SOLUTIONS MANUAL

LASER ELECTRONICS

Third Edition

Joseph T. Verdeyen

Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois

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Contents

1.1

Field quantities are usually specified in terms of peak values, rather than rms ones. Hence, the factor of 2 appears because of the average over an optical cycle.

1.2

The various operations $\nabla, \nabla\cdot$, and $\nabla\times$ become: $\nabla \rightarrow = j\mathbf{k}; \nabla\cdot \rightarrow -j\mathbf{k}\cdot; \nabla\times \rightarrow -j\mathbf{k}\times$ as can be verified by direct expansion.

1.3

$\mathbf{k}\times\mathbf{H} = -\omega\mathbf{D}$; $\mathbf{k}\cdot(\mathbf{k}\times\mathbf{H}) \equiv 0$; (a) $\therefore$ $\mathbf{k}\cdot\mathbf{D} \equiv 0$; (b) $\mathbf{D} = (1/\omega)\mathbf{k}\times\mathbf{H}$;

$\mathbf{B} = (1/\omega)\,\mathbf{k}\times\mathbf{E}$, use $(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = \mathbf{C}[\mathbf{A}\cdot(\mathbf{B}\times\mathbf{D})] - \mathbf{D}[\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})]$

$\therefore\ \mathbf{D}\times\mathbf{B} = -(1/\omega^2)\,\{\mathbf{k}(\mathbf{k}\cdot(\mathbf{H}\times\mathbf{E})) - \mathbf{E}(\mathbf{k}\cdot(\mathbf{H}\times\mathbf{k})\}$;

Now $\mathbf{H}\times\mathbf{E} = -\mathbf{E}\times\mathbf{H}$ and $\mathbf{H}\times\mathbf{k} = \omega\mathbf{D}$ $\therefore$ $\mathbf{D}\times\mathbf{B} = (1/\omega^2)\,\mathbf{k}[\mathbf{k}\cdot(\mathbf{E}\times\mathbf{H})]$;

$\mathbf{k}\times(\mathbf{k}\times\mathbf{E}) = \omega\mu_0(\mathbf{k}\times\mathbf{H}) = -\omega^2\mu_0\mathbf{D}$;

Now $\mathbf{D}\cdot[\mathbf{k}(\mathbf{k}\cdot\mathbf{E}) - k^2\mathbf{E}] = -\omega^2\mu_0\mathbf{D}\cdot\mathbf{D}$, but $\mathbf{D}\cdot\mathbf{k} = 0$;

$$\therefore\ \boxed{k^2 = \omega^2\mu_0\,\frac{\mathbf{D}\cdot\mathbf{D}}{\mathbf{E}\cdot\mathbf{D}}}$$

1.4

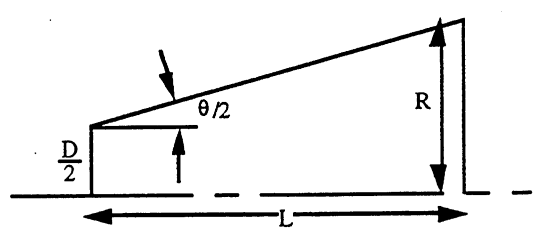
Consider the sampled by the beam shown in the diagram at the right.

The radius is given by:

$$R = \frac{D}{2} + \frac{L\theta}{2};\ \frac{\theta}{2} = \frac{2\lambda}{\pi D};\ \therefore\ R = \left[\frac{D}{2} + \frac{2L\lambda}{\pi D}\right]$$

Volume of frustrated cone:

$$V = \frac{1}{3}\pi L\left\{\left[\frac{D}{2}\right]^2 + \left[\frac{RD}{2}\right] + R^2\right\}:$$



$$\text{Minimize } V(D/L);\ \left[\frac{D}{L}\right]^4 = \frac{4}{3}\left[\frac{2\lambda}{\pi L}\right]^2$$

$$\text{or } D^4 = \frac{4}{3}\left[\frac{2L\lambda}{\pi}\right]^2\ \therefore D = 0.216\text{ cm};$$

$$\frac{\theta}{2} = \frac{2\lambda}{\pi D} = 1.87\times10^{-4}\text{ rad};\ 2R = 0.589$$

cm

1.5

	eV	Å	nm	Hz	cm^{-1}
GaAs	1.47	8434	843.4	3.56^{+14}	11,857
Ar^{+}	2.41	5145	514.5	5.83^{+14}	19,436
He:Ne	1.96	6328	632.8	4.74^{+14}	15,803
CO_2	0.117	λ = 10.6 μm		2.83^{+13}	943
ISM(rf)	5.6^{-8}	λ = 22.1 meters		13.56^{+6}	4.5^{-4}
KrF	4.98	2490	249	1.2^{+15}	40,161

1.6

Covered in the text, sec. 1.11

1.7

$$E(y) = E_0 \exp[-(y/w_0)^2]; \ \therefore \ E(k_y) = E_0 \int_{-\infty}^{-\infty} \exp[-(y/w)^2]\, e^{-jk_y y}\, dy;$$

Complete square in exponent: $E(k_y) = w_0 E_0 \exp -(k_y w_0/2)^2 \ \{ \int_{-\infty}^{+\infty} \exp[-u^2]\, du = \sqrt{\pi} \}$

$$E(k_y) = \sqrt{\pi}\, w_0\, E_0 \exp\left[-\left(\frac{k_y w_0}{2}\right)^2\right]$$

1.8

$$(\Delta x)^2 = \frac{\int_{-\infty}^{\infty} \frac{2x^2}{w^2} \exp\left[-\frac{2x^2}{w_0^2}\right] d\left(\frac{2^{1/2}x}{w_0}\right) \cdot \frac{w_0^3}{2^{3/2}}}{\int_{-\infty}^{\infty} \exp\left[\frac{-2x^2}{w_0^2}\right] d\left(\frac{2^{1/2}x}{w_0}\right) \cdot \frac{w_0}{\sqrt{2}}} = \frac{w_0^2}{2} \frac{\int_{-\infty}^{\infty} u^2 \exp[-u^2]\, du}{\int_{-\infty}^{\infty} \exp[-u^2]}$$

Now $\int_0^{\infty} x^m \exp[-ax^2 dx] = \frac{\Gamma[(m+1)/2]}{2a^{(m+1)/2}}$

with $\Gamma(1/2) = \sqrt{\pi}$; $\Gamma(3/2)) = \sqrt{\pi}/2$; $\Gamma(5/2) = 3\sqrt{\pi}/4$; Thus

$\Delta x^2 = \frac{w_0^2}{2} \cdot \frac{\Gamma(3/2)}{\Gamma(1/2)} = \frac{w_0^2}{4}$, or $\boxed{\Delta x = \frac{w_0}{2}}$

$$(\Delta k_x)^2 = \frac{\left\{\frac{w_0 k_x}{2}\right\}^2 \int_{-\infty}^{\infty} \exp -\left[\frac{w_0^2 k_x^2}{2}\right] d\left[\frac{w_0 k_x}{\sqrt{2}}\right]\cdot\frac{2^{3/2}}{w_0^3}}{\int_{-\infty}^{\infty} \exp -\left[\frac{w_0^2 k_x^2}{2}\right] d\left[\frac{w_0 k_x}{\sqrt{2}}\right]\cdot\frac{2^{1/2}}{w_0}} = \frac{2}{w_0^2}\,\frac{\int_{-\infty}^{\infty} u^2 \exp[-u^2]\,du}{\int_{-\infty}^{\infty} \exp[-u^2]\,du}$$

The product leads to an equality in the uncertainty relations

$$(\Delta k_x)^2 = \frac{2}{w_0^2}\,\frac{\Gamma(3/2)}{\Gamma(1/2)} = \frac{1}{w_0^2} \qquad \boxed{\Delta k_x \cdot \Delta x = \frac{1}{2}};$$

The same procedure is used for the $TEM_{1,0}$ field distribution.

(b) For $TEM_{1,0}$ $$(\Delta x)^2 = \frac{w_0^2}{2}\,\frac{\int_{-\infty}^{\infty} u^4 \exp[-u^2]\,du}{\int_{-\infty}^{\infty} u^2 \exp[-u^2]\,du} = \frac{w_0^2}{2}\,\frac{\Gamma(5/2)}{\Gamma(3/2)} = \frac{3}{2}\,\frac{w_0^2}{2}$$

$$E(k_x) = E_0 \int_{-\infty}^{\infty} \frac{\sqrt{2}x}{w_0} \exp\left\{-\left[\frac{x}{w}\right]^2\right\} \exp[-jk_x x]\,dx; \text{ Let } u = \left[\frac{x}{w_0} + j\,\frac{k_x w_0}{z}\right]$$

and thus $\frac{\sqrt{2}x}{w_0} = \sqrt{2}u - j\,\frac{k_x w_0}{\sqrt{2}}$

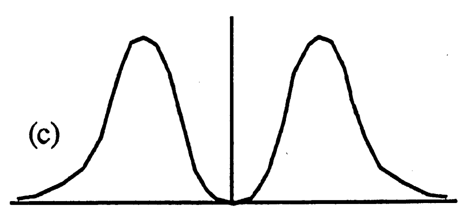
$$E(k_x) = w_0 \exp-\left[\frac{k_x w_0}{2}\right]^2 \left\{ \int_{-\infty}^{\infty} \sqrt{2}u \exp[-u^2]du - j\,\frac{k_x w_0}{\sqrt{2}} \int_{-\infty}^{\infty} \exp[-u^2]du \right\}$$

$$E(k_x) = \left[\frac{jk_x w_0}{\sqrt{2}}\right] \exp -\left[\frac{k_x w_0}{2}\right]^2 \sqrt{\pi}$$

$$(\Delta k_x)^2 = \frac{2}{w_0^2}\,\frac{\int u^4 \exp[-u^2]\,du}{\int u^2 \exp[-u^2]\,du} = \frac{2}{w_0^2}\,\frac{\Gamma(5/2)}{\Gamma(3/2)}$$

Hence $\boxed{\Delta k_x \cdot \Delta x \cdot = 3/2}$ Part (b)

(c)



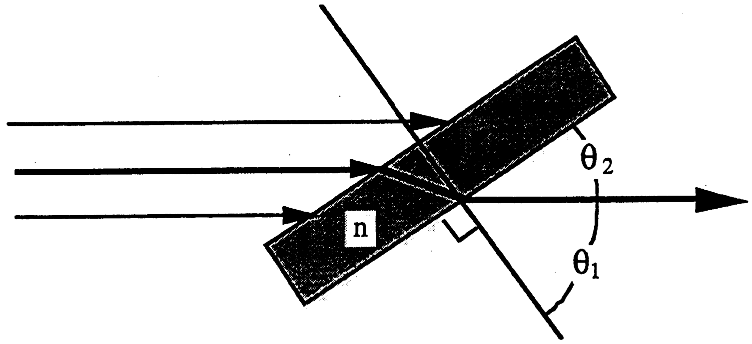
1.9

$E_T = E_0\{e^{-j[kz+\phi]}\} + e^{+j[kz]}\}$; The first term is the incidentwave, second is the reflected.

$$E_T = 2j\,E_0\,e^{+j(\phi/2)}\left\{\frac{\exp\{+j[kz+(\phi/2)]\} - \exp\{-j[kz+(\phi/2)]\}}{2j}\right\};$$

$$\therefore\ E_T \cdot E_T^* = 4E_0^2 \sin^2\left[kz + \frac{\phi}{2}\right]$$

1.10



$\tan \theta_1 = n = 1.43$

$\therefore \theta_1 = 55°; \theta_2 = 35°$

2.1

Consider the geometry shown below: $r_2 = r_1$ ∴ $A = 1$ and $B = 0$ since the planes 1 and 2 are spaced an infinitesimal distance apart. Now use Snell's Law:

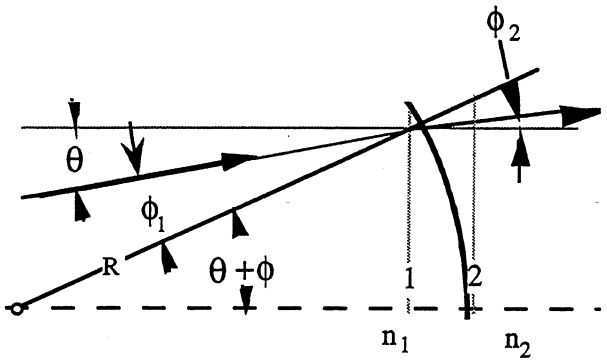
$$\frac{\omega}{c} n_1 \sin\phi_1 = \frac{\omega}{c} n_2 \sin\phi_2; \text{ For small angles: } n_1\phi_1 = n_2\phi_2$$

and $r_2' = (\theta + \phi_1) - \phi_2 = \theta + \phi_1\,[1 - (n_1/n_2)]$;

Since $r_1/R = \sin(\theta + \phi_1) \approx \theta + \phi_1$

$$\therefore\ \phi_1 = r_1/R - r_1';\ \text{Hence, } r_2' = r_1' + [1 - \tfrac{n_1}{n_2}]\,[r_1/R - r_1'] = \left\{\left[1 - \frac{n_1}{n_2}\right]\frac{1}{R}\right\} r_1 + \frac{n_1}{n_2} r_1';$$

Thus, the ray matrix is as shown on the side of the diagram.



$$\begin{bmatrix} 1 & 0 \\ [1 - n_1/n_2]\frac{1}{R} & \frac{n_2}{n_1} \end{bmatrix}$$

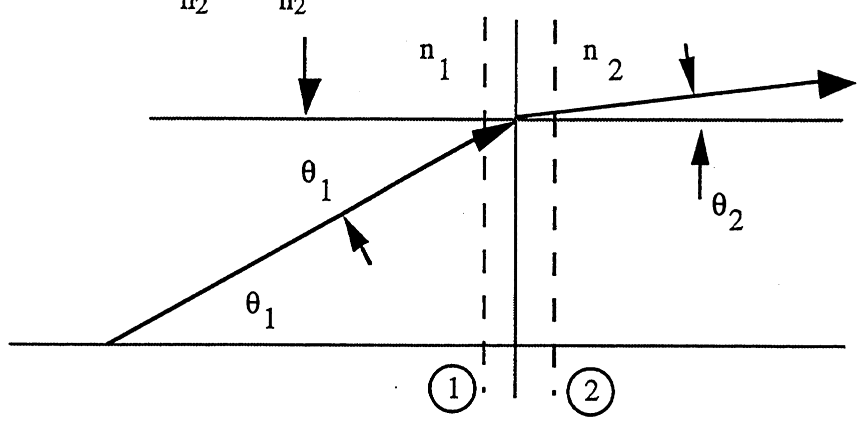
2.2

Consider the geometry shown below:

Since the distance between the two planes → 0, then $r_2 = r_1$ and $A = 1, B = 0$

Use Snell's law for the interface: $\frac{\omega}{c} n_1 \sin\theta_1 = \frac{\omega}{c} \sin\theta_2$ and now use the small angle approximation:

$$\therefore\ \theta_2 = r_2' = \frac{n_1}{n_2}\theta_1 = \frac{n_1}{n_2} r_1' \quad \text{or:}$$



$$T = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

Notice that AD–BC ≠1 because of the different indices of refraction.

2.3

Consider the diagram shown below and apply the results of problem 2.2 to reduce the number of matrices to be derived. The matrix for the last interface comes first with the interchange of n_1 and n_2, followed by the matrix for the length d, and finally by the matrix for the first interface.

$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_2}{n_1} \end{bmatrix} \cdot \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_2}{n_1} \end{bmatrix} \begin{bmatrix} 1 & \frac{n_1}{n_2} d \\ 0 & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{n_1}{n_2} d \\ 0 & 1 \end{bmatrix}$$

Notice that the determinant, AD –BC=1 even though the optical path does include a different index.

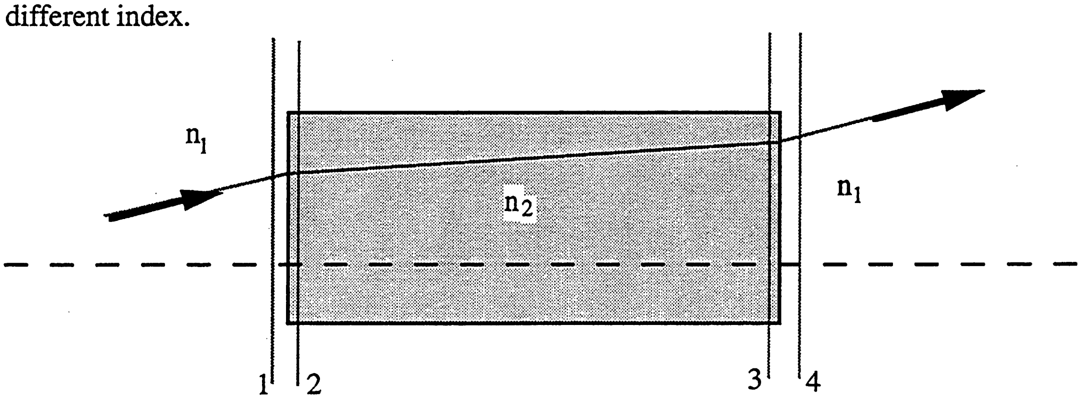


Figure for problem 2.3

2.4

Combine problems 2.1 and 2.3

$$T = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_2}{n_1} \end{bmatrix} \cdot \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R} & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & \frac{n_2}{n_1} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R} & \frac{n_1}{n_2} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 + \left(1 - \frac{n_1}{n_2}\right)\frac{d}{R} & \frac{n_1}{n_2} d \\ \left(\frac{n_2}{n_1} - 1\right)\frac{1}{R} & 1 \end{bmatrix}$$ Note: AD – BC = 1

2.5

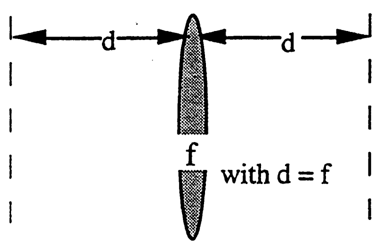
For the purpose of this solution, we will use "t" rather than l ("el") to avoid confusing it with one (1). We include the GRIN-to-air exit interface as the first matrix, then

Eq.2.12.11 for the GRIN lens, and the last matrix represent the air-to-entrance interface where the results of Prob. 2.2 has been used.

$$T = \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix} \cdot \begin{bmatrix} \cos(d/t) & t\sin(d/t) \\ -\frac{1}{t}\sin(d/t) & \cos(d/t) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n_0} \end{bmatrix}$$

$$T = \begin{bmatrix} \cos(d/t) & \frac{t}{n_0}\sin(d/t) \\ -\frac{n_0}{t}\sin(d/t) & \cos(d/t) \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & \frac{t}{n_0} \\ -\frac{n_0}{t} & 0 \end{bmatrix} \text{for } d = \pi t/2$$

Now consider the following simple lens centered between the input and output planes with d=f and use Eq. 2.3.2 to represent the two components so as to minimize the chore of matrix multiplication.



$$T = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & d \\ -\frac{1}{f} & \left(1 - \frac{d}{f}\right) \end{bmatrix}_{d=f} = \begin{bmatrix} 0 & f \\ -\frac{1}{f} & 0 \end{bmatrix}$$

Thus the focal length is $f = t/n_0$

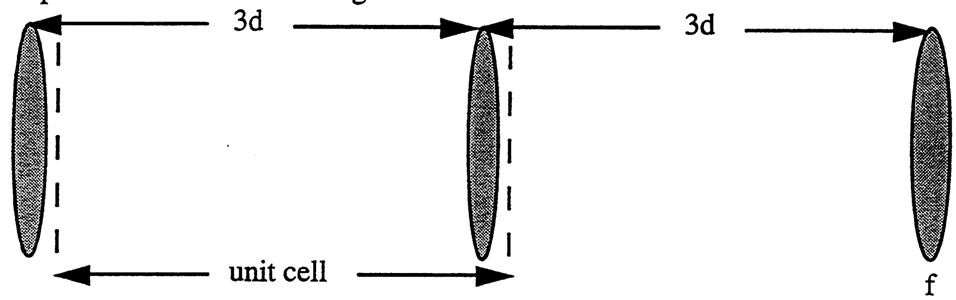
2.6

The only way to have difficulties with this problem is to arrange the matrices in wrong order. Let's evaluate Eq. 2.3.2 for the negative lens + distance d combination (i.e. change the sign on f), multiply by the matrix for the postive lens, and evaluate for d=f.

$$T = \begin{bmatrix} 1 & d \\ +\frac{1}{f} & \left(1 + \frac{d}{f}\right) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}_{d=f} = \begin{bmatrix} 0 & f \\ -\frac{1}{f} & 2 \end{bmatrix}$$

2.7

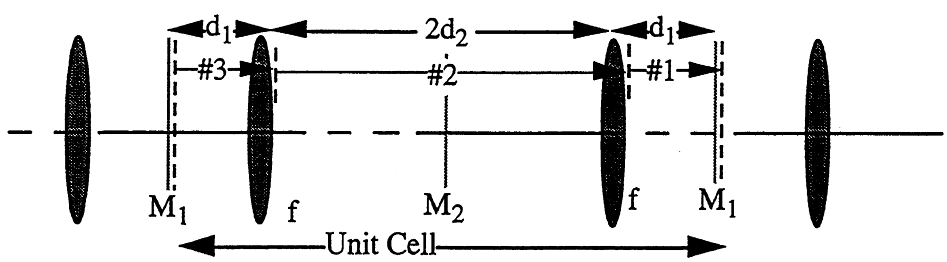
The ray matrix for a flat mirror is A=D=1 and B=C=0 which is the limit for that of a curved mirror with $R \rightarrow \infty$. Hence one could insert three extra matrices in the unit cell and go through the excercise of matrix multiplication to prove the altermative of ignoring the flat mirrors, as being just re-directors of the optic axis, and measuring distances along that line. This viewpoint leads to the following:



$$T = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3d \\ -\frac{1}{f} & \left(1-\frac{3d}{f}\right) \end{bmatrix};$$

$$S = \frac{A+D+2}{4} = 1 - \frac{3d}{4f} \qquad \therefore \boxed{0 \leq 1 - \frac{3d}{4f} \leq 1}$$

2.8



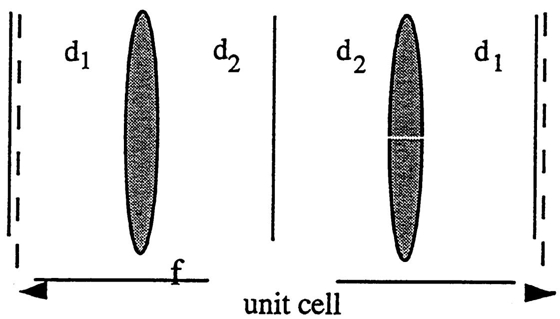
Thus the matrices appear in the order indicated by"#".

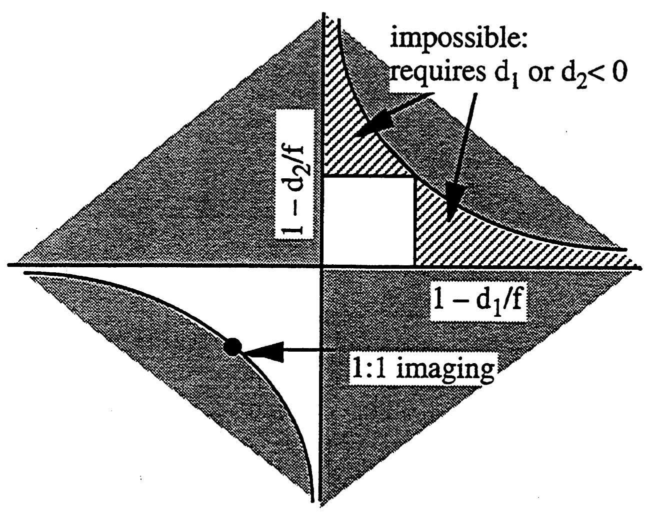
$$T = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2d_2 \\ -\frac{1}{f} & \left(1-\frac{2d_2}{f}\right) \end{bmatrix} \cdot \begin{bmatrix} 1 & d_1 \\ -\frac{1}{f} & \left(1-\frac{d_1}{f}\right) \end{bmatrix}$$

$$T = \begin{bmatrix} 1 - \frac{2d_1}{f} - \frac{2d_2}{f} + \frac{2d_1d_2}{f^2} & \left(1 - \frac{d_1}{f}\right)\left(2d_1 + 2d_2 - \frac{2d_1d_2}{f}\right) \\ -\frac{2}{f}\left(1 - \frac{d_2}{f}\right) & 1 - \frac{2d_1}{f} - \frac{2d_2}{f} + \frac{2d_1d_2}{f^2} \end{bmatrix}$$

Note: $A = D$ as it should since there is a plane of symmetry (M_2).

Stability: $0 \le 1 - \frac{d_1}{f} - \frac{d_2}{f} + \frac{d_1d_2}{f^2} < 1$ or: $0 \le \left(1 - \frac{d_1}{f}\right)\left(1 - \frac{d_2}{f}\right) \le 1$

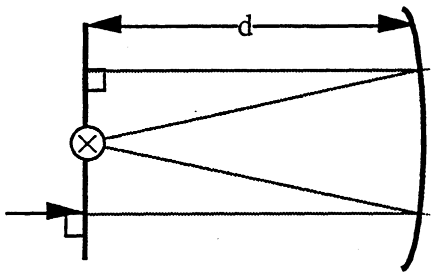


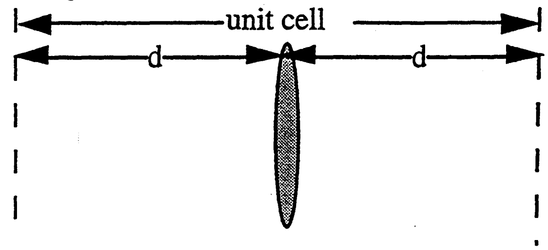


Figures for problem 2.8

2.9

$(d/R) = (1/2)$; $\therefore\ d = f$; 4–Round trips as the figure below indicates





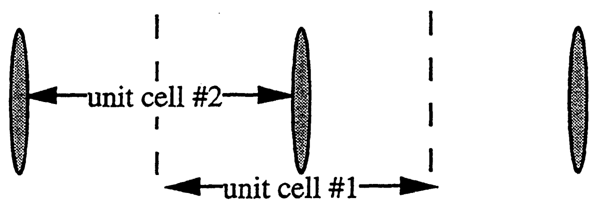
One can do this the hard way using the figure at the right: $T = \begin{bmatrix} 1 & 2d \\ \frac{1}{f} & \left(1 - \frac{2d}{f}\right) \end{bmatrix}$

$$\cos\theta = \frac{A+D}{2} = 0;\ \therefore \theta = \pi/2;\ \alpha = \tan^{-1}\left\{ \frac{a\left[1 - \left(\frac{A+D}{2}\right)^2\right]^{1/2}}{a\left(\frac{A-D}{2}\right) + Bm} \right\}$$

For an input slope $m = 0$; $\frac{A+D}{2} = 0$; $\frac{A-D}{2} = \frac{d}{f} = 1$; $\alpha = \tan^{-1}(1) = \pi/4$;

$r_{initial} = r_{max} \sin\alpha = r_{max}/\sqrt{2}$; $r_{max} = -\sqrt{2}\, r_0 \sin\left[s\frac{\pi}{2} + \frac{\pi}{4} \right]$

2.10



$$T_1 = \begin{bmatrix} 1 - \frac{d}{f} & d\left(2 - \frac{d}{f}\right) \\ -\frac{1}{f} & \left(1 - \frac{d}{f}\right) \end{bmatrix} \qquad T_2 = \begin{bmatrix} 1 - \frac{2d}{f} & 2d \\ -\frac{1}{f} & 1 \end{bmatrix}$$

$$F_{1,2} = \left[\frac{A+D}{2}\right] \pm \left\{ \left[\frac{A+D}{2}\right]^2 - 1 \right\}^{1/2};$$

$\left[\frac{A+D}{2}\right] = 1 - \frac{d}{f}$ for both cases;

Now $d/R = 1.01$; $d/f = 2.02$ $\therefore \left[\frac{A+D}{2}\right] = -1.02$;

$F_1 = -0.8190;\ F_2 = -1.2210;$

Let $r = r_a(F_1)^s + r_b(F_2)^s;$

Unit Cell #1: $r_b = \dfrac{1}{F_1 - F_2}\{a(F_1 - A)\} = 0.5\times10^{-2};$

$r_a = \dfrac{1}{F_1 - F_2}\{a(F_2 - A)\} = 0.5\times10^{-2};$

Thus the position of the ray after s round-trips is:

$r_s = 0.5\times10^{-2}\{(0.819)^s + (-1.221)^s\};\ r_s > 1$ cm after $s = 15$.

Unit Cell #2: $r_b = 0.5525;\ r_a = -0.4525;$

$r_s = -0.4525\,(0.819)^s + 0.5525(-1.221)^s$

at $s = 6$; $r = 1.694$ cm $\leftarrow$ misses spherical mirror after 12 round-trips plus 1 more pass to the spherical mirror; $\therefore\ P_{out} = 1\ \mu W \times G^{13} = 1220$ watts!

2.11

The effective focal lengths are:

$f_x = \dfrac{R}{2}\cos\theta = \dfrac{\sqrt{3}}{4}R\ (\theta = 30°);$ $f_y = \dfrac{R}{2\cos\theta} = \dfrac{R}{\sqrt{3}};$

Stability: $0 < 1 - 4d/3f_{x,y} < 1;$

$d < \dfrac{R}{\sqrt{3}}$ or $\dfrac{4}{3}\dfrac{R}{\sqrt{3}} =$ 0.577 R or 0.7698 R;

$\therefore\ d < 0.577\,R$