Instructor Manual

Foundations of MEMS

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Chapter 5

Visit http://www.memscentral.com, a companion website of the book for additional teaching materials.

Problems

For homework exercises in this chapter, use the parameters in this table unless instructed specifically otherwise.

Category	Parameter	Value
Young's Modulus	Silicon	120 Gpa
	Silicon nitride	385 Gpa
	Gold	57 GPa
Fracture strain	Silicon	0.9%
	Silicon nitride	2.0 %
Thermal expansion coefficient	Gold (Au)	14.2 ppm/°C
	Aluminum (Al)	25 ppm/°C
	Nickel	13 ppm/°C
	Silicon and polysilicon	2.33 ppm/°C
Density (kg/m^3)	Silicon	2330
	Silicon nitride	3100
ε_0	8.854×10^{-12} F/m	

Problem 1: Review

Hint:

Use the definition of thermal resistance, which is the ratio of temperature difference and thermal flux.

Problem 2: Design

For a 1-mm long gold-silicon composite beam (fixed-free boundary conditions), the vertical displacement at the end of the beam is 20 μm when the temperature of the beam is heated by 10 degrees above the room temperature. Estimate the angular and vertical displacement if the temperature is raised to 30 degrees. The formula for calculating the bending curvature is $k = \frac{1}{r} = \frac{6w_1w_2E_1E_2t_1t_2(t_1+t_2)(\alpha_1-\alpha_2)\Delta T}{(w_1E_1t_1^2)^2+(w_2E_2t_2^2)^2+2w_1w_2E_1E_2t_1t_2(2t_1^2+3t_1t_2+2t_2^2)}$.

Show analysis steps in answer. (Hint: use Taylor series expansion to approximate the function Cos(θ) when θ is small.)

(1) vertical displacement = 60 μm, angular displacement = 6.9 degrees
(2) vertical displacement = 49 μm, angular displacement = 2.3 degrees
(3) vertical displacement = 49 μm, angular displacement = 15 degrees
(4) vertical displacement = 60 μm, angular displacement = 15 degrees

Answer: ______________________

Solution:

When the temperature is raised by 3 times, the curvature is three times greater. For the same length, the angle of bending is three times greater.

Also, according to $d = r - r(1-\frac{1}{2}\theta^2 + O(\theta^4)) \approx \frac{1}{2}r\theta^2$, the displacement is three times greater, making it 60 micrometers.

Once the length and the vertical displacement is known, the angle can be calculated using $\theta = \frac{l}{r}$. We can plug

$$r = \frac{l}{\theta}$$

into $d = r - r(1-\frac{1}{2}\theta^2 + O(\theta^4)) \approx \frac{1}{2}\frac{l}{\theta}\theta^2 = 0.5l\theta$

The angle is $\frac{120\times10^{-6}}{1\times10^{-3}} = 0.12rad = 6.87\deg$.

Problem 3: Design

Solution:

The radius of curvature is 0.010265 m. The end displacement is 48.7 micrometers.

Problem 4: Design

First estimate the tip displacement under a given temperature difference. Then use the linear spring model to extrapolate the force acting on the tip to create the equivalent displacement.

For small angle displacement, $d = \frac{1}{2}r\theta^2$.

This can be simplified to $d = \frac{1}{2}r\left(\frac{l}{r}\right)^2 = \frac{1}{2}\frac{l^2}{r}$

The output force is $F = kd = \left(\frac{3EI}{l^3}\right)\left(\frac{1}{2}\frac{l^2}{r}\right) = \frac{3}{2}\frac{EI}{lr}$

Problem 5: Design

Solution:

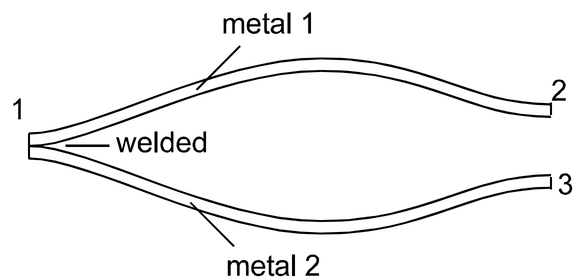
For a uniform temperature rise, the elongation is $l\times\beta\times\Delta T = 100\times10^{-6}m\times25\times10^{-6}\times20 = 5\times10^{-8}m$

If the temperature rise is linearly proportional to the position according to $\Delta T(x) = 20 - \frac{x}{l}20$, then the total elongation is

$$\int_0^l dx \times \beta \times \Delta T(x) = \int_0^l dx \times \beta \times (20 - \frac{x}{l}20) = 20\beta l - 20\beta \frac{1}{l}\frac{l^2}{2} = 10\beta l = 2.5 \times 10^{-8} m$$

Problem 6: Design

Two pieces of metal wires are connected in a thermal couple configuration as follows. The temperature at point 1 is higher than the temperatures at points 2 and 3. There are three possible metals. (1) Chromel, with Seeback coefficient = 30 μV/K), (2) Alumel, with Seeback coefficient = -11 μV/K, (3) Iron, with Seeback coefficient = 10 μV/K. Which of the following statement would be correct? Which one gives the best temperature sensitivity? Show your analysis steps.



(figure name: fig5-prb3)

(1) Metal 1 = Chromel, Metal 2 = Chromel, produces maximum possible thermal couple sensitivity of 60 μV/K;
(2) Metal 1 = Alumel, Metal 2 = Chromel, produces maximum possible thermal couple sensitivity of 30 μV/K
(3) Metal 1 = Iron, Metal 2 = Alumel, produces maximum possible thermal couple sensitivity of 21 μV/K
(4) Metal 1 = Chromel, Metal 2 = Alumel, produces maximum possible thermal couple sensitivity of 41 μV/K.

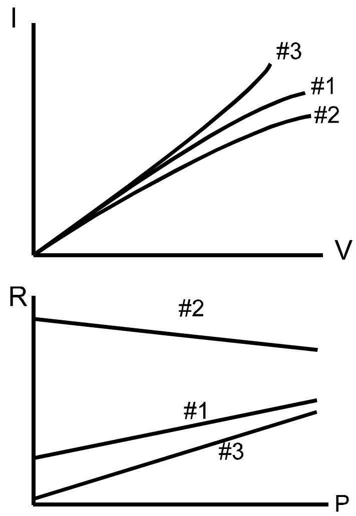
Answer: ____________________

Solution:

The biggest difference that is available from the set of materials is between Chromel and Alumel, with the possible thermal couple sensitivity of 41 *μV/K*.

Problem 7: Design

Three polysilicon thermal resistors with the TCR values of (1) Resistor 1, α=1000 PPM/ºC, (2) resistor 2, α=2000 PPM/ºC, and (3) resistor 3, α=-1000 PPM/ºC. An I-V curve measurement is conducted on these three resistors. Which of the following I-V and R-P curves (A through F) are likely to be true? Explain your analysis.


I
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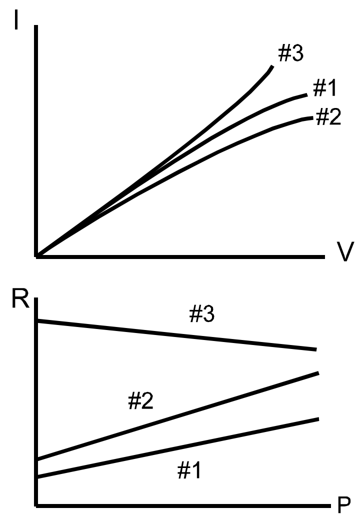
[A]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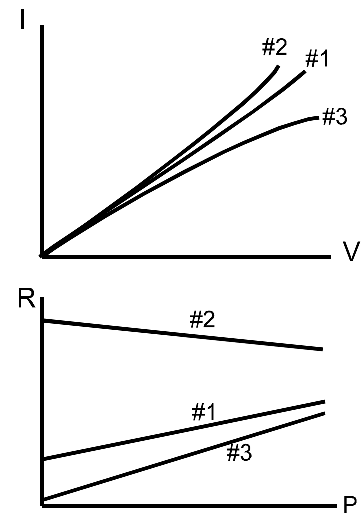
[B]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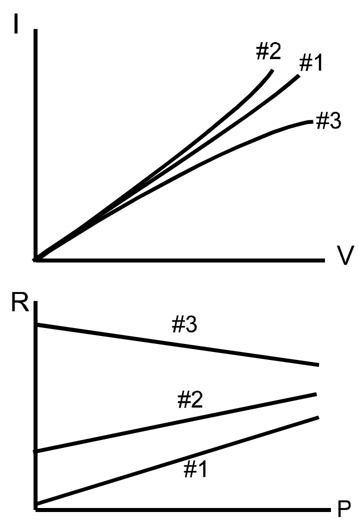
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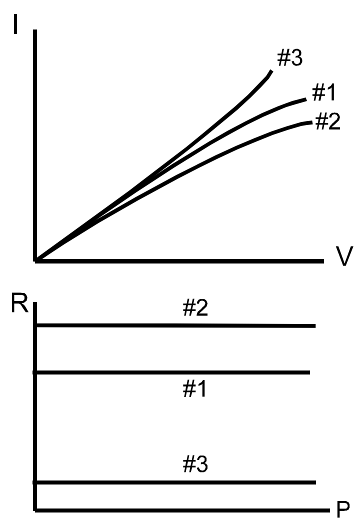
[D]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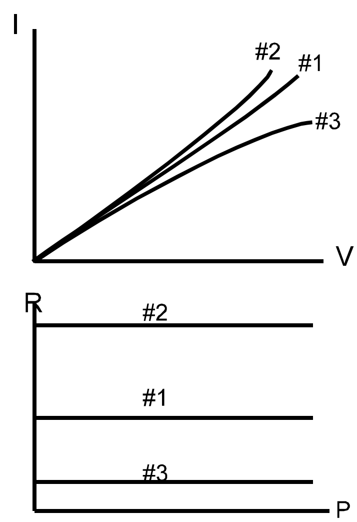
[E]


I
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P
[F]

(1) A
(2) B
(3) C
(4) D
(5) E
(6) F

Answer _____________

Answer:

The correct answer is (2) B

Problem 11: Design

Hint:
Estimate the amount of tip displacement when the intrinsically-bent beams are held down. Use the linear spring model to estimate the holding force. Then find the amount of electrostatic voltages that can be used to generate that force.

Problem 15: Fabrication

Solution:

The total resistance consists of two parts: the resistance of the long leg and of the short leg. The resistance of the long leg is

$$R_{long} = \rho \frac{l}{wt} = 5\times10^{-6}\,\Omega cm \times \frac{500\times10^{-4}\,cm}{2.8\times10^{-4}\,cm\times 2\times10^{-4}} = 4.46\Omega$$

The resistance of the short leg is

$$R_{long} = \rho \frac{l}{wt} = 5\times10^{-6}\,\Omega cm \times \frac{300\times10^{-4}\,cm}{2.8\times10^{-4}\,cm\times 2\times10^{-4}} = 2.68\Omega$$

The total power applied to the arm at 14 V bias voltage is

$$\frac{V^2}{(R_{long} + R_{short})} = 27.45W$$

Problem 17: Design

Solution:

The two composite beams are made of a number of layers, including polyimide (2.7 μm thick), electroplated Permalloy (4 μm thick), Pt (20 nm thick), and Nickel (80 nm thick). The width of the polyimide beam is 6 μm. The width of all metal layers are 4 μm. We assume the thermal conductivity of Ni and NiFe are identical.

For each leg, the thermal resistances are

$$R_{T,polyimide} = \frac{1}{0.001W/cm^o c}\frac{0.01}{0.0006\times 0.00027} = 6.17\times10^6\,k/W$$

$$R_{T,platinum} = \frac{1}{0.716W/cm^{o}c}\frac{0.01}{0.0004\times 20\times 10^{-7}} = 1.74\times 10^{6}\,k/W$$
$$R_{T,nickle} = \frac{1}{0.91W/cm^{o}c}\frac{0.01}{0.0004\times 80\times 10^{-7}} = 3.43\times 10^{5}\,k/W$$
$$R_{T,permalloy} = \frac{1}{0.91W/cm^{o}c}\frac{0.01}{0.0004\times 0.0004} = 6.86\times 10^{4}\,k/W$$

As one can see, the Permalloy layer plays an important role of reducing the overall thermal resistance. The total thermal resistance consists of polyimide resistors (2), platinum resistors (4), nickel resistors (2), and Permalloy resistors (2). The total resistance is $5.48\text{x}10^{4}$ k/W.

Problem 19: Fabrication

Determine the reason why polysilicon is used as the thermal resistor. If the resistor were made of metal, what changes would be necessary in terms of design and fabrication?

Answer:

If metal is used instead of polycrystalline silicon, the performance (sensitivity) may be reduced slightly because the TCR value of metals is not as high as the highest TCR from polysilicon resistors.

Metal would have to be compatible with the sacrificial layer etching using HF.

Problem 21: Design

Solution:

Since the end displacement is 10 μm and the overall length of the cantilever is 200 μm, the radius of curvature of the bent beam can be found. The end displacement and the radius of curvature is related by

$$10\times 10^{-6} = \frac{1}{2}r\theta^{2} = \frac{1}{2}r(\frac{l}{r})^{2} = \frac{1}{2}\frac{l^{2}}{r}$$

Hence

$$r = \frac{l^{2}}{2\times 10\times 10^{-6}} = 0.002m$$

therefore, the value is σ_1 can be calculated as

$$\sigma_1 = 4.18\times 10^{6}\,Pa$$

The Mathematica ® code for this calculation is shown below:

```
e1=279.*10^6;
e2=385.*10^6;
t1=10*10^-9;
t2=1*10^-6;
y1=0.5*(e1 t1 t1+e2 t2 t2)+e2 t1 t2;
y=y1/(e1 t1+e2 t2);
```

```
w=3*10^-6;
EI=w((e1 t1((t1 t1)/12.+(0.5 t1-y)^2))+e2 t2((t2 t2)/12.+(0.5 t2+t1-y)^2));
s2=0
n1=0.21;
n2=0.29;
M1=0.5 t1 t1 ((s1(1-n1)-e1(t1 s1(1-n1)+t2 s2(1-n2))/(e1 t1+e2 t2)));
M2=0.5(t2 t2+t2 t1)((s2(1-n2)-e2(t1 s1(1-n1)+t2 s2(1-n2))/(e1 t1+e2 t2)));
M=w M1+w M2;
R=EI/M
N[R]
```