


Bond Pricing

Inputs
Rate Convention
Annual Coupon Rate
Yield to Maturity (Annualized)
Number of Payments / Year
Number of Periods to Maturity ( $T$ )
Face Value (PAR)

## Outputs

Discount Rate / Period (r)
Coupon Payment (PMT)
Duration and Convexity

## Bond Duration using a Timeline

| Period | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time (Years) | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
| Cash Flows |  | \$20.00 | \$20.00 | \$20.00 | \$20.00 | \$20.00 |
| Present Value of Cash Flows |  | \$19.83 | \$19.66 | \$19.49 | \$19.32 | \$19.15 |
| Bond Price using a Timeline | \$1,086.96 |  |  |  |  |  |
| Weight |  | 1.8 | 1.8\% | 1.8\% | 1.8\% | 1.8\% |
| Weight * Time |  |  | 0.02 | 0.03 | 0.04 | 0.04 |
| Duration using a Timeline |  |  |  |  |  |  |
| Modified Duration using a Timeline |  |  |  |  |  |  |
| Bond Duration using a Formula |  |  |  | $\begin{aligned} & \text { V of Cas } \\ & \text { nter }=\mathrm{C} \end{aligned}$ | $\begin{aligned} & \text { Flow ol } \\ & \text { BB\$20 } \end{aligned}$ | tet $/$ Tot copy act |
| Duration (D) using a Formula |  |  |  |  |  |  |
| Modified Duration using a Formula |  |  |  | $\begin{aligned} & \text { Veight * } \\ & \text { nter } \end{aligned}$ | 17 an | py acros |
| Bond Duration using a Function (under APR)Duration using a Function |  |  |  | Sum of all the Weight * Times Enter =SUM(C22:J22) |  |  |
|  |  |  |  |  |  |  |
| Modified Duration using a Function | 3.74 -... |  |  | Duration / (1+(Discount Rate / P Enter $=\mathrm{B} 23 /(1+\$ \mathrm{~B} \$ 12)$ and cop |  |  |
| (7) DURATION (Settlement Date, Maturity Date, Annual Coupon Rate, Yield to Maturity, Number of Periods) |  |  |  |  |  |  |

Coupon Rate, Yield to Maturity, Number of Periods)
Enter $=\operatorname{IF}\left(\$ C \$ 4=1, "{ }^{\prime \prime}\right.$, DURATION(DATE $(2000,1,1)$, DATE(2000+B8/B7,1,1),B5,B6,B7))
(8) MDURATION (Settlement Date, Maturity Date, Annual Coupon Rate, Yield to Maturity, Number of Periods) Enter $=\mathrm{IF}\left(\$ \mathrm{C} \$ 4=1,{ }^{\text {" } " ~}{ }^{\prime}, \mathrm{MDURATION}(\mathrm{DATE}(2000,1,1)\right.$, DATE(2000+B8/B7,1,1),B5,B6,B7))
(6) The Duration Formula is:
$D=\frac{1+r}{r \cdot N O P}-\frac{1+r+T \cdot(C R}{C R \cdot\left((1+r)^{T}-\right.}$
Enter $=(1+\mathrm{B} 12) /\left(\mathrm{B} 12^{*} \mathrm{~B} 7\right)-(1+\mathrm{B} 12$ $/\left(\mathrm{B} 5^{*}\left((1+\mathrm{B} 12)^{\wedge} \mathrm{B} 8-1\right)+\mathrm{B} 12^{\star}\right.$

## Bond Convexity

Weight * (Time^2+Time)
Convexity using a Timeline Convexity using a Formula


$$
\frac{\left(\begin{array}{l}
C R \cdot(1+r)^{1+T} \cdot(r \cdot(N O P+1)+2) \\
-C R \cdot\left(r^{2} \cdot(N O P+T+1) \cdot(T+1)+r \cdot(N O P+2 \cdot T+3)+2\right)+r^{3} \cdot N O P \cdot T \cdot(N O P+T
\end{array}\right.}{r^{2} \cdot N O P^{2} \cdot\left(C R \cdot(1+r)^{T}-C R+r \cdot N O P\right)}
$$

Enter $=\left(\left(\mathrm{B} 5^{*}\left((1+\mathrm{B} 12)^{\wedge}(1+\mathrm{B} 8)\right)^{\star}\left(\mathrm{B} 12^{\star}(\mathrm{B} 7+1)+2\right)-\mathrm{B} 5^{\star}\left(\mathrm{B} 12^{\wedge} 2^{\star}(\mathrm{B} 7+\mathrm{B} 8+1)^{\star}(\mathrm{B} 8+1)+\mathrm{B} 12^{\star}\left(\mathrm{B} 7+2^{\star} \mathrm{B} 8+3\right)\right.\right.\right.$ $\left.\left./\left(\mathrm{B} 12^{\wedge} 2^{\star} \mathrm{B} 7^{\wedge} 2^{\star}\left(\mathrm{B} 5^{*}(1+\mathrm{B} 12)^{\wedge} \mathrm{B} 8-\mathrm{B} 5+\mathrm{B} 12^{\star} \mathrm{B} 7\right)\right)\right) /\left((1+\mathrm{B} 12)^{\wedge} 2\right)\right)$

US Dollar $\quad \begin{gathered}\text { Exch Rate } \\ \$ 1.00=\end{gathered}$

| Currency Number | 4 |
| :--- | ---: |
| (Select from below) |  |
| 1 = Chinese Yuan | $¥ 7.3790$ |
| 2 = European Euro | $€ 0.6805$ |
| 3 = Indian Rupee | IDR 39.30 |
| $4=$ US Dollar | $\$ 1.00$ |

previous sheet ious sheet to B12

| 6 | 7 | 8 |
| ---: | ---: | ---: |
| 3.0 | 3.5 | 4.0 |
| $\$ 20.00$ | $\$ 20.00$ | $\$ 1,020.00$ |
| $\$ 18.99$ | $\$ 18.82$ | $\$ 951.71$ |
|  |  |  |
| $1.7 \%$ | $1.7 \%$ | $87.6 \%$ |
| 0.05 | 0.06 | 3.50 |




## Bond Pricing

## Price Sensitivity




$$
\begin{aligned}
& \text { Exch Rate } \$ 1.00 \\
& \$ 1.00=\$ 1.00
\end{aligned}
$$



```
F(2) New YTM - Current YTM
    Enter =B21-$B$6 and copy across
```

PV(Actual Discount Rate / Period,
Number of Periods to Maturity,
Coupon Payment, Face Value)
inter $=-\mathrm{PV}(\mathrm{B} 22, \$ \mathrm{~B} \$ 8, \$ \mathrm{~B} \$ 13, \$ \mathrm{~B} \$ 9)$
nd copy across
/Current Discount Rate / Period,
Number of Periods to Maturity,
Coupon Payment, Face Value)
er $=-\mathrm{PV}(\$ \mathrm{~B} \$ 12, \$ \mathrm{~B} \$ 8, \$ \mathrm{~B} \$ 13, \$ \mathrm{~B} \$ 9)$
d copy across
fied Duration
ange in YTM
$=-$ B27*B23
ppy across


| 7.00\% | $7.50 \%$ | $8.00 \%$ | $8.50 \%$ | $9.00 \%$ | $9.50 \%$ | $10.00 \%$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $3.5 \%$ | $3.8 \%$ | $4.0 \%$ | $4.3 \%$ | $4.5 \%$ | $4.8 \%$ | $5.0 \%$ |
| $5.3 \%$ | $5.8 \%$ | $6.3 \%$ | $6.8 \%$ | $7.3 \%$ | $7.8 \%$ | $8.3 \%$ |
| $\$ 863$ | $\$ 847$ | $\$ 832$ | $\$ 817$ | $\$ 802$ | $\$ 788$ | $\$ 774$ |
| $\$ 1,048$ | $\$ 1,048$ | $\$ 1,048$ | $\$ 1,048$ | $\$ 1,048$ | $\$ 1,048$ | $\$ 1,048$ |
| $-17.7 \%$ | $-19.2 \%$ | $-20.7 \%$ | $-22.1 \%$ | $-23.5 \%$ | $-24.9 \%$ | $-26.2 \%$ |
| 3.77 | 3.77 | 3.77 | 3.77 | 3.77 | 3.77 | 3.77 |
| $-19.8 \%$ | $-21.7 \%$ | $-23.6 \%$ | $-25.5 \%$ | $-27.4 \%$ | $-29.3 \%$ | $-31.2 \%$ |
| 18.41 | 18.41 | 18.41 | 18.41 | 18.41 | 18.41 | 18.41 |
| $-17.3 \%$ | $-18.7 \%$ | $-20.0 \%$ | $-21.3 \%$ | $-22.5 \%$ | $-23.7 \% \mid$ | $-24.9 \%$ |

## Bond Pricing

Immunization

Inputs
Rate Convention
Yield to Maturity (Annualized)
Number of Payments / Year

Bond 1
Bond 2
Bond 3
Bond 4
Bond 5
Bond 6
Bond 7
Bond 8

| O EAR O APR | 2 | Annual Percentage Rate |  |
| :---: | :---: | :---: | :---: |
| 1.74\% |  |  |  |
| 2 |  |  |  |
| Annual Coupon Rate | Number of Periods to Maturity (T) | Face Value <br> (PAR) | Number of Bonds |
| 1.50\% | 4 | \$1,000 | 1,783 |
| 2.00\% | 8 | \$1,000 | 2,042 |
| 0.90\% | 2 | \$1,000 | 0 |
| 1.50\% | 4 | \$1,000 | 0 |
| 1.90\% | 6 | \$1,000 | 0 |
| 2.30\% | 8 | \$1,000 | 0 |
| 1.90\% | 6 | \$1,000 | 0 |
| 2.30\% | 8 | \$1,000 | 0 |

Outputs

Discount Rate / Period (r)
(3) Copy the Present Value \& Duratic

Copy the range B19: J 24 from the
Bond Present Value, Duration, and Convexity using a Timeline

| Period | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Time (Years) | 0.0 | 0.5 | 1.0 | 1.5 |
| Liabilities |  | $\$ 0$ | $\$ 0$ | 50 |
| Present Value of Liabilities |  | $\$ 0$ | $\$ 0$ | 50 |

Total Present Value of Liabilities
Weight
Weight * Time
Duration of Liabilities
Modified Duration of Liabilities
Weight * (Time^2+Time)
Convexity of Liabilities

| $0.0 \%$ | $0.0 \%$ | $0.0 \%$ |
| ---: | ---: | ---: |
| 0.00 | 0.00 | 0.00 |



Differences
Total Assets - Liabilities
PV of Assets - PV of Liabilities
Duration of Assets - Duration of Liab
Convexity of Assets - Convexity of Liab


To solve the first problem when there is a single liability to immunize


To solve the second problem when there is a series of liabilities to immunize



To solve the third problem when there is a series of liabilities to immunize with cash flow matching


Coupon
Payment (PMT)

on formulas from the Duration and Convexity sheet Duration and Convexity sheet to B24

| 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| \$0 | \$0 | \$4,000,000 | \$0 | \$0 |
| \$0 | \$0 | \$3,797,413 | \$0 | \$0 |
| 0.0\% | 0.0\% | 100.0\% | 0.0\% | 0.0\% |
| 0.00 | 0.00 | 3.00 | 0.00 | 0.00 |


| 0.00 | 0.00 |  | 12.00 | 0.00 |
| ---: | ---: | ---: | ---: | ---: | ---: |


| 2.76 | 0.03 | 0.05 | 0.06 | 10.11 |
| :--- | :--- | :--- | :--- | :--- |


| Duration, and Convexity formulas from above <br> to B44 |
| :--- |
| $\$ 1,811,540$ $\$ 15,313$ $(\$ 3,984,687)$ $\$ 15,313$ <br> yy across    <br> and Liabilities in Present Value, Duration, and Convexity    <br> and $=$ B51-B31 in B57    |




| Currency Number <br> (Select from below) | 4 |
| :--- | ---: |
| 1 = Chinese Yuan | $¥ 7.3790$ |
| 2 = European Euro | $€ 0.6805$ |
| 3 = Indian Rupee | IDR 39.30 |
| 4 = US Dollar | $\$ 1.00$ |

$\left.\begin{array}{l}\begin{array}{c}\text { Annual } \\ \text { Coupon Rate }\end{array} \begin{array}{c}\text { Number of } \\ \text { Periods to } \\ \text { Maturity (T) }\end{array} \\ \begin{array}{|r|r|r|r|}\text { Face Value } \\ \text { (PAR) }\end{array} \\ \hline 1.50 \%\end{array} \quad 4 \begin{array}{c}\text { Number of } \\ \text { Bonds }\end{array}\right]$

| 00.0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
|  | 0.5 | 1.0 | 1.5 |
|  | \$0 | \$0 | \$0 |
|  | \$2,000,000 | \$2,200,000 | \$2,500,000 |


| Annual <br> Coupon <br> Rate |
| :---: |
|  Number of <br> Periods to <br> Maturity (T) Face Value <br> $($ PAR) Number of <br> Bonds <br> $0.00 \%$ 1 $\$ 1,000$ 2,000 <br> $0.00 \%$ 2 $\$ 1,000$ 2,200 <br> $0.00 \%$ 3 $\$ 1,000$ 2,500 <br> $0.00 \%$ 4 $\$ 1,000$ 3,200 <br> $0.00 \%$ 5 $\$ 1,000$ 3,700 <br> $0.00 \%$ 6 $\$ 1,000$ 4,300 <br> $0.00 \%$ 7 $\$ 1,000$ 4,700 <br> $0.00 \%$ 8 $\$ 1,000$ 5,100 |


| 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| \$0 | \$0 | \$4,000,000 | \$0 | \$0 |
|  |  |  |  |  |
| \$3,200,000 | \$3,700,000 | \$4,300,000 | \$4,700,000 | \$5,100,000 |






## Bond Pricing

Inputs
Rate Convention
Annual Coupon Rate
Yield to Maturity (Annualized)
Number of Payments / Year
Number of Periods to Maturity (T)
Face Value (PAR)

## Outputs

Discount Rate / Period (r)
Coupon Payment (PMT)


## 2 Annual Percentage Rate

6
5

2

8
20
(1) Copy the Outputs \& Timeline from the Copy the range B12: J20 from the previ

Bond Duration using a Timeline


(6) The Duration Formula is:
$D=\frac{1+r}{r \cdot N O P}-\frac{1+r+T \cdot(C R}{C R \cdot\left((1+r)^{T}-\right.}$
Enter $=(1+\mathrm{B} 12) /\left(\mathrm{B} 12^{*} \mathrm{~B} 7\right)-(1+\mathrm{B} 12$ $/\left(\mathrm{B}^{\star}\left((1+\mathrm{B} 12)^{\wedge} \mathrm{B} 8-1\right)+\mathrm{B} 12^{\star}\right.$

## Bond Convexity

Weight * (Time^2+Time)
Convexity using a Timeline Convexity using a Formula
18.177
18.17

$$
\frac{\left(\begin{array}{l}
C R \cdot(1+r)^{1+T} \cdot(r \cdot(N O P+1)+2) \\
-C R \cdot\left(r^{2} \cdot(N O P+T+1) \cdot(T+1)+r \cdot(N O P+2 \cdot T+3)+2\right)+r^{3} \cdot N O P \cdot T \cdot(N O P+T
\end{array}\right.}{r^{2} \cdot N O P^{2} \cdot\left(C R \cdot(1+r)^{T}-C R+r \cdot N O P\right)}
$$

Enter $=\left(\left(\mathrm{B} 5^{*}\left((1+\mathrm{B} 12)^{\wedge}(1+\mathrm{B} 8)\right)^{\star}\left(\mathrm{B} 12^{\star}(\mathrm{B} 7+1)+2\right)-\mathrm{B} 5^{\star}\left(\mathrm{B} 12^{\wedge} 2^{\star}(\mathrm{B} 7+\mathrm{B} 8+1)^{\star}(\mathrm{B} 8+1)+\mathrm{B} 12^{\star}\left(\mathrm{B} 7+2^{\star} \mathrm{B} 8+3\right)\right.\right.\right.$ $\left.\left./\left(\mathrm{B} 12^{\wedge} 2^{\star} \mathrm{B} 7^{\wedge} 2^{\star}\left(\mathrm{B} 5^{*}(1+\mathrm{B} 12)^{\wedge} \mathrm{B} 8-\mathrm{B} 5+\mathrm{B} 12^{\star} \mathrm{B} 7\right)\right)\right) /\left((1+\mathrm{B} 12)^{\wedge} 2\right)\right)$

US Dollar $\quad \begin{gathered}\text { Exch Rate } \\ \$ 1.00=\end{gathered}$

Currency Number $\square$
(Select from below)

| 1 = Chinese Yuan | $¥ 7.3790$ |
| :--- | ---: |
| 2 = European Euro | $€ 0.6805$ |
| 3 = Indian Rupee | IDR 39.30 |
| 4 = US Dollar | $\$ 1.00$ |

previous sheet ious sheet to B12

| 6 | 7 | 8 |
| ---: | ---: | ---: |
| 3.0 | 3.5 | 4.0 |
| $\$ 16.00$ | $\$ 16.00$ | $\$ 1,016.00$ |
| $\$ 14.84$ | $\$ 14.65$ | $\$ 918.80$ |
|  |  |  |
| $1.4 \%$ | $1.4 \%$ | $89.6 \%$ |
| 0.04 | 0.05 | 3.58 |




## Bond Pricing

Inputs
Rate Convention
Annual Coupon Rate
Yield to Maturity (Annualized)
Number of Payments / Year
Number of Periods to Maturity ( T )
Face Value (PAR)

## Outputs

Discount Rate / Period (r)
Coupon Payment (PMT)


1
6
5

2
8
20

## Effective Annual Rate

(1) Copy the Outputs \& Timeline from the Copy the range $\mathrm{B} 12: \mathrm{J} 20$ from the prev

Bond Duration using a Timeline


(6) The Duration Formula is:
$D=\frac{1+r}{r \cdot N O P}-\frac{1+r+T \cdot(C R}{C R \cdot\left((1+r)^{T}-\right.}$
Enter $=(1+\mathrm{B} 12) /\left(\mathrm{B} 12^{*} \mathrm{~B} 7\right)-(1+\mathrm{B} 12$ $/\left(\mathrm{B} 5^{\star}\left((1+\mathrm{B} 12)^{\wedge} \mathrm{B} 8-1\right)+\mathrm{B} 12^{\star}\right.$

## Bond Convexity

Weight * (Time^2+Time)
Convexity using a Timeline Convexity using a Formula
0.01

$$
\frac{\left(\begin{array}{l}
C R \cdot(1+r)^{1+T} \cdot(r \cdot(N O P+1)+2) \\
-C R \cdot\left(r^{2} \cdot(N O P+T+1) \cdot(T+1)+r \cdot(N O P+2 \cdot T+3)+2\right)+r^{3} \cdot N O P \cdot T \cdot(N O P+T
\end{array}\right.}{r^{2} \cdot N O P^{2} \cdot\left(C R \cdot(1+r)^{T}-C R+r \cdot N O P\right)}
$$

Enter $=\left(\left(\mathrm{B} 5^{*}\left((1+\mathrm{B} 12)^{\wedge}(1+\mathrm{B} 8)\right)^{\star}\left(\mathrm{B} 12^{\star}(\mathrm{B} 7+1)+2\right)-\mathrm{B} 5^{\star}\left(\mathrm{B} 12^{\wedge} 2^{\star}(\mathrm{B} 7+\mathrm{B} 8+1)^{\star}(\mathrm{B} 8+1)+\mathrm{B} 12^{\star}\left(\mathrm{B} 7+2^{\star} \mathrm{B} 8+3\right)\right.\right.\right.$ $\left.\left./\left(\mathrm{B} 12^{\wedge} 2^{\star} \mathrm{B} 7^{\wedge} 2^{\star}\left(\mathrm{B} 5^{*}(1+\mathrm{B} 12)^{\wedge} \mathrm{B} 8-\mathrm{B} 5+\mathrm{B} 12^{\star} \mathrm{B} 7\right)\right)\right) /\left((1+\mathrm{B} 12)^{\wedge} 2\right)\right)$

US Dollar $\quad \begin{gathered}\text { Exch Rate } \\ \$ 1.00=\end{gathered}$

Currency Number $\square$
(Select from below)

| 1 = Chinese Yuan | $¥ 7.3790$ |
| :--- | ---: |
| 2 = European Euro | $€ 0.6805$ |
| 3 = Indian Rupee | IDR 39.30 |
| 4 = US Dollar | $\$ 1.00$ |

previous sheet ious sheet to B12

| 6 | 7 | 8 |
| ---: | ---: | ---: |
| 3.0 | 3.5 | 4.0 |
| $\$ 16.00$ | $\$ 16.00$ | $\$ 1,016.00$ |
| $\$ 14.84$ | $\$ 14.66$ | $\$ 919.37$ |
|  |  |  |
| $1.4 \%$ | $1.4 \%$ | $89.6 \%$ |
| 0.04 | 0.05 | 3.58 |




## Bond Pricing

## Price Sensitivity




$$
\begin{aligned}
& \text { Exch Rate } \$ 1.00 \\
& \$ 1.00=\$ 1.00
\end{aligned}
$$

## Sensitivity


? in Yield To Maturity

| $3.00 \%$ | $3.50 \%$ | $4.00 \%$ | $4.50 \%$ | $5.00 \%$ | $5.50 \%$ | $6.00 \%$ | $6.50 \%$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $1.5 \%$ | $1.8 \%$ | $2.0 \%$ | $2.3 \%$ | $2.5 \%$ | $2.8 \%$ | $3.0 \%$ | $3.3 \%$ |
| $-1.3 \%$ | $-0.8 \%$ | $-0.3 \%$ | $0.2 \%$ | $0.7 \%$ | $1.2 \%$ | $1.7 \%$ | $2.2 \%$ |
| $\$ 1,105$ | $\$ 1,085$ | $\$ 1,066$ | $\$ 1,047$ | $\$ 1,029$ | $\$ 1,011$ | $\$ 993$ | $\$ 976$ |
| $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ |
| $4.7 \%$ | $2.9 \%$ | $1.0 \%$ | $-0.7 \%$ | $-2.5 \%$ | $-4.2 \%$ | $-5.9 \%$ | $-7.5 \%$ |
| 3.56 | 3.56 | 3.56 | 3.56 | 3.56 | 3.56 | 3.56 | 3.56 |
| $4.6 \%$ | $2.8 \%$ | $1.0 \%$ | $-0.7 \%$ | $-2.5 \%$ | $-4.3 \%$ | $-6.1 \%$ | $-7.9 \%$ |
| 16.93 | 16.93 | 16.93 | 16.93 | 16.93 | 16.93 | 16.93 | 16.93 |
| $4.7 \%$ | $2.9 \%$ | $1.0 \%$ | $-0.7 \%$ | $-2.5 \%$ | $-4.2 \%$ | $-5.8 \%$ | $-7.5 \%$ |

```
F(2) New YTM - Current YTM
    Enter =B21-$B$6 and copy across
```

PV(Actual Discount Rate / Period,
Number of Periods to Maturity,
Coupon Payment, Face Value)
inter $=-\mathrm{PV}(\mathrm{B} 22, \$ \mathrm{~B} \$ 8, \$ \mathrm{~B} \$ 13, \$ \mathrm{~B} \$ 9)$
nd copy across
/Current Discount Rate / Period,
Number of Periods to Maturity,
Coupon Payment, Face Value)
er $=-\mathrm{PV}(\$ \mathrm{~B} \$ 12, \$ \mathrm{~B} \$ 8, \$ \mathrm{~B} \$ 13, \$ \mathrm{~B} \$ 9)$
d copy across
fied Duration
ange in YTM
=-B27*B23
ppy across


| 7.00\% | $7.50 \%$ | $8.00 \%$ | $8.50 \%$ | $9.00 \%$ | $9.50 \%$ | $10.00 \%$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $3.5 \%$ | $3.8 \%$ | $4.0 \%$ | $4.3 \%$ | $4.5 \%$ | $4.8 \%$ | $5.0 \%$ |
| $2.7 \%$ | $3.2 \%$ | $3.7 \%$ | $4.2 \%$ | $4.7 \%$ | $5.2 \%$ | $5.7 \%$ |
| $\$ 959$ | $\$ 942$ | $\$ 926$ | $\$ 910$ | $\$ 894$ | $\$ 879$ | $\$ 864$ |
| $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ | $\$ 1,055$ |
| $-9.1 \%$ | $-10.7 \%$ | $-12.2 \%$ | $-13.7 \%$ | $-15.2 \%$ | $-16.7 \%$ | $-18.1 \%$ |
| 3.56 | 3.56 | 3.56 | 3.56 | 3.56 | 3.56 | 3.56 |
| $-9.7 \%$ | $-11.4 \%$ | $-13.2 \%$ | $-15.0 \%$ | $-16.8 \%$ | $-18.6 \%$ | $-20.3 \%$ |
| 16.93 | 16.93 | 16.93 | 16.93 | 16.93 | 16.93 | 16.93 |
| $-9.0 \%$ | $-10.6 \%$ | $-12.1 \%$ | $-13.5 \%$ | $-14.9 \%$ | $-16.3 \%$ | $-17.6 \%$ |

## Bond Pricing

Immunization

Inputs
Rate Convention
Yield to Maturity (Annualized)
Number of Payments / Year

Bond 1
Bond 2
Bond 3
Bond 4
Bond 5
Bond 6
Bond 7
Bond 8

| $\bigcirc$ EAR $\bigcirc^{\text {O APR }}$ | 2 | Annual Percentage Rate |  |
| :---: | :---: | :---: | :---: |
| 3.17\% |  |  |  |
| 2 |  |  |  |
| Annual Coupon Rate | Number of Periods to Maturity (T) | Face Value <br> (PAR) | Number of Bonds |
| 3.25\% | 4 | \$1,000 | 2,838 |
| 4.25\% | 8 | \$1,000 | 3,789 |
| 0.90\% | 2 | \$1,000 | 0 |
| 1.50\% | 4 | \$1,000 | 0 |
| 1.90\% | 6 | \$1,000 | 0 |
| 2.30\% | 8 | \$1,000 | 0 |
| 1.90\% | 6 | \$1,000 | 0 |
| 2.30\% | 8 | \$1,000 | 0 |

Outputs

Discount Rate / Period (r)
(3) Copy the Present Value \& Duratic

Copy the range B19:J24 from the
Bond Present Value, Duration, and Convexity using a Timelline

| Period | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Time (Years) | 0.0 | 0.5 | 1.0 | 1.5 |
| Liabilities |  | $\$ 0$ | $\$ 0$ | 50 |
| Present Value of Liabilities |  | $\$ 0$ | $\$ 0$ | 50 |

Total Present Value of Liabilities
Weight
Weight * Time
Duration of Liabilities
Modified Duration of Liabilities
Weight * (Time^2+Time)
Convexity of Liabilities

| $0.0 \%$ | $0.0 \%$ | $0.0 \%$ |
| ---: | ---: | ---: |
| 0.00 | 0.00 | 0.00 |


| 3.00 |
| ---: |
| 2.95 |
| 11.60 |$\quad 0.00_{4-20}$



Differences
Total Assets - Liabilities
PV of Assets - PV of Liabilities
Duration of Assets - Duration of Liab
Convexity of Assets - Convexity of Liab


To solve the first problem when there is a single liability to immunize


To solve the second problem when there is a series of liabilities to immunize



To solve the third problem when there is a series of liabilities to immunize with cash flow matching


Coupon
Payment (PMT)

in formulas from the Duration and Convexity sheet Duration and Convexity sheet to B24

| 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| \$0 | \$0 | \$7,300,000 | \$0 | \$0 |
| \$0 | \$0 | \$6,642,711 | \$0 | \$0 |
| 0.0\% | 0.0\% | 100.0\% | 0.0\% | 0.0\% |
| 0.00 | 0.00 | 3.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 12.00 | 0.00 | 0.00 |


| ents)^ 2) | (4) Weight * $\left(\right.$ Time ${ }^{\wedge} 2+$ Time) <br> Enter $=\mathrm{C} 26^{*}\left(\mathrm{C} 22^{\wedge} 2+\mathrm{C} 22\right)$ and copy across |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| \$2,884,451 | \$0 | \$0 | \$0 | \$0 |
| \$61,567 | \$61,567 | \$61,567 | \$61,567 | \$3,850,280 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$2,946,017 | \$61,567 | \$61,567 | \$61,567 | \$3,850,280 |
| \$2,766,413 | \$56,911 | \$56,023 | \$55,149 | \$3,395,124 |


| $41.6 \%$ | $0.9 \%$ | $0.8 \%$ | $0.8 \%$ | $51.1 \%$ |
| ---: | ---: | ---: | ---: | ---: |
| 0.83 | 0.02 | 0.03 | 0.03 | 2.04 |

0.07
0.10
0.13
10.22




| Currency Number <br> (Select from below) | 4 |
| :--- | ---: |
| 1 = Chinese Yuan | $¥ 7.3790$ |
| 2 = European Euro | $€ 0.6805$ |
| 3 = Indian Rupee | IDR 39.30 |
| 4 = US Dollar | $\$ 1.00$ |

$\left.\begin{array}{l}\begin{array}{c}\text { Annual } \\ \text { Coupon Rate }\end{array} \begin{array}{c}\text { Number of } \\ \text { Periods to } \\ \text { Maturity (T) }\end{array} \\ \begin{array}{|r|r|r|r|}\text { Face Value } \\ \text { (PAR) }\end{array} \\ \hline 3.25 \%\end{array} \quad 4 \begin{array}{c}\text { Number of } \\ \text { Bonds }\end{array}\right]$

| 00.0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
|  | 0.5 | 1.0 | 1.5 |
|  | \$0 | \$0 | \$0 |
|  | \$2,000,000 | \$2,200,000 | \$2,500,000 |


| Annual <br> Coupon <br> Rate |
| :---: |
|  Number of <br> Periods to <br> Maturity (T) Face Value <br> $($ PAR) Number of <br> Bonds <br> $0.00 \%$ 1 $\$ 1,000$ 2,000 <br> $0.00 \%$ 2 $\$ 1,000$ 2,200 <br> $0.00 \%$ 3 $\$ 1,000$ 2,500 <br> $0.00 \%$ 4 $\$ 1,000$ 3,200 <br> $0.00 \%$ 5 $\$ 1,000$ 3,700 <br> $0.00 \%$ 6 $\$ 1,000$ 4,300 <br> $0.00 \%$ 7 $\$ 1,000$ 4,700 <br> $0.00 \%$ 8 $\$ 1,000$ 5,100 |


| 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| \$0 | \$0 | \$4,000,000 | \$0 | \$0 |
|  |  |  |  |  |
| \$3,200,000 | \$3,700,000 | \$4,300,000 | \$4,700,000 | \$5,100,000 |

## Bond Pricing

Immunization

Inputs
Rate Convention
Yield to Maturity (Annualized)
Number of Payments / Year

Bond 1
Bond 2
Bond 3
Bond 4
Bond 5
Bond 6
Bond 7
Bond 8

| O EAR O APR | 2 | Annual Percentage Rate |  |
| :---: | :---: | :---: | :---: |
| 3.17\% |  |  |  |
| 2 |  |  |  |
| Annual Coupon Rate | Number of Periods to Maturity (T) | Face Value <br> (PAR) | Number of Bonds |
| 1.50\% | 2 | \$1,000 | 0 |
| 2.70\% | 4 | \$1,000 | 34,996 |
| 2.90\% | 6 | \$1,000 | 5,073 |
| 3.20\% | 8 | \$1,000 | 10,046 |
| 1.90\% | 6 | \$1,000 | 0 |
| 2.30\% | 8 | \$1,000 | 0 |
| 1.90\% | 6 | \$1,000 | 0 |
| 2.30\% | 8 | \$1,000 | 0 |

Outputs

Discount Rate / Period (r)
(3) Copy the Present Value \& Duratic

Copy the range B19:J24 from the
Bond Present Value, Duration, and Convexity using a Timeline

| Period | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| Time (Years) | 0.0 | 0.5 | 1.0 | 1.5 |
| Liabilities |  | 54,500,000 | \$5,100,000 | \$5,600,000 |
| Present Value of Liabilities |  | \$4,429.788 | \$4,942,094 | \$5,341,943 |
| Total Present Value of Liabilities | \$48,127, 151 |  |  |  |
| Weight |  | 9.2\% | 10.3\% | 11.1\% |
| Weight * Time |  | 0.05 | 0.10 | 0.17 |

Duration of Liabilities
Modified Duration of Liabilities
Weight * (Time^2+Time)
Convexity of Liabilities
0.10
0.17

| 2.44 |
| ---: |
| 2.40 |

0.05
0.074 -- - - 0.212
(5) (Sum of Weight * (Time ${ }^{\wedge} 2+$ Time)) / ((1 + Yield to Maturity / Number of Paym Enter $=\mathrm{SUM}(\mathrm{C} 30: \mathrm{J} 30) /\left((1+\mathrm{B} 18)^{\wedge} 2\right)$

| Assets | Enter $=$ | )/((1) |  |
| :---: | :---: | :---: | :---: |
| Bond 1 | \$0 | \$0 | \$0 |
| Bond 2 | \$262,469 | \$262,469 | \$262,469 |
| Bond 3 | \$38,044 | \$38,044 | \$38,044 |
| Bond 4 | \$75,346 | \$75,346 | \$75,346 |
| Bond 5 | \$0 | \$0 | \$0 |
| Bond 6 | \$0 | \$0 | \$0 |
| Bond 7 | \$0 | \$0 | \$0 |
| Bond 8 | \$0 | \$0 | \$0 |
| Total Assets | \$375,859 | \$375,859 | \$375,859 |
| Present Value of Assets |  | \$364,222 | \$358,539 |
|  |  |  |  |
| Weight |  | 0.8\% | 0.7\% |
| Weight * Time |  | 0.01 | 0.01 |
| Duration of Assets | ct |  |  |
| Modified Duration of Assets |  |  |  |
| Weight * (Time^2+Time) |  | 0.02 | 0.03 |

Differences

Total Assets - Liabilities
PV of Assets - PV of Liabilities
Duration of Assets - Duration of Liab
Convexity of Assets - Convexity of Liab
$(\$ 4,124,141) \quad(\$ 4,724,141) \quad(\$ 5,224,141)$
(7) Total Assets - Liabilities Enter $=$ C43-C23 and cop
8) Compute the differences between Assets Enter $=\mathrm{B} 45-\mathrm{B} 25$ in B55, $=\mathrm{B} 48-\mathrm{B} 28$ in B56,

To solve the first problem when there is a single liability to immunize


To solve the second problem when there is a series of liabilities to immunize



To solve the third problem when there is a series of liabilities to immunize with cash flow matching


Coupon

in formulas from the Duration and Convexity sheet Duration and Convexity sheet to B24

| 4 | 5 | 6 | 7 | 8 |
| ---: | ---: | ---: | ---: | ---: |
| 2.0 | 2.5 |  |  |  |
| $\$ 6,300,000$ | $\$ 6,800,000$ | $\$ 7,200,000$ | $\$ 7,900,000$ | $\$ 8,600,000$ |
| $\$ 5,915,919$ | $\$ 6,285,806$ | $\$ 6,551,715$ | $\$ 7,076,524$ | $\$ 7,583,362$ |


| $12.3 \%$ | $13.1 \%$ | $13.6 \%$ | $14.7 \%$ | $15.8 \%$ |
| ---: | ---: | ---: | ---: | ---: |
| 0.25 | 0.33 | 0.41 | 0.51 | 0.63 |

0.74
$1.14 \quad 1.63$
2.32
3.15

| 4.14 | 0.02 | 1.18 | 0.02 | 3.71 |
| :--- | :--- | :--- | :--- | :--- |

Duration, and Convexity formulas from above to B44

| $\$ 29,071,700$ | $(\$ 6,686,610)$ | $(\$ 2,014,074)$ | $(\$ 7,824,654)$ | $\$ 1,521,479$ |
| :--- | :--- | :--- | :--- | :--- |

jy across
and Liabilities in Present Value, Duration, and Convexity and =B51-B31 in B57



| Currency Number <br> (Select from below) | 4 |
| :--- | ---: |
| 1 = Chinese Yuan | $¥ 7.3790$ |
| 2 = European Euro | $€ 0.6805$ |
| 3 = Indian Rupee | IDR 39.30 |
| 4 = US Dollar | $\$ 1.00$ |

$\left.\begin{array}{l}\begin{array}{c}\text { Annual } \\ \text { Coupon Rate }\end{array} \begin{array}{c}\text { Number of } \\ \text { Periods to } \\ \text { Maturity (T) }\end{array} \\ \begin{array}{|r|r|r|r|}\text { Face Value } \\ \text { (PAR) }\end{array} \\ \hline 1.50 \%\end{array} \quad 4 \begin{array}{c}\text { Number of } \\ \text { Bonds }\end{array}\right]$

| 00.0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
|  | 0.5 | 1.0 | 1.5 |
|  | \$0 | \$0 | \$0 |
|  | \$2,000,000 | \$2,200,000 | \$2,500,000 |


| Annual <br> Coupon <br> Rate |
| :---: |
|  Number of <br> Periods to <br> Maturity (T) Face Value <br> $($ PAR) Number of <br> Bonds <br> $0.00 \%$ 1 $\$ 1,000$ 2,000 <br> $0.00 \%$ 2 $\$ 1,000$ 2,200 <br> $0.00 \%$ 3 $\$ 1,000$ 2,500 <br> $0.00 \%$ 4 $\$ 1,000$ 3,200 <br> $0.00 \%$ 5 $\$ 1,000$ 3,700 <br> $0.00 \%$ 6 $\$ 1,000$ 4,300 <br> $0.00 \%$ 7 $\$ 1,000$ 4,700 <br> $0.00 \%$ 8 $\$ 1,000$ 5,100 |


| 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| \$0 | \$0 | \$4,000,000 | \$0 | \$0 |
|  |  |  |  |  |
| \$3,200,000 | \$3,700,000 | \$4,300,000 | \$4,700,000 | \$5,100,000 |

## Bond Pricing

Immunization

Inputs

Rate Convention
Yield to Maturity (Annualized)
Number of Payments / Year


2 Annual Percentage Rate

Bond 1
Bond 2
Bond 3
Bond 4
Bond 5
Bond 6
Bond 7
Bond 8
Outputs

Discount Rate / Period (r)
(3) Copy the Present Value \& Duratic

Copy the range B19:J24 from the
Bond Present Value, Duration, and Convexity using a Timeline

| Period | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| Time (Years) | 0.0 | 0.5 | 1.0 | 1.5 |
| Liabilities |  | S4,500,000 | \$5,100,000 | \$5,600,000 |
| Present Value of Liabilities |  | \$4,429.788 | \$4.942,094 | \$5,341,943 |
| Total Present Value of Liabilities | \$48,127,151 |  |  |  |
| Weight |  | 9.2\% | 10.3\% | 11.1\% |
| Weight * Time |  | 0.05 | 0.10 | 0.17 |

Duration of Liabilities
Modified Duration of Liabilities
Weight * (Time^2+Time)
Convexity of Liabilities

| 2.44 |
| ---: |
| 2.40 |

Differences
Total Assets - Liabilities
PV of Assets - PV of Liabilities
Duration of Assets - Duration of Liab
Convexity of Assets - Convexity of Liab


To solve the first problem when there is a single liability to immunize


To solve the second problem when there is a series of liabilities to immunize



To solve the third problem when there is a series of liabilities to immunize with cash flow matching


Coupon

in formulas from the Duration and Convexity sheet Duration and Convexity sheet to B24

| 4 | 5 | 6 | 7 | 8 |
| ---: | ---: | ---: | ---: | ---: |
| 2.0 | 2.5 |  |  |  |


| $12.3 \%$ | $13.1 \%$ | $13.6 \%$ | $14.7 \%$ | $15.8 \%$ |
| ---: | ---: | ---: | ---: | ---: |
| 0.25 | 0.33 | 0.41 | 0.51 | 0.63 |

0.74
1.14
1.63
2.32
3.15

| 0.74 | 1.14 | 1.63 | 2.32 | 3.15 |
| :---: | :---: | :---: | :---: | :---: |
| ents)^ 2) | (4) Weight * (Tim <br> Enter $=\mathrm{C} 26{ }^{*}$ | $\begin{aligned} & + \text { Time) } \\ & 2+\mathrm{C} 22) \text { an } \end{aligned}$ | across |  |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$0 |
| \$6,300,000 | \$0 | \$0 | \$0 | \$0 |
| \$0 | \$6,800,000 | \$0 | \$0 | \$0 |
| \$0 | \$0 | \$7,200,000 | \$0 | \$0 |
| \$0 | \$0 | \$0 | \$7,900,000 | \$0 |
| \$0 | \$0 | \$0 | \$0 | \$8,600,000 |
| \$6,300,000 | \$6,800,000 | \$7,200,000 | \$7,900,000 | \$8,600,000 |
| \$5,915,919 | \$6,285,806 | \$6,551,715 | \$7,076,524 | \$7,583,362 |


| $12.3 \%$ | $13.1 \%$ | $13.6 \%$ | $14.7 \%$ | $15.8 \%$ |
| ---: | ---: | ---: | ---: | ---: |
| 0.25 | 0.33 | 0.41 | 0.51 | 0.63 |
|  |  |  |  |  |
| 0.74 | 1.14 | 1.63 | 2.32 | 3.15 |


| Duration, and Convexity formulas from above to B44 |  |  |
| :---: | :---: | :---: |
| \$0 \$0 \$0 | \$0 | \$0 |
| y a across |  |  |
| and Liabilities in Present Value, Duration, and Convexity and $=$ B51-B31 in B57 |  |  |




| Currency Number <br> (Select from below) | 4 |
| :--- | ---: |
| 1 = Chinese Yuan | $¥ 7.3790$ |
| 2 = European Euro | $€ 0.6805$ |
| 3 = Indian Rupee | IDR 39.30 |
| 4 = US Dollar | $\$ 1.00$ |

$\left.\begin{array}{l}\begin{array}{c}\text { Annual } \\ \text { Coupon Rate }\end{array} \begin{array}{c}\text { Number of } \\ \text { Periods to } \\ \text { Maturity (T) }\end{array} \\ \begin{array}{|r|r|r|r|}\text { Face Value } \\ \text { (PAR) }\end{array} \\ \hline 1.50 \%\end{array} \quad 4 \begin{array}{c}\text { Number of } \\ \text { Bonds }\end{array}\right]$

| 00.0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
|  | 0.5 | 1.0 | 1.5 |
|  | \$0 | \$0 | \$0 |
|  | \$2,000,000 | \$2,200,000 | \$2,500,000 |


| Annual <br> Coupon <br> Rate |
| :---: |
|  Number of <br> Periods to <br> Maturity (T) Face Value <br> $($ PAR) Number of <br> Bonds <br> $0.00 \%$ 1 $\$ 1,000$ 2,000 <br> $0.00 \%$ 2 $\$ 1,000$ 2,200 <br> $0.00 \%$ 3 $\$ 1,000$ 2,500 <br> $0.00 \%$ 4 $\$ 1,000$ 3,200 <br> $0.00 \%$ 5 $\$ 1,000$ 3,700 <br> $0.00 \%$ 6 $\$ 1,000$ 4,300 <br> $0.00 \%$ 7 $\$ 1,000$ 4,700 <br> $0.00 \%$ 8 $\$ 1,000$ 5,100 |


| 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| \$0 | \$0 | \$4,000,000 | \$0 | \$0 |
|  |  |  |  |  |
| \$3,200,000 | \$3,700,000 | \$4,300,000 | \$4,700,000 | \$5,100,000 |










