Chapter 2
An Introduction to Linear Programming

Learning Objectives

1. Obtain an overview of the kinds of problems linear programming has been used to solve.

2. Learn how to develop linear programming models for simple problems.

3. Be able to identify the special features of a model that make it a linear programming model.

4. Learn how to solve two variable linear programming models by the graphical solution procedure.

5. Understand the importance of extreme points in obtaining the optimal solution.

6. Know the use and interpretation of slack and surplus variables.

7. Be able to interpret the computer solution of a linear programming problem.

8. Understand how alternative optimal solutions, infeasibility and unboundedness can occur in linear programming problems.

9. Understand the following terms:

problem formulation	feasible region
constraint function	slack variable
objective function	standard form
solution	redundant constraint
optimal solution	extreme point
nonnegativity constraints	surplus variable
mathematical model	alternative optimal solutions
linear program	infeasibility
linear functions	unbounded
feasible solution	

Solutions:

1. a, b, and e, are acceptable linear programming relationships.

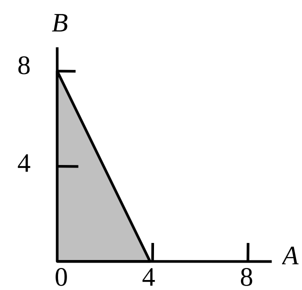
   c is not acceptable because of $-2B^2$

   d is not acceptable because of $3\sqrt{A}$

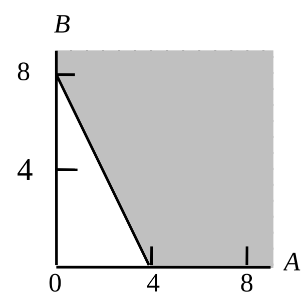
   f is not acceptable because of *1AB*

   c, d, and f could not be found in a linear programming model because they have the above nonlinear terms.

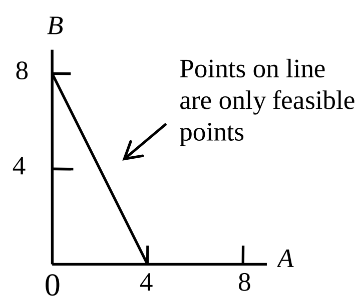
2. a.



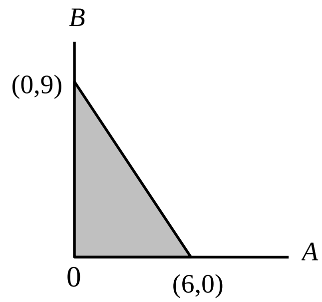
b.



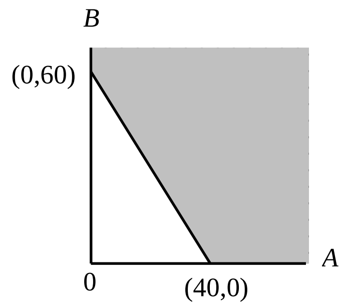
c.



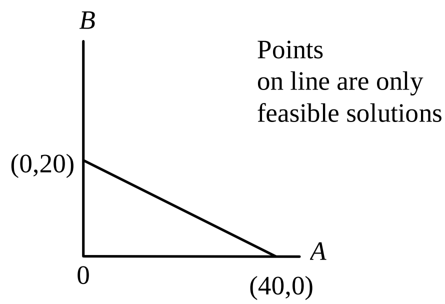
3. a.


B
(0,9)
A
0
(6,0)

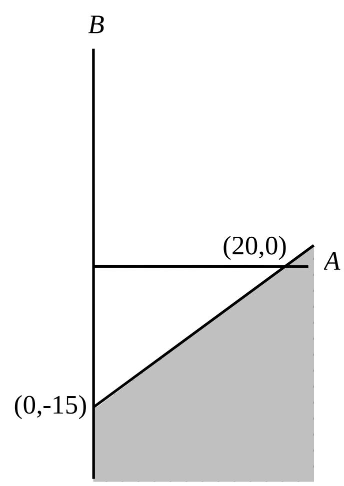
b.


B
(0,60)
A
0
(40,0)

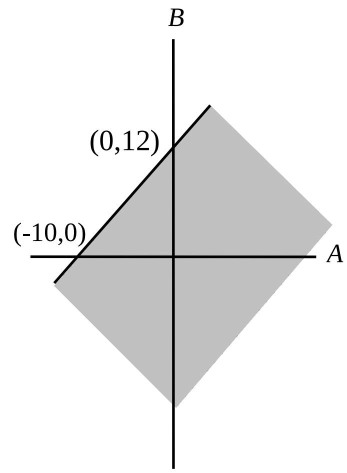
c.


B
Points
on line are only
feasible solutions
(0,20)
A
0
(40,0)

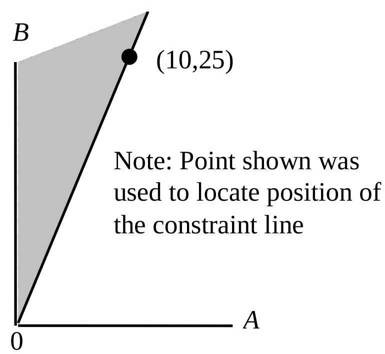
4. a.


B
(20,0)
A
(0,-15)

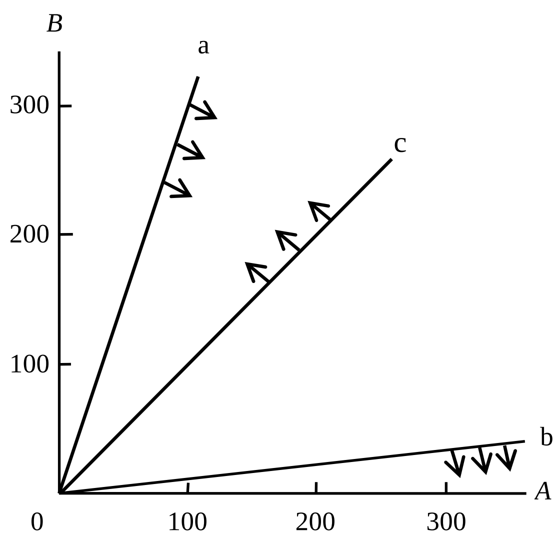
b.


B
(0,12)
(-10,0)
A

c.


B
(10,25)
Note: Point shown was used to locate position of the constraint line
A
0

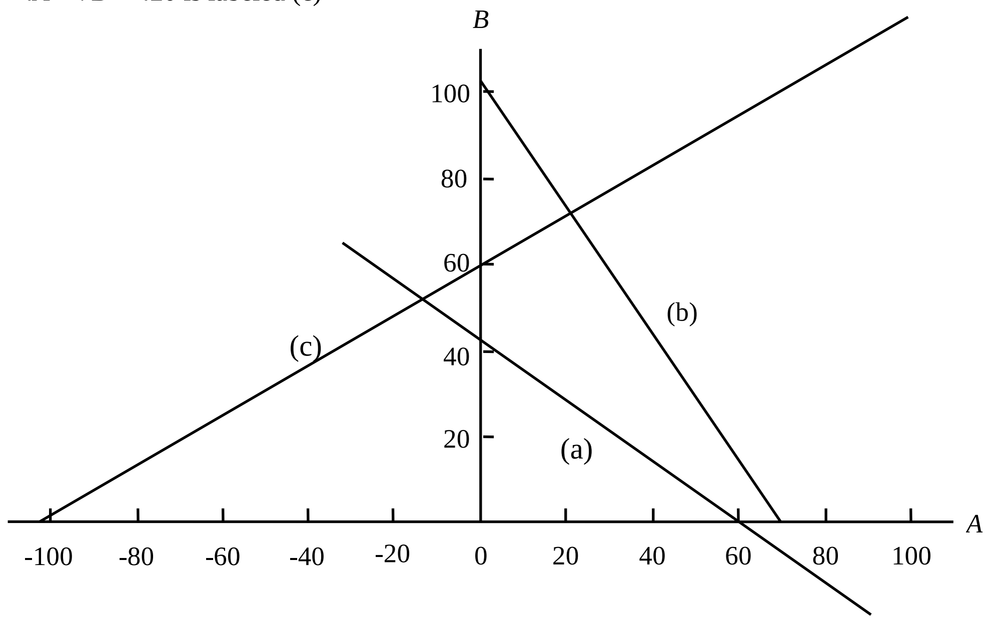
5.


B
a
c
b
300
200
100
0
100
200
300
A

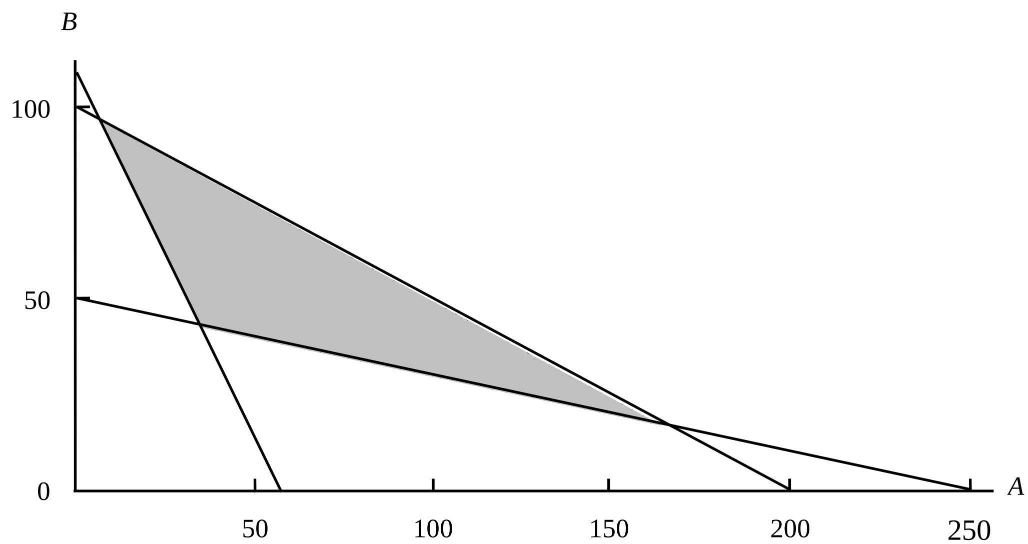
6. $7A + 10B = 420$ is labeled (a)

$6A + 4B = 420$ is labeled (b)

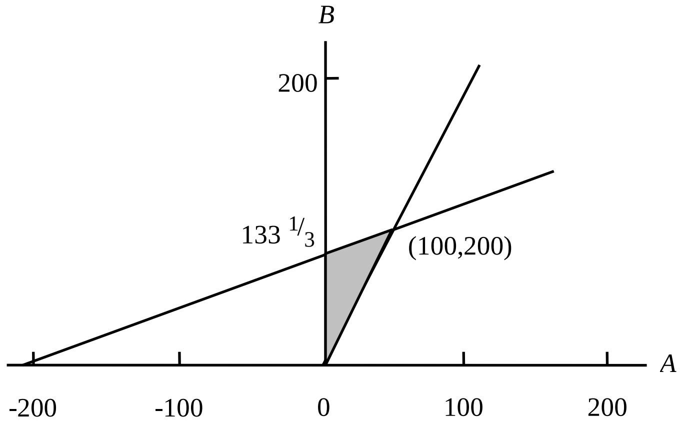
$-4A + 7B = 420$ is labeled (c)



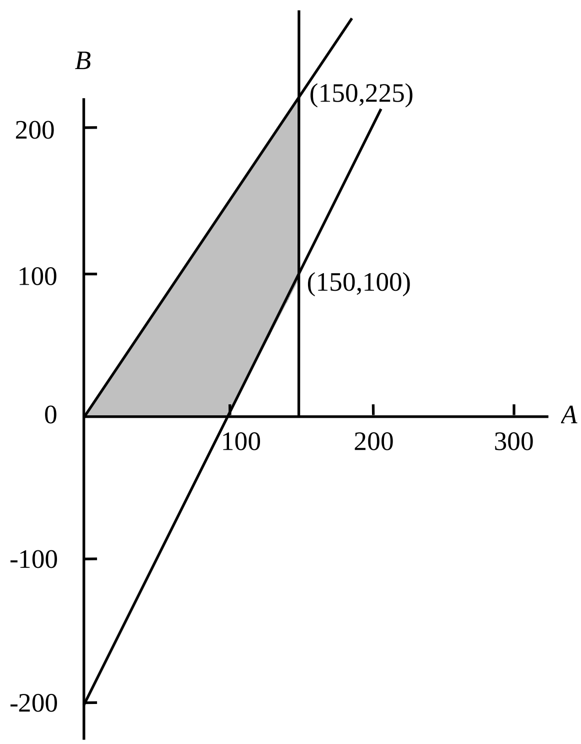
7.



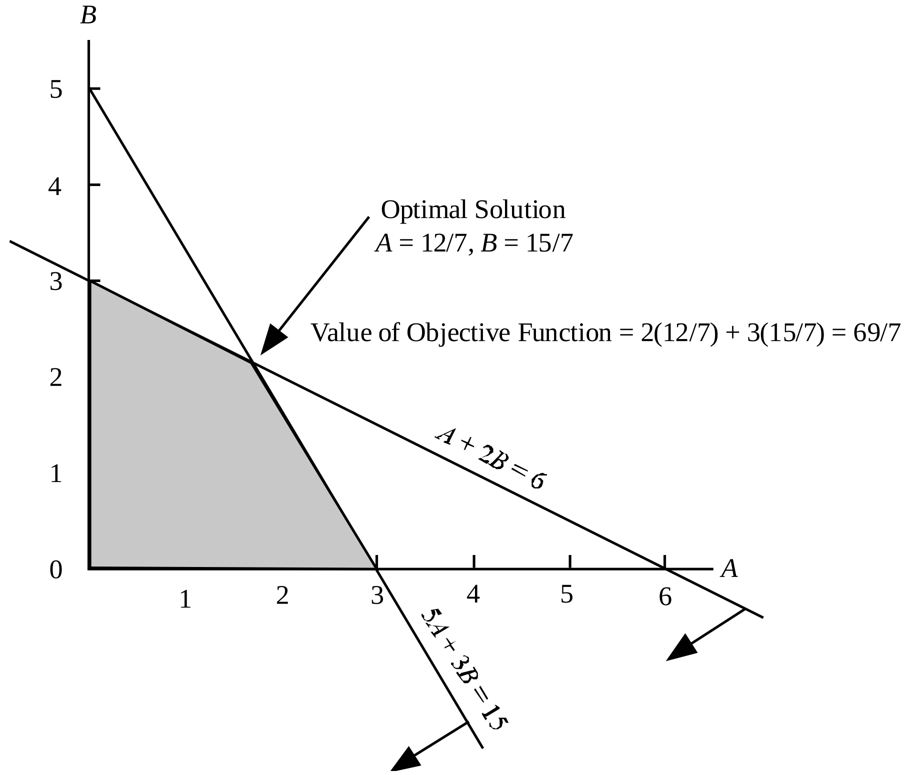
8.


B
200
133 1/3
(100,200)
A
-200
-100
0
100
200

9.


B
200
(150,225)
100
(150,100)
0
A
100
200
300
-100
-200

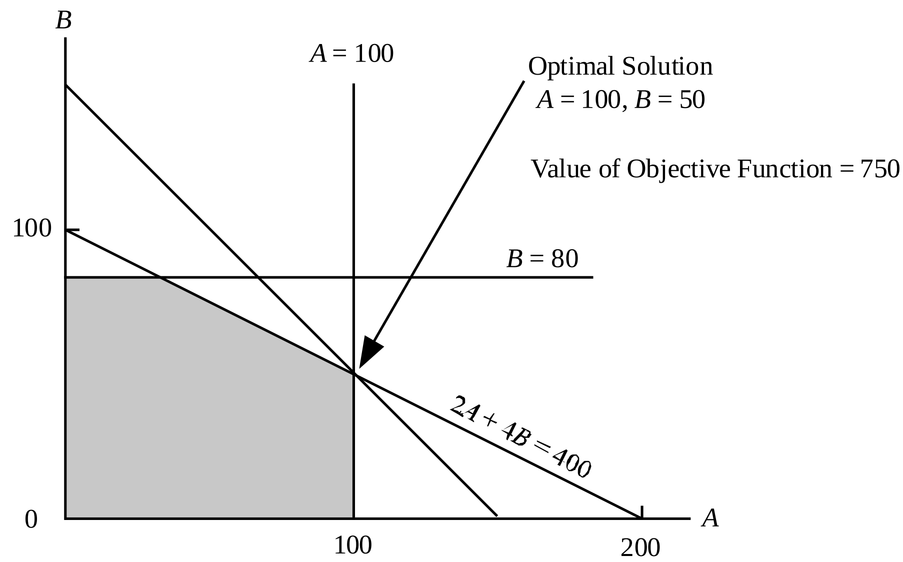
10.



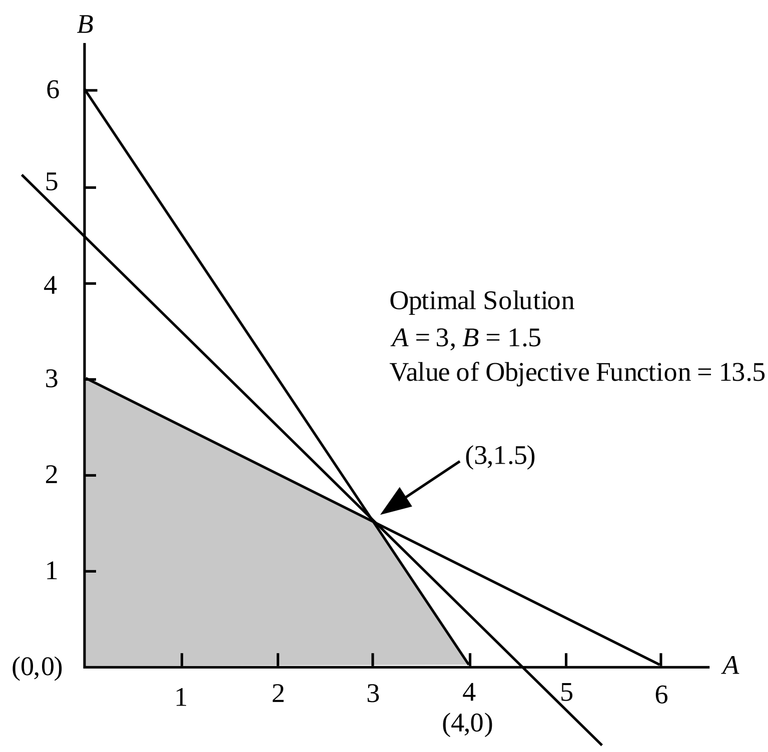
	A	+	$2B$	=	6	(1)
	$5A$	+	$3B$	=	15	(2)
(1) × 5	$5A$	+	$10B$	=	30	(3)
(2) - (3)		-	$7B$	=	-15	
			B	=	15/7	

From (1), $A = 6 - 2(15/7) = 6 - 30/7 = 12/7$

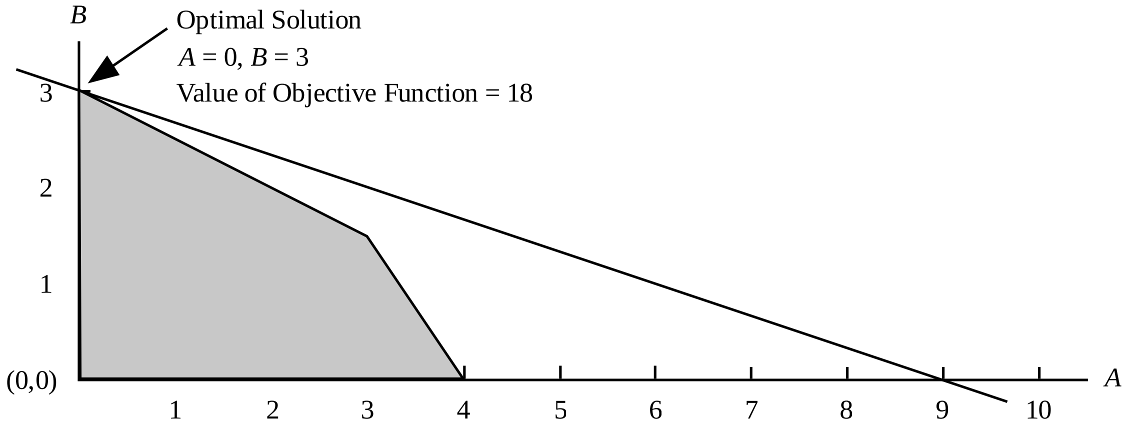
11.


B
A = 100
Optimal Solution
A = 100, B = 50
Value of Objective Function = 750
100
B = 80
2A + 4B = 400
0
100
200
A

12. a.

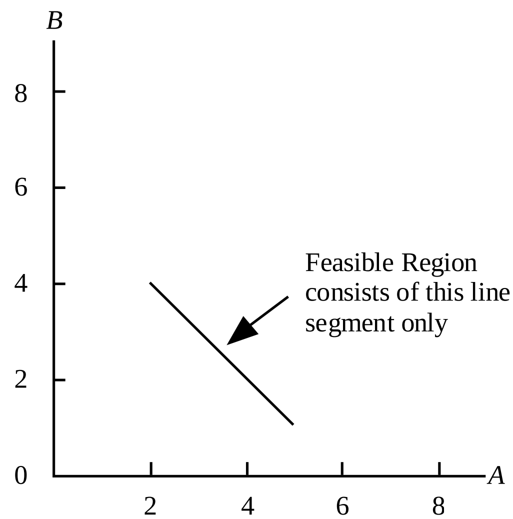

B
6
5
4
3
2
1
Optimal Solution
A = 3, B = 1.5
Value of Objective Function = 13.5
(3,1.5)
(0,0)
1
2
3
4
5
6
A
(4,0)

b.



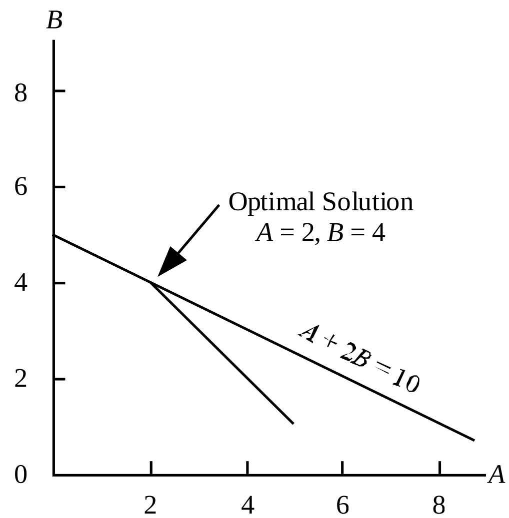
c. There are four extreme points: (0,0), (4,0), (3,1,5), and (0,3).

13. a.



b. The extreme points are (5, 1) and (2, 4).

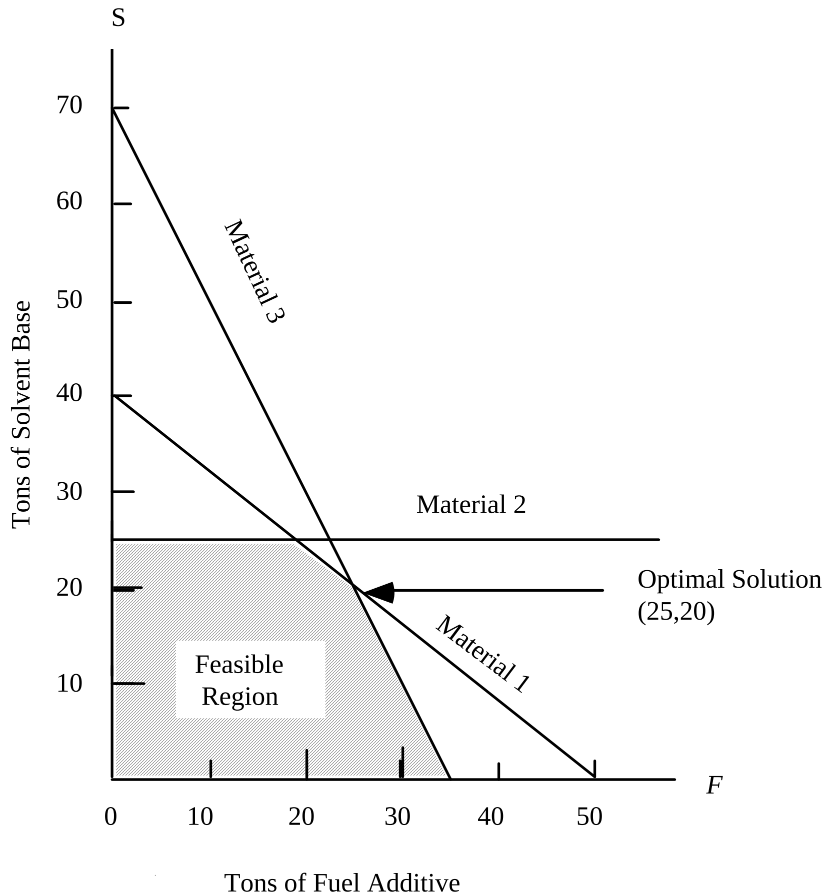
c.



14. a. Let F = number of tons of fuel additive
S = number of tons of solvent base

$$
\begin{aligned}
\text{Max} \quad & 40F + 30S \\
\text{s.t.} \quad & \\
& 2/5F + {}^{1}/_{2}\,S \le 200 \quad \text{Material 1} \\
& {}^{1}/_{5}\,S \le 5 \quad \text{Material 2} \\
& {}^{3}/_{5}\,F + {}^{3}/_{10}\,S \le 21 \quad \text{Material 3} \\
& F,\ S \ge 0
\end{aligned}
$$

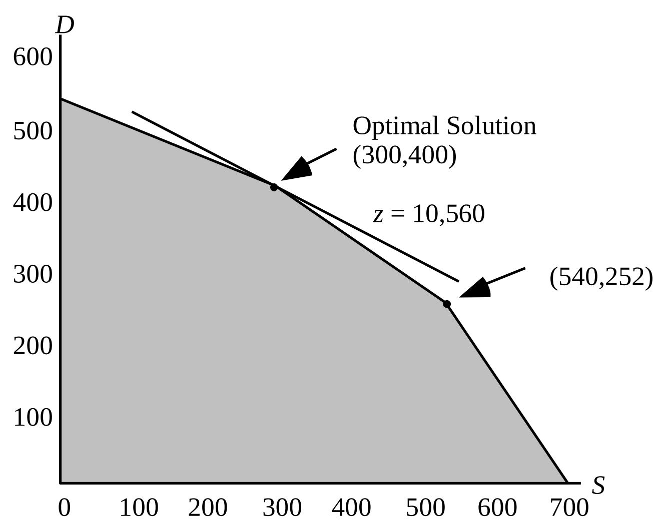
b.



c. Material 2: 4 tons are used, 1 ton is unused.

d. No redundant constraints.

15. a.



b. Similar to part (a): the same feasible region with a different objective function. The optimal solution occurs at (708, 0) with a profit of $z = 20(708) + 9(0) = 14{,}160$.

c. The sewing constraint is redundant. Such a change would not change the optimal solution to the original problem.

16. a. A variety of objective functions with a slope greater than -4/10 (slope of I & P line) will make extreme point (0, 540) the optimal solution. For example, one possibility is $3S + 9D$.

b. Optimal Solution is $S = 0$ and $D = 540$.

c.

Department	Hours Used	Max. Available	Slack
Cutting and Dyeing	1(540) = 540	630	90
Sewing	$^5/_6$(540) = 450	600	150
Finishing	$^2/_3$(540) = 360	708	348
Inspection and Packaging	$^1/_4$(540) = 135	135	0

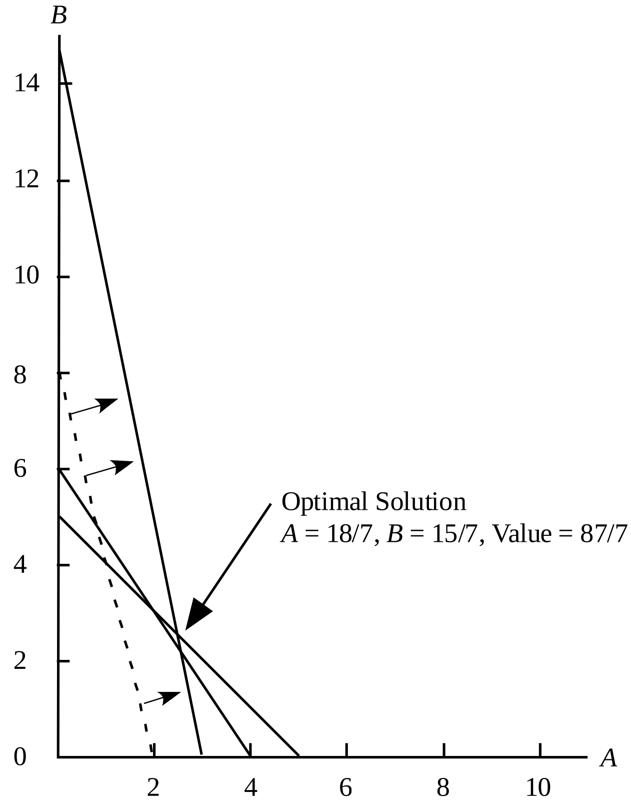
17.

$$\begin{array}{llllllll} \text{Max} & 5A + 2B + 0S_1 + 0S_2 + 0S_3 & & \\ \text{s.t.} & & & \\ & 1A - 2B + 1S_1 & = & 420 \\ & 2A + 3B + 1S_2 & = & 610 \\ & 6A - 1B + 1S_3 & = & 125 \\ & A, B, S_1, S_2, S_3 \geq 0 & & \end{array}$$

18. a.

$$\begin{array}{llll} \text{Max} & 4A + 1B + 0S_1 + 0S_2 + 0S_3 & & \\ \text{s.t.} & & & \\ & 10A + 2B + 1S_1 & = & 30 \\ & 3A + 2B + 1S_2 & = & 12 \\ & 2A + 2B + 1S_3 & = & 10 \\ & A, B, S_1, S_2, S_3 \geq 0 & & \end{array}$$

b.



c. $S_1 = 0$, $S_2 = 0$, $S_3 = 4/7$

19. a.

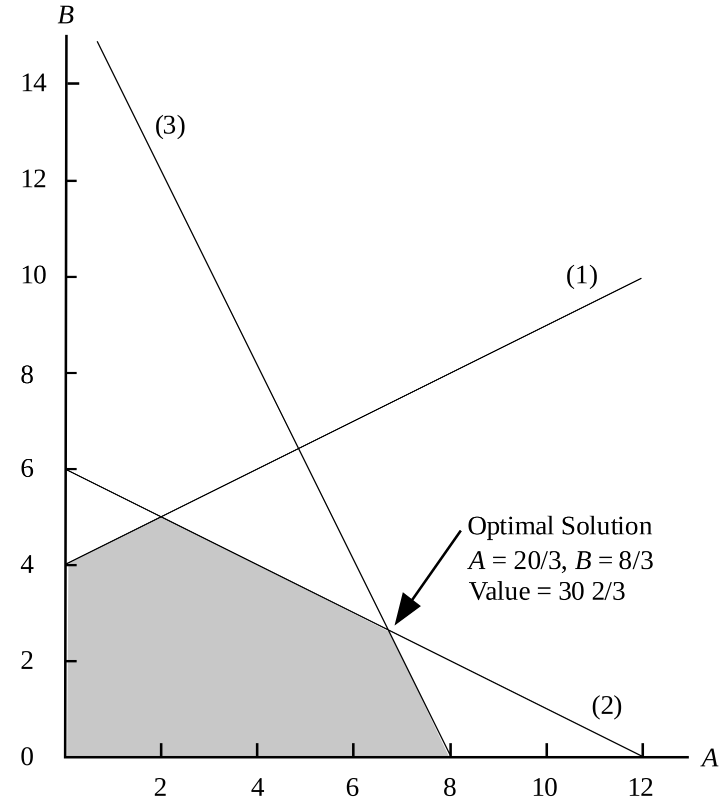
$$\text{Max} \quad 3A + 4B + 0S_1 + 0S_2 + 0S_3$$

s.t.

$$\begin{aligned} -1A + 2B + 1S_1 \quad &= 8 \quad (1) \\ 1A + 2B + 1S_2 \quad &= 12 \quad (2) \\ 2A + 1B + 1S_3 \quad &= 16 \quad (3) \end{aligned}$$

$$A, B, S_1, S_2, S_3 \geq 0$$

b.



c. $S_1 = 8 + A - 2B = 8 + 20/3 - 16/3 = 28/3$

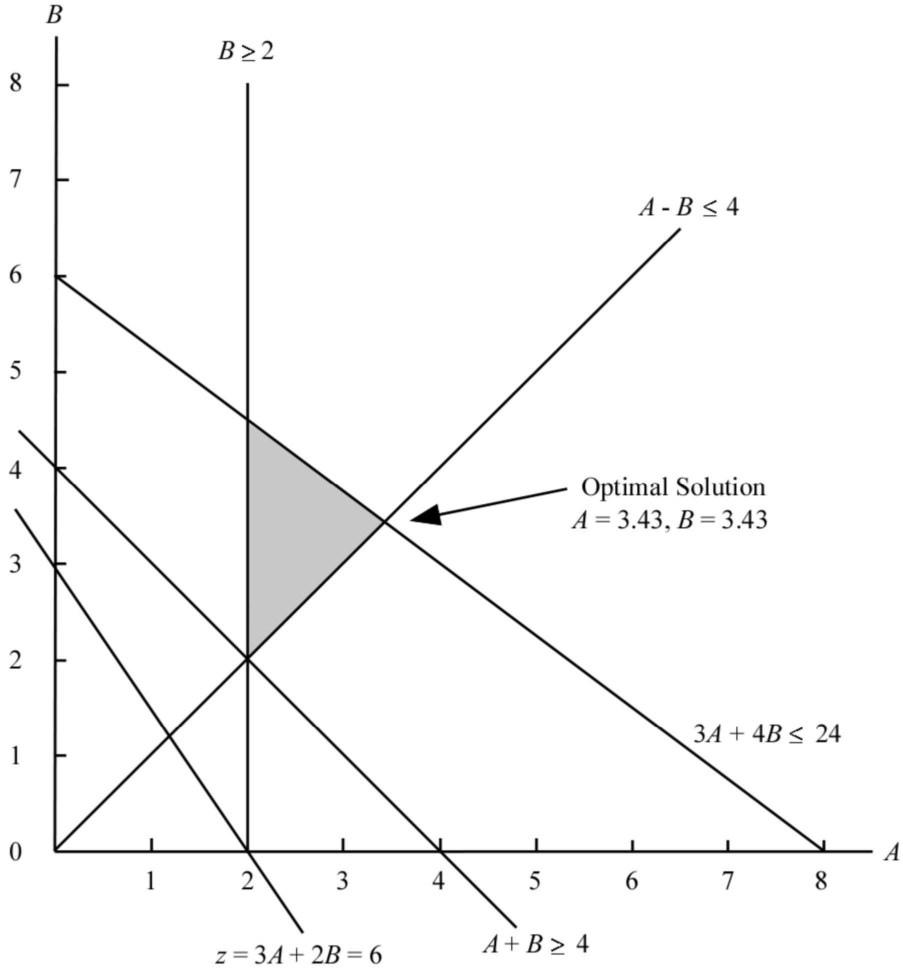
$S_2 = 12 - A - 2B = 12 - 20/3 - 16/3 = 0$

$S_3 = 16 - 2A - B = 16 - 40/3 - 8/3 = 0$

20. a.

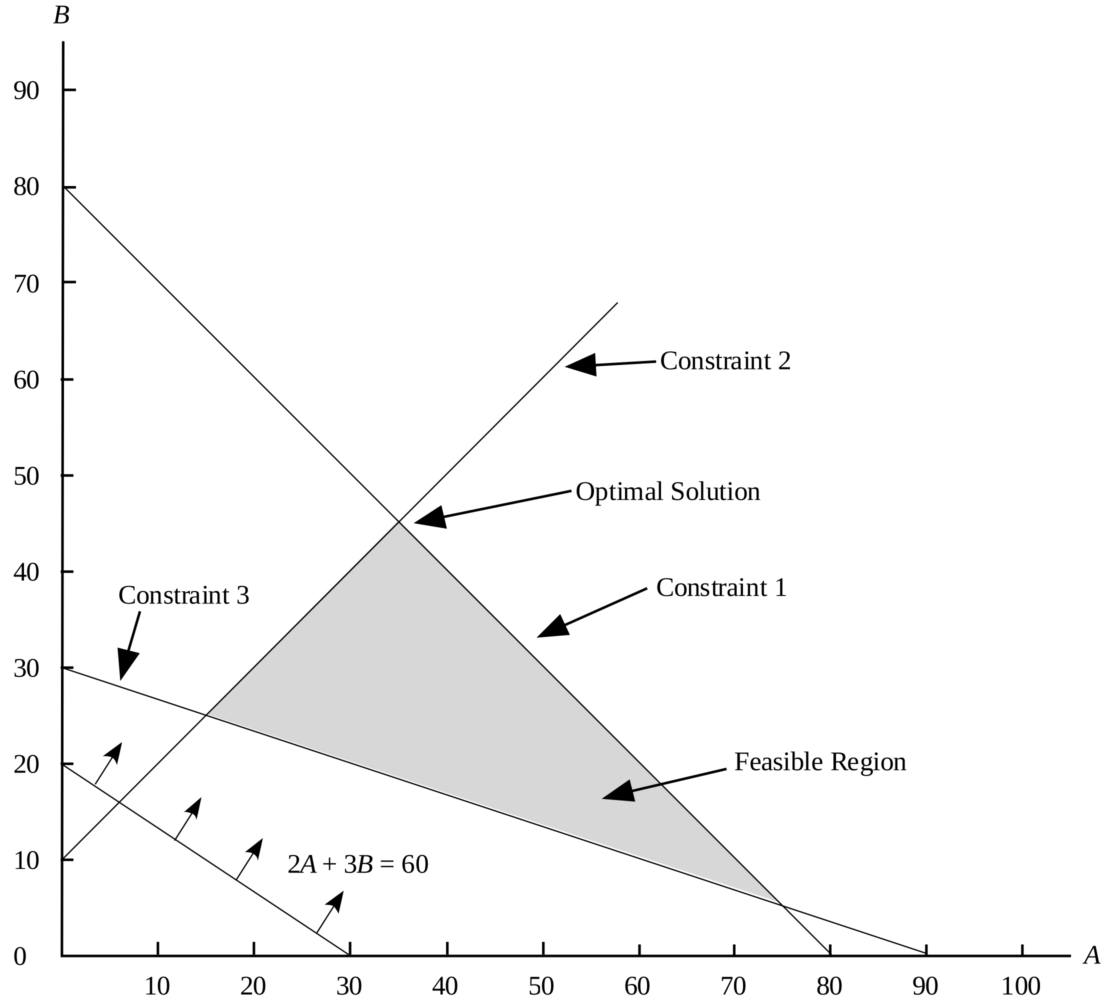
$$
\begin{array}{llllllll}
\text{Max} & 3A + 2B & & & & & & \\
\text{s.t.} & & & & & & & \\
& A + B & - S_1 & & & & = & 4 \\
& 3A + 4B & & + S_2 & & & = & 24 \\
& A & & & - S_3 & & = & 2 \\
& A - B & & & & - S_4 & = & 0 \\
& & A, B, S_1, S_2, S_3, S_4 \geq 0 & & & & &
\end{array}
$$

b.



c. S_1 = (3.43 + 3.43) - 4 = 2.86
S_2 = 24 - [3(3.43) + 4(3.43)] = 0
S_3 = 3.43 - 2 = 1.43
S_4 = 0 - (3.43 - 3.43) = 0

21. a. and b.



c. Optimal solution occurs at the intersection of constraints 1 and 2. For constraint 2,

$$B = 10 + A$$

Substituting for B in constraint 1 we obtain

$$\begin{aligned} 5A + 5(10 + A) &= 400 \\ 5A + 50 + 5A &= 400 \\ 10A &= 350 \\ A &= 35 \end{aligned}$$

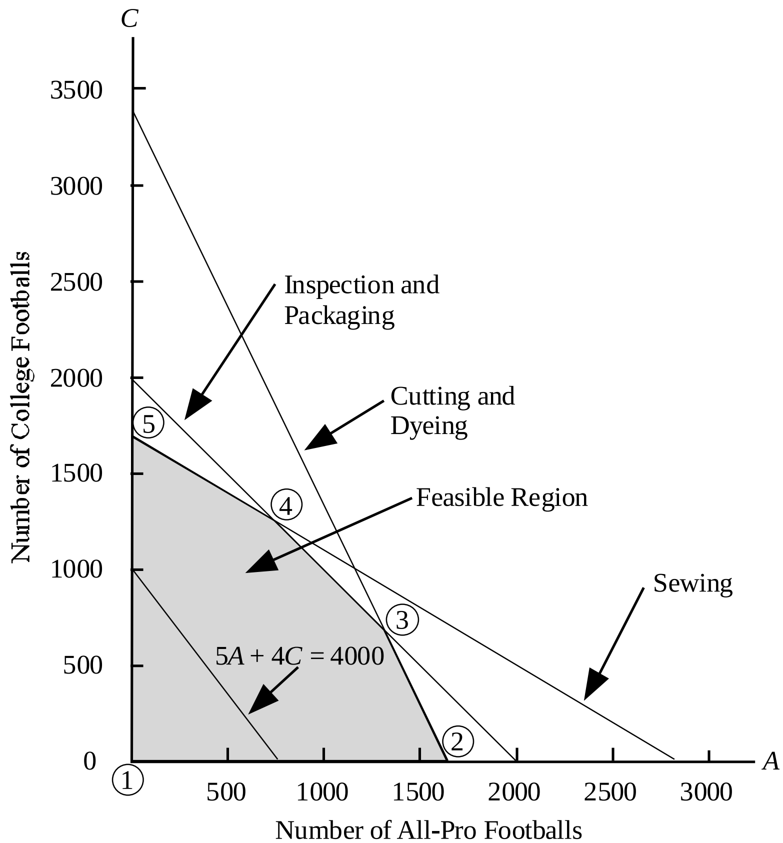
$$B = 10 + A = 10 + 35 = 45$$

Optimal solution is $A = 35$, $B = 45$

d. Because the optimal solution occurs at the intersection of constraints 1 and 2, these are binding constraints.

e. Constraint 3 is the nonbinding constraint. At the optimal solution 1*A* + 3*B* = 1(35) + 3(45) = 170. Because 170 exceeds the right-hand side value of 90 by 80 units, there is a surplus of 80 associated with this constraint.

22. a.



b.

Extreme Point	Coordinates	Profit
1	(0, 0)	5(0) + 4(0) = 0
2	(1700, 0)	5(1700) + 4(0) = 8500
3	(1400, 600)	5(1400) + 4(600) = 9400
4	(800, 1200)	5(800) + 4(1200) = 8800
5	(0, 1680)	5(0) + 4(1680) = 6720

Extreme point 3 generates the highest profit.

c. Optimal solution is *A* = 1400, *C* = 600

d. The optimal solution occurs at the intersection of the cutting and dyeing constraint and the inspection and packaging constraint. Therefore these two constraints are the binding constraints.

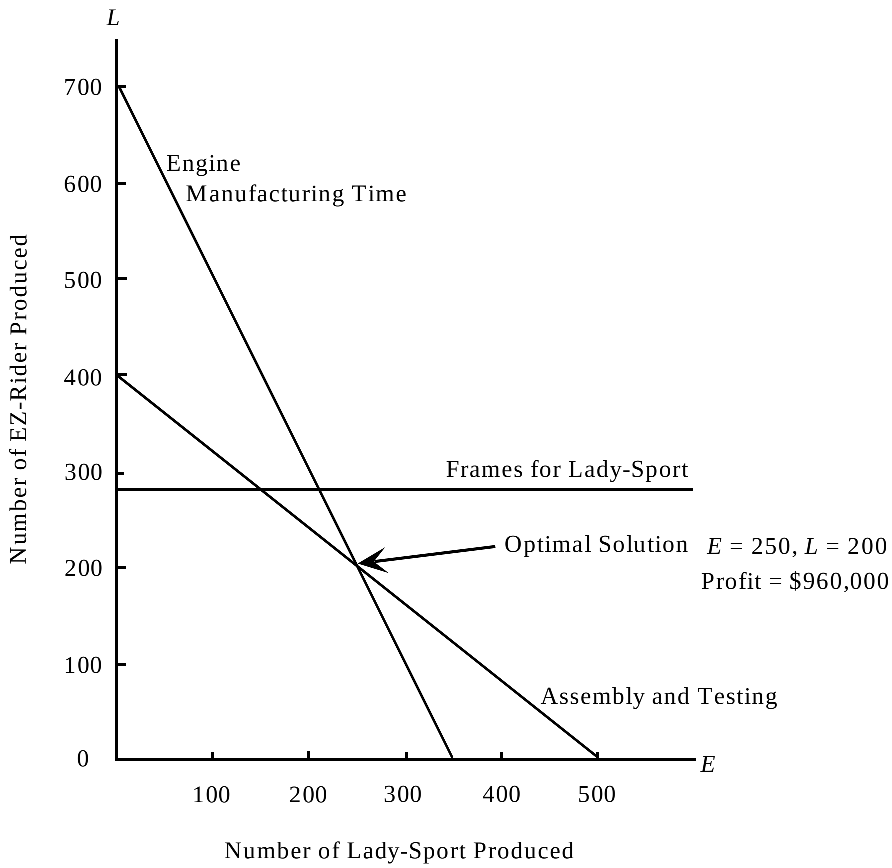
e. New optimal solution is *A* = 800, *C* = 1200

Profit = 4(800) + 5(1200) = 9200

23. a. Let E = number of units of the EZ-Rider produced
L = number of units of the Lady-Sport produced

$$\begin{array}{lrcrcrl} \text{Max} & 2400E & + & 1800L & & & \\ \text{s.t.} & & & & & & \\ & 6E & + & 3L & \leq & 2100 & \text{Engine time} \\ & & & L & \leq & 280 & \text{Lady-Sport maximum} \\ & 2E & + & 2.5L & \leq & 1000 & \text{Assembly and testing} \\ & & & E, L \geq 0 & & & \end{array}$$

b.

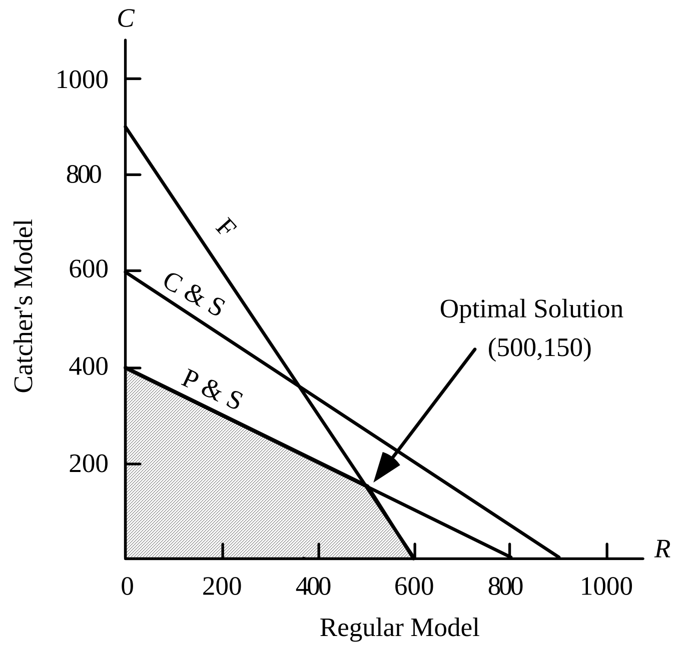


c. The binding constraints are the manufacturing time and the assembly and testing time.

24. a. Let R = number of units of regular model.
C = number of units of catcher's model.

$$\begin{array}{lrcrcrl} \text{Max} & 5R & + & 8C & & & \\ \text{s.t.} & & & & & & \\ & 1R & + & {}^3/_2\, C & \leq & 900 & \text{Cutting and sewing} \\ & {}^1/_2\, R & + & {}^1/_3\, C & \leq & 300 & \text{Finishing} \\ & {}^1/_8\, R & + & {}^1/_4\, C & \leq & 100 & \text{Packing and Shipping} \\ R,\ C \geq 0 & & & & & & \end{array}$$

b.



c. 5(500) + 8(150) = $3,700

d. C & S 1(500) + $^{3}/_{2}$(150) = 725

F $^{1}/_{2}$(500) + $^{1}/_{3}$(150) = 300

P & S $^{1}/_{8}$(500) + $^{1}/_{4}$(150) = 100

e.

Department	Capacity	Usage	Slack
C & S	900	725	175 hours
F	300	300	0 hours
P & S	100	100	0 hours

25. a. Let B = percentage of funds invested in the bond fund
S = percentage of funds invested in the stock fund

$$\begin{aligned} \text{Max} \quad & 0.06B + 0.10S \\ \text{s.t.} \quad & \\ & B \geq 0.3 \quad \text{Bond fund minimum} \\ & 0.06B + 0.10S \geq 0.075 \quad \text{Minimum return} \\ & B + S = 1 \quad \text{Percentage requirement} \end{aligned}$$

b. Optimal solution: $B = 0.3$, $S = 0.7$

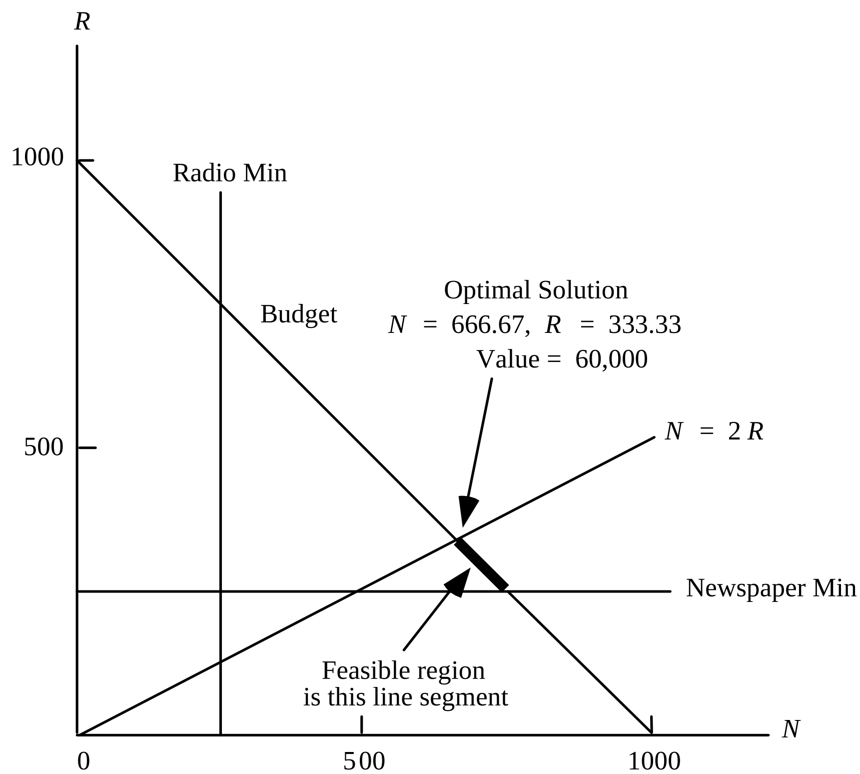
Value of optimal solution is 0.088 or 8.8%

26. a. Let N = amount spent on newspaper advertising
R = amount spent on radio advertising

Max	$50N$	+	$80R$				
s.t.							
	N	+	R	=	1000	Budget
	N			≥	250	Newspaper min.
			R	≥	250	Radio min.
	N		$-2R$	≥	0	News ≥ 2 Radio

$N, R \geq 0$

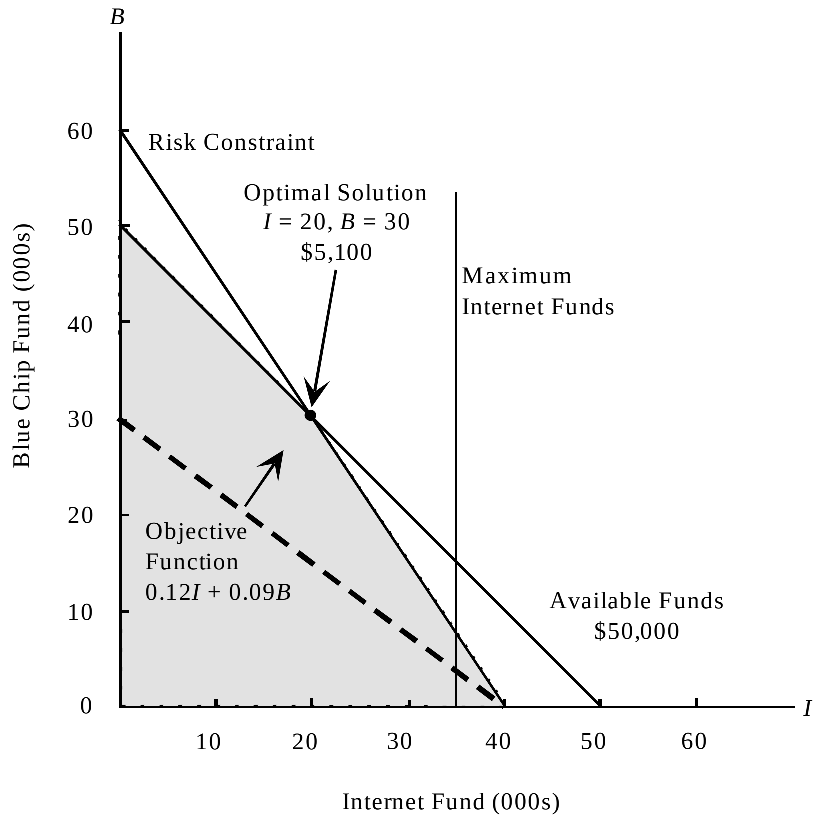
b.



27. a. Let I = Internet fund investment in thousands
B = Blue Chip fund investment in thousands

Max	$0.12I$	+	$0.09B$			
s.t.						
	$1I$	+	$1B$	≤	50	Available investment funds
	$1I$			≤	35	Maximum investment in the internet fund
	$6I$	+	$4B$	≤	240	Maximum risk for a moderate investor

$I, B \geq 0$



Internet fund	\$20,000
Blue Chip fund	\$30,000
Annual return	\$ 5,100

b. The third constraint for the aggressive investor becomes

$$6I + 4B \leq 320$$

This constraint is redundant; the available funds and the maximum Internet fund investment constraints define the feasible region. The optimal solution is:

Internet fund	\$35,000
Blue Chip fund	\$15,000
Annual return	\$ 5,550

The aggressive investor places as much funds as possible in the high return but high risk Internet fund.

c. The third constraint for the conservative investor becomes

$$6I + 4B \leq 160$$

This constraint becomes a binding constraint. The optimal solution is

Internet fund	\$0
Blue Chip fund	\$40,000
Annual return	\$ 3,600

The slack for constraint 1 is $10,000. This indicates that investing all $50,000 in the Blue Chip fund is still too risky for the conservative investor. $40,000 can be invested in the Blue Chip fund. The remaining $10,000 could be invested in low-risk bonds or certificates of deposit.

28. a. Let W = number of jars of Western Foods Salsa produced
M = number of jars of Mexico City Salsa produced

$$\begin{aligned} \text{Max} \quad & 1W + 1.25M \\ \text{s.t.} \quad & \\ & 5W \quad 7M \leq 4480 \quad \text{Whole tomatoes} \\ & 3W + 1M \leq 2080 \quad \text{Tomato sauce} \\ & 2W + 2M \leq 1600 \quad \text{Tomato paste} \\ & W,\ M \geq 0 \end{aligned}$$

Note: units for constraints are ounces

b. Optimal solution: $W = 560$, $M = 240$

Value of optimal solution is 860

29. a. Let B = proportion of Buffalo's time used to produce component 1
D = proportion of Dayton's time used to produce component 1

Maximum Daily Production

	Component 1	Component 2
Buffalo	2000	1000
Dayton	600	1400

Number of units of component 1 produced: $2000B + 600D$

Number of units of component 2 produced: $1000(1 - B) + 600(1 - D)$

For assembly of the ignition systems, the number of units of component 1 produced must equal the number of units of component 2 produced.

Therefore,

$$2000B + 600D = 1000(1 - B) + 1400(1 - D)$$

$$2000B + 600D = 1000 - 1000B + 1400 - 1400D$$

$$3000B + 2000D = 2400$$

Note: Because every ignition system uses 1 unit of component 1 and 1 unit of component 2, we can maximize the number of electronic ignition systems produced by maximizing the number of units of subassembly 1 produced.

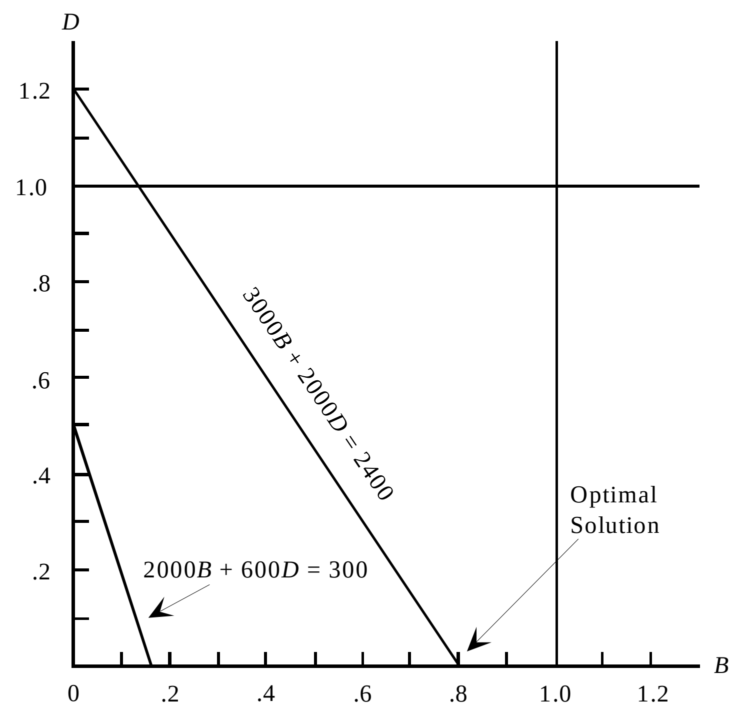
$$\text{Max } 2000B + 600D$$

In addition, $B \leq 1$ and $D \leq 1$.

The linear programming model is:

Max	$2000B$	$+\ 600D$	
s.t.			
	$3000B$	$+\ 2000D$	$= 2400$
	B		≤ 1
		D	≤ 1
		B, D	≥ 0

b. The graphical solution is shown below.



Optimal Solution: $B = .8$, $D = 0$

Optimal Production Plan

Buffalo - Component 1	.8(2000) = 1600
Buffalo - Component 2	.2(1000) = 200
Dayton - Component 1	0(600) = 0
Dayton - Component 2	1(1400) = 1400

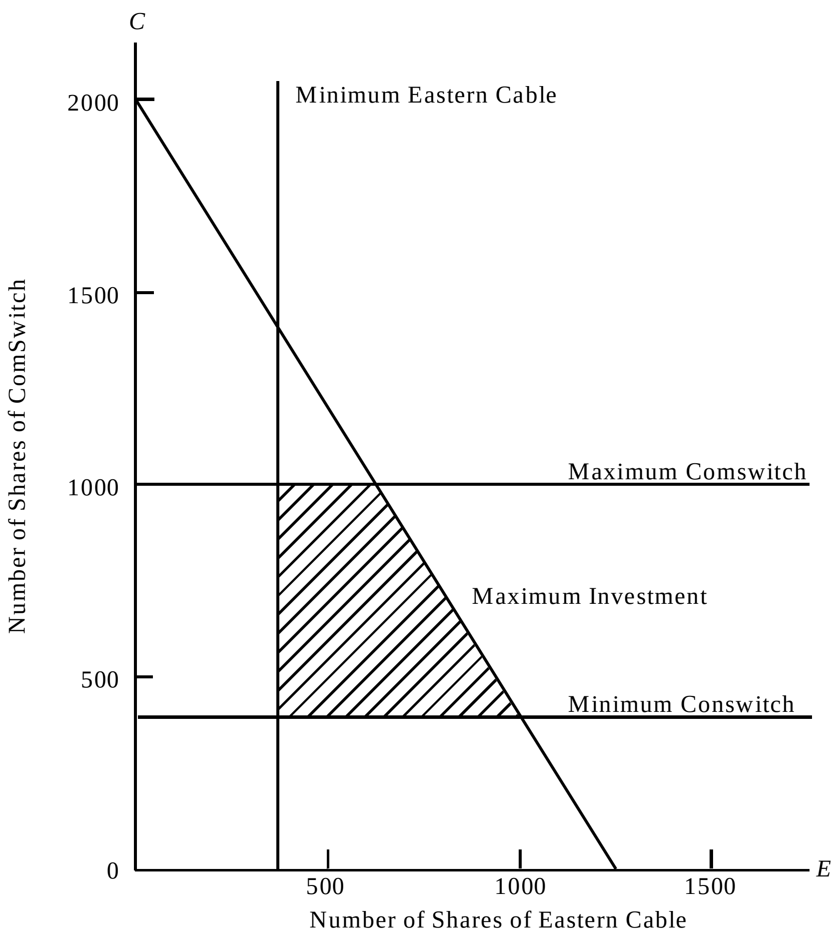
Total units of electronic ignition system = 1600 per day.

30. a. Let E = number of shares of Eastern Cable
C = number of shares of ComSwitch

Max	$15E + 18C$			
s.t.				
	$40E + 25C$	$\leq$	50,000	Maximum Investment
	$40E$	$\geq$	15,000	Eastern Cable Minimum
	$25C$	$\geq$	10,000	ComSwitch Minimum
	$25C$	$\leq$	25,000	ComSwitch Maximum

$E, C \geq 0$

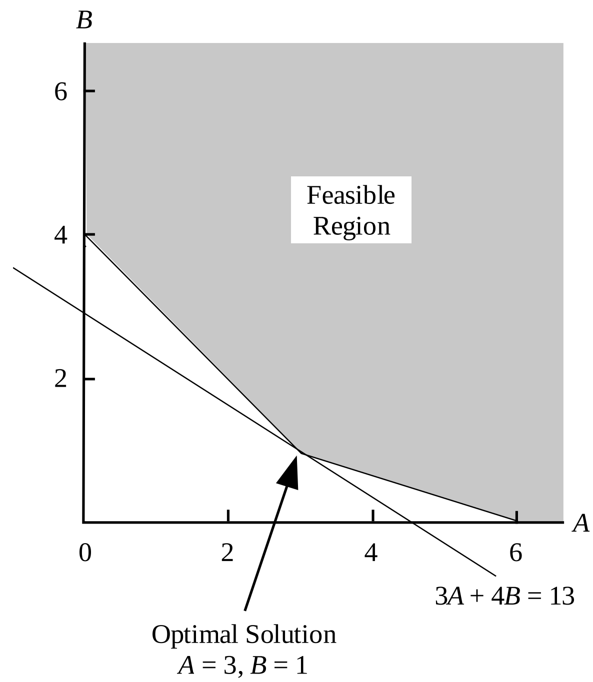
b.



c. There are four extreme points: (375,400); (1000,400);(625,1000); (375,1000)

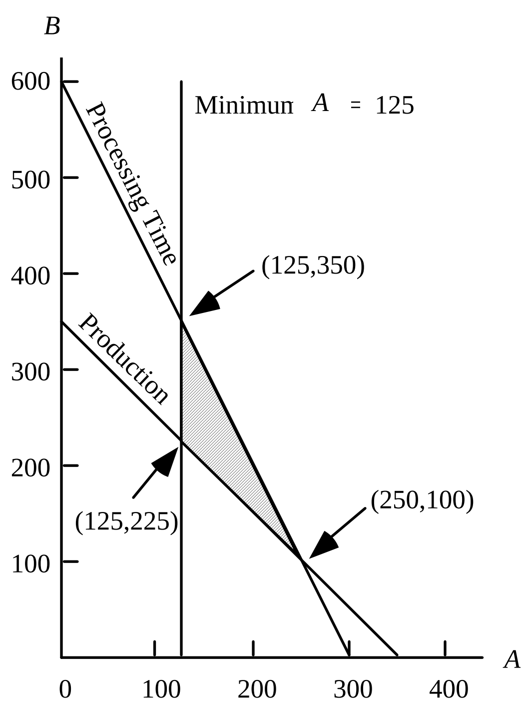
d. Optimal solution is $E = 625$, $C = 1000$
Total return = $27,375

31.



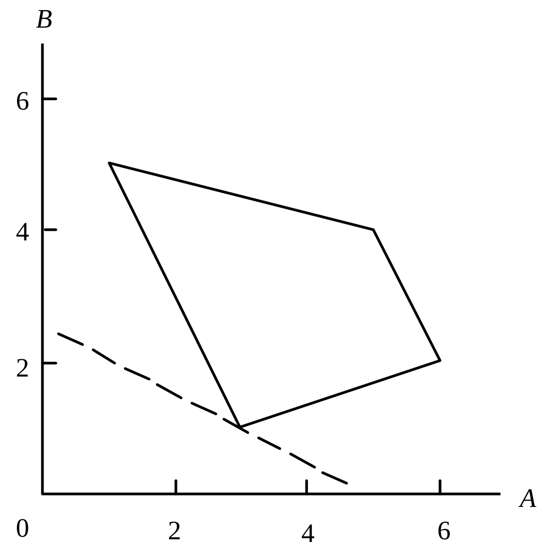
Objective Function Value = 13

32.



Extreme Points	Objective Function Value	Surplus Demand	Surplus Total Production	Slack Processing Time
(A = 250, B = 100)	800	125	—	—
(A = 125, B = 225)	925	—	—	125
(A = 125, B = 350)	1300	—	125	—

33. a.

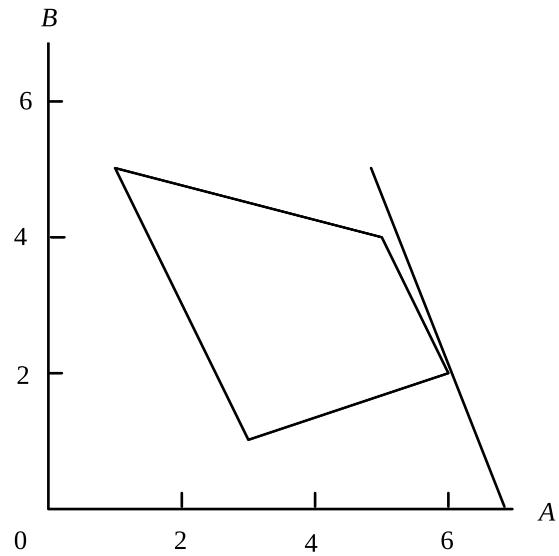


Optimal Solution: A = 3, B = 1, value = 5

b.

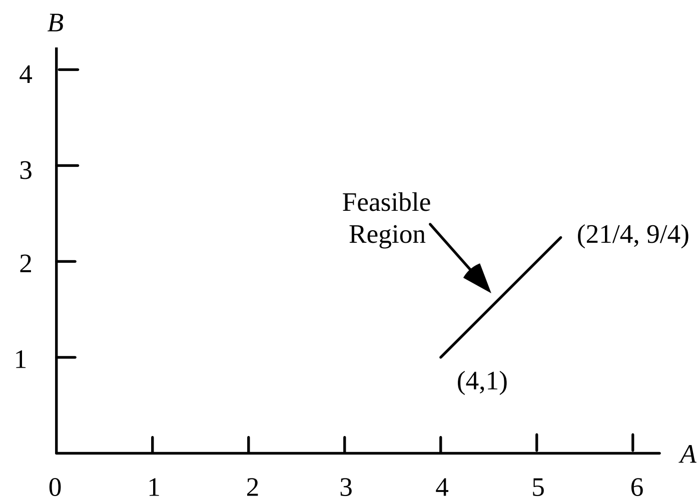
(1)	3 + 4(1) = 7	Slack = 21 - 7 = 14
(2)	2(3) + 1 = 7	Surplus = 7 - 7 = 0
(3)	3(3) + 1.5 = 10.5	Slack = 21 - 10.5 = 10.5
(4)	-2(3) +6(1) = 0	Surplus = 0 - 0 = 0

c.



Optimal Solution: *A* = 6, *B* = 2, value = 34

34. a.



b. There are two extreme points: (*A* = 4, *B* = 1) and (*A* = 21/4, *B* = 9/4)

c. The optimal solution is *A* = 4, *B* = 1

35. a.

$$\begin{array}{lrcrcrcrcrcl}
\text{Min} & 6A & + & 4B & + & 0S_1 & + & 0S_2 & + & 0S_3 & & \\
\text{s.t.} & & & & & & & & & & & \\
& 2A & + & 1B & - & S_1 & & & & & = & 12 \\
& 1A & + & 1B & & & - & S_2 & & & = & 10 \\
& & & 1B & & & & & + & S_3 & = & 4
\end{array}$$

$$A, B, S_1, S_2, S_3 \geq 0$$

b. The optimal solution is $A = 6$, $B = 4$.

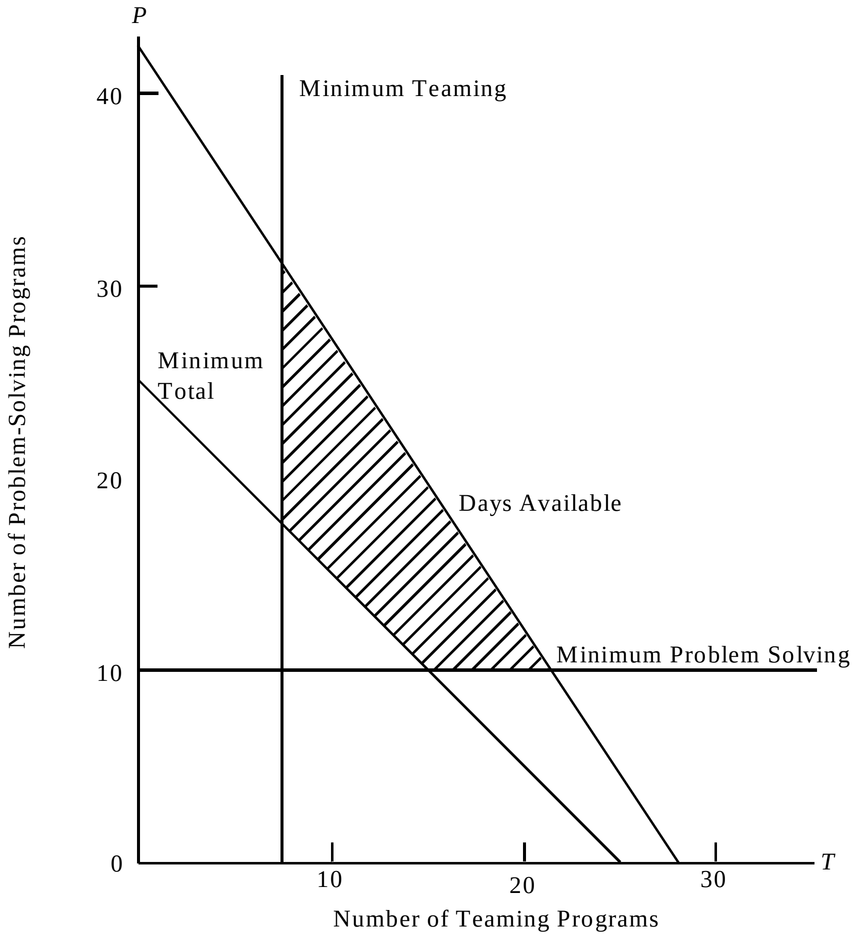
c. $S_1 = 4$, $S_2 = 0$, $S_3 = 0$.

36. a. Let T = number of training programs on teaming
P = number of training programs on problem solving

$$\begin{array}{llcrcrl}
\text{Max} & 10{,}000T & + & 8{,}000P & & & \\
\text{s.t.} & & & & & & \\
& T & & & \geq & 8 & \text{Minimum Teaming} \\
& & & P & \geq & 10 & \text{Minimum Problem Solving} \\
& T & + & P & \geq & 25 & \text{Minimum Total} \\
& 3T & + & 2P & \leq & 84 & \text{Days Available}
\end{array}$$

$$T, P \geq 0$$

b.



c. There are four extreme points: (15,10); (21.33,10); (8,30); (8,17)

d. The minimum cost solution is $T = 8$, $P = 17$
Total cost = $216,000

37.

	Regular	Zesty	
Mild	80%	60%	8100
Extra Sharp	20%	40%	3000

Let R = number of containers of Regular
Z = number of containers of Zesty

Each container holds 12/16 or 0.75 pounds of cheese

Pounds of mild cheese used $= 0.80\ (0.75)\ R + 0.60\ (0.75)\ Z$
$= 0.60\ R + 0.45\ Z$

Pounds of extra sharp cheese used $= 0.20\ (0.75)\ R + 0.40\ (0.75)\ Z$
$= 0.15\ R + 0.30\ Z$

Cost of Cheese	=	Cost of mild + Cost of extra sharp
	=	$1.20\ (0.60\ R + 0.45\ Z) + 1.40\ (0.15\ R + 0.30\ Z)$
	=	$0.72\ R + 0.54\ Z + 0.21\ R + 0.42\ Z$
	=	$0.93\ R + 0.96\ Z$

Packaging Cost = $0.20\ R + 0.20\ Z$

Total Cost	=	$(0.93\ R + 0.96\ Z) + (0.20\ R + 0.20\ Z)$
	=	$1.13\ R + 1.16\ Z$

Revenue = $1.95\ R + 2.20\ Z$

Profit Contribution	=	Revenue - Total Cost
	=	$(1.95\ R + 2.20\ Z) - (1.13\ R + 1.16\ Z)$
	=	$0.82\ R + 1.04\ Z$

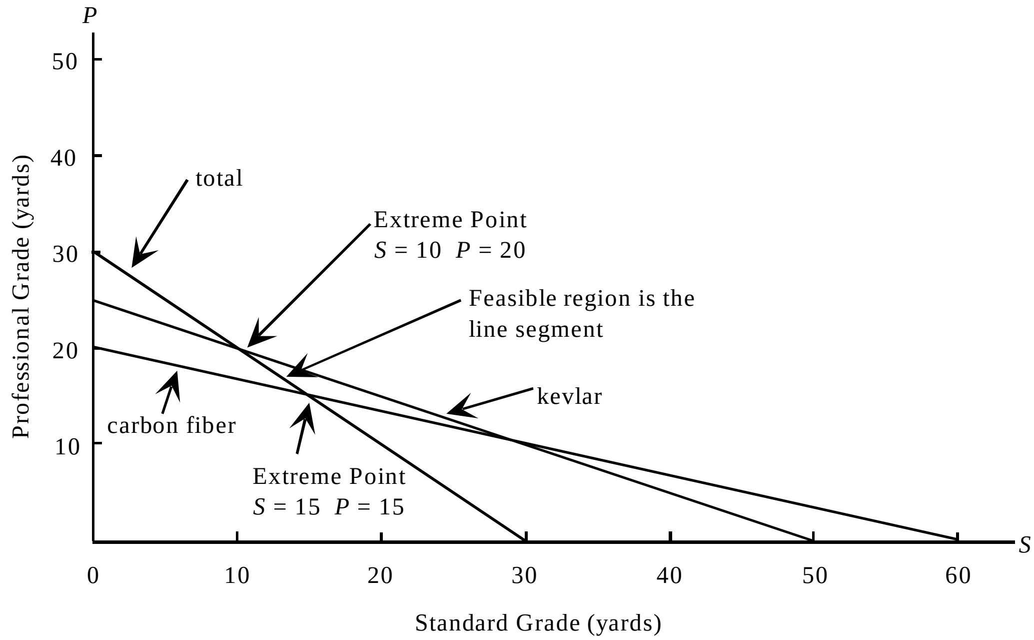
$$
\begin{array}{lrcrcrl}
\text{Max} & 0.82\ R & + & 1.04\ Z & & & \\
\text{s.t.} & & & & & & \\
 & 0.60\ R & + & 0.45\ Z & \leq & 8100 & \text{Mild} \\
 & 0.15\ R & + & 0.30\ Z & \leq & 3000 & \text{Extra Sharp} \\
 & & & R, Z \geq 0 & & &
\end{array}
$$

Optimal Solution: $R = 9600$, $Z = 5200$, profit = $0.82(9600) + 1.04(5200) = \$13,280$

38. a. Let S = yards of the standard grade material per frame
P = yards of the professional grade material per frame

$$
\begin{array}{lrcrcrl}
\text{Min} & 7.50S & + & 9.00P & & & \\
\text{s.t.} & & & & & & \\
 & 0.10S & + & 0.30P & \geq & 6 & \text{carbon fiber (at least 20\% of 30 yards)} \\
 & 0.06S & + & 0.12P & \leq & 3 & \text{kevlar (no more than 10\% of 30 yards)} \\
 & S & + & P & = & 30 & \text{total (30 yards)} \\
 & S, P \geq 0 & & & & &
\end{array}
$$

b.



c.

Extreme Point	Cost
(15, 15)	7.50(15) + 9.00(15) = 247.50
(10, 20)	7.50(10) + 9.00(20) = 255.00

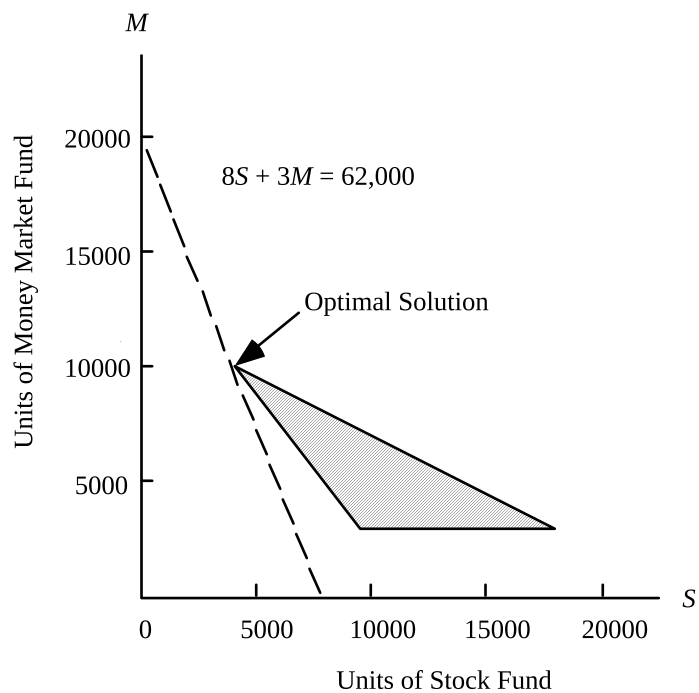
The optimal solution is $S = 15$, $P = 15$

d. Optimal solution does not change: $S = 15$ and $P = 15$. However, the value of the optimal solution is reduced to 7.50(15) + 8(15) = $232.50.

e. At $7.40 per yard, the optimal solution is $S = 10$, $P = 20$. The value of the optimal solution is reduced to 7.50(10) + 7.40(20) = $223.00. A lower price for the professional grade will not change the $S = 10$, $P = 20$ solution because of the requirement for the maximum percentage of kevlar (10%).

39. a. Let S = number of units purchased in the stock fund
M = number of units purchased in the money market fund

$$\begin{array}{llcrcrl} \text{Min} & 8S & + & 3M & & & \\ \text{s.t.} & & & & & & \\ & 50S & + & 100M & \leq & 1{,}200{,}000 & \text{Funds available} \\ & 5S & + & 4M & \geq & 60{,}000 & \text{Annual income} \\ & & & M & \geq & 3{,}000 & \text{Minimum units in money market} \\ & & & S, M, & \geq 0 & & \end{array}$$



Optimal Solution: $S = 4000$, $M = 10000$, value = 62000

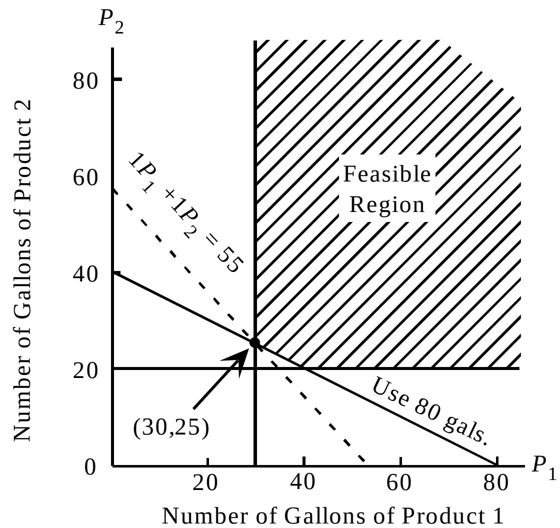
b. Annual income = 5(4000) + 4(10000) = 60,000

c. Invest everything in the stock fund.

40. Let P_1 = gallons of product 1
P_2 = gallons of product 2

Min	$1P_1$	+	$1P_2$			
s.t.						
	$1P_1$	+		$\geq$	30	Product 1 minimum
			$1P_2$	$\geq$	20	Product 2 minimum
	$1P_1$	+	$2P_2$	$\geq$	80	Raw material

$P_1, P_2 \geq 0$



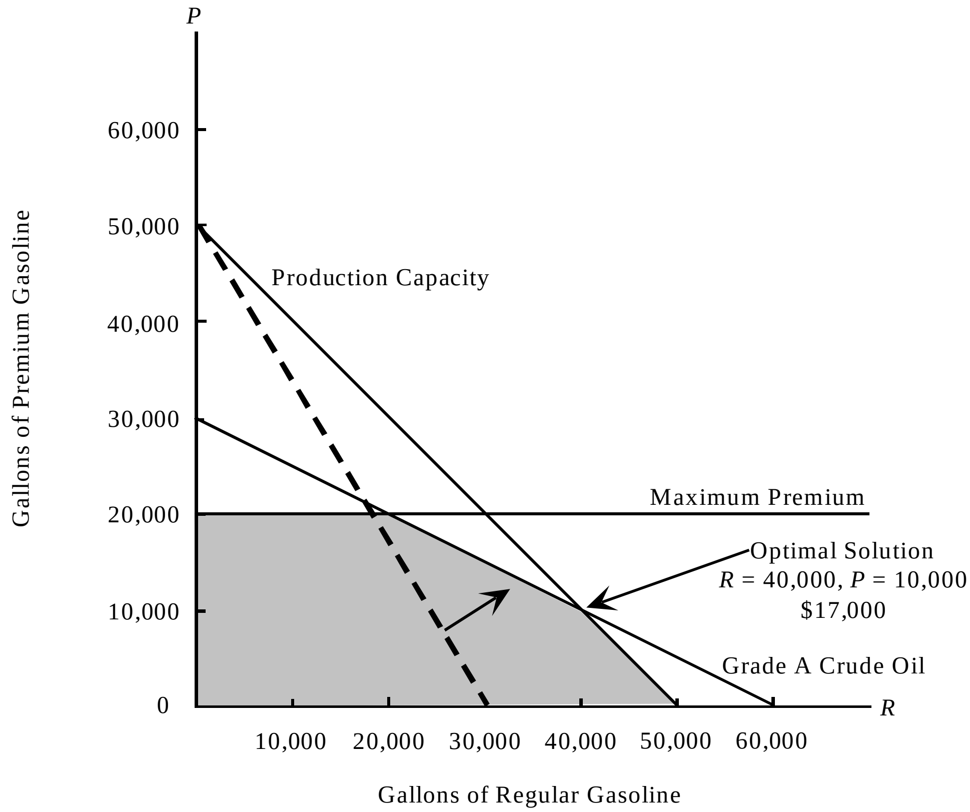
Optimal Solution: $P_1 = 30$, $P_2 = 25$ Cost = $55

41. a. Let R = number of gallons of regular gasoline produced
P = number of gallons of premium gasoline produced

Max	$0.30R$	+	$0.50P$			
s.t.						
	$0.30R$	+	$0.60P$	$\leq$	18,000	Grade A crude oil available
	$1R$	+	$1P$	$\leq$	50,000	Production capacity
			$1P$	$\leq$	20,000	Demand for premium

$R, P \geq 0$

b.



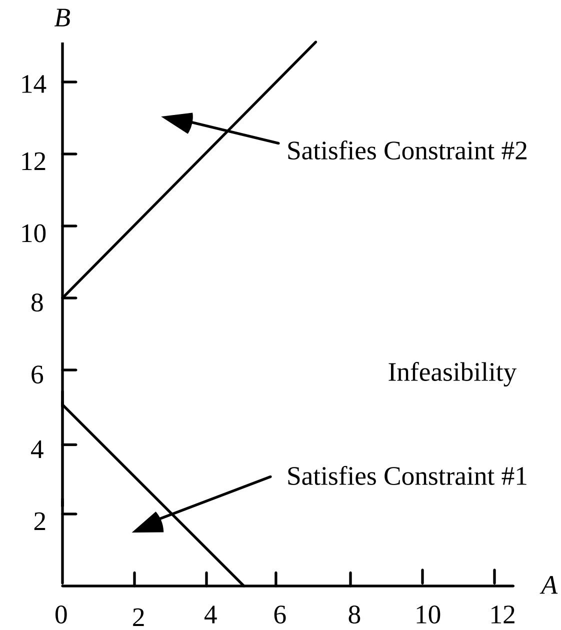
Optimal Solution:
40,000 gallons of regular gasoline
10,000 gallons of premium gasoline
Total profit contribution = $17,000

c.

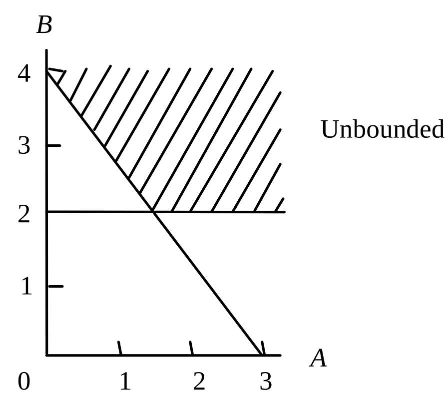
Constraint	Value of Slack Variable	Interpretation
1	0	All available grade A crude oil is used
2	0	Total production capacity is used
3	10,000	Premium gasoline production is 10,000 gallons less than the maximum demand

d. Grade A crude oil and production capacity are the binding constraints.

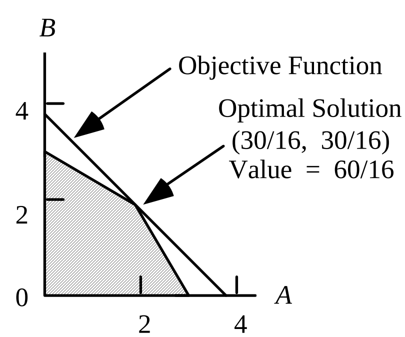
42.



43.

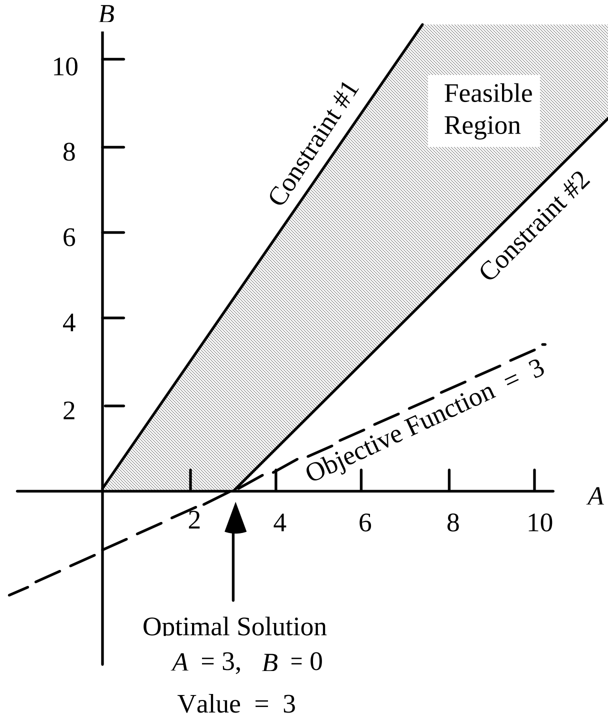


44. a.



b. New optimal solution is $A = 0$, $B = 3$, value = 6.

45. a.



b. Feasible region is unbounded.

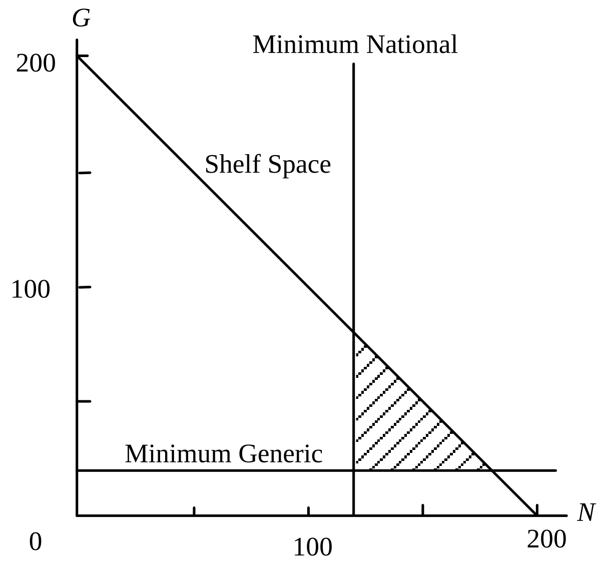
c. Optimal Solution: $A = 3$, $B = 0$, $z = 3$.

d. An unbounded feasible region does not imply the problem is unbounded. This will only be the case when it is unbounded in the direction of improvement for the objective function.

46. Let N = number of sq. ft. for national brands
G = number of sq. ft. for generic brands

Problem Constraints:

N	+	G	$\leq$	200	Space available
N			$\geq$	120	National brands
		G	$\geq$	20	Generic



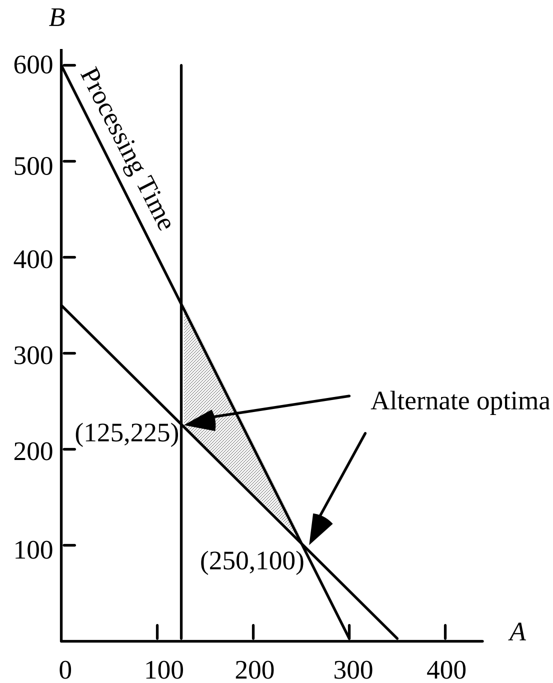
Extreme Point	N	G
1	120	20
2	180	20
3	120	80

a. Optimal solution is extreme point 2; 180 sq. ft. for the national brand and 20 sq. ft. for the generic brand.

b. Alternative optimal solutions. Any point on the line segment joining extreme point 2 and extreme point 3 is optimal.

c. Optimal solution is extreme point 3; 120 sq. ft. for the national brand and 80 sq. ft. for the generic brand.

47.



Alternative optimal solutions exist at extreme points (*A* = 125, *B* = 225) and (*A* = 250, *B* = 100).

Cost = 3(125) + 3(225) = 1050

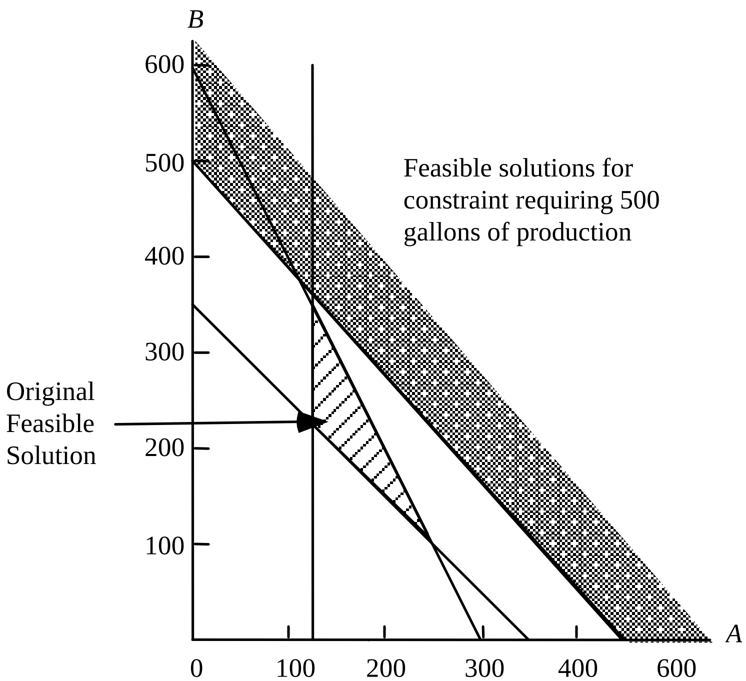
or

Cost = 3(250) + 3(100) = 1050

The solution (*A* = 250, *B* = 100) uses all available processing time. However, the solution (*A* = 125, *B* = 225) uses only 2(125) + 1(225) = 475 hours.

Thus, (*A* = 125, *B* = 225) provides 600 - 475 = 125 hours of slack processing time which may be used for other products.

48.



Possible Actions:

i. Reduce total production to *A* = 125, *B* = 350 on 475 gallons.

ii. Make solution *A* = 125, *B* = 375 which would require 2(125) + 1(375) = 625 hours of processing time. This would involve 25 hours of overtime or extra processing time.

iii. Reduce minimum *A* production to 100, making *A* = 100, *B* = 400 the desired solution.

49. a. Let P = number of full-time equivalent pharmacists
T = number of full-time equivalent physicians

The model and the optimal solution obtained using The Management Scientist is shown below:

```
MIN 40P+10T

     S.T.
       1)  1P+1T>250
       2)  2P-1T>0
       3)  1P>90

OPTIMAL SOLUTION

Objective Function Value =        5200.000

   Variable              Value            Reduced Costs
--------------     ---------------     ------------------
      P                     90.000                 0.000
      T                    160.000                 0.000
```

```
  Constraint       Slack/Surplus         Dual Prices
--------------    ---------------     ------------------
      1                  0.000               -10.000
      2                 20.000                 0.000
      3                  0.000               -30.000
```

The optimal solution requires 90 full-time equivalent pharmacists and 160 full-time equivalent technicians. The total cost is $5200 per hour.

b.

	Current Levels	**Attrition**	**Optimal Values**	**New Hires Required**
Pharmacists	85	10	90	15
Technicians	175	30	160	15

The payroll cost using the current levels of 85 pharmacists and 175 technicians is 40(85) + 10(175) = $5150 per hour.

The payroll cost using the optimal solution in part (a) is $5200 per hour.

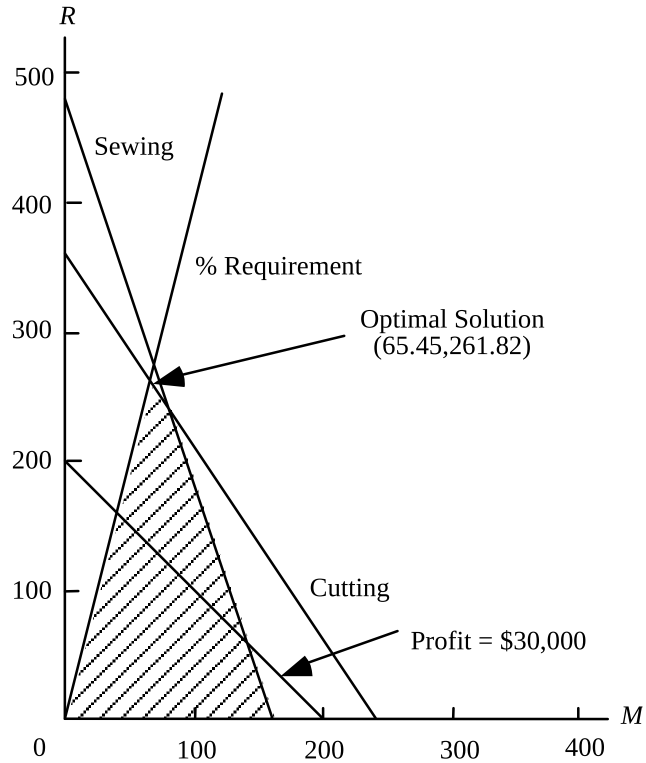
Thus, the payroll cost will go up by $50

50. Let M = number of Mount Everest Parkas
R = number of Rocky Mountain Parkas

Max	$100M$	+	$150R$			
s.t.						
	$30M$	+	$20R$	$\leq$	7200	Cutting time
	$45M$	+	$15R$	$\leq$	7200	Sewing time
	$0.8M$	-	$0.2R$	$\geq$	0	% requirement

Note: Students often have difficulty formulating constraints such as the % requirement constraint. We encourage our students to proceed in a systematic step-by-step fashion when formulating these types of constraints. For example:

M must be at least 20% of total production
$M \geq 0.2$ (total production)
$M \geq 0.2\,(M + R)$
$M \geq 0.2M + 0.2R$
$0.8M - 0.2R \geq 0$



The optimal solution is M = 65.45 and R = 261.82; the value of this solution is z = 100(65.45) + 150(261.82) = $45,818. If we think of this situation as an on-going continuous production process, the fractional values simply represent partially completed products. If this is not the case, we can approximate the optimal solution by rounding down; this yields the solution M = 65 and R = 261 with a corresponding profit of $45,650.

51. Let C = number sent to current customers
N = number sent to new customers

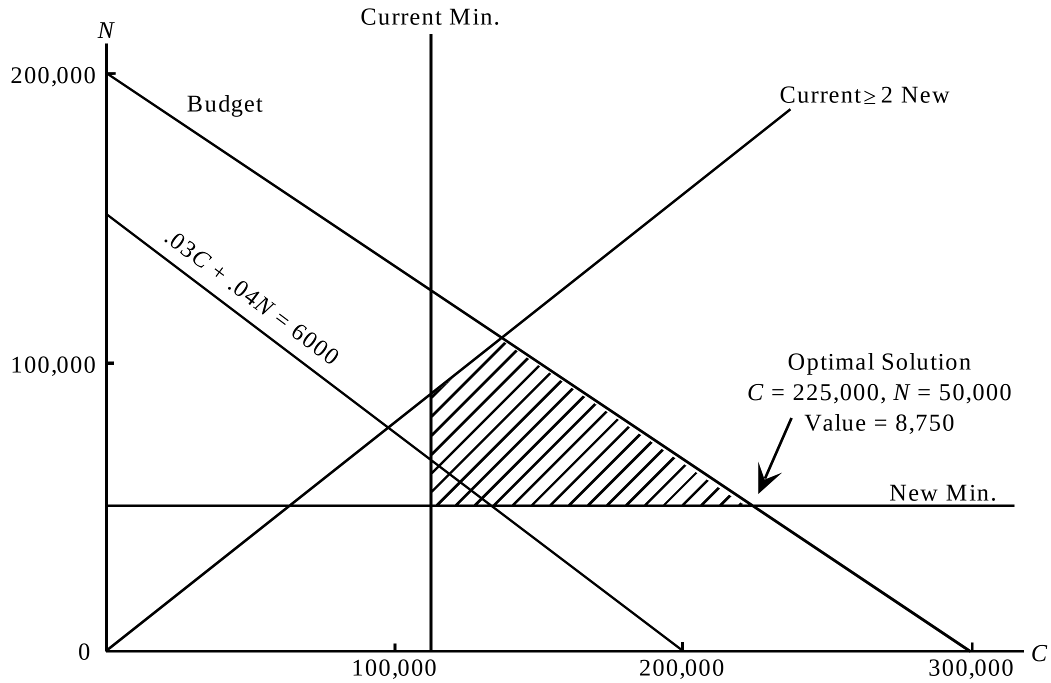
Note:

Number of current customers that test drive = .25 C

Number of new customers that test drive = .20 N

Number sold = .12 (.25 C) + .20 (.20 N)
= .03 C + .04 N

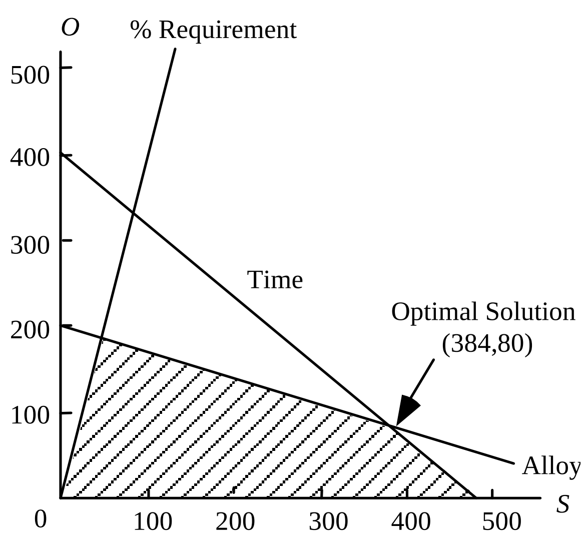
Max	.03C	+	.04N			
s.t.						
	.25 C			≥	30,000	Current Min
			.20 N	≥	10,000	New Min
	.25 C	-	.40 N	≥	0	Current vs. New
	4 C	+	6 N	≤	1,200,000	Budget
	C, N, ≥ 0					



52. Let S = number of standard size rackets
O = number of oversize size rackets

Max	$10S$	+	$15O$			
s.t.						
	$0.8S$	-	$0.2O$	$\geq$	0	% standard
	$10S$	+	$12O$	$\leq$	4800	Time
	$0.125S$	+	$0.4O$	$\leq$	80	Alloy

$S, O, \geq 0$



53. a. Let R = time allocated to regular customer service
N = time allocated to new customer service

$$\begin{array}{llllll} \text{Max} & 1.2R & + & N & & \\ \text{s.t.} & & & & & \\ & R & + & N & \leq & 80 \\ & 25R & + & 8N & \geq & 800 \\ & -0.6R & + & N & \geq & 0 \end{array}$$

$$R,\ N,\ \geq 0$$

b.

```
OPTIMAL SOLUTION

Objective Function Value =          90.000

     Variable           Value           Reduced Costs
  --------------   ---------------   ------------------
        R                 50.000                0.000
        N                 30.000                0.000


    Constraint        Slack/Surplus         Dual Prices
  --------------   ---------------   ------------------
        1                  0.000                1.125
        2                690.000                0.000
        3                  0.000               -0.125
```

Optimal solution: R = 50, N = 30, value = 90

HTS should allocate 50 hours to service for regular customers and 30 hours to calling on new customers.

54. a. Let M_1 = number of hours spent on the M-100 machine
M_2 = number of hours spent on the M-200 machine

Total Cost
$6(40)M_1 + 6(50)M_2 + 50M_1 + 75M_2 = 290M_1 + 375M_2$

Total Revenue
$25(18)M_1 + 40(18)M_2 = 450M_1 + 720M_2$

Profit Contribution
$(450 - 290)M_1 + (720 - 375)M_2 = 160M_1 + 345M_2$

$$
\begin{array}{llcllll}
\text{Max} & 160\,M_1 & + & 345M_2 & & & \\
\text{s.t.} & & & & & & \\
 & M_1 & & & \leq & 15 & \text{M-100 maximum} \\
 & & & M_2 & \leq & 10 & \text{M-200 maximum} \\
 & M_1 & & & \geq & 5 & \text{M-100 minimum} \\
 & & & M_2 & \geq & 5 & \text{M-200 minimum} \\
 & 40\,M_1 & + & 50\,M_2 & \leq & 1000 & \text{Raw material available}
\end{array}
$$

$$M_1,\ M_2 \geq 0$$

b.

```
OPTIMAL SOLUTION

Objective Function Value =        5450.000

      Variable              Value             Reduced Costs
   --------------    ---------------      ------------------
         M1                 12.500                  0.000
         M2                 10.000                  0.000

     Constraint         Slack/Surplus          Dual Prices
   --------------    ---------------      ------------------
         1                   2.500                  0.000
         2                   0.000                145.000
         3                   7.500                  0.000
         4                   5.000                  0.000
         5                   0.000                  4.000
```

The optimal decision is to schedule 12.5 hours on the M-100 and 10 hours on the M-200.

Chapter 1
Introduction

Case Problem: Scheduling a Golf League

Note to Instructor: This case problem illustrates the value of the rational management science approach. The problem is easy to understand and, at first glance, appears simple. But, most students will have trouble finding a solution. The solution procedure suggested involves decomposing a larger problem into a series of smaller problems that are easier to solve. The case provides students with a good first look at the kinds of problems where management science is applied in practice. The problem is a real one that one of the authors was asked by the Head Professional at Royal Oak Country Club for help with.

Solution: Scheduling problems such as this occur frequently, and are often difficult to solve. The typical approach is to use trial and error. An alternative approach involves breaking the larger problem into a series of smaller problems. We show how this can be done here using what we call the Red, White, and Blue algorithm.

Suppose we break the 18 couples up into 3 divisions, referred to as the Red, White, and Blue divisions. The six couples in the Red division can then be identified as R1, R2, R3, R4, R5, R6; the six couples in the White division can be identified as W1, W2,…, W6; and the six couples in the Blue division can be identified as B1, B2,…, B6. We begin by developing a schedule for the first 5 weeks of the season so that each couple plays every other couple in its own division. This can be done fairly easily by trial and error. Shown below is the first 5-week schedule for the Red division.

Week 1	**Week 2**	**Week 3**	**Week 4**	**Week 5**
R1 vs. R2	R1 vs. R3	R1 vs. R4	R1 vs. R5	R1 vs. R6
R3 vs. R4	R2 vs. R5	R2 vs. R6	R2 vs. R4	R2 vs. R3
R5 vs. R6	R4 vs. R6	R3 vs. R5	R3 vs. R6	R4 vs. R5

Similar 5-week schedules can be developed for the White and Blue divisions by replacing the R in the above table with a W or a B.

To develop the schedule for the next 3 weeks, we create 3 new six-couple divisions by pairing 3 of the teams in each division with 3 of the teams in another division; for example, (R1, R2, R3, W1, W2, W3), (B1, B2, B3, R4, R5, R6), and (W4, W5, W6, B4, B5, B6). Within each of these new divisions, matches can be scheduled for 3 weeks without any couples playing a couple they have played before. For instance, a 3-week schedule for the first of these divisions is shown below:

Week 6	**Week 7**	**Week 8**
R1 vs. W1	R1 vs. W2	R1 vs. W3
R2 vs. W2	R2 vs. W3	R2 vs. W1
R3 vs. W3	R3 vs. W1	R3 vs. W2

A similar 3-week schedule can be easily set up for the other two new divisions. This will provide us with a schedule for the first 8 weeks of the season.

For the final 9 weeks, we continue to create new divisions by pairing 3 teams from the original Red, White and Blue divisions with 3 teams from the other divisions that they have not yet been paired with. Then a 3-week schedule is developed as above. Shown below is a set of divisions for the next 9 weeks.

Weeks 9-11

(R1, R2, R3, W4, W5, W6)	(W1, W2, W3, B1, B2, B3)	(R4, R5, R6, B4, B5, B6)

Weeks 12-14

(R1, R2, R3, B1, B2, B3)	(W1, W2, W3, B4, B5, B6)	(W4, W5, W6, R4, R5, R6)

Weeks 15-17

(R1, R2, R3, B4, B5, B6)	(W1, W2, W3, R4, R5, R6)	(W4, W5, W6, B1, B2, B3)

This Red, White and Blue scheduling procedure provides a schedule with every couple playing every other couple over the 17-week season. If one of the couples should cancel, the schedule can be modified easily. Designate the couple that cancels, say R4, as the Bye couple. Then whichever couple is scheduled to play couple R4 will receive a Bye in that week. With only 17 couples a Bye must be scheduled for one team each week.

This same scheduling procedure can obviously be used for scheduling sports teams and or any other kinds of matches involving 17 or 18 teams. Modifications of the Red, White and Blue algorithm can be employed for 15 or 16 team leagues and other numbers of teams.

Chapter 2
An Introduction to Linear Programming

Case Problem 1: Workload Balancing

1.

	Production Rate (minutes per printer)		
Model	Line 1	Line 2	Profit Contribution ($)
DI-910	3	4	42
DI-950	6	2	87

Capacity: 8 hours × 60 minutes/hour = 480 minutes per day

Let D_1 = number of units of the DI-910 produced
D_2 = number of units of the DI-950 produced

$$\begin{aligned} \text{Max} \quad & 42D_1 + 87D_2 \\ \text{s.t.} \quad & \\ & 3D_1 + 6D_2 \leq 480 \quad \text{Line 1 Capacity} \\ & 4D_1 + 2D_2 \leq 480 \quad \text{Line 2 Capacity} \\ & D_1, D_2 \geq 0 \end{aligned}$$

Using *The Management Scientist*, the optimal solution is $D_1 = 0$, $D_2 = 80$. The value of the optimal solution is $6960.

Management would not implement this solution because no units of the DI-910 would be produced.

2. Adding the constraint $D_1 \geq D_2$ and resolving the linear program results in the optimal solution D_1 = 53.333, D_2 = 53.333. The value of the optimal solution is $6880.

3. Time spent on Line 1: 3(53.333) + 6(53.333) = 480 minutes

Time spent on Line 2: 4(53.333) + 2(53.333) = 320 minutes

Thus, the solution does not balance the total time spent on Line 1 and the total time spent on Line 2. This might be a concern to management if no other work assignments were available for the employees on Line 2.

4. Let T_1 = total time spent on Line 1
T_2 = total time spent on Line 2

Whatever the value of T_2 is,

$$T_1 \leq T_2 + 30$$
$$T_1 \geq T_2 - 30$$

Thus, with $T_1 = 3D_1 + 6D_2$ and $T_2 = 4D_1 + 2D_2$

$$3D_1 + 6D_2 \leq 4D_1 + 2D_2 + 30$$
$$3D_1 + 6D_2 \geq 4D_1 + 2D_2 - 30$$

Hence,

$$-1D_1 + 4D_2 \leq 30$$
$$-1D_1 + 4D_2 \geq -30$$

Rewriting the second constraint by multiplying both sides by -1, we obtain

$$-1D_1 + 4D_2 \leq 30$$
$$1D_1 - 4D_2 \leq 30$$

Adding these two constraints to the linear program formulated in part (2) and resolving using *The Management Scientist*, we obtain the optimal solution D_1 = 96.667, D_2 = 31.667. The value of the optimal solution is $6815. Line 1 is scheduled for 480 minutes and Line 2 for 450 minutes. The effect of workload balancing is to reduce the total contribution to profit by $6880 - $6815 = $65 per shift.

5. The optimal solution is D_1 = 106.667, D_2 = 26.667. The total profit contribution is

$$42(106.667) + 87(26.667) = \$6800$$

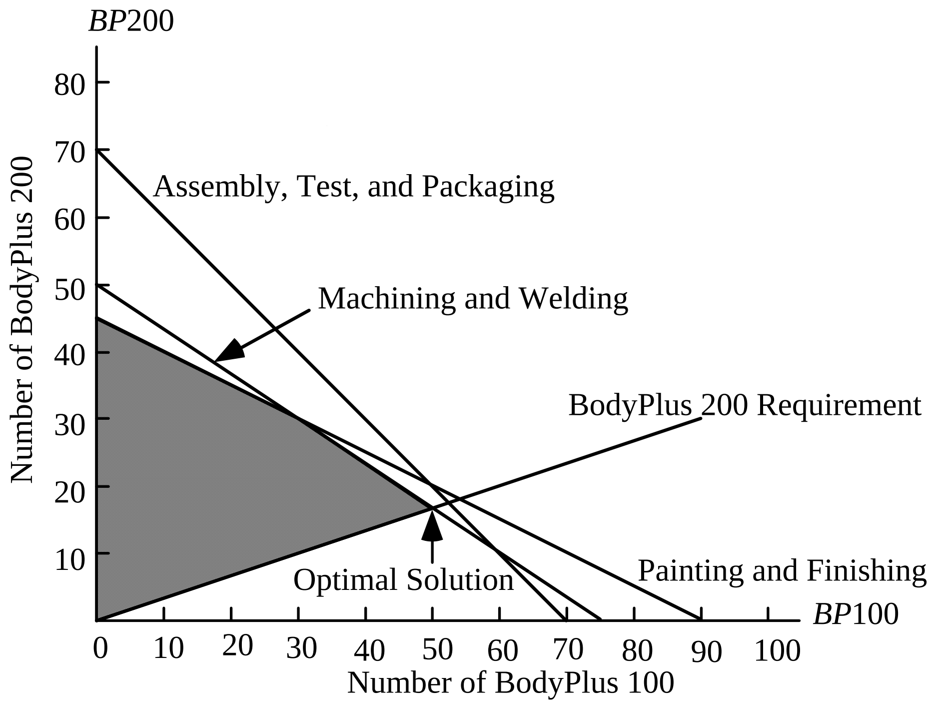
Comparing the solutions to part (4) and part (5), maximizing the number of printers produced (106.667 + 26.667 = 133.33) has increased the production by 133.33 - (96.667 + 31.667) = 5 printers but has reduced profit contribution by $6815 - $6800 = $15. But, this solution results in perfect workload balancing because the total time spent on each line is 480 minutes.

Case Problem 2: Production Strategy

1. Let $BP100$ = the number of BodyPlus 100 machines produced
$BP200$ = the number of BodyPlus 200 machines produced

Max	$371BP100$	+	$461BP200$			
s.t.						
	$8BP100$	+	$12BP200$	$\leq$	600	Machining and Welding
	$5BP100$	+	$10BP200$	$\leq$	450	Painting and Finishing
	$2BP100$	+	$2BP200$	$\leq$	140	Assembly, Test, and Packaging
	$-0.25BP100$	+	$0.75BP200$	$\geq$	0	BodyPlus 200 Requirement

$BP100, BP200 \geq 0$



Optimal solution: *BP*100 = 50, *BP*200 = 50/3, profit = $26,233.33. Note: If the optimal solution is rounded to *BP*100 = 50, *BP*200 = 16.67, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package such as The Management Scientist.

2. In the short run the requirement reduces profits. For instance, if the requirement were reduced to at least 24% of total production, the new optimal solution is *BP*100 = 1425/28, *BP*200 = 225/14, with a total profit of $26,290.18; thus, total profits would increase by $56.85. Note: If the optimal solution is rounded to *BP*100 = 50.89, *BP*200 = 16.07, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package such as The Management Scientist.

3. If management really believes that the BodyPlus 200 can help position BFI as one of the leader's in high-end exercise equipment, the constraint requiring that the number of units of the BodyPlus 200 produced be at least 25% of total production should not be changed. Since the optimal solution uses all of the available machining and welding time, management should try to obtain additional hours of this resource.

Case Problem 3: Hart Venture Capital

1. Let S = fraction of the Security Systems project funded by HVC
   M = fraction of the Market Analysis project funded by HVC

Max	$1{,}800{,}000S$	+	$1{,}600{,}000M$			
s.t.						
	$600{,}000S$	+	$500{,}000M$	$\leq$	800,000	Year 1
	$600{,}000S$	+	$350{,}000M$	$\leq$	700,000	Year 2
	$250{,}000S$	+	400,000M	$\leq$	500,000	Year 3
	S			$\leq$	1	Maximum for S
			M	$\leq$	1	Maximum for M
	S,M	$\geq$	0			

The solution obtained using The Management Scientist software package is shown below:

```
OPTIMAL SOLUTION

Objective Function Value =     2486956.522

      Variable            Value            Reduced Costs
   --------------    ---------------    ------------------
         S                   0.609                  0.000
         M                   0.870                  0.000


     Constraint         Slack/Surplus          Dual Prices
   --------------    ---------------    ------------------
         1                   0.000                  2.783
         2               30434.783                  0.000
         3                   0.000                  0.522
         4                   0.391                  0.000
         5                   0.130                  0.000


OBJECTIVE COEFFICIENT RANGES

   Variable       Lower Limit       Current Value     Upper Limit
 ------------   ---------------   ---------------   ---------------
      S         No Lower Limit       1800000.000    No Upper Limit
      M         No Lower Limit       1600000.000    No Upper Limit


RIGHT HAND SIDE RANGES

  Constraint      Lower Limit       Current Value     Upper Limit
 ------------   ---------------   ---------------   ---------------
       1        No Lower Limit        800000.000        822950.820
       2            669565.217        700000.000    No Upper Limit
       3            461111.111        500000.000    No Upper Limit
       4                 0.609             1.000    No Upper Limit
       5                 0.870             1.000    No Upper Limit
```

Thus, the optimal solution is *S* = 0.609 and *M* = 0.870. In other words, approximately 61% of the Security Systems project should be funded by HVC and 87% of the Market Analysis project should be funded by HVC.

The net present value of the investment is approximately $2,486,957.

2.

	Year 1	**Year 2**	**Year 3**
Security Systems	$365,400	$365,400	$152,250
Market Analysis	$435,000	$304,500	$348,000
Total	$800,400	$669,900	$500,250

Note: The totals for Year 1 and Year 3 are greater than the amounts available. The reason for this is that rounded values for the decision variables were used to compute the amount required in each year. To see why this situation occurs here, first note that each of the problem coefficients is an integer value. Thus, by default, when The Management Scientist prints the optimal solution, values of the decision variables are rounded and printed with three decimal places. To increase the number of decimal places shown in the output, one or more of the problem coefficients can be entered with additional digits to the right of the decimal point. For instance, if we enter the coefficient of 1 for *S* in constraint 4 as 1.000000 and resolve the problem, the new optimal values for *S* and *D* will be rounded and printed with six decimal places. If we use the new values in the computation of the amount required in each year, the differences observed for year 1 and year 3 will be much smaller than we obtained using the values of *S* = 0.609 and *M* = 0.870.

3. If up to $900,000 is available in year 1 we obtain a new optimal solution with *S* = 0.689 and *M* = 0.820. In other words, approximately 69% of the Security Systems project should be funded by HVC and 82% of the Market Analysis project should be funded by HVC.

The net present value of the investment is approximately $2,550,820.
The solution obtained using The Management Scientist software package follows:

```
OPTIMAL SOLUTION

Objective Function Value =     2550819.672

      Variable             Value            Reduced Costs
   --------------     ---------------     ------------------
         S                     0.689                  0.000
         M                     0.820                  0.000
     Constraint        Slack/Surplus          Dual Prices
   --------------     ---------------     ------------------
         1                 77049.180                  0.000
         2                     0.000                  2.098
         3                     0.000                  2.164
         4                     0.311                  0.000
         5                     0.180                  0.000

OBJECTIVE COEFFICIENT RANGES

   Variable       Lower Limit       Current Value     Upper Limit
 ------------   ---------------   ---------------  ---------------
      S          No Lower Limit       1800000.000   No Upper Limit
      M          No Lower Limit       1600000.000   No Upper Limit
```

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	822950.820	900000.000	No Upper Limit
2	No Lower Limit	700000.000	802173.913
3	No Lower Limit	500000.000	630555.556
4	0.689	1.000	No Upper Limit
5	0.820	1.000	No Upper Limit

4. If an additional $100,000 is made available, the allocation plan would change as follows:

	Year 1	Year 2	Year 3
Security Systems	$413,400	$413,400	$172,250
Market Analysis	$410,000	$287,000	$328,000
Total	$823,400	$700,400	$500,250

5. Having additional funds available in year 1 will increase the total net present value. The value of the objective function increases from $2,486,957 to $2,550,820, a difference of $63,863. But, since the allocation plan shows that $823,400 is required in year 1, only $23,400 of the additional $100,00 is required. We can also determine this by looking at the slack variable for constraint 1 in the new solution. This value, 77049.180, shows that at the optimal solution approximately $77,049 of the $900,000 available is not used. Thus, the amount of funds required in year 1 is $900,000 - $77,049 = $822,951. In other words, only $22,951 of the additional $100,000 is required. The differences between the two values, $23,400 and $22,951, is simply due to the fact that the value of $23,400 was computed using rounded values for the decision variables. The value of $22,951 is computed internally in The Management Scientist output and is not subject to this rounding. Thus, the most accurate value is $22,951.