## Chapter 2 <br> Organizing and Graphing Data

## Section 2.1

2.1 Data in their original form are often too large and unmanageable. It is easier to make sense of grouped data than ungrouped data and easier to make decisions and draw conclusions using grouped data.
2.2 The relative frequency for a category is obtained by dividing the frequency of that category by the sum of the frequencies of all categories. The percentage for a category is obtained by multiplying the relative frequency of that category by 100 . Example $2-2$ in the text is an example which shows how relative frequencies and percentages are calculated.
2.3 a. and b.

| Category | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| A | 8 | $8 / 30=.267$ | 26.7 |
| B | 8 | $8 / 30=.267$ | 26.7 |
| C | 14 | $14 / 30=.467$ | 46.7 |

c. 26.7 \% of the elements in this sample belong to category B.
d. $22 / 30=73.3 \%$ of the elements in this sample belong to category A or C.
e.

2.4
a. and b.

| Category | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | ---: | :---: |
| Y | 23 | $23 / 40=.575$ | 57.5 |
| N | 13 | $13 / 40=.325$ | 32.5 |
| D | 4 | $4 / 40=.100$ | 10.0 |

c. $57.5 \%$ of the elements belong to category Y.
d. $17 / 40=42.5 \%$ of the elements belong to categories N or D .
e.

2.6
a. and b.

| Category | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| B | 14 | $14 / 45=.311$ | 31.1 |
| H | 19 | $19 / 45=.422$ | 42.2 |
| W | 5 | $5 / 45=.111$ | 11.1 |
| N | 7 | $7 / 45=.156$ | 15.6 |

c. $\mathrm{B}+\mathrm{W}=14+5=19 ; 19 / 45 \approx .422=42.2 \%$

About $42.2 \%$ of the respondents mentioned Major League Baseball or Breakfast at Wimbledon.
d.


Best Fourth of July Weekend Sports
Tradition
a. and b.

| Category | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | ---: | :---: |
| T | 4 | $4 / 30=.133$ | 13.3 |
| R | 10 | $10 / 30=.333$ | 33.3 |
| A | 7 | $7 / 30=.233$ | 23.3 |
| P | 8 | $8 / 30=.267$ | 26.7 |
| M | 1 | $1 / 30=.033$ | 3.3 |

c. $(10+7) / 30=56.7 \%$ of the adults ranked refrigerators or air conditioning as the convenience that they would find most difficult to do without.
d.


Convenience
a. and b.

| Category | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| PI | 9 | $9 / 36=.25$ | 25 |
| S | 8 | $8 / 36=.222$ | 22.2 |
| V | 13 | $13 / 36=.361$ | 36.1 |
| PO | 3 | $3 / 36=.083$ | 8.3 |
| B | 1 | $1 / 36=.028$ | 2.8 |
| C | 2 | $2 / 36=.056$ | 5.6 |

c. $\mathrm{V}+\mathrm{PO}+\mathrm{C}=13+3+2=18 ; 18 / 36=.5=50 \%$
$50 \%$ of the respondents mentioned vegetables and fruits, poultry, or cheese.
d.

2.8
a. and b.

| Category | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| C | 4 | $4 / 16=.250$ | 25.0 |
| CK | 5 | $5 / 16=.313$ | 31.3 |
| CC | 4 | $4 / 16=.250$ | 25.0 |
| D | 2 | $2 / 16=.125$ | 12.5 |
| O | 1 | $1 / 16=.063$ | 6.3 |

c.

2.9 Let the four categories listed in the table be denoted by V, S, NTS, and NAS respectively, and let DK/NA represent "did not know or had no opinion."

2.10 Let the seven categories listed in the table be denoted by CA, EC, D, SS, EL, DSK and O respectively.


## Section 2.2

2.11 The three decisions that have to be made to group a data set in the form of a frequency distribution table are 1. The number of classes to be used to group the given data.
2. The width of each class.
3. The lower limit of the first class.
2.12 The relative frequency for a class is obtained by dividing the frequency of that class by the sum of frequencies of all classes. The percentage for a class is obtained by multiplying the relative frequency of that class by 100. Example 2-4 is an example that illustrates the calculation of relative frequencies and percentages.
2.13 A data set that does not contain fractional values is usually grouped by using classes with limits. Example $2-4$ is an example of the writing classes using limits method. A data set that contains fractional values is grouped by using the less than method. Example 2-5 is an example of the less than method. Single-valued classes are used to group a data set that contains only a few distinct (integer) values. Example 2-6 is an example of the single-valued classes method.
2.14 a. $31+78+49+81+117+13=369$ customers were served.
b. Each class has a width of 4 .

| Gallons of Gas <br> (Class Limits) | Class Width | Class Midpoint |
| :---: | :---: | :---: |
| 0 to less than 4 | 4 | $\frac{0+4}{2}=2$ |
| 4 to less than 8 | 4 | $\frac{4+8}{2}=6$ |
| 8 to less than 12 | 4 | $\frac{8+12}{2}=10$ |
| 12 to less than 16 | 4 | $\frac{12+16}{2}=14$ |
| 16 to less than 20 | 4 | $\frac{20+24}{2}=22$ |
| 20 to less than 24 | 4 | 48 |

c.

| Gallons of Gas | Number of Customers | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 0 to less than 4 | 31 | $31 / 369 \approx .084$ | 8.4 |
| 4 to less than 8 | 78 | $78 / 369 \approx .211$ | 21.1 |
| 8 to less than 12 | 49 | $49 / 369 \approx .133$ | 13.3 |
| 12 to less than 16 | 81 | $81 / 369 \approx .220$ | 22.0 |
| 16 to less than 20 | 117 | $117 / 369 \approx .317$ | 31.7 |
| 20 to less than 24 | 13 | $13 / 369 \approx .035$ | 3.5 |

d. $22.0+31.7+3.5=57.2 \%$ of the customers purchased 12 gallons or more .
e. The number of customers who purchased 10 gallons or less cannot be determined exactly because 10 is not a boundary value.
2.15
a. $32+67+44+20+11=174$ containers of yogurt were inspected.
b. Each class has a width of 6 .

| Number of Days <br> (Class Limits) | Class Boundary | Class Width | Class Midpoint |
| :---: | :---: | :---: | :---: |
| 0 to 5 | -0.5 to less than 5.5 | $5.5-(-0.5)=6$ | $\frac{0+5}{2}=2.5$ |
| 6 to 11 | 5.5 to less than 11.5 | $11.5-5.5=6$ | $\frac{6+11}{2}=8.5$ |
| 12 to 17 | 11.5 to less than 17.5 | $17.5-11.5=6$ | $\frac{12+17}{2}=14.5$ |
| 18 to 23 | 17.5 to less than 23.5 | $23.5-17.5=6$ | $\frac{18+23}{2}=20.5$ |
| 24 to 29 | 23.5 to less than 29.5 | $29.5-23.5=6$ | $\frac{24+29}{2}=26.5$ |

c.

| Number of Days <br> (Class Limits) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 0 to 5 | 32 | $32 / 174 \approx .184$ | 18.4 |
| 6 to 11 | 67 | $67 / 174 \approx .385$ | 38.5 |
| 12 to 17 | 44 | $44 / 174 \approx .253$ | 25.3 |
| 18 to 23 | 20 | $20 / 174 \approx .115$ | 11.5 |
| 24 to 29 | 11 | $11 / 174 \approx .063$ | 6.3 |

d. $25.3+38.5+18.4=82.2 \%$ of the containers would expire in less than 18 days.
e. The exact number of containers that have already expired cannot be determined because 0 is included in the class 0 to 5 .
f. The largest number of containers that could already have expired is 32 .
a. and b.

| Median Household <br> Income | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 37,000 to 41,999 | 8 | .157 | 15.7 |
| 42,000 to 46,999 | 12 | .235 | 23.5 |
| 47,000 to 51,999 | 13 | .255 | 25.5 |
| 52,000 to 56,999 | 9 | .176 | 17.6 |
| 57,000 to 61,999 | 5 | .098 | 9.8 |
| 62,000 to 66,999 | 4 | .078 | 7.8 |

c. The data are skewed slightly to the right.
d. $(9+5+4) / 51=35.3 \%$ of these states had a median household income of $\$ 52,000$ or more.
2.19
a. and b.

| Class Limits |  | Class Boundaries |
| ---: | ---: | ---: |
| 1 to 200 | .5 to less than 200.5 | 100.5 |
| 201 to 400 | 200.5 to less than 400.5 | 300.5 |
| 401 to 600 | 400.5 to less than 600.5 | 500.5 |
| 601 to 800 | 600.5 to less than 800.5 | 700.5 |
| 801 to 1000 | 800.5 to less than 1000.5 | 900.5 |
| 1001 to 1200 | 1000.5 to less than 1200.5 | 1100.5 |

2.17 a., b., and c.

| Class Limits |  | Class Boundaries | Class Width |
| ---: | ---: | :---: | :---: |
| 1 to 25 | .5 to less than 25.5 | 25 | Class Midpoint |
| 26 to 50 | 25.5 to less than 50.5 | 25 | 13 |
| 51 to 75 | 50.5 to less than 75.5 | 25 | 38 |
| 76 to 100 | 75.5 to less than 100.5 | 25 | 63 |
| 101 to 125 | 100.5 to less than 125.5 | 25 | 88 |
| 126 to 150 | 125.5 to less than 150.5 | 25 | 113 |

a. and b.

| Number of Births per <br> 1000 People | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 2 to less than 5 | 3 | .054 | 5.4 |
| 5 to less than 8 | 8 | .143 | 14.3 |
| 8 to less than 11 | 23 | .411 | 41.1 |
| 11 to less than 14 | 7 | .125 | 12.5 |
| 14 to less than 17 | 9 | .161 | 16.1 |
| 17 to less than 20 | 3 | .054 | 5.4 |
| 20 to less than 23 | 3 | .054 | 5.4 |

c.

d. $(8+3+23) / 56=60.7 \%$ of the counties had a birth rate of less than 11 births per 1000 people.

| and b.Fatal Motorcycle <br> Accidents | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | :---: | :---: | :---: |
| to 10 | 15 | .326 | 32.6 |
| 11 to 20 | 17 | .370 | 37.0 |
| 21 to 30 | 7 | .152 | 15.2 |
| 31 to 40 | 4 | .087 | 8.7 |
| 41 to 50 | 1 | .022 | 2.2 |
| 51 to 60 | 2 | .043 | 4.3 |

c.


Fatal Motorcycle Accidents
d. $(7+4) / 46=23.9 \%$ of the counties had between 21 and 40 fatal motorcycle accidents during 2009 .

| Charitable Contributions <br> (millions of dollars) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 25 to less than 65 | 25 | .625 | 62.5 |
| 65 to less than 105 | 8 | .200 | 20.0 |
| 105 to less than 145 | 4 | .025 | 10.0 |
| 145 to less than 185 | 1 | .000 | 2.5 |
| 185 to less than 225 | 0 | .000 | 0.0 |
| 225 to less than 265 | 0 | .025 | 0.0 |
| 265 to less than 305 | 1 | .025 | 2.5 |
| 305 to less than 345 | 1 |  | 2.5 |

c.

d. The donation amounts $332.0,279.2$, and 162.5 stand out because they are much larger than the rest of the donation amounts.
a. and $\mathbf{b}$. The minimum colon and rectum cancer rate for women is 40.4 , and the maximum rate is 48.9 . The following table groups these data into six classes of equal width (1.5) with a starting point of 39.5 .

| Colon \& Rectum Cancer <br> Rates (Females) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 39.5 to less than 41 | 3 | .111 | 11.1 |
| 41 to less than 43.5 | 9 | .222 | 33.3 |
| 43.5 to less than 45 | 5 | .185 | 18.5 |
| 45 to less than 46.5 | 4 | .148 | 14.8 |
| 46.5 to less than 48 | 4 | .148 | 14.8 |
| 48 to less than 49.5 | 2 | .074 | 7.4 |

2.23 a. and $\mathbf{b}$. The minimum colon and rectum cancer rate for men is 49.4 , and the maximum rate is 68 . The following table groups these data into six classes of equal width (4) with a starting point of 46.0.

| Colon \& Rectum Cancer <br> Rates (Males) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 46.0 to less than 50.0 | 1 | .037 | 3.7 |
| 50.0 to less than 54.0 | 1 | .037 | 3.7 |
| 54.0 to less than 58.0 | 1 | .296 | 29.6 |
| 58.0 to less than 62.0 | 4 | .407 | 40.7 |
| 62.0 to less than 66.0 | 2 | .148 | 14.8 |
| 66.0 to less than 70.0 |  | .074 | 7.4 |

2.24 a. and $\mathbf{b}$. The minimum lung and bronchus cancer rate for women is 46.3 , and the maximum rate is 78.2. The following table groups these data into six classes of equal width (6) with a starting point of 44.0.

| Lung \& Bronchus Cancer <br> Rates (Females) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 44.0 to less than 50.0 | 1 | .037 | 3.7 |
| 50.0 to less than 56.0 | 8 | .296 | 29.6 |
| 56.0 to less than 62.0 | 10 | .370 | 37.0 |
| 62.0 to less than 68.0 | 5 | .185 | 18.5 |
| 68.0 to less than 74.0 | 2 | .074 | 7.4 |
| 74.0 to less than 80.0 | 1 | .037 | 3.7 |

c.

2.25 a. and $\mathbf{b}$. The minimum lung and bronchus cancer rate for men is 76.8 , and the maximum rate is 131.3. The following table groups these data into six classes of equal width (10) with a starting point of 72.0.

| Lung \& Bronchus Cancer <br> Rates (Males) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 72.0 to less than 82.0 | 6 | .222 | 22.1 |
| 82.0 to less than 92.0 | 8 | .296 | 29.6 |
| 92.0 to less than 102.0 | 8 | .296 | 29.6 |
| 102.0 to less than 112.0 | 2 | .074 | 7.4 |
| 112.0 to less than 122.0 | 2 | .074 | 7.4 |
| 122.0 to less than 132.0 | 1 | .037 | 3.7 |

c.

2.26 a. and $\mathbf{b}$. The minimum non-Hodgkin lymphoma cancer rate for women is 13.4 , and the maximum rate is 19.1. The following table groups these data into four classes of equal width (2) with a starting point of 12.0 .

| Non-Hodgkin Lymphoma <br> Cancer Rates (Females) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 12.0 to less than 14.0 | 3 | .111 | 11.1 |
| 14.0 to less than 16.0 | 6 | .222 | 22.2 |
| 16.0 to less than 18.0 | 12 | .444 | 44.4 |
| 18.0 to less than 20.0 | 6 | .222 | 22.2 |

c.


Non-Hodgkin Lymphoma Cancer Rates for Women
a. and b.

| Strikeouts per Game | Frequency | Relative <br> Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 5.50 to less than 6.30 | 4 | .133 | 13.3 |
| 6.30 to less than 7.10 | 12 | .400 | 40.0 |
| 7.10 to less than 7.90 | 11 | .367 | 36.7 |
| 7.90 to less than 8.70 | 2 | .067 | 6.7 |
| 8.70 to less than 9.50 | 1 | .033 | 3.3 |

a. and b.

| Turnovers | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 1 | 4 | .160 | 16.0 |
| 2 | 5 | .200 | 20.0 |
| 3 | 3 | .120 | 12.0 |
| 4 | 3 | .120 | 12.0 |
| 5 | 7 | .280 | 28.0 |
| 6 | 2 | .080 | 8.0 |
| 7 | 0 | .000 | 0.0 |
| 8 | 1 | .040 | 4.0 |

c. $3+7=10$ games had four or five turnovers. The relative frequency is $10 / 25=.400$.
d.

a. and b.

| Number of Hot Dogs | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 0 | 4 | 0.167 | 16.7 |
| 1 | 4 | 0.167 | 16.7 |
| 2 | 7 | 0.292 | 29.2 |
| 3 | 4 | 0.167 | 16.7 |
| 4 | 3 | 0.125 | 12.5 |
| 5 | 1 | 0.042 | 4.2 |
| 6 | 1 | 0.042 | 4.2 |

c. $4+4+7+4=19$ patrons ate fewer than 4 hot dogs. The relative frequency is $19 / 24=.792$

2.30


Number of Tickets


Number of Tickets
The truncated graph exaggerates the difference in the number of students with different numbers of tickets.
2.31


The graph with the truncated frequency axis exaggerates the differences in the frequencies of the various time intervals.

## Section 2.3

2.32 The cumulative frequency distribution gives the total number of values that fall below the upper boundary of each class. The cumulative relative frequencies are obtained by dividing the cumulative frequencies by the total number of observations in the data set. The cumulative percentages are obtained by multiplying the cumulative relative frequencies by 100.
2.33 An ogive is drawn for a cumulative frequency distribution, a cumulative relative frequency distribution, or a cumulative percentage distribution. An ogive can be used to find the approximate cumulative frequency (cumulative relative frequency or cumulative percentage) for any class interval.

| a. and b. | Cumulative Frequency | Cumulative <br> Relative Frequency | Cumulative <br> Percentage |
| :--- | :---: | :---: | :---: |
| (allons of Gasoline to less than 4 | 31 | $31 / 369=.084$ | 8.4 |
| 0 to less than 8 | $31+78=109$ | $109 / 369=.295$ | 29.5 |
| 0 to less than 12 | $31+78+49=158$ | $158 / 369=.428$ | 42.8 |
| 0 to less than 16 | $31+78+49+81=239$ | $239 / 369=.648$ | 64.8 |
| 0 to less than 20 | $31+78+49+81+117=356$ | $356 / 369=.965$ | 96.5 |
| 0 to less than 24 | $31+78+49+81+117+13=369$ | 1.000 | 100.0 |

c. $64.8 \%$ of the customers purchased less than 16 gallons.
d.


Gallons of Gasoline
e. Approximately $38 \%$ of customers purchased less than 10 gallons of gasoline, as indicated on the ogive in part d.
a. and b.

| Number <br> of Days | Cumulative Frequency | Cumulative <br> Relative <br> Frequency | Cumulative <br> Percentage |
| :---: | :---: | :---: | :---: |
| 0 to 5 | 32 | .184 | 18.4 |
| 0 to 11 | $32+67=99$ | .569 | 56.9 |
| 0 to 17 | $32+67+44=143$ | .822 | 82.2 |
| 0 to 23 | $32+67+44+20=163$ | .937 | 93.7 |
| 0 to 29 | $32+67+44+20+11=174$ | 1.000 | 100.0 |

c. $100-56.9=43.1 \%$ of the containers will expire in 12 or more days.
d.


## Number of Days to Expiry Date

e. Approximately $85 \%$ of the containers will expire in less than 20 days, as indicated on the ogive in part d.

2.42

| Charitable Contributions <br> (millions of dollars) | Cumulative <br> Frequency | Cumulative Relative <br> Frequency | Cumulative <br> Percentage |
| :---: | :---: | :---: | :---: |
| 25 to less than 65 | 25 | .625 | 62.5 |
| 25 to less than 105 | 33 | .825 | 82.5 |
| 25 to less than 145 | 37 | .925 | 92.5 |
| 25 to less than 185 | 38 | .950 | 95.0 |
| 25 to less than 225 | 38 | .950 | 95.0 |
| 25 to less than 265 | 38 | .950 | 95.0 |
| 25 to less than 305 | 39 | .975 | 97.5 |
| 25 to less than 345 | 40 | 1.000 | 100.0 |



Approximately 29 individuals made charitable contributions of $\$ 85$ million or less.
2.43

| Points Scored | Cumulative <br> Frequency | Cumulative Relative <br> Frequency | Cumulative <br> Percentage |
| :---: | :---: | :---: | :---: |
| 5.50 to less than 6.30 | 4 | .133 | 13.3 |
| 5.50 to less than 7.10 | 16 | .533 | 53.3 |
| 5.50 to less than 7.90 | 27 | .900 | 90.0 |
| 5.50 to less than 8.70 | 29 | .967 | 96.7 |
| 5.50 to less than 9.50 | 30 | 1.000 | 100.0 |



Approximately 12 of the teams had 6.8 or fewer strikouts per game.

## Section 2.4

2.44 To prepare a stem-and-leaf display for a data set, each value is divided into two parts; the first part is called the stem and the second part is called the leaf. The stems are written on the left side of a vertical line and the leaves for each stem are written on the right side of the vertical line next to the corresponding stem. Example 2-9 is an example of a stem-and-leaf display.
2.45 The advantage of a stem-and-leaf display over a frequency distribution is that by preparing a stem-and-leaf display we do not lose information on individual observations. From a stem-and-leaf display we can obtain the original data. However, we cannot obtain the original data from a frequency distribution table. Consider the stem-andleaf display from Example 2-8:

| 5 | 2 | 0 | 7 |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 5 | 9 | 1 | 8 | 4 |  |  |  |  |
| 7 | 5 | 9 | 1 | 2 | 6 | 9 | 7 | 1 | 2 |
| 8 | 0 | 7 | 1 | 6 | 3 | 4 | 7 |  |  |
| 9 | 6 | 3 | 5 | 2 | 2 | 8 |  |  |  |

The data that were used to make this stem-and-leaf display are: $52,50,57,65,69,61$, $68,64,75,79,71,72,76,79,77,71,72,80$, $87,81,86,83,84,87,96,93,95,92,92,98$
2.46 The data that were used to make this stem-and-leaf display are: $43,46,50,51,54,55$, $63,64,66,67,67,67,68,69,72,72,73,75$, 76, 76, 79, 80, 87, 88, 89
2.47 The data that were used to make this stem-and-leaf display are: $218,245,256,329$, 367, 383, 397, 404, 427, 433, 471, 523, 537, $551,563,581,592,622,636,647,655,678$, 689, 810, 841

$\mathbf{2 . 4 8}$|  | 0 | 3 | 5 | 5 | 6 | 8 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 0 | 4 | 5 | 6 | 7 | 7 | 9 |
|  | 2 | 1 | 2 | 3 | 5 |  |  |  |
|  | 3 | 0 | 1 | 1 | 4 |  |  |  |


| 2.49 | 7 | 45 | 75 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 8 | 00 | 48 | 57 |  |
|  | 9 | 21 | 33 | 67 | 95 |
|  | 10 | 09 | 24 |  |  |
|  | 11 | 33 | 45 |  |  |
|  | 12 | 75 |  |  |  |
|  |  |  |  |  |  |

2.50

| 0 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 6 | 6 | 7 | 7 | 8 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 4 | 4 | 7 | 7 | 7 | 7 | 8 | 8 |
| 2 | 0 | 0 | 1 | 3 | 3 | 8 | 9 | 9 |  |  |  |  |  |  |  |  |
| 3 | 0 | 4 | 5 | 8 |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 0 | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

2.51

a. |  | 0 | 2 | 3 | 5 | 5 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 9 | 9 | 9 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9 | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
|  | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 5 | 6 | 8 | 9 |  |
|  | 2 | 0 | 0 | 2 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

b. | 0 | 2 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 5 | 5 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 9 | 9 | 9 | 9 | 9 | 9 |

$\begin{array}{llllllllllllllllll}0 & 5 & 5 & 6 & 6 & 7 & 7 & 7 & 8 & 8 & 8 & 8 & 9 & 9 & 9 & 9 & 9 & 9 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 2\end{array}$

|  | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 5 | 5 | 5 | 5 | 5 | 6 | 8 | 9 |
| 2 | 0 | 0 | 2 | 3 |  |  |  |  |

c. The stem-and-leaf of part (b) is better because for 56 values, five stems are easier to read than three stems.
$\begin{array}{lllllllllllllllllllllll}\text { a. } & 0 & 27 & 28 & 29 & 30 & 30 & 30 & 30 & 30 & 32 & 32 & 33 & 35 & 39 & 40 & 41 & 42 & 45 & 49 & 50 & 50 & 50\end{array}$ $\begin{array}{llllllll}50 & 53 & 59 & 62 & 67 & 80 & 84 & 88\end{array}$ $\begin{array}{lllllllll}00 & 00 & 01 & 01 & 10 & 17 & 18 & 20 & 63\end{array}$ 279 32

b. |  | 0 | 27 | 28 | 29 | 30 | 30 | 30 | 30 | 30 | 32 | 32 | 33 | 35 | 39 | 40 | 41 | 42 | 45 | 49 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

| 0 | 50 | 50 | 50 | 50 | 53 | 59 | 62 | 67 | 80 | 84 | 88 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| 1 | 00 | 00 | 01 | 01 | 10 | 17 | 18 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

1.63
79

32
c. For 40 values, the stem-and-leaf of part (b) is easier to read than that of part (a).
2.53

| 0 | 5 | 7 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 5 | 7 | 9 |  |
| 2 | 1 | 2 | 3 | 6 | 6 | 9 |
| 3 | 2 | 3 | 9 |  |  |  |
| 4 | 3 | 8 |  |  |  |  |
| 5 | 0 |  |  |  |  |  |
| 6 | 5 |  |  |  |  |  |

a. | 0 | 6 | 5 | 8 | 9 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 2 | 5 | 3 | 8 |  |  |  |
| 2 | 0 | 5 | 4 | 0 | 4 | 6 | 2 | 8 |
| 3 | 7 | 0 | 3 | 6 | 6 | 2 | 8 |  |
| 4 | 0 | 5 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |
| 7 | 2 | 0 | 2 |  |  |  |  |  |
| 8 | 0 | 4 | 6 |  |  |  |  |  |
| 9 | 6 | 2 | 0 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

b. | $0-2$ | 6 | 5 | 8 | 9 | $*$ | 0 | 2 | 5 | 3 | 8 | $*$ | 0 | 5 | 4 | 0 | 4 | 6 | 2 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $3-5$ | 7 | 0 | 3 | 6 | 6 | 2 | 8 | $*$ | 0 | 5 | $*$ |  |  |  |  |  |  |  |
|  | $6-9$ | $*$ | 2 | 0 | 2 | $*$ | 0 | 4 | 6 | $*$ | 6 | 2 | 0 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

2.55

| a. | 2 | 58 |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 20 | 45 |  |  |  |  |  |  |  |
| 4 | 30 | 38 | 60 | 90 |  |  |  |  |  |
| 5 | 05 | 30 | 38 | 50 | 60 | 65 | 70 | 75 |  |
| 6 | 10 | 17 | 20 | 35 | 38 |  |  |  |  |
| 7 | 02 | 05 | 06 | 20 | 21 | 28 | 65 | 87 |  |
| 8 | 40 | 45 | 68 | 70 | 90 |  |  |  |  |
| 9 | 57 | 68 |  |  |  |  |  |  |  |

b. $2-4 \left\lvert\, \begin{array}{lllllllll} & 28 & * & 20 & 45 & * & 30 & 38 & 60 \\ 90\end{array}\right.$
$\begin{array}{llllllllllllllllll}5-6 & 05 & 30 & 38 & 50 & 60 & 65 & 70 & 75 & * & 10 & 17 & 20 & 35 & 38 & & & \\ 7-9 & 02 & 05 & 06 & 20 & 21 & 28 & 65 & 87 & * & 40 & 45 & 68 & 70 & 90 & * & 57 & 68\end{array}$

## Section 2.5

In order to prepare a dotplot, first we draw a horizontal line with numbers that cover the given data set. Then we place a dot above the value on the number line that represents each measurement in the data set. Example 2-12 illustrates this procedure.
2.57 A stacked dotplot is used to compare two or more data sets by creating a dotplot for each data set with numbers lines for all data sets on the same scale. The data sets are placed on top of each other.

2.59

2.60

2.61



There are two clusters in the data; most of the values lie in the cluster between zero and five, with only three data points between seven and nine. The value 15 appears to be an outlier.


The data for males is clustered in two groups with the first group having values from zero to five, and the second having values from seven to 10 .


The data for females is also clustered in two groups with the first group having values from zero to two, and the second having values from four to six; 10 appears to be an outlier. With these clusters in different areas, it appears that the female students ate at fast-food restaurants less often than did males during a seven-day period.


The data set contains a cluster from zero to four. The values 28 and 31 are outliers for number.

## Supplementary Exercises

2.66
a. and b.

| Political Party | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| D | 9 | .300 | 30.0 |
| DR | 4 | .133 | 13.3 |
| F | 2 | .067 | 6.7 |
| R | 11 | .367 | 36.7 |
| W | 4 | .133 | 13.3 |

c.


d. $13.3 \%$ of these presidents were Whigs.
2.67

| a. and b. | Response | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: | :---: |
|  | W | 22 | .500 | 50.0 |
|  | 16 | .364 | 36.4 |  |
|  | N | 6 | .136 | 13.6 |

c.

d. $50.0 \%$ of these respondents said "wrong priorities".
a. and b.

| TV sets owned | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 0 | 1 | .025 | 2.5 |
| 1 | 14 | .350 | 35.0 |
| 2 | 14 | .350 | 35.0 |
| 3 | 8 | .200 | 20.0 |
| 4 | 3 | .075 | 7.5 |

c.

d. $(14+8+3) / 40=62.5 \%$ of the households own two or more television sets.
a. and b.

| Correct Names | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 0 | 1 | .042 | 4.2 |
| 1 | 3 | .125 | 12.5 |
| 2 | 4 | .167 | 16.7 |
| 3 | 6 | .250 | 25.0 |
| 4 | 4 | .167 | 16.7 |
| 5 | 6 | .250 | 25.0 |

c. $(1+3) / 24=16.7 \%$ of the students named fewer than two of the representatives correctly.
d.

a. and b.

| Number of Text Messages | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| $32-37$ | 10 | .250 | 25.0 |
| $38-43$ | 9 | .225 | 22.5 |
| $44-49$ | 13 | .325 | 32.5 |
| $50-55$ | 6 | .150 | 15.0 |
| $56-61$ | 2 | .050 | 5.0 |

c.

d. On $(13+6+2) / 40=52.5 \%$ of the 40 days, the student sent more than 44 text messages.
a. and b.

| Number of Orders | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| $23-29$ | 4 | .133 | 13.3 |
| $30-36$ | 9 | .300 | 30.0 |
| $37-43$ | 6 | .200 | 20.0 |
| $44-50$ | 8 | .267 | 26.7 |
| $51-57$ | 3 | .100 | 10.0 |

c. For $(6+8+3) / 30=56.7 \%$ of the hours in this sample, the number of orders was more than 36 .
2.72
a. and b.

| Concession (dollars) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| 0 to less than 6 | 9 | .300 | 30.0 |
| 6 to less than 12 | 10 | .333 | 33.3 |
| 12 to less than 18 | 5 | .167 | 16.7 |
| 18 to less than 24 | 4 | .133 | 13.3 |
| 24 to less than 30 | 2 | .067 | 6.7 |

c.

a. and b.

| Car Repair Costs (dollars) | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| $1-1400$ | 11 | .367 | 36.7 |
| $1401-2800$ | 10 | .333 | 33.3 |
| $2801-4200$ | 3 | .100 | 10.0 |
| $4201-5600$ | 2 | .067 | 6.7 |
| $5601-7000$ | 4 | .133 | 13.3 |

c.

d. The class boundaries of the fourth class are $\$ 4200.50$ and $\$ 5600.50$. The width of this class is $\$ 1400$.
a. \& b.

| Number of Text <br> Messages | Cumulative <br> Frequency | Cumulative <br> Relative <br> Frequency | Cumulative <br> Percentage |
| :---: | :---: | :---: | :---: |
| $32-37$ | 10 | .250 | 25.0 |
| $32-43$ | 19 | .475 | 47.5 |
| $32-49$ | 32 | .800 | 80.0 |
| $32-55$ | 38 | .950 | 95.0 |
| $32-61$ | 40 | 1.000 | 100.0 |

2.75

| Number of <br> Orders | Cumulative Frequency | Cumulative Relative <br> Frequency | Cumulative <br> Percentage |
| :---: | :---: | :---: | :---: |
| $23-29$ | 4 | .133 | 13.3 |
| $23-36$ | 13 | .433 | 43.3 |
| $23-43$ | 19 | .633 | 63.3 |
| $23-50$ | 27 | .900 | 90.0 |
| $23-57$ | 30 | 1.000 | 100.0 |

2.76

| Concession <br> (dollars) | Cumulative Frequency | Cumulative Relative <br> Frequency | Cumulative <br> Percentage |
| :--- | :---: | :---: | :---: |
| 0 to less than 6 | 9 | .300 | 30.0 |
| 0 to less than 12 | 19 | .633 | 63.3 |
| 0 to less than 18 | 24 | .800 | 80.0 |
| 0 to less than 24 | 28 | .933 | 93.3 |
| 0 to less than 30 | 30 | 1.000 | 100.0 |


| Car Repair <br> Costs (dollars) | Cumulative Frequency | Cumulative Relative <br> Frequency | Cumulative <br> Percentage |
| :---: | :---: | :---: | :---: |
| $1-1400$ | 11 | .367 | 36.7 |
| $1-2800$ | 21 | .700 | 70.0 |
| $1-4200$ | 24 | .800 | 80.0 |
| $1-5600$ | 26 | .867 | 86.7 |
| $1-7000$ | 30 | 1.000 | 100.0 |

2.78

| 3 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 7 | 7 | 7 | 8 | 9 |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 5 | 7 | 7 | 7 | 7 | 7 | 8 | 8 | 9 |
| 5 | 0 | 0 | 1 | 2 | 3 | 4 | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

2.79

| 2 | 8 | 4 | 7 | 7 |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 4 | 1 | 8 | 5 | 2 | 9 | 3 | 7 | 0 | 8 | 4 | 6 | 0 |
| 4 | 4 | 1 | 7 | 6 | 1 | 9 | 5 | 6 | 7 |  |  |  |  |
| 5 | 2 | 3 | 7 | 0 |  |  |  |  |  |  |  |  |  |



The truncated graph exaggerates the differences in the number of girls with the given names.


Region


Region

The truncated graph exaggerates the differences in average price per gallon for the period.
2.82

2.83




a. | Age | Frequency | Relative Frequency |
| :---: | :---: | :---: |
| 18 to less than 20 | 7 | .060 |
| 20 to less than 25 | 12 | .103 |
| 25 to less than 30 | 18 | .154 |
| 30 to less than 40 | 14 | .120 |
| 40 to less than 50 | 15 | .128 |
| 50 to less than 60 | 16 | .137 |
| 60 and over | 35 | .299 |


b. and $\mathbf{c}$. This histogram is misleading because the class widths differ. If you were to change the frequency distribution to reflect equal class widths, the resulting histogram would give a clearer picture.
2.87 The greater relative frequency of accidents in the older age group does not imply that they are more accident-prone than the younger group. For instance, the older group may drive more miles during a week than the younger group.
2.88
a. Using Sturge's formula:

$$
\begin{aligned}
c & =1+3.3 \log n=1+3.3 \log 135 \\
& =1+3.3(2.13033377) \\
& =1+7.03=8.03 \approx 8
\end{aligned}
$$

b. Approximate class width

$$
\begin{aligned}
& =\frac{\text { Largest value }- \text { smallest value }}{\text { Number of classes }} \\
& =\frac{53-20}{8}=4.125
\end{aligned}
$$

Use a class width of 5 .
a. The top money winners on the men's
tour tend to make more money per tournament than those on the women's tour. Earnings on the men's tour begin at $\$ 2300$, and more of the data points are toward the higher end of the scale. Earnings on the women's tour begin at $\$ 800$, and more of the data points are toward the lower end of the scale.
b. Typical earnings per tournament played for the women's tour would be around \$2500; typical earnings per tournament played for the men's tour would be around $\$ 3650$.
c. The data do not appear to have similar spreads for the two tours. Earnings on the men's tour begin at $\$ 2300$, the largest grouping is between $\$ 2300$ and $\$ 4800$, and go up to $\$ 9500$. Earnings on the women's tour begin at $\$ 800$, the largest grouping is between $\$ 1100$ and $\$ 2600$, and only go up to $\$ 7500$.
d. On the women's tour, the $\$ 7500$ earnings level appears to be an outlier; on the men's tour, both the $\$ 8700$ and $\$ 9500$ earnings levels appear to be outliers.
2.90
b. i.

| 9 | 9 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | 2 | 8 | 8 |  |  |  |
| 11 | 0 | 4 | 5 | 5 | 6 | 9 |
| 12 | 3 | 3 | 3 | 5 | 8 | 8 |
| 13 | 2 | 3 | 8 |  |  |  |
| 14 | 6 | 7 | 7 | 8 |  |  |
| 15 | 5 | 9 |  |  |  |  |
| 16 | 1 | 2 | 4 | 8 |  |  |
| 17 | 4 | 4 | 5 | 9 | 9 | 9 |
| 18 | 0 | 2 | 3 | 9 |  |  |
| 19 | 3 | 3 | 5 |  |  |  |
| 20 | 2 | 4 |  |  |  |  |

ii. The display shows a bimodal distribution, due to the presence of both females and males in the sample. The males tend to be heavier, so their weights are concentrated in the larger values, while the females' weights are found primarily in the smaller values.

2.91 a. Top Histogram - Endpoints: - $0.5,0.5$, $1.5,2.5,3.5,4.5,5.5,6.5,7.5,8.5,9.5$, 10.5 ; width $=1$

Bottom Histogram - Endpoints: 0, 1, 2, $3,4,5,6,7,8,9,10$; width $=1$
b. There is one observation between the left endpoint of the interval and 8 . This can be seen by overlaying the histograms and determining the counts for each interval of .5 on the $x$ axis starting at the far left. The following table displays these frequencies:

| Interval | Frequency |
| :---: | :---: |
| 0.0 to less than 0.5 | 2 |
| 0.5 to less than 1.0 | 2 |
| 1.0 to less than 1.5 | 5 |
| 1.5 to less than 2.0 | 6 |
| 2.0 to less than 2.5 | 6 |
| 2.5 to less than 3.0 | 3 |
| 3.0 to less than 3.5 | 4 |
| 3.5 to less than 4.0 | 5 |
| 4.0 to less than 4.5 | 1 |
| 4.5 to less than 5.0 | 4 |
| 5.0 to less than 5.5 | 3 |
| 5.5 to less than 6.0 | 1 |
| 6.0 to less than 6.5 | 0 |
| 6.5 to less than 7.0 | 1 |
| 7.0 to less than 7.5 | 2 |
| 7.5 to less than 8.0 | 1 |
| 8.0 to less than 8.5 | 2 |
| 8.5 to less than 9.0 | 0 |
| 9.0 to less than 9.5 | 0 |
| 9.5 to less than 10.0 | 2 |
| 10.0 to les than 10.5 | 0 |

c. The leftmost bar in the first histogram is misleading because it makes it appear as though there are values in the data set less than zero.


The distribution of all weights is bimodal. The distribution of weights for males is skewed to the left while the distribution for females is skewed to the right. You cannot distinguish between the lightest males and heaviest females in the dotplot of all weights as the distributions overlap in the area between 130 and 170 pounds.
2.96 a. Answers may include 10.0 because it is in the center of the data.
b. There is one outlier in the data. The value 4.8 is an outlier as it is in the tail of the distribution with a large gap preceding it.
c. The distribution is skewed right as the majority of the values are between 7.3 and 12.3 with more values further to the right than to the left.
d. We cannot conclude that Oklahoma had the highest obesity rate nor Alaska the lowest as these data represent the change in the obesity rates, not the actual rates.
2.97 a. The West has the least variability as the data are clustered together. The South has the most variability as the data are the most widely spread.
b. The South tends to have the highest obesity rates as a large number of the data points are above 27.0. The West and Northeast tend to have the lowest obesity rates with most of the data points below 25 and only one each above 27.
c. The West appears to have an outlier at approximately 21.2. The South also appears to have outliers at approximately 22.5 and 34. The Northwest has one outlier at about 28.5. The Midwest has an outlier at about 25 .
2.98 a. The ACC received more than $25 \%$ of the vote. The section of the pie chart representing the ACC is more than one-quarter of the whole pie.
b. Southeastern and Big East
c. $\mathrm{a}=$ Conference USA, $\mathrm{b}=\operatorname{Pac} 10, \mathrm{c}=$ Others, $\mathrm{d}=$ Southeastern, $\mathrm{e}=\operatorname{Big}$ East, $\mathrm{f}=\operatorname{Big} 12, \mathrm{~g}=\operatorname{Big}$ Ten, $\mathrm{h}=\mathrm{ACC}$.

## Self-Review Test

1. An ungrouped data set contains information on each member of a sample or population individually. The first part of Example 2-1 in the text, listing the responses of each of the 30 employees, is an example of ungrouped data. Data presented in the form of a frequency table are called grouped data. Table 2.4 in the solution of Example 2-1 is an example of grouped data.
2. a. 5
b. 7
c. 17
d. 6.5
e. 13.5
f. 90
g. 30
3. A histogram that is identical on both sides of its central point is called a symmetric histogram. A histogram that is skewed to the right has a longer tail on the right side, and a histogram that is skewed to the left has a longer tail on the left side. The following three histograms present these three cases. Figure 2.8 in the text provides graphs of symmetric histograms, Figure 2.9a displays a histogram skewed to the right, and Figure 2.9 b displays a histogram that is skewed to the left.
4. a. and b.

| Category | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| B | 8 | .40 | 40 |
| F | 4 | .20 | 20 |
| M | 7 | .35 | 35 |
| S | 1 | .05 | 5 |

c. $35 \%$ of the children live with their mothers only.
d.


5.
a. and b.

| Number of False Alarms | Frequency | Relative Frequency | Percentage |
| :---: | :---: | :---: | :---: |
| $1-3$ | 5 | .208 | 20.8 |
| $4-6$ | 6 | .250 | 25.0 |
| $7-9$ | 6 | .250 | 25.0 |
| $10-12$ | 4 | .167 | 16.7 |
| $13-15$ | 3 | .125 | 12.5 |

c. $(5+6+6) / 24=70.8 \%$ of the weeks had 9 or fewer false alarms.
d.

6.

| Number of False <br> Alarms | Cumulative Frequency | Cumulative Relative Frequency | Cumulative <br> Percentage |
| :---: | :---: | :---: | :---: |
| $1-3$ | 5 | .208 | 20.8 |
| $1-6$ | 11 | .458 | 45.8 |
| $1-9$ | 17 | .708 | 70.8 |
| $1-12$ | 21 | .875 | 87.5 |
| $1-15$ | 24 | 1.000 | 100.0 |



7. $0 |$|  | 0 | 6 | 7 |
| :--- | :--- | :--- | :--- |

| 1 | 0 | 2 | 2 | 3 | 4 | 4 | 5 | 6 | 6 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 0 | 1 | 2 | 2 | 5 | 9 |  |  |  |  |  |  |  |
| 3 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |

8. $\quad 3 \quad 30$
9. 



