

INSTRUCTOR'S MANUAL

to accompany

*Linear Algebra:
4th Edition*

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Contents

1	Vector Spaces	1
1.1	Introduction	1
1.2	Vector Spaces	1
1.3	Subspaces	1
1.4	Linear Combinations and Systems of Linear Equations	2
1.5	Linear Dependence and Linear Independence	2
1.6	Bases and Dimension	2
2	Linear Transformations and Matrices	4
2.1	Linear Transformations, Null Spaces, and Ranges	4
2.2	The Matrix Representation of a Linear Transformation	4
2.3	Composition of Linear Transformations and Matrix Multiplication	5
2.4	Invertibility and Isomorphisms	5
2.5	The Change of Coordinate Matrix	5
2.6	Dual Spaces	6
2.7	Homogeneous Linear Differential Equations with Constant Coefficients	6
3	Elementary Matrix Operations and Systems of Linear Equations	7
3.1	Elementary Matrix Operations and Elementary Matrices	7
3.2	The Rank of a Matrix and Matrix Inverses	7
3.3	Systems of Linear Equations—Theoretical Aspects	8
3.4	Systems of Linear Equations—Computational Aspects	8
4	Determinants	10
4.1	Determinants of Order 2	10
4.2	Determinants of Order n	10
4.3	Properties of Determinants	10
4.4	Summary—Important Facts about Determinants	11
4.5	A Characterization of the Determinant	11
5	Diagonalization	12
5.1	Eigenvalues and Eigenvectors	12
5.2	Diagonalizability	13
5.3	Matrix Limits and Markov Chains	13
5.4	Invariant Subspaces and the Cayley-Hamilton Theorem	14

6	Inner Product Spaces	15
6.1	Inner Products and Norms	15
6.2	The Gram-Schmidt Orthogonalization Process and Orthogonal Complements	15
6.3	The Adjoint of a Linear Operator	16
6.4	Normal and Self-Adjoint Operators	16
6.5	Unitary and Orthogonal Operators and Their Matrices	17
6.6	Orthogonal Projections and the Spectral Theorem	17
6.7	The Singular Value Decomposition and the Pseudoinverse	18
6.8	Bilinear and Quadratic Forms	19
6.10	Conditioning and the Rayleigh Quotient	19
6.11	The Geometry of Orthogonal Operators	19
7	Canonical Forms	20
7.1	Jordan Canonical Form I	20
7.2	Jordan Canonical Form II	20
7.3	The Minimal Polynomial	21
7.4	Rational Canonical Form	21

Vector Spaces

1.1 INTRODUCTION

2. (b) $x = (2, 4, 0) + t(-5, -10, 0)$ (d) $x = (-2, -1, 5) + t(5, 10, 2)$
 3. (b) $x = (3, -6, 7) + s(-5, 6, -11) + t(2, -3, -9)$
 (d) $x = (1, 1, 1) + s(4, 4, 4) + t(-7, 3, 1)$
 4. (0, 0)

1.2 VECTOR SPACES

2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
 4. (b) $\begin{pmatrix} 1 & -1 \\ 3 & -5 \\ 3 & 8 \end{pmatrix}$ (d) $\begin{pmatrix} 30 & -20 \\ -15 & 10 \\ -5 & -40 \end{pmatrix}$
 (f) $-x^3 + 7x^2 + 4$ (h) $3x^5 - 6x^3 + 12x + 6$
 5. $\begin{pmatrix} 8 & 3 & 1 \\ 3 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 9 & 1 & 4 \\ 3 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 17 & 4 & 5 \\ 6 & 0 & 0 \\ 4 & 1 & 0 \end{pmatrix}$
 16. Yes 18. No, (VS 1) fails. 19. No, (VS 8) fails.

1.3 SUBSPACES

2. (b) $\begin{pmatrix} 0 & 3 \\ 8 & 4 \\ -6 & 7 \end{pmatrix}$ (d) $\begin{pmatrix} 10 & 2 & -5 \\ 0 & -4 & 7 \\ -8 & 3 & 6 \end{pmatrix}$ (f) $\begin{pmatrix} -2 & 7 \\ 5 & 0 \\ 1 & 1 \\ 4 & -6 \end{pmatrix}$
 The trace is 12.
 (h) $\begin{pmatrix} -4 & 0 & 6 \\ 0 & 1 & -3 \\ 6 & -3 & 5 \end{pmatrix}$
 The trace is 2.
 8. (b) No (d) Yes (f) No
 9. $W_1 \cap W_3 = \{0\}$, $W_1 \cap W_4 = W_1$,
 $W_3 \cap W_4 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = -11a_3 \text{ and } a_2 = -3a_3\}$

1.4 LINEAR COMBINATIONS AND SYSTEMS OF LINEAR EQUATIONS

2. (b) $(-2, -4, -3)$
 (d) $\{x_3(-8, 3, 1, 0) + (-16, 9, 0, 2) : x_3 \in R\}$
 (f) $(3, 4, -2)$
3. (a) $(-2, 0, 3) = 4(1, 3, 0) - 3(2, 4, -1)$
 (b) $(1, 2, -3) = 5(-3, 2, 1) + 8(2, -1, -1)$
 (d) No
 (f) $(-2, 2, 2) = 4(1, 2, -1) + 2(-3, -3, 3)$
4. (a) $x^3 - 3x + 5 = 3(x^3 + 2x^2 - x + 1) - 2(x^3 + 3x^2 - 1)$
 (b) No
 (c) $-2x^3 - 11x^2 + 3x + 2 = 4(x^3 - 2x^2 + 3x - 1) - 3(2x^3 + x^2 + 3x - 2)$
 (d) $x^3 + x^2 + 2x + 13 = -2(2x^3 - 3x^2 + 4x + 1) + 5(x^3 - x^2 + 2x + 3)$
 (f) No
5. (b) No (d) Yes (f) No (h) No
11. The span of $\{x\}$ is $\{0\}$ if $x = 0$ and is the line through the origin of R^3 in the direction of x if $x \neq 0$.
17. if W is finite

1.5 LINEAR DEPENDENCE AND LINEAR INDEPENDENCE

2. (b) Linearly independent (d) Linearly dependent
 (f) Linearly independent (h) Linearly independent
 (j) Linearly dependent
10. $(1, 0, 0), (0, 1, 0), (1, 1, 0)$

1.6 BASES AND DIMENSION

2. (b) Not a basis (d) Basis
3. (b) Basis (d) Basis
4. No, $\dim(P_3(R)) = 4$. 5. No, $\dim(R^3) = 3$.
8. $\{u_1, u_3, u_5, u_7\}$
10. (b) $12 - 3x$ (d) $-x^3 + 2x^2 + 4x - 5$
14. $\{(0, 1, 0, 0, 0), (0, 0, 0, 0, 1), (1, 0, 1, 0, 0), (1, 0, 0, 1, 0)\}$ and
 $\{(-1, 0, 0, 0, 1), (0, 1, 1, 1, 0)\}$; $\dim(W_1) = 4$ and $\dim(W_2) = 2$.
16. $\dim(W) = \frac{1}{2}n(n+1)$