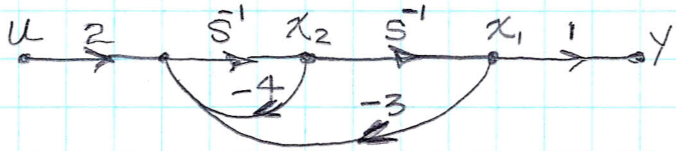
CHAPTER 3 PROBLEM SOLUTIONS

3.1 (a) $\ddot{y} + 4\dot{y} + 3y = 2u$

LET: $x_1 = y$, $\dot{x}_1 = \dot{y} = x_2$, $\dot{x}_2 = -3x_1 - 4x_2 + 2u$

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \underline{u}, \quad y = [1 \quad 0]\underline{x} + [0]u$$

(b)

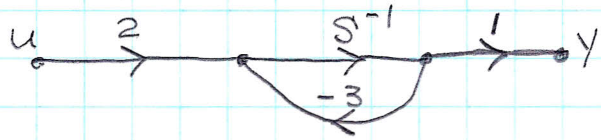


(c) $\dot{y} + 3y = 2u$; LET $x = y$

$$\dot{x} = -3x + 2u, \quad y = x$$

(d)

(CONTROL CANONICAL)



(observer CANONICAL)

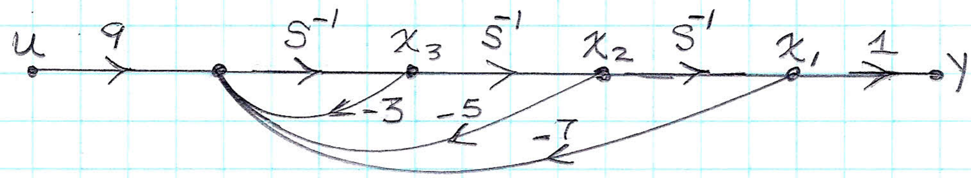
3.2 (a) $\dot{x}_3 + 3x_3 + 5x_2 + 7x_1 = 9u$

$\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, $\dot{x}_3 = -7x_1 - 5x_2 - 3x_3 + 9u$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 9 \end{bmatrix} u$$

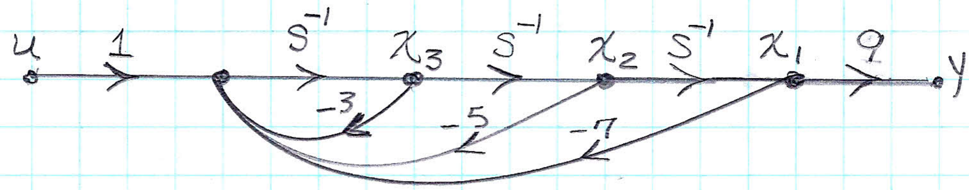
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$

(b)



(C) OBSERVER CANONICAL FORM (SEE (b))

CONTROL CANONICAL FORM

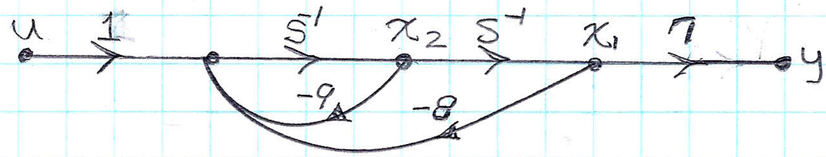


3.3 (a) $$\frac{Y(s)}{U(s)} = \frac{7s^{-2}}{1-(-9s^{-1}-8s^{-2})}$$

(SEE MASON'S GAIN FORMULA)

$M_1 \Delta_1 = 7s^{-2}$

$\Delta = 1-(-9s^{-1}-8s^{-2})$



(b) $\dot{x}_1 = x_2$, $\dot{x}_2 = -8x_1 - 9x_2 + u$

$y = 7x_1$

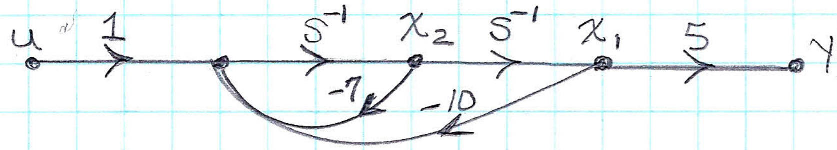
$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -8 & -9 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

(CONTROL CANONICAL FORM)

$$y = \begin{bmatrix} 7 & 0 \end{bmatrix} \underline{x} + [0] u$$

3.4 (a)

$$\frac{Y(s)}{U(s)} = \frac{5s^{-2}}{1-(-7s^{-1}-10s^{-2})}$$



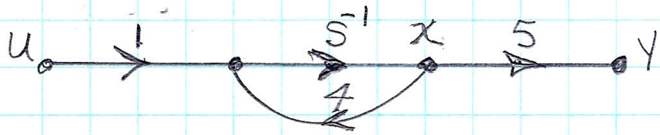
(b) $\dot{x}_1 = x_2$, $\dot{x}_2 = -10x_1 - 7x_2 + u$

$y = 5x_1$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [5 \quad 0]\underline{x} + [0]u$$

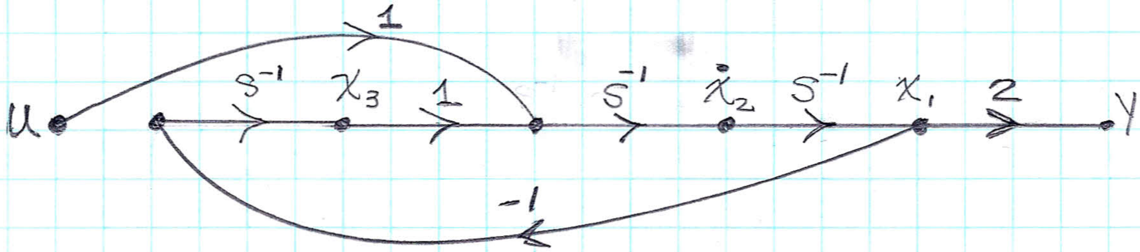
(c) $$\frac{Y(s)}{U(s)} = \frac{5s^{-1}}{1-(4s^{-1})}$$



$\dot{x} = 4x + u$; $y = 5x$

$y = 5x$

3.4 (d) $\frac{Y(s)}{U(s)} = \frac{2s^{-2}}{1-(-s^{-3})}$



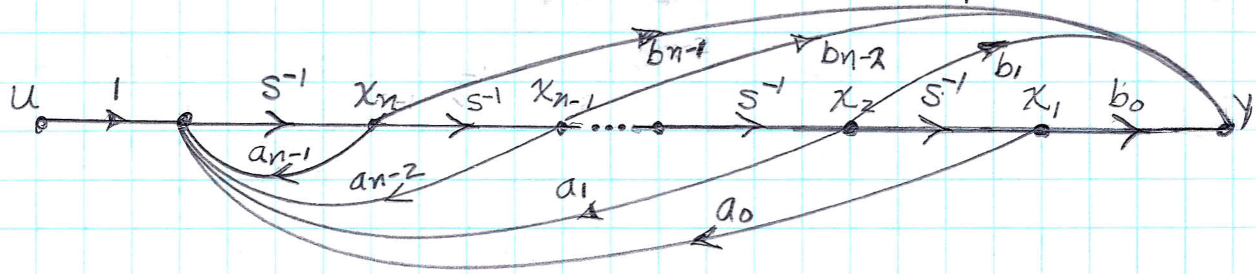
$\dot{x}_1 = x_2$; $\dot{x}_2 = x_3 + u$; $\dot{x}_3 = -x_1$

$y = 2x_1$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} \underline{x}$$

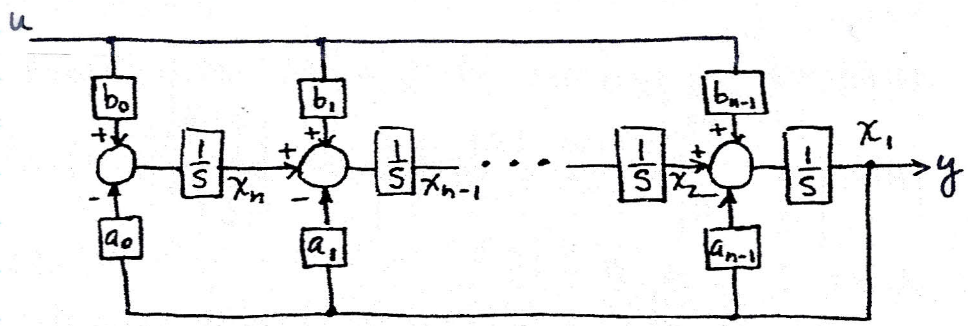
3.5 (a) $G(s) = \dfrac{b_{n-1}s^{-1} + \cdots + b_1 s^{-(n-1)} + b_0 s^{-n}}{1 - (-a_{n-1}s^{-1} - \cdots - a_1 s^{-(n-1)} - a_0 s^{-n})}$



$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & & 0 & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-2} & -a_{n-1} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \underline{u}$$

$$y = \begin{bmatrix} b_0 & b_1 & \cdots & b_{n-2} & b_{n-1} \end{bmatrix} \underline{x} + [0] u$$

(b)



$$\dot{\underline{x}} = \begin{pmatrix} -a_{n-1} & 1 & 0 & \cdots & 0 \\ -a_{n-2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_1 & 0 & 0 & \cdots & 1 \\ -a_0 & 0 & 0 & \cdots & 0 \end{pmatrix} \underline{x} + \begin{pmatrix} b_{n-1} \\ b_{n-2} \\ \vdots \\ b_1 \\ b_0 \end{pmatrix} u$$

$$y = (1 \; 0 \; 0 \; \cdots \; 0)\, \underline{x}$$

3.6 (a) $$\dot{\underline{x}} = \begin{bmatrix} -5-4 & 8-12 \\ -1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$[SI-A]^{-1} = \begin{bmatrix} S+9 & 4 \\ 1 & S+3 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} S+3 & -4 \\ -1 & S+9 \end{bmatrix}}{S^2+12S+23}$$

(b)

$$[SI-A]^{-1} = \begin{bmatrix} \frac{S+3}{S^2+12S+23} & \frac{-4}{S^2+12S+23} \\ \frac{-1}{S^2+12S+23} & \frac{S+9}{S^2+12S+23} \end{bmatrix} = \underline{\Phi}(S)$$

$$|SI-A| = S^2+12S+23 \simeq (S+2.4)(S+9.6)$$

(c)

$$\phi(t) = \mathcal{L}^{-1}\{[SI-A]^{-1}\} = \begin{bmatrix} 0.084e^{-2.4t} + 0.916e^{-9.6t} & -0.555(e^{-2.4t} - e^{-9.6t}) \\ -0.139(e^{-2.4t} - e^{-9.6t}) & 0.916e^{-2.4t} + 0.083e^{-9.6t} \end{bmatrix}$$

3.7 (a)

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -4-5 & -5-10 & 15 \\ -1 & -2 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \underline{x}$$

(b)

$$[sI-A] = \begin{bmatrix} s & -1 & 0 \\ 9 & s+15 & -15 \\ 1 & 2 & s+2 \end{bmatrix}$$

$$\Phi(s) = [sI-A]^{-1}$$

$$\Phi(s) = \frac{\begin{bmatrix} s^2+17s+60 & s+2 & 15 \\ -(9s+33) & s(s+2) & 15s \\ -(s-3) & -(2s+1) & s^2+15s+9 \end{bmatrix}}{s^2+17s^2+69s+3}$$

(c) $\phi(t) = \mathcal{L}^{-1}\{\Phi(s)\}$

$$\phi_{11}(t) = 0.995e^{-0.55t} + 1.083e^{-5.45t} - 1.034e^{-11t}$$

$$\Phi_{12}(t) = 0.0283e^{-0.55t} + 0.1269e^{-5.45t} - 0.1552e^{-11t}$$

$$\phi_{13}(t) = 0.293e^{-0.55t} - 0.5516e^{-5.45t} + 0.2586e^{-11t}$$

$$\phi_{21}(t) = -0.5478e^{-0.55t} - 0.5901e^{-5.45t} + 1.1379e^{-11t}$$

$$\phi_{22}(t) = -0.0156e^{-0.55t} - 0.6913e^{-5.45t} + 1.7069e^{-11t}$$

$$\phi_{23}(t) = -0.1613e^{-0.55t} + 3.006e^{-5.45} - 2.8448e^{-11t}$$

$$\phi_{31}(t) = 0.0694e^{-0.55t} - 0.3107e^{-5.45t} + 2.414e^{-11t}$$

$$\phi_{32}(t) = 0.0020e^{-0.55t} - 0.3640e^{-5.45t} + 0.3621e^{-11t}$$

$$\phi_{33}(t) = 0.0202e^{-0.55t} + 1.5830e^{-5.45t} - 0.6034e^{-11t}$$

3.8 (a) $[sI-A] = s+3$; $B = 4$

$$\Phi(s) = [sI-A]^{-1} = \frac{1}{s+3}$$

(b) $\phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = e^{-3t}$

(c) (3-24): $x(t) = \phi(t)x(0) + \int_0^t \phi(\tau)Bu(t-\tau)d\tau$

$$x(t) = \int_0^t 4e^{-3\tau}d\tau = -\frac{4}{3}e^{-3\tau}\Big|_0^t$$

$$x(t) = 4/3\,(1-e^{-3t})u(t)$$

(d) $x(0) = -1 \Rightarrow x(t) = -e^{-3t} + \frac{4}{3}(1-e^{-3t})$

$$x(t) = \left(\frac{4}{3} - \frac{7}{3}e^{-3t}\right)u(t)$$

(e) $sX(s) = -3X(s) + 4U(s) \Rightarrow U(s) = \frac{(s+3)X(s)}{4}$

$Y(s) = X(s)$

$$\frac{Y(s)}{U(s)} = \frac{4}{s+3} \Rightarrow Y(s) = \frac{4U(s)}{s+3}$$

$\mathcal{L}\{u(t)\} = 1/s \quad \therefore \quad Y(s) = \frac{4}{s(s+3)}$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{4}{s(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{4/3}{s} - \frac{4/3}{s+3}\right\}$$

$$y(t) = \frac{4}{3}(1-e^{-3t})u(t)$$

(f) (3-20): $X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s)$

$x(0) = -1$

$$X(s) = \frac{-1}{s+3} + \frac{1}{s+3}(4)\left(\frac{1}{s}\right)$$

$$x(t) = \left(-e^{-3t} + \frac{4}{3}(1-e^{-3t})\right)u(t)$$

$$x(t) = \left(4/3 - 7/3\,e^{-3t}\right)u(t)$$

3.9 $A=-1$, $B=1$, $C=1$, $D=1$

$$sI-A = s+1$$

(a) $\Phi(s) = [sI-A]^{-1} = \frac{1}{s+1}$

(b) $\phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = e^{-t}$

(c) $x(t) = \int_0^t \phi(\tau) B\, u(t-\tau)\, d\tau = \int_0^t e^{-\tau} d\tau$

$$x(t) = (1-e^{-t}),\ t \geq 0$$

$$y(t) = x(t) + u(t) = 2 - e^{-t},\ t \geq 0$$

(d) $x(0) = -1$

$$x(t) = \phi(t)x(0) + \int_0^t \phi(\tau) B\, u(\tau - t)\, d\tau$$

$$x(t) = -1e^{-t} + 1 - e^{-t} = 1 - 2e^{-t},\ t \geq 0$$

$$y(t) = x(t) + u(t) = 2 - 2e^{-t},\ t \geq 0$$

(e) $sX(s) = -X(s) + U(s) \Rightarrow X(s) = \frac{U(s)}{s+1}$

$$Y(s) = X(s) + U(s)$$

$$Y(s) = \frac{U(s)}{s+1} + U(s) = \left(\frac{1}{s+1} + 1\right) U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{s+2}{s+1} \Rightarrow Y(s) = \frac{s+2}{s(s+1)}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s} + \frac{-1}{s+1}\right\} = 2 - e^{-t},\ t \geq 0$$

(f) $X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}B\,U(s)$

$$= \frac{1}{s+1}(-1) + \frac{1}{s(s+1)} = \frac{-s+1}{s(s+1)}$$

$$Y(s) = X(s) + U(s) = \frac{-s+1}{s(s+1)} + \frac{1}{s} = \frac{-s+1+s+1}{s(s+1)}$$

$$Y(s) = \frac{2}{s(s+1)} \Rightarrow y(t) = 2 - 2e^{-t},\ t \geq 0$$

3.10 $[sI - A] = \begin{bmatrix} s & -2 \\ 2 & s+5 \end{bmatrix}$

(a) $\Phi(s) = [sI - A]^{-1} = \dfrac{\begin{bmatrix} s+5 & 2 \\ -2 & s \end{bmatrix}}{s^2 + 5s + 4}$

$$\Phi(s) = \begin{bmatrix} \dfrac{s+5}{(s+1)(s+4)} & \dfrac{2}{(s+1)(s+4)} \\ \dfrac{-2}{(s+1)(s+4)} & \dfrac{s}{(s+1)(s+4)} \end{bmatrix}$$

$$\Phi(s) = \begin{bmatrix} \dfrac{4/3}{s+1} + \dfrac{-1/3}{s+4} & \dfrac{2/3}{s+1} + \dfrac{-2/3}{s+4} \\ \dfrac{-2/3}{s+1} + \dfrac{2/3}{s+4} & \dfrac{-1/3}{s+1} + \dfrac{4/3}{s+4} \end{bmatrix}$$

(b) $\phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = \begin{bmatrix} \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t} & \frac{2}{3}e^{-t} - \frac{2}{3}e^{-4t} \\ -\frac{2}{3}e^{-t} + \frac{2}{3}e^{-4t} & -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t} \end{bmatrix}$

(c) $\underline{x}(t) = \phi(t)\underline{x}(0) + \int_0^t \phi(\tau) B\, u(t-\tau)\, d\tau$

$\underline{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\underline{x}(t) = \int_0^t \phi(\tau) \begin{bmatrix} 0 \\ 1 \end{bmatrix} 1\, d\tau$$

$$= \int_0^t \begin{bmatrix} 2/3\, e^{-\tau} - 2/3\, e^{-4\tau} \\ -1/3\, e^{-\tau} + 4/3\, e^{-4\tau} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} \int_0^t 2/3\,(e^{-\tau} - e^{-4\tau})\, d\tau \\ \int_0^t 1/3\,(-e^{-\tau} + 4e^{-4\tau})\, d\tau \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} 1/2 - 2/3\, e^{-t} + \frac{1}{6} e^{-4t} \\ 1/3\, e^{-t} - 1/3\, e^{-4t} \end{bmatrix}$$

(d) $y(t) = [1 \;\; 0]\, \underline{x}(t) = \left(1/2 - 2/3\, e^{-t} + 1/6\, e^{-4t}\right), \; t \geq 0$

3.10 (d) $\underline{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\phi(t)\underline{x}(0) = \phi(t)\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{3}e^{-t} - \frac{5}{3}e^{-4t} \\ -\frac{4}{3}e^{-t} + \frac{10}{3}e^{-4t} \end{bmatrix}$$

$$\underline{x}(t) = \phi(t)\underline{x}(0) + \int_0^t \phi(\tau) B\, u(t-\tau)\, d\tau$$

from (c)

$$= \begin{bmatrix} \frac{8}{3}e^{-t} - \frac{5}{3}e^{-4t} \\ -\frac{4}{3}e^{-t} + \frac{10}{3}e^{-4t} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} - \frac{2}{3}e^{-t} + \frac{1}{6}e^{-4t} \\ \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t} \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} 0.5 + 2e^{-t} - 1.5e^{-4t} \\ -1e^{-t} + 3e^{-4t} \end{bmatrix}$$

$$y(t) = [1 \quad 0]\underline{x}(t) = 0.5 + 2e^{-t} - 1.5e^{-4t}, \quad t \geq 0$$

(e) $\frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D$

$$= [1 \quad 0]\,\Phi(s)\begin{bmatrix} 0 \\ 1 \end{bmatrix} = [1 \quad 0]\begin{bmatrix} \frac{2}{(s+1)(s+4)} \\ \frac{-\frac{1}{3}s}{(s+1)(s+4)} \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{2}{(s+1)(s+4)}, \quad U(s) = 1/s$$

$$Y(s) = \frac{2}{s(s+1)(s+4)} = \frac{0.5}{s} - \frac{2/3}{s+1} + \frac{1/6}{s+4}$$

$$y(t) = 0.5 - \tfrac{2}{3}e^{-t} + \tfrac{1}{6}e^{-4t}, \quad t \geq 0$$

3.11 (a) $[sI-A] = \begin{bmatrix} s & -1 \\ 3 & s+6 \end{bmatrix} \Rightarrow [sI-A]^{-1} = \dfrac{\begin{bmatrix} s+6 & 1 \\ -3 & s \end{bmatrix}}{s^2+6s+3}$

$$\Phi(s) = \begin{bmatrix} \dfrac{s+6}{s^2+6s+3} & \dfrac{1}{s^2+6s+3} \\ \dfrac{-3}{s^2+6s+3} & \dfrac{s}{s^2+6s+3} \end{bmatrix}$$

(b) $\phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} \dfrac{s+6}{(s+0.55)(s+5.45)} & \dfrac{1}{(s+0.55)(s+5.45)} \\ \dfrac{-3}{(s+0.55)(s+5.45)} & \dfrac{s}{(s+0.55)(s+5.45)} \end{bmatrix}\right\}$

$$\phi(t) = \begin{bmatrix} 1.1124e^{-0.55t} - 0.1124e^{-5.45t} & 0.2041(e^{-0.55t} - e^{-5.45t}) \\ 0.6124(e^{-0.55t} - e^{-5.45t}) & -0.1124e^{-0.55t} + 1.1124e^{-5.45t} \end{bmatrix}$$

(c) $\underline{x}(t) = \phi(t)\underline{x}(0) + \int_0^t \phi(\tau) B u(t-\tau)\,d\tau$

$\therefore \underline{x}(t) = \int_0^t \phi(t)\begin{bmatrix} 0 \\ 1 \end{bmatrix} 1\,d\tau$

$$= \begin{bmatrix} \int_0^t 0.2041(e^{-0.55\tau} - e^{-5.45\tau})\,d\tau \\ \int_0^t (-0.1124e^{-0.55\tau} + 1.1124e^{-5.45\tau})\,d\tau \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} -0.371(e^{-0.55t} - 1) \; .0375(e^{-5.45t} - 1) \\ 0.2042(e^{-0.55t} - 1) - 0.2042(e^{-5.45t} - 1) \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} 0.3335 - 0.371e^{-0.55t} + 0.0375e^{-5.45t} \\ 0.2042(e^{-0.55t} - e^{-5.45t}) \end{bmatrix}$$

3.11 (d) $\underline{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\underline{x}(t) = \phi(t)\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \phi(\tau)\begin{bmatrix} 0 \\ 1 \end{bmatrix}(1)\,d\tau$$

$$= \begin{bmatrix} 1.1124e^{-0.55t} - 0.1124e^{-5.45t} \\ 0.6124(e^{-0.55t} - e^{-5.45t}) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.2041\int_0^t (e^{-0.55\tau} - e^{-5.45\tau})\,d\tau \\ \int_0^t (-0.5505e^{-0.55\tau} + 8.7866e^{-5.45\tau})\,d\tau \end{bmatrix}$$

$$= \begin{bmatrix} 0.7414e^{-0.55t} - 0.0749e^{-5.45t} \\ 1.6124e^{-0.55t} - 2.2244e^{-5.45t} \end{bmatrix}$$

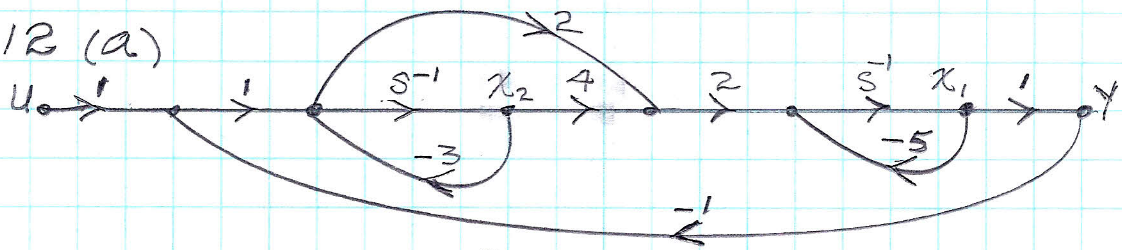
(e) $$\underline{X}(s) = [sI-A]^{-1}\underline{x}(0) + [sI-A]^{-1}BU(s) \qquad [(3\text{-}20)]$$

$$= \frac{\begin{bmatrix} s+6 & 1 \\ -3 & s \end{bmatrix}}{(s+0.55)(s+5.45)}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(\frac{1}{s}\right)$$

$$= \begin{bmatrix} \dfrac{1}{s(s+0.55)(s+5.45)} \\ \dfrac{1}{s(s+0.55)(s+5.45)} \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} 0.3335 - 0.371e^{-0.55t} + 0.0375e^{-5.45t} \\ 0.2042e^{-0.55t} - 0.2042e^{-5.45t} \end{bmatrix}$$

3.12 (a)



$$\underline{\dot{x}} = \begin{bmatrix} -5-4 & 8-12 \\ -1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} u = \begin{bmatrix} -9 & -4 \\ -1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$

(b) $$G_c(s) = \frac{2+4s^{-1}}{1+3s^{-1}} = \frac{2s+4}{s+3}$$

$$G_p(s) = \frac{2s^{-1}}{1+5s^{-1}} = \frac{2}{s+5}$$

(c) $$\frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} = \frac{\dfrac{4s+8}{(s+3)(s+5)}}{1+\dfrac{4s+8}{(s+3)(s+5)}}$$

$$= \frac{4s+8}{s^2+12s+23}$$

(d) $$|sI-A| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -9 & -4 \\ -1 & -3 \end{bmatrix} \right|$$

$$= \begin{vmatrix} s+9 & 4 \\ 1 & s+3 \end{vmatrix} = (s+9)(s+3) - (4)(1)$$

$$|sI-A| = s^2 + 12s + 23$$

(e) $$\Delta = 1 - (-3s^{-1} - 5s^{-1} - 8s^{-2} - 4s^{-1}) + (-3s^{-1})(-5s^{-1})$$

$$= 1 - (-12s^{-1} - 8s^{-2}) + 15s^{-2}$$

$$\Delta = s^{-2}(s^2 + 12s + 23) = |sI - A|$$