Chapter 1 End of Chapter Problem Solutions

1.1

n = 4×10^{20} molecules/in^3

$\bar{v} = \sqrt{kgRT} = 1.32 \times 10^4$ in./s

$A = \frac{\pi}{4}(10^{-3}\text{ in})^2$

$NA = \frac{1}{4} n\bar{v}A = 1.04 \times 10^8$ m/s

1.2

Flow Properties: Velocity, Pressure Gradient, Stress

Fluid Properties: Pressure, Temperature, Density, Speed of Sound, Specific Heat

1.3

mass of solid= $\rho_s v_s$
mass of fluid= $\rho_f v_f$

$$X = \frac{\rho_s v_s}{\rho_{s v_s} + \rho_f v_f}$$

$$\Rightarrow \frac{v_f}{v_s} = \frac{1-x}{x} \frac{\rho_s}{\rho_f}$$

$$\rho_{\text{mix}} = \frac{\rho_s v_s + \rho_f v_f}{v_s + v_f} = \frac{\rho_s + \rho_f (\frac{v_f}{v_s})}{1 + \frac{v_f}{v_s}}$$

$$= \frac{\rho_s \rho_f}{x \rho_f + (1-x) \rho_f}$$

1.4

Given $\frac{P+B}{P_1+B} = \left(\frac{\rho}{\rho_1}\right)^7$

For P_1= 1 atm $\frac{\rho}{\rho_1} = 1.01$

P= $3001(1.01)^7 - 3000 = 217$ atm

1.5

At Constant Temperature

$\frac{P}{\rho_T}$ = constant $\Longrightarrow$ $\frac{P}{\rho}$ = constant

For 10% increase in ρ

P must also increase by 10 %

1.6

Since density varies as $\rho = \kappa P$

$$\rho_{250{,}000\text{ ft}} = \rho_{S.L.} \cdot \frac{P_{250{,}000\ ft}}{P_{S.L.}}$$

& ρ=nM   (M=Molecular wt.)

$$\therefore \quad n_{250{,}000} = n_{S.L.} \left[\frac{1.5\ x\ 10^{-7}}{2.378\ x\ 10^{-3}}\right] = 4 \text{ x } 10^{20} \left[\frac{1.5\ x\ 10^{-7}}{2.378\ x\ 10^{-3}}\right] = 2.5 \text{ x } 10^{16}$$

1.7

$\vec{e}_r = |\vec{e}_r|_{\mathrm{x}}\ \vec{e}_x + |\vec{e}_r|_{\mathrm{y}}\ \vec{e}_y$

$= \cos\theta\ \vec{e}_x\ + \sin\theta\ \vec{e}_r$

$\vec{e}_\theta = |\vec{e}_\theta|_{\mathrm{x}}\ \vec{e}_x + |\vec{e}_\theta|_{\mathrm{y}}\ \vec{e}_y$

$= -\sin\theta\ \vec{e}_x + \cos\theta\ \vec{e}_y$

Q.E.D.

1.8

$$\frac{d\vec{e}_r}{d\theta} = -\sin\theta \ \vec{e}_x + \cos\theta \ \vec{e}_y = \vec{e}_\theta$$

$$\frac{d\vec{e}_r}{d\theta} = -\cos\theta \ \vec{e}_x \ -\sin\theta \ \vec{e}_r = -\vec{e}_r$$

Q.E.D.

1.9 Transformation from (x,y) to (r,θ)

$$\frac{d}{dx}=\frac{dr}{dx}\frac{d}{dr}+\frac{d\theta}{dx}\frac{d}{d\theta}$$

$$\frac{d}{dy}=\frac{dr}{dy}\frac{d}{dr}+\frac{d\theta}{dy}\frac{d}{d\theta}$$

$r^2 = x^2+y^2$

$$\theta=\tan^{-1}\left({}^{y}/_{x}\right)$$

so: $\dfrac{dr}{dx}=\dfrac{x}{(x^2+y^2)^{1/2}}=\dfrac{r\cos\theta}{r}=\cos\theta$

$$\frac{d\theta}{dx}=-\frac{y}{x^2+y^2}=-\frac{r\sin\theta}{r^2}=-\frac{\sin\theta}{r}$$

$$\frac{dr}{dy}=\sin\theta \qquad \frac{d\theta}{dy}=\frac{\cos\theta}{r}$$

$$\Rightarrow \frac{d}{dx}=\cos\theta\frac{d}{dr}-\frac{\sin\theta}{r}\frac{d}{d\theta}$$

$$\frac{d}{dy}=\sin\theta\frac{d}{dr}+\frac{\cos\theta}{r}\frac{d}{d\theta}$$

1.10

$$\nabla = \frac{d}{dx}\vec{e}_x + \frac{d}{dy}\vec{e}_y + \frac{d}{dz}\vec{e}_z$$

$$= (\cos\theta \frac{d}{dr} - \frac{\sin\theta}{r}\frac{d}{d\theta})\,\vec{e}_x + (\sin\theta \frac{d}{dr} + \frac{\cos\theta}{r}\frac{d}{d\theta})\,\vec{e}_y + \frac{d}{dz}\vec{e}_z$$

$$= (\vec{e}_x \cos\theta + \vec{e}_y \sin\theta)\frac{d}{dr} + \frac{1}{r}(-\vec{e}_x \sin\theta + \vec{e}_y \cos\theta)\frac{d}{d\theta} + \vec{e}_z \frac{d}{dz}$$

Thus: $\nabla = \vec{e}_r \frac{d}{dr} + \frac{1}{r}\vec{e}_\theta \frac{d}{d\theta} + \vec{e}_z \frac{d}{dz}$

1.11

∇P= $\frac{dP}{dx}\vec{e}_x + \frac{dP}{dy}\vec{e}_y$

∇P(a,b)= $\rho_\infty \upsilon_\infty{}^2\{[\frac{1}{a}\cos 1 \sin 1 + 2]\,\vec{e}_x + \frac{1}{b}(\sin 1 \cos 1)\,\vec{e}_y\}$

= $\rho_\infty \upsilon_\infty{}^2\{[\frac{1}{a}\frac{\sin 2}{2} + 2)\,\vec{e}_x + \frac{1}{b}(\frac{\sin 2}{2})\,\vec{e}_y\}$

1.12

$$\nabla T(x,y) = T_o e^{-1/4} \left[\frac{1}{a} \left(\cos\frac{x}{a} \cosh\frac{y}{b}\right) \vec{e}_x + \frac{1}{b} \left(\sin\frac{x}{a} \sinh\frac{y}{b}\right) \vec{e}_y \right]$$

$$\nabla T(a,b) = T_o e^{-1/4} \left[\frac{1}{a} (\cos 1 \cosh 1) \vec{e}_x + \frac{1}{b} (\sin 1 \sinh 1) \vec{e}_y \right]$$

$$= T_o e^{-1/4} \left[\frac{cos1(e+e^{-1})}{2a} \vec{e}_x + \frac{sin1(e+e^{-1})}{2b} \vec{e}_y \right]$$

$$= \frac{T_o e^{-5/4}}{2} \left[\frac{cos1}{a} (1+e^{-2}) \vec{e}_x + \frac{sin1}{b} (1 - e^{-2}) \vec{e}_y \right]$$

1.13

In problem 1.12 T(x,y) is dimensionally homogeneous (D.H.)

P(x,y) in Prob 1.11 will be D.H. if

$P_\infty \sim \frac{P}{\upsilon_\infty^2}$ $L_{Bf}\ s^2/\ ft^4$

or using the conversion factor g_c

1.14 ϕ= $3x^2y+4y^2$

A scalar field is given by the function: $\emptyset = 3x^2y + 4y^2$

(a) Find $\nabla\emptyset$ at the point (3,5)

$$\emptyset = 3x^2y + 4y^2$$

$$\nabla\emptyset = \frac{\partial\emptyset_i}{\partial x} + \frac{\partial\emptyset_j}{\partial y} = (6xy)i + (3x^2 + 8y)j$$

For the value of $\nabla\emptyset$ at the point (3,5)

$$\nabla\emptyset = (6xy)i + (3x^2 + 8y)j = (6)(3)(5)i + [(3)(3)^2 + (8)(5)]j = \mathbf{90}\boldsymbol{i} + \mathbf{67}\boldsymbol{j}$$

(b) Find the component of $\nabla\emptyset$ that makes a -60∘ angle with the axis at the point (3,5)

Let the unit vector be represented by $e_s = \cos\theta_i + \sin\theta_j$

$$\nabla\emptyset \cdot e_s = [(6xy)i + (3x^2 + 8y)j] \cdot \left[\cos\theta_i + \sin\theta_j\right]$$

At the point (3,5) this becomes:

$$\nabla\emptyset \cdot e_s = [90i + 67j] \cdot [\cos(-60)i + \sin(-60)j] = 90(0.5) + (67)(-0.866) = \mathbf{-13.02}$$

1.15

For an ideal gas

P=$\frac{\rho RT}{M}$

From Prob 1.3: $\rho = \frac{\rho_m(1-x)}{1-\frac{\rho_m}{\rho_s}x}$

$\therefore \quad P = \frac{\rho_m(1-x)}{1-\frac{\rho_m}{\rho_s}x}\frac{RT}{M}$

1.16

ψ = Arsinθ(1-$\frac{a^2}{r^2}$)

a) $\nabla\,\psi = \frac{d\psi}{dr}\,\vec{e}_r + \frac{1}{r}\frac{d\psi}{d\theta}\,\vec{e}_\theta$= Asin$\theta$(1-$\frac{a^2}{r^2}$) $\vec{e}_\theta$

b) $|\nabla\,\psi\,|$= A$\left[\sin^2\theta(1+\frac{a^2}{r^2})^2 + \cos^2\theta(1-\frac{a^2}{r^2})^2\right]^{1/2}$

$|\nabla\,\psi\,|_{max}$ is given by d$|\nabla\,\psi\,|$= 0 or $\frac{d}{dr}|\nabla\,\psi\,|$dr+ $\frac{d}{d\theta}|\nabla\,\psi\,|d\theta$= 0

Requiring $\frac{d}{dr}|\nabla\,\psi\,| = \frac{d}{d\theta}|\nabla\,\psi\,|$=0

For $\frac{d}{dr}|\nabla\,\psi|$= 0: -$\sin^2\theta$(1+$\frac{a^2}{r^2}$) + $\cos^2\theta$(1- $\frac{a^2}{r^2}$) = 0 (1)

And for $\frac{d}{d\theta}|\nabla\,\psi|$=0: $sin\theta cos\theta\left[(1+\frac{a^2}{r^2})^2-(1-\frac{a^2}{r^2})^2\right]$ (2)

From Eq. 2: $sin\theta cos\theta$ $4a^2/r^2=0$

If a≠0, r≠ 0 then $sin\theta cos\theta = 0$ for which θ = 0, $\frac{\pi}{2}$ (3)

Subst. into Eq. 1 θ=0, $1-a^2/r^2=0$

Giving a = r

For θ= $\frac{\pi}{2}$ $1+ a^2/r^2 =0$ ~ impossible

Thus conditions for $|\nabla\,\psi|_{max}$ are $\theta = 0$ r = a

1.17

$$P = P_o + \frac{1}{2}\rho v_\infty{}^2\left[\frac{2xyz}{L^3} + 3\left(\frac{x}{L}\right)^2 + \frac{v_\infty t}{L}\right]$$

$$\frac{dP}{dx}\,\vec{e}_x = \frac{1}{2}\rho v_\infty{}^2\left[\frac{2yz}{L^3} + \frac{6x}{L^2}\right]\vec{e}_x$$

$$\frac{dP}{dy}\,\vec{e}_y = +\frac{1}{2}\rho v_\infty{}^2\left[\frac{2xz}{L^3}\right]\vec{e}_y$$

$$\frac{dP}{dz}\,\vec{e}_z = +\frac{1}{2}\rho v_\infty{}^2\left[\frac{2xy}{L^3}\right]\vec{e}_z$$

$$\nabla P = \frac{1}{2}\rho v_\infty{}^2\left[\left(\frac{2yz}{L^3} + \frac{6x}{L^2}\right)\vec{e}_x + \frac{2xz}{L^3}\vec{e}_y + \frac{2xy}{L^3}\vec{e}_z\right]$$

1.18

Vertical cylinder d=10m, h=6m

V= $\frac{\pi}{4}$ (10m)^2(6m)= 471.2 m^3

@ 20℃ ρ_w=998.2 kg/m^3

m= ρ_wV= (998.2)(471.2)= 470350 kg
@ 80℃ ρ_w=971.8 kg/m^3

m=(971.8)(471.2)= 457910 kg

$\Delta m =$ 12440 kg

1.19

Liquid V= 1200 cm^3 @ 1.25 MPa
V=1188 cm^3 @ 2.5 MPa

$$\beta = -V\left(\frac{dP}{dV}\right)_T \cong -V\frac{\Delta P}{\Delta V}$$

V= 1194 cm^3= 1.194 x 10^{-3} m^3

ΔV= -12 cm^3= -1.2 x 10^{-7} m^3

$$\beta = -1.194 \times 10^{-3} \left[\frac{1.25\ MPa}{-1.2x10^{-7}}\right]$$

= +12440 MPa

1.20

β= -V$\left(\frac{dP}{dV}\right)$T $\cong$ -V$\frac{\Delta P}{\Delta V}$

V=0.25 m^3

ΔV= -0.005 m^3

ΔP= 10 mPa

β= -0.25$\left[\frac{10}{-0.005}\right]$= 500 MPa

1.21

For H_20: β=2.205 GPa

$\frac{\Delta V}{V}$ = -0.0075

$\beta \cong -V\frac{\Delta P}{\Delta V}$ or $\Delta P = \beta \frac{\Delta V}{V}$

ΔP= (2.205 GPa)(0.0075) = 0.0165 GPa= 16.5 MPa

1.22

For $H_2 0$: P_1=100kPa P_2=120 MPa $\beta = 2.205$

β = -V$\frac{\Delta P}{\Delta V}$ or $\frac{\Delta V}{V} = \frac{\Delta P}{\beta}$

$\frac{\Delta V}{V} = \frac{\Delta P}{\beta} = \frac{(120000-100)kPa}{120x10^6 kPa}$ = 0.999 x 10^{-3} = 0.0999 percent

1.23

H_20@ 68℃ (341 K)

$\sigma = 0.123\ [1-0.00139(341)] = 0.0647$ N/m

In a clean tube- θ=0°

$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{2(.0647)}{979(981)\left(\frac{0.2875x10^{-2}}{2}\right)} = 9.37 \text{ x } 10^{-3}\text{m} = 9.37 \text{ mm}$$

1.24

Parallel Glass Plates

Gap=1.625 mm

σ= 0.0735 N/m

For a unit depth:

Surface Tension Force = 2(1) $\sigma\cos\theta$

Weight of H_20 = $\rho g h(1)(1.625\text{x}10^{-3})$

For clean glass cos θ=1

Equating Forces:

2(1) $\sigma = \rho g h(1)(1.625\text{x}10^{-3})$

h = $\frac{2(0.0735)}{1000(9.81)(1.625x10^{-3})}$ = 0.00922m= 9.22 mm

1.25

Glass Tube: d_i=0.25 mm
d_o=0.35 mm
θ=130°

Surface Tension Force-
Inside: $2\pi r_i \sigma\cos\theta$
Outside: $2\pi r_o \sigma\cos\theta$

Total Upward Force
$F= 2\pi\sigma\cos\theta(r_i+r_o) = 2\pi(0.44)(\cos130°)(\frac{0.25+0.35}{2}\text{x } 10^{-3}) = 5.33\text{x } 10^{-4}$ N

1.26

$H_2 0$-Air-Glass Interface @$40°C$
Tube Radius= 1 mm

h= $\frac{2\sigma cos\theta}{\rho g r}$ $\cos\theta = 1$

σ= 0.123[1-0.0139(313)]=0.0695 N/m

h = $\frac{2(0.0695)}{993(9.81)(1x10^{-3})}$ = 0.0143m (1.43 cm)

1.27

Soap Bubble- T=20$°C$ d=4mm

σ= 0.025 N/m (Table 1.2)

Force Balance for Bubble:

$\pi r^2 \Delta P = 2\pi r\, \sigma$

so $\Delta P = \frac{2\sigma}{r} = \frac{2(0.025)}{2x10^{-3}}$ = 25 N/m^2 = 25 Pa

1.28

Glass Tube in Hg (S.G.= 13.6)

For Hg/ Glass: σ=0.44 N/m

$\theta = 130°$

h= $\frac{2\sigma}{\rho g r}$ r= 3mm

= $\frac{2(0.44)}{13.6(1000)(1.5x10^{-3})}$

= -0.0277 m

= 2.77 cm Depression

1.29

@60℃ σ_{H20}= 0.0662 N/m
σ_{Hg}= 0.44 N/m

Tube Diameter= 0.55 mm

h= $\frac{2\sigma cos\theta}{\rho g r}$

For $H_2$0: $h = \frac{2(0.0662)\cos(0)}{983(9.81)\left(\frac{0.55x10^{-3}}{2}\right)}$ = 0.0499m (4.99 cm Rise)

For Hg: $h = \frac{2(0.44)(\cos 130°)}{13.6(983)(9.81)\left(\frac{0.55x10^{-3}}{2}\right)}$ = -0.0157 m (1.57 cm Depression)

1.30

H_20/ Glass Interface

T=30℃

σ = 0.123[1-0.0139(303)] = 0.0712 N/m

ρ = 996 kg/m^3

h $\leq$ 1 mm

h = $\frac{2\sigma cos\theta}{\rho gr}$ $cos\theta = 1$

r = $\frac{2\sigma}{\rho gh}$ = $\frac{2(0.0712)}{996(9.81)(1x10^{-3})}$ = 0.0146 m (1.457 cm)

d = 2r = 2.915 cm

1.31

Bubble Diameter = 0.25 cm = 0.0025 m, and so Radius = 0.00125 m
Capillary Tube: Diameter = 0.2cm = 0.002 m, and so Radius = 0.001 m

Beginning with:

$$\Delta P = \frac{2\sigma}{R}$$

Rearrange and remember the unit conversion Pa=kg/ms^2,

$$\sigma = \frac{(P - P_0)R}{2} = \frac{(101453 \text{ Pa} - 101325 \text{ Pa})(0.00125\, m)}{2} = 0.08\, Pa \cdot m = 0.08\, kg/s^2$$

Next, we can calculate the height of the fluid in the tube:

$$h = \frac{2\sigma cos\theta}{\rho g r} = \frac{2(0.08\, kg/s^2)\cos(30)}{\left(750\frac{kg}{m^3}\right)\left(9.8\frac{m}{s}\right)(0.001m)} = 0.01885m = \mathbf{1.885\ cm}$$

1.32

First, calculate the surface tension of water using the temperature of the water:

$T = 80℃ = 353K$

$\sigma = 0.123(1 - 0.00139T)$

$\sigma = 0.123\big(1 - 0.00139(353)\big) = 0.06265\ N/m$

Next, using the equation for the height of a fluid in a capillary,

$$h = \frac{2\sigma cos\theta}{\rho gr}$$

Rearranging and solving for the radius:

$$r = \frac{2\sigma cos\theta}{\rho gh}$$

$$= \frac{2(0.06265\ N/m)\cos(0)}{\left(\left(\frac{97.18\ g}{100\ mls}\right)\left(\frac{kg}{1000g}\right)\left(\frac{1000\ ml}{liter}\right)\left(\frac{28.32\ l}{0.028317\ m^3}\right)\right)(9.81\ m/s^2)(17.5mm)\left(\frac{m}{1000mm}\right)}$$

$$= 7.509x10^{-4}\ meters = 0.7509\ mm$$

Diameter is 2r so D=1.50 mm

1.32

First, calculate the surface tension of water using the equation for the height:

$$h = \frac{2\sigma cos\theta}{\rho g r}$$

Rearrange, solving for σ,

$$\sigma = \frac{h\rho g r}{2cos\theta} = \frac{(1.88\ cm)\left(\frac{m}{100\ cm}\right)(987\ kg/m^3)(9.81\ m/s^2)\left(\frac{1.5\ mm}{2}\right)\left(\frac{m}{1000\ mm}\right)}{2\cos 0}$$

$$= 0.0683\frac{kg}{s^2} = 0.0683\ N/m$$

Now use the surface tension to calculate the temperature:

$\sigma = 0.123(1 - 0.00139T)$

$0.0683\ N/m = 0.123(1 - 0.00139T)$

$T = 320.2\ K = 47.02$ ℃