CHAPTER 2

LINEAR PROGRAMMING MODELS: GRAPHICAL AND COMPUTER METHODS

SOLUTIONS TO DISCUSSION QUESTIONS

2-1. The requirements for an LP problem are listed in Section 2.2. It is also assumed that conditions of *certainty* exist; that is, coefficients in the objective function and constraints are known with certainty and do not change during the period being studied. Another basic assumption that mathematically sophisticated students should be made aware of is *proportionality* in the objective function and constraints. For example, if one product uses 5 hours of a machine resource, then making 10 of that product uses 50 hours of machine time.

LP also assumes *additivity*. This means that the total of all activities equals the sum of each individual activity. For example, if the objective function is to maximize Profit = $6X_1 + 4X_2$, and if $X_1 = X_2 = 1$, the profit contributions of 6 and 4 must add up to produce a sum of 10.

2-2. If we consider the feasible region of an LP problem to be continuous (i.e., we accept non-integer solutions as valid), there will be an infinite number of feasible combinations of decision variable values (unless of course, only a single solution satisfies all the constraints). In most cases, only one of these feasible solutions yields the *optimal* solution.

2-3. A problem can have alternative optimal solutions if the level profit or level cost line runs parallel to one of the problem's binding constraints (refer to Section 2.6 in the chapter).

2-4. A problem can be unbounded if one or more constraints are missing, such that the objective value can be made infinitely larger or smaller without violating any constraints (refer to Section 2.6 in the chapter).

2-5. This question involves the student using a little originality to develop his or her own LP constraints that fit the three conditions of (1) unbounded solution, (2) infeasibility, and (3) redundant constraints. These conditions are discussed in Section 2.6, but each student's graphical displays should be different.

2-6. The manager's statement indeed has merit if he/she understood the deterministic nature of LP input data. LP assumes that data pertaining to demand, supply, materials, costs, and resources are known with certainty and are constant during the time period being analyzed. If the firm operates in a very unstable environment (for example, prices and availability of raw materials change daily or even hourly) the LP model's results may be too sensitive and volatile to be trusted. The application of sensitivity analysis might, however, be useful to determine whether LP would still be a good approximating tool in decision making in this environment.

2-7. The objective function is not linear because it contains the product of X1 and X2, making it a second-degree term. The first, second, and fourth constraints are okay as is. The third and fifth constraints are nonlinear because they contain terms to the second degree and one-half degree, respectively.

2-8. The computer is valuable in (1) solving LP problems quickly and accurately; (2) solving large problems that might take days or months by hand; (3) performing extensive sensitivity analysis automatically; and (4) allowing a manager to try several ideas, models, or data sets.

2-9. Most managers probably have Excel (or another spreadsheet software) available in their companies, and use it regularly as part of their regular activities. As such, they are likely to be familiar with its usage. In addition, a lot of the data (such as parameter values) required for developing LP models is likely to be available either in some Excel file or in a database file (such as Microsoft Access) from which it is easy to import to Excel. For these reasons, a manager may find the ability to use Excel to set up and solve LP problems very beneficial.

2-10. The three components are: target cell (objective function), changing cells (decision variables), and constraints.

2-11. Slack is defined as the RHS minus the LHS value for a ≤ constraint. It may be interpreted as the amount of unused resource described by the constraint. Surplus is defined as the LHS minus the RHS value for a ≥ constraint. It may be interpreted as the amount of over satisfaction of the constraint.

2-12. An unbounded solution occurs when the objective of an LP problem can go to infinity (negative infinity for a minimization problem) while satisfying all constraints. Solver indicates an unbounded solution by the message “The Set Cell values do not converge”.