SOLUTIONS MANUAL

KINEMATICS AND DYNAMICS OF MACHINERY

THIRD EDITION

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A Guide for Instructors

In solving the problems, Dr. Sadler and I used various analytical and graphical methods, aided by various types of software. It is expected that professors assigning problems from the text will select solution methods and software in accordance with goals they have set for their students. Although we have used reasonable care in solving the problems, errors always creep in. We will be grateful for any corrections or comments related to the text or this guide.

If you recently taught kinematics and dynamics of machinery, you have probably already decided on the course content based on the needs and abilities of your students. And, you may have a wealth of material to supplement the course based on your teaching experience, industrial experience, research, or consulting. The comments that follow are for professors who have not taught the course recently, or wish to revise the content.

Goals for Your Students; Encouraging Students to Think

Thomas Edison said that "All progress, all success, springs from thinking." But in his laboratory, Edison posted a quote from Sir Joshua Reynolds: "There is no expedient to which a man will not resort to avoid the real labor of thinking." We often face a similar problem. Some of our students calculate the solution to an academic exercise without understanding why the problem was assigned, and with little understanding to the meaning and significance of their answer. It may be impossible to teach our students how to think. But we can assign tasks that encourage them to think as engineers must think. We can ask students to

- identify a need
- propose a linkage or some other system to meet that need
- perform some of the tasks required to design that component or system
- analyze a tentative design: determine motion, velocity, acceleration and forces including inertial effects
- interpret the results of their analysis
- propose changes to improve that design
- communicate their results through written and oral reports, graphs, and motion simulations and field questions related to the significance of their analysis

Each chapter has a few homework problems designed to encourage in-depth analysis and thinking. Motion simulation software and mathematics software relieve the user of repetitive calculations, and allow a more thorough presentation of results. Students can examine linkages through a full cycle of motion, or evaluate the effect of an array of possible design changes. For example, we can ask students to design a crank-rocker linkage to produce a given range of output motion, while optimizing transmission angle. We can ask students to look into a series of reverted gear trains for producing a range of speed reductions, while using minimum tooth numbers consistent with avoiding interference. Or they can plot and examine a large number of coupler curves in an attempt to design a linkage with specified motion requirements.

Developing a Syllabus for a Course in Kinematics and Dynamics of Machinery

Every topic in the text was added or retained on the recommendation of one or more reviewers. Nevertheless, a typical course in kinematics and dynamics of machinery does not allow enough time to cover all of the topics in the text. Obviously, the desired outcomes of your course will govern your selection of topics to emphasize, topics to cover quickly, and topics to delete. I can only offer a few suggestions based on my own goals for a course of kinematics and dynamics of machinery and my interpretation of the criteria of the Accrediting Board for Engineering Technology (ABET).

A Few Comments on Selection of Topics

Chapter 1

You may find the following topics important as a basis for further study: computer use; terminology and definitions; degrees of freedom; Grashof criterion; transmission angle. If motion simulation software is available, students can simulate the motion of various classes of four-bar linkages, verifying the Grashof criterion. You may want to assign one of the homework problems that requires a contour plot showing an envelope of acceptable linkage proportions based on range of motion and transmission angle. If time is short, you may want to delete topics like limiting position of offset slider crank linkages, and put the section on mechanisms for specific applications in a "read only" category. Numerical procedures are now incorporated in various software packages; there is no need for students to write numerical method programs unless programming is a specific goal of the course.

Chapter 2

Important items include unit vectors, dot and cross product, and vector differentiation. Vectors are useful for solving planar linkages, and the only practical way to solve spatial linkages. For students already proficient in simple vector operations a quick review is all that is needed. If graphical methods are emphasized, position analysis of planar linkages is a trivial exercise. If you intend to rely on motion simulation software for planar linkage analysis, then position calculations are not absolutely necessary. But, I prefer to have the students spot-check results obtained with motion simulation software. Although it seems complicated, I prefer the cross-product method for position analysis of planar four-bar linkages. If the cross-product method is selected, it is not necessary to teach the dot product method. Complex number methods offer no advantages over other methods of position analysis. But complex number methods can be introduced at this point if you intend to use them for velocity and acceleration analysis.

A graphical method can be used to check analytical position analysis of a spatial linkage for one instant in time. But it is not an easy task. I prefer to skip graphical analysis of spatial linkages entirely, relying on verification tests that can be built into a computer solution.

Chapter 3

Important topics include the vector cross product equations for velocity, particularly for spatial linkages. Matrix methods for solving a set of linear differential equations are important too, but this will be a quick review for some.

I think that analytical velocity analysis should be included, even though it is not absolutely necessary if you intend to rely on motion simulation software for planar linkage analysis. My personal preference is the complex number method, but there is strong support for vector methods as well. If your students use mathematics software that solves matrices directly, they will not need determinant methods, except possibly for use on tests where computers are unavailable.

A velocity polygon can be used to spot-check analytical results and motion simulation plots at one instant in time. Unless you want to concentrate on graphical methods, you will probably cover velocity polygons briefly, and eliminate centro methods entirely. Kinematics analysis using spreadsheets will probably be eliminated unless you want to introduce spreadsheets for use in other courses.

Chapter 4

I think that analytical acceleration analysis should be included, even though it is not absolutely necessary if you intend to rely on motion simulation software for planar linkage analysis. Again, I prefer the complex number method, but if you specified vector methods for velocity analysis, you will want to specify vectors for acceleration as well. Unless you want to concentrate on graphical methods, you may want to skip acceleration polygons. Results obtained from analytical acceleration calculations and motion simulation software can be checked by numerical differentiation of the results of analytical velocity analysis. Acceleration analysis using spreadsheets will probably be eliminated unless you plan to use spreadsheets for other courses as well.

Chapter 5

Important points include the boundary conditions required to generate "good" cams. You will probably want to emphasize cycloidal motion and 5th order and 8th order polynomial motion. You may want to skip graphical construction of cam profiles, since it is not a step in the generation of actual cams. You will probably allot a few minutes to harmonic, parabolic, and constant acceleration follower motion, showing why these motion forms are inferior. The theory of envelopes is an advanced topic—its inclusion depends on how much time you have to cover cams.

Chapter 6

Gear nomenclature, tooth proportions, and standard pressure angles are essential topics. Interference and contact ratio are also important, as are free-body diagrams of individual gears showing forces and torques. Ask your students to evaluate their results and make design changes where indicated. For example, if a tentative design results in interference, have them suggest changes to correct this problem. Gear topics should be coordinated with machine design courses to ensure adequate coverage without excessive repetition.

Chapter 7

Helical gears on parallel shafts and worm drives deserve the most emphasis. Thrust forces on helical gears, and balancing of thrust forces in helical gear countershafts are important topics. If time is limited, other types of gears may be placed in the "read only" category. Again, topics should be coordinated with machine design courses to ensure adequate coverage without excessive repetition.

Chapter 8

Speed ratios in planetary and non-planetary gear trains are important. The superposition method for analyzing planetary trains is nice because its tabular form allows for adding gear dimensions, forces, torques, and power. But the formula method for analyzing planetary trains is best for analyzing differentials. If you do not have the luxury of teaching both methods, the formula method is probably the best choice. Important also are free body diagrams of individual gears, and a torque balance of planetary train. You will probably want to assign a study showing the speed ratio of a series of proposed planetary train designs, and the number of planets that will produce a balanced train in each case. If time is short, you may have to skip chain drives, friction drives, and gear train diagnostics based on noise and vibration frequencies.

Chapter 9

Important topics include analytical static-force analysis and computer-aided simulations. Unless you intend to emphasize graphical methods throughout, graphical examples can be treated as demonstrations and as a means to develop analytical models.

Chapter 10

Important topics include analytical dynamic-force analysis and computer-aided simulations. D'Alembert's principle is the key because it transforms a dynamics problem into a statics-type problem. Unless you intend to emphasize graphical methods throughout, graphical examples can again be treated as demonstrations and as a means to develop analytical models. Motion simulation software will be particularly helpful in determining dynamic motion analysis for an assumed input torque. You may choose to skip balancing, particularly if this topic is covered in another course.

Chapter 11

Important topics include two- and three-position synthesis, design of a function generator, and coupler curves. The results of three-position synthesis can be checked with motion simulation software. If you have used complex numbers for velocity and acceleration analysis, your students will probably prefer the complex matrix method for design of a function generator. Design of a function generator may involve many attempts and a long time in front of a computer if the end-result is to have continuous motion and acceptable transmission angles. If motion simulation software is available, students can generate a large number of coupler curves before selecting the best one for a specified application. You may want to skip velocity and acceleration synthesis by the complex number method. It is an interesting exercise, but has little practical value.

Chapter 12

Important topics include degrees of freedom and transformation matrices. Motion simulation software may be used to analyze simple manipulators with planar motion. If a separate course in robot design is offered, you will probably assign only a small part of this chapter, if any.

Projects

Projects can be rewarding if time allows. They can approximate real-world engineering design practice, and allow for more imagination and creativity than standard homework problems. A few project suggestions follow the problem sections in some chapters. Additional projects can be developed from your research or consulting. Or, you can base projects on articles in engineering periodicals. If you use group projects, oral reports and questions to individual members of the group will help you evaluate each student's degree of participation level of understanding.

General Comments

Working smart

Encourage your students to work smart by becoming familiar with mathematics software as early as possible. Tell them to include titles and descriptive comments in their work so that they can refer to it later. Do not let them lose sight of the underlying engineering principles and mathematical concepts, and the implications of their results. If our students do not understand what they are doing and why they are doing it, they are wasting their time and our time as well.

Work smart yourself by including self-verifying steps in problems. For example, consider analysis of a planar or spatial linkage. Require the students to check for closure of the vector loop at some instant in time. Can they check their acceleration analysis by numerical differentiation?

Problems, answers, and examinations

In most cases, a given concept is evaluated by two or three problems so that you do not have to assign the same homework problem term after term. Partial answers are given for most of the odd-numbered problems. If you give open-book examinations that include text problems, you might select even-numbered problems for the examinations, and odd-numbered problems for homework. In each chapter, those problems near the end of the problem set are likely to involve detailed analysis and plotting and include self-verification of some results.

I hope that your course in kinematics and dynamics of machinery is challenging and rewarding to your students. And, may you find satisfaction in sharing your knowledge with them.

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Chapter 1
Mechanisms and Machines: Basic Concepts

1.1 a. The ball-joint (spherical pair) has three degrees-of-freedom, the prismatic pair one, and the cylindrical pair two. The number of degrees-of-freedom of the spatial linkage is given by

$$DF_{\langle spatial \rangle} = 6(n_L - n_J - 1) + \Sigma f_i$$

$$= 6(4 - 3 - 1) + 3 + 1 + 2$$

$$= 6 \text{ degrees-of-freedom}$$

(We do not need the inequality sign for this open-loop chain).

b. Treating the construction equipment schmatic as a spatial linkage:

$$DF_{\langle spatial \rangle} \geq 6(n_L - n_J - 1) + \Sigma f_i$$

$$\geq 6(9 - 11 - 1) + 9 + 2 \times 2$$

$$DF_{\langle spatial \rangle} \geq -5$$

c. Treating the construction equipment schematic as a planar linkage, where a sliding pair has only one degree-of-freedom in plane motion:

$$DF_{\langle planar \rangle} = 3(n_L - n_J - 1) + \Sigma f_i$$

$$DF_{\langle planar \rangle} = 3(9 - 11 - 1) + 9 + 2$$

$$DF_{\langle planar \rangle} = 2$$

d. The planar motion assumption applies if motion occurs in a plane or in a set of parallel planes. The planes of motion of the links must all be parallel. The revolute joint axes must be perpendicular to those planes. These conditions do apply to the construction machinery. The operator controls the linkage via the two hydraulic cylinders.

1.2(a) For the linkage as sketched originally,

DF = $3(n_L-1) - 2n_{J'} = 3(9-1) - 2\times12 = 0$

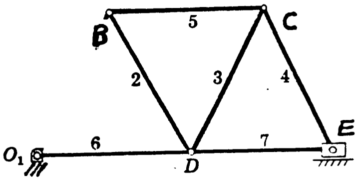
a) Removing any single link (one through seven) reduces n_L by one and $n_{J'}$ by two from which

DF = $3(8-1) - 2\times10 = 1$.

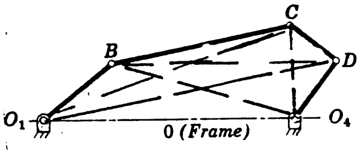
b) Also, we obtain DF = 1 by removing links 1, 2 and 5; or 3, 4 and 5; or 1, 2, 3, 4, and 5.

c) DF = 1 if all links are removed except link 1, or 6 or the slider.

d) DF = 1 if the slider is removed, but the joint at E retained.



1.2(b) Add one link extending from O_1 to C, or B to D, or C to O_4, or B to O_4, or O_1 to D. Then, DF = $3(n_L-1) - 2n_J' = 3(6-1) - 2\times7 = 1$.



1.3 Stroke length S = 2R.

$v_{(avg)} = 2Sn = 2(4 \text{ in})(3000 \text{ rev/min}) = 24{,}000$ in/min or 400 in/sec).

1.4 R = 2, L = 4, E = 1 in. $\omega = \frac{2\pi}{60}(3000) = 314$ rad/sec

Referring to fig. 1.16

$\phi_1 = \arcsin E/(L-R) = 30^\circ$.

$\phi_2 = \arcsin E/(L+R) = 9.6^\circ$.

$\alpha = 180^\circ + \phi_1 - \phi_2 = 200.4^\circ$ or 3.50 rad.

$\beta = 180^\circ - \phi_1 + \phi_2 = 159.6^\circ$ or 2.79 rad.

Stroke $S = [(L+R)^2 - E^2]^{1/2} - [(L-R)^2 - E^2]^{1/2} = 4.184$ in.

"Forward" stroke time $t_1 = \alpha/\omega = 0.01114$ sec. $v_{1(avg)} = S/t_1 = 375.5$ in/sec.

"Return" stroke time $t_2 = \beta/\omega = 0.00889$ sec. $v_{2(avg)} = S/t_2 = 470.6$ in/sec.

1.5 (see 1.4) $\phi_1 = 48.5^\circ$, $\phi_2 = 14.5^\circ$, $\alpha = 3.73$ rad, $\beta = 2.55$ rad.
$S = 4.49$ in, $t_1 = 0.01187$ sec, $v_{1(avg)} = 378$ in/sec.
$t_2 = 0.00812$ sec, $v_{2(avg)} = 553$ in/sec.

1.6 $\omega = 3000\ \pi/30 = 314.16$ rad/s

$$\phi_1 = \arcsin (E/(L-R))$$
$$= \arcsin (50/(200-100)) = .5236 \text{ rad}$$
$$\phi_2 = \arcsin (E/(L+R))$$
$$= \arcsin (50/(200+100)) = .1674 \text{ rad}$$

$\alpha = \pi + \phi_1 - \phi_2 = 3.4977$ rad $\quad \beta = \pi - \phi_1 + \phi_2 = 2.7854$ rad

Forward stroke time $t_1 = \alpha/\omega = .01113$ s.

Return stroke time $t_2 = \beta/\omega = .008866$ s.

Stroke $S = \sqrt{(L+R)^2 - E^2} - \sqrt{(L-R)^2 - E^2} = 209.20$ mm

Avg. vel. forward $v_{1(avg)} = S/t_1 = 18790$ mm/s

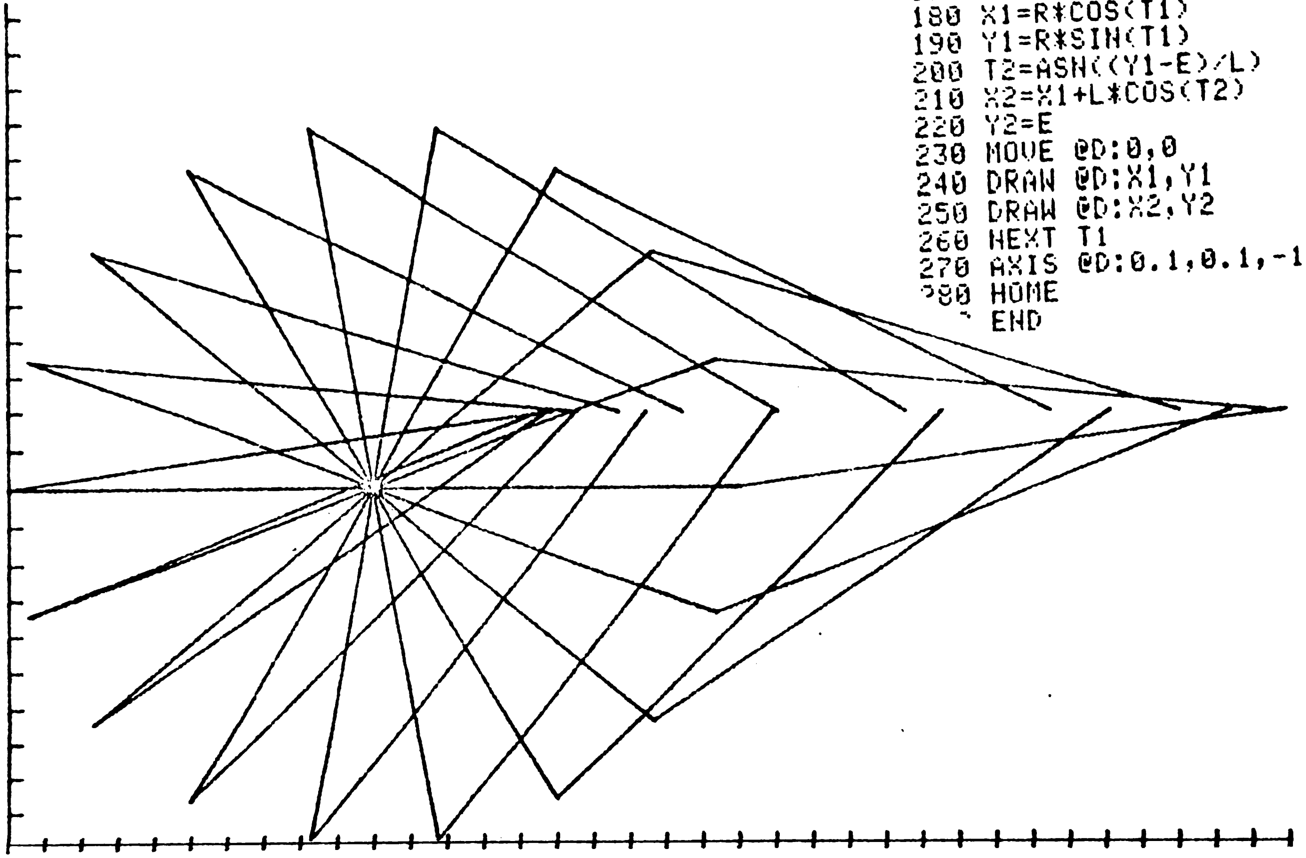
Avg. vel. return $v_{2(avg)} = S/t_2 = 23595$ mm/s

1.7 The crank and connecting rod positions are determined as in the flow chart. The linkage skeleton diagram is shown on the sketch for L/R=1.5, E/R=0.2 where crank angle T_1 varies from 0° to 340° in 20° steps.

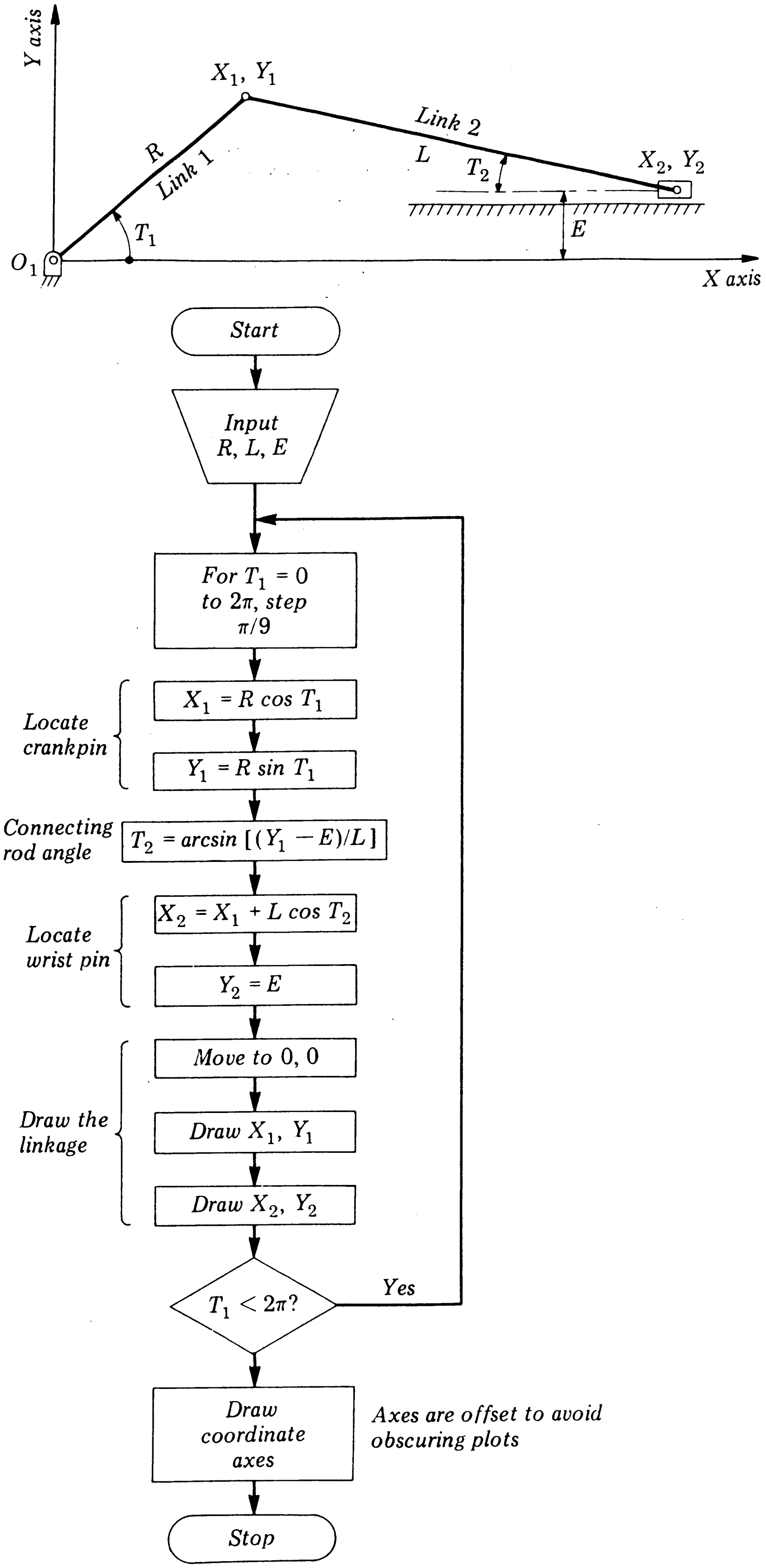
Angles T_1 and T_2 and slider location X_2 are tabulated below for R=1:

```
     T1                 T2                X2
 0              -7.66225566877     2.48660687473
 20              5.43290764513     2.43295424518
 40             17.1690352815      2.19920149468
 60             26.3634597467      1.84402758957
 80             31.5474944957      1.45195830359
 100            31.5474944957      1.12466195325
 120            26.3634597467      0.844027589566
 140            17.1690352815      0.667112607638
 160             5.43290764513     0.553569803629
 180            -7.66225566876     0.486602874732
 200           -21.1829283574      0.438954641475
 220           -34.1844166885      0.474805686816
 240           -45.2985623587      0.555267662011
 260           -52.1735359216      0.746259740205
 280           -52.1735359216      1.09355610154
 300           -45.2985623587      1.55526766201
 320           -34.1844166885      2.80689457325
 340           -21.1829283574      2.38833968305
 360            -7.66225566877     2.48660687473
```

```
120 R=1
130 L=1.5
140 E=0.2
150 WINDOW -1,2.5,-1.346,1.346
160 VIEWPORT 0,130,0,100
170 FOR T1=0 TO 2*PI STEP PI/9
180 X1=R*COS(T1)
190 Y1=R*SIN(T1)
200 T2=ASN((Y1-E)/L)
210 X2=X1+L*COS(T2)
220 Y2=E
230 MOVE @D:0,0
240 DRAW @D:X1,Y1
250 DRAW @D:X2,Y2
260 NEXT T1
270 AXIS @D:0.1,0.1,-1,-1
280 HOME
    END
```



1.7



1.7 Flow chart.