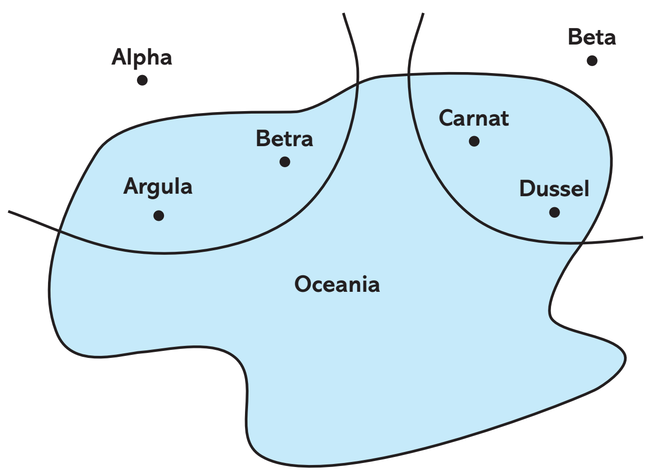
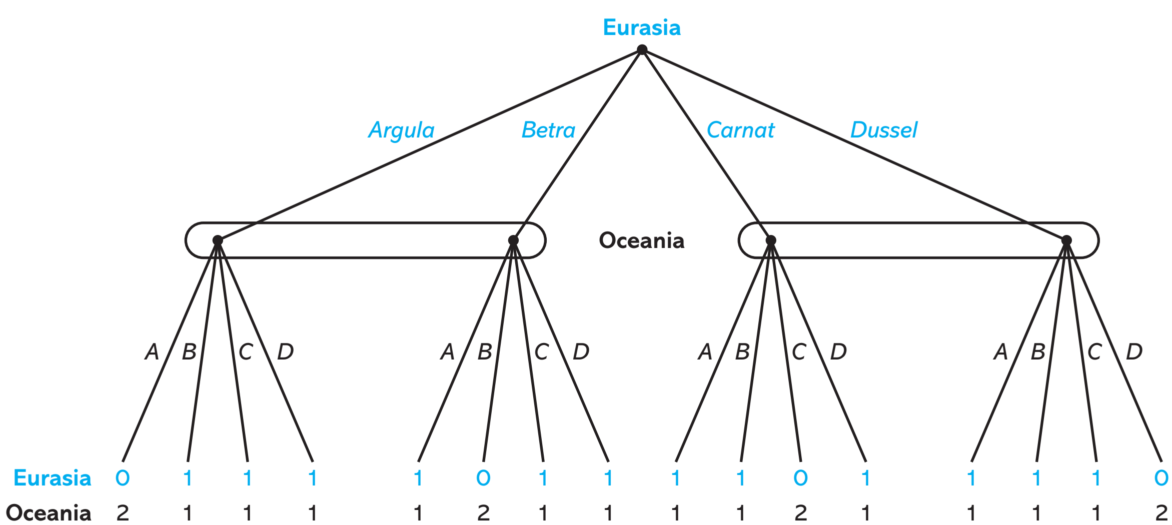
2

Building a Model of a Strategic Situation

1. The countries of Oceania and Eurasia are at war.[5] As depicted in the figure, Oceania has four cities—Argula, Betra, Carnat, and Dussel—and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra; or from base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn't know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate. Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. Write down the extensive form game.

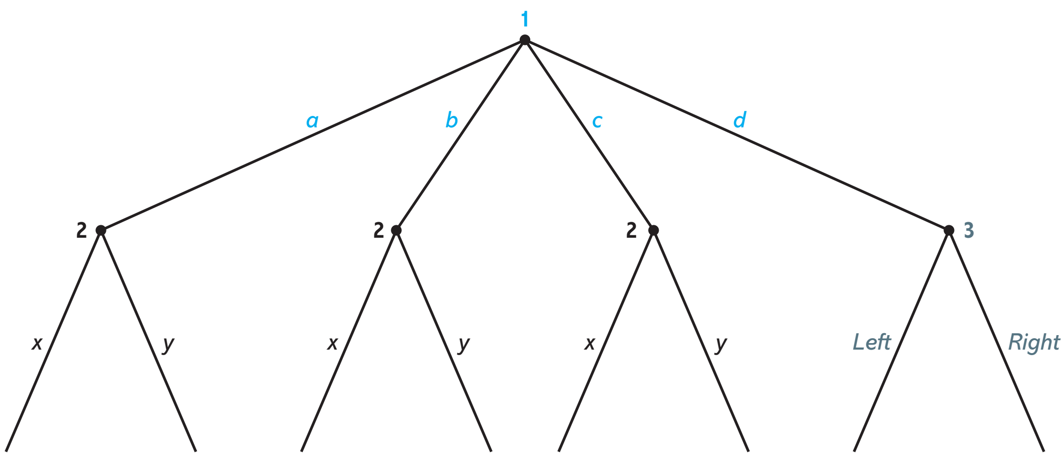


ANSWER: In the figure below, *A* stands for Argula, *B* for Betra, *C* for Carnat, and *D* for Dussel.



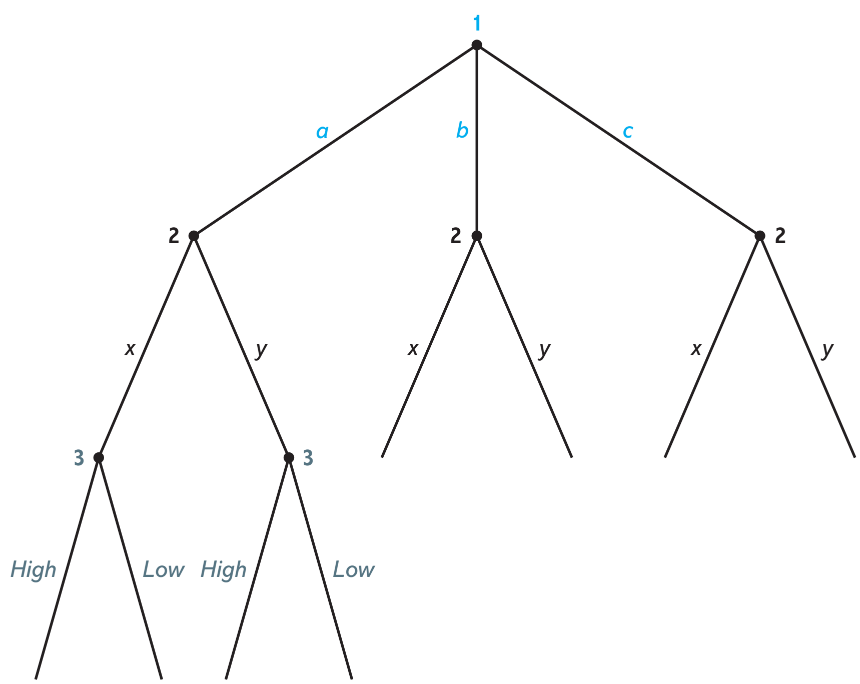
2. Player 1 moves initially by choosing among four actions: *a*, *b*, *c*, and *d*. If player 1 chose anything but *d*, then player 2 chooses between *x* and *y*. Player 2 gets to observe the choice of player 1. If player 1 chose *d*, then player 3 moves by choosing between *left* and *right*. Write down the extensive form of this setting. (You can ignore payoffs.)

ANSWER:



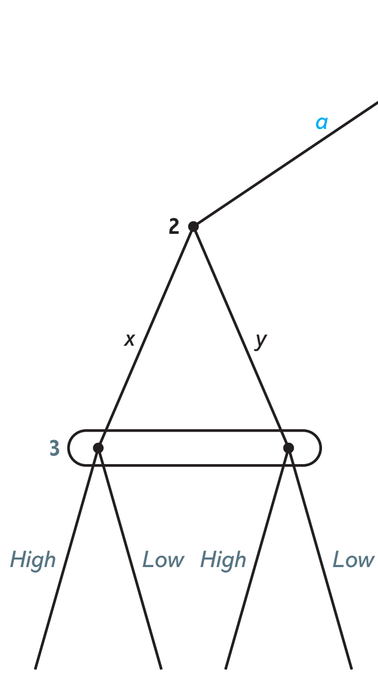
3. Consider a setting in which player 1 moves first by choosing among three actions: *a*, *b*, and *c*. After observing the choice of player 1, player 2 chooses among two actions: *x* and *y*. Consider the following three variants as to what player 3 can do and what she knows when she moves:
   a. If player 1 chose *a*, then player 3 selects among two actions: *high* and *low*. Player 3 knows player 2's choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)

ANSWER:



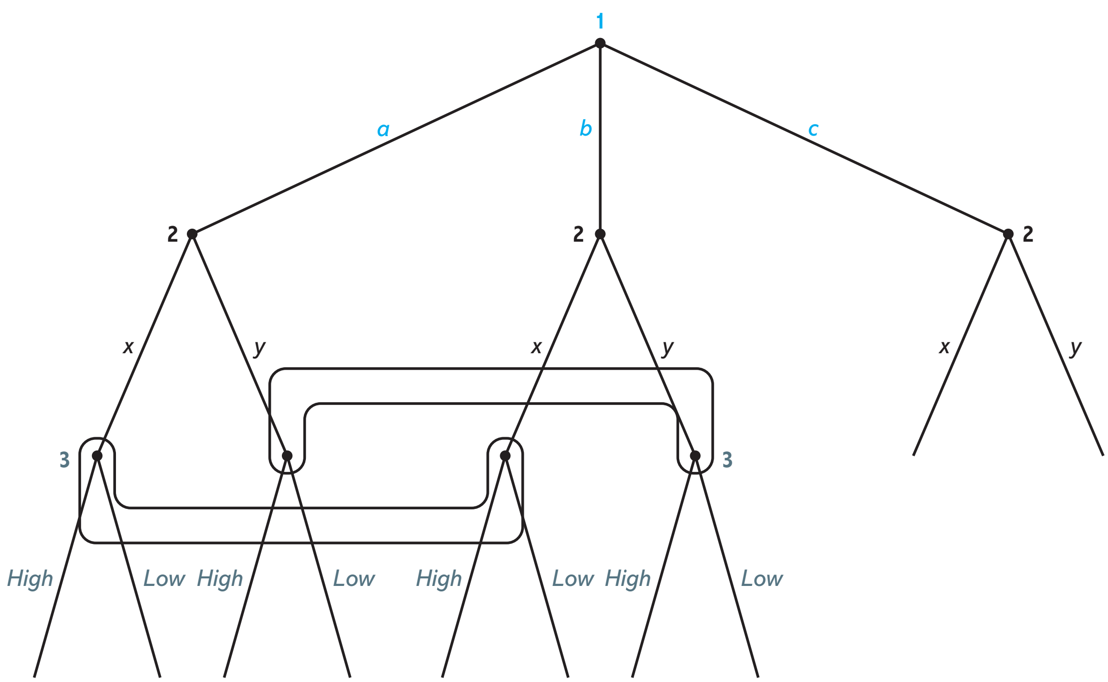
   b. If player 1 chose *a*, then player 3 selects among two actions: *high* and *low*. Player 3 does not know player 2's choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)

ANSWER:



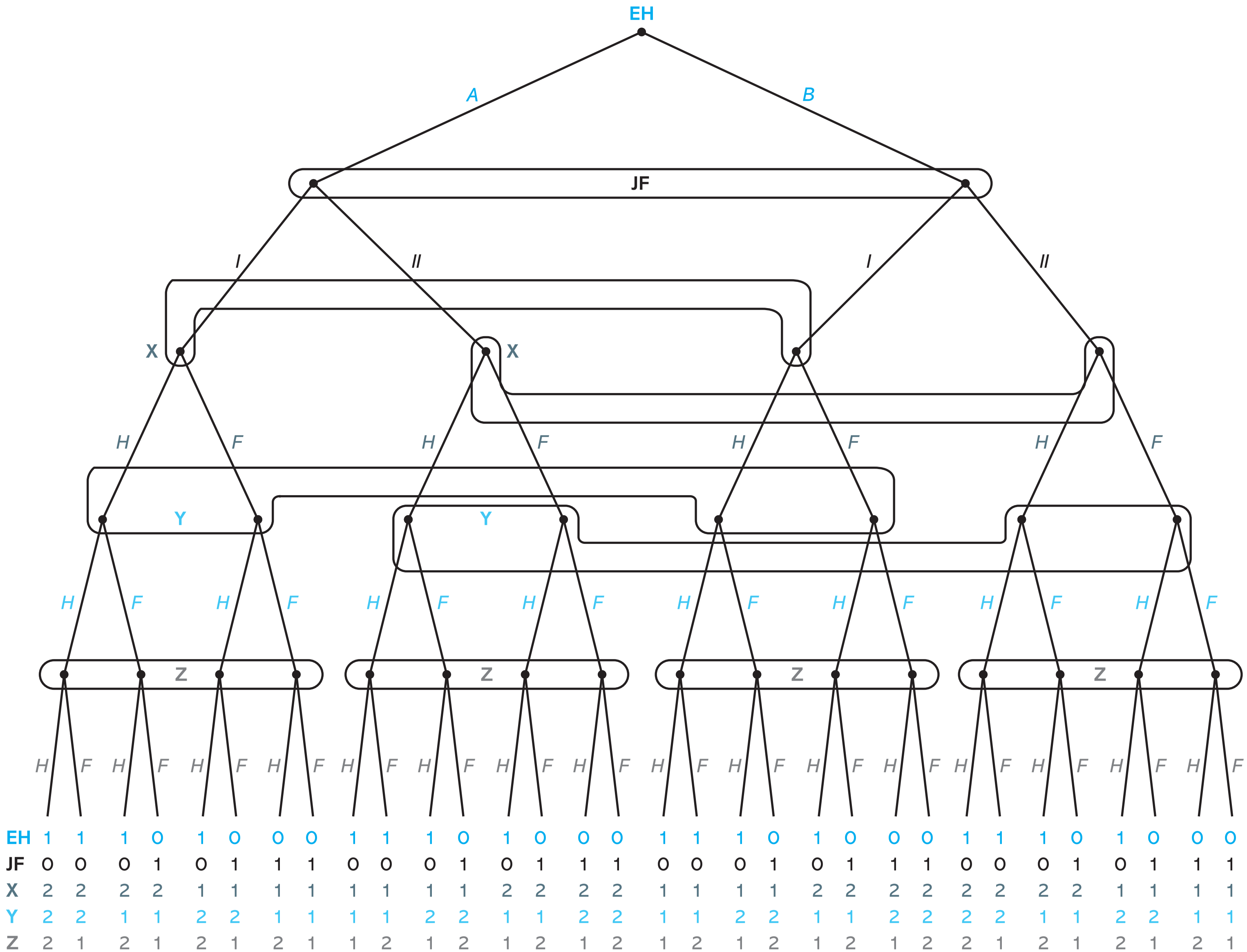
c. If player 1 chose either *a* or *b*, then player 3 selects among two actions: *high* and *low*. Player 3 observes the choice of player 2, but not that of player 1. Write down the extensive form of this setting. (You can ignore payoffs.)

ANSWER:



4. Return to the game involving the U.S. Court of Appeals in Section 2.2. Suppose, at the start of the game, it is known by all that Judge Z will read only the brief of Ms. Hasenpfeffer. Write down the corresponding extensive form game. You may exclude payoffs.

ANSWER TO 4:


EH
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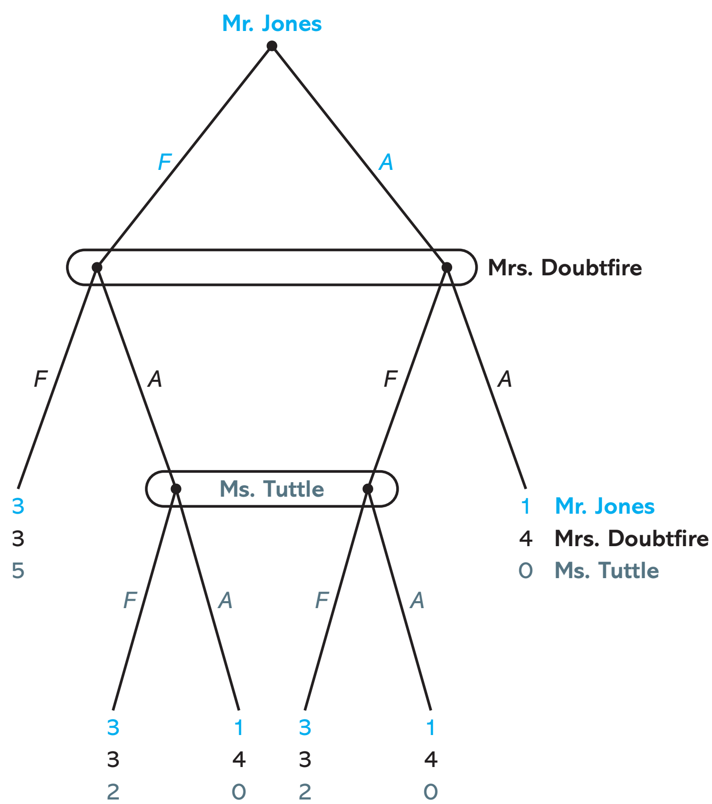
JF 0 0 0 1 0 1 1 1 0 0 0 1 0 1 1 1 0 0 0 1 0 1 1 1 0 0 0 1 0 1 1 1
X 2 2 2 2 1 1 1 1 1 1 1 1 2 2 2 2 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1
Y 2 2 1 1 2 2 1 1 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 2 2 1 1 2 2 1 1
Z 2 1 2 1 2 1 2 1 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1

5. The city council is to decide on a proposal to raise property taxes. Suppose Ms. Tuttle is the chair and the council's other two members are Mr. Jones and Mrs. Doubtfire. The voting procedure works as follows: Excluding the chair, Mr. Jones and Mrs. Doubtfire simultaneously write down their votes on slips of paper. Each writes either *for* or *against* the tax increase. The secretary of the city council then opens the slips of paper and announces the vote tally. If the secretary reports that both slips say *for*, then the tax increase is implemented and the game is over. If both vote *against*, then the tax increase is not implemented and, again, the game is over. However, if it is reported that the vote is one *for* and one *against*, then Ms. Tuttle has to vote. If she votes *for,* then the tax increase is implemented, and if she votes *against,* then it is not. In both cases, the game is then over. As to payoffs, if the tax increase is implemented, then Mrs. Doubtfire and Mr. Jones each receive a payoff of 3. If the tax increase proposal fails, then Mrs. Doubtfire has a payoff of 4 and Mr. Jones's payoff is 1. As for Ms. Tuttle, she prefers to have a tax increase—believing that it will provide the funds to improve the city's schools—but would prefer not to be on record as voting for higher taxes. Her payoff from a tax increase when her vote is not required is 5, her payoff from a tax increase when her *for* vote is required is 2, and her payoff from taxes not being increased is zero (regardless of whether or not she voted). Write down the extensive form of the game composed of Ms. Tuttle, Mr. Jones, and Mrs. Doubtfire.

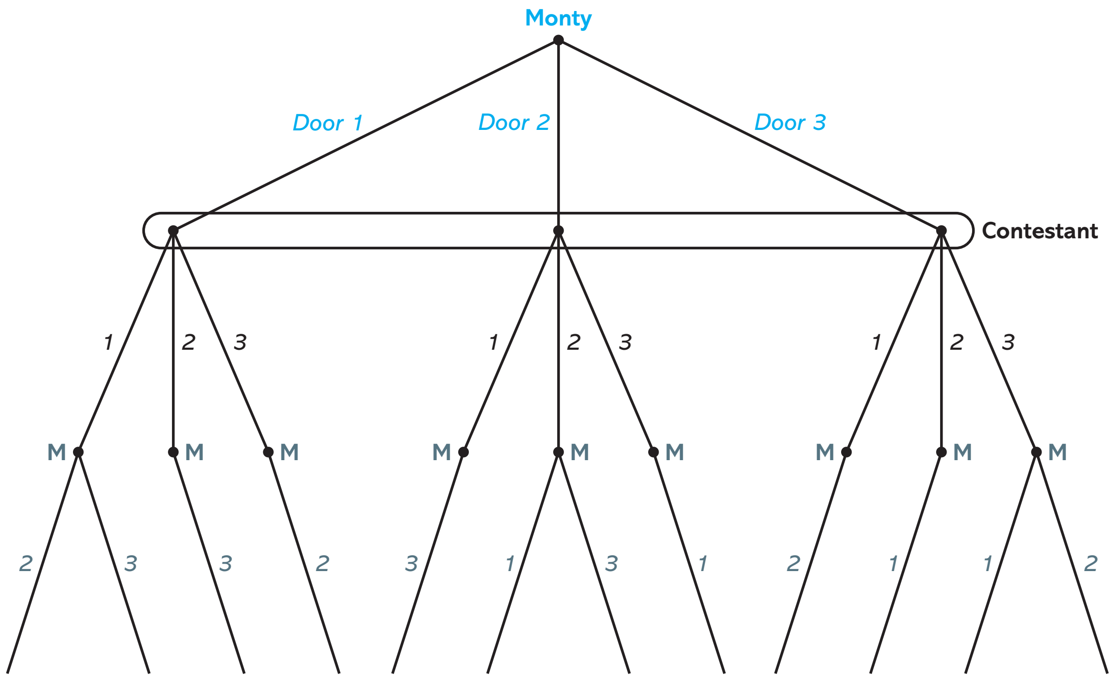
ANSWER:



6. Consider a contestant on the legendary game show *Let's Make a Deal*. There are three doors, and behind two doors is a booby prize (i.e., a prize of little value), while behind one door is a prize of considerable value, such as an automobile. The doors are labeled 1, 2, and 3. The strategic situation starts when, prior to the show, the host, Monty Hall, selects one of the three doors behind which to place the good prize. Then, during the show, a contestant selects one of the three doors. After its selection, Monty opens up one of the two doors not selected by the contestant. In opening up a door, a rule of the show is that Monty is prohibited from opening the door with the good prize. After Monty opens a door, the contestant is then given the opportunity to continue with the door originally selected or switch to the other unopened door. After the contestant's decision, the remaining two doors are opened.

a. Write down an extensive form game of *Let's Make a Deal* up to (but not including) the stage at which the contestant decides whether to maintain his original choice or switch to the other unopened door. Thus, you are to write down the extensive form for when (1) Monty Hall chooses the door behind which the good prize is placed; (2) the contestant chooses a door; and (3) Monty Hall chooses a door to open. You may exclude payoffs.

ANSWER:



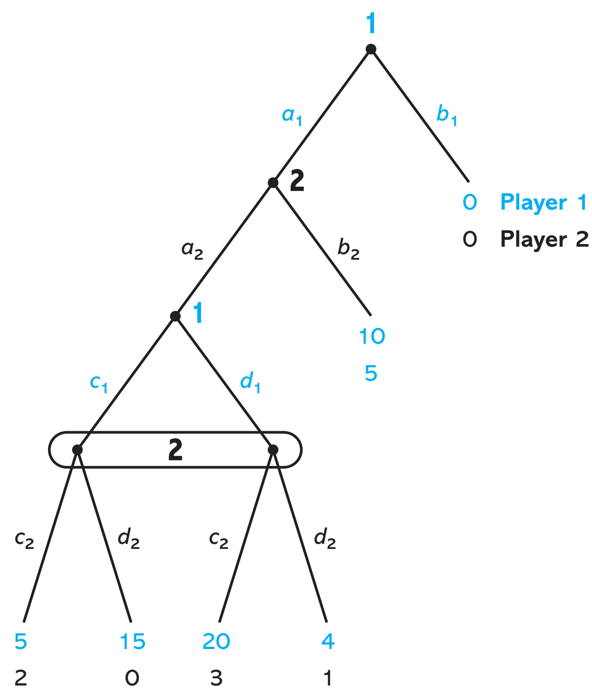
b. For the stage at which the contestant decides whether or not to switch, write down the contestant's collection of information sets. In doing so, denote a node by a triple, such as 3/2/1, which describes the sequence of play leading up to that node. 3/2/1 would mean that Monty Hall put the good prize behind door 3, the contestant initially selected door 2, and Monty Hall opened door 1.

ANSWER: There are six information sets for the contestant at the point when he has to decide whether or not to switch doors: (1) nodes 1/1/2 and 3/1/2; (2) nodes 1/1/3 and 2/1/3; (3) nodes 1/2/3 and 2/2/3; (4) nodes 1/3/2 and 3/3/2; (5) nodes 2/2/1 and 3/2/1; and (6) nodes 2/3/1 and 3/3/1. For example, the first information set comprises nodes 1/1/2 and 3/1/2. At node 1/1/2, Monty put the prize behind door 1, the contestant chose door 1, and Monty opened door 2. At node 3/1/2, Monty put the prize behind door 3, the contestant chose door 1, and Monty opened door 2. The contestant cannot discriminate between those two nodes since they entail the same sequence of observed actions—the contestant chose door 1 and Monty opened door 2—and differ only in terms of where Monty put the prize. That the information set includes both nodes 1/1/2 and 3/1/2 means that the contestant doesn't know whether the good prize is behind door 1 or door 3.

7. For the Iraq War game in Figure 2.11, write down the strategy sets for the three players.

ANSWER: Iraq has three information sets: (1) the initial node; (2) the set in which it does not have WMD and the UN requested inspections; and (3) the set in which it does have WMD and the UN requested inspections. A strategy for Iraq is then a triple of actions. At two of those information sets it has two feasible actions and at the other one it has three actions. The total number of strategies for Iraq is then 12 strategies. The United States has four information sets: (1) the UN did not request inspections; (2) the UN requested inspections and Iraq rejected the request; (3) the UN requested inspections, Iraq acquiesced to the request, and WMD were not found (that is, either Iraq doesn't have them or has them and hid them); and (4) the UN requested inspections, Iraq acquiesced to the request,, and WMD were found (that is, Iraq had them and did not hide them). The first three information sets each comprise two nodes, one corresponding to Iraq's having WMD and one to its not having WMD. The final information set is a singleton because of the implicit assumption that a UN inspection will reveal that Iraq has WMD when Iraq does not attempt to hide them. A strategy for the U.S. is then a 4-tuple of actions. Since at each of its four information sets the U.S. has two feasible actions—attacking Iraq or not attacking Iraq—the U.S. has 16 strategies. Finally, the UN has one information set. Like the U.S., it has two nodes with one corresponding to Iraq's having WMD and one not. A strategy for the UN is then a single action; its strategy set is composed of *request inspections* and *do not request inspections*.

8. Derive the corresponding strategic form for the extensive form game in the figure below.

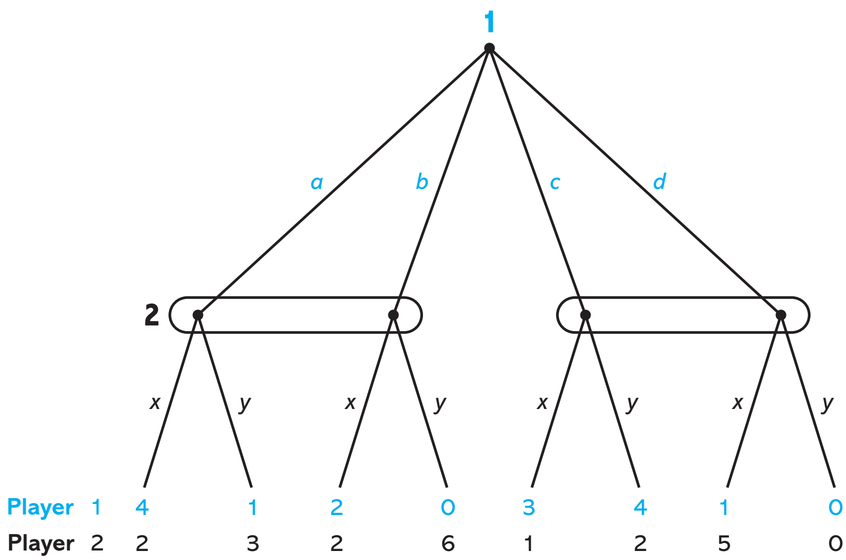


ANSWER: Player 1 has two information sets, the initial node and the information set associated with a_1 and a_2 having been played. Let x/y denote a strategy for player 1 that assigns action x to the initial node and action y to the other information set. Player 1's strategy then contains four elements: a_1/c_1, a_1/d_1, b_1/c_1, and b_1/d_1. Player 2 also has two information sets, the singleton associated with 1 having used a_1 and the information set with two nodes—one when the path is $a_1 \rightarrow a_2 \rightarrow c_1$ (read as "a_1 is chosen then a_2 is chosen then c_1 is chosen") and one when the path is $a_1 \rightarrow a_2 \rightarrow d_1$. If strategy x/y assigns action x to the first information set and action y to the second one, then player 2 has four strategies: a_2/c_2, a_2/d_2, b_2/c_2, and b_2/d_2. The payoff matrix associated with these strategies is shown in the figure below.

Player 2

Player 1		a_2/c_2	a_2/d_2	b_2/c_2	b_2/d_2
	a_1/c_1	5,2	15,0	10,5	10,5
	a_1/d_1	20,3	4,1	10,5	10,5
	b_1/c_1	0,0	0,0	0,0	0,0
	b_1/d_1	0,0	0,0	0,0	0,0

9. Write down the strategic form game for the extensive form game below.

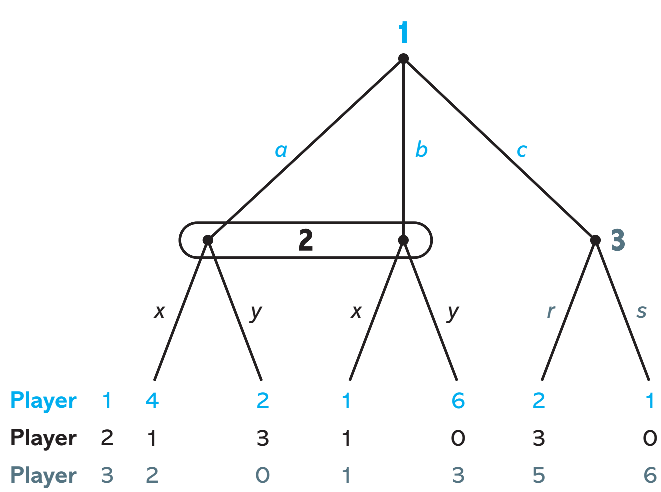


ANSWER: Player 1 has only one information set, which is the initial node. Player 2 has two information sets. Her first information set is the information set associated with player 1 having chosen either *a* or *b*. Her second information set is associated with player 1 having chosen *c* or *d*. Strategy *x/y* for player 2 assigns action *x* to the first information set and action *y* to the second information set. The strategic form game for this game is shown in the figure below.

Player 2

Player 1	*x/x*	*x/y*	*y/x*	*y/y*
a	4,2	4,2	1,3	1,3
b	2,2	2,2	0,6	0,6
c	3,1	4,2	3,1	4,2
d	1,5	0,0	1,5	0,0

10. Write down the strategic form game for the extensive form game in the game below.



ANSWER: Each player has only one information set. The strategic form game is shown in the figure below.

Player 3: *r*

Player 2

Player 1		*x*	*y*
	a	4,1,2	2,3,0
	b	1,1,1	6,0,3
	c	2,3,5	2,3,5

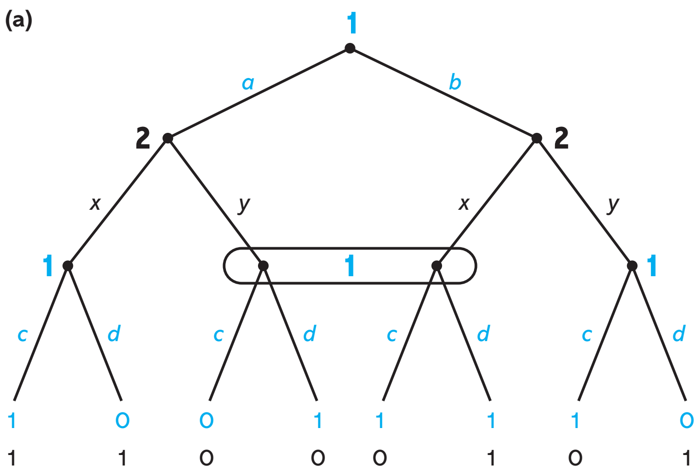
Player 3: *s*

Player 2

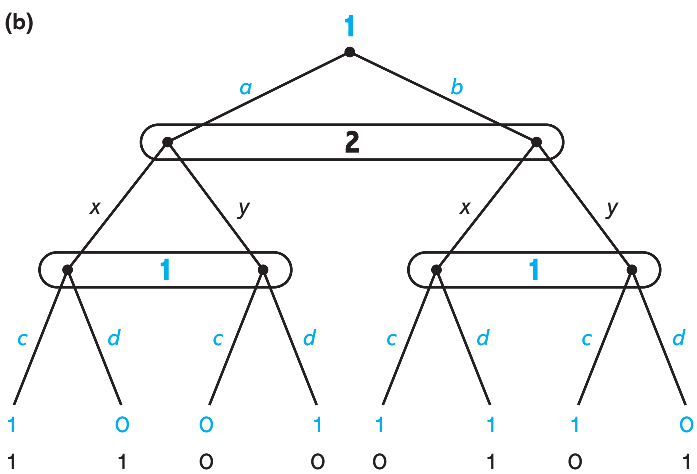
Player 1		*x*	*y*
	a	4,1,2	2,3,0
	b	1,1,1	6,0,3
	c	1,0,6	1,0,6

11. Three extensive form games are shown in the following figure. State which of them, if any, violate the assumption of perfect recall. Explain your answer.

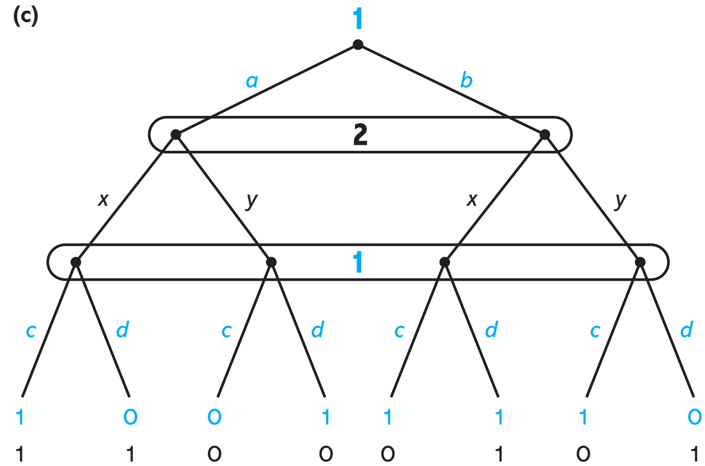
(a)



(b)



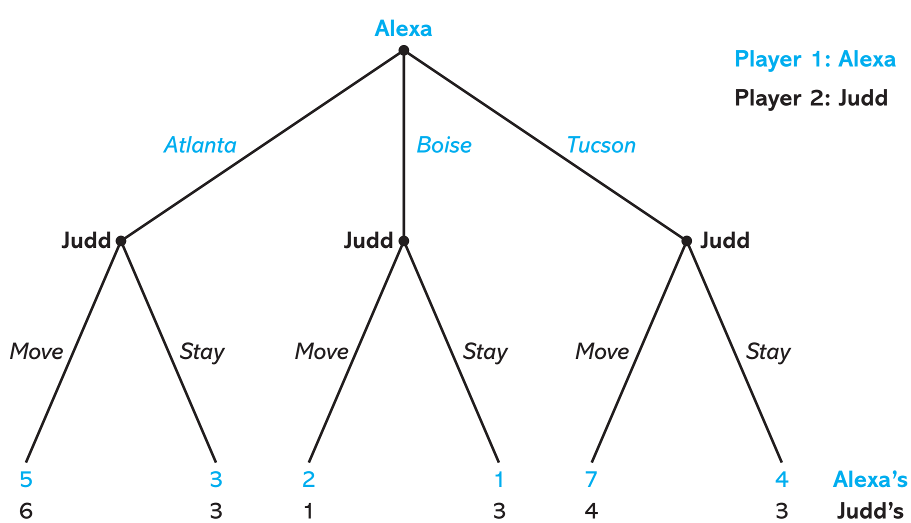
(*Continued*) **(c)**



ANSWER: Only game (b) satisfies perfect recall. In game (a), consider the information set for player 1 that includes two nodes. One node is associated with 1 having chosen a and 2 having chosen y. The other is associated with 1 having chosen b and 2 having chosen x. At this information set, 1 is then unsure whether she chose a or b. That violates perfect recall. As to game (c), the information set for player 1 which includes four nodes captures the property that, when 1 chooses between actions c and d, she doesn't know what player 2 chose (which is not in violation of perfect recall) nor what she originally chose at the initial node (which is in violation of perfect recall). Game (b) satisfies perfect recall. When 1 chooses between actions c and d, she cannot discriminate between the nodes in which play was $b \rightarrow x$ and play was $a \rightarrow y$, nor between the nodes in which play was $b \rightarrow x$ and play was $b \rightarrow y$. The former reflects 1's uncertainty over 2's action but knowledge that she originally chose a. The latter reflects 1's uncertainty over 2's action but knowledge that she originally chose b.

12. Alexa and Judd live in Boston and have been dating for about a year and are fairly serious. Alexa has been promoted to Regional Manager and been given the choice of assignments in Atlanta, Boise, and Tucson. After she makes her choice (and this is observed by Judd), he'll decide whether to stay in Boston or follow Alexa. The payoffs associated with the six possible outcomes are in the accompanying table.
    a. Derive the extensive form game.

ANSWER:



b. Derive the strategic form game.

Alexa's choice	Judd's choice	Alexa's payoff	Judd's payoff
Atlanta	Move	5	6
Atlanta	Stay	3	3
Boise	Move	2	1
Boise	Stay	1	3
Tucson	Move	7	4
Tucson	Stay	4	3

ANSWER: The strategy set of Alexa is {A,B,T}. A strategy for Judd is a 3-tuple of actions; what to do if Alexa moves to Atlanta, what to do if she moves to Boise, and what to do if she moves to Tucson. His strategy set is {M/M/M,M/M/S,M/S/M,M/S/S, S/M/M,S/M/S,S/S/M,S/S/S}.

	A	*B*	*T*
M/M/M	5,6	2,1	7,4
M/M/S	5,6	2,1	4,3
M/S/M	5,6	1,3	7,4
M/S/S	5,6	1,3	4,3
S/M/M	3,3	2,1	7,4
S/M/S	3,3	2,1	4,3
S/S/M	3,3	1,3	7,4
S/S/S	3,3	1,3	4,3

13. When he released his new novel *The Plant*, the best-selling author Stephen King chose to make early chapters downloadable for free on his website www.stephenking.com but he also asked readers to make voluntary contributions. Furthermore, he stated that he would not release subsequent chapters unless people contributed: "Remember: Pay and the story rolls. Steal and the story folds." In modeling this approach to selling a book, suppose there are just three readers: Abigail, Carrie, and Danny. All chapters have been released except for the final one, which, of course, has the climax. For Abigail or Carrie, if the final chapter is released then each receives a payoff of 5 minus how much money she contributed. For Danny, if the final chapter is released then he receives a payoff of 10 minus how much money he contributed. If the final chapter is not released then each reader receives a payoff of 2 minus how much he or she contributed. Abigail and Carrie are deciding between contributing nothing and $2. Danny is deciding between $2 and $4. For the final chapter to be released, at least $6 must be raised.

a. Assume all three readers make simultaneous contribution decisions. Write down the strategic form game.

ANSWER: The set of players is Abigail, Carrie, and Danny. The strategy set for Abigail and Carrie is {0,2}, and for Danny is {2,4}. The strategic form is shown below where the first payoff in a cell is for Abigail, the second payoff is for Carrie, and the third is for Danny.

Danny: 2

Abigail \ Carrie	*0*	*2*
0	2,2,0	2,0,0
2	0,2,0	3,3,8

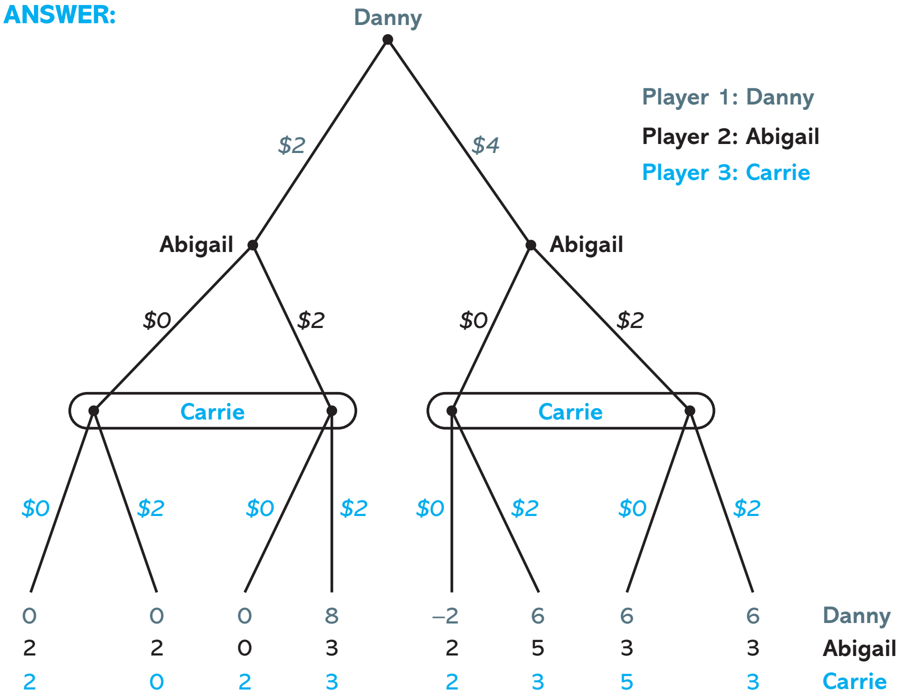
Danny: 4

Abigail \ Carrie	*0*	*2*
0	2,2,–2	5,3,6
2	3,5,6	3,3,6

Now suppose Danny contributes first, and then Abigail and Carrie make simultaneous contribution decisions after observing Danny's contribution.

b. Write down the extensive form game.

ANSWER:



c. Write down each player's strategy set.

ANSWER: A strategy for Danny is a single action and his strategy set is {2,4}. A strategy for Abigail or Carrie is a pair of actions; what to do when Danny chooses 2 and what to do when Danny chooses 4. The strategy set for Abigail and Carrie is {0/0,0/2,2/0,2/2}.

d. Write down the strategic form game.

ANSWER:

Danny: 2

Carrie

Abigail	0/0	0/2	2/0	2/2
0/0	2,2,0	2,2,0	0,2,0	0,2,0
0/2	2,2,0	2,2,0	0,2,0	0,2,0
2/0	0,2,0	0,2,0	3,3,8	3,3,8
2/2	0,2,0	0,2,0	3,3,8	3,3,8

Danny: 4

Carrie

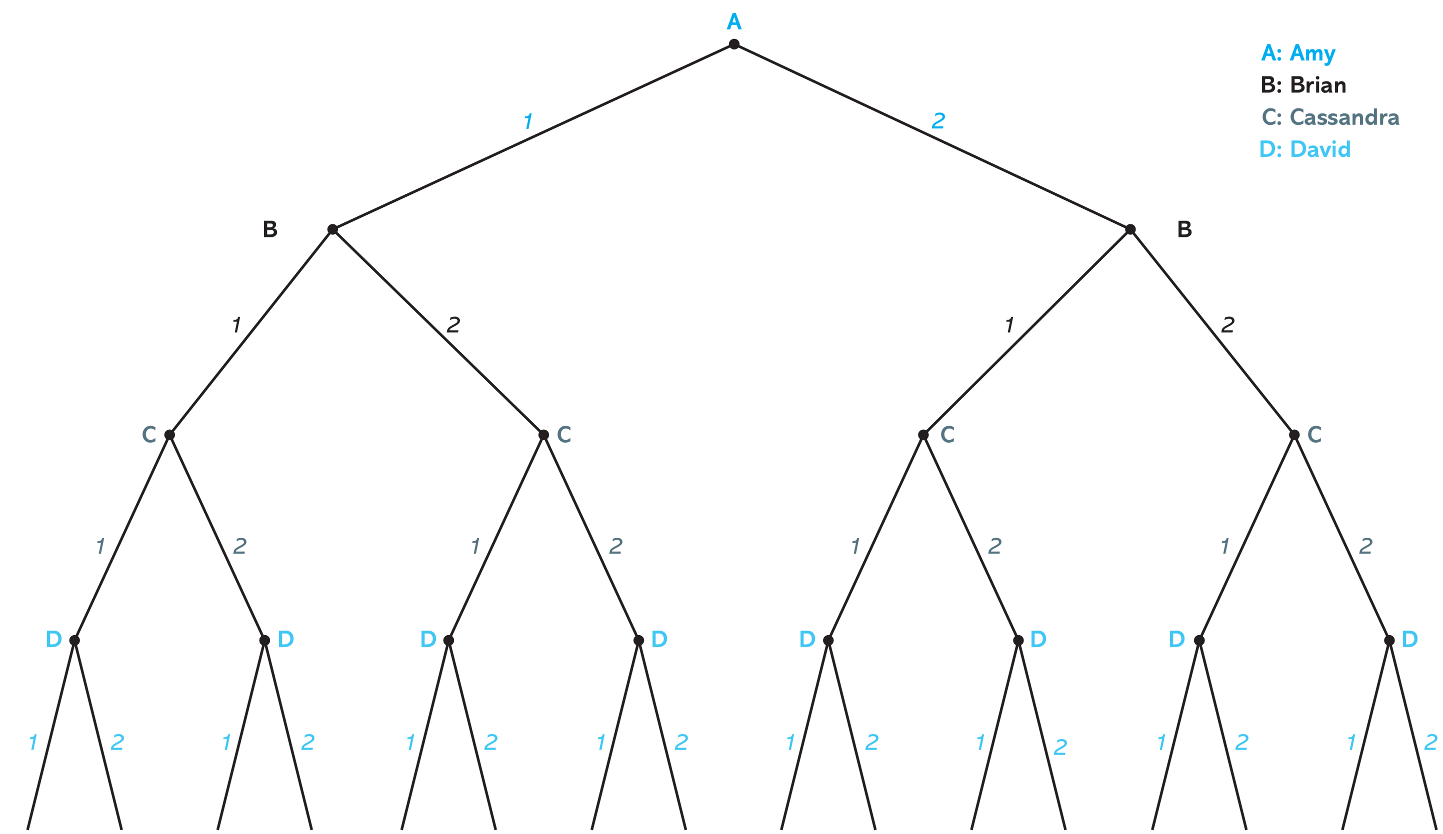
Abigail	0/0	0/2	2/0	2/2
0/0	2,2,–2	5,3,6	2,2,–2	5,3,6
0/2	3,5,6	3,3,6	3,5,6	3,3,6
2/0	2,2,–2	3,5,6	2,2,–2	5,3,6
2/2	3,5,6	3,3,6	3,5,6	3,3,6

14. There are four shoppers in a store—Amy, Brian, Cassandra, and David—who sequentially show up at two checkout lines: line number 1 and line number 2. Initially, both checkout lines are empty. Amy shows up first, sees no one in either line, and chooses one of the two lines. Brian shows up next, sees which line Amy entered, and chooses between the two lines. Next, Cassandra shows up. She sees Amy and Brian in line and chooses between the two lines. Finally, David arrives, sees Amy, Brian, and Cassandra waiting in line, and enters one of the two lines. Assume a player's payoff equals 5 minus the number of shoppers in line ahead of him or her.

a. Derive the extensive form game.

b. Now suppose that by the time that David shows up, Amy has already checked out. Thus, only Brian and Cassandra are in line. Derive the extensive form game.

ANSWER:

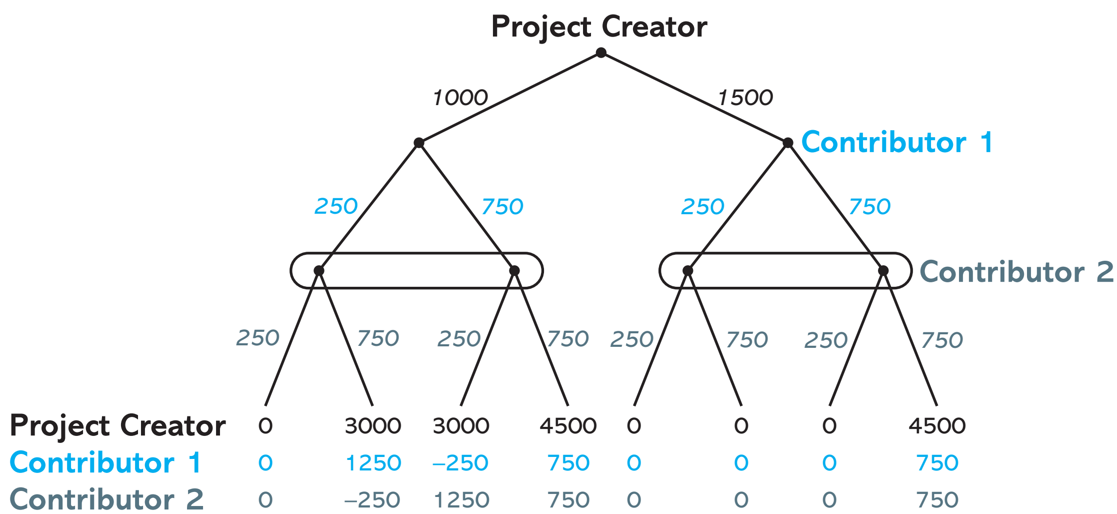


Part (a)	A	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	B	4	4	4	4	5	5	5	5	5	5	5	5	4	4	4	4
	C	3	3	5	5	4	4	4	4	4	4	4	4	5	5	3	3
	D	2	5	3	4	3	4	4	3	3	4	4	3	4	3	5	2
Part (b)	A	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	B	4	4	4	4	5	5	5	5	5	5	5	5	4	4	4	4
	C	3	3	5	5	4	4	4	4	4	4	4	4	5	5	3	3
	D	3	5	4	4	4	4	5	3	3	5	4	4	4	4	5	3

15. Kickstarter (www.kickstarter.com) provides a platform for raising venture capital through crowdsourcing. A project creator sets a funding target and posts the project at Kickstarter. People then decide how much money to pledge. If the total pledges are at least as great as the funding target, then the pledges are converted into contributions and the project is funded. Though the contributors do not own a share of the project, they can receive rewards from the project creator. If the pledges fall short, then the project is not funded. Assume there are three players: one project creator and two potential contributors. The project creator chooses between a funding target of $1,000 and $1,500. With the funding target posted at Kickstarter, the two contributors simultaneously decide whether to pledge $250 or $750. Assume the project creator's payoff equals three times the amount of funding (which is zero if contributions are less than the funding target). A contributor's payoff is zero when the project is not funded (irrespective of the pledge made), and is two times the total amount of pledges minus three times the contributor's own pledge when it is funded.

a. Write down the extensive form game.

ANSWER:



b. Write down each player's strategy set.

ANSWER: The project creator's strategy is a target level of funding target and the strategy set is {1000,1500}. A contributor's strategy is a pair of actions that specifies an amount of pledge for each funding target chosen by the project creator. A contributor' strategy set is {250/250,250/750,750/250,750/750}.

c. Write down the strategic form game.

ANSWER:

Project creator: 1000

Contributor #2

Contributor #1	250/250	250/750	750/250	750/750
250/250	0,0,0	0,0,0	1250,–250,3000	1250,–250,3000
250/750	0,0,0	0,0,0	1250,–250,3000	1250,–250,3000
750/250	–250,1250,3000	–250,1250,3000	750,750,4500	750,750,4500
750/750	–250,1250,3000	–250,1250,3000	750,750,4500	750,750,4500

(*Continued*)

Project creator: 1500

Contributor #2

Contributor #1	250/250	250/750	750/250	750/750
250/250	0,0,0	0,0,0	0,0,0	0,0,0
250/750	0,0,0	750,750,4500	0,0,0	750,750,4500
750/250	0,0,0	0,0,0	0,0,0	0,0,0
750/750	0,0,0	750,750,4500	0,0,0	750,750,4500

16. Consider drivers who commonly traverse a major highway. Each driver is deciding whether to buy E-ZPass. E-ZPass electronically charges a driver for going through a toll, which avoids having to stop and hand over money. E-ZPass costs \$4 and allows a driver to go through the E-ZPass lane. Without E-ZPass, a driver goes through the Cash lane. With either lane, the toll is \$6. The average time it takes for a car to get through the E-ZPass line is 10 seconds multiplied by the number of cars in the E-ZPass lane (which is assumed to equal the number of cars with E-ZPass). For the Cash lane, the average time it takes for a car to get through is 30 seconds multiplied by the number of cars in the Cash lane (which is assumed to equal the number of cars without E-ZPass). The value of a driver's time is 30 cents per minute. Assume there are 100 drivers, each of whom has a payoff equal to 20 minus the value of time spent in line minus expenditure (the latter is \$4 without E-ZPass and \$10 with E-ZPass). Drivers make simultaneous decisions about whether or not to buy E-ZPass.

a. The strategy set for a driver is (*E-ZPass, No E-ZPass*). Derive a driver's payoff function depending on his choice and the choices of the other 99 drivers.

ANSWER: Let m denote the number of other drivers that choose E-ZPass. A driver's payoff from buying E-ZPass is $20 - .3(\frac{m+1}{6}) - 10$, and from not buying E-ZPass is $20 - .3(\frac{100-m}{2}) - 6$

b. Now suppose a driver with E-ZPass can use either lane. Assume that it takes the same amount of time to go through the Cash lane whether a driver has E-ZPass or not. Drivers without E-ZPass can still go through the Cash-only lane. The strategy set for a driver is (*E-ZPass & E-ZPass lane, E-ZPass & Cash lane, No E-ZPass & Cash lane*). Derive a driver's payoff function, depending on her choice and the choices of the other 99 drivers.

ANSWER: Let m denote the number of other drivers that choose E-ZPass & E-ZPass lane and n denote the number of other drivers that choose E-ZPass & Cash lane. A driver's payoff from E-ZPass & E-ZPass lane is $20 - .3(\frac{m+1}{6}) - 10$, from E-ZPass & Cash lane is $20 - .3(\frac{100-m}{6}) - 10$, and from not buying E-ZPass is $20 - .3(\frac{100-m}{2}) - 6$

Chapter 2: Building a Model of a Strategic Situation

Notes to the Instructor to accompany
Games, Strategies, and Decision Making, Second Edition
by Joseph E. Harrington, Jr.

I. Ideas for Class Discussion

Model Building

- Start by describing a situation—that is, develop a narrative—and then talk about how to distill key elements from that narrative for the purpose of constructing a model.
- Describe a situation and then develop an extensive form of representation, but leave out the payoffs. At that point, ask: What more is needed to make predictions about behavior? Answer: We need a description of what it is that the person in the situation described cares about, which requires specifying payoffs or preferences.
- Describe alternative preferences for the kidnapper in the Kidnapping game based on the character Sam Stone in the film 1986 film *Ruthless People*.
  - Sam Stone, a businessman in the clothing industry, married his wife, Barbara, for her money and now plans to kill her. However, when he gets home to commit the murder he receives a call from some people who have kidnapped her. They demand a ransom and threaten to kill Barbara if he informs the police, which he happily does in the hope that the kidnappers will stick to their promise and kill her so he can inherit her money.
  - What are Sam Stone's preferences? Ordering the outcomes from most to the least preferred, his preferences are: Kidnap–No Ransom–Kill, Kidnap–Pay Ransom–Kill, Do Not Kidnap, Kidnap–No Ransom–Release, and Kidnap–Pay Ransom–Release.

Thoughts Related to the Concept of a Strategy

- Write on the board: "Victorious warriors win first and then go to war, while defeated warriors go to war first and then seek to win." Then ask the class: What do you think this quotation means? Relate it to the way a strategy is conceived in game theory.
- In principle, a strategy—as a completely described decision rule—could be written down as a set of instructions and given to another player to use or could be programmed so that a computer could play it. In this sense, a strategy is "cognitively penetrable" in that a player is conscious of the strategy he or she is using.
- Intelligence and judgment are used in selecting a strategy, but those skills are not needed to execute that strategy. Given a strategy, playing a game is just a matter of following a set of instructions.
- Timing of the selection of a strategy is important in that each player chooses his or her strategy before the game begins. The outcome of the game follows from the strategy profile. What is interesting in a game is related to the selection of strategies.

Cardinality of the Strategy Set and the Abstract Nature of a Strategy

- On abstraction**.** All skill lies in choosing a strategy because once the strategy is selected using it just means following step-by-step instructions. This is clearly an abstraction, however, in that people do reason over the course of complicated games as it may be too complicated (and take too long) to figure out before playing the game what to do under all contingencies. Though this abstraction sweeps a lot of complexity under the rug, it is a highly useful abstraction. Ultimately, the theory will be judged by how well it predicts or how successful are its recommendations.
- On cardinality**.** Let's count how many strategies there are in some games. Consider a simple extensive form game of perfect information in which player 1 moves first and selects among k possible actions. After observing what player 1 did, player 2 then chooses among n possible actions. For example, suppose $k = 3$ and n = 2. There are then three strategies in player 1's strategy set corresponding to the three feasible actions, which we'll denote *a, b,* and *c.* Player 2 has two actions, denoted x and y, and three information sets, which correspond to the three possible things that player 1 could have done. A strategy for player 2 is then a triple of actions. Let a triple—such as *xyx*—denote player 2's strategy when she chooses the first action in that triple when at information set *a,* the second action when at information set *b,* and the third action when at information set *c.* Player 2 then has eight strategies:

$$xxx, xyx, xxy, xyy, yxx, yxy, yyx, yyy.$$

This strategy set was derived by considering all the ways in which to create a triple from the two elements x and y. Note that the number of strategies for player 2 (which is 8) equals the number of actions available to player 2 raised to a power equal to the number of information sets (which is 3). When a player has k information sets and n possible actions (at each of those information sets), then that player has n^k strategies. Here's the proof. Denote the actions for player 1 as $\{a_1, a_2, ..., a_k\}$ and the available actions for player 2 as $\{x_1, x_2, ..., x_n\}$. A strategy template for this player is then: If a_1 then ___, if a_2 then ___, ... , if a_k then ___; and each of those blanks is filled with an element from $\{x_1, x_2, ..., x_n\}$. There are then n possible actions one can put in the first blank (which is what to do after player 1 chose a_1). For each of those n values, there are n actions one can put in the second blank (which is what to do after player 1 chose a_2). That means there are n (or n^2) strategies that differ in the first two blanks. For each of those n^2 strategies, there are n actions that can be placed in the third blank (which is what to do after player 1 chose a_3). That means there are $n^2 \times n$ (or n^3) strategies that differ in the first three blanks. Continuing by this logic there are then n^k strategies that differ in terms of all k blanks. Hence, if player 1 has k information sets and n feasible actions at each of them, then there are n^k strategies in player 2's strategy set.

So why do we go through this monotonous multiplication? It is to make the point that it doesn't take many actions before player 2 is sifting through a lot of strategies trying to figure out which one to use. For example, if player 1 has 5 actions and player 2 has 5 actions, then player 2 has 5^5 or 3,125 strategies. If player 1 has 10 actions and player 2 has 10 actions, then player 2 has 10^{10} or 10 million strategies! Being a lover of large numbers, we've only begun to boggle the mind.

Suppose the two players were to play a game of chess but, to keep the calculation manageable, each is allowed to make only one move. White moves first and has 20 possible moves as there are two ways to move each of his eight pawns and two ways to move each of his two knights. Black moves second and similarly has 20 moves. By our formula, Black then has 20^{20} strategies, which is on the order of one hundred septillion. How many strategies does each player have in a game of chess that can go for a maximum of 40 moves? Well, a googol (the name of the famous search engine is a respelling of this term) is 1 followed by 100 zeros, or 10^{100}, and the number of strategies in 40-move chess is a googol multiplied by itself 10 times or $(10^{100})^{10}$.

So have we just made the case that what we're doing is absurd? Fortunately, no. One doesn't have to believe that a person goes through every strategy; indeed, many strategies are absurd and can easily be dismissed. Furthermore, we don't really believe that real people do all their thinking before they play. They do a lot of learning and rethinking as they play the game or as they play it again and again. When it comes to modeling behavior, our interest is in what people do after that learning has settled down. So, one can think of the assumption that a player considers all strategies before the start of a game as an abstraction for all sorts of real-time learning. This will prove to be an immensely productive abstraction that will allow us to draw a lot of interesting, subtle, and often enlightening conclusions.

Ordinality of the Payoffs

- Emphasize the ordinality of payoffs. What matters is not whether they are positive or negative, but their ordering. Because students often see zero as a special quantity, it is worth emphasizing that zero as a payoff has no intrinsic significance. Adding a constant to each payoff has no effect on the choices that are eventually made (according to the usual solution concepts). Multiplying each payoff by a positive constant similarly has no effect. In fact, any transformation that leaves the ordering unaltered will not affect the conclusions we draw.

Anecdotes on Common Knowledge

- A literary example of failure of common knowledge is O. Henry's short story *The Gift of the Magi*. A poor husband and wife shop for Christmas presents for each other. The wife finds a chain for her husband's prized pocket watch, while the husband finds a comb his wife can use for her long, beautiful hair. But the husband sells his watch in order to buy the comb, and the wife sells her hair in order to buy the chain, so both gifts are useless in practical terms (though not in terms of the love they bear each other).

II. Games to Play in Class

NONE

III. Multimedia Presentation

Show a video clip of a strategic situation and ask: How do we model this situation? A good source for clips is at www.gametheory.net/popular. Some suggestions include the following:

- The truel in the 1966 western film *The Good, the Bad and the Ugly* starring Clint Eastwood.
- The bar scene in the 2001 movie *A Beautiful Mind,* which is based on the life of the troubled genius John Nash (portrayed by Russell Crowe.)
- A clip from the show *FRIENDS* in which Chandler and Monica plot to play a prank on their friends who suffer from a lack of common knowledge.
- The scene from the film *Ransom* in which Tom Mullen (played by Mel Gibson) announces during a news broadcast that he has converted the ransom money into a bounty on the kidnapper. (A description of the maneuver is provided in Chapter 2, Section 2.9 of the text.)

Games, Strategies, and Decision Making by Joseph E. Harrington 2nd Edition

Chapter 2: Building a Model of a Strategic Interaction

Prepared by

Philip Heap

2.1 Introduction

- Our goal in this chapter is to learn how to build a model of strategic situations.

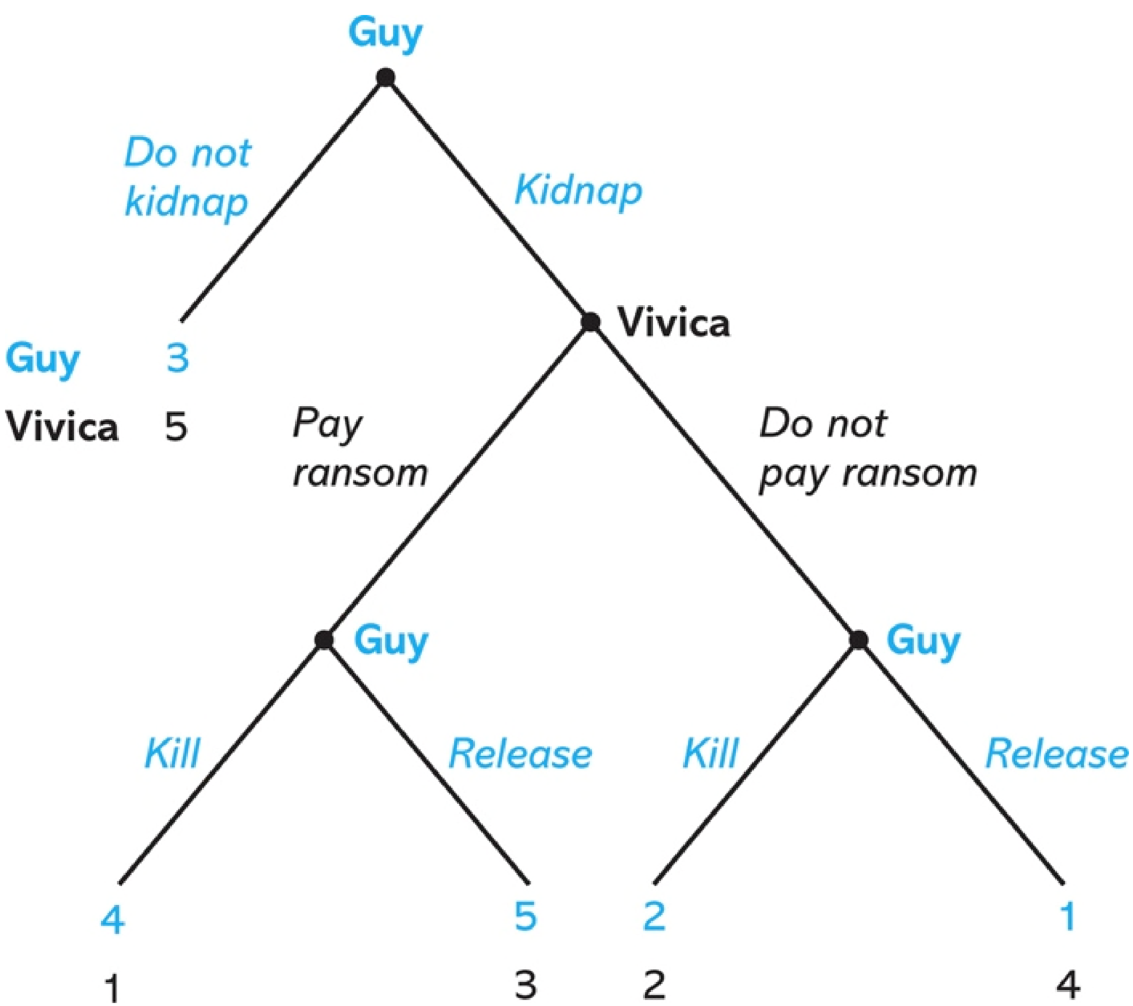
- Most of the models are metaphorical rather than literal.

- Two primary modeling methods: *extensive form* and *strategic form*.

2.2 Extensive Form Games: Perfect Information

- Modeling a Kidnapping game
  - Guy must decide whether to kidnap Orlando (Vivica’s husband) or not kidnap Orlando.
  - If Guy kidnaps Orlando, Vivica must decide whether to pay a ransom or not pay a ransom.
  - Guy may then kill Orlando or release him.

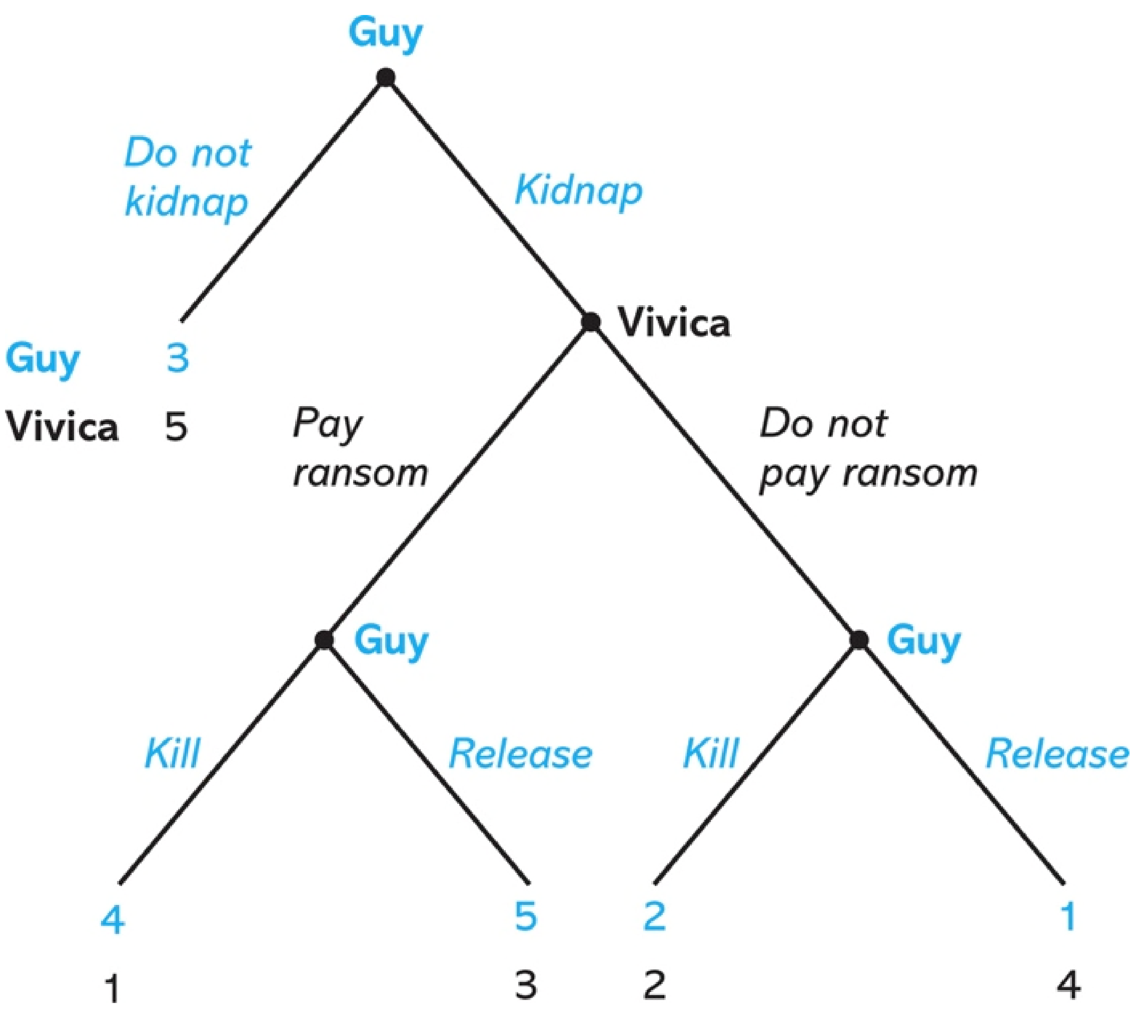
A Decision Tree for the Kidnapping Game



The dots are decision nodes, which are a point in the game when someone makes a decision.

The first node at the top of the tree, (its "root"), is called the initial node.

A Decision Tree for the Kidnapping Game



The branches coming from each node represent the different actions available to the decision maker.

The outcomes of the game correspond to the different paths through the game tree.

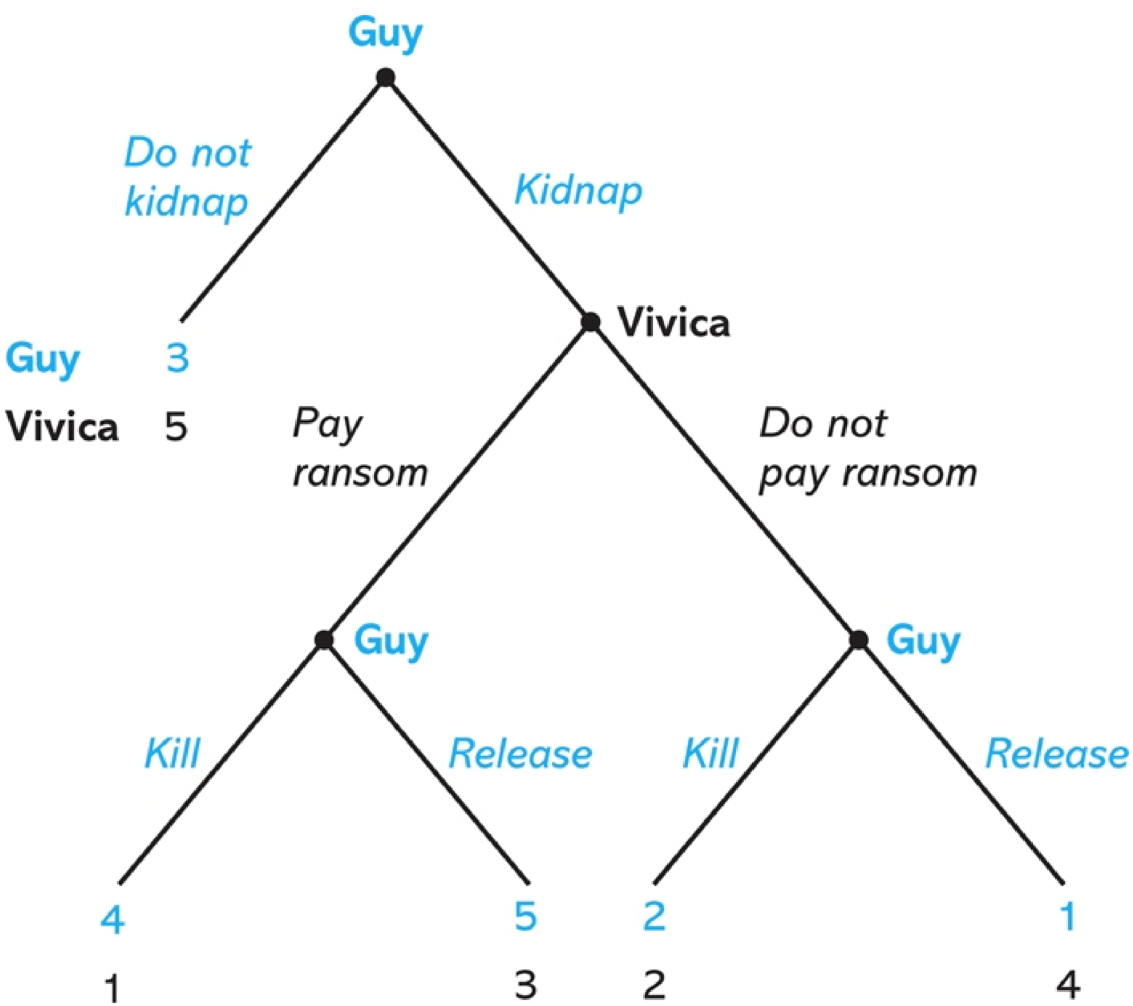
Payoffs for the Kidnapping Game

To make some predictions about how Guy and Vivica will behave, we need to say something about their preferences.

Outcome	Guy	(Violent) Guy	Vivica
No kidnapping	3	3	5
Kidnapping, ransom is paid, Orlando is killed	4	5	1
Kidnapping, ransom is paid, Orlando is released	5	4	3
Kidnapping, ransom is not paid, Orlando is killed	2	2	2
Kidnapping, ransom is not paid, Orlando is released	1	1	4

The numbers in the table represent the payoff or utility Guy and Vivica receive for each outcome.

A Decision Tree for the Kidnapping Game



The payoffs are placed at the terminal node.

Note that in this game, each node is preceded by exactly one node.

Therefore, there is only one sequence of actions that brings the game to any terminal node.

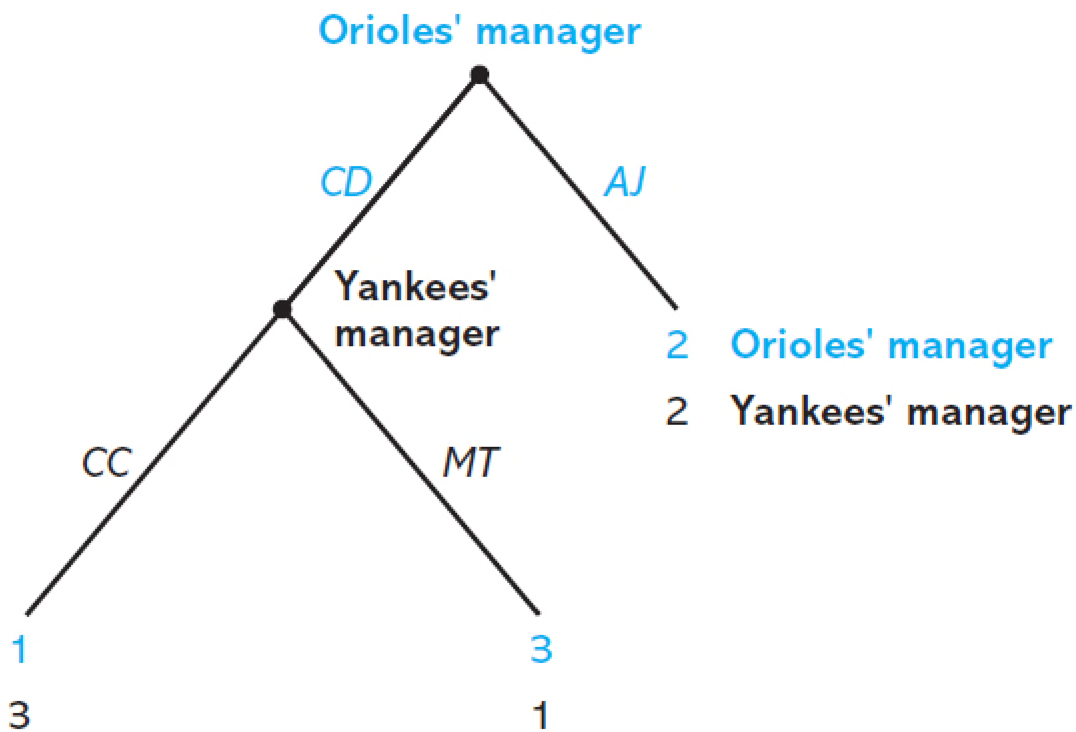
Baseball, I: The Orioles and the Yankees

- How does a baseball manager decide when to substitute a batter or pitcher?

- Statistics show that right-handed (left-handed) batters do better against left-handed (right-handed) pitchers, as shown below.

Batter	Pitcher	Batting Average
Right	Right	.255
Right	Left	.274
Left	Right	.291
Left	Left	.266

Baseball, I: The Orioles and the Yankees

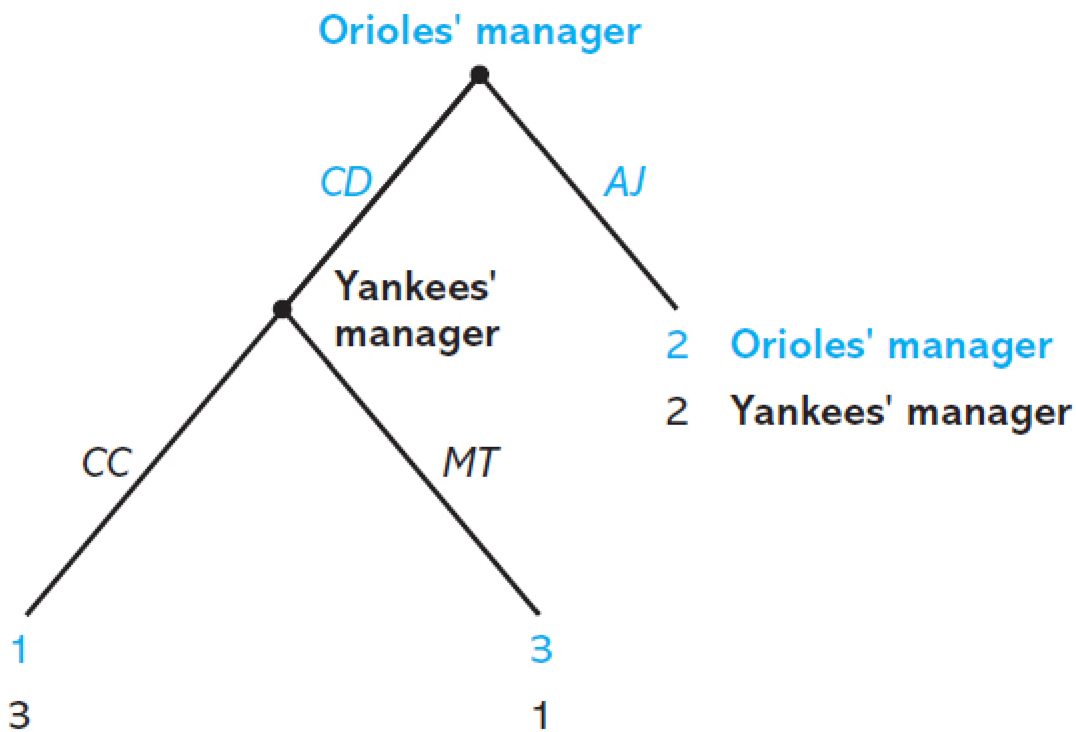


It's the bottom of the ninth, and game is tied.

Mashiro Tanaka (MT) is on the mound for the Yankees.

The Orioles manager can leave Adam Jones (AJ) in or bring in Chris Davis (CD).

Baseball, I: The Orioles and the Yankees



The Yankees manager could then respond to Chris Davis by bringing in Cesar Cabral (CC) to pitch or keeping Mashiro Tanaka.

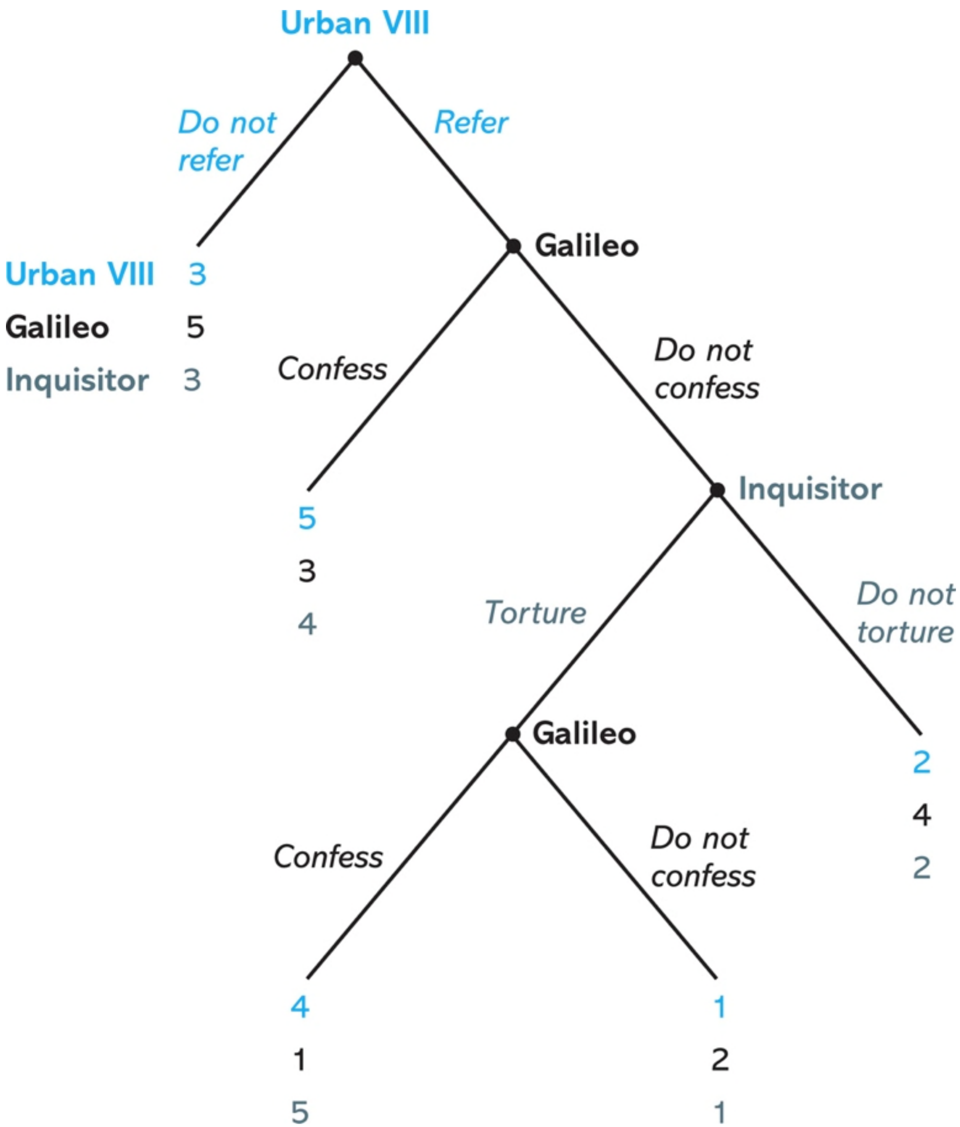
Since each manager cares about winning, what is best for one manager is worst for the other.

Galileo and the Inquisition, I

- In 1633, Galileo was under consideration for interrogation by the Inquisition for teaching the Copernican theory that the earth revolves around the sun.

- Three players:
  - Pope Urban VIII
  - Galileo
  - and the Inquisitors.

Galileo and the Inquisition, I

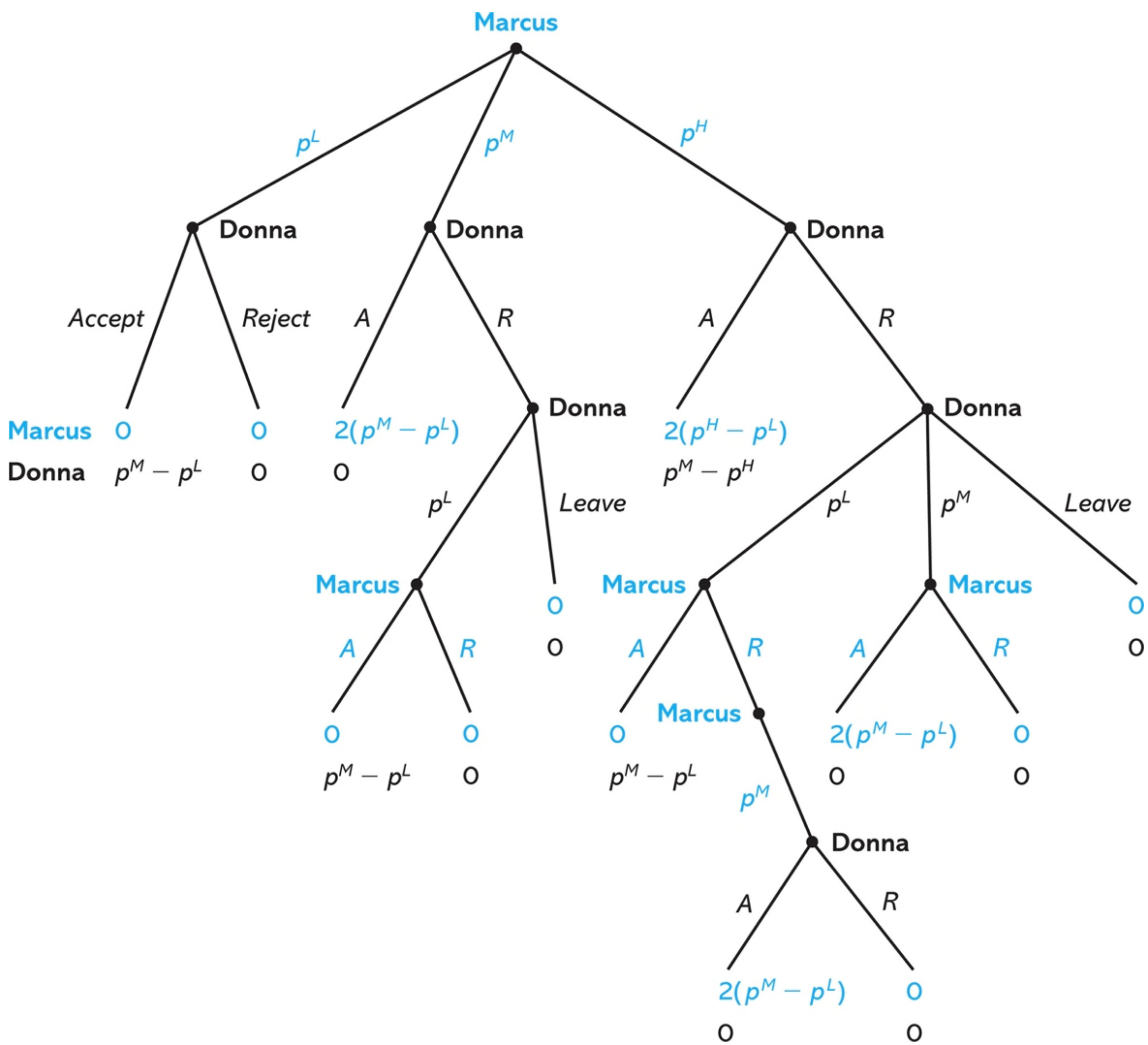


What do the payoffs tell us about each player's preferences?

Haggling at an Auto Dealership, I

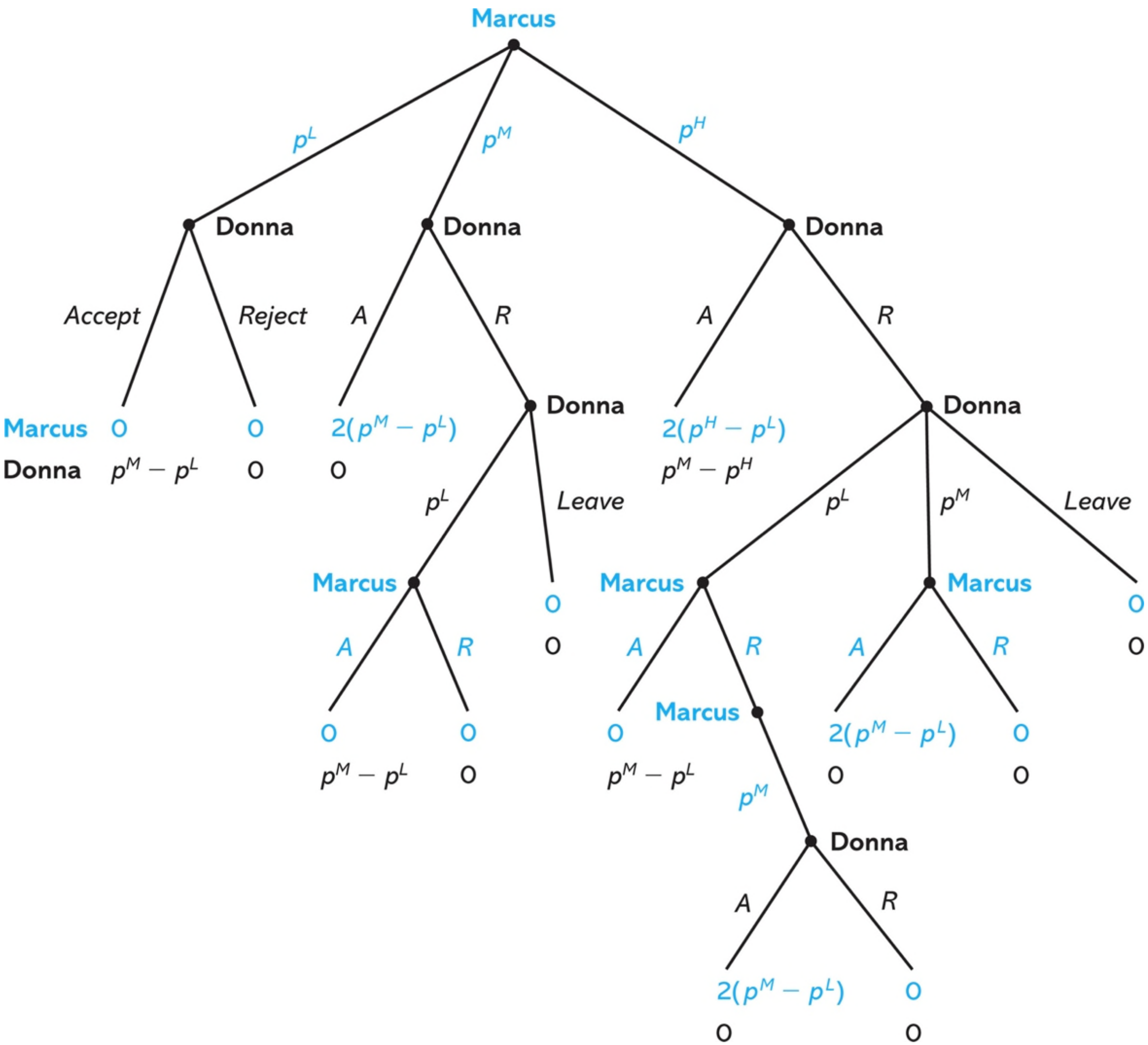
- Two players: Donna (buyer) and Marcus (salesman).
- Marcus offers one of three prices to Donna: $p^H > p^M > p^L$
- Donna can respond by accepting or rejecting. If she rejects, she can leave or make a counteroffer.
- Marcus can accept or reject Donna's counteroffer. If he rejects it, he makes a counteroffer.
- Haggling continues until Donna leaves or an offer is accepted.

Haggling at an Auto Dealership, I



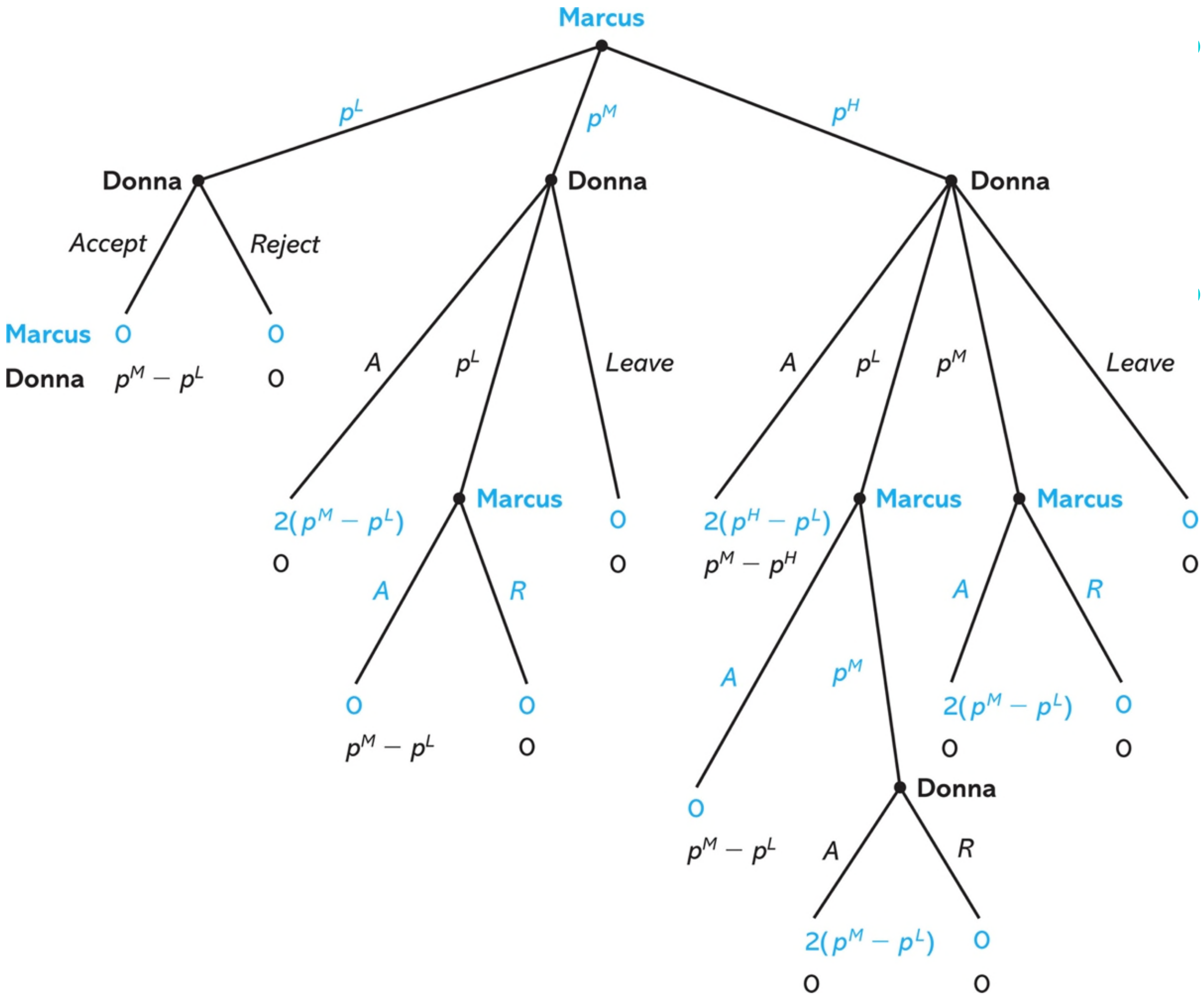
- Payoffs:
- If no sale both get 0.
- If a sale is made at a price, p:
  - Donna gets $p^M - p$
  - Marcus gets $2(p - p^L)$

Haggling at an Auto Dealership, I



- What happens if Marcus starts with an offer of p^L?
- What if he offers p^M?
- What about p^H?

Haggling at an Auto Dealership, I



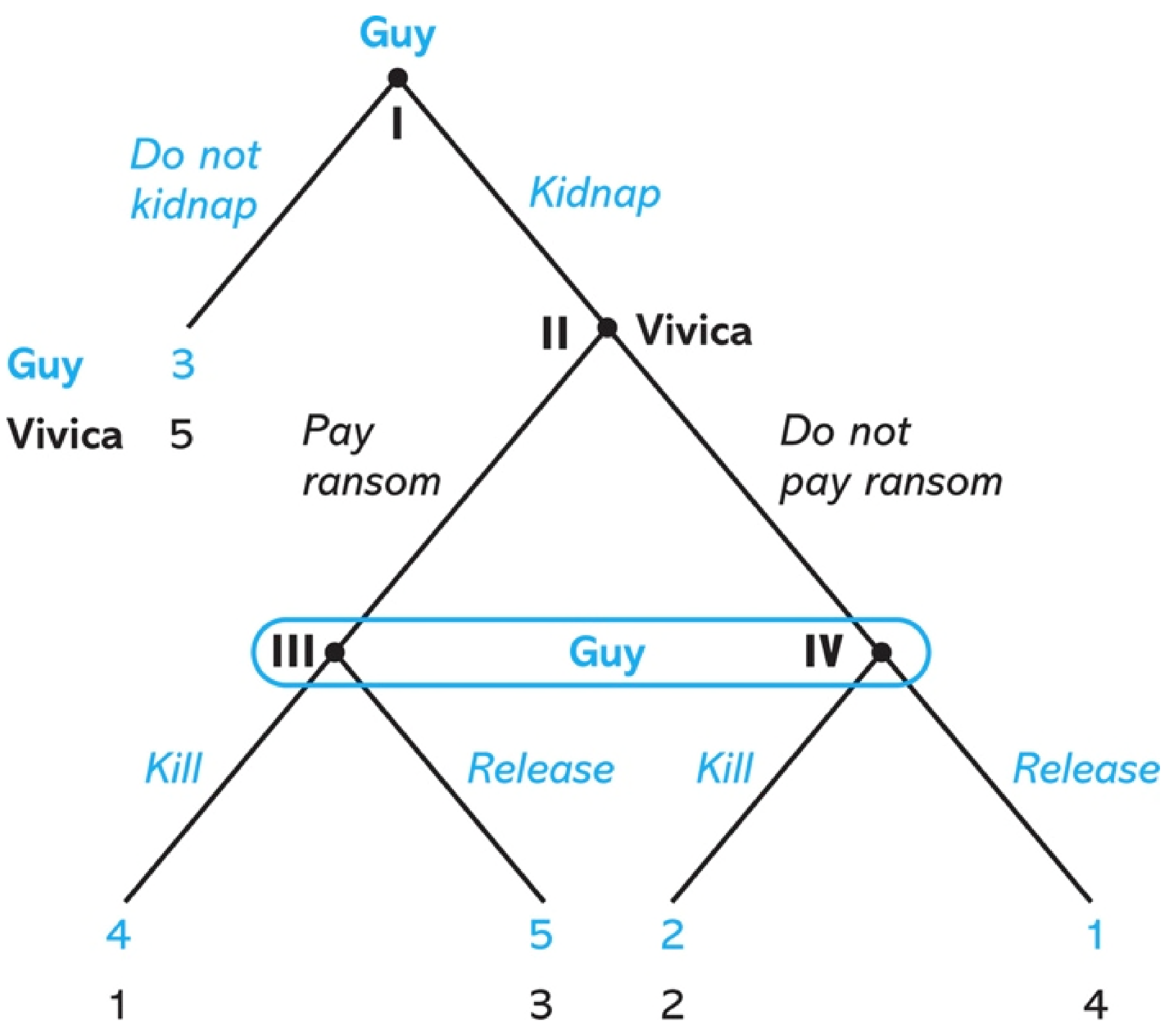
- A different extensive form model.
- Donna can accept, reject and counteroffer with a lower price, or leave.

2.3 Extensive Form Games: Imperfect Information

- How do we model a game when one or both of the players does not know what the other has done when it is her turn to decide?

- Suppose in the Kidnapping game, Guy must decide what to do with Orlando at the same time Vivica must decide whether to pay the ransom or not.

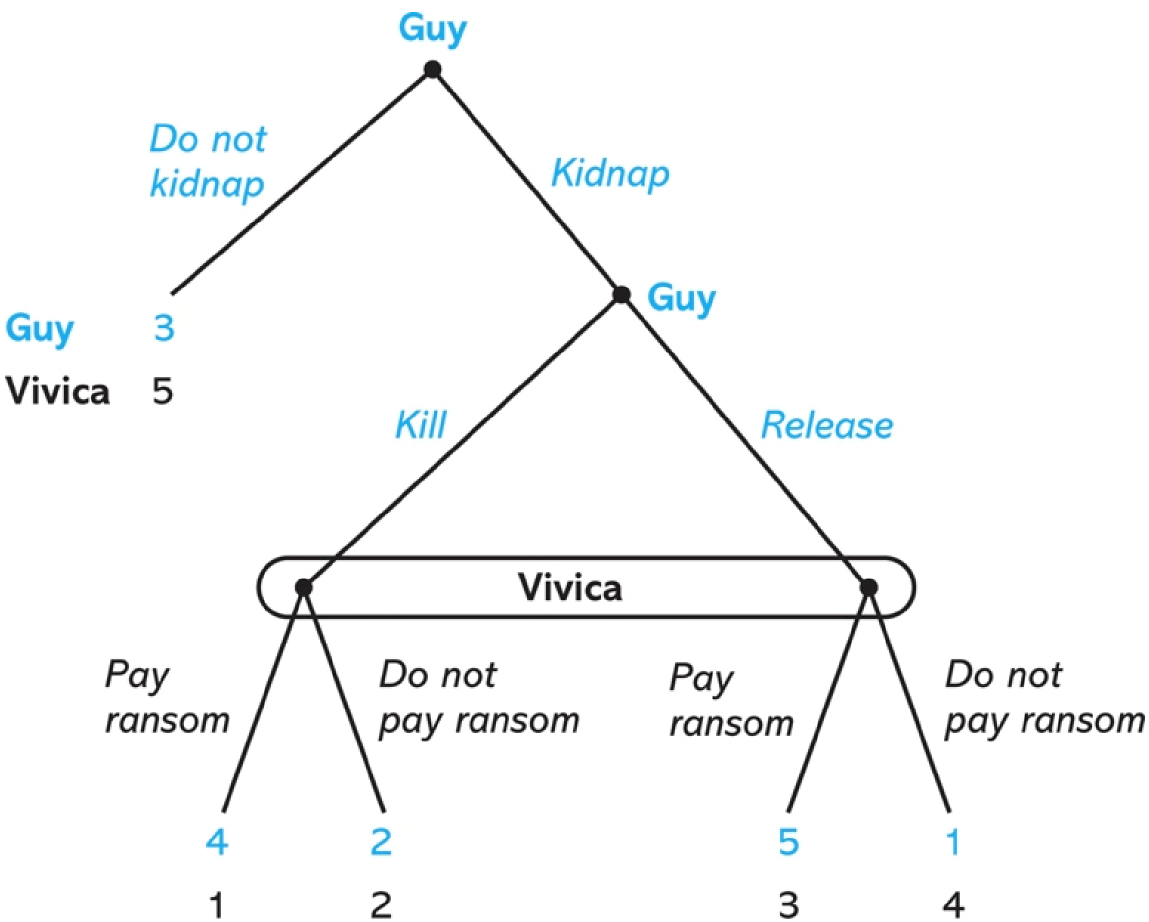
The Kidnapping Game with Imperfect Information



The oval around nodes III and IV indicate that Guy does not know whether Vivica has paid the ransom or not.

Guy has two information sets, and Vivica has one. An information set is made up of all of the decision nodes that a player is incapable of distinguishing among.

The Kidnapping Game with Imperfect Information



We can show the same game this way.

In both versions each player's information sets are the same.

In both games we can assume that Guy and Vivica move simultaneously.

2.3 Extensive Form Games: Imperfect Information

- There are two types of games depending on the information players have.

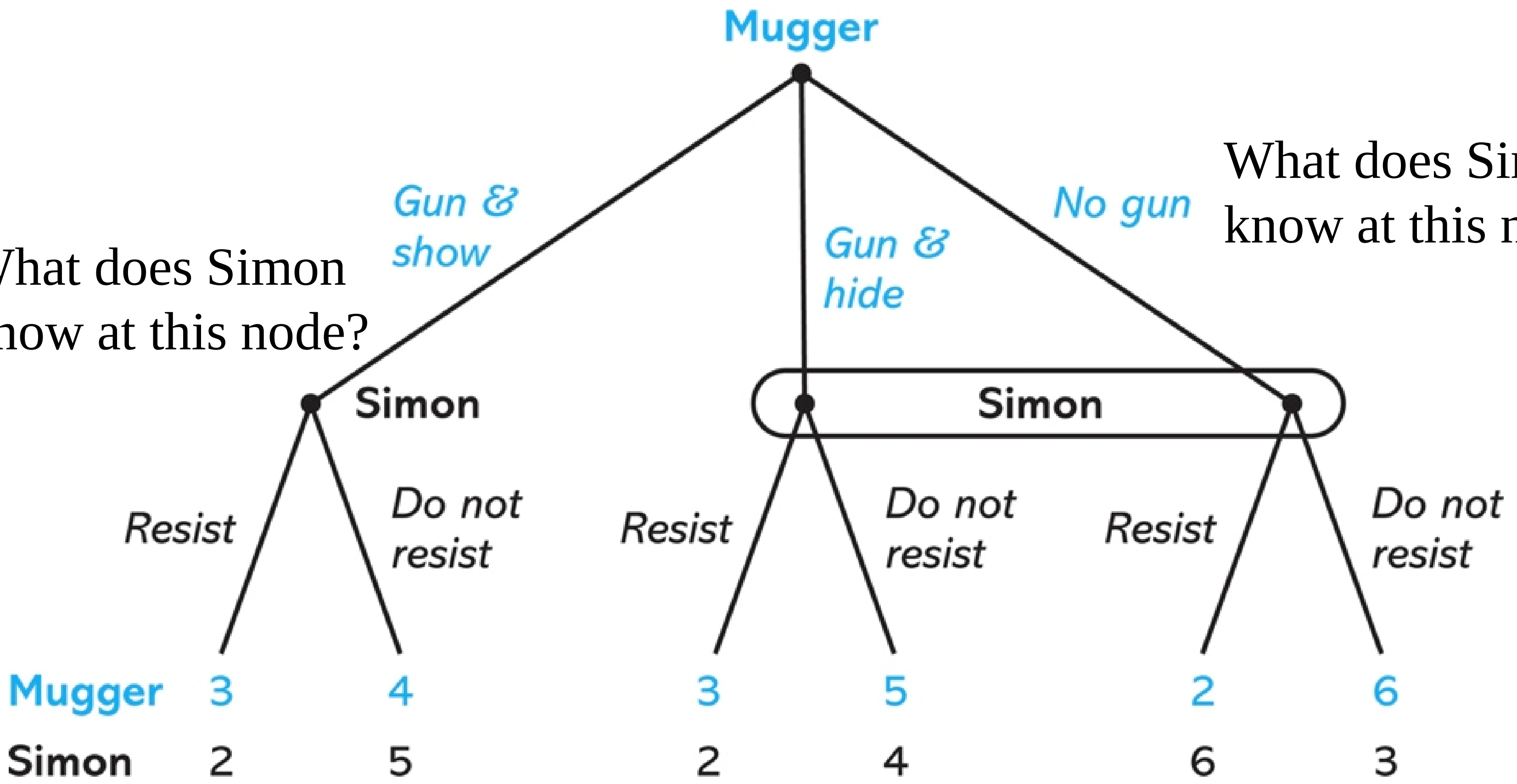
- A game of perfect information is one in which each player knows where they are in the game when they must decide. All the information sets are singletons.

- A game of imperfect information is one in which one or both players do not know where they are in the game when they must decide. One or more information sets are not singletons.

Mugging

What does Simon know at this node?

What does Simon know at this node?



U.S. Court of Appeals

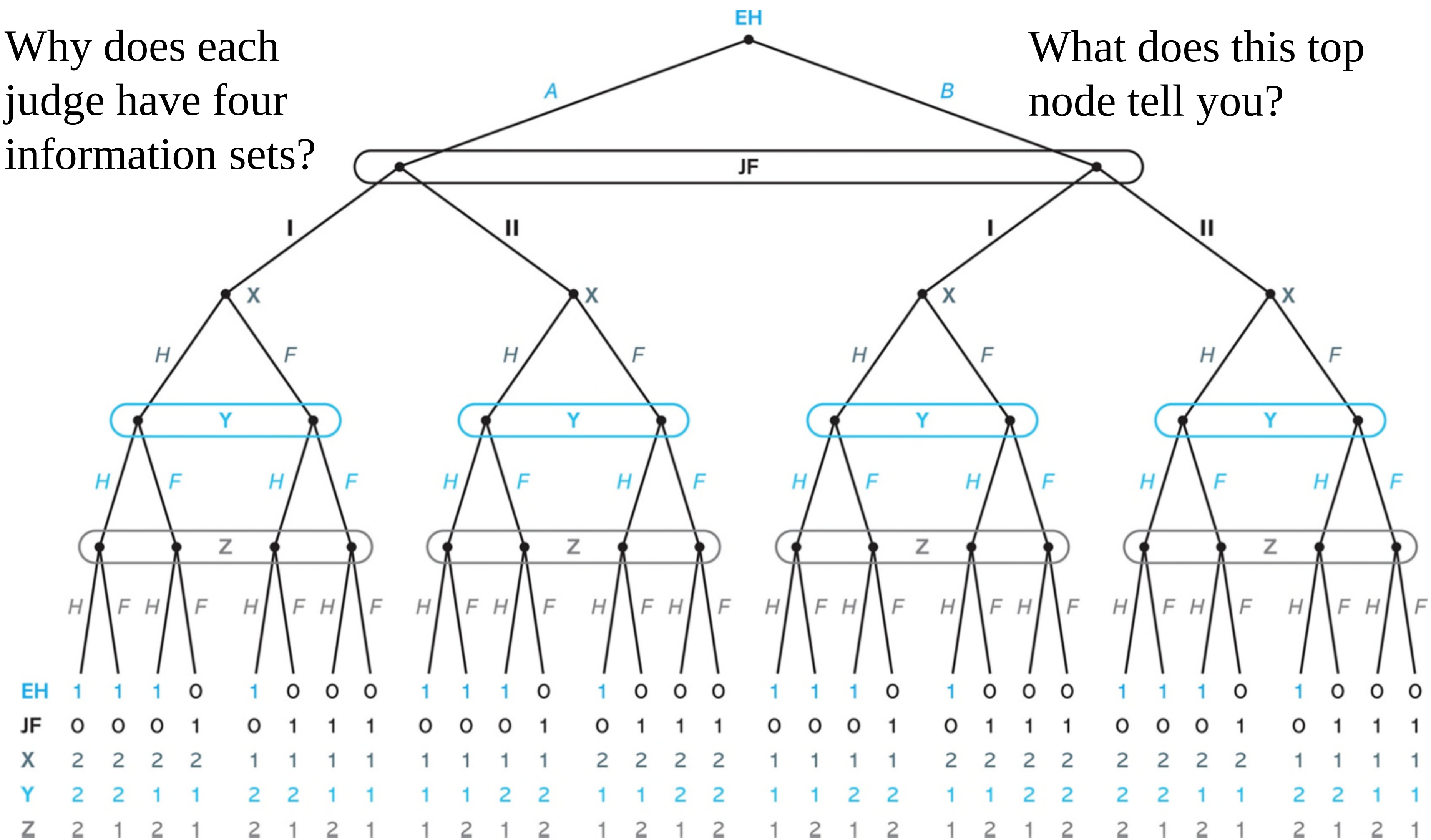
- Two lawyers, Elizabeth Hasenpeffer (EH) and Joseph Fargillio (JF) decide on one of two legal strategies: *A* and *B* for EH, *I* and *II* for JF. They then write a brief, and submit them simultaneously to three judges.

- The three judges – X, Y, and Z – read the briefs, and then simultaneously vote in favor of one of the lawyer's argument.

U.S. Court of Appeals for the Federal Circuit

Why does each judge have four information sets?

What does this top node tell you?



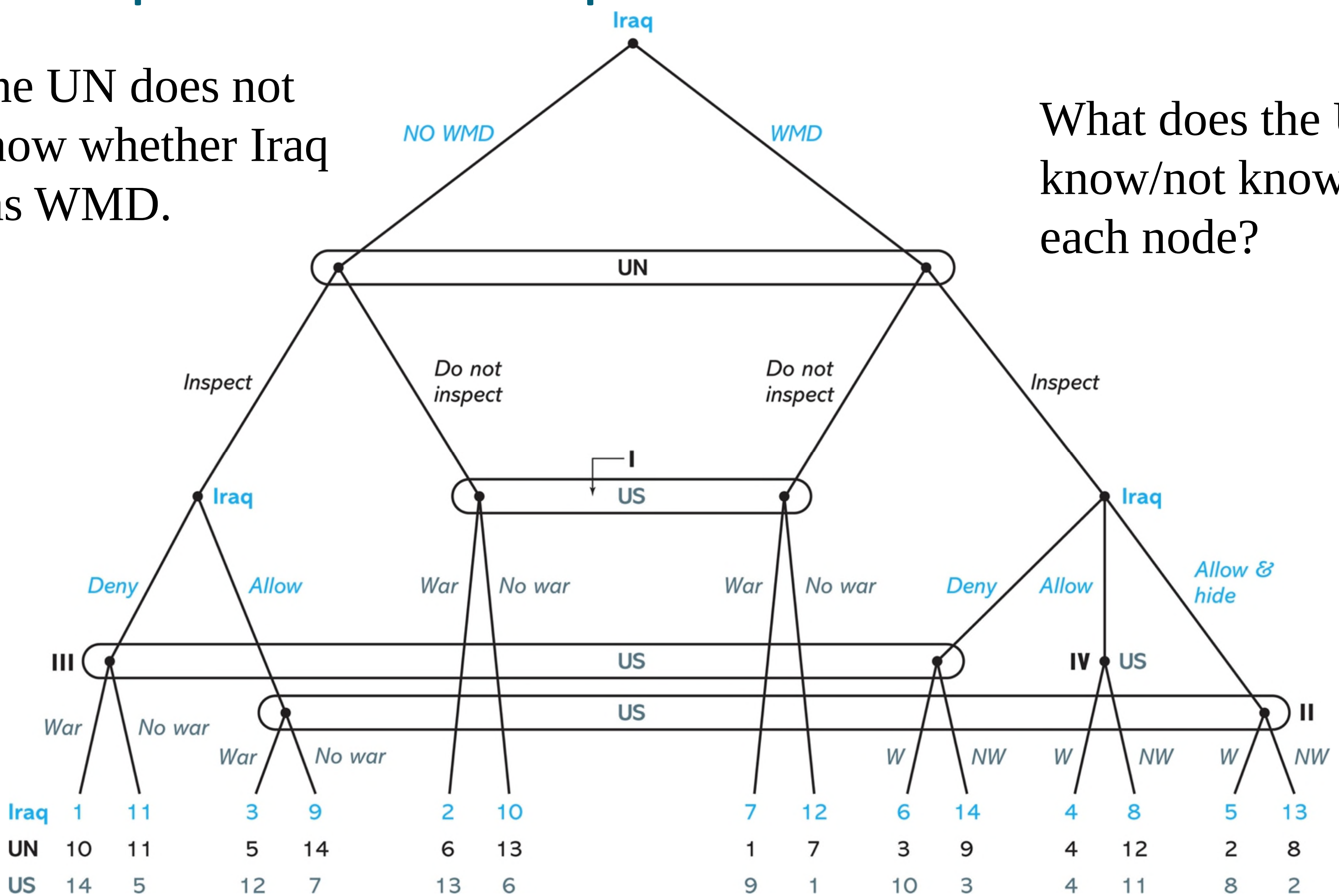
The Iraq War and Weapons of Mass Destruction

- Does Sadaam Hussein have weapons of mass destruction?
- Three players: Iraq, United Nations, and United States.
- Iraq first decides whether to acquire Weapons of Mass Destruction (WMD) or not.
- The UN then decides whether to inspect or not.
- Iraq may respond by allowing inspection or not.
- Finally, the United States may decide to declare war or not.

The Iraq War and Weapons of Mass Destruction

The UN does not know whether Iraq has WMD.

What does the US know/not know at each node?



2.4 What Is a Strategy?

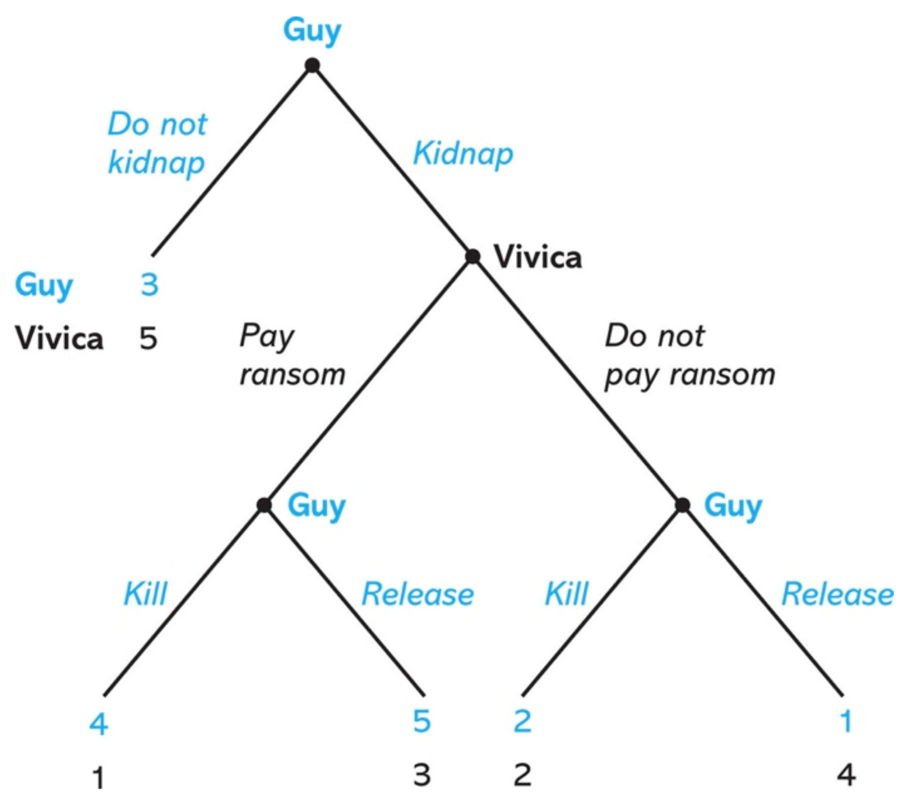
- From extensive form games to strategic form games.
- A strategy is a fully specified decision rule for how to play a game that incorporates every possible contingency.
- Think of a strategy as a *complete* set of instructions that you could give to another person to play.
- A strategy set for a player is the collection of all feasible strategies for that player.

Strategies for the Kidnapping Game

- Guy’s strategy template.

- At the initial node _________.
- If a kidnapping occurred and ransom was paid, then _______.
- If a kidnapping occurred and ransom was not paid, then ____.

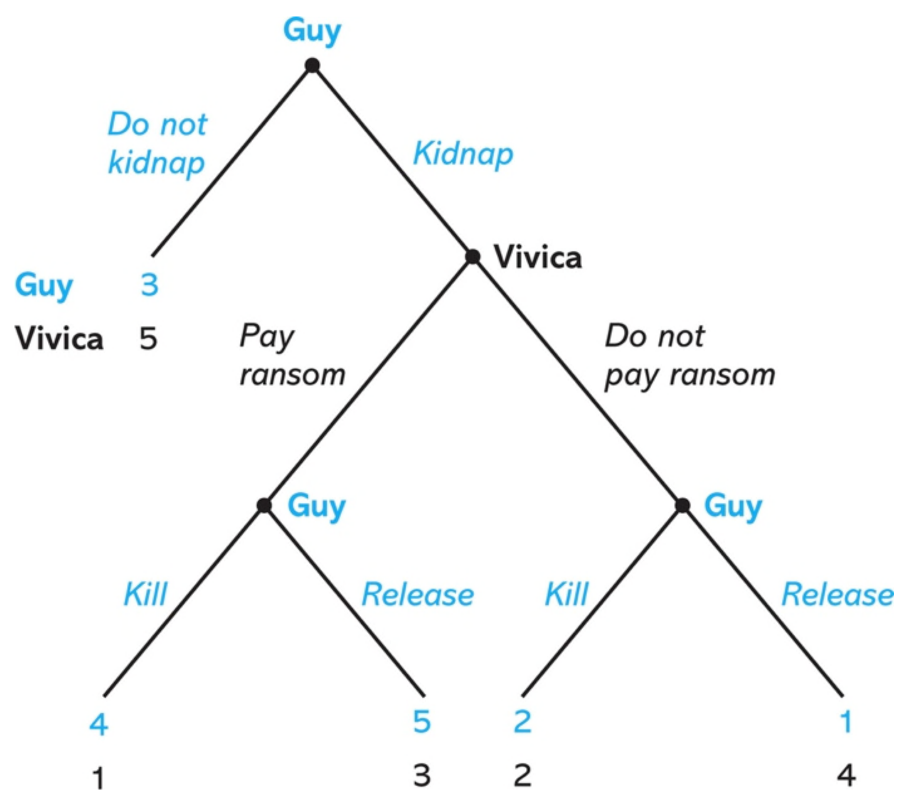
- Guy’s strategy set includes eight strategies (2 × 2 × 2).



Strategies for the Kidnapping Game

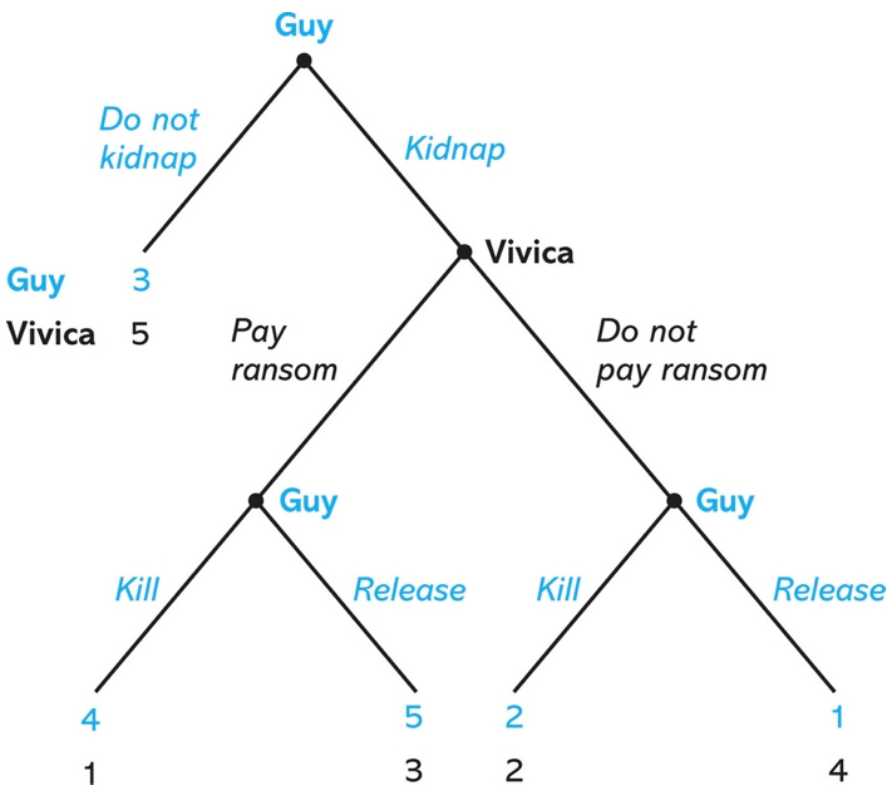
- For example:

- At the initial node *Kidnap*.
- If a kidnapping occurred and ransom was paid, then *Release*.
- If a kidnapping occurred and ransom was not paid, then *Kill*.



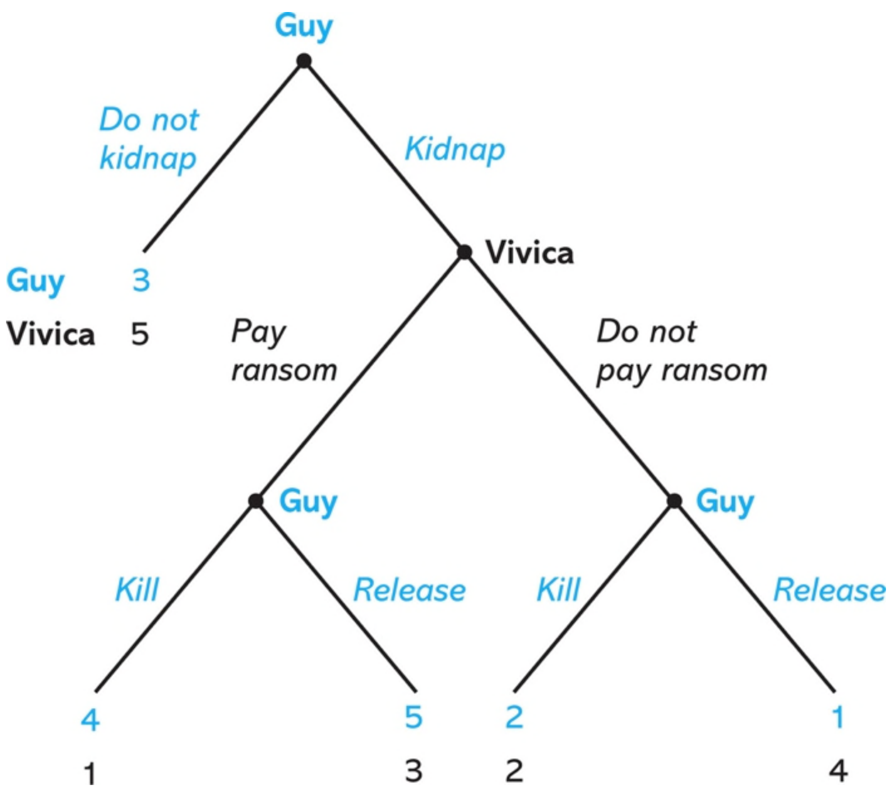
Strategies for the Kidnapping Game

- For example:
- At the initial node *Do not kidnap*.
- If a kidnapping occurred and ransom was paid, then *Release*.
- If a kidnapping occurred and ransom was not paid, then *Kill*.
- Yes, this seems a bit odd. We'll cover this later.



Strategies for the Kidnapping Game

- Vivica's strategy template is simpler.
- If a kidnapping occurred then ______.
- Vivica has only two feasible strategies in her strategy set.



Strategies for the Kidnapping Game

- Once each player chooses a strategy, you know the outcome.

- If Guy chooses:
  - At the initial node *Kidnap*. If a kidnapping occurred and ransom was paid, then *Release*. If a kidnapping occurred and ransom was not paid, then *Kill*.

- And if Vivica chooses:
  - If a kidnapping occurs, then *Pay ransom*.

- Then . . .?

2.5 Strategic Form Games

- A strategic form game addresses the following questions:

1. Who is making the decisions?
   - The set of players.
2. Over what are they making decisions?
   - The players' strategy sets.

2.5 Strategic Form Games

- A strategic form game addresses the following questions:

3. How do they evaluate different decisions?

    - By using a payoff function, which assigns a payoff value to all possible strategy profiles.

Tosca

- The story.
- Two players: Scarpia and Tosca.
- Scarpia has two strategies: use *Real bullets* or use *Blanks*.
- Tosca has two strategies: *Consent* or *Stab*.
- Payoffs to both depend on their preferences over the four strategy profiles.

Tosca

		Scarpia	
		Real	*Blanks*
Tosca	*Stab*	2,2	4,1
	Consent	1,4	3,3

The payoffs on the **left** (right) are for the “**Row**” (“Column”) player.

How well do the payoffs fit the story?

Competition for Elected Office

- Two candidates for office: a Democrat and a Republican.
- The Democrat has three platforms (strategies): *Liberal*, *Moderately liberal*, and *Moderate*.
- The Republican has three platforms (strategies): *Moderate*, *Moderately conservative*, and *Conservative*.
- Candidates simultaneously and independently select their platform.
- A candidate’s payoffs depend both on ideology and the chance of getting elected.

Competition for Elected Office

		Republican candidate		
		Moderate	*Moderately conservative*	*Conservative*
Democratic candidate	*Moderate*	4,4	6,3	8,2
	Moderately liberal	3,6	5,5	9,4
	Liberal	2,8	4,9	7,7

A candidate's payoff is higher the more extreme her rival's position.

If both candidates choose the same platform, their payoffs are higher as they move to their ideal position.

The *Science 84* Game

- Anyone can submit a request for either $20 or $100.
- If no more than 20% of the submissions requested $100, everyone receives what they requested. Otherwise, everybody gets nothing.
- A player's strategy has three elements: *do not send in request*, *request $20, request $100*.
- A player's payoff is equal to amount received minus $1.

The *Science 84* Game

- Let x (y) denote the number of players who request \$20 (\$100), excluding player i.

- Player i's payoff function:
  - 0 if i chooses *do not send in request*
  - 19 if i chooses *request \$20* and $y/(x + y + 1) \leq .2$
  - 99 if i chooses *request \$100* and $(y + 1)/(x + y + 1) \leq .2$
  - –1 if i chooses *request \$20* and $.2 < y/(x + y + 1)$
  - –1 if i chooses *request \$100* and $.2 < (y + 1)/(x + y + 1)$

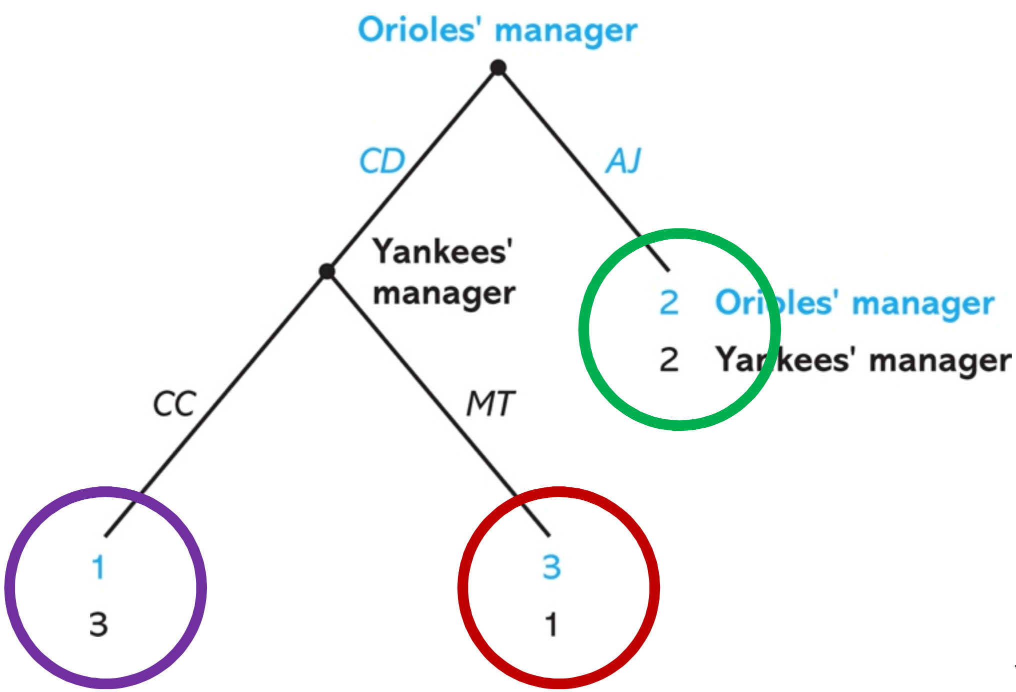
  Example: if i requests \$20 and less than 20% of submissions request \$100, then player i gets \$19.

- What happened in the contest?

2.6 Moving from Extensive and Strategic Form

- For every extensive form game there is a unique strategic form representation.

Baseball, II



If the Orioles retain AJ, both get a payoff of 2.

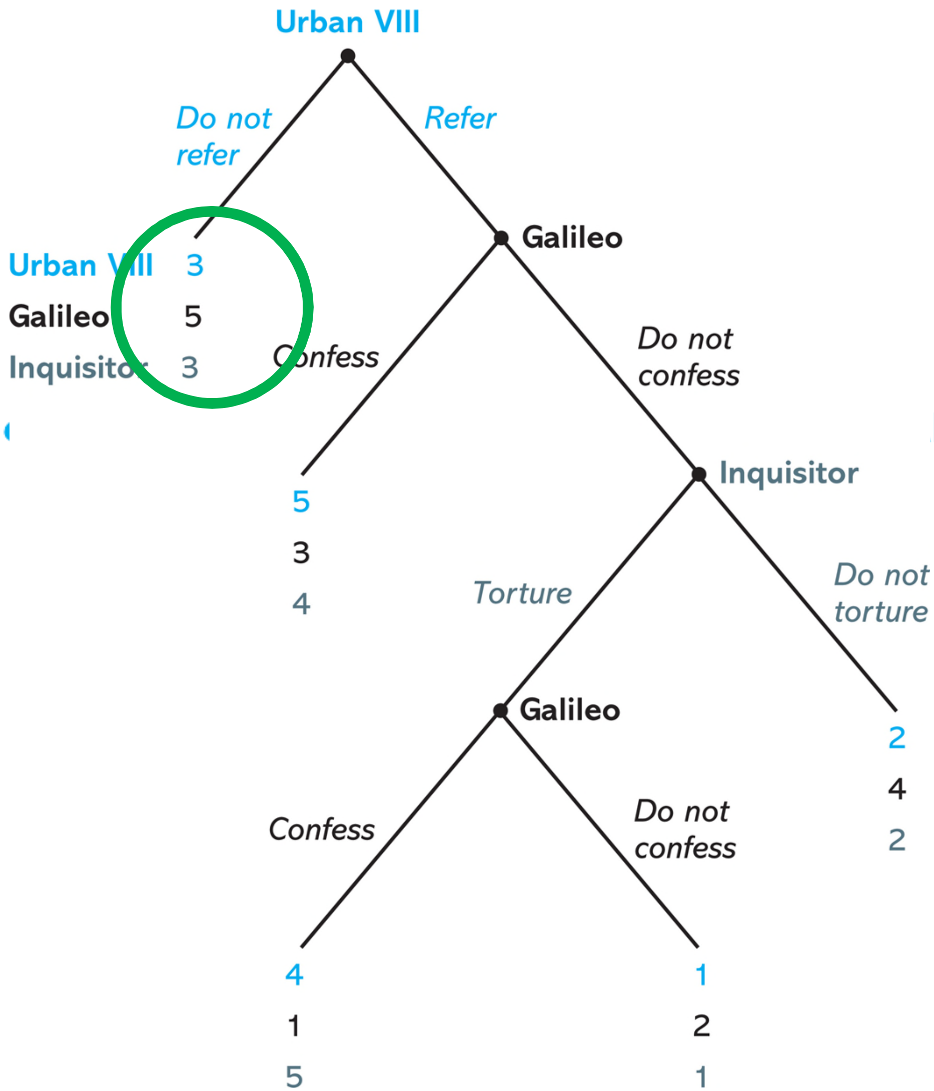
If the Orioles sub CD, and Yankees keep MT, the payoffs are 3 and 1.

If the Orioles sub CD, and Yankees sub CC, the payoffs are 1 and 3.

Yankees' manager

Orioles' manager		Retain Tanaka	Substitute Cabral
	Retain Jones	2,2	2,2
	Substitute Davis	3,1	1,3

Galileo and the Inquisition, II



Pope Urban VIII—*Do Not Refer*

	Inquisitor	
	Torture	*Do not torture*
C/C	5,3,3	5,3,3
C/DNC	5,3,3	5,3,3
DNC/C	5,3,3	5,3,3
DNC/DNC	5,3,3	5,3,3

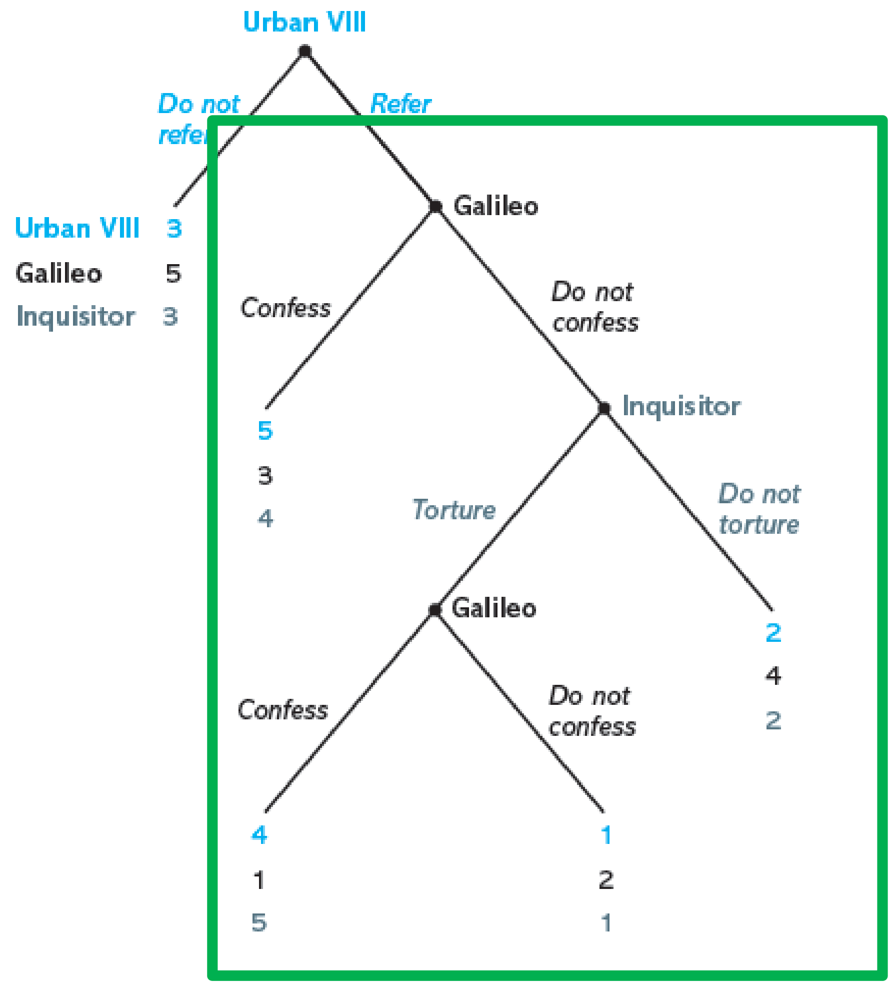
This half of the strategic form matches the left side of the game tree.

Galileo and the Inquisition, II

Pope Urban VIII—*Refer*

Inquisitor

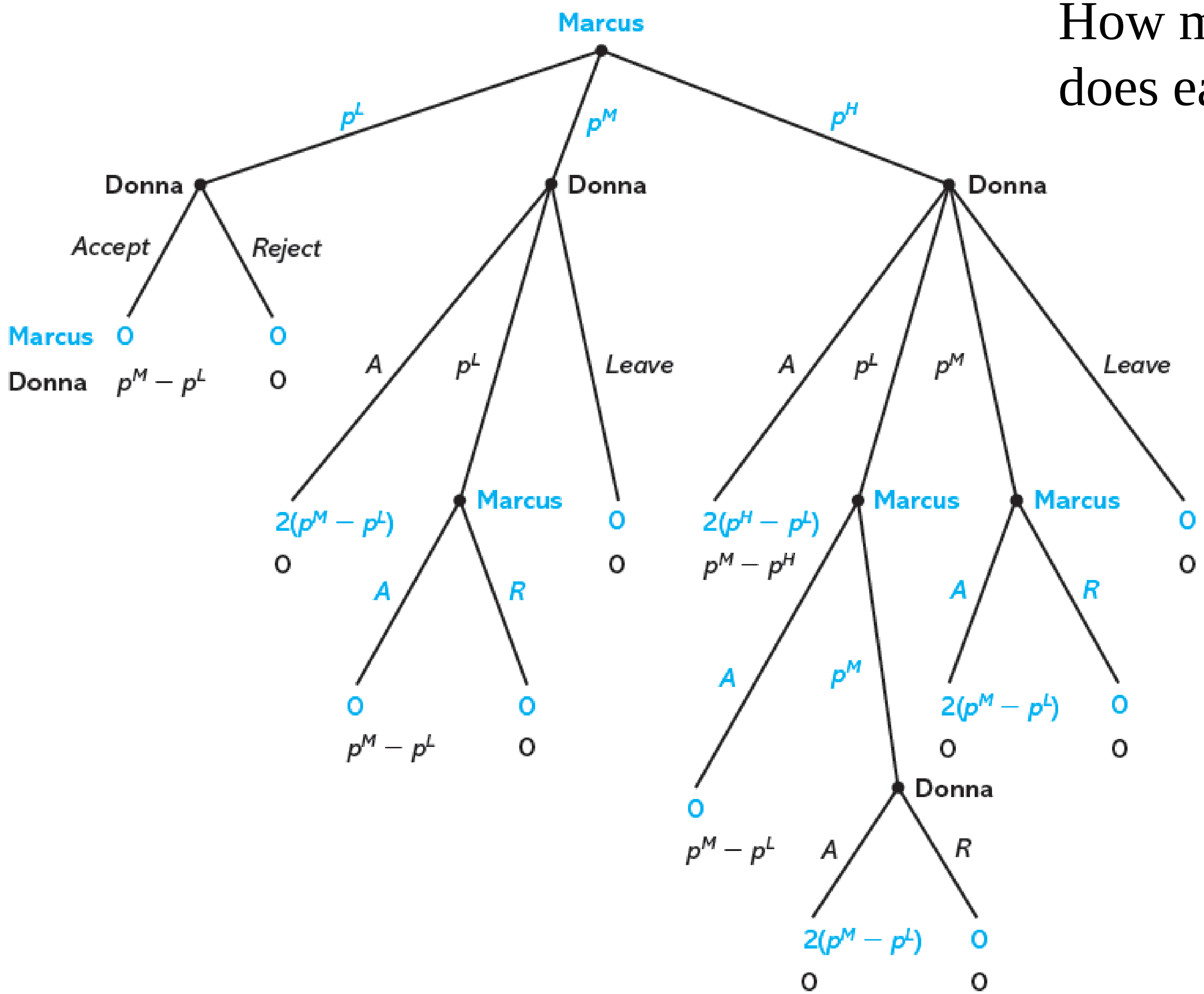
Galileo	*Torture*	*Do not torture*
C/C	3,4,5	3,4,5
C/DNC	3,4,5	3,4,5
DNC/C	1,5,4	4,2,2
DNC/DNC	2,1,1	4,2,2



This half of the strategic form matches the right side of the game tree.

Can you match the individual strategy profiles?

Haggling At An Auto Dealership II



How many information sets does each player have?

Both have four.

Haggling At An Auto Dealership II

- Marcus' strategy template:
  - At the initial node, offer __________ [fill in p^L, p^M, or p^H].
  - If I offered p^M and Donna rejected it and offered p^L, then _________ [fill in *accept* or *reject*].
  - If I offered p^H and Donna rejected it and offered p^M, then _________ [fill in *accept* or *reject*].
  - If I offered p^H and Donna rejected it and offered p^L, then _________ [fill in *accept* or *reject and offer* p^M].

- There are 24 (3x2x2x2) different ways to fill out the template.

Haggling At An Auto Dealership II

- Donna's strategy template:
  - If Marcus offered p^{L}, then __________ [fill in *accept* or *reject*].
  - If Marcus offered p^{M}, then __________ [fill in *accept, reject and offer* p^{L} or *reject and leave*].
  - If Marcus offered p^{H}, then __________ [fill in *accept, reject and offer* p^{L}, *reject and offer* p^{M} or *reject and leave*].
  - If Marcus offered p^{H}, I rejected and offered p^{L}, and Marcus rejected and offered p^{M}, then _________ [fill in *accept* or *reject*].

- There are 48 (2x3x4x2) different ways to fill out the template.

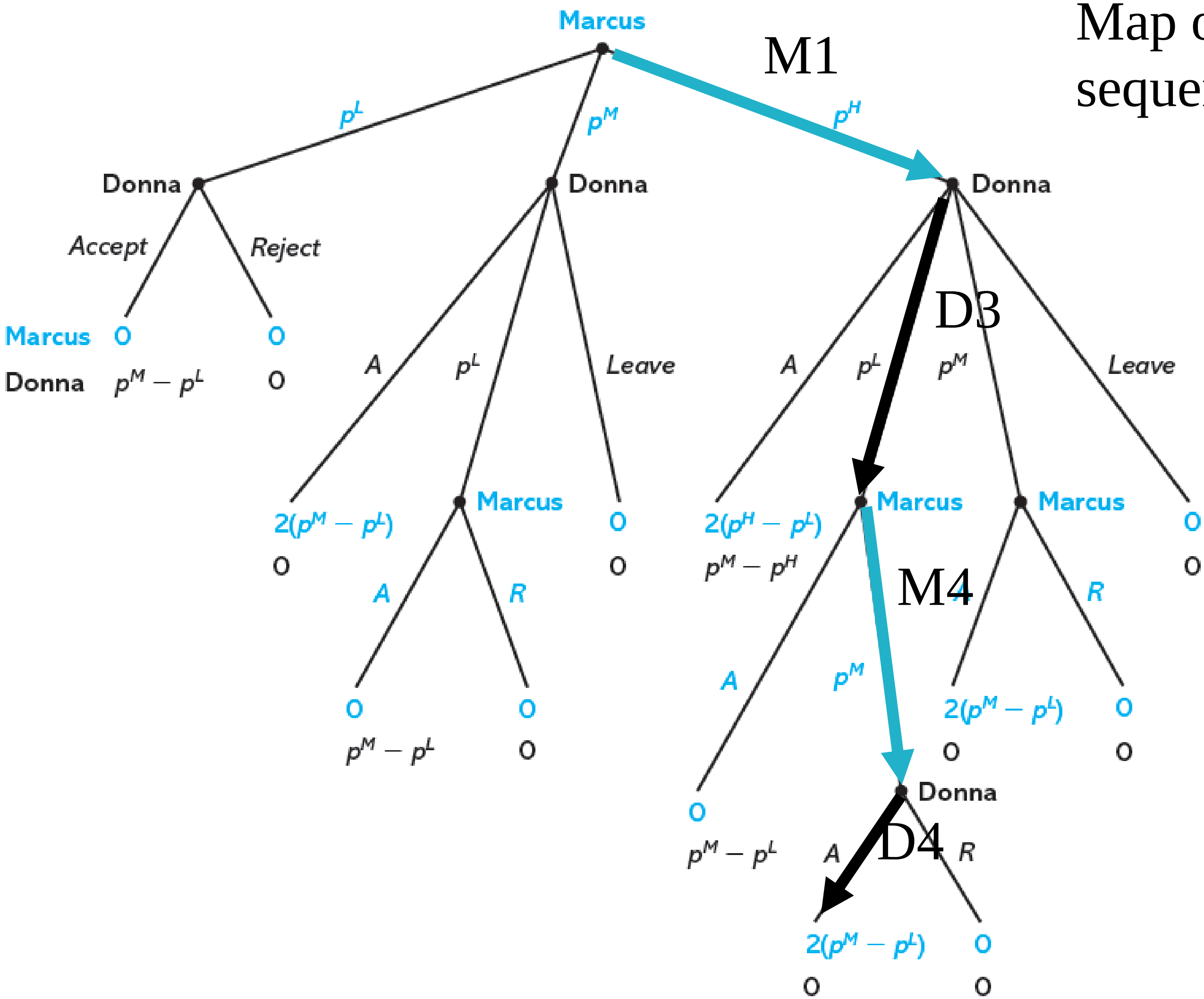
Haggling At An Auto Dealership II

Let's consider a specific strategy profile for each player:

- Marcus
  1. At the initial node offer p^H.
  2. If I offered p^M and Donna rejected it and offered p^L, then *reject*.
  3. If I offered p^H and Donna rejected it and offered p^M, then *accept*.
  4. If I offered p^H and Donna rejected it and offered p^L, then *reject and offer* p^M.

- Donna
  1. If Marcus offered p^L, *accept*.
  2. If Marcus offered p^M, *accept*.
  3. If Marcus offered p^H, then *reject and offer* p^L.
  4. If Marcus offered p^H, I rejected and offered p^L, and Marcus rejected and offered p^M, then *accept*.

Haggling At An Auto Dealership II



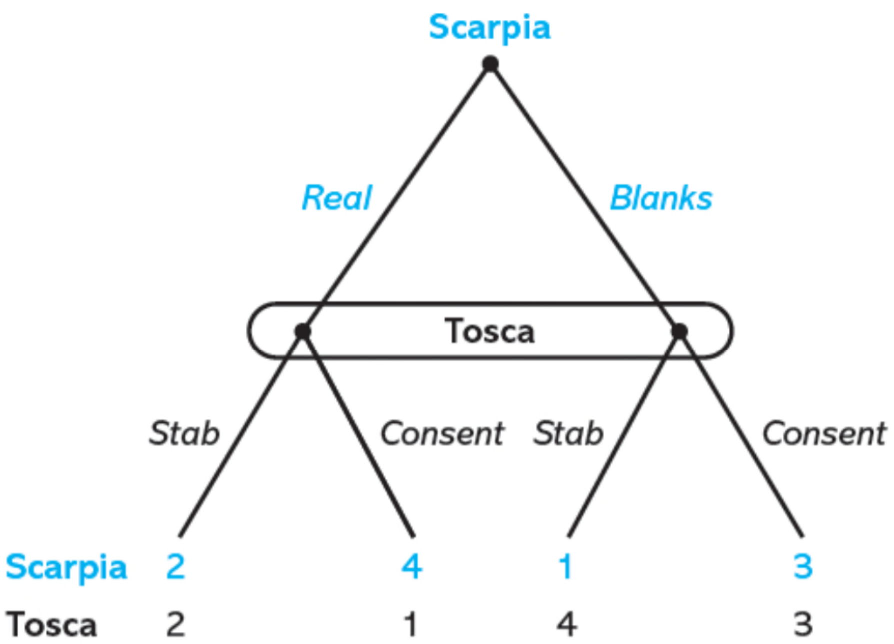
Map out the sequence of play.

2.7 Going From Strategic to Extensive Form

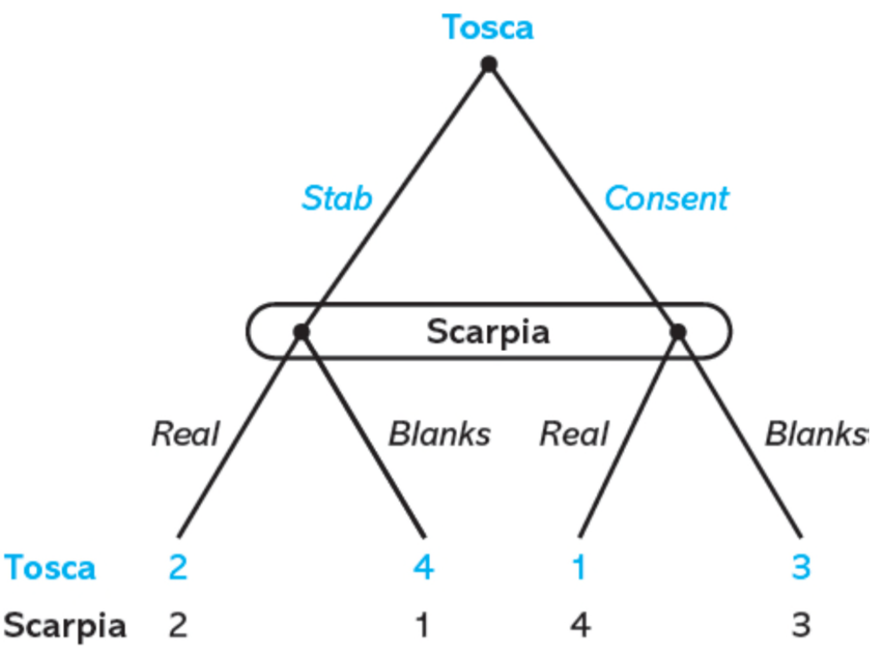
- The same strategic form game can be associated with more than one extensive form game.
- As we move from the extensive form to the strategic form, we lose some information, but it won't matter.
- Let's see this result in the next slide.

Tosca

		Scarpia	
		Real	*Blanks*
Tosca	*Stab*	2,2	4,1
	Consent	1,4	3,3



(a)



(b)

2.8 Common Knowledge

- The story of Jack and Kate.
- Jack and Kate face a problem of a lack of common knowledge.
- An event E is common knowledge to two players if:
  - 1 knows E and 2 knows E.
  - . . .

2.8 Common Knowledge

- An event E is common knowledge to two players if:
  - 1 knows E and 2 knows E.
  - 1 knows that 2 knows E, and 2 knows that 1 knows E.
  - 1 knows that 2 knows that 1 knows E, and 2 knows . . .
  - 1 knows that 2 knows that 1 knows that 2 knows E, and 2 knows . . .

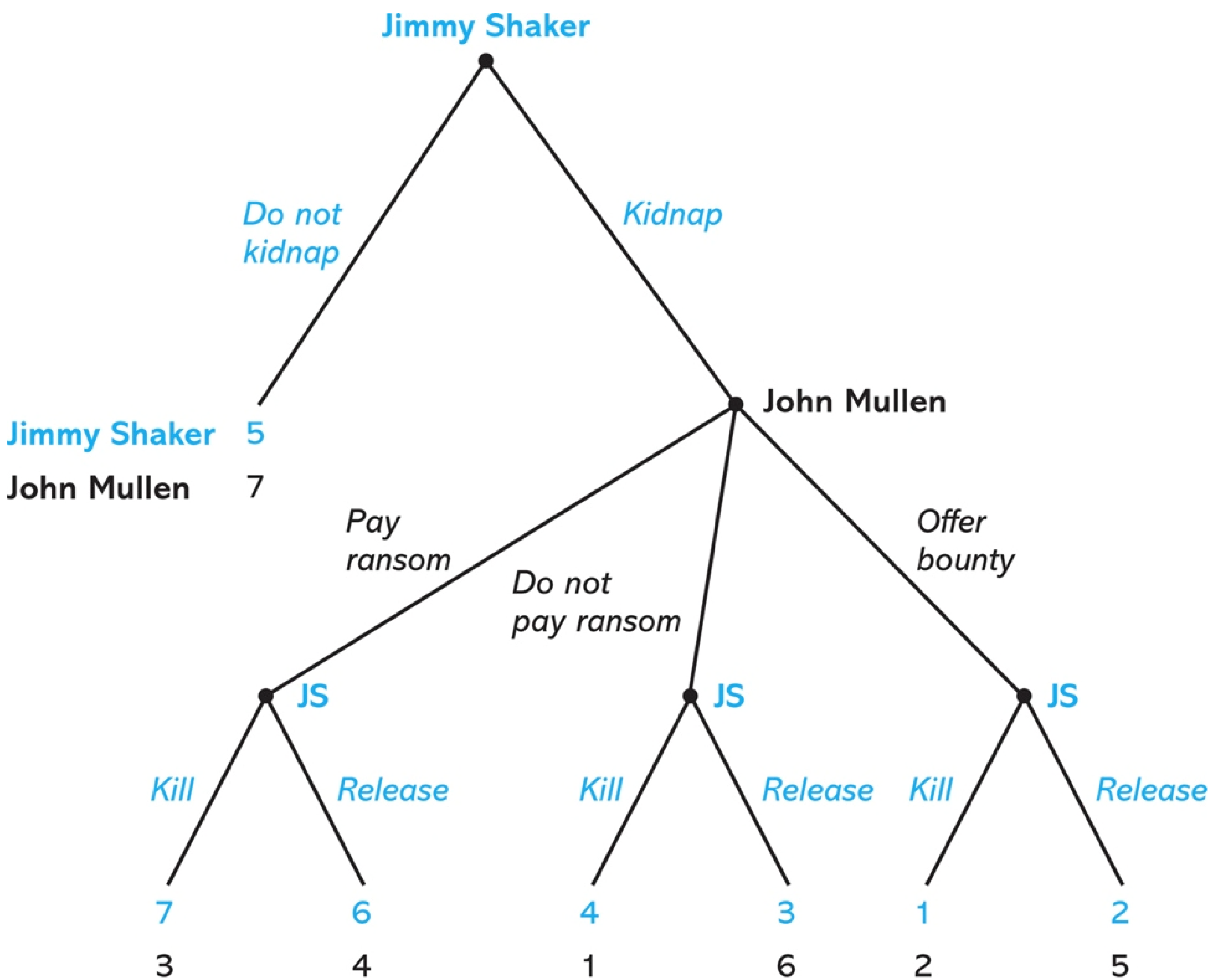
2.8 Common Knowledge

- For most of this course we will assume that the game is common knowledge to the players.

2.9 A Few More Issues in Modeling Games

- Can a player forget?
  - Perfect recall and Imperfect recall
- Can a player change the game?
  - No. The players cannot, but we can give them more options.

Ransom


Jimmy Shaker
Do not kidnap
Kidnap
John Mullen
Jimmy Shaker 5
John Mullen 7
Pay ransom
Do not pay ransom
Offer bounty
JS
JS
JS
Kill
Release
Kill
Release
Kill
Release
7
3
6
4
4
1
3
6
1
2
2
5

2.9 A Few More Issues in Modeling Games

- Can a player forget?
- Can a player change the game?
- Does the game have to be factually accurate?
    - No, what matters are the players' beliefs.

Summary

- In this chapter you have learned two game theory frameworks.
- Extensive Form Game.
- What a player knows can be represented by an information set.
- And a player may have perfect or imperfect information.

Summary

- In this chapter you have learned two game theory frameworks.

- Strategic Form Game.

- A strategic form game is defined by a set of players, the strategy set for each player, and a payoff function.

- Finally, a central assumption underlying most of the games we will consider is that the game is common knowledge.