# An Introduction to Parallel Programming Solutions, Chapter 1 

Jinyoung Choi and Peter Pacheco

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```
1. quotient = n / p;
remainder = n % p;
if (my_rank < remainder) {
    my_n_count = quotient + 1;
    my_first_i = my_rank * my_n_count;
} else {
    my_n_count = quotient;
    my_first_i = my_rank * my_n_count + remainder;
}
my_last_i = my_first_i + my_n_count;
```

2. We are assigning blocks of elements to cores in order (the first $n / p$ elements to core 0 , the next $n / p$ elements to core 1 , so on). So, for example, if $n=12$ and $p=4$, core 0 spends 12 milliseconds in the call to Compute_next_value ( $i=0,1,2$ ), core 1 spends 30 milliseconds ( $i=3,4,5$ ), core 2 spends 48 milliseconds $(i=6,7,8)$, and core 3 spends 66 milliseconds $(i=9,10,11)$. So clearly this assignment will do a very poor job of load balancing.
A better approach uses a cyclic assignment of the work to the cores:
```
/* assign[c][j] is the jth value of i assigned to core c */
/* work[c] is the total amount of work assigned to core c */
c = j = 0;
for (i = 0; i < n; i++) {
    work[c] += 2*(i+1);
    assign[c][j] = i;
    c = (c + 1) % p;
    if (c == 0) j++;
}
```

Prof. Timothy Rolfe of Eastern Washington University came up with a much better approach. He uses a cyclic assignment of the work to the cores, but he starts with the largest amount of work ( $2 n$ ) and works backward through the work ( $2 n-2,2 n-$ $4, \ldots, 4,2)$. However, he alternates between going forward ( $0,1, \ldots, p-1$ ) and backward $(p-1, p-2, \ldots, 1,0)$ through the cores. For example, suppose $p=5$ and $n=23$. Then the cyclic assignment outlined above will assign work as follows:

| Core | Value of $i$ |  |  |  |  | Total Work |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 5 | 10 | 15 | 20 | 110 |
| 1 | 1 | 6 | 11 | 16 | 21 | 120 |
| 2 | 2 | 7 | 12 | 17 | 22 | 130 |
| 3 | 3 | 8 | 13 | 18 |  | 92 |
| 4 | 4 | 9 | 14 | 19 |  | 100 |

On the other hand, Prof. Rolfe's solution assigns the work as follows:

| Core | Value of $i$ |  |  |  |  | Total Work |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 22 | 13 | 12 | 3 | 2 | 114 |
| 1 | 21 | 14 | 11 | 4 | 1 | 112 |
| 2 | 20 | 15 | 10 | 5 | 0 | 110 |
| 3 | 19 | 16 | 9 | 6 |  | 108 |
| 4 | 18 | 17 | 8 | 7 |  | 108 |

His algorithm can be described as follows:

```
j = 0; i = n-1;
while (i >= 0) {
    /* Go forward through cores */
    for (c = 0; c < p && i >= 0; c++) {
        work[c] += 2*(i+1);
        assign[c][j] = i;
        i--;
    }
    j++;
    /* Go backward through cores */
    for (c = p-1; c >= 0 && i >= 0; c-- ) {
        work[c] += 2*(i+1);
        assign[c][j] = i;
```

```
            i--;
        }
        j++;
    }
```

3. divisor $=2$;
core_difference = 1;
sum = my_value;
while ( divisor <= number of cores ) \{
if ( my_rank \% divisor == 0 ) \{
partner = my_rank + core_difference;
receive value from partner core;
sum += received value;
\} else \{
partner = my_rank core_difference;
send my sum to partner core;
\}
divisor *= 2;
core_difference *=2;
\}
4. bitmask $=1$;
divisor = 2;
sum = my_value;
while ( bitmask < number of cores ) \{
partner = my_rank ^ bitmask;
if ( my_rank \% divisor == 0 ) \{
receive value from partner core;
sum += received value;
\} else \{
send my_sum to partner core;
\}
bitmask <<= 1;
divisor *= 2;
\}
5. It could happen that some cores wait for non-existent cores to send values, and this would probably cause the code to hang or crash. We can simply add a condition,
if (partner < number of cores) \{
```
    receive value
    sum += received value
}
```

when a cores tries to receive a value from its partner to make sure the program will handle the case in which the number of cores isn't a power of 2 .
6. (a) The number of receives is $p-1$, and the number of additions is $p-1$.
(b) The number of receives is $\log _{2}(p)$, and the number of additions is $\log _{2}(p)$.

| p | Original | Tree-Structured |
| ---: | ---: | ---: |
| 2 | 1 | 1 |
| 4 | 3 | 2 |
| 8 | 7 | 3 |
| 16 | 15 | 4 |
| 32 | 31 | 5 |
| 64 | 63 | 6 |
| 128 | 127 | 7 |
| 256 | 255 | 8 |
| 512 | 511 | 9 |
| 1024 | 1023 | 10 |

7. The example is a combination of task- and data- parallelism. In each phase of the tree-structured global sum, the cores are computing partial sums. This can be seen as data-parallelism. Also, in each phase, there are two types of tasks. Some cores are sending their sums and some are receiving another cores partial sum. This can be seen as task-parallelism.
8. (a) Cleaning the place for the party, bringing food, scheduling the setup, making party posters, etc.
(b) There are several locations to clean. We can partition them among the faculty.
(c) For instance, we can assign the task of preparing the food and drinks to some of the faculty. Then, this group can be partitioned according to the types of food: some individuals can be responsible for hors d'oeuvres, some for sandwiches, some for the punch, etc.
9. (ESSAY)
