components together to find the resulting vector.

Section 3.3 Relative Motion

28. **INTERPRET** This problem involves relative velocities. We are asked to find the wind velocity relative to the ground given the airplane's velocity and the time it takes for the airplane to make a given displacement.

DEVELOP Equation 3.7, $\vec{v} = \vec{v}' + \vec{V}$ allows us to find the wind velocity $\vec{V}$ relative to the ground. Here, $\vec{v}$ is the velocity of the airplane relative to the ground, and $\vec{v}'$ is the velocity of the airplane relative to the air. Use a coordinate system with the x axis indicating east and the y axis indicating north.

EVALUATE From the problem statement, the velocity of the jetliner relative to the ground is

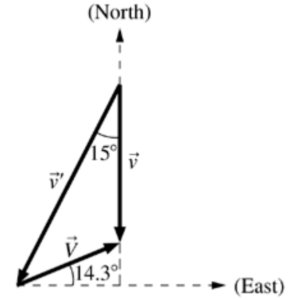
$$\vec{v} = \frac{(-1600\text{ km})\hat{j}}{110\text{ min}}\left(\frac{60\text{ min}}{1\text{ h}}\right) = (-872.73\text{ km/h})\hat{j}$$

Similarly, using the fact that the unit vector in the direction 15° west of south (255° CCW from the positive-x axis) is $\cos(255°)\hat{i} + \sin(255°)\hat{j}$, the velocity of the airplane relative to the air is

$$\vec{v}' = (1200\text{ km/h})\left[\cos(255°)\hat{i} + \sin(255°)\hat{j}\right] = (-310\text{ km/h})\hat{i} + (-1159.11\text{ km/h})\hat{j}$$

Thus, the wind velocity is

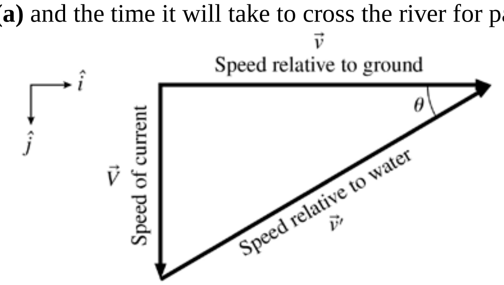
$$\vec{V} = \vec{v} - \vec{v}' = (-310.58\text{ km/h})\hat{i} + \left[(-872.73\text{ km/h}) - (-1159.11\text{ km/h})\right]\hat{j} = (-310.58\text{ km/h})\hat{i} + (286.38\text{ km/h})\hat{j}$$



ASSESS The wind speed is $V=\sqrt{(-310.58\ \text{km/h})^2+(286.38\ \text{km/h})^2}=422.46\ \text{km/h}$, and the angle between $\vec{V}$ and the x axis is $\theta=\text{atan}\left[(65.9\ \text{km/h})/(259\ \text{km/h})\right]=137.3°$ (north of east, see figure above). The wind direction, by convention, is the direction from which the wind is blowing; in this case $137.3°$ S of W.

29. **INTERPRET** This problem involves relative motion. You are asked to find the direction of the velocity with respect to the water so that a boat traverses the current perpendicularly with respect to the shore. You also need to find the time it takes to cross the river.

DEVELOP Choose a coordinate system where $\hat{i}$ is the direction perpendicular to the water current, and $\hat{j}$ is the direction of the water current (see figure below). The velocity of the current $\vec{V}$ relative to the ground is $\vec{V}=(0.57\ \text{m/s})\,\hat{j}$, and the magnitude of the velocity $\vec{v}'$ of the boat relative to the water is $v'=1.3\ \text{m/s}$. We also known that the velocity $\vec{v}$ of the boat relative to the shore is in the $\hat{i}$ direction, so $\vec{v}=v\hat{i}$, and that the river is d = 63 m wide. These three vectors satisfy Equation 3.7, $\vec{v}=\vec{v}'+\vec{V}$, as drawn in the figure below. This allows us to find the direction of $\vec{v}'$ for part **(a)** and the time it will take to cross the river for part **(b)**.



EVALUATE **(a)** From the figure above, we see that

$$\sin(\theta)=V/v'$$
$$\theta=\text{asin}(V/v')=\text{asin}(0.57\ \text{m/s}/1.3\ \text{m/s})=26°$$

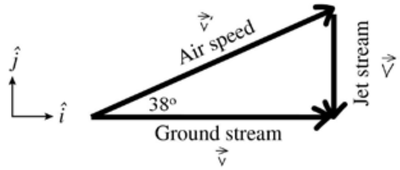
so your heading upstream is 26° above the x axis.

(b) To find the time to traverse the river, we calculate the speed v with respect to the shore. From the figure above, we see that $v=v'\cos(\theta)=(1.3\ \text{m/s})\cos 26°=1.17\ \text{m/s}$, so the crossing time t is $t=d/v=(63\ \text{m})/(1.17\ \text{m/s})=53.9\ \text{s}=54\ \text{s}$ to two significant figures.

ASSESS In this time, you will have rowed a distance d' relative to the water of $d'=v't=(1.3\ \text{m/s})(53.9\ \text{s})=70\ \text{m}$.

30. **INTERPRET** This problem involves relative velocities. We are asked to calculate the speed of the jet stream given the velocity of an airplane relative to the ground and the direction at which the airplane must fly relative to the ground to perpendicularly traverse the jet stream.

DEVELOP Use Equation 3.7, $\vec{v}=\vec{v}'+\vec{V}$ to find $\vec{V}$, the velocity of the jet stream. This relationship is shown graphically in the figure below. Here, $\vec{v}$ is the velocity of the airplane relative to the ground, and $v'=320\ \text{km/h}$ is the speed of the airplane relative to the air. Notice that the angle between $\vec{v}'$ and $\vec{v}$ is 38°, as given in the problem statement.

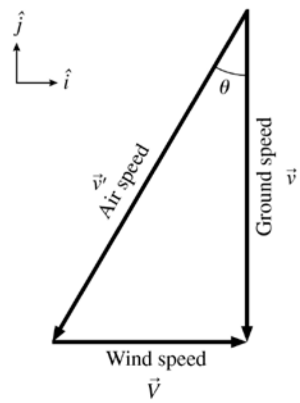


EVALUATE From trigonometry, the magnitude of the jet stream speed is $V = v'\sin(\theta) = (320\ \text{km/h})\sin(38°) = 197.01$ km/h.

ASSESS The speed of the airplane relative to the ground is $v = v'\cos(\theta) = (320\ \text{km/h})\cos(38°) = 252.16$ km/h . The plane's heading of $38°$ north of east is a reasonable compensation for the southward wind blowing at a speed of 197.01 km/h.

31. **INTERPRET** This problem involves relative velocities. We are asked to calculate the direction at which geese should fly to travel due south given the wind velocity and the bird's air speed.

DEVELOP The Equation 3.7, $\vec{v} = \vec{v}' + \vec{V}$ is shown graphically in the figure below. We are given that the magnitude of $\vec{v}'$ is 7.6 m/s, and that the wind velocity is $\vec{V} = (4.9\ \text{m/s})\hat{i}$. This information allows us to calculate the direction θ at which the birds should fly.



EVALUATE From trigonometry, we see that $\theta = \text{asin}(V/v') = \text{asin}(4.9\ \text{m/s}/7.6\ \text{m/s}) = 40°$, so the geese should fly 40° west of south to travel due south.

ASSESS The ground speed of the geese is $v = v'\cos(\theta) = (7.6\ \text{m/s})\cos(40°) = 5.8\ \text{m/s}$.

Section 3.4 Constant Acceleration

32. **INTERPRET** This problem calls for finding the acceleration vector in two dimensions, with the position vector as given.

DEVELOP The acceleration can be found by taking the second derivative of the position,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\frac{d\vec{r}}{dt}\right] = \frac{d^2\vec{r}}{dt^2}$$

EVALUATE Evaluating the first derivative

$$\begin{aligned}\vec{a} &= \frac{d^2}{dt^2}\left[\left(3.2t + 1.8t^2\right)\hat{i} + \left(1.7t - 2.4t^2\right)\hat{j}\ \text{m}\right] \\ &= \frac{d}{dt}\left[\left(3.2 + 3.6t\right)\hat{i} + \left(1.7 - 4.8t\right)\hat{j}\ \text{m/s}\right]\end{aligned}$$

Notice that the units are m/s, since we are taking the derivative with respect to *t*, which is in seconds. Now doing the second derivative

$$\vec{a} = 3.6\hat{i} - 4.8\hat{j}\ \text{m/s}^2$$

The magnitude and direction of the acceleration are then:

$$a = \sqrt{(3.6)^2 + (-4.8)^2}\ \text{m/s}^2 = 6.0\ \text{m/s}^2 \quad ; \quad \theta = \tan^{-1}\left(\frac{-4.8}{3.6}\right) = -53°$$

ASSESS We find the acceleration to be a constant vector. In general, a position vector in two dimensions that is quadratic in t can be related to the velocity and acceleration vectors as

$$\begin{aligned}\vec{r} &= r_x\hat{i} + r_y\hat{j} = \left(r_{x0} + v_{x0}t + \tfrac{1}{2}a_x t^2\right)\hat{i} + \left(r_{y0} + v_{y0}t + \tfrac{1}{2}a_y t^2\right)\hat{j} \\ &= \left(r_{x0}\hat{i} + r_{y0}\hat{j}\right) + \left(v_{x0}\hat{i} + v_{y0}\hat{j}\right)t + \tfrac{1}{2}\left(a_x\hat{i} + a_y\hat{j}\right)t^2 \\ &= \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2\end{aligned}$$

From the above expression, we see that the constant acceleration vector a is equal to twice the coefficient of the t^2 term.

33. **INTERPRET** This problem asks you to determine how far your sailboard goes during a gust of wind that results in a constant acceleration.

DEVELOP We'll assume the initial velocity is in the positive x direction $(\vec{v}_0 = 6.5\hat{i}\ \text{m/s})$. The acceleration can be broken up into x and y components as follows:

$$a_x = a\cos\theta = \left(0.53\ \text{m/s}^2\right)\cos 35° = 0.434\ \text{m/s}^2$$

$$a_y = a\sin\theta = \left(0.53\ \text{m/s}^2\right)\sin 35° = 0.304\ \text{m/s}^2$$

To find the displacement, we can use Equation 2.10 for both the x and y directions.

EVALUATE The displacement in the x direction is

$$\Delta x = v_0 t + \tfrac{1}{2}a_x t^2 = (6.5\ \text{m/s})(5.8\ \text{s}) + \tfrac{1}{2}\left(0.434\ \text{m/s}^2\right)(5.8\ \text{s})^2 = 44.9\ \text{m}$$

The displacement in the y direction is

$$\Delta y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}\left(0.304\ \text{m/s}^2\right)(5.8\ \text{s})^2 = 5.11\ \text{m}$$

The magnitude and direction of the displacement are

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(44.9\ \text{m})^2 + (5.11\ \text{m})^2} = 45.2\ \text{m}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{5.11\ \text{m}}{44.9\ \text{m}}\right) = 6.5°$$

ASSESS The angle of the displacement is less than that of the acceleration. That makes sense because the initial velocity was along the x axis, and therefore there should be a greater displacement in that direction.

Section 3.5 Projectile Motion

34. **INTERPRET** This problem involves two objects moving under the influence of gravity near the Earth's surface, so we can apply the equations of projectile motion. We are asked to find the time at which two objects hit the ground given their initial height and initial velocities.

DEVELOP Recall that the horizontal and vertical motions are independent of each other (see Figure 3.11). The time of flight t for either projectile is determined from the vertical component of the motion, which is the same for both. We are given that $y_0 = 2.8$ m, $y = 0.0$ m, and $v_{y0} = 0.0$ m/s, so we can calculate t by solving Equation 3.13, $y = y_0 + v_{y0}t - gt^2/2$.

EVALUATE From the above equation, the total flight time is

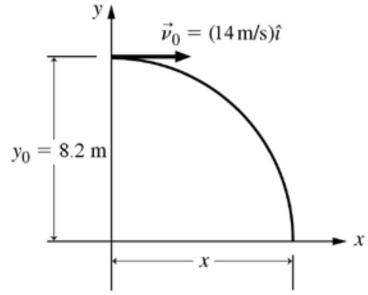
$$t = \sqrt{\frac{2(y_0 - y)}{g}} = \sqrt{\frac{2(2.8\ \text{m} - 0.0\ \text{m})}{9.8\ \text{m/s}^2}} = 0.75\ \text{s}$$

which is the time it takes for both fruit to reach the ground.

ASSESS The apple and the peach both reach the ground at the same time. This is expected because the total flight time is determined by the equation of motion in the vertical direction, which is independent of mass (and fruit type). The nonzero horizontal component of the velocity for the apple does not affect its vertical velocity, but only makes the apple move away from you as it falls.

35. **INTERPRET** This problem involves an object moving under the influence of gravity near the Earth's surface, so we are dealing with projectile motion. We are given the initial velocity and height of the object, and are asked to find the time of flight and the horizontal distance traveled by the object before it hits the ground.

DEVELOP Draw a diagram of the situation (see figure below) to define the coordinate system. The difference $y - y_0 = 0\text{ m} - 8.2\text{ m} = -8.2\text{ m}$, and the initial vertical velocity is $v_{y0} = 0$ m/s. Using this information, we can solve Equation 3.13 for t, which we can then insert into Equation 3.12 to find $x - x_0$, given that $v_{x0} = 14$ m/s.



EVALUATE **(a)** The shingle reaches the ground when

$$y - y_0 = \overbrace{v_{y0}}^{=0}t - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(8.2\text{ m})}{9.8\text{ m/s}^2}} = 1.29\text{ s} = 1.3\text{ s}$$

in which we have retained two significant figures.
(b) The horizontal displacement is $x - x_0 = v_0 t = (14\text{ m/s})(1.29\text{ s}) = 18.1\text{ m}$ to two significant figures.
ASSESS The height of 8.2 m indicates that the building in question is likely a two story building, taking into consideration the height of the worker who throws the tiles. The tiles cover a distance of 18.1 m, which is the length of roughly 6 pick-up trucks.

36. **INTERPRET** This problem involves an object moving under the influence of gravity near the Earth's surface, so it is projectile motion. We are asked to find the horizontal distance traveled by an arrow given its initial horizontal velocity, vertical velocity, and the height from which it is shot.
DEVELOP The horizontal and vertical motions of the arrows are independent of each other, so we can consider them separately. The time of flight t of the arrow can be determined from its range (horizontal motion, Equation 3.12). Once t is found, we can insert it into the equation of motion for the vertical direction (Equation 3.13) to determine the initial height.
EVALUATE From Equation 3.12, the total flight time of the arrow is

$$t = \frac{x - x_0}{v_{0x}} = \frac{23\text{ m}}{35\text{ m/s}} = 0.657\text{ s}$$

Substituting this result into Equation 3.13, and noting that $v_{y0} = 0$, the height from which the arrow was shot is

$$y_0 = y + \frac{1}{2}gt^2 = 0.0\text{ m} + \frac{1}{2}(9.8\text{ m/s}^2)(0.657\text{ s})^2 = 2.116\text{ m}$$

to two significant figures.
ASSESS Dropping a height of 1.5 m in half a second is reasonable for free fall. We may relate y_0 to x as

$$y_0 = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{x - x_0}{v_{0x}}\right)^2$$

From which it is clear that the larger is y_0, the longer it takes for the arrow to reach the ground, and the greater the horizontal distance traveled.

37. **INTERPRET** This problem involves ink drops moving under the influence of gravity near the surface of the Earth, so it is projectile motion. For this problem, we are given the horizontal distance traveled and the initial horizontal and vertical velocities.
DEVELOP The horizontal and vertical motions of the drops are independent of each other, so we can consider them separately. From Equation 3.12 we can find the time of flight t, which we can insert into Equation 3.13 to find

the vertical displacement (i.e., the distance fallen during the time interval t). This initial conditions are $x - x_0 = 1.0\text{ mm} = 1.0\times10^{-3}\text{ m}$, $v_{0x} = 12\text{ m/s}$, $v_{0y} = 0.0\text{ m/s}$, and $y = 0.0$ m.

EVALUATE From Equation 3.12, the total flight time for an ink drop is

$$t = \frac{x - x_0}{v_{0x}} = \frac{1.0\times10^{-3}\text{ m}}{12\text{ m/s}} = 8.33\times10^{-5}\text{ s}$$

Substituting this result into Equation 3.13, we find the distance y_0 that an ink drop falls is

$$y_0 = y + \frac{1}{2}gt^2 = 0.0\text{ m} + \frac{1}{2}\left(9.8\text{ m/s}^2\right)\left(8.33\times10^{-5}\text{ s}\right)^2 = 3.4\times10^{-8}\text{ m} = 34\text{ nm}$$

to two significant figures.

ASSESS This distance is an order of magnitude less than the wavelength of visible light, so it is insignificant on the scale of printed, visual matter.

38. **INTERPRET** This problem involves projectile motion because the objects are moving near the surface of the Earth under the influence of gravity. We are given the distance traveled by the particles and the distance they drop during this trajectory.

DEVELOP Traveling a distance of 1.5 km = 1500 m with a drop of only 1.1×10^{-6} m indicates that the horizontal component of the speed must be much greater than the vertical component. With $v_y \ll v_0$ the average speed can be approximated as $v = \sqrt{v_0^2 + v_y^2} \approx v_0$. We are given the displacement of the particles, $y - y_0 = 1.1\ \mu\text{m} = 1.1\times10^{-6}$ m the initial vertical velocity $v_{0y} = 0.0\text{ m/s}$, from which we can find the time of flight t from Equation 3.13. We can then insert t into Equation 3.12 to estimate the average speed knowing that the horizontal displacement is $x - x_0 = 1500$ m.

EVALUATE From Equation 3.13, we find the time of flight is

$$y - y_0 = -\frac{1}{2}gt^2$$

$$t = \pm\sqrt{\frac{2(y - y_0)}{g}} = \sqrt{\frac{2(y - y_0)}{g}}$$

where we have chose the positive square root because the negative square root is for particles traveling in the opposite direction. Inserting this result into Equation 3.12 gives the initial speed of the particle as

$$x - x_0 = v_{x0}t$$

$$v_{x0} = \frac{x - x_0}{t} = (x - x_0)\sqrt{\frac{g}{2(y - y_0)}} = (1500\text{ m})\sqrt{\frac{9.8\text{ m/s}^2}{2\left(1.1\times10^{-6}\text{ m}\right)}} = 3.2\times10^{6}\text{ m/s}$$

ASSESS The particles are traveling at approximately 1% of the speed of light, so relativistic mechanics may apply here (see Chapter 33).

39. **INTERPRET** This is a problem in projectile motion that asks us to find the horizontal range of a golf ball on the Moon, given the range on Earth.

DEVELOP Equation 3.15 gives us the horizontal range, which is how far a projectile travels over level ground (i.e., $y = y_0 = 0$):

$$x = \frac{v_0^2}{g}\sin 2\theta_0$$

We assume that the initial velocity and angle are the same on Earth and Moon. The Moon's gravity is 1.62 m/s^2 (Appendix E).

EVALUATE The range is inversely proportional to the gravity, so

$$x_\text{M} = x_\text{E}\frac{g_\text{E}}{g_\text{M}} = (170\text{ m})\frac{\left(9.81\text{ m/s}^2\right)}{\left(1.62\text{ m/s}^2\right)} = 1029.44\text{ m}$$

ASSESS In 1971 as part of the Apollo 14 mission, the astronaut Alan Shepard hit a golf ball on the Moon, but it didn't travel a kilometer (more like 200-300 m apparently). To Shepard's credit, he was wearing a bulky spacesuit and could only swing with one arm.

Section 3.6 Uniform Circular Motion

40. **INTERPRET** This problem involves uniform circular motion. We are asked to calculate the magnitude of the centripetal acceleration as the train makes a circular turn at a given speed.

DEVELOP Given the radius r and the speed v, we use Equation 3.16, $a = v^2/r$, to solve for the acceleration a.

EVALUATE With r = 7.0 km = 7000 m and v = 350 km/h = 92.22 m/s, the train's centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(97.22\ \text{m/s})^2}{7000\ \text{m}} = 1.35\ \text{m/s}^2$$

ASSESS If the speed of the train is kept fixed, decreasing the turn radius would the centripetal acceleration.

41. **INTERPRET** The movement of the tip of the minute hand of a clock is a uniform circular motion. We are asked to calculate the magnitude of the centripetal acceleration of the tip.

DEVELOP The minute hand of a clock makes one full revolution in an hour. Therefore, its period is $T = 1\ \text{h} = 3600\ \text{s}$. The speed of the tip is $v = 2\pi r/T$. Given the radius r and the speed v, we use Equation 3.16, $a = v^2/r$, to solve for the acceleration a.

EVALUATE With r = 7.50 cm = 7.5 × 10^{-2} m, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(7.50\times10^{-2}\ \text{m})}{(3600\ \text{s})^2} = 2.28\times10^{-7}\ \text{m/s}^2$$

ASSESS Note that since v is proportional to r in this case $(v = 2\pi r/T)$, the centripetal acceleration varies linearly with r. In other words, a different clock with a longer minute hand will have a greater centripetal acceleration at its tip.

42. **INTERPRET** This problem involves uniform circular motion. We are asked to find with what speed a car must travel through a curve so that the centripetal acceleration is equal to the magnitude of acceleration due to gravity on the surface of the Earth.

DEVELOP Given the radius and the acceleration ($a = g$ = 9.8 m/s^2), we may use Equation 3.16, $a = v^2/r$ to solve for the speed v.

EVALUATE Using Equation 3.16, the speed of the car is

$$v = \sqrt{ar} = \sqrt{\left(9.8\ \text{m/s}^2\right)\left(50\ \text{m}\right)} = 22.1\ \text{m/s} = 79.56\ \text{km/h}$$

ASSESS If the radius of the turn is kept fixed, then the only means to attain a higher centripetal acceleration is to increase the speed. A centripetal acceleration of 1g is just within the capability of autocross tires.

43. **INTERPRET** This problem asks us to estimate the acceleration of the Moon given its orbital radius and its orbital period. Because the Moon's orbit is nearly circular, we can use the formulas for uniform circular motion.

DEVELOP For uniform circular motion, the centripetal (i.e., center-seeking) acceleration is given by Equation 3.16, $a = v^2/r$, where v is the orbital speed and r is the orbital radius. The problem states that r = 3.85 ×105 km and that the orbital period T is T = 27 days = 648 h. The orbital speed is the distance covered in one period divided by the period, or v = $2\pi r/T$.

EVALUATE Inserting the given quantities into Equation 3.16, we find

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(3.85\times10^5\ \text{km}\right)}{\left(648\ \text{h}\right)^2} = \left(36\ \text{km/h}^2\right)\left(\frac{10^6\ \text{mm}}{\text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)^2 = 2.8\ \text{mm/s}^2$$

ASSESS The direction of the acceleration is always towards the center of the Earth.

44. **INTERPRET** We are asked to find the time it takes to complete an orbit, given the centripetal acceleration and the radius of the circle.

DEVELOP The period of the satellite is $T = C/v$, where C is the circumference of the orbit and v is the orbital velocity. Use Equation 3.16, $a = v^2/r$ to find the speed, and $C = 2\pi r$ for the circumference. The acceleration a is provided by gravity, which at 20,000 km above the surface of the Earth is 5.8% of the surface value, or $a = 0.058g$

(with $g = 9.8\ \text{m/s}^2$). Note that the radius r is the Earth's radius R_E *plus* the altitude so $r = R_E + 20{,}000\ \text{km} = R_E + 2 \times 10^7\ \text{m}$. From Appendix E, we find that the Earth's radius is $R_E = 6.37 \times 10^6\ \text{m}$.

EVALUATE Inserting the given quantities in the expression for T gives

$$T = \frac{C}{v} = \pm\frac{2\pi r}{\sqrt{ar}} = \pm\frac{2\pi\sqrt{6.37\times10^6\ \text{m} + 2\times10^7\ \text{m}}}{\sqrt{0.058\left(9.8\ \text{m/s}^2\right)}} = \pm\left(4.28\times10^4\ \text{s}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = \pm 12\ \text{h}$$

ASSESS The fact that the answer has two possible signs reflects the fact that the satellite may travel clockwise or anticlockwise around the Earth.

PROBLEMS

45. **INTERPRET** This problem is an exercise in vector addition.

DEVELOP That the vectors have the same magnitude A and are perpendicular to each other may be expressed mathematically as $\vec{A} = A\hat{i}$ and $\vec{B} = A\hat{j}$. Use Equation 3.1 to find the magnitude of the vector sum.

EVALUATE **(a)** $\vec{A} + 2\vec{B} = A\hat{i} + 2A\hat{j}$, so the magnitude is $\left|\vec{A} + 2\vec{B}\right| = \sqrt{A^2 + (2A)^2} = A\sqrt{5}$. **(b)** $3\vec{A} - \vec{B} = 3A\hat{i} - A\hat{j}$ so the magnitude is $\left|3\vec{A} - \vec{B}\right| = \sqrt{(3A)^2 + (-A)^2} = A\sqrt{10}$.

ASSESS The formula for the vector magnitude is simply the Pythagorean Theorem applied to the orthogonal (i.e., perpendicular) components.

46. **INTERPRET** We interpret this as a problem involving adding two vectors in two dimensions to produce a resultant vector which points in a certain direction.

DEVELOP The two vectors can be decomposed as $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$. Using Equation 3.2, $A_x = A\cos\theta_A$ and $B_x = B\cos\theta_B$. We're told that $A = 1.3\ \text{m}$, $\theta_A = -30°$ (since $\vec{A}$ points clockwise from the x axis), and $B = 1.9\ \text{m}$. Adding the vectors gives

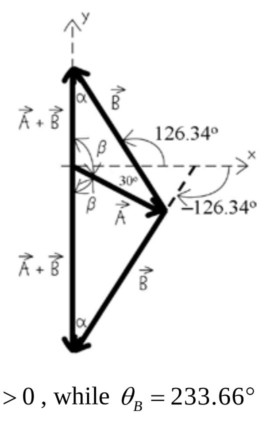
$$\left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

We want the sum to point in the y direction, so $A_x + B_x = 0$. We can solve for θ_B from this equation.

EVALUATE Using $B_x = -A_x$, we can derive the following for θ_B:

$$\theta_B = \cos^{-1}\left(\frac{-A\cos\theta_A}{B}\right) = \cos^{-1}\left(\frac{-(1.3\ \text{m})\cos(-30°)}{(1.9\ \text{m})}\right) = 126.34°$$

This is not the only possible answer. The angle $\theta_B = -126.34°$, or equivalently $360° - 126.34° = 233.66°$, gives the same result. We show how this is possible in the figure below.



Note that $\theta_B = -126.34°$ corresponds to $B_y > 0$, while $\theta_B = 233.66°$ corresponds to $B_y < 0$.

ASSESS Since $\left|\cos\theta_B\right| \le 1$, solutions are possible only if $B \ge A$.

47. **INTERPRET** This problem is an exercise in vector addition.

DEVELOP The vectors are given in component form, so we can perform the indicated algebraic operations on each component individually. The components of $\vec{C}$ are the opposite of the sum of the components of $\vec{A}$ and $\vec{B}$: $\vec{C} = -\left(\vec{A}+\vec{B}\right)$.

EVALUATE Using $\vec{A} = 17\hat{i} - 42\hat{j}$ and $\vec{B} = 31\hat{j} + 17\hat{k}$, and writing the above vector equation in component form gives the components of $\vec{C}$ as

$$C_i = -17\hat{i}$$
$$C_j = -\left(-42\hat{j}\right) - 31\hat{j} = 11\hat{j}$$
$$C_k = -17\hat{k}$$

so the complete vector is $\vec{C} = -17\hat{i} + 11\hat{j} - 17\hat{k}$.

ASSESS The magnitude of this vector is $C = \sqrt{(-17)^2 + (11)^2 + (-17)^2} = 26.43$. This vector is three dimensional, so the angle that its projection onto the *x*-*y* plane makes with the positive-*x* axis is θ = atan(11/−17) = 147.1°. The angle that $\vec{C}$ makes with the positive-*z* axis is

$$\phi = 180° - \text{atan}\left(\frac{\sqrt{(-17)^2 + (11)^2}}{17}\right) = 130°$$

48. **INTERPRET** This problem involves finding the average velocity vector and then finding its magnitude, which is the speed.

DEVELOP The average velocity can be found from Equation 3.3: $\bar{\vec{v}} = \Delta\vec{r}/\Delta t$, where the displacement vector comes from subtracting the bacterium's final position $(\vec{r}_2)$ from its initial position $(\vec{r}_1)$.

EVALUATE **(a)** The displacement is

$$\Delta\vec{r} = \left(4.6\hat{i} + 1.9\hat{k}\ \mu\text{m}\right) - \left(2.2\hat{i} + 3.7\hat{j} - 1.2\hat{k}\ \mu\text{m}\right) = 2.4\hat{i} - 3.7\hat{j} + 3.1\hat{k}\ \mu\text{m}$$

Dividing this by the time, $\Delta t = 6.6$ s gives the average velocity,

$$\bar{\vec{v}} = \frac{2.4\hat{i} - 3.7\hat{j} + 3.1\hat{k}\ \mu\text{m}}{6.6\ \text{s}} = 0.36\hat{i} - 0.56\hat{j} + 0.46\hat{k}\ \mu\text{m/s}$$

(b) The average speed is the magnitude of the average velocity vector. The formula is similar to that of Equation 3.1, but since there are three components (for the three dimensions) there are three squares inside the square root.

$$\bar{v} = \sqrt{(0.36)^2 + (0.56)^2 + (0.46)^2}\ \mu\text{m/s} = 0.81\ \mu\text{m/s}$$

Where we have included an extra significant figure in the computation.

ASSESS The typical size of bacteria is on the order of 1 micron, or 1μm. In this example, the bacterium moves a distance comparable to its own length in one second. This is a reasonable result.

49. **INTERPRET** We are asked to find when the particle is moving in a particular direction. For this, we will need to derive an expression for the particle's velocity.

DEVELOP The velocity can be found from Equation 3.4: $\vec{v} = d\vec{r}/dt$. The particle will be moving in the *x* direction when $v_y = 0$, and it will be moving in the *y* direction when $v_x = 0$.

EVALUATE First taking the derivative of the position vector with respect to time:

$$\vec{v} = \frac{d}{dt}\left[\left(ct^2 - 2dt^3\right)\hat{i} + \left(2ct^2 - dt^3\right)\hat{j}\right] = \left(2ct - 6dt^2\right)\hat{i} + \left(4ct - 3dt^2\right)\hat{j}$$

(a) To move in the *x* direction,

$$v_y = 4ct - 3dt^2 = 0 \quad \rightarrow \quad t = 4c/3d$$

Note that $t = 0$ is also a solution, but at that time $v_x = 0$, so the particle is not moving at all then. **(b)** To move in the *y* direction,

$$v_x = 2ct - 6dt^2 = 0 \quad \rightarrow \quad t = c/3d$$

ASSESS We can plug these times back into the equation for the velocity vector:

$$\vec{v}(c/3d) = \left[4c\left(\frac{c}{3d}\right) - 3d\left(\frac{c}{3d}\right)^2\right]\hat{j} = \frac{c^2}{d}\hat{j}$$

$$\vec{v}(4c/3d) = \left[2c\left(\frac{4c}{3d}\right) - 6d\left(\frac{4c}{3d}\right)^2\right]\hat{i} = -8\frac{c^2}{d}\hat{i}$$

So the particle starts at rest; sometime later it is traveling upwards (+*y* direction), and then even later it is moving to the left (–*x* direction). As *t* gets very large, the particle moves in the direction:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \approx \tan^{-1}\left(\frac{-3dt^2}{-6dt^2}\right) = \tan^{-1}(0.5) = 207^\circ$$

Where we have used the fact that t^2 grows faster than *t*.

50. **INTERPRET** We are asked to find when the particle is at rest and accelerating in the *x* direction.

DEVELOP We already found the velocity in the previous problem. The particle is at rest when both components are zero, $v_x = v_y = 0$. The acceleration is the time derivative of the velocity, from Equation 3.6. The particle will be accelerating in the *x* direction when $a_y = 0$.

EVALUATE **(a)** As found previously, the velocity of the particle is:

$$\vec{v} = \left(2ct - 6dt^2\right)\hat{i} + \left(4ct - 3dt^2\right)\hat{j}$$

Both components are zero, and the particle is at rest, when $t = 0$.

(b) Taking the time derivative of the velocity,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\left(2ct - 6dt^2\right)\hat{i} + \left(4ct - 3dt^2\right)\hat{j}\right] = \left(2c - 12dt\right)\hat{i} + \left(4c - 6dt\right)\hat{j}$$

The particle is accelerating in the *x* direction when

$$a_y = 4c - 6dt = 0 \quad \rightarrow \quad t = 2c/3d$$

ASSESS The acceleration vector in the *x* direction is

$$\vec{a}(t = 2c/3d) = \left[2c - 12d\left(\frac{2c}{3d}\right)\right]\hat{i} = -6c\hat{i}$$

We needed to check this to be sure that both components of the acceleration are not zero, in which case the particle would not be accelerating at all.

51. **INTERPRET** This problem involves uniform circular motion. Given the threshold (maximum) centripetal acceleration, we are asked to calculate the minimum turn radius of the highway interchange.

DEVELOP The car is moving at a speed *v*. Since the centripetal acceleration of the car is not to exceed $a = 0.40g$, where $g = 9.8\text{ m/s}^2$, we use Equation 3.16, $a = v^2/r$, to solve for the minimum radius *r*.

EVALUATE With *v* = 70 km/h = 19.44 m/s, we find the radius to be

$$r = \frac{v^2}{a} = \frac{(19.44\text{ m/s})^2}{(0.40)(9.8\text{ m/s}^2)} = 96\text{ m}$$

ASSESS The radius is about 300 ft. Note that a larger turn radius would be required for higher speed. At 55 mph, or 24.58 m/s, the minimum radius (keeping acceleration unchanged) would be 154 m.

52. **INTERPRET** This problem involves calculating the initial velocity given the acceleration, the final velocity, and the acceleration period. We are also asked to find the change in the speed and direction of the vehicle, and explain why the change in speed is not equivalent to the magnitude of the acceleration multiplied by the time.

DEVELOP Use Equation 3.5, $\bar{\vec{a}} = \Delta\vec{v}/\Delta t$, to calculate the initial velocity, given that the acceleration is $\bar{\vec{a}} = \left(2.3\hat{i} + 3.6\hat{j}\right)\text{ m/s}^2$, the final velocity is $\vec{v} = \left(33\hat{i} + 15\hat{j}\right)\text{ m/s}$, and the time interval is $\Delta t = 10\text{ s}$. To find the speed change, use $\Delta v = v - v_0$ using the result from part **(a)**. To find the change in direction, take the difference between the initial and final angles made by the corresponding velocity vectors with the *x* axis.

EVALUATE **(a)** The initial velocity is

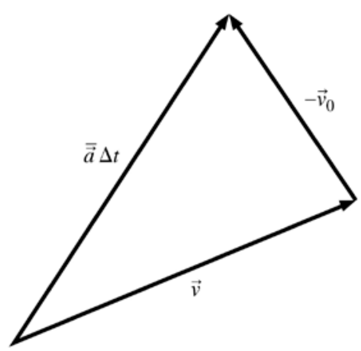
$$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{\Delta t}$$

$$\vec{v}_0 = \vec{v} - \bar{\vec{a}}\Delta t = \left(33\hat{i} + 15\hat{j}\right)\text{ m/s} - \left[\left(2.3\hat{i} + 3.6\hat{j}\right)\text{ m/s}^2\right](10\text{ s}) = \left(10\hat{i} - 21\hat{j}\right)\text{ m/s}$$

(b) The initial speed is $\vec{v}_0 = \sqrt{(10\text{ m/s})^2 + (21\text{ m/s})^2} = 23.26\text{ m/s}$ and the final speed is $v = \sqrt{(33\text{ m/s})^2 + (15\text{ m/s})^2} = 36.25\text{ m/s}$, so the change in speed is $v = 36.25\text{ m/s} - 23.26\text{ m/s} = 13\text{ m/s}$ to two significant figures.

(c) The intial and final angles are $\theta_0 = \text{atan}(-21/10) = 295° = -64.5°$ and $\theta = \text{atan}(15/33) = 24.4°$, so the difference is $\Delta\theta = \theta - \theta_0 = 24.4° - (-64.5°) = 89°$.

(d) The magnitude of the acceleration is $\bar{a} = \sqrt{(2.3\text{ m/s}^2)^2 + (3.6\text{ m/s}^2)^2} = 4.27\text{ m/s}^2$. Multiplying this by the time interval Δt = 10 s gives $\bar{a}t = 47.2\text{ m/s}$ which is not the same as the change in speed from part **(b)**. The reason for this may be seen from the figure below. The change in speed is just the difference in the lengths of the vectors $\vec{v}$ and $\vec{v}_0$. It does not depend on the angle between these two vectors. However, the magnitude of the change in velocity is the length of the vector $\bar{\vec{a}}\Delta t$, which *does* depends on the angle between the initial and final velocities. Only if this angle is zero are the two quantities the same.



ASSESS This problem demonstrates the importance of the vector direction in determining physical quantities.

53. **INTERPRET** In this problem we have to find the average velocity and acceleration by taking the difference in the position vector and the velocity vector, and then dividing by the time.

DEVELOP Let's choose a coordinate system with origin at the center of the Ferris wheel, so that the position vector always has a magnitude of $R = d/2 = (183\text{ m})/2 = 91.5\text{ m}$. The speed is the circumference divided by the rotational period: $v = 2\pi R/T = 2\pi(91.5\text{ m})/(37.3\text{ min}) = 0.257\text{ m/s}$. Let's take the initial position to be at the lowest point, i.e., $\vec{r}_0 = -91.5\hat{j}\text{ m}$, and we'll assume the wheel moves counterclockwise, such that $\vec{v}_0 = 0.257\hat{i}\text{ m/s}$. After $\Delta t = 5.0\text{ min}$, the wheel will have completed 5/37.3 of its rotation, meaning it will have advanced by $\theta = 48.3°$. In component form, the final position and velocity are

$$\vec{r} = R\left(\sin\theta\hat{i} - \cos\theta\hat{j}\right) = (91.5\text{ m})\sin(48.3°)\hat{i} - (91.5\text{ m})\cos(48.3°)\hat{j} = 68.32\hat{i} - 60.87\hat{j}\text{ m}$$

$$\vec{v} = v\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) = (0.257\text{ m/s})\cos(48.3°)\hat{i} + (0.257\text{ m/s})\sin(48.3°)\hat{j} = 0.171\hat{i} + 0.192\hat{j}\text{ m/s}$$

EVALUATE **(a)** The average velocity is change in position divided by the time:

$$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\left(68.32\hat{i} - 60.87\hat{j}\text{ m}\right) - \left(-91.5\hat{j}\text{ m}\right)}{(5.0\text{ min})(60\text{ s/min})} = 0.228\hat{i} + 0.102\hat{j}\text{ m/s}$$

The magnitude of the average velocity is $v = \sqrt{(0.228\text{ m/s})^2 + (0.102\text{ m/s})^2} = 0.249\text{ m/s}$.

(b) The average acceleration is change in velocity divided by the time:

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\left(0.171\hat{i} + 0.192\hat{j}\ \text{m/s}\right) - \left(0.257\hat{i}\ \text{m/s}\right)}{(5.0\ \text{min})(60\ \text{s/min})} = \left(-2.86\hat{i} + 6.39\hat{j}\right) \times 10^{-4}\,\text{m/s}^2$$

The magnitude of the average acceleration is

$$a = \sqrt{(-2.86 \times 10^{-4}\ \text{m/s}^2)^2 + (6.39 \times 10^{-4}\ \text{m/s}^2)^2} = 7.00 \times 10^{-4}\ \text{m/s}^2$$

(c) The instantaneous acceleration is

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = R\left(\frac{d\theta}{dt}\right)^2 \left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)$$

where $d\theta / dt = (2\pi)/(37.3 \times 60\ \text{s}) = 2.81 \times 10^{-3}$ rad/s. Thus, the magnitude of $\vec{a}$ is

$$a = R\left(\frac{d\theta}{dt}\right)^2 = (91.5\ \text{m})(2.81 \times 10^{-3}\ \text{rad/s})^2 = 7.21 \times 10^{-4}\ \text{m/s}^2$$

The difference is about 3%.

ASSESS The magnitude of the average velocity is 0.249 m/s, which is close to the instantaneous velocity of 0.257 m/s. The average velocity is smaller because it doesn't take into account the curved path followed by a point on the rim of the wheel.

54. **INTERPRET** This is a problem of relative velocities. The ferryboat has to head upstream slightly to compensate for the current that drags it downstream.

DEVELOP The river velocity $\vec{V}$ and the ferryboat velocity $\vec{v}'$ with respect to the water are two sides of a right triangle (see the figure in the solution to Exercise 3.27), where the third side is the velocity of the boat relative to the ground.

EVALUATE **(a)** The angle that the boat must head is given by

$$\theta = \sin^{-1}\left(\frac{V}{v'}\right)$$

(b) If V were greater than v', then there would be no solution for the angle (since $|\sin\theta| \le 1$). What this means is that the river is flowing too fast for the ferryboat to be able to get straight across.

ASSESS If $V = v'$, there's no real solution, since that would mean $\theta = 90°$. At that heading (straight upstream), the ferryboat would not be moving towards the other side of the river. It would be motionless relative to the ground.

55. **INTERPRET** This problem is an exercise in vector addition. We are asked to compare the magnitude of two vectors given that their sum is perpendicular to their difference.

DEVELOP Use Equation 3.1 to express vector $\vec{A}$ in component form: $\vec{A} = A_x\hat{i} + A_y\hat{j}$ where $A = \sqrt{A_x^2 + A_y^2}$ and $\theta_A = \text{atan}\left(A_y / A_x\right)$. Similarly, express vector $\vec{B}$ as $\vec{B} = B_x\hat{i} + B_y\hat{j}$. Let $\vec{C}$ be the sum of the two vectors:

$$\vec{C} = \vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j} = C_x\hat{i} + C_y\hat{j}$$

and $\vec{D}$ be the difference of the two vectors:

$$\vec{D} = \vec{A} - \vec{B} = \left(A_x - B_x\right)\hat{i} + \left(A_y - B_y\right)\hat{j} = D_x\hat{i} + D_y\hat{j}\ .$$

If $\vec{C}$ and $\vec{D}$ are to be perpendicular to each other, then we can let them define our coordinate system, so $\vec{C} = C_x\hat{i}$ and $\vec{D} = D_y\hat{j}$.

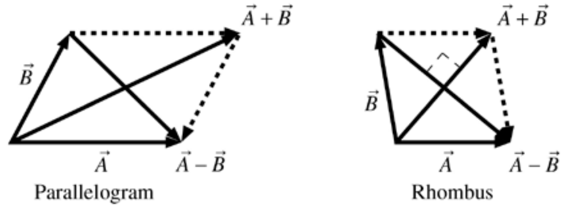
EVALUATE The conditions set for $\vec{C}$ and $\vec{D}$ imply $C_y = 0$ and $D_x = 0$, or

$$C_y = A_y + B_y = 0 \quad \Rightarrow \quad A_y = -B_y$$
$$D_x = A_x - B_x = 0 \quad \Rightarrow \quad A_x = B_x$$

Using these results, we can show that the magnitudes of $\vec{A}$ and $\vec{B}$ are equal as follows:

$$A=\sqrt{A_x^2+A_y^2}=\sqrt{B_x^2+\left(-B_y\right)^2}=\sqrt{B_x^2+B_y^2}=B$$

ASSESS An alternative way to establish the equality between A and B is to note that the vectors $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ form the two diagonals of a parallelogram formed by sides $\vec{A}$ and $\vec{B}$ (see figure below). If the diagonals are perpendicular, the parallelogram is a rhombus, so $A = B$.



56. **INTERPRET** We have to construct a unit vector in the given direction.

DEVELOP The angle $45°$ is halfway between the x and y axes. Let's define an arbitrary vector pointing in this direction:

$$\vec{A}=\hat{i}-\hat{j}$$

To make a unit vector, we divide $\vec{A}$ by its magnitude, A.

EVALUATE The magnitude of $\vec{A}$ is $A=\sqrt{1+1}=\sqrt{2}$, so a unit vector pointing clockwise from the x axis is

$$\hat{n}=\frac{\vec{A}}{A}=\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}$$

ASSESS We can get the same answer by using Equation 3.2 and recalling that angles clockwise from the x axis are defined as negative, so $\theta=-45°$.

$$\hat{n}=\cos\left(-45°\right)\hat{i}+\sin\left(-45°\right)\hat{j}$$

In general, any unit vector in two dimensions can be written as $\hat{n}=\cos\theta\hat{i}+\sin\theta\hat{j}$.

57. **INTERPRET** This problem involves motion in two dimensions. We are asked to find the acceleration of an object given its initial velocities and the distance it moves in a given time interval.

DEVELOP Note that the displacements in the x and y directions are independent of each other. The x component of the displacement is due to the initial velocity, $\Delta\vec{x}=\vec{v}_0\Delta t$, and the y component is due to the acceleration, $\Delta\vec{y}=\vec{a}\Delta t^2/2$.

EVALUATE The condition that the object moves an equal distance in both directions can be expressed mathematically as $\Delta x=\Delta y$ when $\Delta t=22\text{ s}$. This gives

$$v_0\Delta t=\frac{1}{2}a\Delta t^2$$
$$a=\frac{2v_0}{\Delta t}=\frac{2\left(5.5\text{ m/s}\right)}{22\text{ s}}=0.50\text{ m/s}^2$$

ASSESS The answer can be checked by substituting the value of a into Δy:

$$\Delta y=\frac{1}{2}a\Delta t^2=\frac{1}{2}\left(0.5\text{ m/s}^2\right)\left(22\text{ s}\right)^2=121\text{ m}$$

This is equal to $\Delta x=v_0\Delta t=\left(4.5\text{ m/s}\right)\left(22\text{ s}\right)=121\text{ m}$.

58. **INTERPRET** This problem is one of constant acceleration.

DEVELOP The particle starts at the origin, $\vec{r}_0=0$, with an initial velocity, $\vec{v}_0=11\hat{i}+18\hat{j}\text{ m/s}$, and constant acceleration, $\vec{a}=-1.4\hat{i}+0.27\hat{j}\text{ m/s}^2$. We can plug this into Equation 3.9 and break it up into components:

$$x=v_{x0}t+\tfrac{1}{2}a_xt^2=\left(11\text{ m/s}\right)t+\tfrac{1}{2}\left(-1.4\text{ m/s}^2\right)t^2$$
$$y=v_{y0}t+\tfrac{1}{2}a_yt^2=\left(18\text{ m/s}\right)t+\tfrac{1}{2}\left(0.27\text{ m/s}^2\right)t^2$$

We can use these equations to find the time when the particle crosses the *y* axis $(x=0)$, and where it crosses. We can then use Equation 3.8 $(\vec{v}=\vec{v}_0+\vec{a}t)$ to find the velocity at the crossing time.

EVALUATE **(a)** Solving for the time when $x=0$ gives

$$t=\frac{2v_{x0}}{a_x}=\frac{2(11\ \text{m/s})}{(-1.4\ \text{m/s}^2)}=15.7\ \text{s}\simeq 16\ \text{s}$$

(b) Plugging this time in for the *y* position,

$$y=(18\ \text{m/s})(15.7\ \text{s})+\tfrac{1}{2}(0.27\ \text{m/s}^2)(15.7\ \text{s})^2=315.87\ \text{m}$$

(c) Plugging this time in for the velocity,

$$\vec{v}=(11\hat{i}+18\hat{j}\ \text{m/s})+(-1.4\hat{i}+0.27\hat{j}\ \text{m/s}^2)(15.7\ \text{s})=-10.98\hat{i}+22.24\hat{j}\ \text{m/s}$$

The magnitude and direction of this velocity are

$$v=\sqrt{(-10.98\ \text{m/s})^2+(22.24\ \text{m/s})^2}=24.8\ \text{m/s}$$

$$\theta=\tan^{-1}\left(\frac{22.24}{-10.98}\right)=116.27°$$

ASSESS Note that most calculators will compute the angle as $-60°$, but there are actually two answers, since $\tan\theta=\tan(\theta+180°)$. In this case, the *x* component of the velocity is negative, while the *y* component is positive, so the vector points in the upper-left quadrant where the angle $120°\ \ (=-60°+180°)$ is located.

59. **INTERPRET** This problem involves combined horizontal and vertical motion due to the gravity near the Earth's surface, so it is projectile motion. More specifically, it asks for the initial position of the water, so it involves the trajectory of the water.

DEVELOP We are given the initial height of the water, y_0 = 1.7 m, the final height, y = 0.98 m, and the range x = 2.2 m. In addition, the problem states that the water is fired horizontally, so the initial angle at which the water is fired is θ_0 = 0°. These quantities are related by the trajectory Equation 3.14,

$$y-y_0=x\tan(\theta_0)-\frac{g}{2v_0^2\cos^2(\theta_0)}x^2$$

which we can solve to find the initial speed v_0.

EVALUATE With $\theta_0=0$, the above equation simplifies to $y-y_0=-gx^2/(2v_0^2)$, which gives

$$v_0=x\sqrt{\frac{-g}{2(y-y_0)}}=(2.2\ \text{m})\sqrt{\frac{(-9.8\ \text{m/s}^2)}{2(0.98\ \text{m}-1.7\ \text{m})}}=5.74\ \text{m/s}$$

ASSESS We can check the answer by solving the problem in a different way. Because the water was fired horizontally $v_{0y}=0$, so the time it takes to fall from y_0 = 1.7 m to y – 0.98 m is given by Equation 3.13:

$$t=\sqrt{2(y_0-y)/g}=\sqrt{2(1.7\ \text{m}-0.98\ \text{m})/(9.8\text{m/s}^2)}=0.38\ \text{s}$$

Its initial speed, $v_0=v_0x$, can be found from Equation 3.12:

$$v_0=(x-x_0)/t=(2.2\ \text{m})/(0.38\ \text{s})=5.73\,\text{m/s}$$

Both approaches lead to the same answer for v_0.

60. **INTERPRET** We need to find a general equation for the initial speed of a projectile, given the range *R* and maximum height *h*.

DEVELOP If the trajectory is over level ground, then it reaches the maximum height at a time halfway through the flight. The time for the entire trajectory is twice the time it takes to fall from height *h*, so use this time and the range to find the *x*-component of the initial velocity. We can also use the height to find the *y* component of the initial velocity (note that the *x* component of the velocity is constant throughout the trajectory). Thus, use $y=y_0+v_{y0}t-\frac{1}{2}gt^2$, with $y-y_0=h$, to find the time *t*. Next, use $R=v_{x0}t_{\text{tot}}=v_{x0}(2t)$, where t_{tot} is the total flight time of the projectile, to find the *x* component of the initial velocity and $v_y=v_{y0}+a_yt$ to find the *y* component of initial velocity.

EVALUATE The time for the trajectory is

$$y - y_0 = h = +v_{y0}t - \frac{1}{2}gt^2$$
$$t = \sqrt{\frac{2h}{g}}$$

Inserting this time into the expression relating the range to v_{0x} gives

$$v_{x0} = \frac{R}{2t} = \frac{R}{2}\sqrt{\frac{g}{2h}}$$

Finally, the y component of the initial velocity is

$$v_y = v_{y0} + a_y t$$
$$v_{y0} = gt = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$$

so the initial speed of the projectile must be

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{\frac{R^2 g}{8h} + 2gh}$$

ASSESS If the problem did not specify level ground, it would not have been possible to solve. Why?

61. INTERPRET You are asked to calculate the kinematics of two projectiles (the balls) with different starting points.
DEVELOP Let's assume you are at the origin. Your friend's direction is $45°$ above the horizontal from you, which means your friend is located at the same distance from you in the vertical and horizontal directions, i.e., $y_f = h$ and $x_f = h$. Your ball (ball 1) has an initial velocity of:

$$\vec{v}_0 = v_0\cos 45°\hat{i} + v_0 \sin 45°\hat{j} = \tfrac{1}{\sqrt{2}}v_0\left(\hat{i} + \hat{j}\right)$$

This ball therefore follows a trajectory of:

$$x_1 = \tfrac{1}{\sqrt{2}}v_0 t$$
$$y_1 = \tfrac{1}{\sqrt{2}}v_0 t - \tfrac{1}{2}gt^2$$

Your friend's ball (ball 2) has no initial velocity, so it falls straight down from its initial position

$$x_2 = h$$
$$y_2 = h - \tfrac{1}{2}gt^2$$

To verify whether or not the balls collide, we need to first find when ball 1 to reaches the horizontal position of ball 2, i.e. when $x_1 = x_2$. Once we have this time we can compare the vertical position of each ball.

EVALUATE **(a)** Solving for the time when $x_1 = x_2$ gives

$$t = \frac{x_1}{\frac{1}{\sqrt{2}}v_0} = \frac{\sqrt{2}\cdot h}{v_0}$$

Plugging this time in for the vertical position of each ball gives

$$y_1 = \frac{v_0}{\sqrt{2}}\left(\frac{\sqrt{2}h}{v_0}\right) - \tfrac{1}{2}g\left(\frac{\sqrt{2}h}{v_0}\right)^2 = h - \frac{gh^2}{v_0^2}$$
$$y_2 = h - \tfrac{1}{2}g\left(\frac{\sqrt{2}h}{v_0}\right)^2 = h - \frac{gh^2}{v_0^2}$$

The fact that $y_1 = y_2$ implies a collision. But the assumption is that ball 1 has enough initial velocity to reach the horizontal position of ball 2 before hitting the ground. We will now derive an expression for this minimum velocity.
(b) For the collision to occur, ball 1 has to still be in the air $(y_1 \ge 0)$ when it reaches the horizontal position of ball 2:

$$y_1 = h - \frac{gh^2}{v_0^2} \ge 0 \;\rightarrow\; v_0 \ge \sqrt{gh}$$

ASSESS Another way to derive this is by saying that ball 1's horizontal range: $x = v_0^2 \sin 2\theta_0 / g$ (Equation 3.15) has to be large enough to reach the horizontal position of ball 2.

62. **INTERPRET** This problem involves motion under the influence of gravity near the Earth's surface, where we are only interested in the position of an object, not in the time at which it reaches a position. Thus, the problem involves a trajectory, so Equation 3.14 applies.

DEVELOP Because the stuntman runs horizontally off the roof, $\theta_0 = 0°$. Furthermore, we know that $y = 1.9$ m and $x = 4.5$ m. Insert these quantities into Equation 3.14 to find the initial speed v_0.

EVALUATE Using tan(0°) = 0 and cos(0°) = 1, we find that the stuntman's initial speed is

$$y = -\frac{g}{2v_0^2}x^2$$

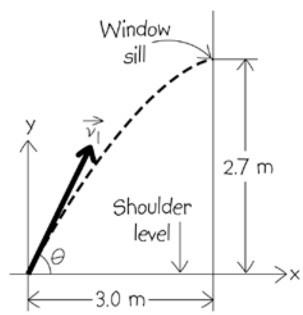
$$v_0 = \pm\sqrt{-\frac{gx^2}{2y}} = \pm\sqrt{-\frac{(9.8\text{ m/s}^2)(4.5\text{ m})^2}{2(-1.9\text{ m})}} = \pm 7.2\text{ m/s}$$

where the two signs indicate that the stuntman may have run off the building going to the left or the right.

ASSESS It is also possible to solve this problem using the equations of projectile motion (from which Equation 3.14 is derived). Using Equations 3.12 and 3.13, we find that the horizontal and vertical distances covered by the stuntman are $x - x_0 = v_0 t$ and $y_0 - y = \frac{1}{2}gt^2$ (because $v_{0x} = v_0$ and $v_{0y} = 0$). Eliminating t, we find $v_0 = \pm(x - x_0)\sqrt{-g/[2(y - y_0)]}$, which is the same expression as above for the initial velocity.

63. **INTERPRET** This problem involves motion under the influence of gravity near the Earth's surface, where we are only interested the velocity of an object. Thus, we will use the equations of projectile motion.

DEVELOP Draw a diagram of the situation (see figure below). Use Equation 3.13, $y = y_0 + v_{y0}t - gt^2/2$, to find the initial velocity in the y direction, with $y - y_0 = 1.5 - 4.2$ m $= -27$ m. This result may be used to find the time of flight to the window sill by inserting it into Equation 3.11, $v_y = v_{y0} - gt$, with $v_y = 0$ m/s because the package presumably attains its maximum height at the window sill. The time t may be inserted into Equation 3.12, $x = x_0 + v_{x0}t$ with $x - x_0 = 3.0$ m.



EVALUATE Solving Equation 3.13 for the initial velocity in the y direction gives

$$v_{0y} = \sqrt{2(9.8\text{ m/s}^2)(2.7\text{ m})} = 7.27\text{ m/s}$$

where we retain more significant figures than warranted because this is an intermediate result. From Equation 3.11, the time of flight to the window sill is

$$t = \frac{v_y - v_{y0}}{g} = \frac{0\text{ m/s} - 7.27\text{ m/s}}{9.8\text{ m/s}^2} = 0.7423\text{ s}$$

where we have again retained excess significant figures. The initial velocity in the x direction is

$$v_{0x} = \frac{x}{t} = \frac{3.0\text{ m}}{0.742\text{ s}} = 4.041\text{ m/s}$$

From these components, we find the magnitude and direction of $\vec{v}_0$ are

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(4.041\text{ m/s})^2 + (7.275\text{ m/s})^2} = 8.3\text{ m/s}$$

$$\theta_0 = \operatorname{atan}\left(\frac{v_{0y}}{v_{0x}}\right) = \operatorname{atan}\left(\frac{7.275}{4.041}\right) = 61°$$

above the x axis. Notice that we have retained only two significant figures in the final result.

ASSESS We can check the answer by inserting the result for the angle into Equation 3.14. At the window sill, Equation 3.14 gives

$$y = x\tan(\theta_0) - \frac{g}{2v_0^2\cos^2(\theta_0)}x^2 = (3.0\text{ m})\tan(60.9°) - \frac{(9.8\text{ m/s}^2)(3.0\text{ m})^2}{2(8.32\text{ m/s})^2\cos^2(60.9°)} = 2.7\text{ m}$$

which agrees with the problem statement. Notice that although we evaluated Equation 3.14 for $x = 3.0$ m (i.e., at the window sill), we still used the initial angle θ_0. This is because the initial angle is a constant, whereas the quantities x and y vary along the trajectory.

64. **INTERPRET** This problem involves motion under the influence of gravity near the Earth's surface, and we are interested only in the position of the object, not its travel time. Therefore, this is projectile motion and Equation 3.14 applies for calculating the trajectory.

DEVELOP Apply Equation 3.14 with $\theta_0 = 0°$ (object starts with horizontal velocity), so $\tan(\theta_0) = 0$ and $\cos(\theta_0) = 1$. Furthermore, we know that the projectile will cover a vertical distance h, so $y = h$. Finally, the initial speed of the projectile is given as v_0.

EVALUATE Equation 3.14 gives

$$h = x\tan(\theta_0) - \frac{g}{2v_0^2\cos^2(\theta_0)}x^2$$

which leads to

$$h = -\frac{g}{2v_0^2}x^2 \quad \Rightarrow \quad x = \pm v_0\sqrt{-\frac{2h}{g}}$$

ASSESS Does this make sense? The units of the right-hand side are m, which is good. Recall as well that y is the vertical distance covered by the trajectory, and in this case $y < 0$ because the final position is less than the initial position (the projectile falls under the influence of gravity). In addition, g is the *magnitude* of the acceleration due to gravity, so $g = 9.8\text{ m/s}^2 > 0$. Thus, the number under the radical is positive. The two signs (±) indicate that the horizontal displacement can be to the left or to the right.

65. **INTERPRET** We are asked to derive a mathematical relation between the time of flight of two projectiles.

DEVELOP In general, the time of flight over level ground is $t = 2v_{y0}/g$, where we have used Equation 3.13 with $y = y_0$. The initial velocities of the two projectiles have the same magnitude but point at different angles, so the vertical components of the velocities can be written as:

$$v_{y0}^+ = v\sin(45° + \alpha)\ ;\ \ v_{y0}^- = v\sin(45° - \alpha)$$

To derive the final equation, we will need some of the trigonometric identities from Appendix A.

EVALUATE Using the vertical components of the initial velocities, the time of flight ratio is

$$\frac{t^+}{t^-} = \frac{v_{y0}^+}{v_{y0}^-} = \frac{\sin(45° + \alpha)}{\sin(45° - \alpha)}$$

Using the equations for $\sin(\alpha - \beta)$ and $\cos(\alpha + \beta)$ from Appendix A and the fact that $\sin 45^{\circ} = \cos 45^{\circ}$, we can rewrite the denominator from above as:

$$\begin{aligned}\sin(45° - \alpha) &= \sin 45°\cos\alpha - \cos 45°\sin\alpha \\ &= \cos 45°\cos\alpha - \sin 45°\sin\alpha \\ &= \cos(45° + \alpha)\end{aligned}$$

Plugging this back into the time of flight ratio equation gives

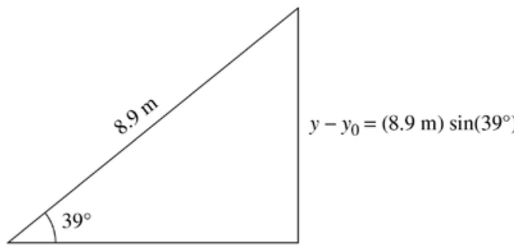
$$\frac{t^+}{t^-} = \frac{\sin(45° + \alpha)}{\cos(45° + \alpha)} = \tan(45° + \alpha)$$

ASSESS We could just as easily have derived $\cot(45° - \alpha)$, where the cotangent is equal to 1/tan.

66. **INTERPRET** This problem involves motion under the influence of gravity near the Earth's surface, so Equations 3.10–3.13 for projectile motion apply. We are asked to determine the initial velocity vector necessary so that an object attains a given height and horizontal distance at the peak of its trajectory.

DEVELOP When the protein bar reaches your friend, it is traveling horizontally, so the final vertical velocity is zero, or $v_y = 0$. Use Equation 3.11, $v_y = v_{y0} - gt$, to find the travel time t, which can then be inserted into Equations 3.12 and 3.13 $(x = x_0 + v_{x0}t$ and $y = y_0 + v_{y0}t - gt^2/2)$ to find the initial x and y velocity. Note that the velocity at the peak of the bar's trajectory (when it reaches your friend) is $v_y = 0$ m/s because the bar is not moving vertically at that point, only horizontally. Furthermore, because our friend is located 8.9 m up a slope inclined at 39°, we must use trigonometry to calculate the horizontal and vertical displacements (see figure below). The result is

$$x - x_0 = (8.6\text{ m})\cos(39°) = 6.68\text{ m}$$
$$y - y_0 = (8.6\text{ m})\sin(39°) = 5.41\text{ m}.$$



EVALUATE From Equation 3.11, we find that the travel time t for the protein bar is

$$\overbrace{v_y}^{=0} = v_{y0} - gt$$
$$t = v_{y0}/g$$

Inserting this result into Equation 3.13 gives an initial vertical velocity of

$$y - y_0 = v_{y0}t - \frac{1}{2}gt^2 = v_{y0}\left(\frac{v_{y0}}{g}\right) - \frac{1}{2}g\left(\frac{v_{y0}}{g}\right)^2$$
$$v_{y0} = \pm\sqrt{2g(y - y_0)} = \pm\sqrt{2(9.8\text{ m/s}^2)(5.41\text{ m})} = 10.30\text{ m/s}$$

Inserting the time t and the vertical velocity v_{y0} just calculated into Equation 3.12 gives an initial horizontal velocity of

$$x - x_0 = v_{x0}t$$
$$v_{x0} = \frac{x - x_0}{t} = (x - x_0)\left(\frac{g}{v_{y0}}\right) = \pm\frac{(9.8\text{ m/s}^2)(6.68\text{ m})}{10.30\text{ m/s}} = 6.36\text{ m/s}$$

Thus, the initial velocity of the bar must be

$$\vec{v}_0 = v_{x0}\hat{i} + v_{y0}\hat{j} = (6.4\text{ m/s})\hat{i} + (10\text{ m/s})\hat{j}$$

where the result is reported to two significant figures.

ASSESS The initial velocity may also be described by its magnitude and direction, which are

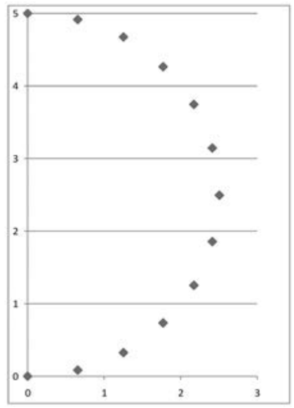
$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{(6.36\text{ m/s})^2 + (10.30\text{ m/s})^2} = 12\text{ m/s}$$
$$\theta_0 = \text{atan}\left(\frac{v_{y0}}{v_{x0}}\right) = \text{atan}\left(\frac{10.30\text{ m/s}}{6.36\text{ m/s}}\right) = 58°$$

Notice that 58° > 39°, which is reasonable because you must throw the bar at an angle greater than the slope, or you will be throwing it into the slope.

67. **INTERPRET** This is a data-analysis problem, where the position of an object in the x-y plane is given at various times . We analyze the data and determine the nature of its path, its velocity and acceleration.

DEVELOP From the table, we note that as *y* increases, *x* first increases and then decreases, indicating that the trajectory is curved. The points (0, 0), (2.5, 2.5) and (0, 5.0) suggests a semi-circular path.

EVALUATE A plot of position *y* versus *x* is given below.



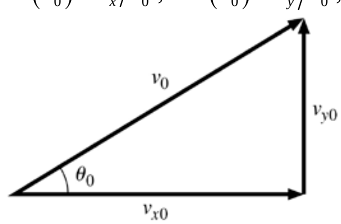
Indeed, the path is a semi-circle with radius 2.5 m. The period is 2.4 s, since it takes 1.2 s to traverse half a circle. Thus, the speed of the object is $v = 2\pi r / T = 2\pi(2.5\text{ m})/(2.4\text{ s}) = 6.54\text{ m/s}$. Similarly, we find the centripetal acceleration to be

$$a = \frac{v^2}{r} = \frac{(6.54\text{ m/s})^2}{2.5\text{ m}} = 17.1\text{ m/s}^2$$

ASSESS An analytic expression for the path given above is $x^2 + (y-2.5)^2 = (2.5)^2$, for positive *x* and *y*.

68. **INTERPRET** This problem involves projectile motion. We are asked to express the maximum horizontal range in terms of the angle at which a projectile is launched and the maximum height it attains.

DEVELOP The expression for the horizontal range (when the initial and final heights are equal) is $x = 2v_0^2\sin(\theta_0)$ (Equation 3.15). The maximum height $h = y_{\text{max}} - y_0$ can be found from Equation 2.11, $v_y^2 = v_{y0}^2 - 2g(y_{\text{max}} - y_0)$, with *v* = 0, which gives $v_{y0} = \sqrt{2gh}$. If you draw a picture of the initial velocity vector and its components (see figure below), it becomes apparent that $\cos(\theta_0) = v_x/v_0$, $\sin(\theta_0) = v_y/v_0$, and $\tan(\theta_0) = v_{y0}/v_{x0}$.



Therefore, from the equation just before Equation 3.15, we have $x = (2v_0^2/g)\sin(\theta_0)\cos(\theta_0) = 2v_{x0}v_{y0}/g$. Combine these equations to solve the problem.

EVALUATE Inserting $\tan(\theta_0) = v_{y0}/v_{x0}$ and $v_{y0} = \sqrt{2gh}$ into the last expression from above for *x* gives

$$x = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2}{g\tan(\theta_0)} = \frac{4h}{\tan(\theta_0)}$$

ASSESS This result reflects a classical geometrical property of the parabola, namely, that the latus rectum is four times the distance from vertex to focus.

69. **INTERPRET** This problem involves projectile motion. You are asked to estimate the initial horizontal speed of the motorcyclist given the range over which he flew.

DEVELOP Imagine the motorcyclist is traveling at the legal speed, 60 km/h = 16.67 m/s. If we find that his range is less than the 39 m reported, we can conclude that he was probably not speeding. If his range is greater than 39 m, then he was probably speeding. Assume that he is deflected upwards off the car's windshield (which we consider to be a frictionless surface), at 45°, which will maximize his range. We can then use Equation 3.15 to find the range over which he would travel before landing on the road.

EVALUATE Inserting the intial speed and angle into Equation 3.15 gives

$$x = \frac{v_0^2}{g}\sin(2\theta_0) = \frac{(16.67\text{ m/s})^2\sin(90°)}{9.8\text{ m/s}^2} = 28\text{ m}$$

Because of our assumptions, this would be the motorcyclist's maximum range. The fact that he flew 39 m before landing implies that he was almost certainly speeding.

ASSESS To estimate the minimum speed at which he was traveling, insert the range of x = 39 m into Equation 3.15 and solve for the initial velocity v_0 (again assuming θ_0 = 45°). This gives

$$x = \frac{v_0^2}{g}\sin(2\theta_0)$$

$$v_0 = \pm\sqrt{\frac{xg}{\sin(2\theta_0)}} = \pm\sqrt{\frac{(39\text{ m})(9.8\text{ m/s}^2)}{\sin(90°)}} = (19.56\text{ m/s})\left(\frac{1\text{ km}}{10^3\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 70\text{ km/h}$$

70. **INTERPRET** We will be comparing the horizontal range of two trajectories that differ in their initial angle.

DEVELOP From Equation 3.15, the range is $x = v_0^2\sin 2\theta_0 / g$. We will show that the sine of $\theta_0 = 45° + \alpha$ is equivalent to the sine of $\theta_0 = 45° - \alpha$.

EVALUATE The trigonometric identity in Appendix A for the sine of the sum of two angles shows that

$$\sin 2(45° \pm \alpha) = \sin(90° \pm 2\alpha) = \sin 90° \cos 2\alpha \pm \cos 90° \sin 2\alpha \ = \cos 2\alpha$$

where we have used the fact that $\sin 90° = 1$ and $\cos 90° = 0$. The horizontal range formula, therefore, gives the same range for either launch angle, assuming the same initial speed.

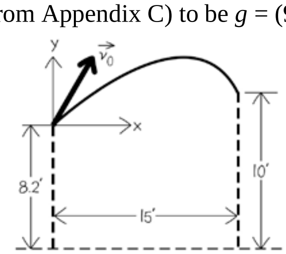
ASSESS As in problem 3.65, we can define the different velocity components as $v_{x0}^{\pm} = v\cos(45° \pm \alpha)$ and $v_{y0}^{\pm} = v\sin(45° \pm \alpha)$. The "+" projectile has less horizontal velocity ($v_{x0}^{+} < v_{x0}^{-}$), but it is shot up higher ($v_{y0}^{+} > v_{y0}^{-}$), so it will be in the air longer ($t^+ > t^-$). These two effects balance each other out to give the same range ($x = v_{x0}^{+}t^{+} = v_{x0}^{-}t^{-}$).

71. **INTERPRET** This problem involves projectile motion. We are asked to calculate the angle at which to aim a basketball to score a basket given the horizontal distance to the basket, the launch speed of the basketball, and the initial height difference between the basketball and the basket.

DEVELOP Draw a diagram of the situation (see figure below). The initial height of the ball is y_0 = 8.2 ft, the final height is y = 10 fs, the initial speed is v_0 = 26 ft/s, and the range is x = 15 ft. These quantities are related by the trajectory Equation 3.14

$$y - y_0 = x\tan(\theta_0) - \frac{g}{2v_0^2\cos^2(\theta_0)}x^2$$

which we shall solve to find the launch angle θ_0. We will also need the acceleration due to gravity in ft/s^2, which we calculate (using the conversion factor from Appendix C) to be g = (9.8 m/s^2)(1 ft/0.3048 m) = 32 ft/s^2.



EVALUATE With origin at the point from which the ball is thrown, the equation of the trajectory, evaluated at the basket, becomes

$$y - y_0 = (10\text{ ft} - 8.2\text{ ft}) = (15\text{ ft})\tan(\theta_0) - \frac{(32\text{ ft/s}^2)(15\text{ ft})^2}{2(26\text{ ft/s})^2\cos^2(\theta_0)}$$

$$1.8\text{ ft} = (15\text{ ft})\tan(\theta_0) - \frac{5.33\text{ ft}}{\cos^2(\theta_0)}$$

Using the trigonometric identity $1 + \tan^2(\theta_0) = \sec^2(\theta_0)$, convert this equation into a quadratic in $\tan(\theta_0)$. The result is

$$7.13 - 15\tan(\theta_0) + 5.33\tan^2(\theta_0) = 0$$

where we have divided out the units of ft, leaving us with a dimensionless equation. The answers are

$$\theta_0 = \text{atan}\left[\frac{15 \pm \sqrt{15^2 - 4(5.33)(7.13)}}{2(5.33)}\right] = 31.2° \text{ or } 65.7°$$

ASSESS Like the horizontal range formula for given v_0, there are two launch angles whose trajectories pass through the basket, although in this case they are not symmetric about 45°. Basketball players know that a higher launch angle gives a better chance of scoring a basket. Can you show why this is so?

72. **INTERPRET** We are asked to prove that two projectiles fired with different velocities cannot land on the same point at the same time.

DEVELOP Let's assume that the starting point is the origin. We're told that $v_{01} \neq v_{02}$ and $\theta_{01} \neq \theta_{02}$. To prove that the projectiles can't land simultaneously at the same time, we will assume that they do (i.e., there exists a time t for which $x_1 = x_2$ and $y_1 = y_2$). If this leads to a contradiction, then we can infer that our assumption was wrong (reductio ad absurdum).

EVALUATE Assuming $x_1 = x_2$ implies that

$$v_{01}\cos\theta_{01} = v_{02}\cos\theta_{02}$$

Where we have used Equation 3.12. Assuming $y_1 = y_2$ and applying Equation 3.13 gives

$$v_{01}\sin\theta_{01}t - \tfrac{1}{2}gt^2 = v_{02}\sin\theta_{02}t - \tfrac{1}{2}gt^2 \quad \rightarrow \quad v_{01}\sin\theta_{01} = v_{02}\sin\theta_{02}$$

Combining the two preceding results gives

$$\tan\theta_{01} = \tan\theta_{02}$$

This implies that $\theta_{01} = \theta_{02}$, which is a contradiction of the given parameters. (Technically, it could also imply that $\theta_{01} = \theta_{02} + 180°$, but then that would mean $v_{01} = -v_{02}$. This is essentially a double reversal, resulting in $\vec{v}_1$ and $\vec{v}_2$ being identical.)

ASSESS This result does not imply that two projectiles can't land in the same spot at different times.

73. **INTERPRET** We have two projectiles that are launched simultaneously from the same point on a horizontal surface, one at $\theta_1 = 45°$ to the horizontal and the other at $\theta_2 = 60°$. Their launch speeds are different but the two projectiles travel the same horizontal distance before landing.

DEVELOP Let's assume that the starting point is the origin. We're told that $v_{01} = v$ and $\theta_{01} = 45°$. The maximum horizontal range can be found from Equation 3.15: $r = v^2/g$, and the total flight time is $t_1 = 2v_{01}\sin\theta_{01}/g$. Similarly, for the second projectile, the range is $x_2 = v_{02}^2\sin 2\theta_{02}/g$. The condition that $x_1 = x_2$ implies $v_{01}^2\sin 2\theta_{01} = v_{02}^2\sin 2\theta_{02}$.

EVALUATE **(a)** With $v_{01} = v$, we find the launch speed of the second projectile to be

$$v_{02} = v_{01}\sqrt{\frac{\sin 2\theta_{01}}{\sin 2\theta_{02}}} = v\sqrt{\frac{\sin 2(45°)}{\sin 2(60°)}} = v\sqrt{\frac{1}{\sqrt{3}/2}} = v\sqrt{\frac{2}{\sqrt{3}}} = 1.07v$$

(b) The total flight time of the first projectile is $t_1 = 2v_{01}\sin\theta_{01}/g = 2v\sin 45°/g = \sqrt{2}v/g$. For the second projectile, we have

$$t_2 = \frac{2v_{02}\sin 60°}{g} = \frac{2}{g}\left(v\sqrt{\frac{2}{\sqrt{3}}}\right)\frac{\sqrt{3}}{2} = \frac{\sqrt{2}v}{g}\sqrt[4]{3} = \sqrt[4]{3}t \approx 1.3t$$

ASSESS Had the two projectiles been launched at the same speed, the trajectyory of the one at a higher launch angle will take longer, and its horizontal range will be shorter.

74. **INTERPRET** The parabolic path of a trajectile near its peak can be approximated as a circle. We want to argue that the radius of curvature in this case is $r = v^2/g$, where v is the speed of the projectile at the peak of the trajectory.

DEVELOP At the top of the trajectory, if the projectile behaves as momentarily as if its path is circular, then the centripetal acceleration of the circular motion would be $a = g$.

EVALUATE Since $a = v^2/r$, we find the radius of curvature to be $r = v^2/g$.

ASSESS Mathematically, the radius of curvature can be expressed as $r = |1/y''|$, where y is given by Eq. 3.14: $y = x\tan\theta_0 - \dfrac{gx^2}{2v_0^2\cos^2\theta_0}$. Differentiating the expression twice with respect to x gives $y'' = -\dfrac{g}{v_0^2\cos^2\theta_0} = -\dfrac{g}{v^2}$, where $v = v_0\cos\theta_0$ at the top of the trajectory.

75. **INTERPRET** The plane's 90° turn is part of a circular arc, undertaken at constant speed, so this problem involves uniform circular motion. We are asked to find the minimum-radius turn that the jet can execute without subjecting the pilot to an acceleration that exceeds $5g$.

DEVELOP The magnitude of the acceleration in circular motion is $a = v^2/r$, which here must not exceed $5g$. The minimum height for starting the turn is the radius r of the circular turn.

EVALUATE The speed is v = 1200 km/h = 333.33 m/s. Setting $a = 5g$ and solving Equation 3.16 for r gives

$$r \geq \frac{v^2}{5g} = \frac{(333.33\ \text{m/s})^2}{5(9.8\ \text{m/s}^2)} = 2268\ \text{m} \approx 2.3\ \text{km}$$

ASSESS To give a margin of safety, the pilot should start the turn at well above this height. Although we ignored gravity, including it would not have changed our 2-significant-figure answer.

76. **INTERPRET** This problem involves projectile motion. We are asked to find the launch speed so that a projectile's trajectory passes through a given point. Because the problem does not involve time, we can use Equation 3.14 that describes a projectile's trajectory.

DEVELOP Let the origin of coordinates system be at the slingshot with the stranded climbers at the point x = 390 m and y = 270 m. Solve Equation 3.14 for the initial speed, given that the initial angle is θ_0 = 70°.

EVALUATE Solving Equation 3.14 for v_0, we obtain

$$v_0 = \frac{x}{\cos(\theta_0)}\sqrt{\frac{g}{2[x\tan(\theta_0) - y]}} = \frac{390\ \text{m}}{\cos(70°)}\sqrt{\frac{9.8\ \text{m/s}^2}{2[(390\ \text{m})\tan(70°) - 270\ \text{m}]}} = 89\ \text{m/s}$$

ASSESS Note that the initial speed depends on the launch angle, as well as the target's position. Can you think of a way to find the launch angle that gives the minimum initial speed?

77. **INTERPRET** You want to know what is the farthest you can throw a stone horizontally, given the height you're able to throw it straight up.

DEVELOP When throwing the stone straight up, $v_{y0} = v_0$. Once it reaches its maximum height, h, its velocity will be zero, so using Equation 2.11 we have $v_0^2 = 2gh$. We assume you throw the ball with the same initial speed, but at a different angle. The maximum horizontal range can be found from Equation 3.15: $x = v_0^2 \sin 2\theta_0 / g$.

EVALUATE Using the expression for the initial velocity from the first throw, the range is

$$x = \frac{2gh\sin 2\theta_0}{g} = 2h\sin 2\theta_0$$

Since the maximum that $\sin 2\theta_0$ can be is 1, the maximum distance you can throw the stone is $2h$.

ASSESS If $\sin 2\theta_0 = 1$, then $\theta_0 = 45°$. We will prove that indeed this is the optimum throwing angle in Problem 3.81.

78. **INTERPRET** This problem involves projectile motion. We are asked to find the maximum height of a projectile given its launch speed, which we must calculate from the given horizontal range.

DEVELOP From Equation 3.15, which gives the horizontal range of a projectile, we see that the maximum range occurs for a launch angle of $\theta_0 = 45°\left[\sin(2\theta_0) = 1\right]$ and that $x_{\max} = v_0^2/g$. In addition, when the projectile reaches its maximum height $v_y = 0$. We can calculate the maximum height from Equation 2.11, $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, which gives

$$y_{\max} = y_0 + \frac{v_{y0}^2}{2g}$$

EVALUATE A maximum range of $x_{\max} = 220\ \text{km} = 2.2\times10^5\ \text{m}$ implies that

$$v_0 = \sqrt{x_{\max} g} = \sqrt{\left(2.2\times10^5\ \text{m}\right)\left(9.8\ \text{m/s}^2\right)} = 1468.33\ \text{m/s}$$

Inserting this into the expression above for $y_{\max}$ (with $y_0 = 0$ because the missile is launched from the ground which we define as 0 m) gives

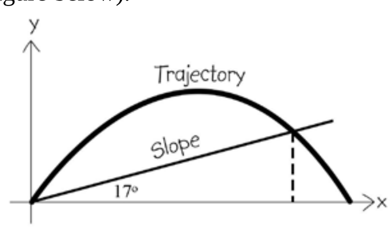
$$y_{\max} = \frac{v_{0y}^2}{2g} = \frac{(1468.33\ \text{m/s})^2}{2\left(9.8\ \text{m/s}^2\right)} = 110{,}000\ \text{m} = 110\ \text{km}$$

ASSESS The maximum height is half the maximum range. To see why, note that the maximum height of a projectile is $y_{\max} = v_0^2 \sin^2(\theta_0)/g$. Thus,

$$\frac{y_{\max}}{x} = \frac{v_0^2 \sin^2(\theta_0)/g}{v_0^2 \sin(2\theta_0)/g} = \frac{\sin^2(\theta_0)}{\sin(2\theta_0)} = \frac{1}{2\tan(\theta_0)}$$

When choosing $\theta_0 = 45°$ so that $x = x_{\max}$, then $\tan(45°) = 1$ and we have $y_{\max}/x_{\max} = 1/2$

79. **INTERPRET** This problem involves projectile motion. We are asked to find the horizontal distance at which a trajectory intercepts a 17° slope (see figure below).



DEVELOP We need to find the intersection of the ball's trajectory (expressed by Equation 3.14) with a 17° line through the same origin, which is mathematically expressed as $y = x\tan(\theta_1)$, where $\theta_1 = 17°$. Equating these two expressions gives

$$y = x\tan(\theta_1) = x\tan(\theta_0) - \frac{g}{2v_0^2\cos^2(\theta_0)}x^2$$

$$x = 2v_0^2\cos^2(\theta_0)\left[\frac{\tan(\theta_0) - \tan(\theta_1)}{g}\right]$$

To solve for x, we need to know the initial velocity, which we can obtain from the fact that the range of the ball on level ground is $x_f = 33$ m. Thus

$$y(x_f = 33\ \text{m}) = 0 = x_f\tan(\theta_0) - \frac{g}{2v_0^2\cos^2(\theta_0)}x_f^2$$

$$v_0 = \pm\sqrt{\frac{g x_f}{2\tan(\theta_0)\cos^2(\theta_0)}}$$

Insert this result into the previous expression for the intersection of the trajectory to find the horizontal distance x that the player can kick the ball up a 17° slope.

EVALUATE The expression for the x coordinate of the intersection of the trajectory and the 17°C slope gives

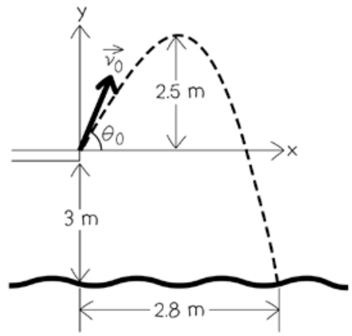
$$\begin{aligned} x &= 2\left(\frac{gx_f}{2\tan(\theta_0)\cos^2(\theta_1)}\right)\cos^2(\theta_0)\left[\frac{\tan(\theta_0)-\tan(\theta_1)}{g}\right] \\ &= x_f\frac{\tan(\theta_0)-\tan(\theta_1)}{\tan(\theta_0)} = (33\text{ m})\frac{\tan(37°)-\tan(17°)}{\tan(37°)} = 19.6\text{ m}. \end{aligned}$$

Thus, the player can kick the ball a horizontal distance of 19.6 m up a 17° slope.

ASSESS The horizontal distance that the player can kick the ball up a slope is linear in the range he has on level ground, and quadratic in the initial velocity he can impart to the ball.

80. **INTERPRET** This problem involves projectile motion. We are asked to find the diver's initial speed and take-off angle given the height of her trajectory and the horizontal distance covered when she enters the pool.

DEVELOP Draw a diagram of the situation (see figure below). Because we are given the maximum height (at which point $v_y = 0$), Equation 2.11, $v^2 = v_0^2 + 2a(y-y_0)$, can be used to find the y component of the diver's initial velocity, with $y - y_0 = h$ and $a = -9.8\text{ m/s}^2$. The x component of v_0 can be found from Equation 3.12, $x = x_0 + v_{x0}t$, once the time of flight is known.



EVALUATE Solving Equation 2.11 gives

$$0 = v_{0y}^2 - 2g\overbrace{(y-y_0)}^{=h} \quad \Rightarrow \quad v_{0y} = \sqrt{2(9.8\text{ m/s}^2)(2.5\text{ m})} = 7.00\text{ m/s}$$

where we retain more significant figures than warranted because this is an intermediate result. To find the total flight time, we solve Equation 3.13 $y - y_0 = v_{0y}t - gt^2/2$ with $y - y_0 = -3$ m (a 3-m board is 3 m above the water level). This gives

$$t = \frac{v_{0y} \pm \sqrt{v_{0y}^2 + 2g(y_0 - y)}}{g} = \frac{7\text{ m/s} + \sqrt{(7\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(3\text{ m})}}{9.8\text{ m/s}^2} = 1.77\text{ s}$$

where we take the positive square root because the diver springs upward off the board. Thus, the x component of the velocity is

$$v_{0x} = \frac{x - x_0}{t} = \frac{2.8\text{ m}}{1.77\text{ s}} = 1.58\text{ m/s}$$

From v_{x0} and v_{y0} we find the magnitude of $\vec{v}_0$ is

$$\begin{aligned} v_0 &= \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(1.58\text{ m/s})^2 + (7.0\text{ m/s})^2} \\ &= 7.2\text{ m/s} \end{aligned}$$

and direction is $\theta_0 = \text{atan}(v_{0y}/v_{0x}) = \text{atan}(7.0\text{ m/s}/1.58\text{ m/s}) = 77.3°$.

ASSESS It takes the diver 1.77 s to complete the dive. The result is reasonable. The greater the value of θ_0, the closer the diver will be to the diving board.

81. **INTERPRET** This problem is an exercise in calculus. We are asked to use calculus to show that the maximum range of a projectile occurs for a launch angle of $\theta_0 = 45°$.

DEVELOP Equation 3.15, $x = v_0^2 \sin(2\theta_0)/g$, gives the range of a projectile over level ground. Differentiate it with respect to θ_0 and set the result to zero. Solving this expression will give the value(s) of θ_0 where Equation 3.15 is at an extremum (maximum or minimum). Interpret the result to determine if the result is a maximum or minimum.

EVALUATE Differentiating Equation 3.15 with respect to θ_0 gives

$$\frac{dx}{d\theta_0} = \frac{2v_0^2}{g}\cos(2\theta_0) = 0$$
$$\theta_0 = 45°, 135°, 225°, 315°$$

The angles 225° and 315° are below the horizontal and so are unphysical. The angles 45° and 135° correspond to launching a projectile at 45° either to the left or to the right. Thus, the angle for maximum range is 45°.

ASSESS Looking at Equation 3.15, we see that at θ_0 = 45°, sin(2θ_0) = 1, which is the maximum value for the sine function. Thus, the answer appears reasonable.

82. **INTERPRET** This is a problem involving projectile motion. The physical quantity of interest is the slope of the ground in the landing zone of the ski jump.

DEVELOP We first note that the direction of the skier's velocity is $\theta = \tan^{-1}(v_y / v_x)$ where angles are measured CCW from the x axis, chosen horizontal to the right in Fig. 3.25 with the y axis upward. In the landing zone, θ is in the fourth quadrant, which can be represented by a negative angle below the x axis. The slope of the ground at this point can be represented by a similar angle θ_g, and for the safety of ski jumpers, $\theta_g - \theta = 3.0°$.

EVALUATE The ratio of the skier's final velocity components can be evaluated with Equations 3.10 and 3.11

$$\frac{v_y}{v_x} = \frac{v_{y0} - gt}{v_{x0}} = \tan\theta_0 - \frac{g(x - x_0)}{v_0^2\cos^2\theta_0}$$

Where we have used $v_{y0} = v_0\sin\theta_0$, $v_{x0} = v_0\cos\theta_0$, and the time of flight, $t = (x - x_0)/v_{x0}$, from Equation 3.12. Plugging in the given values: $x - x_0 = 55$ m, $v_0 = 28$ m/s, $\theta_0 = -9.5°$, the ground's slope must be

$$\theta_g = 3.0° + \tan^{-1}\left[\tan(-9.5°) - \frac{(9.8\ \text{m/s}^2)(55\ \text{m})}{(28\ \text{m/s})^2\cos^2(-9.5°)}\right] = 3.0° - 41.2° = -38°$$

ASSESS An angle of 38° below the x axis is reasonable, seeing as typical values are between 35° and 40°.

83. **INTERPRET** We are given the problem of showing that the *slope* of an equation for a projectile's trajectory is in the direction of the projectile's velocity. The slope of an equation is the derivative. We also need to show that the components of velocity that we find are the same as the components given for projectile motion.

DEVELOP The trajectory Equation 3.14 is

$$y = x\tan\theta_0 - \frac{g}{2v_0^2\cos^2(\theta_0)}x^2$$

Use the derivative with respect to x to find the direction of the slope (tanθ = dy/dx). If we think of the derivative as a fraction, the numerator corresponds to the y component of the velocity, and the denominator corresponds to the x component. The ratio (i.e., slope) that we find should match Equations 3.10($v_x = v_{x0}$) and 3.11 ($v_y = v_{y0} - gt$).

EVALUATE Differentiating Equation 3.14 gives

$$\frac{dy}{dx} = \tan(\theta_0) - \frac{g}{v_0^2\cos^2(\theta_0)}x = \frac{\sin(\theta_0)}{\cos(\theta_0)} - \frac{gx}{v_0^2\cos^2(\theta_0)} = \frac{v_0^2\sin(\theta_0)\cos(\theta_0) - gx}{v_0^2\cos^2(\theta)_0}$$

The initial components of velocity are $v_{x0} = v_0\cos(\theta_0)$ and $v_{x0} = v_0\sin(\theta_0)$, which we insert into the above expression for dy/dx to obtain

$$\frac{dy}{dx} = \frac{[v_0\sin(\theta_0)][v_0\cos(\theta_0)] - gx}{v_0^2\cos^2(\theta_0)} = \frac{v_{y0}v_{x0} - gx}{v_{x0}^2} = \frac{v_{y0} - gx/v_{x0}}{v_{x0}}$$

Distance divided by velocity is time, so $x/v_{x0} = t$. Inserting this into the expression above gives

$$\frac{dy}{dx} = \frac{v_{y0} - gt}{v_{x0}}$$

Comparing this result with Equations 3.10 and 3.11, we see that the numerator is v_y and the denominator is v_x.

ASSESS Note that dy/dx is not the velocity itself, but it is a dimensionless ratio that is the same as the dimensionless ratio $\tan\theta = v_x/v_y$.

84. **INTERPRET** This problem asks you to find the initial angle, θ_0, that gives the maximum range, x, for the trebuchet.

DEVELOP The general case of a projectile launched with speed v_0 from a height h is tackled in Problem 3.85. As this is a rather complicated derivation, we will not reproduce it here, but instead use the result:

$$\theta_{\max} = \tfrac{1}{2}\cos^{-1}\left(\frac{1}{1 + v_0^2/gh}\right)$$

EVALUATE Plugging in the launching speed and height of the cliff:

$$\theta_{\max} = \tfrac{1}{2}\cos^{-1}\left(\frac{1}{1 + (36\text{ m/s})^2/(9.8\text{ m/s}^2)(75\text{ m})}\right) = 34°$$

ASSESS We can plug $\theta_0 = \theta_{\max}$ and $y = -h$ into Equation 3.14:

$$(-75\text{ m}) = x\tan 34° - \frac{(9.8\text{ m/s}^2)}{2(36\text{ m/s})^2\cos^2 34°}x^2$$

Using the quadratic formula, we find a range of $x \approx 190\text{ m}$. If we instead had chosen $\theta_0 = 45°$, the range would have been slightly smaller, $x \approx 180\text{ m}$.

85. **INTERPRET** This problem asks you to find the initial angle, θ_0, that gives the maximum range, x, for a projectile launched with speed v_0 from a height h. Recall that the maximum occurs when the derivate, $dx/d\theta_0$, is zero.

DEVELOP We need to find an equation that relates x and θ_0. Let's assume the projectile is launched from the origin, so that it lands at a vertical position of $y = -h$. We can find the range from Equation 3.14,

$$y = -h = x\tan\theta_0 - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$

Let's rearrange this equation by multiplying through by $\cos^2\theta_0$ and defining $H = v_0^2/2g$ (which is the maximum height of the stone's trajectory using Equation 2.11)

$$x^2 - 4xH\sin\theta_0\cos\theta_0 - 4hH\cos^2\theta_0 = 0$$

Now using the trigonometric identities: $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$, we have

$$x^2 - 2xH\sin 2\theta_0 - 2hH(\cos 2\theta_0 + 1) = 0$$

We could solve for x using the quadratic formula, but that will get messy. Instead, we will leave the equation like this and take the derivative with respect to θ_0. We can then set $dx/d\theta_0$ equal to zero and then solve for the angle that gives the maximum range.

EVALUATE In taking the derivative of the above equation, we are careful to apply the chain rule and product rule from Appendix A:

$$2x\cdot\frac{dx}{d\theta_0} - 2H\left[\frac{dx}{d\theta_0}\cdot\sin 2\theta_0 + 2x\cos 2\theta_0\right] - 2hH\left[-2\sin 2\theta_0\right] = 0$$

If we then assume $dx/d\theta_0 = 0$ for the maximum range, we are left with

$$-4Hx_{\max}\cos 2\theta_{\max} - 4hH\sin 2\theta_{\max} = 0 \quad\rightarrow\quad x_{\max} = h\tan\theta_{\max}$$

where $\theta_{\max}$ is the angle that gives the maximum range, $x_{\max}$. Notice that $\theta_{\max} = 45°$ is undefined except for $h = 0$, which would be the normal case of a trajectory over level ground (see Equation 3.75). To solve for $\theta_{\max}$ generally, we plug it and the expression for $x_{\max}$ into the trajectory equation that we derived above:

$$h^2\tan^2 2\theta_{\max} - 2hH\tan 2\theta_{\max}\sin 2\theta_{\max} - 2hH\left(\cos 2\theta_{\max} + 1\right) = 0$$
$$h\sin^2 2\theta_{\max} - 2H\sin^2 2\theta_{\max}\cos 2\theta_{\max} - 2H\cos^2 2\theta_{\max}\left(\cos 2\theta_{\max} + 1\right) = 0$$

Using the fact that $\sin^2\alpha = 1 - \cos^2\alpha = (1-\cos\alpha)(1+\cos\alpha)$, the above equation reduces to:

$$\cos 2\theta_{\max} = \frac{1}{1 + 2H/h}$$

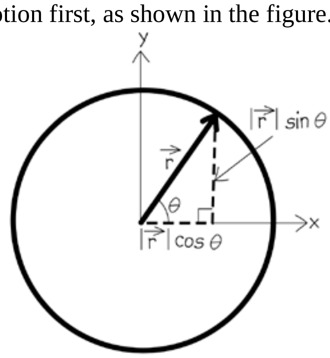
Or equivalently

$$\theta_{\max} = \tfrac{1}{2}\cos^{-1}\left(\frac{1}{1 + v_0^2/gh}\right)$$

ASSESS If we assume the ground is level $(h = 0)$, then the argument in the $\cos^{-1}$ function goes to zero, which means $\theta_{\max} = 45°$, as it should when the trajectory is over level ground.

86. **INTERPRET** In the first part, we are asked to show that for circular motion the given equation tells us the position. Next, we find the angle between the position vector and the *x* axis. Finally, we use the second derivative to find the equation for centripetal acceleration.

DEVELOP We draw a diagram of the motion first, as shown in the figure.



From the diagram we can see the position vector and its components. To relate θ to time *t* and period *T*, we use the definition of speed $v = \frac{\text{distance}}{\text{time}}$. Finally, we can use derivatives, twice, to obtain the acceleration since $\vec{a} = \frac{d}{dt}\vec{v} = \frac{d^2}{dt^2}\vec{r}$. But note that the unit vectors $\hat{i}$ and $\hat{j}$ are constants that do not change with time.

EVALUATE **(a)** From the figure, we see that the *x* component of position is $R\cos\theta$, where $R = |\vec{r}|$. Similarly, the *y* component is $R\sin\theta$. The position vector, $\vec{r}$, is the sum of these two components, so

$$\vec{r} = R\cos\theta\hat{i} + R\sin\theta\hat{j} = R\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

(b) The distance that the particle moves in going around one complete lap is the circumference of the circle, $2\pi R$, while the time it takes for one complete lap is the period *T*. This tells us that the speed of the particle is $v = 2\pi R/T$. The angle θ, in radians, is the arc distance divided by the radius, so

$$\theta = \frac{vt}{R} = \frac{2\pi t}{T}$$

(c) In performing the time derivatives, it is worth noting that the unit vectors $\hat{i}$ and $\hat{j}$ are constants that do not change with time.

$$\vec{a} = \frac{d^2}{dt^2}\left[R\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)\right] = R\frac{d^2}{dt^2}\left[\cos\left(\tfrac{2\pi}{T}t\right)\hat{i} + \sin\left(\tfrac{2\pi}{T}t\right)\hat{j}\right]$$
$$= R\frac{d}{dt}\left[-\frac{2\pi}{T}\sin\left(\tfrac{2\pi}{T}t\right)\hat{i} + \frac{2\pi}{T}\cos\left(\tfrac{2\pi}{T}t\right)\hat{j}\right]$$

To evaluate the first derivative, we have used the chain rule and other formulas from Appendix A. Continuing with the second derivative:

$$\begin{aligned}\vec{a} &= R\left(\frac{2\pi}{T}\right)^2\left[-\cos\left(\tfrac{2\pi}{T}t\right)\hat{i} - \sin\left(\tfrac{2\pi}{T}t\right)\hat{j}\right]\\ &= \frac{R}{R^2}\left(\frac{2\pi R}{T}\right)^2(-1)\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)\\ &= \frac{v^2}{R}(-\hat{r})\end{aligned}$$

We have used the unit vector, $\hat{r} = \vec{r}/R$, which points in the direction of $\vec{r}$. The fact that there is a negative sign above signifies that $\vec{a}$ points in the opposite direction of $\vec{r}$, i.e., towards the center of the circle. Note, as well, that the magnitude of this vector agrees with Equation 3.16 for the centripetal acceleration.

ASSESS This is a different derivation of the equation for the centripetal acceleration than the one from the text, but it gives identical results.

87. **INTERPRET** We're asked to find the relation betweeen the magnitude of tangential acceleration and the rate of change of radial acceleration.

DEVELOP For a particle undergoing nonuniform circular motion, its radial acceleration is $a_r = v^2/r$, where v is its speed and r is the radius of the circle. In addition, there will also be tangential acceleration $a_t = dv/dt$. The rate of change of a_r can be related to a_t.

EVALUATE Differentiating a_r with respect to t gives

$$\frac{da_r}{dt} = \frac{d}{dt}\left(\frac{v^2}{r}\right) = \frac{2v}{r}\frac{dv}{dt} = \frac{2v}{r}a_t$$

where r is assumed to be constant.

ASSESS Tangential acceleration arises from the change in magnitude of the speed, whereas radial acceleration arises from the change in direction of the speed.

88. **INTERPRET** As in Problem 87, we explore the relation betweeen the magnitude of tangential acceleration and the rate of change of radial acceleration, assuming that the radius can also change with time.

DEVELOP For a particle undergoing nonuniform circular motion, its radial acceleration is $a_r = v^2/r$, where v is its speed and r is the radius of the circle. In addition, there will also be tangential acceleration $a_t = dv/dt$. The rate of change of a_r can be related to a_t.

EVALUATE Differentiating a_r with respect to t gives

$$\frac{da_r}{dt} = \frac{d}{dt}\left(\frac{v^2}{r}\right) = \frac{2v(dv/dt)r - v^2(dr/dt)}{r^2} = \frac{2v}{r}a_t - \frac{v^3}{r^2}$$

ASSESS Tangential acceleration arises from the change in magnitude of the speed, whereas radial acceleration arises from the change in direction of the speed. Tangential acceleration points in a direction tangent to the circle, while radial acceleration points toward the center of the circle.

89. **INTERPRET** We're asked to interpret a map of three trajectories.

DEVELOP The shortest distance between two points is a straight line, so Alice has the shortest path. And Bob takes a shorter circular arc than Carrie.

EVALUATE The distances traveled are ordered length C>B>A.

The answer is **(c)**.

ASSESS Carrie's path appears to be a semicircle, which is the largest circular arc without starting in a direction opposite of one's goal.

90. **INTERPRET** We're asked to interpret a map of three trajectories.
DEVELOP The displacement is the distance between the initial point (Dorm) and final point (Library).

EVALUATE The displacements are all equal.

The answer is **(a)**.

ASSESS Despite their very different trajectories, the three students all start and end at the same places.

91. **INTERPRET** We're asked to interpret a map of three trajectories.
DEVELOP The average speed is the distance traveled divided by the time, which were told is the same for the three walkers.

EVALUATE Since Alice had the shortest distance, she must have the slowest average speed. The fastest must have been Carrie, since she had the farthest distance to walk.

The answer is **(c)**.

ASSESS Notice, if the question had asked for the average velocity: $\bar{\vec{v}} = \Delta\vec{r}\,/\,\Delta t$ (Equation 3.3), the answer would be they are all equal, since the displacement $\Delta\vec{r}$ is the same for the three students.

92. **INTERPRET** We're asked to interpret a map of three trajectories.
DEVELOP We are told that the three students walk at constant speed, and Alice walks straight, so her acceleration is zero. However, the velocity is changing for Bob and Carrie as they follow their respective circular paths, so they accelerate according to Equation 3.16: $a = v^2\,/\,r$. Since Bob's circle has a larger radius, his acceleration is less than Carrie's.

EVALUATE From the arguments above, C>B>A.

The answer is **(d)**.

ASSESS You could argue that all three students follow a circular path, but Alice's circle has an infinite radius, so her acceleration is zero.