Chapter 2
Risk and Return
ANSWERS TO BEGINNING-OF-CHAPTER QUESTIONS

Our students have had an introductory finance course, and many have also taken a course on investments and/or capital markets. Therefore, they have seen the Chapter 2 material previously. However, we use the Beginning of Chapter (BOC) questions to review the chapter because our students need a refresher.

With students who have not had as much background, it is best to go through the chapter on a point-by-point basis, using the PowerPoint slides. With our students, this would involve repeating too much of the intro course. Therefore, we just discuss the questions, including the model for Question 6. Before the class, we tell our students that the chapter is a review and that we will call on them to discuss the BOC questions in class. We expect students to be able to give short, incomplete answers that demonstrate that they have read the chapter, and then we provide more complete answers as necessary to make sure the key points are covered.

Our students have mainly taken multiple-choice exams, so they are uncomfortable with essay tests. Also, we cover the chapters they were exposed to in the intro course rather quickly, so our assignments often cover a lot of pages. We explain that much of the material is a review, and that if they can answer the BOC questions (after the class discussion) they will do OK on the exams. We also tell them, partly for motivation and partly to reduce anxiety, that our exams will consist of 5 slightly modified BOC questions, of which they must answer 3. We also tell them that they can use a 4-page "cheat sheet," two sheets of paper, front and back. They can put anything they want on it—formulas, definitions, outlines of answers to the questions, or complete answers.

The better students write out answers to the questions before class, and then extend them after class and before the exams. This helps them focus and get better prepared. Writing out answers is a good way to study, and outlining answers to fit them on the cheat sheet (in really small font!) also helps them learn. We try to get students to think in an integrated manner, relating topics covered in different chapters to one another. Studying all of the BOC questions in a fairly compressed period before the exams helps in this regard. They tell us that they learn a great deal when preparing their cheat sheets.

We initially expected really excellent exams, given that the students had the questions and could use cheat sheets. Some of the exams were indeed excellent, but we were surprised and disappointed at the poor quality of many of the midterm exams. Part of the problem is that our students were not used to taking essay exams. Also, they would have done better if they had taken the exam after we covered cases (in the second half of the semester), where we apply the text material to real-world cases. While both points are true, it's also true that some students are just better than others.

The students who received low exam grades often asked us what they did wrong. That's often a hard question to answer regarding an essay exam. What we ended up doing was make copies of the best 2 or 3 student answers to each exam question, and then when students came in to see why they did badly, we made them read the good answers before we talked with them. 95% of the time, they told us they understand why their grade was low, and they resolved to do better next time. Finally, since our students are all graduating seniors, we graded rather easily.

Answers

2-1 Stand-alone risk is the risk faced by an investor who holds just one asset, versus the risk inherent in a diversified portfolio.

Stand-alone risk is measured by the standard deviation (SD) of expected returns or the coefficient of variation (CV) of returns = SD/expected return.

A portfolio's risk is measured by the SD of its returns, and the risk of the individual stocks in the portfolio is measured by their beta coefficients. Note that unless returns on all stocks in a portfolio are perfectly positively correlated, the portfolio's SD will be less than the average of the SD's of the individual stocks. Diversification reduces risk.

In theory, investors should be concerned only with portfolio risk, but in practice many investors are not well diversified, hence are concerned with stand-alone risk. Managers or other employees who have large stockholdings in their companies are an example. They get stock (or options) as incentive compensation or else because they founded the company, and they are often constrained from selling to diversify. Note too that years ago brokerage costs and administrative hassle kept people from diversifying, but today mutual funds enable small investors to diversify efficiently. Also, the Enron and WorldCom debacles and their devastating effects on 401k plans heavily in those stocks illustrated the importance of diversification.

2-2 Diversification can eliminate <u>unsystematic risk</u>, but <u>market risk</u> will remain. See Figure 2-8 for a picture of what happens as stocks are added to a portfolio. The graph shows that the risk of the portfolio as measured by its SD declines as more and more stocks are added. This is the situation if randomly selected stocks are added, but if stocks in the same industry are added, the benefits of diversification will be lessened.

Conventional wisdom says that 40 to 50 stocks from a number of different industries is sufficient to eliminate most unsystematic risk, but in recent years the markets have become increasingly volatile, so now it takes somewhat more, perhaps 60 or 70. Of course, the more stocks, the closer the portfolio will be to having zero unsystematic risk. Again, this assumes that stocks are randomly selected. Note, however, that the more stocks the portfolio contains, the greater the administrative costs. Mutual funds can help here.

Different diversified portfolios can have different amounts of risk. First, if the portfolio concentrates on a given industry or sector (as sector mutual funds do), then the portfolio will not be well diversified even if it contains 100 stocks. Second, the betas of the individual stocks affect the risk of the portfolio. If the stock with the highest beta in each industry is selected, then the portfolio may not have much unsystematic risk, but it will have a high beta and thus have a lot of market risk. **(Note: The market risk of a portfolio is measured by the beta of the portfolio, and that beta is a weighted average of the betas of the stocks in the portfolio.)**

2-3 a. Note: This question is covered in more detail in Chapter 5, but students should remember this material from their first finance course, so it is a review.

<u>Expected:</u> The rate of return someone expects to earn on a stock. It's typically measured as $D_1/P_0 + g$ for a constant growth stock.

<u>Required:</u> The minimum rate of return that an investor must expect on a stock to induce him or her to buy or hold the stock. It's typically measured as $r_s = r_{rf} + b(MRP)$, where MRP is the market risk premium or the risk premium required for an average stock.

<u>Historical:</u> The average rate of return earned on a stock during some past period. The historical return on an average large stock varied from –3% to +37% during the 1990s, and the average annual return was about 15%. The worm turned after 1999— the average return was negative in 2000, 2001, and 2002, with the S&P 500 down 23.4% in 2002. The Nasdaq average of mostly tech stock did even worse, falling 31.5% in 2002 alone. Of course, the bottom fell out of the market with the Global Economic Crisis in 2008 and 2009! The variations for individual stocks were much greater—the best performer on the NYSE in 2000 gained 413% and the worst performer lost 100% of its value. Stocks improved in the mid 2000s only to crash again in 2008 with the financial crisis. In short, stock returns are highly variable!

b. Are the 3 types of return equal? 1) Expected = required?. The answer is, "maybe."
    For the market to be in equilibrium, the expected and required rate of return as seen
    by "the marginal investor" must be equal for any given stock and therefore for the
    entire market. If the expected return exceeded the required return, then investors
    would buy, pushing the price up and the expected return down, and thus produce an
    equilibrium. Note, though, that any individual investor may believe that a given
    stock's expected and required returns differ, so individuals may think there are
    bargains to be bought or dogs to be sold. Also, new information is constantly hitting
    the market and changing the opinions of marginal investors, and this leads to swings
    in the market. New technology is causing new information to be disseminated ever
    more rapidly, and that is leading to more rapid and violent market swings.

    2) Historical = expected and/or required? There is no reason whatever to think
    that the historical rate of return for any given year for either one stock or for all stocks
    on average will be equal to the expected and/or required rate of return. Rational
    people don't expect abnormally good or bad performance to continue. On the other
    hand, people do argue that investors expect to earn returns in the future that
    approximate average past returns. For example, if stocks returned 9% on average in
    the past (from 1926 to 2011, which is as far back as good data exist), then they may
    expect to earn about 9% on stocks in the future. Note, though, that this is a
    controversial issue—the period 1926-2011 covers a lot of very different economic
    environments, and investors may not expect the future to replicate the past. Certainly
    investors didn't expect future returns to equal distant past returns during the height of
    the 1999 bull market or to lose money as they did in 2002 and 2008.

2-4 To be **risk averse** means to dislike risk. Most investors are risk averse. Therefore, if
     Securities A and B both have an expected return of say 10%, but Security A has less risk
     than B, then most investors will prefer A. As a result, A's price will be bid up, and B's
     price bid down, and in the resulting equilibrium A's expected rate of return will be below
     that of B. Of course, A's required rate of return will also be less than B's, and in
     equilibrium the expected and required returns will be equal.

     One issue here is the type of risk investors are averse to—unsystematic, market, or
     both? According to CAPM theory, only market risk as measured by beta is relevant and
     thus only market risk requires a premium. However, empirical tests indicate that
     investors also require a premium for bearing unsystematic risk as measured by the stock's
     SD.

2-5   CAPM = Capital Asset Pricing Model. The CAPM establishes a metric for measuring the market risk of a stock (beta), and it specifies the relationship between risk as measured by beta and the required rate of return on a stock. Its principal developers (Sharpe and Markowitz) won the Nobel Prize in 1990 for their work.

The key assumptions are spelled out in Chapter 3, but they include the following: (1) all investors focus on a single holding period, (2) investors can lend or borrow unlimited amounts at the risk-free rate, (3) there are no brokerage costs, and (4) there are no taxes. The assumptions are not realistic, so the CAPM may be incorrect. Empirical tests have neither confirmed nor refuted the CAPM with any degree of confidence, so it may or may not provide a valid formula for measuring the required rate of return.

The SML, or Security Market Line (see Figure 2-10), specifies the relationship between risk as measured by beta and the required rate of return, $r_s = r_{rf} + b(MRP)$. MRP = Expected rate of return on the market $-$ Risk-free rate $= r_m - r_{fr}$.

The data requirements are beta, the risk-free rate, and the rate of return expected on the market. Betas are easy to get (by calculating them or from some source such as Value Line or Yahoo!, but a beta shows how volatile a stock was in the past, not how volatile it will be in the future. Therefore, historical betas may not reflect investors' perceptions about a stock's future risk, which is what's relevant. The risk-free rate is based on either T-bonds or T-bills; these rates are easy to get, but it is not clear which should be used, and there can be a big difference between bill and bond rates, depending on the shape of the yield curve. Finally, it is difficult to determine the rate of return investors expect on an average stock. Some argue that investors expect to earn the same average return in the future that they earned in the past, hence use historical MRPs, but as noted above, that may not reflect investors' true expectations.

The bottom line is that we cannot be sure that the CAPM-derived estimate of the required rate of return is actually correct.

2-6   a.   Given historical returns on X, Y, and the Market, we could calculate betas for X and Y. Then, given r_{rf} and the MRP, we could use the SML equation to calculate X and Y's required rates of return. We could then compare these required returns with the given expected returns to determine if X and Y are bargains, bad deals, or in equilibrium.

We assumed a set of data and then used an Excel model to calculate betas for X and Y, and the SML required returns for these stocks. Note that in our Excel model (ch02-M) we also show, for the market, how to calculate the total return based on stock price changes plus dividends. $b_x = 0.69$; $b_y = 1.66$ and $r_x = 10.7\%$; $r_y = 14.6\%$. Since Y has the higher beta, it has the higher required return.

In our examples, the returns all fall on the trend line. Thus, the two stocks have essentially no diversifiable, unsystematic risk—all of their risk is market risk. If these were real companies, they might have the indicated trend lines and betas, but the points would be scattered about the trend line. See Figure 3-8 in Chapter 3, where data for General Electric are plotted. Although the situation for our Stocks X and Y would never occur for individual stocks, it would occur (approximately) for index funds, if Stock X were an index fund that held stocks with betas that averaged 0.69 and Stock Y were an index fund with b = 1.66 stocks.

b. Here we drop Year 1 and add Year 6, then calculate new betas and r's. For Stock X, the beta and required return would be reasonably stable. However, Y's beta would fall, given its sharp decline in a year when the market rose. In our Excel model, Y's beta falls from 1.66 to 0.19, and its required return as calculated with the SML falls to 8.8%.
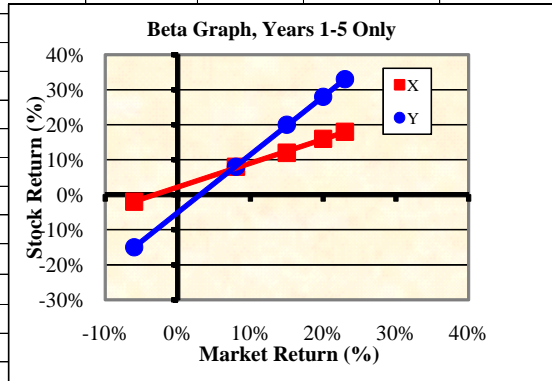
The results for Y make little sense. The stock fell sharply because investors became worried about its future prospects, which means that it fell **because it became riskier**. Yet its beta fell. As a riskier stock, its required return should rise, yet the calculated return fell from 14.6% to 8.8%, which is only a little above the riskless rate.

The problem is that Y's low return tilted the regression line down—the point for Year 6 is in the lower right quadrant of the Excel graph. The low R^2 and the large standard error as seen in the Excel regression make it clear that the beta, and thus the calculated required return, are not to be trusted.

Note that in April 2001, the same month that PG&E declared bankruptcy, its beta as reported by Finance.Yahoo was only 0.05, so our hypothetical Stock Y did what the real PG&E actually did. The moral of the story is that the CAPM, like other cost of capital estimating techniques, can be dangerous if used without care and judgment.

One final point on all this: The utilities are regulated, and regulators estimate their cost of capital and use it as a basis for setting electric rates. If the estimated cost of capital is low, then the companies are only allowed to earn a low rate of return on their invested capital. At times, utilities like PG&E become more risky, have resulting low betas, and are then in danger of having some squirrelly finance "expert" argue that they should be allowed to earn an improper CAPM rate of return. In the industrial sector, a badly trained financial analyst with a dumb supervisor could make the same mistake, estimate the cost of capital to be below the true cost, and cause the company to make investments that should not be made.

	A	B	C	D	E	F	G	H	I
1	Worksheet for Chapter 2 BOC Questions (ch02boc-model.xls)							1/25/2011	
2									
3	We use BOC Question 2-6 to illustrate some points about the CAPM, the SML, and Excel. For additional								
4	information on Excel, see the Tool Kit for Chapter 2.								
8									Rate of Return Ca
9	The following returns were earned on the market and on Stocks X and Y during the last 5 years:								For the Market:
10									Ending
11			Percentage Returns						Price
12	Year	Market	Stock X	Stock Y					$100.00
13	1	20%	16%	28%					$118.00
14	2	8%	8%	8%					$124.94
15	3	15%	12%	20%					$140.68
16	4	-6%	-2%	-15%					$128.74
17	5	23%	18%	33%					$154.35
18	6	20%	16%	-70%					$180.72
19	Avg 1-5	12.0%	10.4%	14.8%					
20									
21	Beta X:		0.69	From					
22	Beta Y:		1.66	below					
23									
24	Could get betas by regression, but an easier way is to use the LINEST function. Click fx > Statistical >								
25	LINEST and then follow the menu to get beta X = 0.69 and beta Y = 1.66. Here's the completed dialog box for X. You								
26	can use the data to find beta to Y as an exercise, and also to find the revised beta based on years 2-6.								
27									
28									
29	Beta X:		0.69						
30	Beta Y:		1.66						

Beta Graph, Years 1-5 Only

LINEST

Known_y's	C13:C17 = {0.16;0.08;0.12;-0.
Known_x's	B13:B17 = {0.2;0.08;0.15;-0.0
Const	= logical
Stats	= logical

= {0.685393258426960

Returns an array that describes a straight line that best fits your data, calculated by usin the least squares method.

Known_x's is an optional set of x-values that you may already know in the relationship y = mx + b.

Formula result = 0.69   OK   Cance

	A	B	C	D	E	F	G	H	I
34	SML Analysis:								
35	Risk-free rate:		8.0%						
36	Market return:		12.0%						
37									
38									
39									
40									
41	r(X) = r(rf) +		b(r(Market) - r(fr))						
42		8.0%	+	2.7%	=	10.7%	= Predicted return for X.		
43									
44	r(Y) = r(rf) +		b(r(Market) - r(fr))						
45		8.0%	+	6.6%	=	14.6%	= Predicted return for Y.		
46									
47	New Beta Y:		0.19						
48									
49	New r(y):		8.8%						
50									
51	We could also use the statistical function RSQ to calculate the R-squares for the betas.								
52	For Y R-square dropped from 1.0 to .0029. This indicates that the beta, and the CAPM								
53	required return, are being measured with a lot of error. So, we cannot trust the accuracy								
54	of the new estimated required return.								

Answers and Solutions: **2 - 7**

ANSWERS TO END-OF-CHAPTER QUESTIONS

2-1   a. Stand-alone risk is only a part of total risk and pertains to the risk an investor takes by holding only one asset. Risk is the chance that some unfavorable event will occur. For instance, the risk of an asset is essentially the chance that the asset's cash flows will be unfavorable or less than expected. A probability distribution is a listing, chart or graph of all possible outcomes, such as expected rates of return, with a probability assigned to each outcome. When in graph form, the tighter the probability distribution, the less uncertain the outcome.

           b. The expected rate of return ($\hat{r}$) is the expected value of a probability distribution of expected returns.

           c. A continuous probability distribution contains an infinite number of outcomes and is graphed from $-\infty$ and $+\infty$.

           d. The standard deviation (σ) is a statistical measure of the variability of a set of observations. The variance (σ^2) of the probability distribution is the sum of the squared deviations about the expected value adjusted for deviation. The coefficient of variation (CV) is equal to the standard deviation divided by the expected return; it is a standardized risk measure which allows comparisons between investments having different expected returns and standard deviations.

           e. A risk averse investor dislikes risk and requires a higher rate of return as an inducement to buy riskier securities. A realized return is the actual return an investor receives on their investment. It can be quite different than their expected return.

           f. A risk premium is the difference between the rate of return on a risk-free asset and the expected return on Stock i which has higher risk. The market risk premium is the difference between the expected return on the market and the risk-free rate.

           g. CAPM is a model based upon the proposition that any stock's required rate of return is equal to the risk free rate of return plus a risk premium reflecting only the risk remaining after diversification.

           h. The expected return on a portfolio. $\hat{r}_p$, is simply the weighted-average expected return of the individual stocks in the portfolio, with the weights being the fraction of total portfolio value invested in each stock. The market portfolio is a portfolio consisting of all stocks.

i. Correlation is the tendency of two variables to move together. A correlation coefficient (ρ) of +1.0 means that the two variables move up and down in perfect synchronization, while a coefficient of -1.0 means the variables always move in opposite directions. A correlation coefficient of zero suggests that the two variables are not related to one another; that is, they are independent.

j. Market risk is that part of a security's total risk that cannot be eliminated by diversification. It is measured by the beta coefficient. Diversifiable risk is also known as company specific risk, that part of a security's total risk associated with random events not affecting the market as a whole. This risk can be eliminated by proper diversification. The relevant risk of a stock is its contribution to the riskiness of a well-diversified portfolio.

k. The beta coefficient is a measure of a stock's market risk, or the extent to which the returns on a given stock move with the stock market. The average stock's beta would move on average with the market so it would have a beta of 1.0.

l. The security market line (SML) represents in a graphical form, the relationship between the risk of an asset as measured by its beta and the required rates of return for individual securities. The SML equation is essentially the CAPM, $r_i = r_{RF} + b_i(r_M - r_{RF})$.

m. The slope of the SML equation is ($r_M - r_{RF}$), the market risk premium. The slope of the SML reflects the degree of risk aversion in the economy. The greater the average investors aversion to risk, then the steeper the slope, the higher the risk premium for all stocks, and the higher the required return.

2-2 a. The probability distribution for complete certainty is a vertical line.

b. The probability distribution for total uncertainty is the X axis from $-\infty$ to $+\infty$.

2-3 Security A is less risky if held in a diversified portfolio because of its lower beta and negative correlation with other stocks. In a single-asset portfolio, Security A would be more risky because $\sigma_A > \sigma_B$ and $CV_A > CV_B$.

2-4   a. No, it is not riskless. The portfolio would be free of default risk and liquidity risk, but inflation could erode the portfolio's purchasing power. If the actual inflation rate is greater than that expected, interest rates in general will rise to incorporate a larger inflation premium (IP) and the value of the portfolio would decline.

      b. No, you would be subject to reinvestment rate risk. You might expect to "roll over" the Treasury bills at a constant (or even increasing) rate of interest, but if interest rates fall, your investment income will decrease.

      c. A U.S. government-backed bond that provided interest with constant purchasing power (that is, an indexed bond) would be close to riskless.

2-5   The risk premium on a high beta stock would increase more.

$$RP_j = \text{Risk Premium for Stock } j = (r_M - r_{RF})b_j.$$

      If risk aversion increases, the slope of the SML will increase, and so will the market risk premium $(r_M - r_{RF})$. The product $(r_M - r_{RF})b_j$ is the risk premium of the jth stock. If b_j is low (say, 0.5), then the product will be small; RP_j will increase by only half the increase in RP_M. However, if b_j is large (say, 2.0), then its risk premium will rise by twice the increase in RP_M.

2-6   According to the Security Market Line (SML) equation, an increase in beta will increase a company's expected return by an amount equal to the market risk premium times the change in beta. For example, assume that the risk-free rate is 6 percent, and the market risk premium is 5 percent. If the company's beta doubles from 0.8 to 1.6 its expected return increases from 10 percent to 14 percent. Therefore, in general, a company's expected return will not double when its beta doubles.

2-7   Yes, if the portfolio's beta is equal to zero. In practice, however, it may be impossible to find individual stocks that have a nonpositive beta. In this case it would also be impossible to have a stock portfolio with a zero beta. Even if such a portfolio could be constructed, investors would probably be better off just purchasing Treasury bills, or other zero beta investments.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

2-1

Investment	Beta
$20,000	0.7
35,000	1.3
Total $55,000	

$(\$20,000/\$55,000)(0.7) + (\$35,000/\$55,000)(1.3) = 1.08$.

2-2 $r_{RF} = 4\%$; $r_M = 12\%$; $b = 0.8$; $r_s = ?$

$$\begin{aligned} r_s &= r_{RF} + (r_M - r_{RF})b \\ &= 4\% + (12\% - 4\%)0.8 \\ &= 10.4\%. \end{aligned}$$

2-3 $r_{RF} = 5\%$; $RP_M = 7\%$; $r_M = ?$

$r_M = 5\% + (7\%)1 = 12\% = r_s$ when $b = 1.0$.

r_s when $b = 1.7 = ?$

$r_s = 5\% + 7\%(1.7) = 16.9\%$.

2-4 $\hat{r} = (0.1)(-50\%) + (0.2)(-5\%) + (0.4)(16\%) + (0.2)(25\%) + (0.1)(60\%)$
$= 11.40\%$.

$$\begin{aligned} \sigma^2 &= (-50\% - 11.40\%)^2(0.1) + (-5\% - 11.40\%)^2(0.2) + (16\% - 11.40\%)^2(0.4) \\ &\quad + (25\% - 11.40\%)^2(0.2) + (60\% - 11.40\%)^2(0.1) \end{aligned}$$
$\sigma^2 = 712.44$; $\sigma = 26.69\%$.

$$CV = \frac{26.69\%}{11.40\%} = 2.34.$$

2-5   a. $\hat{r}_m = (0.3)(15\%) + (0.4)(9\%) + (0.3)(18\%) = 13.5\%$.

          $\hat{r}_j = (0.3)(20\%) + (0.4)(5\%) + (0.3)(12\%) = 11.6\%$.

   b. $\sigma_M = [(0.3)(15\% - 13.5\%)^2 + (0.4)(9\% - 13.5\%)^2 + (0.3)(18\% - 13.5\%)^2]^{1/2}$

            $= \sqrt{14.85\%} = 3.85\%$.

       $\sigma_J = [(0.3)(20\% - 11.6\%)^2 + (0.4)(5\% - 11.6\%)^2 + (0.3)(12\% - 11.6\%)^2]^{1/2}$

            $= \sqrt{38.64\%} = 6.22\%$.

   c. $CV_M = \dfrac{3.85\%}{13.5\%} = 0.29$.

       $CV_J = \dfrac{6.22\%}{11.6\%} = 0.54$.

2-6   a. $r_A = r_{RF} + (r_M - r_{RF})b_A$
         $12\% = 5\% + (10\% - 5\%)b_A$
         $12\% = 5\% + 5\%(b_A)$
         $7\% = 5\%(b_A)$
         $1.4 = b_A$.

   b. $r_A = 5\% + 5\%(b_A)$
         $r_A = 5\% + 5\%(2)$
         $r_A = 15\%$.

2-7 a. $r_i = r_{RF} + (r_M - r_{RF})b_i = 5\% + (12\% - 5\%)1.4 = 14.8\%$.

b. 1. r_{RF} increases to 6%:

r_M increases by 1 percentage point, from 12% to 13%.

$r_i = r_{RF} + (r_M - r_{RF})b_i = 6\% + (12\% - 5\%)1.4 = 15.8\%$.

2. r_{RF} decreases to 4%:

r_M decreases by 1%, from 12% to 11%.

$r_i = r_{RF} + (r_M - r_{RF})b_i = 4\% + (12\% - 5\%)1.4 = 13.8\%$.

c. 1. r_M increases to 14%:

$r_i = r_{RF} + (r_M - r_{RF})b_i = 5\% + (14\% - 5\%)1.4 = 17.6\%$.

2. r_M decreases to 11%:

$r_i = r_{RF} + (r_M - r_{RF})b_i = 5\% + (11\% - 5\%)1.4 = 13.4\%$.

2-8 Old portfolio beta $= \dfrac{\$70,000}{\$75,000}(b) + \dfrac{\$5,000}{\$75,000}(0.8)$

$1.2 = 0.9333b + 0.0533$
$1.1467 = 0.9333b$
$1.229 = b$.

New portfolio beta $= 0.9333(1.229) + 0.0667(1.6) = 1.25$.

Alternative Solutions:

1. Old portfolio beta $= 1.2 = (0.0667)b_1 + (0.0667)b_2 + ... + (0.0667)b_{20}$

$1.2 = (\Sigma b_i)(0.0667)$
$\Sigma b_i = 1.2/0.0667 = 18.0$.

New portfolio beta $= (18.0 - 0.8 + 1.6)(0.0667) = 1.253 = 1.25$.

2. Σb_i excluding the stock with the beta equal to 0.8 is $18.0 - 0.8 = 17.2$, so the beta of the portfolio excluding this stock is $b = 17.2/14 = 1.2286$. The beta of the new portfolio is:

$$1.2286(0.9333) + 1.6(0.0667) = 1.1575 = 1.253.$$

2-9   Portfolio beta $= \dfrac{\$400,000}{\$4,000,000}(1.50) + \dfrac{\$600,000}{\$4,000,000}(-0.50)$

$+ \dfrac{\$1,000,000}{\$4,000,000}(1.25) + \dfrac{\$2,000,000}{\$4,000,000}(0.75)$

$= 0.1)(1.5) + (0.15)(-0.50) + (0.25)(1.25) + (0.5)(0.75)$

$= 0.15 - 0.075 + 0.3125 + 0.375 = 0.7625.$

$r_p = r_{RF} + (r_M - r_{RF})(b_p) = 6\% + (14\% - 6\%)(0.7625) = 12.1\%.$

Alternative solution: First compute the return for each stock using the CAPM equation $[r_{RF} + (r_M - r_{RF})b]$, and then compute the weighted average of these returns.

$r_{RF} = 6\%$ and $r_M - r_{RF} = 8\%$.

Stock	Investment	Beta	$r = r_{RF} + (r_M - r_{RF})b$	Weight
A	$ 400,000	1.50	18%	0.10
B	600,000	(0.50)	2	0.15
C	1,000,000	1.25	16	0.25
D	2,000,000	0.75	12	0.50
Total	$4,000,000			1.00

$r_p = 18\%(0.10) + 2\%(0.15) + 16\%(0.25) + 12\%(0.50) = 12.1\%.$

2-10   First, calculate the beta of what remains after selling the stock:

$b_p = 1.1 = (\$100,000/\$2,000,000)0.9 + (\$1,900,000/\$2,000,000)b_R$
$1.1 = 0.045 + (0.95)b_R$
$b_R = 1.1105.$

$b_N = (0.95)1.1105 + (0.05)1.4 = 1.125.$

2-11   We know that $b_R = 1.50$, $b_S = 0.75$, $r_M = 13\%$, $r_{RF} = 7\%$.

$r_i = r_{RF} + (r_M - r_{RF})b_i = 7\% + (13\% - 7\%)b_i$.

$r_R = 7\% + 6\%(1.50) = 16.0\%$
$r_S = 7\% + 6\%(0.75) = \underline{11.5}$
$\underline{4.5}\%$

2-12   The answers to a, b, c, and d are given below:

	$\bar{r}_A$	$\bar{r}_B$	Portfolio
2008	(20.00%)	(5.00%)	(12.50%)
2009	42.00	15.00	28.50
2010	20.00	(13.00)	3.50
2011	(8.00)	50.00	21.00
2012	25.00	12.00	18.50
Mean	11.80	11.80	11.80
Std Dev	25.28	24.32	16.34
CV	2.14	2.06	1.38

   e. A risk-averse investor would choose the portfolio over either Stock A or Stock B alone, since the portfolio offers the same expected return but with less risk. This result occurs because returns on A and B are not perfectly positively correlated (ρ_{AB} = -0.13).

2-13   a. $b_X = 1.3471$; $b_Y = 0.6508$. These can be calculated with a spreadsheet.

   b. $r_X = 6\% + (5\%)1.3471 = 12.7355\%$.
      $r_Y = 6\% + (5\%)0.6508 = 9.2540\%$.

   c. $b_p = 0.8(1.3471) + 0.2(0.6508) = 1.2078$.
      $r_p = 6\% + (5\%)1.2078 = 12.04\%$.
      Alternatively,
      $r_p = 0.8(12.7355\%) + 0.2(9.254\%) = 12.04\%$.

   d. Stock X is undervalued, because its expected return exceeds its required rate of return.

SOLUTION TO SPREADSHEET PROBLEM

2-14   The detailed solution for the spreadsheet problem is available in the file ***Solution to Ch02-P14 Build a Model.xls*** at the textbook's Web site.

MINI CASE

Assume that you recently graduated with a major in finance, and you just landed a job as a financial planner with Barney Smith Inc., a large financial services corporation. Your first assignment is to invest $100,000 for a client. Because the funds are to be invested in a new business the client plans to start at the end of one year, you have been instructed to plan for a one-year holding period. Further, your boss has restricted you to the investment alternatives shown in the table below. (Disregard for now the items at the bottom of the data; you will fill in the blanks later.)

| | | | Returns On Alternative Investments | | | | |
| | | | Estimated Rate Of Return | | | | |
State of the economy	Prob.	T-Bills	Alta Inds	Repo Men	Am. Foam	Market portfolio	2-stock portfolio
Recession	0.1	8.0%	-22.0%	28.0%	10.0%*	-13.0%	3.0%
Below avg	0.2	8.0	-2.0	14.7	-10.0	1.0	
Average	0.4	8.0	20.0	0.0	7.0	15.0	10.0
Above avg	0.2	8.0	35.0	-10.0	45.0	29.0	
Boom	0.1	8.0	50.0	-20.0	30.0	43.0	15.0
r-hat ($\hat{r}$)		8.0%		1.7%	13.8%	15.0%	
Std dev (σ)		0.0		13.4	18.8	15.3	
Coef of var (cv)				7.9	1.4	1.0	
beta (b)				-0.87	0.89		

*Note that the estimated returns of American Foam do not always move in the same direction as the overall economy. For example, when the economy is below average, consumers purchase fewer mattresses than they would if the economy were stronger. However, if the economy is in a flat-out recession, a large number of consumers who were planning to purchase a more expensive inner spring mattress may purchase, instead, a cheaper foam mattress. Under these circumstances, we would expect American Foam's stock price to be higher if there is a recession than if the economy was just below average.

> **Barney Smith's economic forecasting staff has developed probability estimates for the state of the economy, and its security analysts have developed a sophisticated computer program that was used to estimate the rate of return on each alternative under each state of the economy. Alta Industries is an electronics firm; Repo Men collects past-due debts; and American Foam manufactures mattresses and various other foam products. Barney Smith also maintains an "index fund" which owns a market-weighted fraction of all publicly traded stocks; you can invest in that fund, and thus obtain average stock market results. Given the situation as described, answer the following questions.**

---

a.   What are investment returns? What is the return on an investment that costs $1,000 and is sold after one year for $1,100?

Answer: Investment return measures the financial results of an investment. They may be expressed in either dollar terms or percentage terms.
        The dollar return is $1,100 - $1,000 = $100. The percentage return is $100/$1,000 = 0.10 = 10%.

---

b.   1. Why is the t-bill's return independent of the state of the economy? Do t-bills promise a completely risk-free return?

Answer: The 8 percent t-bill return does not depend on the state of the economy because the treasury must (and will) redeem the bills at par regardless of the state of the economy.
        The t-bills are risk-free in the default risk sense because the 8 percent return will be realized in all possible economic states. However, remember that this return is composed of the real risk-free rate, say 3 percent, plus an inflation premium, say 5 percent. Since there is uncertainty about inflation, it is unlikely that the realized <u>real</u> rate of return would equal the expected 3 percent. For example, if inflation averaged 6 percent over the year, then the realized real return would only be 8% - 6% = 2%, not the expected 3%. Thus, in terms of purchasing power, t-bills are not riskless.
        Also, if you invested in a portfolio of T-bills, and rates then declined, your nominal income would fall; that is, t-bills are exposed to <u>reinvestment rate risk</u>. So, we conclude that there are no truly risk-free securities in the United States. If the treasury sold inflation-indexed, tax-exempt bonds, they would be truly riskless, but all actual securities are exposed to some type of risk.

b.   **2. Why are Alta Ind.'s returns expected to move with the economy whereas Repo Men's are expected to move counter to the economy?**

Answer: Alta Industries' returns move with, hence are positively correlated with, the economy, because the firm's sales, and hence profits, will generally experience the same type of ups and downs as the economy. If the economy is booming, so will Alta. On the other hand, Repo Men is considered by many investors to be a hedge against both bad times and high inflation, so if the stock market crashes, investors in this stock should do relatively well. Stocks such as Repo Men are thus <u>negatively correlated</u> with (move counter to) the economy. (note: in actuality, it is almost impossible to find stocks that are <u>expected</u> to move counter to the economy. Even Repo Men shares have positive (but low) correlation with the market.)

c.   **Calculate the expected rate of return on each alternative and fill in the blanks on the row for $\hat{r}$ in the table.**

Answer: The expected rate of return, $\hat{r}$, is expressed as follows:

$$\hat{r} = \sum_{i=1}^{n} P_i\, r_i.$$

Here P_i is the probability of occurrence of the ith state, r_i is the estimated rate of return for that state, and n is the number of states. Here is the calculation for Alta Inds.:

$$\hat{r}_{\text{Alta Inds}} = 0.1(-22.0\%) + 0.2(-2.0\%) + 0.4(20.0\%) + 0.2(35.0\%) + 0.1(50.0\%)$$
$$= 17.4\%.$$

We use the same formula to calculate r's for the other alternatives:

$$\hat{r}_{\text{T-bills}} = 8.0\%.$$
$$\hat{r}_{\text{Repo Men}} = 1.7\%.$$
$$\hat{r}_{\text{Am Foam}} = 13.8\%.$$
$$\hat{r}_{M} = 15.0\%.$$

d. **You should recognize that basing a decision solely on expected returns is appropriate only for risk-neutral individuals. Because your client, like virtually everyone, is risk averse, the riskiness of each alternative is an important aspect of the decision. One possible measure of risk is the standard deviation of returns.**

1. Calculate this value for each alternative, and fill in the blank on the row for σ in the table.

Answer: The standard deviation is calculated as follows:

$$\sigma = \sqrt{\sum_{i=1}^{n} (r_i - \hat{r}_i)^2 P_i} \; .$$

$$
\begin{aligned}
\sigma_{Alta} &= [(-22.0 - 17.4)^2(0.1) + (-2.0 - 17.4)^2(0.2) + (20.0 - 17.4)^2(0.4) \\
&\quad + (35.0 - 17.4)^2(0.2) + (50.0 - 17.4)^2(0.1)]^{0.5} \\
&= \sqrt{401.4} = 20.0\%.
\end{aligned}
$$

Here are the standard deviations for the other alternatives:

$$\sigma_{\text{T-bills}} = 0.0\%.$$

$$\sigma_{\text{Repo}} = 13.4\%.$$

$$\sigma_{\text{Am Foam}} = 18.8\%.$$

$$\sigma_{\text{M}} = 15.3\%.$$

d. 2. What type of risk is measured by the standard deviation?

Answer: The standard deviation is a measure of a security's (or a portfolio's) <u>stand-alone risk</u>. The larger the standard deviation, the higher the probability that actual realized returns will fall far below the expected return, and that losses rather than profits will be incurred.

d.   3. Draw a graph that shows *roughly* the shape of the probability distributions for Alta Industries, American Foam, and T-bills.

Answer:
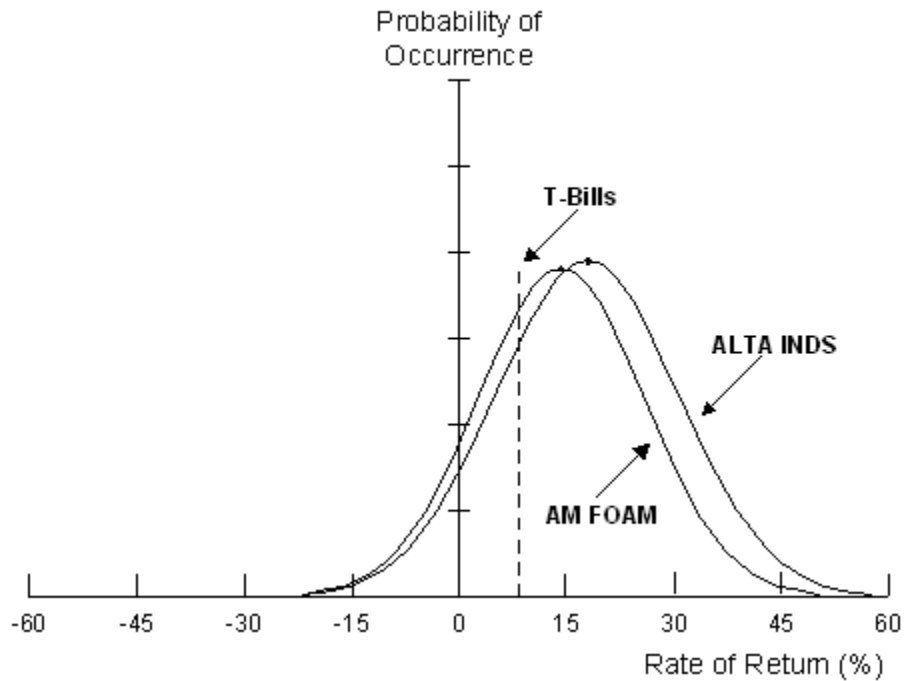
Based on these data, Alta Inds is the most risky investment, t-bills the least risky.

e.	**Suppose you suddenly remembered that the coefficient of variation (CV) is generally regarded as being a better measure of stand-alone risk than the standard deviation when the alternatives being considered have widely differing expected returns. Calculate the missing CVs, and fill in the blanks in the row for CV in the table. Does the CV produce the same risk rankings as the standard deviation?**

Answer: The coefficient of variation (CV) is a standardized measure of dispersion about the expected value; it shows the amount of risk per unit of return.

$$CV = \frac{\sigma}{\hat{r}}.$$

$$CV_{\text{T-bills}} = 0.0\%/8.0\% = 0.0.$$

$$CV_{\text{Alta Inds}} = 20.0\%/17.4\% = 1.1.$$

$$CV_{\text{Repo Men}} = 13.4\%/1.7\% = 7.9.$$

$$CV_{\text{Am Foam}} = 18.8\%/13.8\% = 1.4.$$

$$CV_{\text{M}} = 15.3\%/15.0\% = 1.0.$$

When we measure risk per unit of return, Repo Men, with its low expected return, becomes the most risky stock. The CV is a better measure of an asset's stand-alone risk than σ because CV considers both the expected value and the dispersion of a distribution--a security with a low expected return and a low standard deviation could have a higher chance of a loss than one with a high σ but a high $\hat{r}$.

f.	**Suppose you created a 2-stock portfolio by investing $50,000 in Alta Industries and $50,000 in Repo Men.**
	1. Calculate the expected return ($\hat{r}_p$), the standard deviation (σ_p), and the coefficient of variation (CV_p) for this portfolio and fill in the appropriate blanks in the table.

Answer: To find the expected rate of return on the two-stock portfolio, we first calculate the rate of return on the portfolio in each state of the economy. Since we have half of our money in each stock, the portfolio's return will be a weighted average in each type of economy. For a recession, we have: $r_p = 0.5(-22\%) + 0.5(28\%) = 3\%$. We would do similar calculations for the other states of the economy, and get these results:

State	Portfolio
Recession	3.0%
Below Average	6.4
Average	10.0
Above Average	12.5
Boom	15.0

Now we can multiply probabilities times outcomes in each state to get the expected return on this two-stock portfolio, 9.6%.

Alternatively, we could apply this formula,

$$R = w_i \times r_i = 0.5(17.4\%) + 0.5(1.7\%) = 9.6\%,$$

Which finds r as the weighted average of the expected returns of the individual securities in the portfolio.

It is tempting to find the standard deviation of the portfolio as the weighted average of the standard deviations of the individual securities, as follows:

$$\sigma_p \neq w_i(\sigma_i) + w_j(\sigma_j) = 0.5(20\%) + 0.5(13.4\%) = 16.7\%.$$

However, this is not correct--it is necessary to use a different formula, the one for σ that we used earlier, applied to the two-stock portfolio's returns.

The portfolio's σ depends jointly on (1) each security's σ and (2) the correlation between the securities' returns. The best way to approach the problem is to estimate the portfolio's risk and return in each state of the economy, and then to estimate σ_p with the σ formula. Given the distribution of returns for the portfolio, we can calculate the portfolio's σ and CV as shown below:

$$\begin{aligned}
\sigma_p &= [(3.0 - 9.6)^2(0.1) + (6.4 - 9.6)^2(0.2) + (10.0 - 9.6)^2(0.4) \\
&\quad + (12.5 - 9.6)^2(0.2) + (15.0 - 9.6)^2(0.1)]^{0.5} \\
&= 3.3\%.
\end{aligned}$$

$$CV_p = 3.3\%/9.6\% = 0.3.$$

---

f. 2. How does the risk of this 2-stock portfolio compare with the risk of the individual stocks if they were held in isolation?

Answer: Using either σ or CV as our stand-alone risk measure, the stand-alone risk of the portfolio is significantly less than the stand-alone risk of the individual stocks. This is because the two stocks are negatively correlated--when Alta Inds is doing poorly, Repo Men is doing well, and vice versa. Combining the two stocks diversifies away some of the risk inherent in each stock if it were held in isolation, i.e., in a 1-stock portfolio.

g.	**Suppose an investor starts with a portfolio consisting of one randomly selected stock. What would happen (1) to the risk and (2) to the expected return of the portfolio as more and more randomly selected stocks were added to the portfolio? What is the implication for investors? Draw a graph of the two portfolios to illustrate your answer.**

Answer:
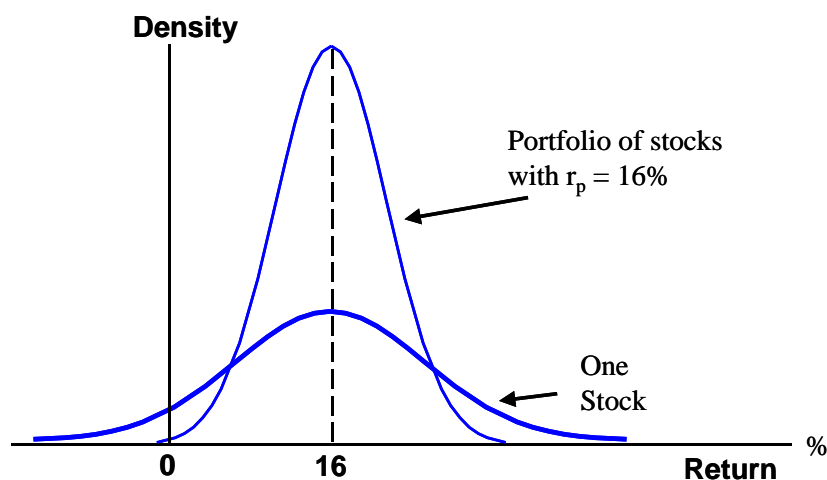
The standard deviation gets smaller as more stocks are combined in the portfolio, while r_p (the portfolio's return) remains constant. Thus, by adding stocks to your portfolio, which initially started as a 1-stock portfolio, risk has been reduced.

In the real world, stocks are positively correlated with one another--if the economy does well, so do stocks in general, and vice versa. Correlation coefficients between stocks generally range from +0.5 to +0.7. The average correlation between stocks is about 0.35. A single stock selected at random would on average have a standard deviation of about 35 percent. As additional stocks are added to the portfolio, the portfolio's standard deviation decreases because the added stocks are not perfectly positively correlated. However, as more and more stocks are added, each new stock has less of a risk-reducing impact, and eventually adding additional stocks has virtually no effect on the portfolio's risk as measured by σ. In fact, σ stabilizes at about 20 percent when 40 or more randomly selected stocks are added. Thus, by combining stocks into well-diversified portfolios, investors can eliminate almost one-half the riskiness of holding individual stocks. (Note: it is not completely costless to diversify, so even the largest institutional investors hold less than all stocks. Even index funds generally hold a smaller portfolio which is highly correlated with an index such as the S&P 500 rather than hold all the stocks in the index.)

The implication is clear: <u>investors should hold well-diversified portfolios of stocks rather than individual stocks.</u> (In fact, individuals can hold diversified portfolios through mutual fund investments.) By doing so, they can eliminate about half of the riskiness inherent in individual stocks.

---

h.   1. Should portfolio effects impact the way investors think about the risk of individual stocks?

Answer: Portfolio diversification does affect investors' views of risk. A stock's stand-alone risk as measured by its σ or CV, may be important to an undiversified investor, but it is not relevant to a well-diversified investor. A rational, risk-averse investor is more interested in the impact that the stock has on the riskiness of his or her portfolio than on the stock's stand-alone risk. Stand-alone risk is composed of diversifiable risk, which can be eliminated by holding the stock in a well-diversified portfolio, and the risk that remains is called market risk because it is present even when the entire market portfolio is held.

h.   2. **If you decided to hold a 1-stock portfolio, and consequently were exposed to more risk than diversified investors, could you expect to be compensated for all of your risk; that is, could you earn a risk premium on that part of your risk that you could have eliminated by diversifying?**

Answer: If you hold a one-stock portfolio, you will be exposed to a high degree of risk, but you won't be compensated for it. If the return were high enough to compensate you for your high risk, it would be a bargain for more rational, diversified investors. They would start buying it, and these buy orders would drive the price up and the return down. Thus, you simply could not find stocks in the market with returns high enough to compensate you for the stock's diversifiable risk.

i. **How is market risk measured for individual securities?   How are beta coefficients calculated?**

Answer: Market risk, which is relevant for stocks held in well-diversified portfolios, is defined as the contribution of a security to the overall riskiness of the portfolio. It is measured by a stock's beta coefficient, which measures the stock's volatility relative to the market.

Run a regression with returns on the stock in question plotted on the y axis and returns on the market portfolio plotted on the x axis.

The slope of the regression line, which measures relative volatility, is defined as the stock's beta coefficient, or b.

j. **Suppose you have the following historical returns for the stock market and for another company, P.Q. Unlimited. Explain how to calculate beta, and use the historical stock returns to calculate the beta for PQU. Interpret your results.**

YEAR	MARKET	PQU
1	25.7%	40.0%
2	8.0%	-15.0%
3	-11.0%	-15.0%
4	15.0%	35.0%
5	32.5%	10.0%
6	13.7%	30.0%
7	40.0%	42.0%
8	10.0%	-10.0%
9	-10.8%	-25.0%
10	-13.1%	25.0%

Answer: Betas are calculated as the slope of the "characteristic" line, which is the regression line showing the relationship between a given stock and the general stock market.

$$r_{PQU} = 0.83r_M + 0.03$$
$$R^2 = 0.36$$
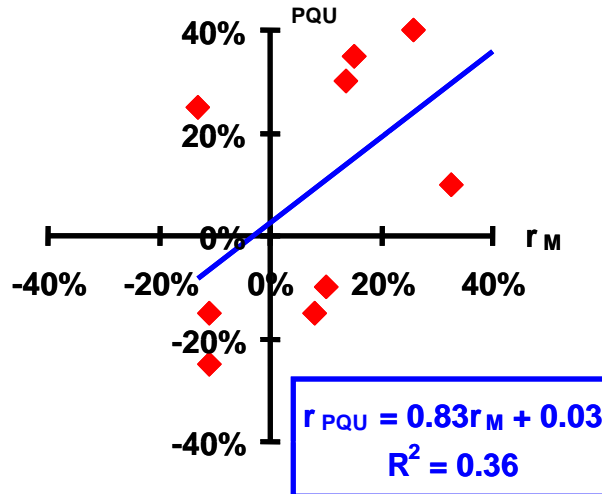
Show the graph with the regression results. Point out that the beta is the slope coeeficient, which is 0.83. State that an average stock, by definition, moves with the market. Beta coefficients measure the relative volatility of a given stock relative to the stock market. The average stock's beta is 1.0. Most stocks have betas in the range of 0.5 to 1.5. Theoretically, betas can be negative, but in the real world they are generally positive.

In practice, 4 or 5 years of monthly data, with 60 observations, would generally be used. Some analysts use 52 weeks of weekly data. Point out that the r^2 of 0.36 is slightly higher than the typical value of about 0.29. A portfolio would have an r^2 greater than 0.9.

k. The expected rates of return and the beta coefficients of the alternatives as supplied by Barney Smith's computer program are as follows:

Security	Return ($\hat{r}$)	Risk (Beta)
Alta Inds	17.4%	1.30
Market	15.0	1.00
Am. Foam	13.8	0.89
T-Bills	8.0	0.00
Repo Men	1.7	(0.87)

(1) Do the expected returns appear to be related to each alternative's market risk?
(2) Is it possible to choose among the alternatives on the basis of the information developed thus far?

Answer: The expected returns are related to each alternative's market risk--that is, the higher the alternative's rate of return the higher its beta. Also, note that t-bills have 0 risk.

We do not yet have enough information to choose among the various alternatives. We need to know the required rates of return on these alternatives and compare them with their expected returns.

l.   **1. Write out the Security Market Line (SML) equation, use it to calculate the required rate of return on each alternative, and then graph the relationship between the expected and required rates of return.**

Answer: Here is the SML equation:

$$r_i = r_{rf} + (r_m - r_{rf})b_i.$$

If we use the t-bill yield as a proxy for the risk-free rate, then $r_{RF} = 8\%$. Further, our estimate of $r_m = \hat{r}_m$ is 15%. Thus, the required rates of return for the alternatives are as follows:

Alta Inds: $8\% + (15\% - 8\%)1.30 = 17.1\%$.

Market: $8\% + (15\% - 8\%)1.00 = 15.0\%$.

Am Foam : $8\% + (15\% - 8\%)0.89 = 14.2\%$.

T-Bills: $8\% + (15\% - 8\%)0.00 = 8.0\%$.

Repo Men: $8\% + (15\% - 8\%)-0.87 = 1.9\%$.

l.   **2. How do the expected rates of return compare with the required rates of return?**

Answer: We have the following relationships:

SECURITY	Expected Return $(\hat{r})$	Required Return (r)	CONDITION
Alta Inds	17.4%	17.1%	Undervalued: $\hat{r} > R$
Market	15.0	15.0	Fairly Valued (Market Equilibrium)
Am Foam	13.8	14.2	Overvalued: $R > \hat{r}$
T-Bills	8.0	8.0	Fairly Valued
Repo Men	1.7	1.9	Overvalued: $R > \hat{r}$

SML: $r_i = r_{RF} + RP_M \ b_i$
$= 8\% + 7\% \ (b)$

Required and Expected Rates of Return
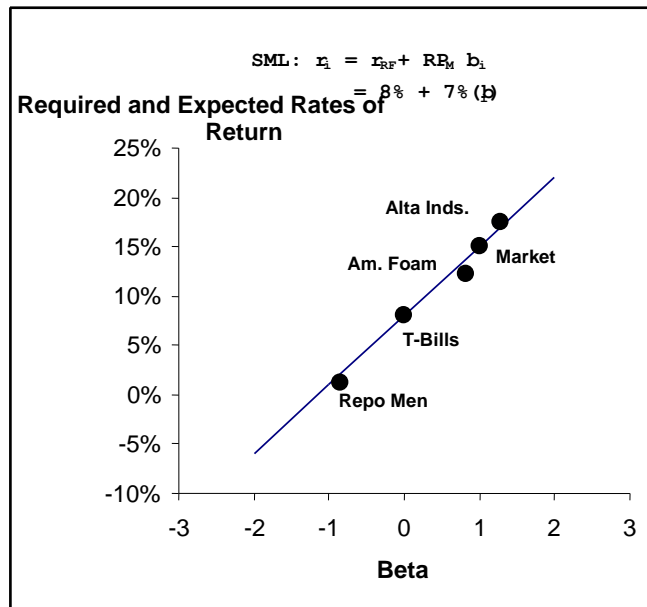
(Note: the plot looks somewhat unusual in that the x axis extends to the left of zero. We have a negative beta stock, hence a required return that is less than the risk-free rate.) The t-bills and market portfolio plot on the SML, Alta Inds. plots above it, and Am. Foam and Repo Men plot below it. Thus, the t-bills and the market portfolio promise a fair return, Alta Inds is a good deal because it has an expected return above its required return, and Am. Foam and Repo Men have expected returns below their required returns.

---

l. 3. Does the fact that Repo Men has an expected return that is less than the T-bill rate make any sense?

Answer: Repo Men is an interesting stock. Its negative beta indicates negative market risk--including it in a portfolio of "normal" stocks will lower the portfolio's risk. Therefore, its required rate of return is below the risk-free rate. Basically, this means that Repo Men is a valuable security to rational, well-diversified investors. To see why, consider this question: would any rational investor ever make an investment which has a <u>negative</u> expected return? The answer is "yes"--just think of the purchase of a life or fire insurance policy. The fire insurance policy has a negative expected return because of commissions and insurance company profits, but businesses buy fire insurance because they pay off at a time when normal operations are in bad shape. Life insurance is similar--it has a high return when work income ceases. A negative beta stock is conceptually similar to an insurance policy.

l.   4.   What would be the market risk and the required return of a 50-50 portfolio of Alta Industries and Repo Men? Of Alta Industries and American Foam?

Answer: Note that the beta of a portfolio is simply the weighted average of the betas of the stocks in the portfolio. Thus, the beta of a portfolio with 50 percent Alta Inds and 50 percent Repo Men is:

$$b_p = \sum_{i=1}^{n} w_i b_i .$$

$$b_p = 0.5(b_{Alta}) + 0.5(b_{Repo}) = 0.5(1.30) + 0.5(-0.87)$$
$$= 0.215,$$

$$r_p = r_{RF} + (r_M - r_{RF})b_p = 8.0\% + (15.0\% - 8.0\%)(0.215)$$
$$= 8.0\% + 7\%(0.215) = 9.51\% \approx 9.5\%.$$

For a portfolio consisting of 50% Alta Inds plus 50% Am. Foam, the required return would be 14.9%:

$$b_p = 0.5(1.30) + 0.5(0.89) = 1.095.$$

$$r_p = 8.0\% + 7\%(1.095) = 15.7\%.$$

m.   1.   Suppose investors raised their inflation expectations by 3 percentage points over current estimates as reflected in the 8 percent T-bill rate. What effect would higher inflation have on the SML and on the returns required on high- and low-risk securities?

Answer:
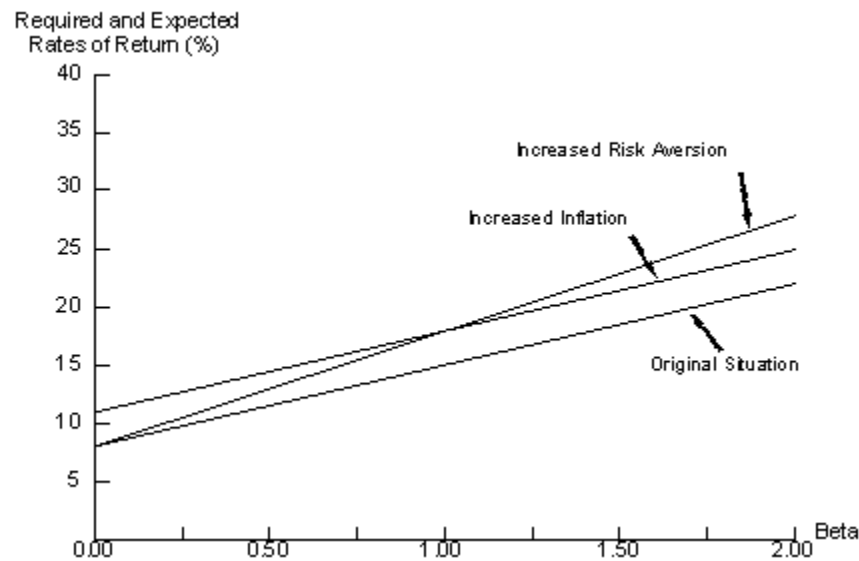
Here we have plotted the SML for betas ranging from 0 to 2.0. The base case SML is based on $r_{RF} = 8\%$ and $r_M = 15\%$. If inflation expectations increase by 3 percentage points, with no change in risk aversion, then the entire SML is shifted upward (parallel to the base case SML) by 3 percentage points. Now, $r_{RF} = 11\%$, $r_M = 18\%$, and all securities' required returns rise by 3 percentage points. Note that the market risk premium, $r_m - r_{RF}$, remains at 7 percentage points.

m.   2.   Suppose instead that investors' risk aversion increased enough to cause the market risk premium to increase by 3 percentage points. (Inflation remains constant.) What effect would this have on the SML and on returns of high- and low-risk securities?

Answer: When investors' risk aversion increases, the SML is rotated upward about the y-intercept (r_{RF}). r_{RF} remains at 8 percent, but now r_M increases to 18 percent, so the market risk premium increases to 10 percent. The required rate of return will rise sharply on high-risk (high-beta) stocks, but not much on low-beta securities.

Solutions to Problems

2B-1   a.

$\bar{r}_Y(\%)$
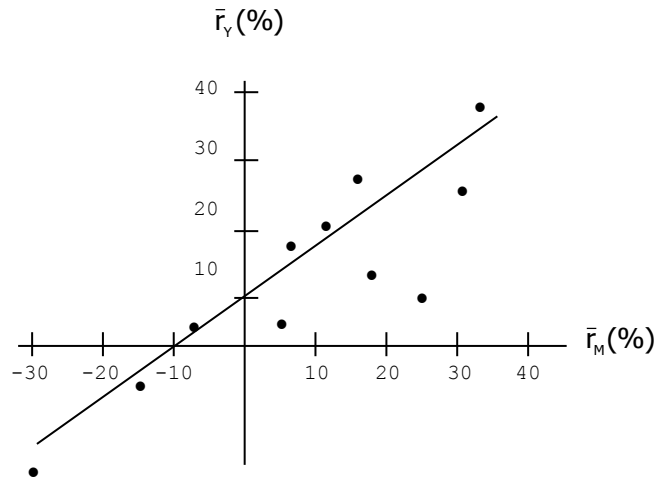
b = Slope = 0.62. However, b will depend on students' freehand line. Using a calculator, we find b = 0.6171 ≈ 0.62.

b.   Because b = 0.62, Stock Y is about 62% as volatile as the market; thus, its relative risk is about 62% that of an average stock.

c.   1.   Stand-alone risk as measured by σ would be greater, but beta and hence systematic (relevant) risk would remain unchanged. However, in a 1-stock portfolio, Stock Y would be riskier under the new conditions.

2.   CAPM assumes that company-specific risk will be eliminated in a portfolio, so the risk premium under the CAPM would not be affected. However, if the scatter were wide, we would not have as much confidence in the beta, and this could increase the stock's risk and thus its risk premium.

d.   1.   The stock's variance and σ would not change, but the risk of the stock to an investor holding a diversified portfolio would be greatly reduced, because it would now have a negative correlation with r_M.

2.   Because of a relative scarcity of such stocks and the beneficial net effect on portfolios that include it, its "risk premium" is likely to be very low or even negative. Theoretically, it should be negative.

e. The following figure shows a possible set of probability distributions. We can be reasonably sure that the 100-stock portfolio comprised of b = 0.62 stocks as described in Condition 2 will be less risky than the "market." Hence, the distribution for Condition 2 will be more peaked than that of Condition 3.
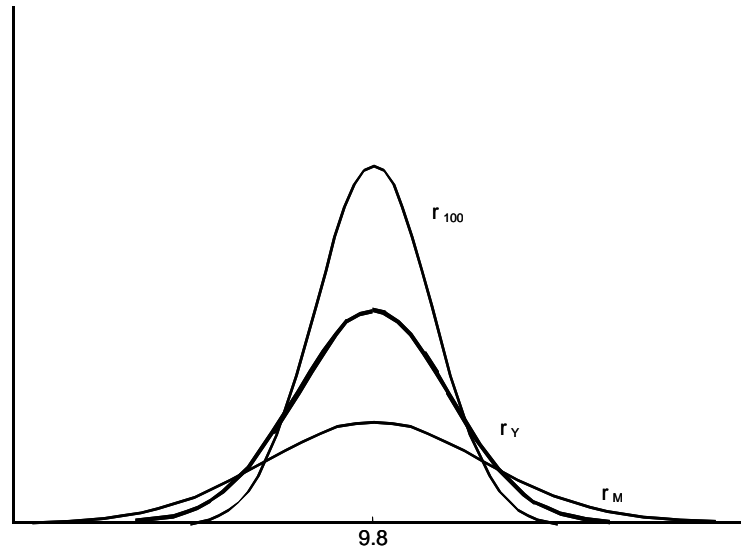
We can also say on the basis of the available information that σ_Y is smaller than σ_M; Stock Y's *market* risk is only 62% of the "market," but it does have company-specific risk, while the market portfolio does not, because it has been diversified away. However, we know from the given data that $\sigma_Y = 13.8\%$, while $\sigma_M = 19.6\%$. Thus, we have drawn the distribution for the single stock portfolio more peaked than that of the market. The relative rates of return are *not* reasonable. The return for any stock should be

$$r_i = r_{RF} + (r_M - r_{RF})b_i.$$

Stock Y has b = 0.62, while the average stock (M) has b = 1.0; therefore,

$$r_y = r_{RF} + (r_M - r_{RF})0.62 < r_M = r_{RF} + (r_M - r_{RF})1.0.$$

A disequilibrium exists—Stock Y should be bid up to drive its yield down. More likely, however, the data simply reflect the fact that past returns are not an exact basis for expectations of future returns.

f. The expected return could not be predicted with the historical characteristic line because the increased risk should change the beta used in the characteristic line.

g. The beta would decline to 0.53. A decline indicates that the stock has become less risky; however, with the change in the debt ratio the stock has actually become more risky. In periods of transition, when the risk of the firm is changing, the beta can yield conclusions that are exactly opposite to the actual facts. Once the company's risk stabilizes, the calculated beta should rise and should again approximate the true beta.

2B-2 a.
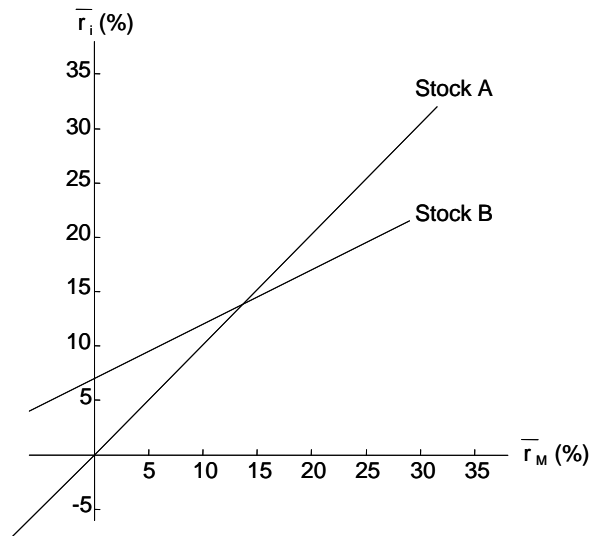
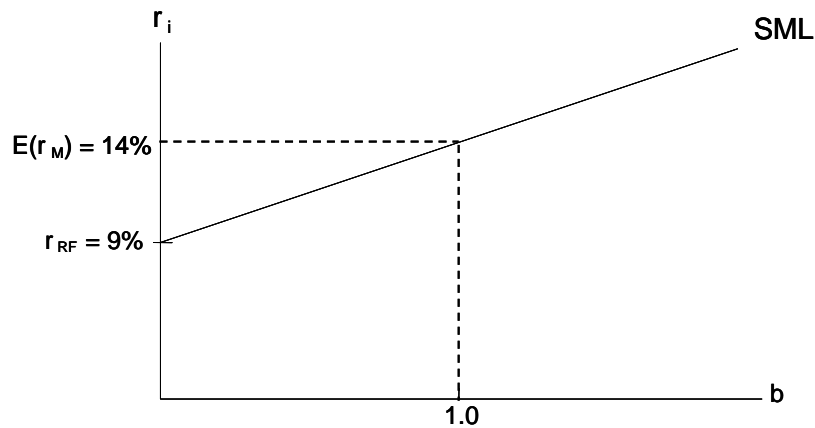The slope of the characteristic line is the stock's beta coefficient.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta r_i}{\Delta \bar{r}_M} .$$

$$\text{Slope}_A = \text{Beta}_A = \frac{29.00 - 15.20}{29.00 - 15.20} = 1.0.$$

$$\text{Slope}_B = \text{Beta}_B = \frac{20.00 - 13.10}{29.00 - 15.20} = 0.5.$$

b. $\hat{r}_M = 0.1(-14\%) + 0.2(0\%) + 0.4(15\%) + 0.2(25\%) + 0.1(44\%)$
       $= -1.4\% + 0\% + 6\% + 5\% + 4.4\% = 14\%.$

The graph of the SML is as follows:

The equation of the SML is thus:
$$r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (14\% - 9\%)b_i = 9\% + (5\%)b_i.$$

c. Required rate of return on Stock A:
$$r_A = r_{RF} + (r_M - r_{RF})b_A = 9\% + (14\% - 9\%)1.0 = 14\%.$$

Required rate of return on Stock B:
$$r_B = 9\% + (14\% - 9\%)0.5 = 11.50\%.$$

d. Expected return on Stock C $= \hat{r}_c = 18\%$.

Return on Stock C if it is in equilibrium:
$$r_C = r_{RF} + (r_M - r_{RF})b_C = 9\% + (14\% - 9\%)2 = 19\% \neq 18\% = \hat{r}_c.$$

A stock is in equilibrium when its required return is equal to its expected return. Stock C's required return is greater than its expected return; therefore, Stock C is not in equilibrium. Equilibrium will be restored when the expected return on Stock C is driven up to 19%. With an expected return of 18% on Stock C, investors should sell it, driving its price down and its yield up.