

**2.1 (7<sup>th</sup> edition Prob. 2.1)**

(a) The work done on the water is equal to the work done by gravity on the weight, which is equal to force times distance. The force is  $mg = 35 \text{ kg} \cdot 9.8 \text{ m s}^{-2} = 343 \text{ N}$ . The distance is 5 m, so the work is  $343 \text{ N} \cdot 5 \text{ m} = 1715 \text{ N m} = 1715 \text{ J}$ .

(b) The internal energy change is equal to the work done, 1715 J.

(c) The internal energy change (1715 J) is equal to the total heat capacity times the temperature change ( $\Delta U^t = mC_p\Delta T$ ) or  $1715 \text{ J} = 25 \text{ kg} \cdot 4180 \text{ J kg}^{-1} \text{ K} \cdot \Delta T \text{ K}$ , from which

$$\Delta T = 1715 / (25 \cdot 4180) = 0.016 \text{ K}$$

So, the final temperature of the water is 20.02°C (the initial temperature was 20°C).

(d) To return the water to its original state, one must remove 1715 J as heat. (e) Zero, zero, zero! The total energy change of the universe is always zero (in the absence of nuclear reactions, of course).

**2.2 (7<sup>th</sup> edition Prob. 2.2)**

The work done on the water is equal to the work done by gravity on the weight, which is equal to force times distance. The force is  $mg = 35 \text{ kg} \cdot 9.8 \text{ m s}^{-2} = 343 \text{ N}$ . The distance is 5 m, so the work is  $343 \text{ N} \cdot 5 \text{ m} = 1715 \text{ N m} = 1715 \text{ J}$ .

a) The internal energy change of the system (water plus container) is equal to the work done, 1715 J. However, not all of this is change in the internal energy of the water. The heat capacity of the water is  $(25/30) = 5/6$  of the total heat capacity. Thus, when the temperature of the water and container both increase,  $5/6$  of the energy goes into the water, and  $1/6$  goes into the container. So, the internal energy of the water increases by  $5/6 \cdot 1715 = 1429 \text{ J}$ . The internal energy of the container changes by 286 J.

b) The internal energy change (1715 J) is equal to the total heat capacity times the temperature change ( $\Delta U^t = mC_p\Delta T$ ) or  $1715 \text{ J} = 30 \text{ kg} \cdot 4180 \text{ J kg}^{-1} \text{ K} \cdot \Delta T \text{ K}$ , from which

$$\Delta T = 1715 / (30 \cdot 4180) = 0.0137 \text{ K}$$

So, the final temperature of the water is 20.014°C (the initial temperature was 20 °C).

- c) To return the water to its original state, one must remove 1715 J as heat (assuming the container as well as the water cools down). If only the water cools down, and not the container, then only 1429 J must be removed.
- d) Zero, zero, zero! The total energy change of the universe is always zero (in the absence of nuclear reactions, of course).

### 2.3 (7<sup>th</sup> edition Prob. 2.3)

$W$  is positive – work is done by gravity on the egg

- (a)  $\Delta E_p$  is negative – the potential energy of the egg decreases when it falls.
- (b)  $\Delta E_k$  is zero – the egg starts out with zero velocity (and therefore zero kinetic energy, relative to the earth) and ends with zero velocity.
- (c) If the egg returns to its original temperature, then  $\Delta U^t$  for the egg is zero.  $U$  is a state function, so if the egg starts and ends at the same  $T$  and  $P$  (and there are no chemical changes in the egg), then  $U$  doesn't change.
- (d)  $Q$  is negative – the work done on the egg is converted to heat when the egg (irreversibly) breaks on the pavement. If the egg returns to its original temperature, this heat must all flow into the concrete, so that  $\Delta U^t = Q + W = 0$ .

### 2.4 (7<sup>th</sup> edition Prob. 2.4)

At steady state, the electrical power input to the motor has to be balanced by the work and heat output of the motor. At steady-state, for a closed system, the internal energy is constant:

$$\frac{dU^t}{dt} = Q + W = 0$$

The work is equal to the difference between the electrical work done on the system (positive) and the mechanical work done by the system (negative). Thinking way, way, back to first-year physics, we remember that electrical power (work per time) is the product of current and voltage ( $P = IV$ ), and that the product of volts times amps is equal to watts. Thus the energy input to the motor (electrical work) is  $P = 9.7 \text{ A} \cdot 110 \text{ V} = 1067 \text{ V} \cdot \text{A} = 1067 \text{ W}$ . The work output is 1.25 hp, and from table A1 we see that  $1 \text{ kW} = 1000 \text{ W} = 1.341 \text{ hp}$ . So, the work output is  $1.25 \text{ hp} / (1.341 \text{ kW/hp}) = 0.932 \text{ kW} = 932 \text{ W}$ . The heat transfer rate (the energy removed from the motor as heat) is equal to the difference between the electrical energy input and the mechanical work output:

$$Q = -W = -1067 - 932 = -135 \text{ W}$$

135 W is removed from the motor as heat.

## 2.5 (New)

The electrical power supplied to the mixer is  $1.5 \text{ A} \cdot 110 \text{ V} = 165 \text{ V} \cdot \text{A} = 165 \text{ W}$ . The total energy supplied over a period of 5 minutes is then  $5 \cdot 60 \cdot 165 = 49500 \text{ J} = 49.5 \text{ kJ}$ . The increase in internal energy of the cookie dough is given by  $\Delta U' = m C_v \Delta T = 1 \text{ kg} \cdot 4.2 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} \cdot 5 \text{ }^\circ\text{C} = 21 \text{ kJ}$ . Thus, the fraction of the electrical energy that went into heating the cookie dough is  $21/49.5 = 0.42$ ; 42% of the energy input has appeared as increased internal energy of the cookie dough. The rest of the energy input has been transferred as heat to the surroundings, or remains as internal energy of the electric mixer, which is probably also at a higher temperature at the end of the process than at the beginning.

From the data given, we know the work input only if the system includes the mixer itself (because we know the current and voltage supplied to it, from which we compute electrical work). Taking the mixer AND the dough as the system, we know  $W = 49.5 \text{ kJ}$ , but we do not separately know  $Q$ , the amount of heat transferred between the system and surroundings, or  $\Delta U$ , the change in internal energy of the mixer and dough together.

On the other hand, if we take only the dough as the system, we know that it has  $\Delta U' = Q + W = 21 \text{ kJ}$ , but we do not know  $Q$  and  $W$  individually. Here the work done by the mixer on the dough

should be some amount smaller than the electrical work provided to the mixer, because the mixer should have some inherent irreversibilities that prevent it from fully converting the incoming electrical work into mechanical work done on the dough.

From the given data, we cannot evaluate all terms in the energy balance for either the dough alone or the mixer and dough together as the system.

### 2.6 (7<sup>th</sup> edition Prob. 2.5)

For this problem, we need to use two principles:  $\Delta U^f = Q + W$ , which gives us the 3<sup>rd</sup> number in any row in which we already have 2 numbers, and  $U^f$  is a state function, so its value doesn't change in a cyclic process (if we end up at the same conditions as where we started,  $U^f$  will return to its initial value).

Step	$\Delta U^f$ (J)	$Q$ (J)	$W$ (J)
12	-200	?	-6000
23	?	-3800	?
34	?	-800	300
41	4700	?	?
12341	?	?	-1400

So, we just go through and fill in the numbers by adding and subtracting things.  $Q$  for the first step has to be  $Q = \Delta U^f - W = 5800$  J. Likewise,  $\Delta U^f = Q + W = -500$  for the third step. Also,  $\Delta U^f$  for the whole cycle is 0, as stated above. Filling these in gives

Step	$\Delta U^f$ (J)	$Q$ (J)	$W$ (J)
12	-200	5800	-6000

23	?	-3800	?
34	-500	-800	300
41	4700	?	?
12341	0	?	-1400

Now, we can fill in  $Q = 1400$  J for the overall cycle (so  $Q + W = \Delta U^f$ ). Then we know all but one number in the columns for  $\Delta U^f$  and  $Q$ , and the sum of the 4 steps in each case has to add up to the value for the overall cycle, so we have  $\Delta U^f_{23} = 0 + 200 + 500 - 4700 = -4000$  J and  $Q_{41} = 1400 - 5800 + 3800 + 800 = 200$ . Filling these in gives:

Step	$\Delta U^f$ (J)	$Q$ (J)	$W$ (J)
12	-200	5800	-6000
23	-4000	-3800	?
34	-500	-800	300
41	4700	200	?
12341	0	1400	-1400

Finally, we now know two of the three entries in the 2<sup>nd</sup> and 4<sup>th</sup> rows, so we can fill in these last two numbers. In each case, we have  $W = \Delta U^f - Q$ , which gives  $W = -200$  J for step 23 and  $W = 4500$  J for step 41. Filling these in completes the table:

Step	$\Delta U^{\prime}$ (J)	$Q$ (J)	$W$ (J)
12	-200	5800	-6000
23	-4000	-3800	-200
34	-500	-800	300
41	4700	200	4500
12341	0	1400	-1400

As a check, we can add up the work values for the four steps and make sure the sum comes out to be  $-1400$  (it does).

### 2.7 (7<sup>th</sup> edition Prob. 2.6)

If the refrigerator is inside the kitchen, then the electrical energy entering the refrigerator (from outside the kitchen) must inevitably appear in the kitchen. The only mechanism is by heat transfer (from the condenser of the refrigerator, usually located behind the unit or in its walls). This raises, rather than lowers, the temperature of the kitchen. The only way to make the refrigerator double as an air conditioner is to place the condenser of the refrigerator outside the kitchen (outdoors). If, for example, one mounted the refrigerator in an exterior doorway and left the refrigerator door open, one would have the (bulky, expensive, and relatively inefficient) equivalent of a window air conditioner.

### 2.8 (7<sup>th</sup> edition Prob. 2.11)

The total heat capacity of the water is  $20 \text{ kg} \cdot 4.18 \text{ kJ}/(\text{kg} \cdot ^{\circ}\text{C}) = 83.6 \text{ kJ}/^{\circ}\text{C} = 83600 \text{ J}/^{\circ}\text{C}$ . So, to increase the temperature by  $10^{\circ}\text{C}$  requires an energy input of  $836 \text{ kJ}$ . If we are doing work on the water at a rate of  $0.25 \text{ kW} = 0.25 \text{ kJ/s}$ , then we will have to do so for  $836/0.25 = 3344 \text{ s} = 55.7 \text{ minutes} = 0.929 \text{ hr}$ .

**2.9 (7<sup>th</sup> edition Prob. 2.12)**

The first law of thermodynamics tells us that

$$\Delta U^f = Q + W$$

So, if  $Q = 7.5$  kJ and  $\Delta U^f = -12$  kJ, we see that  $W = -19.5$  kJ. That is, 19.5 kJ was transferred from the system to the surroundings as work.

If the same change of state occurred (which tells us that  $\Delta U^f$  was the same), but no work was done ( $W = 0$ ) we would have  $-12$  kJ =  $Q + 0$ . That is, if no work was done, then 12 kJ would have to be removed from the system to cause the same change in the state of the system.

**2.10 (7<sup>th</sup> edition Prob. 2.13)**

An energy balance shows us that the internal energy decrease of the casting must equal the internal energy increase of the water and steel tank, since all of the heat that flows out of the casting flows into the water and tank (and none leaves the overall system). So, we have

$$m_{\text{casting}} C_{p,\text{casting}} (T_{\text{casting}} - T_{\text{final}}) = m_{\text{water}} C_{p,\text{water}} (T_{\text{final}} - T_{\text{water}}) + m_{\text{tank}} C_{p,\text{tank}} (T_{\text{final}} - T_{\text{tank}})$$

$$2 \cdot 0.5 \cdot (500 - T_{\text{final}}) = (40 \cdot 4.18 + 5 \cdot 0.5)(T_{\text{final}} - 25)$$

$$1 + 2.5 + 167.2 T_{\text{final}} = 500 + 4180 + 62.5$$

$$T_{\text{final}} = 27.8^\circ\text{C}$$

**2.11 (7<sup>th</sup> edition Prob. 2.14)**

If there is no change in volume of the fluid, no work can be done. Work is  $dW = -PdV$ . As long as  $V$  doesn't change, it doesn't matter what  $P$  does. If the work is zero and (since the piston is

insulated) the heat is also zero, the change in internal energy of the fluid is also zero. The internal energy of an incompressible fluid is independent of pressure.

### 2.12 (7<sup>th</sup> edition Prob. 2.15)

In the previous problems, the heat capacity of water was given as  $4.18 \text{ kJ kg}^{-1} \text{ K}^{-1}$ . For liquids,  $C_p$  and  $C_v$  are very nearly the same, so it doesn't matter whether this change occurs at constant  $P$  or at constant  $V$ . The change in internal energy is thus

$$\Delta U^i = mC_p\Delta T = 1 \text{ kg} \cdot 4.18 \text{ kJ kg}^{-1} \text{ K}^{-1} \cdot 1 \text{ K} = 4.18 \text{ kJ}$$

(a) The change in gravitational potential energy is

$$\Delta E_p = mg\Delta z$$

So, to have the same change in potential energy as the change in internal energy in part (a), we need

$$4180 \text{ J} = 1 \text{ kg} \cdot 9.8 \text{ m s}^{-2} \cdot \Delta z$$

from which  $\Delta z = 427 \text{ m}$  (or 1400 feet)

(b) This time, we need  $\Delta E_k = \frac{1}{2}mv^2 = 4180 \text{ J}$ , from which  $v^2 = 8360 \text{ (m/s)}^2$  or  $v = 91.4 \text{ m/s}$  (or 205 miles per hour).

What have we learned? We have discovered again that what corresponds, in our everyday experience, to a small amount of thermal energy corresponds to a relatively large amount of mechanical energy. The amount of energy required to heat a quart of water (1 kg of water is about 1 liter or 1 quart) from room temperature to its boiling point (increasing the temperature by about 75 K) would be enough to raise the water to a height of about 20 miles (in a frictionless world, or one in which we lifted the object very slowly so that the drag of the air on it was negligible).

### 2.13 (7<sup>th</sup> edition Prob. 2.16)



Electrical and mechanical irreversibilities cause an increase in the internal energy, and therefore the temperature, of the motor (they convert some of the input electrical energy to heat rather than mechanical work). The temperature of the motor rises until a steady state is established such that heat transfer from the motor to the surroundings is equal to the rate at which heat is produced within the motor by these irreversibilities. Insulating the motor does nothing to decrease the irreversibilities in the motor and merely causes the temperature of the motor to rise until steady-state heat-transfer is reestablished with the surroundings. Insulating the motor will just make it run hotter and probably damage it.

### 2.14 (7<sup>th</sup> edition Prob. 2.17)

A general energy balance on the hydroturbine can be written as  $\dot{Q} + \dot{W} = \dot{m} \Delta (h + \frac{1}{2}u^2 + zg)$

$$\frac{d(mU)_{cv}}{dt} + \Delta \left( (H + \frac{1}{2}u^2 + zg)m \right)_{fs} = Q + W$$

The system can reasonably be assumed to operate at steady-state, so the first term is zero. The change in kinetic energy will also be negligible, because the inlet and outlet pipes have the same diameter, and therefore the water will have the same velocity at the inlet and outlet. A somewhat stronger assumption is that the change in enthalpy of the water is negligible. All real processes, including hydroturbines, have some degree of irreversibility. However, it turns out that a well-designed hydroturbine can be quite efficient, and can convert more than 90% of the potential energy of the water into mechanical energy (to be used to drive a generator, which will have its own inefficiencies, so that the final electrical energy output will be smaller). Thus, for the hydroturbine, it is reasonable to neglect the change in enthalpy of the water, as well as heat flow to or from the system. Doing so gives simply

$$W = \Delta mgz = mg\Delta z$$

This will be an upper limit for the mechanical power (rate of work) produced by the turbine. To compute it, we need to know the mass flow rate of the water through the turbine (which is the same at the inlet and outlet, by a trivial mass balance). The cross-sectional area of the 2 m diameter inlets and outlets is  $\pi \cdot (1 \text{ m})^2 = 3.14 \text{ m}^2$ . Multiplying this by the water velocity of  $5 \text{ m s}^{-1}$  gives a volumetric flow rate of  $15.7 \text{ m}^3/\text{s}$ . Multiplying this by the density of water ( $1000 \text{ kg/m}^3$ ) gives the mass flow rate as  $15,700 \text{ kg/s}$ . Thus, the mechanical work is estimated as

$$W = 15700 \text{ kg/s} \cdot 9.8 \text{ m/s}^2 \cdot 50 \text{ m} = 7.7 \cdot 10^6 \text{ kg m}^2 \text{ s}^{-3} = 7.7 \text{ MW}$$

A crude estimate is that the average power consumption rate of a U.S. household is around 1 kW. Allowing for an overall efficiency (turbine plus generator) of 85%, this is still enough to power about 6500 households.

### 2.15 (New)

If we take a cylinder of 40 m diameter and wind speed of 8 m/s, then the volumetric flow rate of the air impinging on the turbine is  $8 \text{ m/s} \cdot (40 \text{ m})^2 / 4 \cdot \pi = 10050 \text{ m}^3/\text{s}$ . The corresponding mass flow rate is  $10050 \text{ m}^3/\text{s} \cdot 1.2 \text{ kg/m}^3 = 12060 \text{ kg/s}$ . The kinetic energy per unit mass is  $8^2/2 = 32 \text{ m}^2/\text{s}^2 = 32 \text{ J/kg}$ . Thus, the total kinetic energy per time is  $12060 \text{ kg/s} \cdot 32 \text{ J/kg} = 386000 \text{ J/s} = 386 \text{ kW}$ . The fraction of this converted to electrical power is  $90/368 = 0.233$ . That is, 23.3% of the kinetic energy of the oncoming wind is captured.

### 2.16 (New)

If the total capacity of the battery is 56 Watt-hours, and it is drained in 4 hours, then the average discharge rate is  $56/4 = 14 \text{ W}$ , or  $14 \text{ J/s}$ . Remembering that power is current times voltage, the average current drawn by the laptop is  $14 \text{ W}/11.1 \text{ V} = 1.26 \text{ A}$ .

Taking the computer (not including the battery) as the system, and neglecting any change in internal energy of the computer, all of the electrical work supplied to the computer by the battery (at an average rate of  $24.4 \text{ J/s}$ ) is ultimately converted to heat. That is, we can write the first law as

$$\frac{dU^t}{dt} = Q + W = 0$$

If  $U$  remains constant, then  $Q = -W = -14 \text{ W}$ . When the computer is at steady state (not changing temperature)  $14 \text{ W}$  must be dissipated into the surroundings as heat.

If we instead considered a system that includes the battery as well as the rest of the computer, then the work would be zero (the battery is now part of the system, rather than the surroundings). We could write the electrical potential energy of the battery (when fully charged) as

$$E_{EP} = 56 \text{ W-hr}$$

Thus, the total energy of the system would include this as well as the internal energy of the computer and we could write the first law as

$$\Delta(U^i + E_{EP}) = Q + W$$

Taking  $W = 0$ ,  $\Delta U^i = 0$ , and  $\Delta E_{EP} = -56 \text{ W-hr}$  (when the battery is fully discharged,  $E_{EP} = 0$ ), we have

$$-56 \text{ W-hr} = Q$$

The total heat removal is 56 W-hr over a period of 4 hours, so the average rate of heat flow is

$$Q = -56/4 = -14 \text{ W}$$

### 2.17 (New)

If we take the laptop and bag as the system, then there is no exchange of heat or work between the system and surroundings, and the first law tells us that  $\Delta U^i = 0$ . The internal energy does not change, but the chemical energy stored in the battery is released as sensible heat, which raises the temperature of the computer. The total amount of energy stored in the battery is  $56 \text{ W}\cdot\text{h} = 56 \text{ J/s}\cdot\text{h}\cdot 3600 \text{ s/h} = 201600 \text{ J} = 201.6 \text{ kJ}$ . The total heat capacity of the laptop is  $2.3\cdot 0.8 = 1.84 \text{ kJ/K}$ . Thus, energy release of 201.6 kJ would increase the temperature by  $201.6 \text{ kJ}/1.84 \text{ kJ/K} = 110 \text{ K} = 110^\circ\text{C}$  (because this is a temperature DIFFERENCE, and not an absolute temperature, units of K and  $^\circ\text{C}$  are the same). If the computer started off at a cool  $20^\circ\text{C}$ , the estimated final temperature is  $130^\circ\text{C}$  (about  $270^\circ\text{F}$ ). At that point, the foam briefcase would probably start melting onto the surface of the laptop. Fortunately, in real life, this is unlikely to happen, and even if it did, the bag would not be perfectly insulating. Nonetheless, a typical laptop battery does store sufficient energy to cook the laptop in which it is installed if no heat could be removed.

**2.18 (New)**

At steady state, the net energy flow into the solar panel is zero. The rate of energy transfer *to* the panel as sunlight is  $0.8 \cdot 1 \text{ kW/m}^2 \cdot 3 \text{ m}^2 = 2.4 \text{ kW}$  (80% of the total, since 20% is reflected). The rate at which energy leaves as electrical work is  $0.17 \cdot 1 \text{ kW/m}^2 \cdot 3 \text{ m}^2 = 0.51 \text{ kW}$ . At steady state, the remaining energy must leave as heat. Written as an energy balance,

$\Delta U' = 0 = 2.4 \text{ kW} - 0.51 \text{ kW} + Q = 0$  from which  $Q = -1.89 \text{ kW}$ . The rate of heat removal is 1.89 kW. Because solar cells inherently convert a minority of the absorbed energy into electrical work, they inevitably require removal of substantial amounts of energy as heat. This can lead them to operate at high temperature, which may further reduce their efficiency.

**2.19 (7<sup>th</sup> edition Prob. 2.18)**

(a) By definition,  $H = U + PV$ , so  $H = 762.0 \text{ kJ/kg} + 1002.7 \text{ kPa} \cdot 0.001128 \text{ m}^3/\text{kg} = 763.1 \text{ kJ/kg}$ .

(Note that we converted the specific volume to  $\text{m}^3$  per kg, and that  $\text{Pa} \cdot \text{m}^3 = \text{N/m}^2 \cdot \text{m}^3 = \text{N} \cdot \text{m} = \text{J}$ , so  $\text{kPa} \cdot \text{m}^3 = \text{kJ}$ ).(b)  $\Delta U = 2784.4 - 762 = 2022.4 \text{ kJ/kg}$ . The enthalpy of the vapor is

$$H = 2784.4 \text{ kJ/kg} + 1500 \text{ kPa} \cdot 0.1697 \text{ m}^3/\text{kg} = 3039.0 \text{ kJ/kg}. \text{ So,}$$

$\Delta H = 3039.0 - 763.1 = 2275.9 \text{ kJ/kg}$ .  $\Delta H$  is significantly larger than  $\Delta U$  because of the large increase in specific volume during the process.

**2.20 (7<sup>th</sup> edition Prob. 2.19)**

Let symbols without subscripts refer to the solid and symbols with subscript *w* refer to the water. Heat transfer from the solid to the water results in changes in internal energy of both. Because energy is conserved and there are no energy flows other than the transfer of heat from the solid to

the water,  $\Delta U^t = -\Delta U_w^t$ . If total heat capacity of the solid is  $C^t (= mC)$  and total heat capacity of the water is  $C_w (= m_w C_w)$ , then:

$$\Delta U^t = C^t(T - T_0) = -\Delta U_w^t = -C_w(T_w - T_{w0})$$

or

$$T_w = T_{w0} - \frac{C^t}{C_w}(T - T_0)$$

This equation relates instantaneous values of  $T_w$  and  $T$ .

The heat-transfer rate is given as  $Q = K(T_w - T)$ . Thus, writing an energy balance (the first law) with the solid as the system and  $W = 0$ , yields

$$\frac{dU^t}{dt} = Q = K(T_w - T)$$

This change in internal energy is related to the change in temperature by the heat capacity:

$$\frac{dU^t}{dt} = C^t \frac{dT}{dt}$$

Combining these equations, we have

$$C^t \frac{dT}{dt} = K(T_w - T)$$

Substituting into this our expression for  $T_w$  in terms of  $T$  gives a single equation for  $T$  that is separable:

$$C^t \frac{dT}{dt} = K \left( T_{w0} - \frac{C^t}{C_w} T - T_0 - T \right)$$

$$\frac{dT}{dt} = K \left( \frac{T_{w0}}{C^t} + \frac{T_0}{C_w^t} - \left( \frac{1}{C^t} + \frac{1}{C_w^t} \right) T \right)$$

$$\frac{dT}{\left( \frac{T_{w0}}{C^t} + \frac{T_0}{C_w^t} - \left( \frac{1}{C^t} + \frac{1}{C_w^t} \right) T \right)} = K dt$$

Integrating from  $t = 0$  (where  $T = T_0$ ) to  $t$  gives:

$$\frac{-1}{\left( \frac{1}{C^t} + \frac{1}{C_w^t} \right)} \ln \left( \frac{\frac{T_{w0}}{C^t} + \frac{T_0}{C_w^t} - \left( \frac{1}{C^t} + \frac{1}{C_w^t} \right) T}{\frac{T_{w0}}{C^t} + \frac{T_0}{C_w^t} - \left( \frac{1}{C^t} + \frac{1}{C_w^t} \right) T_0} \right) = Kt$$

$$\frac{\frac{T_{w0}}{C^t} + \frac{T_0}{C_w^t} - \left( \frac{1}{C^t} + \frac{1}{C_w^t} \right) T}{\frac{T_{w0}}{C^t} + \frac{T_0}{C_w^t} - \left( \frac{1}{C^t} + \frac{1}{C_w^t} \right) T_0} = \exp \left( -Kt \left( \frac{1}{C^t} + \frac{1}{C_w^t} \right) \right)$$

$$\frac{T_0 C^t + C_w^t T_{w0} - (C^t + C_w^t) T}{C^t C_w^t} = \frac{T_{w0} - T_0}{C^t} \exp \left( \frac{-K(C^t + C_w^t)t}{C^t C_w^t} \right)$$

$$T_0 C^t + C_w^t T_{w0} - (C^t + C_w^t) T = C_w^t T_{w0} - T_0 \exp \left( \frac{-K(C^t + C_w^t)t}{C^t C_w^t} \right)$$

$$T = \frac{T_0 C^t + C_w^t T_{w0} - C_w^t T_{w0} + T_0 \exp \left( \frac{-K(C^t + C_w^t)t}{C^t C_w^t} \right)}{C^t + C_w^t}$$

### 2.21 (7<sup>th</sup> edition Prob. 2.20)

The general equation applicable here is Eq. (2.29):

$$\Delta \left[ \left( H + \frac{1}{2} u^2 + gz \right) m \right]_{fs} = Q + W_s$$

- (a) We write this for the single stream flowing within the pipe, neglect potential and kinetic energy changes, and set the work term equal to zero. This yields:

$$\Delta mH = m\Delta H = m H_{out} - H_{in} = Q$$

In the second and third versions of the LHS, we have used the fact that there is a single inlet and outlet, and that mass conservation requires that the mass flow rate at the inlet and outlet are equal.

- (b) The equation is here written for the two streams (1 and 2) flowing in the two pipes, again neglecting any potential- and kinetic-energy changes. There is no work, and the heat transfer is internal to the system, between the two streams, making  $Q = 0$ . Thus,

$$\Delta mH = m_1\Delta H_1 + m_2\Delta H_2 = 0$$

- (c) For a pump operating on a single liquid stream, the assumption of negligible potential- and kinetic energy changes is reasonable, as is the assumption of negligible heat transfer to the surroundings. Whence,

$$\Delta mH = m\Delta H = m H_{out} - H_{in} = W$$

- (d) For a properly designed gas compressor the result is the same as in Part (c).

$$\Delta mH = m\Delta H = m H_{out} - H_{in} = W$$

- (e) For a properly designed turbine the result is the same as in Part (c).

$$\Delta mH = m\Delta H = m H_{out} - H_{in} = W$$

- (f) The purpose of a throttle is to reduce the pressure on a flowing stream. One usually assumes adiabatic operation with negligible potential- and kinetic-energy changes. Since there is no work, the equation is:

$$\Delta mH = m\Delta H = m H_{out} - H_{in} = 0$$

(g) The whole purpose of a nozzle is to produce a stream of high velocity. The kinetic-energy change must therefore be taken into account. However, one usually assumes negligible potential-energy change. Then, for a single stream, adiabatic operation, and no work:

$$\Delta[(H + \frac{1}{2}u^2)m] = 0$$

$$\text{Or } \Delta(H + \frac{1}{2}u^2) = 0$$

The usual case is for a negligible inlet velocity. The equation then reduces to:

$$\Delta H = -\frac{1}{2}u_2^2$$

where  $u_2$  is the exit velocity.

### 2.22 (7<sup>th</sup> edition Prob. 2.21)

The mass flowrate,  $m$ , is related to the density, diameter, and average velocity by

$$m = \rho u \frac{\pi D^2}{4}$$

Thus,

$$\rho u D = \frac{4m}{\pi D}$$

And

$$Re = \frac{\rho u D}{\mu} = \frac{4m}{\pi D \mu}$$

Thus, the Reynolds number is directly proportional to the mass flow rate.



- (a) The expression in part (a) shows that in this case the Reynolds number is inversely proportional to the diameter,  $D$ .

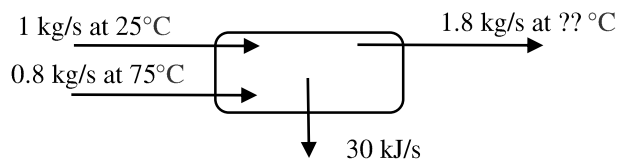
### 2.23 (7<sup>th</sup> edition Prob. 2.22)

(a) For an incompressible fluid flowing steadily, a mass balance reduces to flow in = flow out, or  $\rho u_{in} A_{in} = \rho u_{out} A_{out}$ , or  $u_{in} A_{in} = u_{out} A_{out}$  or  $u_{out} = u_{in} (A_{in}/A_{out})$ . In this case, the diameter increases by a factor of two, so the cross-sectional area increases by a factor of 4, so  $A_{in}/A_{out} = 1/4$ , and  $u_{out} = 2 \text{ m s}^{-1}/4 = 0.5 \text{ m s}^{-1}$ .

(b) The change in kinetic energy (per unit mass) is just the change in  $1/2 u^2 = 1/2 \cdot (0.25 \text{ m}^2 \text{ s}^{-2} - 4 \text{ m}^2 \text{ s}^{-2}) = -1.875 \text{ m}^2 \text{ s}^{-2} = -1.875 \text{ J kg}^{-1}$ .

### 2.24 (7<sup>th</sup> edition Prob. 2.23)

Here it may be helpful to draw a very simple schematic:



The above diagram assumes that you can do the mass balance ‘in your head’: 1 kg/s in plus 0.8 kg/s in = 1.8 kg/s out at steady state. We can write the enthalpy balance as

$$\Delta H = Q + W$$

We are given that the heat removal rate is  $-30 \text{ kJ/s}$  and that no work is done. So, we simply have

$$\Delta H = -30 \text{ kJ/s}$$

The enthalpy change is the sum of the mass flow rates of the input streams each multiplied by their respective temperature change and by the heat capacity of water:

$$\Delta H = 1 \text{ kg/s} \cdot (T_{out} - 25) \cdot 4.18 \text{ kJ/(kg K)} + 0.8 \text{ kg/s} \cdot (T_{out} - 75) \cdot 4.18 \text{ kJ/(kg K)} = -30 \text{ kJ/s}$$

$$\Delta H = 7.524 T_{out} - 355.3 = -30$$

$$\text{from which } T_{out} = 43.2 \text{ }^\circ\text{C}$$

Note that because only temperature differences were involved in the problem (and not absolute temperatures) using  $^\circ\text{C}$  for the units of temperature worked fine. A temperature difference is the same in  $^\circ\text{C}$  as in K.

### 2.25 (7<sup>th</sup> edition Prob. 2.24)

If we take the tank as our control volume, then the mass balance gives us

$$\frac{dm}{dt} = -m'$$

where  $m$  is the mass of fluid still in the tank and  $m'$  is the mass flow rate of the stream leaving the tank.

Likewise, the energy balance gives

$$\frac{d mU}{dt} = -m' H'$$

where  $H'$  is the specific enthalpy of the fluid leaving the tank and  $U$  is the specific internal energy of the fluid still in the tank. In the energy balance we have neglected contributions of kinetic and gravitational potential energy.  $Q$ , the rate of heat transfer in the energy balance is zero because the tank is insulated.  $W$  is zero because no work is being done on or by the system other than the flow work that is already included in  $H'$ .

We can expand the derivative in the energy balance to get

$$m \frac{dU}{dt} + U \frac{dm}{dt} = -m' H'$$

We can use the mass balance to eliminate  $m'$  from this equation to get

$$m \frac{dU}{dt} + U \frac{dm}{dt} = \frac{dm}{dt} H'$$

Rearranging this gives

$$m \frac{dU}{dt} = \frac{dm}{dt} (H' - U)$$

From which we can eliminate time as a variable and get the desired relationship

$$\frac{dU}{H' - U} = \frac{dm}{m}$$

If conditions within the tank are uniform (no gradients in pressure, temperature, etc. between the main part of the tank and the tank exit) then  $H' = H$ .

### 2.26 (7<sup>th</sup> edition Prob. 2.25)

Mass balance at steady state requires that the mass flow (and therefore volumetric flow for water with nearly constant density) be the same before and after the diameter change. The mass flowrate in the 2.5 cm diameter pipe is

$$m = \rho_{H_2O} \text{ kg m}^{-3} \cdot 14 \text{ m s}^{-1} \cdot 0.025 \text{ m}^2 / 4$$

and the mass flow rate after the expansion is

$$m = \rho_{H_2O} \text{ kg m}^{-3} \cdot v_{out} \text{ m s}^{-1} \cdot d_{out}^2 \text{ m}^2 / 4$$

Setting these equal shows that the velocity in the outlet pipe is

$$v_{out} = 14 \text{ m/s} \cdot (2.5 \text{ cm} / d_{out} \text{ cm})^2$$

So, if  $d_{out} = 3.8 \text{ cm}$ , then  $v_{out} = 6.06 \text{ m/s}$

The change in kinetic energy per unit mass is then

$$\Delta E_K/m = \frac{1}{2} (6.06^2 - 14^2) = -79.6 \text{ m}^2 \text{ s}^{-2} = -79.6 \text{ J kg}^{-1}$$

Since no heat is exchanged with the surroundings, there is no change in elevation (no change in gravitational potential energy) and no non-flow work is done on the surroundings, the energy balance is simply

$$\Delta H + \Delta E_K = 0$$

From which  $\Delta H = 79.6 \text{ J kg}^{-1}$

This enthalpy change corresponds to a temperature increase given by

$$\Delta H = C_p \Delta T = 4180 \text{ J kg}^{-1} \text{ K}^{-1} \Delta T = 79.6 \text{ J kg}^{-1}$$

from which  $\Delta T = 0.019 \text{ K}$ .

If the downstream diameter were 7.5 cm, then we would have  $v_{out} = 1.556 \text{ m/s}$ , and the change in kinetic energy per unit mass would be  $-96.8 \text{ J kg}^{-1}$ . The corresponding temperature increase would be  $96.8/4180 = 0.0232 \text{ K}$ .

The maximum temperature increase would occur if the downstream velocity went to zero, so that all of the kinetic energy of the flow were converted to internal energy. In this case, we would have  $\Delta E_K/m = \frac{1}{2} (0^2 - 14^2) = -98 \text{ J kg}^{-1}$ , leading to a temperature increase of  $98/4180 = 0.0234 \text{ K}$ . For the downstream diameter of 7.5 cm, we had very nearly reached this limit.

### 2.27 (7<sup>th</sup> edition Prob. 2.26)

The energy balance, including kinetic energy contributions, for a steady flow system gives us

$$m \left( H_{out} - H_{in} + \frac{1}{2} u_{out}^2 - \frac{1}{2} u_{in}^2 \right) = Q + W$$

The mass flow rate (in and out) is  $50 \text{ kmol hr}^{-1} \cdot 29 \text{ kg/kmol} = 1450 \text{ kg/hr} = 0.40278 \text{ kg/s}$

The enthalpy change is

$$H_{out} - H_{in} = C_p T_{out} - T_{in} = \frac{3.5 \cdot 8.314 \text{ kJ kmol}^{-1} \text{ K}^{-1}}{29 \text{ kg kmol}^{-1}} 520\text{K} - 300\text{K} = 220.75 \text{ kJ kg}^{-1}$$

The kinetic energy change is

$$\frac{1}{2}u_{out}^2 - \frac{1}{2}u_{in}^2 = 0.5(12.25 \text{ m}^2 \text{ s}^{-2} - 100 \text{ m}^2 \text{ s}^{-2}) = -43.9 \text{ m}^2 \text{ s}^{-2} = -0.0439 \text{ kJ kg}^{-1}$$

So, as is so often the case, the change in kinetic energy is negligible.

So, the energy balance becomes

$$0.40278 \text{ kg s}^{-1} (220.71 \text{ kJ kg}^{-1}) = 88.9 \text{ kJ s}^{-1} = 88.9 \text{ kW} = Q + W = Q + 98.8 \text{ kW}$$

from which  $Q = -9.9 \text{ kW}$ . That is, 9.9 kW of heat flows from the system (the air) to the surroundings, or 9.9 kW of the input mechanical energy leaves the compressor as heat.

### 2.28 (7<sup>th</sup> edition Prob. 2.27)

Because the pipe is insulated, we can assume  $Q$  is zero, and because the pipe and valve presumably have no moving parts,  $W$  is also zero. Furthermore, the pipe is horizontal, so there is no change in gravitational potential energy between the inlet and the outlet streams. Thus, we can write the energy balance for an open system with one inlet and one outlet like it is written in equation 2.31. Note that this equation, as written, is per unit mass.

$$\Delta H + \frac{\Delta u^2}{2g_c} + \frac{g}{g_c} \Delta z = Q + W$$

$$\Delta H + \frac{\Delta u^2}{2g_c} = 0$$

The difference in enthalpy between the inlet and outlet streams can be written in terms of the temperature change:  $\Delta H = C_p \Delta T$ . Substituting that in, we have

$$C_p \Delta T + \frac{\Delta u^2}{2} = 0$$

The mass flow rates in and out must be equal (at steady state, with no chemical reactions),  $m_{in} = m_{out}$ , and the mass flow rate is the velocity times the cross-sectional area divided by the specific volume (volumetric flow rate divided by specific volume):  $m = uA/V$ . So, we have

$$\frac{u_{in} A_{in}}{V_{in}} = \frac{u_{out} A_{out}}{V_{out}} \text{ and } u_{out} = \frac{u_{in} A_{in} V_{out}}{A_{out} V_{in}}$$

$A_{in}$  and  $A_{out}$  are the same, and because we have  $PV/T = \text{a constant}$ , we can write

$$\frac{V_{out}}{V_{in}} = \frac{T_{out} P_{in}}{T_{in} P_{out}}$$

So, we can then write

$$u_{out} = \frac{u_{in} P_{in} T_{out}}{P_{out} T_{in}}$$

Putting in the numbers, we have  $P_{in}/P_{out} = 5$  and  $u_{in} = 20 \text{ ft s}^{-1} = 6.096 \text{ m s}^{-1}$ ,  $T_{in} = 120 \text{ }^\circ\text{F} = 322.04 \text{ K}$ , so

$$u_{out} (\text{m} \cdot \text{s}^{-1}) = 5 \cdot \frac{6.096 \cdot 322.04 + \Delta T}{322.04} = 30.48 + 0.09465 \Delta T$$

Where  $\Delta T$  is in K or  $^\circ\text{C}$ .

Also, we have  $C_p = 3.5 \cdot R = 3.5 \cdot 8.314 \text{ J/(mol K)} = 29.10 \text{ J/(mol K)}$ . As given, this is the molar heat capacity. To get the specific heat, we divide by the molecular weight of nitrogen (0.02801 kg/mol) to get  $C_p = 1039 \text{ J/(kg K)}$ . Putting this all together, we see that

$$1039 \cdot \Delta T + \frac{30.48 + 0.09465\Delta T^2 - 6.096^2}{2} = 0$$

where both terms have units of J/kg, and  $\Delta T$  has units of K or °C. Multiplying things out gives

$$0.004479 \cdot \Delta T^2 + 1041.8\Delta T + 445.9 = 0$$

Applying the quadratic formula to this gives  $\Delta T = -0.428 \text{ K} = -0.77^\circ\text{F}$ , so the downstream temperature is  $119.23^\circ\text{F}$

## 2.29 (New)

Because the pipe is insulated, we can assume  $Q$  is zero, and because the pipe and valve presumably have no moving parts,  $W$  is also zero. Furthermore, the pipe is horizontal, so there is no change in gravitational potential energy between the inlet and the outlet streams. Thus, we can write the energy balance for an open system with one inlet and one outlet like it is written in equation 2.31. Note that this equation, as written, is per unit mass.

$$\Delta H + \frac{\Delta u^2}{2g_c} + \frac{g}{g_c} \Delta z = Q + W$$

$$\Delta H + \frac{\Delta u^2}{2g_c} = 0$$

The difference in enthalpy between the inlet and outlet streams can be written in terms of the temperature change:  $\Delta H = C_p \Delta T$ . Substituting that in, we have

$$C_p \Delta T + \frac{\Delta u^2}{2} = 0$$

The mass flow rates in and out must be equal (at steady state, with no chemical reactions),  $m_{in} = m_{out}$ , and the mass flow rate is the velocity times the cross-sectional area divided by the specific volume (volumetric flow rate divided by specific volume):  $m = uA/V$ . So, we have

$$\frac{u_{in}A_{in}}{V_{in}} = \frac{u_{out}A_{out}}{V_{out}} \text{ and } u_{out} = \frac{u_{in}A_{in}V_{out}}{A_{out}V_{in}}$$

$A_{in}$  and  $A_{out}$  are the same, and because we have  $PV/T = \text{a constant}$ , we can write

$$\frac{V_{out}}{V_{in}} = \frac{T_{out}P_{in}}{T_{in}P_{out}}$$

So, we can then write

$$u_{out} = \frac{u_{in}P_{in}T_{out}}{P_{out}T_{in}}$$

Putting in the numbers, we have  $P_{in}/P_{out} = 5.385$  and  $u_{in} = 20 \text{ m s}^{-1}$ ,  $T_{in} = 45 \text{ }^\circ\text{C} = 318.15 \text{ K}$ , so

$$u_{out}(\text{m} \cdot \text{s}^{-1}) = \frac{5.385 \cdot 20(318.15 + \Delta T)}{318.15} = 107.69 + 0.3385\Delta T$$

Where  $\Delta T$  is in K or  $^\circ\text{C}$ .

Also, we have  $C_p = 3.5 \cdot R = 3.5 \cdot 8.314 \text{ J/(mol K)} = 29.10 \text{ J/(mol K)}$ . As given, this is the molar heat capacity. To get the specific heat, we divide by the average molecular weight of nitrogen (0.02897 kg/mol) to get  $C_p = 1004.5 \text{ J/(kg K)}$ . Putting this all together, we see that

$$1004.5 \cdot \Delta T + \frac{107.69 + 0.3385\Delta T}{2} - 20^2 = 0$$



where both terms have units of J/kg, and  $\Delta T$  has units of K or °C. Multiplying things out gives

$$0.057291 \cdot \Delta T^2 + 1040.9\Delta T + 5398.8 = 0$$

Applying the quadratic formula to this gives  $\Delta T = -5.19 \text{ K} = -5.19^\circ\text{C}$ , so the downstream temperature is  $39.8^\circ\text{C}$ .

### 2.30 (7<sup>th</sup> edition Prob. 2.28)

We can write the energy balance (per unit mass) as

$$\Delta H + \Delta E_K = Q + W_s$$

From the problem statement, we have  $\Delta H = 2726.5 - 334.9 = 2391.6 \text{ kJ kg}^{-1}$ , and no shaft work is done. Since there is a large change in velocity from inlet to outlet, we will take into account the change in kinetic energy, which is  $\Delta E_K/m = \frac{1}{2} (200^2 - 3^2) \text{ m}^2 \text{ s}^{-2} = 19996 \text{ J kg}^{-1} = 20.0 \text{ kJ kg}^{-1}$ . So, we have

$$Q = \Delta H + \Delta E_K = 2391.6 + 20.0 = 2411.6 \text{ kJ kg}^{-1}$$

Even for this huge change in velocity, the change in kinetic energy is a very small part of the overall heat requirement.

### 2.31 (7<sup>th</sup> edition Prob. 2.29)

Because the nozzle is insulated,  $Q$  is zero, and because it is a nozzle (with no moving parts) the nonflow work,  $W$ , is zero, so from the steady-state energy balance we have

$$m \left( H_{out} - H_{in} + \frac{1}{2} u_{out}^2 - \frac{1}{2} u_{in}^2 \right) = 0$$

or

$$H_{out} - H_{in} = \frac{1}{2} (u_{in}^2 - u_{out}^2)$$

from which

$$H_{out} - H_{in} = \frac{1}{2} (u_{in}^2 - u_{out}^2)$$

$$u_{out}^2 = 2 H_{in} - H_{out} + u_{in}^2 = 2(3112500 \text{ J kg}^{-1} - 2945700 \text{ J kg}^{-1}) + 900 \text{ J kg}^{-1}$$

$$u_{out}^2 = 334500 \text{ J kg}^{-1} = 334500 \text{ m}^2 \text{ s}^{-2}$$

$$u_{out} = 578 \text{ m s}^{-1}$$

The mass balance simply tells us that the mass out equals the mass in, or

$$\frac{A_{in} u_{in}}{V_{in}} = \frac{A_{out} u_{out}}{V_{out}}$$

from which

$$\frac{A_{out}}{A_{in}} = \frac{V_{out} u_{in}}{V_{in} u_{out}} = \frac{667.75 \text{ cm}^3 \text{ g}^{-1} \cdot 30 \text{ m s}^{-1}}{388.61 \text{ cm}^3 \text{ g}^{-1} \cdot 578 \text{ m s}^{-1}} = 0.0892$$

Therefore

$$\frac{d_{out}}{d_{in}} = \sqrt{\frac{A_{out}}{A_{in}}} = \sqrt{0.0892} = 0.299$$

$$\text{so } d_{out} = 0.299 \cdot 5 \text{ cm} = 1.49 \text{ cm}$$

**2.32 (7<sup>th</sup> edition Prob. 2.30)**

(a) At constant volume, the total heat input will be  $Q = \Delta U' = n \Delta U = nC_v \Delta T$  (for constant heat capacity – if the heat capacity were not constant, we would have to do an integral). So, we have  $Q = 3 \text{ moles} \cdot 20.8 \text{ J}/(\text{mol K}) \cdot 220 \text{ K} = 13728 \text{ J}$ , to heat only the gas. To heat the vessel, with a total heat capacity of  $100 \text{ kg} \cdot 500 \text{ J}/(\text{kg K}) = 50000 \text{ J/K}$  requires  $Q = 50000 \text{ J/K} \cdot 220 \text{ K} = 1.1 \times 10^7 \text{ J}$ . The energy required to heat up the vessel is roughly 1000 times larger than the energy required to heat only the gas. This is fairly typical for gases: the vessel that contains them has greater mass, and therefore greater heat capacity, than the gas itself.

(b) At constant pressure,  $Q = \Delta H' = n \Delta H = nC_p \Delta T$  (again for constant heat capacity; if the heat capacity were not constant, we would have to do an integral). So, we have  $Q = 4 \text{ moles} \cdot 29.1 \text{ J}/(\text{mol K}) \cdot -160 \text{ K} = -18624 \text{ J}$ . About 19 kJ has to be removed.

Note that in both parts of this problem, we only needed to know the total heat capacity (number of moles and molar heat capacity) and did not need to know the actual pressure or volume.

**2.33 (7<sup>th</sup> edition Prob. 2.31)**

(a) For this constant-volume process, we have (neglecting the heat capacity of the vessel)

$$Q = \Delta U' = nC_v \Delta T = 3 (\text{lb mole}) \cdot 5 (\text{Btu}) (\text{lb mole})^{-1} (\text{°F})^{-1} \cdot 280 (\text{°F}) = 4200 \text{ Btu}$$

If we add in the heat capacity of the vessel, then we have

$$Q = \Delta U' = ((nC_v)_{\text{gas}} + (mC_v)_{\text{vessel}}) \Delta T$$

$$Q = \Delta U' = (15 + 24) (\text{Btu}) (\text{°F})^{-1} \cdot 280 (\text{°F}) = 10920 \text{ Btu}$$

(as is likely to be the case in real life, the heat capacity of the vessel is greater than that of the gas).

(b) For this constant-pressure process, we have (neglecting the heat capacity of the piston and cylinder)

$$Q = \Delta H^f = nC_p\Delta T = 4 \text{ (lb mole)} \cdot 7 \text{ (Btu)} (\text{lb mole})^{-1} (\text{°F})^{-1} \cdot -250 \text{ (°F)} = -7000 \text{ Btu}$$

We must extract 7000 Btu from the system.

### 2.34 (7<sup>th</sup> edition Prob. 2.32)

The work of a reversible expansion or compression is given by

$$W = - \int PdV$$

where the integral is over the path from the initial to final state. If the temperature is fixed and the pressure and volume are related by

$$V = \frac{RT}{P} + b$$

Then we can substitute  $P$  in terms of  $V$  or  $dV$  in terms of  $P$  into the integral (depending whether we want to specify the limits of the integral in terms of  $P$  or  $V$ ). If we want substitute for  $dV$  in terms of  $P$ , we have to differentiate the relationship between  $V$  and  $P$ , to get

$$dV = \frac{-RT}{P^2} dP$$

Substituting this into the integral for the work, we have

$$W = \int_{P_1}^{P_2} \frac{RT}{P} dP = RT \ln \left( \frac{P_2}{P_1} \right)$$

Taking the alternative approach, we could substitute for  $P$  in terms of  $V$ , writing

$$P = \frac{RT}{V - b}$$

$$W = - \int_{V_1}^{V_2} \frac{RT}{V - b} dV = RT \ln \left( \frac{V_1 - b}{V_2 - b} \right)$$

Substitution for  $V_1$  and  $V_2$  in terms of  $P_1$  and  $P_2$  shows that this result is identical to the first one.

### 2.35 (7<sup>th</sup> edition Prob. 2.33)

The volumetric flow rate into the pipe is the velocity ( $u = 10 \text{ ft s}^{-1} = 3.048 \text{ m s}^{-1}$ ) times the cross-sectional area ( $A = \pi(0.25 \text{ ft})^2/4 = 0.04909 \text{ ft}^2$ ), so the volumetric flowrate is  $0.4909 \text{ ft}^3 \text{ s}^{-1}$ .

Because the specific volume at these conditions is  $3.058 \text{ ft}^3 \text{ lb}_m^{-1}$ , the mass flowrate in is  $m_m = 0.4909 \text{ ft}^3 \text{ s}^{-1} / 3.058 \text{ ft}^3 \text{ lb}_m^{-1} = 0.1605 \text{ lb}_m \text{ s}^{-1} = 0.07280 \text{ kg s}^{-1}$ . At steady-state, the mass flow rate out must be the same as the mass flowrate in. So, if the specific volume at the outlet conditions is  $78.14 \text{ ft}^3 \text{ lb}_m^{-1}$ , then the volumetric flowrate out is  $78.14 \text{ ft}^3 \text{ lb}_m^{-1} \cdot 0.1605 \text{ lb}_m \text{ s}^{-1} = 12.543 \text{ ft}^3 \text{ s}^{-1}$ . The cross-sectional area of the 10-inch diameter exit pipe is  $\pi(10/12 \text{ ft})^2/4 = 0.5454 \text{ ft}^2$ , so the velocity is  $u_{out} = 12.543 \text{ ft}^3 \text{ s}^{-1} / 0.5454 \text{ ft}^2 = 23.00 \text{ ft s}^{-1} = 7.0104 \text{ m s}^{-1}$ . Now, we can use this in the energy balance for an open system as written in equation 2.30.

$$\left( \Delta H + \frac{\Delta u^2}{2} + g\Delta z \right) m = Q + W$$

We will assume that the heat loss from the turbine is negligible ( $Q = 0$ ) and that the change in elevation from inlet to outlet is negligible ( $\Delta z = 0$ ), so the work output of the turbine is given by:

$$W = \left( \Delta H + \frac{\Delta u^2}{2g_c} \right) m$$

$$W = \left( 1148.6 - 1322.6 \text{ Btu lb}_m^{-1} + \frac{(7.0104 \text{ m s}^{-1})^2 - (3.048 \text{ m s}^{-1})^2}{2} \right) 0.07280 \text{ kg s}^{-1}$$

$$W = (-174.0 \text{ Btu lb}_m^{-1} + 19.93 \text{ J kg}^{-1}) 0.07280 \text{ kg s}^{-1}$$

$$W = (-404724 \text{ J kg}^{-1} - 19.93 \text{ J kg}^{-1}) 0.07280 \text{ kg s}^{-1} = -29462 \text{ J s}^{-1}$$

$W = -29.46 \text{ kJ s}^{-1} = -29.46 \text{ kW} = -27.92 \text{ Btu s}^{-1}$ . Our sign convention for  $W$  is that it is work done on the fluid, so the work output from the system is  $27.92 \text{ Btu s}^{-1} = 39.5 \text{ hp}$ .

### 2.36 (New)

The volumetric flow rate into the pipe is the mass flow rate times the specific volume ( $0.1 \text{ kg s}^{-1} \cdot 0.20024 \text{ m}^3 \text{ kg}^{-1} = 0.020024 \text{ m}^3 \text{ s}^{-1}$ ). The cross-sectional area of the inlet pipe is  $A = \pi(0.08 \text{ m})^2/4 = 0.005027 \text{ m}^2$ , so the average flow velocity is  $0.020024/0.005027 = 3.984 \text{ m s}^{-1}$ . At steady-state, the mass flow rate out must be the same as the mass flowrate in. So, if the specific volume at the outlet conditions is  $3.4181 \text{ m}^3 \text{ kg}^{-1}$ , then the volumetric flowrate out is  $3.4181 \text{ m}^3 \text{ kg}^{-1} \cdot 0.1 \text{ kg s}^{-1} = 0.34181 \text{ m}^3 \text{ s}^{-1}$ . The cross-sectional area of the 25 cm diameter exit pipe is  $\pi(0.25 \text{ m})^2/4 = 0.04909 \text{ m}^2$ , so the velocity is  $u_{out} = 0.34181 \text{ m}^3 \text{ s}^{-1} / 0.04909 \text{ m}^2 = 6.963 \text{ m s}^{-1}$ . Now, we can use this in the energy balance for an open system as written in equation 2.30.

$$\left( \Delta H + \frac{\Delta u^2}{2} + g\Delta z \right) m = Q + W$$

We will assume that the heat loss from the turbine is negligible ( $Q = 0$ ) and that the change in elevation from inlet to outlet is negligible ( $\Delta z = 0$ ), so the work output of the turbine is given by:

$$W = \left( \Delta H + \frac{\Delta u^2}{2g_c} \right) m$$

$$W = \left( (2682.6 - 3150.7 \text{ kJ kg}^{-1}) + \frac{(6.963 \text{ m s}^{-1})^2 - (3.984 \text{ m s}^{-1})^2}{2} \right) 0.1 \text{ kg s}^{-1}$$

$$W = (-468.1 \text{ kJ kg}^{-1} + 16.31 \text{ J kg}^{-1})0.1 \text{ kg s}^{-1} = -46.81 \text{ kJ s}^{-1}$$

$W = -46.81 \text{ kJ s}^{-1} = -46.81 \text{ kW}$ . Our sign convention for  $W$  is that it is work done on the fluid, so the work output from the system is 46.8 kW.

### 2.37 (New)

First, we need to determine the velocity of the discharge,  $u_2$ . To do this the mass flow rate must be determined by using

$$m = \frac{\pi D_1^2 u_1}{V_1} = 3354.70 \frac{\text{cm}^3 \cdot \text{mol}}{\text{L} \cdot \text{s}} \text{ or } 3.3547 \frac{\text{mol}}{\text{s}}.$$

Using this  $m$ ,

$$u_2 = m \frac{V_2}{\pi D_2^2} = 3.6026 \text{ m/s}.$$

Now that  $u_2$  has been determined, using Eqn. 2.31,

$$Q = H_2 - H_1 + \frac{u_2^2 - u_1^2}{2} - \frac{W_s}{mW_{CO_2}} = 2.07 \frac{\text{kJ}}{\text{mol}} - 87.02 \frac{\text{m}^2}{\text{s}^2} - 5681.6 \frac{\text{kJ}}{\text{mol}}$$

and converting  $87.02 \text{ m}^2/\text{s}^2$  by using  $1 \text{ kJ/kg} = 1000 \text{ m}^2/\text{s}^2$  you obtain  $Q = -127.064 \text{ kJ/mol}$ . To get the heat transfer rate,

$$Q = Q \cdot m = -19053 \text{ kJ/s}.$$

### 2.38 (7<sup>th</sup> edition Prob. 2.34)

First, we need to determine the velocity of the discharge,  $u_2$ . To do this, the mass flow rate must be determined by using

$$m = \frac{\pi D_1^2 u_1}{V_1} = 679.3 \text{ lb}_m \text{ hr}^{-1}.$$

Using this  $m$ ,

$$u_2 = m \frac{V_2}{\pi D_2^2} = 9.686 \text{ ft/s}.$$

Now that  $u_2$  has been determined, using Eqn. 2.31,

$$Q = H_2 - H_1 + \frac{u_2^2 - u_1^2}{2} - \frac{W_s}{mW_{CO_2}} = 23 \frac{\text{Btu}}{\text{lb}_m} + 153.09 \frac{\text{ft}^2}{\text{s}^2} - 121.8 \frac{\text{Btu}}{\text{lb}_m}$$

and converting  $153.09 \text{ ft}^2/\text{s}^2$  by using  $25036 \text{ lb}_m \cdot \text{ft}^2/\text{s}^2 \cdot \text{Btu}$ , you obtain  $Q = -98.8 \text{ Btu}/\text{lb}_m$ . To get the heat transfer rate,

$$\dot{Q} = Q \cdot m = -67115 \text{ Btu/hr}$$

### 2.39 (7<sup>th</sup> edition Prob. 2.35)

Starting with  $W$ , we know that:  $dPV = VdP + PdV$  and that in Eqn. 1.3,  $dW = -PdV$ .

Substituting in for  $PdV$  we get:  $dW = VdP - dPV$  and integrating we get  $W = \int VdP - \Delta(PV)$ .

From Eqn. 2.4,  $dQ = dU - dW$  and from Eqn. 2.10,  $U = H - PV$ .

Taking the integral of  $U$  we get  $dU = dH - PdV - VdP$  and substituting this in for  $dU$ , we get

$dQ = dH - PdV - VdP - dW$ . Knowing that  $dW =$

$-PdV$  from earlier and substituting we get  $dQ = dH -$

$VdP$ . Lastly, by integrating this we obtain  $Q = \Delta H - \int VdP$ .

### 2.40 (7<sup>th</sup> edition Prob. 2.36)



If  $PV/T$  is constant then at constant pressure, the volume will triple when the temperature triples, so the final state will be 900 K and 1 bar. The enthalpy change for this constant volume process with constant heat capacity is given by

$$\Delta H = C_p \Delta T = 29 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 900 \text{ K} - 300 \text{ K} = 17400 \text{ J mol}^{-1} = 600 \text{ J g}^{-1} = 600 \text{ kJ kg}^{-1}$$

where we've used the molecular weight of air of about  $29 \text{ g mol}^{-1}$  to convert from molar enthalpy change to specific enthalpy change.

By the definition of  $U$ , we have

$$\Delta U = \Delta H - \Delta PV = 17400 \text{ J mol}^{-1} - 83.14 \text{ bar cm}^3 \text{ mol}^{-1} \text{ K}^{-1} (900 \text{ K} - 300 \text{ K}) \frac{10^5 \frac{\text{Pa}}{\text{bar}}}{10^6 \frac{\text{cm}^3}{\text{m}^3}}$$

$$\Delta U = 17400 \text{ J mol}^{-1} - 4988 \text{ Pa m}^{-3} \text{ mol}^{-1} = 26100 \text{ J mol}^{-1} - 4988 \text{ J mol}^{-1} = 12412 \text{ J mol}^{-1}$$

$$\Delta U = 12412 \text{ J mol}^{-1} = 428 \text{ J g}^{-1} = 428 \text{ kJ kg}^{-1}$$

The work is given by  $-P\Delta V = -\Delta PV$  at constant  $P$ , which we just computed to be  $-4988 \text{ J mol}^{-1} = -172 \text{ J g}^{-1} = -172 \text{ kJ kg}^{-1}$ .

For a constant pressure process,  $Q$  is equal to the enthalpy change, so  $Q = 600 \text{ kJ/kg}$

### 2.41 (7<sup>th</sup> edition Prob. 2.37)

This is a two-step process, going first from  $20^\circ\text{C}$  to  $60^\circ\text{C}$ , and then 1000 kPa to 100 kPa.

So for temperature:  $T_1 = 293.16 \text{ K}$  and  $T_2 = 333.16 \text{ K}$ , and using  $PV = kT$  and  $R = 8.314 \frac{\text{J}}{\text{mol K}}$  to determine the intermediate temperature and the volume changes we have

$$T^I = T_1 \cdot \frac{P_2}{P_1} = 293.16 \text{ K} \cdot \left(\frac{1}{10}\right) = 29.316 \text{ K} \quad \text{and}$$

$$V_1 = R \frac{T_1}{P_1} = 8.314 \cdot 10^{-3} \frac{\text{m}^3 \text{kPa}}{\text{mol k}} \cdot \frac{293.16 \text{ k}}{1000 \text{ kPa}} = 2.437 \cdot 10^{-3} \frac{\text{m}^3}{\text{mol}}$$

$$V_2 = R \frac{T_2}{P_2} = 2.770 \cdot 10^{-2} \frac{\text{m}^3}{\text{mol}}$$

Using the intermediate temperature to determine the temperature change in both steps we have

$$\Delta T_a = T^I - T_1 = -263.844 \text{ k} \text{ and } \Delta T_b = T_2 - T^I = 303.844 \text{ k}.$$

Using  $\gamma = 8.314 \frac{\text{J}}{\text{mol k}}$ , and plugging the temperatures into Eqn. 2.16 and Eqn. 2.20, we have for Step A:

$$\Delta H_a = \Delta U_a + V_1 P_2 - P_1 = -5483.10 \frac{\text{J}}{\text{mol}} + 2.437 \cdot 10^{-3} \frac{\text{m}^3}{\text{mol}} (100 \text{ kPa} - 1000 \text{ kPa}) = -7676.40 \frac{\text{J}}{\text{mol}}$$

$$\Delta U_a = C_v \Delta T_a = \frac{5}{2} \cdot 8.314 \frac{\text{J}}{\text{mol k}} \cdot -263.844 \text{ k} = -5483.10 \frac{\text{J}}{\text{mol}}$$

And for Step B:

$$\Delta H_b = C_p \Delta T_b = \frac{7}{2} \cdot 8.314 \frac{\text{J}}{\text{mol k}} \cdot -303.844 \text{ k} = 8841.56 \frac{\text{J}}{\text{mol}}$$

$$\Delta U_b = \Delta H_b - P_2 V_2 - V_1 = 8841.56 \frac{\text{J}}{\text{mol}} - 100 \text{ kPa} \left( 2.770 \cdot 10^{-2} \frac{\text{m}^3}{\text{mol}} - 2.437 \cdot 10^{-3} \frac{\text{m}^3}{\text{mol}} \right) = 6315.26 \frac{\text{J}}{\text{mol}}$$

Now using these to determine the total  $\Delta U$  and  $\Delta H$

$$\Delta U = \Delta U_a + \Delta U_b = 832.16 \frac{\text{J}}{\text{mol}}$$

$$\Delta H = \Delta H_a + \Delta H_b = 1165.16 \frac{\text{J}}{\text{mol}}$$

## 2.42 (New)

For this problem, we need to find the specific heat or the heat capacity over a range of temperatures. From the text, it is given that

$$Q = \Delta H = {}^{\circ}\text{C } m \Delta T$$

Given the power supplied at the Q values and the flow rate, solving for the heat capacity gives:

$${}^{\circ}\text{C} = \frac{Q}{m\Delta T}$$

Given this the specific heat from 0°C to 10°C is

$${}^{\circ}\text{C} = \frac{5.5 \text{ J/sec}}{0.333 \text{ g/sec} \cdot 10 \text{ k}} = 1.650 \frac{\text{J}}{\text{g k}}$$

And going from 90°C to 100°C is

$${}^{\circ}\text{C} = 1.715 \frac{\text{J}}{\text{g k}}$$

The key here is that the initial temperature for each step is 0 C and that the  $\Delta T$  for each is the difference between 0°C and the new temperature. Over the whole range the °C values are.

$\frac{T_2}{^{\circ}\text{C}}$	10	20	30	40	50	60	70	80	90	100
$\frac{P}{W}$	5.5	11.0	16.6	22.3	28.0	33.7	39.6	45.4	51.3	57.3
$\frac{{}^{\circ}\text{C}}{J/(g k)}$	1.650	1.650	1.660	1.673	1.680	1.685	1.697	1.703	1.710	1.719

The average is  ${}^{\circ}\text{C} = 1.683 \text{ J/(g k)}$ .

### 2.43 (New)

The relevant steady-state energy balance is

$$\Delta H = Q + W$$

Whether we regard the energy input as heat or work depends on where we draw the system boundary. If we draw the system boundary to only contain the fluid, then  $W = 0$ , and  $\Delta H = Q$ , where  $Q$  is the flow of heat from the heating element to the water. The total change in enthalpy of the 237 g of water is  $\Delta H' = mC_p\Delta T = 237 \text{ g} \cdot 4.18 \text{ J g}^{-1}\text{ }^\circ\text{C}^{-1} \cdot 66 \text{ }^\circ\text{C} = 65384 \text{ J} = 65.4 \text{ kJ}$ . This is also the total heat transferred to the water in a period of 60 s. The rate of heat transfer is therefore  $65.4 \text{ kJ}/60 \text{ s} = 1.09 \text{ kJ/s} = 1.09 \text{ kW}$ . We can reasonably assume that the heating element fully converts electrical energy to heat, so at steady state, this would also be the power requirement for the heater. The actual power requirement might be slightly higher, because of heat losses to the surroundings (not all of the heat generated in the heating element actually enters the water). If the coffee maker is plugged into a regular U.S. 110 volt outlet, then it will draw at least 10 amps of current during the heating cycle. One wouldn't want to run this along with much of anything else on a regular 15 amp household circuit.

#### 2.44 (7<sup>th</sup> edition Prob. 2.38)

(a) Based on Equ. 2.23a:  $m =$

$u A \rho$ , if the  $\rho$  is constant, the  $A$  is constant, and the flow is at steady state, which means it remains unchanged  $u A$ , then that would mean  $q$  is constant as well.

(b) Due to the law of mass conservation,  $m$  must be constant.  $n$  will change due to reaction in the stream; the moles of A will go towards 0 as it travels to the end of the pipe. Due to the temperature and pressure changes in the pipe both  $q$  and  $u$  will also vary.

#### 2.45 (7<sup>th</sup> edition Prob. 2.39)

(a) First, the Reynolds number must be determined:

$$\text{Re} = \frac{D \rho u}{\mu} = \frac{2.0 \text{ cm} \cdot 996 \frac{\text{kg}}{\text{m}^3} \cdot 1.0 \frac{\text{m}}{\text{s}}}{9.0 \cdot 10^{-4} \frac{\text{kg}}{\text{m s}}} = 22133$$

Next, using the equation for the fanning friction factor given, we get

$$f_F = 0.3305 \left\{ \ln \left[ 0.27 \cdot 0.0001 + \left( \frac{7}{22133} \right)^{0.9} \right]^{-2} \right\} = 0.00635$$

With the friction factor, the  $m$  and  $\Delta P \Delta L$  can be determined:

$$m = \rho \cdot u \cdot \frac{\pi}{4} \cdot D^2 = 996 \frac{\text{kg}}{\text{m}^3} \cdot 1.0 \frac{\text{m}}{\text{s}} \cdot \frac{\pi}{4} \cdot 0.004 \text{ m}^2 = 0.313 \frac{\text{kg}}{\text{s}}$$

$$\Delta P \Delta L = \frac{-2}{D} \cdot \rho \cdot f_F \cdot u^2 = \frac{-2}{0.02 \text{ m}} \cdot 996 \frac{\text{kg}}{\text{m}^3} \cdot 0.00635 \cdot 1.0 \frac{\text{m}^2}{\text{s}^2} = -0.632 \frac{\text{kPa}}{\text{m}}$$

$$(b) Re = 55333, f_F = 0.00517, m = 1.956 \frac{\text{kg}}{\text{s}}, \Delta P \Delta L = -0.206 \frac{\text{kPa}}{\text{m}}$$

$$(c) Re = 110667, f_F = 0.00452, m = 1.564 \frac{\text{kg}}{\text{s}}, \Delta P \Delta L = -11.255 \frac{\text{kPa}}{\text{m}}$$

$$(d) Re = 276667, f_F = 0.00390, m = 9.778 \frac{\text{kg}}{\text{s}}, \Delta P \Delta L = -3.879 \frac{\text{kPa}}{\text{m}}$$

#### 2.46 (7<sup>th</sup> edition Prob. 2.42)

Making the assumption that the compressor is adiabatic, meaning that the  $Q = 0$ , then the

$$|W| = m \cdot H_2 - H_1 = 1.009 \text{ kW}$$

and then the

$$\text{Cost } \$ = 15200 \cdot \left( \frac{|W|}{\text{kW}} \right)^{0.573} = \$ 799,969.13$$

#### 2.47 (7<sup>th</sup> edition Prob. 2.43)

Performing an energy and material balance on a home, the only sources of energy loss are from the air, enthalpy, and internal energy and the only material loss is the air. This gives:

$$Q = n_{air} H + \frac{d(nU)}{dt} = n_{air} H + n \frac{dU}{dt} + U \frac{dn}{dt} \quad \text{and} \quad n_{air} = -\frac{dn}{dt}$$

By substituting, we obtain

$$Q = -H \frac{dn}{dt} + n \frac{dU}{dt} + U \frac{dn}{dt} = -(H - U) \frac{dn}{dt} + n \frac{dU}{dt} \text{ and know that } U = H - PV$$

We can obtain

$$Q = -PV \frac{dn}{dt} + n \frac{dU}{dt}$$

### 2.48 (7<sup>th</sup> edition Prob. 2.44)

(a) Starting with equation (2.31) we know that:

$$\Delta H + \frac{u^2}{2} + g\Delta z = Q + W_s$$

With the system being adiabatic, steady flow, assuming no height change, and that no work done by the shaft we get:

$$H_2 - H_1 + \frac{u_2^2 - u_1^2}{2} = 0$$

Determining the area for u gives:

$$u = \frac{4m}{\pi \rho D^2}$$

Substituting this in for  $u_1$  and  $u_2$  gives:

$$u_2^2 - u_1^2 = \left(\frac{4}{\pi}\right)^2 \frac{m^2}{\rho^2} \left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right)$$

We know that

$$H_2 - H_1 = \frac{1}{\rho} P_2 - P_1$$

Substituting these both in for the second equation we obtain:

$$\frac{1}{\rho} P_2 - P_1 + \frac{1}{2} \left(\frac{4}{\pi}\right)^2 \frac{m^2}{\rho^2} \left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right) = 0$$

And by solving for  $m$  we get :

$$m = \left[ 2\rho P_2 - P_1 \left(\frac{\pi}{4}\right)^2 \left(\frac{D_1^4 D_2^4}{D_1^4 - D_2^4}\right) \right]^{\frac{1}{2}}$$

(b) The only addition to the  $m$  equation from above would be the  $C(T_2 - T_1)$  term, as so:

$$m = \left[ 2[\rho P_2 - P_1 - C\rho^2(T_2 - T_1)] \left(\frac{\pi}{4}\right)^2 \left(\frac{D_1^4 D_2^4}{D_1^4 - D_2^4}\right) \right]^{1/2}$$