Chapter 2: Sets

EXCURSION EXERCISES, SECTION 2.1

1. a. Erica. Since B, C, D, and F were assigned membership value 0, Erica is certain they don't belong to the set good grade.

   b. Larry. A, B, C, and D were assigned membership value 1 so he is certain they belong to the set good grade.

   c. Answers will vary.

2. a. Locate $x = 15$ and note (15, 0) represents a membership value of 0.

   b. 0.75. Locate $x = 50$ and note (50, 0.75) represents a membership value of 0.75.

   c. 1. Locate $x = 65$ and note (65, 1) represents a membership value of 1.

   d. 30 since 30 is paired with 0.25 in the ordered pair (30, 0.25).

   e. (40, 0.5) since 40 is paired with 0.5 in the ordered pair (40, 0.5).

3. a. 0 since (2, 0) is on the graph.

   b. 0.5 since (3.5, 0.5) is on the graph.

   c. 0 since (7, 0) is on the graph.

   d. (3.5, 0.5) and (4.5, 0.5) since 0.5 is the membership value.

4. a. 0.5 since (40, 0.5) is on the WARM graph.

   b. 1 since (50, 1) is on the WARM graph.

   c. (40, 0.5) and (60, 0.5).

5. Answers will vary.

EXERCISE SET 2.1

1. {penny, nickel, dime, quarter}

2. {January, February, May, July}

3. {Mercury, Mars}

4. {Bashful, Dopey, Doc, Grumpy, Happy, Sleepy, Sneezy}

5. {George W. Bush, Barack Obama}

6. {April, June, September, November}

7. The negative integers greater than –6 are –5, –4, –3, –2, –1. Using the roster method, write the set as {–5, –4, –3, –2, –1}.

8. {0, 1, 2, 3, 4, 5, 6, 7}

9. Adding 4 to each side of the equation produces $x = 7$. {7} is the solution set.

10. Solving:
$$\begin{aligned} 2x - 1 &= -11 \\ 2x &= -10 \\ x &= -5 \end{aligned}$$
So {–5} is the solution set.

11. Solving:
$$\begin{aligned} x + 4 &= 1 \\ x &= -3 \end{aligned}$$
But –3 is not a counting number so the solution set is empty, Ø.

12. Solving:
$$\begin{aligned} x - 1 &< 4 \\ x &< 5 \end{aligned}$$
The set of whole numbers less than 5 is {0, 1, 2, 3, 4}.

In exercises 13 – 20, only one possible answer is given. Your answers may vary from the given answers.

13. {a, e, i, o, u}

14. {1, 2, 3, 4}

15. the set of days of the week that begin with the letter T

16. the set of the signs of the zodiac that begin with the letter L

17. the set consisting of the two planets in our solar system that are closest to the sun

18. the set of U.S. coins with a value less than 25¢

19. the set of single digit natural numbers

20. the set of even natural numbers less than 9

21. the set of natural numbers less than or equal to 7

22. the set of whole numbers less than 5

23. the set of odd natural numbers less than 10

24. the set of negative integers greater than –5

25. Because b is an element of the given set, the statement is true.

26. True; 0 is not a natural number.

27. False; although b $\in$ {a, b, c}, {b} $\notin$ {a, b, c}.

28. True; both sets have 3 elements.

29. False; {0} contains 1 element but Ø contains no elements.

30. False; “large” is a relative term.

31. False; “good” is subjective.

32. True; we can determine whether any number is in the set.

33. True; the set of natural numbers is equal to the set of whole numbers greater than 0

34. False; the empty set has no elements.

35. True; both sets contain the same elements

36. True; both sets have 3 elements

37. $\{x \mid x \in N$ and $x < 13\}$

38. $\{x \mid x$ is a multiple of 5 that ends with a 5, and x is between 40 and 80$\}$

39. $\{x \mid x$ is a multiple of 5 and $4 < x < 16\}$

40. $\{x \mid x$ is a positive square number less than or equal to 81$\}$

41. $\{x \mid x$ is the name of a month that has 31 days$\}$

42. $\{x \mid x$ is the state with a name that has exactly four letters$\}$

43. $\{x \mid x$ is the name of a U.S. state that begins with the letter A$\}$

44. $\{x \mid x$ is a country that shares a boundary with the United States$\}$

45. $\{x \mid x$ is a season that starts with the letter s$\}$

46. $\{x \mid x \in N$ and $1900 \le x \le 1999\}$

47. {February, April, June, September, November}

48. the set of natural numbers, or $\{x|x$Î $N\}$

49. a. {2013, 2014, 2015}
    b. {2008, 2009}
    c. {2010, 2011, 2012}

50. a. {2011, 2012}
    b. {2010, 2013, 2014}
    c. {2007, 2008, 2009}

51. a. {May, June, July, August}
    b. {March, April, September}
    c. {January, November}

52. a. {2010, 2014, 2015}
    b. {2015}
    c. {2006}

53. 11 since set A has 11 elements

54. 8

55. The cardinality of the empty set is 0.

56. 50

57. 4 since 4 states border Minnesota.

58. 13, since the U.S. flag has 13 stripes.

59. 16. There are 16 baseball teams in the league.

60. 32. There are 32 pieces.

61. 121

62. 101

63. Neither. The sets are not equal, nor do they have the same number of elements.

64. Neither. The first set has 9 elements and the second set has 10 elements, so the sets are not equal and are not equivalent.

65. Both.

66. Neither. The sets are not equal, nor do they have the same number of elements.

67. Equivalent. The sets are not equal but each has 3 elements.

68. Equivalent. Each set has 4 elements.

69. Equivalent. Each set has 2 elements.

70. Neither. The first set has 0 elements and the second set has 1 element.

71. Not well-defined since the word “good” is not precise.

72. Well-defined since the populations can be determined.

73. Not well-defined since “tall” is not precise.

74. Well-defined.

75. Well-defined.

76. Well-defined.

77. Well-defined.

78. Well-defined.

79. Not well-defined; “small” is not precise.

80. Not well-defined; “great” is not precise.

81. Not well-defined; “best” is not precise.

82. Not well-defined; “fine” is not precise.

83. Identify the natural numbers less than 5, which are 1, 2, 3, and 4.
Replace x with those numbers and simplify.
When $x = 1$,
$3(1)^2 - 1 = 3(1) - 1 = 3 - 1 = 2.$
When $x = 2$,
$3(2)^2 - 1 = 3(4) - 1 = 12 - 1 = 11.$
When $x = 3$,
$3(3)^2 - 1 = 3(9) - 1 = 27 - 1 = 26.$
When $x = 4$,
$3(4)^2 - 1 = 3(16) - 1 = 48 - 1 = 47.$
Therefore, $D = \{2, 11, 26, 47\}$.

84. Identify the natural numbers less than 7, which are 1, 2, 3, 4, 5, and 6.
Replace x with those numbers and simplify.
When $x = 1$,
$2(1)^2 - 1 = 2(1) - 1 = 2 - 1 = 1.$
When $x = 2$,
$2(2)^2 - 2 = 2(4) - 2 = 8 - 2 = 6.$
When $x = 3$,
$2(3)^2 - 3 = 2(9) - 3 = 18 - 3 = 15.$
When $x = 4$,
$2(4)^2 - 4 = 2(16) - 4 = 32 - 4 = 28.$
When $x = 5$,
$2(5)^2 - 5 = 2(25) - 5 = 50 - 5 = 45.$
When $x = 6$,
$2(6)^2 - 6 = 2(36) - 6 = 72 - 6 = 66.$
Therefore, $E = \{1, 6, 15, 28, 45, 66\}$.

85. Identify the natural numbers greater than or equal to 2, which are 2, 3, 4, 5, 6, 7, …
Replace x with those numbers and simplify.
When $x = 2$,
$(-1)^2(2)^3 = (1)(8) = 8.$
When $x = 3$,
$(-1)^3(3)^3 = (-1)(27) = -27.$
When $x = 4$,
$(-1)^4(4)^3 = (1)(64) = 64.$
When $x = 5$,
$(-1)^5(5)^3 = (-1)(125) = -125.$
When $x = 6$,
$(-1)^6(6)^3 = (1)(216) = 216.$
When $x = 7$,
$(-1)^7(7)^3 = (-1)(343) = -343.$
Therefore,
$F = \{8, -27, 64, -125, 216, -343, \ldots\}$.

86. Identify the natural numbers less than 7, which are 1, 2, 3, 4, 5, and 6.
Replace x with those numbers and simplify.
When $x = 1$,
$(-1)^1\left(\frac{2}{1}\right) = (-1)(2) = -2.$
When $x = 2$,
$(-1)^2\left(\frac{2}{2}\right) = (1)(1) = 1.$
When $x = 3$,
$(-1)^3\left(\frac{2}{3}\right) = (-1)\left(\frac{2}{3}\right) = -\frac{2}{3}.$
When $x = 4$,
$(-1)^4\left(\frac{2}{4}\right) = (1)\left(\frac{1}{2}\right) = \frac{1}{2}.$
When $x = 5$,
$(-1)^5\left(\frac{2}{5}\right) = (-1)\left(\frac{2}{5}\right) = -\frac{2}{5}.$
When $x = 6$,
$(-1)^6\left(\frac{2}{6}\right) = (1)\left(\frac{1}{3}\right) = \frac{1}{3}.$
Therefore, $G = \left\{-2, 1, -\frac{2}{3}, \frac{1}{2}, -\frac{2}{5}, \frac{1}{3}\right\}$.

87. $A = B$. Replacing n with whole numbers, starting with 0, $A = \{1, 3, 5, ...\}$. Replacing n with natural numbers, starting with one, $B = \{1, 3, 5, ...\}$.

88. $A = B$. In set A, when $n = 1$,

$$16\left(\frac{1}{2}\right)^{1-1} = 16\left(\frac{1}{2}\right)^{0} = 16(1) = 16.$$

When $n = 2$,

$$16\left(\frac{1}{2}\right)^{2-1} = 16\left(\frac{1}{2}\right) = 8.$$

When $n = 3$,

$$16\left(\frac{1}{2}\right)^{3-1} = 16\left(\frac{1}{2}\right)^{2} = 16\left(\frac{1}{2}\right) = 4.$$

In set B, when $n = 0$, $16\left(\frac{1}{2}\right)^{0} = 16$.

When $n = 1$, $16\left(\frac{1}{2}\right)^{1} = 8$.

When $n = 2$, $16\left(\frac{1}{2}\right)^{2} = 4$. Both sets contain $\{16, 8, 4, ...\}$.

89. $A \neq B$.

$$A = \{2(1)-1,\ 2(2)-1,\ 2(3)-1,\ ...\}$$
$$= \{1,\ 3,\ 5,\ ...\}$$
$$B = \left\{\frac{1(1+1)}{2}, \frac{2(2+1)}{2}, \frac{3(3+1)}{2},\ ...\right\}$$
$$= \{1,\ 3,\ 6,\ ...\}$$

90. $A = B$.

$$A = \{3(0)+1,\ 3(1)+1,\ 3(2)+1,\ ...\}$$
$$= \{1,\ 4,\ 7,\ ...\}$$
$$B = \{3(1)-2,\ 3(2)-2,\ 3(3)-2,\ ...\}$$
$$= \{1,\ 4,\ 7,\ ...\}$$

EXCURSION EXERCISES, SECTION 2.2

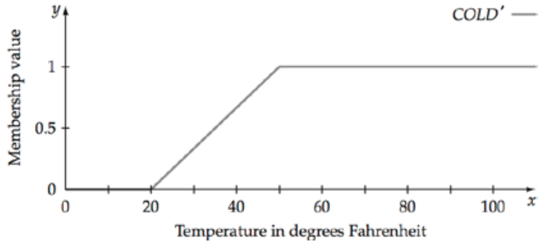
1. Yes, because the membership value of each element of J is less than or equal to its membership value in set K. $(0.3 \le 0.4,\ 0.6 \le 0.6,\ 0.5 \le 0.8,\ 0.1 \le 1)$.

2. Yes, because the graph of the fuzzy set ADOLESCENT is always below or at the same height as the graph of the fuzzy set YOUNG.

3. G' = {(A, 1 – 1), (B, 1 – 0.7), (C, 1 – 0.4), (D, 1 – 0.1), (F, 1 – 0)}
= {(A, 0), (B, 0.3), (C, 0.6), (D, 0.9), (F, 1)}

4. C' = {(Ferrari, 1 – 0.9),
(Ford Mustang, 1 – 0.6),
(Dodge Neon, 1 – 0.5),
(Hummer, 1 – 0.7)}
={(Ferrari, 0.1), (Ford Mustang, 0.4),
(Dodge Neon, 0.5), (Hummer, 0.3)}

5. The dashed line is COLD. The solid line is COLD’.



EXERCISE SET 2.2

1. The complement of {2, 4, 6, 7} contains elements in U but not in the set: {0, 1, 3, 5, 8}.

2. $\{3, 6\}' = \{0, 1, 2, 4, 5, 7, 8\}$

3. $\varnothing' = U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

4. $\{4, 5, 6, 7, 8\}' = \{0, 1, 2, 3\}$

5. $\{x \mid x \in W \text{ and } x < 4\} = \{0,\ 1,\ 2,\ 3\}$

$\{x \mid x \in W \text{ and } x < 4\}' = \{4,\ 5,\ 6,\ 7,\ 8\}$

6. $\{x \mid x \in N \text{ and } x < 5\} = \{1,\ 2,\ 3,\ 4\}$

$\{x \mid x \in N \text{ and } x < 5\}' = \{0,\ 5,\ 6,\ 7,\ 8\}$

7. The set of odd natural numbers less than 8 = {1, 3, 5, 7}.

$\{1,\ 3,\ 5,\ 7\}' = \{0,\ 2,\ 4,\ 6,\ 8\}$

8. The set of even natural numbers less than 6 = {2, 4}.

$\{2,\ 4\}' = \{0,\ 1,\ 3,\ 5,\ 6,\ 7,\ 8\}$

9. $\{x \mid x \in I \text{ and } -3 \le x \le 7\}$
$= \{-3, -2, -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$

Then $\{-2,\ 0,\ 2,\ 4,\ 6,\ 7\}' = \{-3, -1,\ 1,\ 3,\ 5\}$

10. $\{x \mid x \in I \text{ and } -3 \le x \le 7\}$
$= \{-3, -2, -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$

Then $\{-3,\ 1,\ 4,\ 5,\ 6\}' = \{-2, -1,\ 0,\ 2,\ 3,\ 7\}$

11. $\{x \mid x \in I \text{ and } -3 \le x \le 7\}$
$= \{-3, -2, -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$

$\{x \mid x \in I \text{ and } -2 \le x < 3\} = \{-2, -1,\ 0,\ 1,\ 2\}$

Then

$\{x \mid x \in I \text{ and } -2 \le x < 3\}' = \{-3,\ 3,\ 4,\ 5,\ 6,\ 7\}$

12. $\{x \mid x \in I \text{ and } -3 \le x \le 7\}$
 $= \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$
 $\{x \mid x \in W \text{ and } x < 5\} = \{0, 1, 2, 3, 4\}$
 Then
 $\{x \mid x \in W \text{ and } x < 5\}' = \{-3, -2, -1, 5, 6, 7\}$

13. {a, b, c, d} $\subseteq$ {a, b, c, d, e, f, g} since all elements of the first set are contained in the second set.

14. {3, 5, 7} $\not\subseteq$ {3, 4, 5, 6}. 7 is not an element of the second set.

15. {big, small, little} $\not\subseteq$ {large, petite, short}

16. {red, white, blue} $\subseteq$ {the colors in the American flag}

17. I $\subseteq$ Q

18. $\not\subseteq$, since the set of real numbers is not a subset of the set of integers. Real numbers such as $\frac{1}{2}$ and $\sqrt{2}$ are not integers.

19. $\subseteq$ since the empty set is a subset of every set.

20. $\not\subseteq$since not all sandwiches are hamburgers.

21. $\subseteq$ since every element of the first set is an element of the second set.

22. $\not\subseteq$ since the set of rational numbers less than 10 includes numbers that are not integers.

23. True; every element of *F* is an element of *D*.

24. False; *r* and *s* are not elements of *F*.

25. True; $F \neq D$.

26. False; *q* and *s* are not elements of *F*.

27. True; *s* is an element of *E* and $G \neq E$.

28. False; *q* is not an element of *D*.

29. $G' = \{p, q, r, t\}$. Since *q* is not an element of *D*, the statement is false.

30. $F' = \{q, r, s\} \neq E$ so the statement is false.

31. True; the empty set is a subset of every set.

32. False; Ø = Ø so cannot be a proper subset of itself.

33. True; $D' = \{q\}$ and $D' \neq E$.

34. False; *E* does not contain sets, only letters.

35. False; *D* does not contain sets.

36. True; *s* is not an element of *F*.

37. False; *D* has 4 elements, $2^4 = 16$ subsets and $2^4 - 1 = 15$ proper subsets.

38. True; *U* has 5 elements, $2^5 = 32$ subsets.

39. False; $F' = \{q, r, s\}$ so F' has $2^3 = 8$ subsets.

40. False; {0} has 1 element but Ø has no elements.

41. $2^{16} = 65536$ subsets.
 65536 seconds
 = 65536 ÷ 60 seconds per minute
 ≈ 1092 minutes ÷ 60 minutes per hour
 = 18 hours (to the nearest hour).

42. $2^{32} = 4294967296$ ÷ 3600 seconds per hour
 ≈ 1193046 hours ÷ 24 hours per day
 ≈ 49710 days ÷ 365 days per year
 = 136 years (to the nearest year).

43. Ø, {α}, {β}, {α, β}

44. Ø, {α}, {β}, {Γ}, {Δ}, {α, β}, {α, Γ}, {α, Δ}, {β, Γ}, {β, Δ}, {Γ, Δ}, {α, β, Γ}, {α, β, Δ}, {α, Γ, Δ}, {β, Γ, Δ}, {α, β, Γ, Δ}

45. Ø, {I}, {II}, {III}, {I, II}, {I, III}, {II, III}, {I, II, III}

46. Ø

47. The number of subsets is 2^n where *n* is the number of elements in the set. $2^2 = 4$.

48. $2^3 = 8$

49. List the elements in the set:
 {8, 10, 12, 14, 16, 18, 20}.
 $2^7 = 128$.

50. List the elements in the set: {–3, –1, 1, 3, 5, 7}.
 $2^6 = 64$.

51. $2^{11} = 2{,}048$

52. $2^{26} = 67{,}108{,}864$

53. There are no negative whole numbers. $2^0 = 1$.

54. The set consists of {1, 2, 3, 4, 5, 6, 7, 8, 9}.
 $2^9 = 512$.

55. a. This is equivalent to finding the number of proper subsets for a set with 4 elements.
$2^4 - 1 = 16 - 1 = 15$.

b. The sets that contain the 1976 dime or the 1992 dime produce duplicate amounts of money.
There are 8 sets that contain one dime producing 4 sets with the same value.
15 – 4 = 11. The set containing the nickel and two dimes has the same value as the set containing the quarter. 11 – 1 = 10 different sums.

c. Two different sets of coins can have the same value.

56. a. $2^{18} = 262{,}144$
$2^{19} = 524{,}288$
$2^{20} = 1{,}048{,}576$

b. Answers will vary
2^{33} for the TI-83 calculator
2^{3000} for the TI-85 calculator

57. a. $2^8 = 256$ different types

b. Solve $2^x > 2000$ by guessing and checking.
$2^{10} = 1{,}024$
$2^{11} = 2{,}048$
So at least 11 condiments.

58. Solve $2^x = 256$. Since $2^8 = 256$, there must be 8 upgrade options offered.

59. a. $2^{10} = 1{,}024$ types of omelets

b. Solve $2^x > 4{,}000$ by guessing and checking.
$2^{11} = 2{,}048$
$2^{12} = 4{,}096$
At least 12 ingredients must be available.

60. a. $2^{12} = 4{,}096$ different versions

b. Solve $2^x > 14{,}000$ by guessing and checking.
$2^{13} = 8{,}192$
$2^{14} = 16{,}384$
At least 14 upgrade options must be provided.

61. a. {2} is the set containing 2, not the element 2. {1, 2, 3} has only three elements, namely 1, 2, and 3. Because {2} is not equal to 1, 2, or 3, $\{2\} \notin \{1, 2, 3\}$.

b. 1 is not a set, so it cannot be a subset.

c. The given set has the elements 1 and {1}. Because $1 \neq \{1\}$, there are exactly two elements in {1, {1}}.

62. a. Solve $2^x = 1024$. Since $2^{10} = 1024$, there must be 10 elements in the set.

b. Solve $2^x - 1 = 255$.
$2^x = 256 \Rightarrow x = 8$
There must be 8 elements in the set.

c. Yes. The empty set has exactly one subset.

63. a. {A, B, C}, {A, B, D}, {A, B, E}, {A, C, D}, {A, C, E}, {A, D, E}, {B, C, D}, {B, C, E}, {B, D, E}, {C, D, E}, {A, B, C, D}, {A, B, C, E}, {A, B, D, E}, {A, C, D, E}, {B, C, D, E}, {A, B, C, D, E}

b. {A}, {B}, {C}, {D}, {E}, {A,B}, {A, C}, {A, D}, {A, E}, {B, C}, {B, D}, {B, E}, {C, D}, {C, E}, {D, E}

64. a. Using row 5, the number of subsets with exactly:
1 subset has 0 elements
5 subsets have 1 element
10 subsets have 2 elements
10 subsets have 3 elements
5 subsets have 4 elements
1 subset has 5 elements

b. Row 6: 1, 6, 15, 20, 15, 6, 1. The 20 subsets of $\{a, b, c, d, e, f\}$ that have exactly three elements are: $\{a, b, c,\}$, $\{a, b, d\}$, $\{a, b, e\}$, $\{a, b, f\}$, $\{a, c, d\}$, $\{a, c, e\}$, $\{a, c, f\}$, $\{a, d, e\}$, $\{a, d, f\}$, $\{a, e, f\}$, $\{b, c, d\}$, $\{b, c, e\}$, $\{b, c, f\}$, $\{b, d, e\}$, $\{b, d, f\}$, $\{b, e, f\}$, $\{c, d, e\}$, $\{c, d, f\}$, $\{c, e, f\}$, $\{d, e, f\}$

EXCURSION EXERCISES SECTION 2.3

1. To form the union, use the maximum membership value.
$M \cup J$
$= \{(A, 1), (B, 0.8), (C, 0.6), (D, 0.5), (F, 0)\}$

2. To form the intersection, use the minimum membership value.
$M \cap J$
$= \{(A, 1), (B, 0.75), (C, 0.5), (D, 0.1), (F, 0)\}$

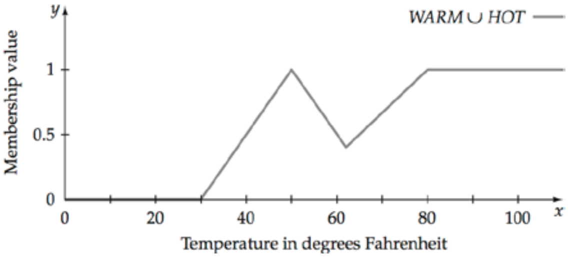
3. $J' = \{(A, 1- 1), (B, 1- 0.8), (C, 1- 0.6), (D, 1- 0.1), (F, 1- 0)\}$
$= \{(A, 0), (B, 0.2), (C, 0.4), (D, 0.9), (F, 1)\}$

$E \cup J'$
$= \{(A, 1), (B, 0.2), (C, 0.4), (D, 0.9), (F, 1)$

4. $L' = \{(A, 1-1), (B, 1-1), (C, 1-1), (D, 1-1), (F, 1-0)\}$
$= \{(A, 0), (B, 0), (C, 0), (D, 0), (F, 1)\}$
$J \cap L' = \{(A, 0), (B, 0), (C, 0), (D, 0), (F, 0)\}$

5. $M' = \{(A, 1-1), (B, 1-0.75), (C, 1-0.5), (D, 1-0.5), (F, 1-0)\}$
$= \{(A, 0), (B, 0.25), (C, 0.5), (D, 0.5), (F, 1)\}$
Since $M' \cup L'$ is in parentheses, form the union first.
$L' = \{(A, 1-1), (B, 1-1), (C, 1-1), (D, 1-1), (F, 1-0)\}$
$= \{(A, 0), (B, 0), (C, 0), (D, 0), (F, 1)\}$
so $M' \cup L' = M'$. Now, $J \cap M' = \{(A, 0), (B, 0.25), (C, 0.5), (D, 0.1), (F, 0)\}$.

6. The line is WARM $\cup$ HOT.



7. $A \cap B$
$= \{(a, 0.3), (b, 0.4), (c, 0.9), (d, 0.2), (e, 0.45)\}$.
$(A \cap B)' = \{(a, 1-0.3), (b, 1-0.4), (c, 1-0.9), (d, 1-0.2), (e, 1-0.45)\}$
$= \{(a, 0.7), (b, 0.6), (c, 0.1), (d, 0.8), (e, 0.55)\}$
$A' = \{(a, 0.7), (b, 0.2), (c, 0), (d, 0.8), (e, 0.25)\}$
$B' = \{(a, 0.5), (b, 0.6), (c, 0.1), (d, 0.3), (e, 0.55)\}$
$A' \cup B'$
$= \{(a, 0.7), (b, 0.6), (c, 0.1), (d, 0.8), (e, 0.55)\}$
Since $(A \cap B)' = A' \cup B'$ De Morgan's Law holds.

EXERCISE SET 2.3

1. $A \cup B = \{2, 4, 6\} \cup \{1, 2, 5, 8\}$
$= \{1, 2, 4, 5, 6, 8\}$

2. $A \cap B = \{2, 4, 6\} \cap \{1, 2, 5, 8\} = \{2\}$

3. $A \cap B' = \{2, 4, 6\} \cap \{3, 4, 6, 7\} = \{4, 6\}$

4. $B \cap C' = \{1, 2, 5, 8\} \cap \{2, 4, 5, 6, 8\}$
$= \{2, 5, 8\}$

5. $(A \cup B)' = (\{1, 2, 4, 5, 6, 8\})' = \{3, 7\}$

6. $A' = \{1, 3, 5, 7, 8\}$.
$A' \cap B = \{1, 3, 5, 7, 8,\} \cap \{1, 2, 5, 8\}$
$= \{1, 5, 8\}$.
$(\{1, 5, 8\})' = \{2, 3, 4, 6, 7\}$

7. $B \cup C = \{1, 2, 5, 8\} \cup \{1, 3, 7\}$
$= \{1, 2, 3, 5, 7, 8\}$.
$A \cup (B \cup C) = \{2, 4, 6\} \cup \{1, 2, 3, 5, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} = U$

8. $A \cap (B \cup C) = \{2, 4, 6\} \cap \{1, 2, 3, 5, 7, 8\}$
$= \{2\}$

9. $B \cap C = \{1, 2, 5, 8\} \cap \{1, 3, 7\} = \{1\}$.
$A \cap (B \cap C) = \{2, 4, 6\} \cap \{1\} = \varnothing$

10. $A' = \{1, 3, 5, 7, 8\}$
$A' \cup (B \cap C) = \{1, 3, 5, 7, 8\} \cup \{1\}$
$= \{1, 3, 5, 7, 8\} = A'$

11. $B \cap (B \cup C) = \{1, 2, 5, 8\} \cap \{1, 2, 3, 5, 7, 8\}$
$= \{1, 2, 5, 8\} = B$

12. $A \cap A' = \{2, 4, 6\} \cap \{1, 3, 5, 7, 8\} = \varnothing$

13. $B \cup B' = \{1, 2, 5, 8\} \cup \{3, 4, 6, 7\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} = U$

14. Note that $A \cap (B \cup C) = \{2\}$ from exercise 8. Then $(\{2\})' = \{1, 3, 4, 5, 6, 7, 8\}$

15. $A \cup C' = \{2, 4, 6\} \cup \{2, 4, 5, 6, 8\}$
$= \{2, 4, 5, 6, 8\}$.
$B \cup A' = \{1, 2, 5, 8\} \cup \{1, 3, 5, 7, 8\}$
$= \{1, 2, 3, 5, 7, 8\}$
$(A \cup C') \cap (B \cup A')$
$= \{2, 4, 5, 6, 8\} \cap \{1, 2, 3, 5, 7, 8\}$
$= \{2, 5, 8\}$

16. $(A \cup C') \cup (B \cup A')$
$= \{2, 4, 5, 6, 8\} \cup \{1, 2, 3, 5, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$
$= U$

17. $C \cup B' = \{1, 3, 7\} \cup \{3, 4, 6, 7\}$
$= \{1, 3, 4, 6, 7\}$.
$(C \cup B') \cup \varnothing = \{1, 3, 4, 6, 7\} \cup \{\}$
$= \{1, 3, 4, 6, 7\} = (C \cup B')$

18. $A' \cup B = \{1, 3, 5, 7, 8\} \cup \{1, 2, 5, 8\}$
$= \{1, 2, 3, 5, 7, 8\}$
$(A' \cup B) \cap \varnothing = \{1, 2, 3, 5, 7, 8\} \cap \{\} = \{\} = \varnothing$

19. $A \cup B = \{1, 2, 4, 5, 6, 8\}$
$B \cap C' = \{1, 2, 5, 8\} \cap \{2, 4, 5, 6, 8\}$
$= \{2, 5, 8\}$
$(A \cup B) \cap (B \cap C')$
$= \{1, 2, 4, 5, 6, 8\} \cap \{2, 5, 8\}$
$= \{2, 5, 8\}$

20. $B \cap A' = \{1, 2, 5, 8\} \cap \{1, 3, 5, 7, 8\}$
$= \{1, 5, 8\}$
$B' \cup C = \{3, 4, 6, 7,\} \cup \{1, 3, 7\}$
$= \{1, 3, 4, 6, 7\}$
$(B \cap A') \cup (B' \cup C) = \{1, 5, 8\} \cup \{1, 3, 4, 6, 7\}$
$= \{1, 3, 4, 5, 6, 7, 8\}$

In Exercises 21-28, one possible answer is given. Your answers may vary from the given answers.

21. The set of all elements that are not in *L* or are in *T*.

22. The set of all elements that are in *K* and are not in *J*.

23. The set of all elements that are in *A*, or are in *C*, but not in *B*.

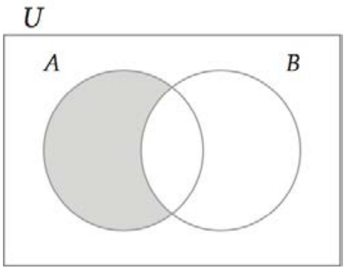
24. The set of all elements that are in either *A* or *B*, and are not in *C*.

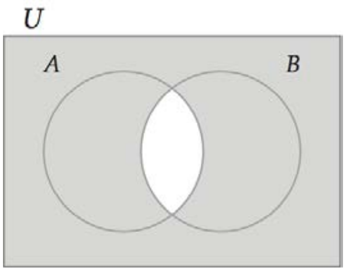
25. The set of all elements that are in *T*, and are also in *J* or not in *K*.

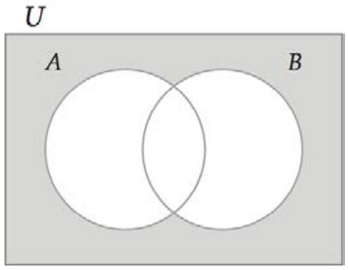
26. The set of all elements that are in both *A* and *B*, or in *C*.

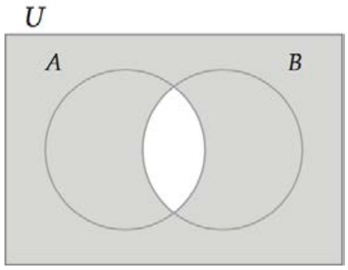
27. The set of all elements that are in both *W* and *V*, or are in both *W* and *Z*.

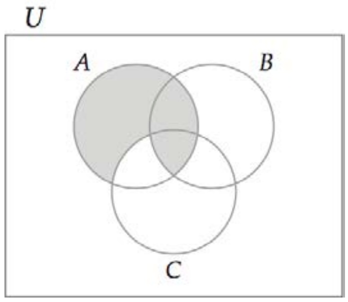
28. The set of all elements that are in *D*, but are not in either *E* or *F*.

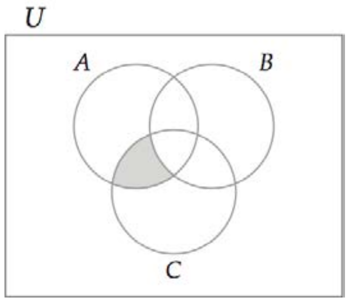
29.


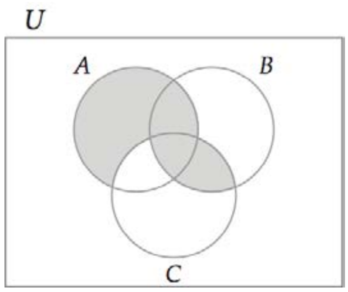
30.


31.


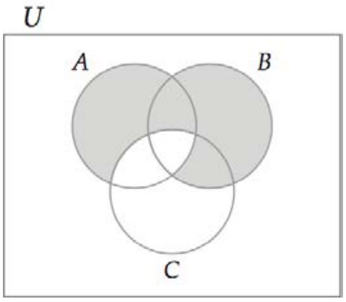
32.


33.


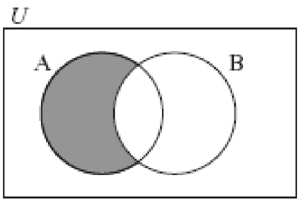
34.


35.


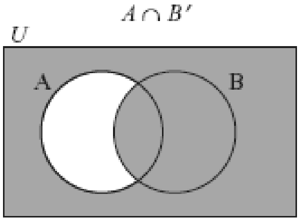
36.



37.



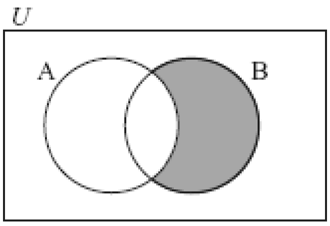
$A \cap B'$



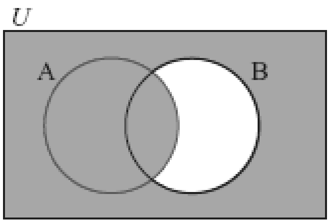
$A' \cup B$

Because the sets $A \cap B'$ and $A' \cup B$ are represented by different regions, $(A \cup B') \neq (A' \cup B)$.

38.



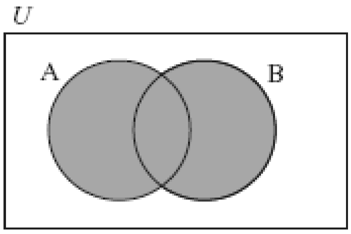
$A' \cap B$



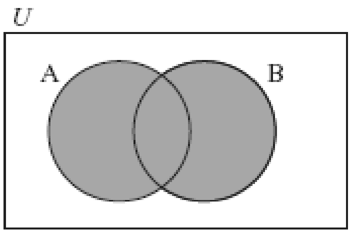
$A \cup B'$

$A' \cap B \neq A \cup B'$

39.



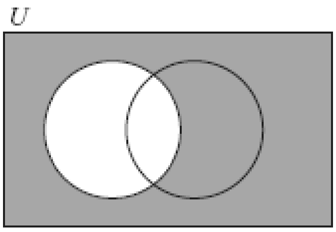
$A \cup (A' \cap B)$



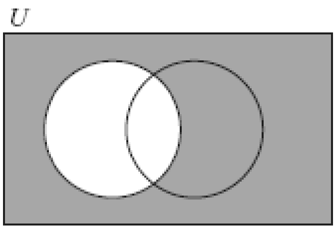
$A \cup B$

$A \cup (A' \cap B) = A \cup B$

40.



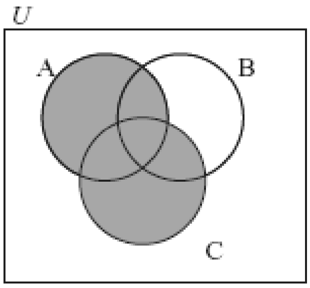
$A' \cap (B \cup B')$



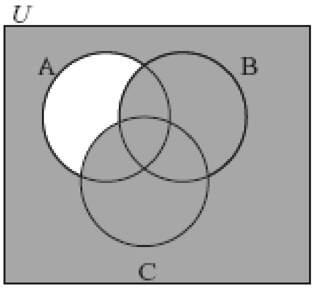
$A' \cup (B \cap B')$

$A' \cap (B \cup B') = A' \cup (B \cap B')$

41.



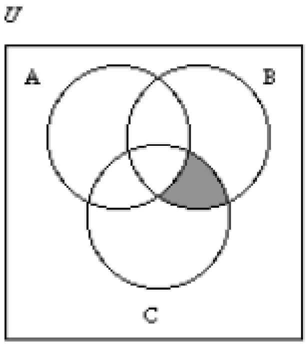
$(A \cup C) \cap B'$



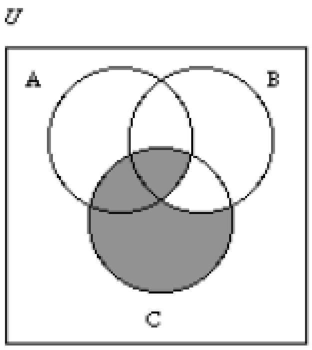
$A' \cup (B \cup C)$

$(A \cup C) \cap B' \neq A' \cup (B \cup C)$

42.

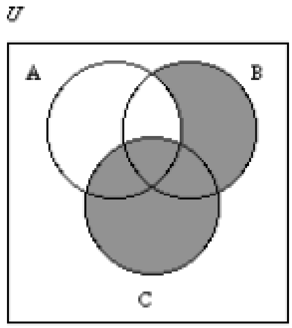


$A' \cap (B \cap C)$

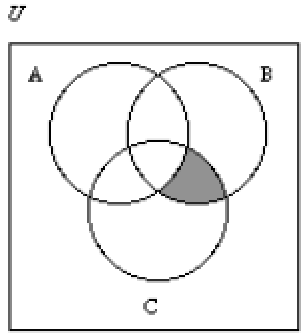


$A' \cap (B \cap C) \neq (A \cup B') \cap C$

43.



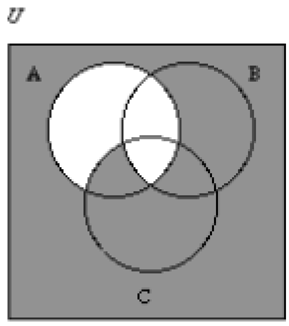
$(A' \cap B) \cup C$



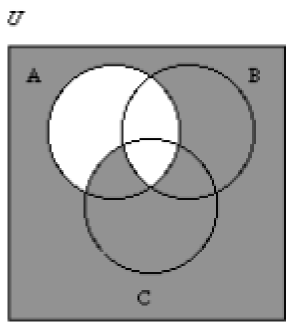
$(A' \cap C) \cap (A' \cap B)$

$(A' \cap B) \cup C \neq (A' \cap C) \cap (A' \cap B)$

44.



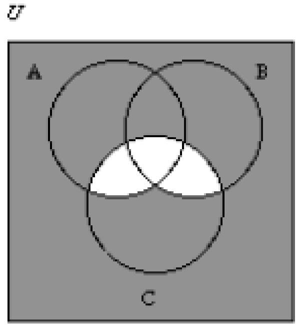
$A' \cup (B' \cap C)$



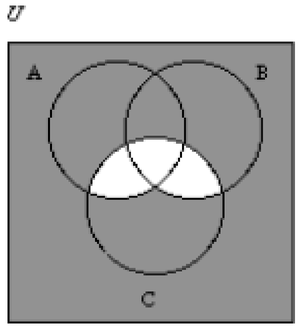
$(A' \cup B') \cap (A' \cup C)$

$A' \cup (B' \cap C) = (A' \cup B') \cap (A' \cup C)$

45.



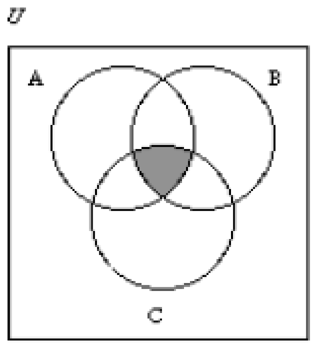
$((A \cup B) \cap C)'$



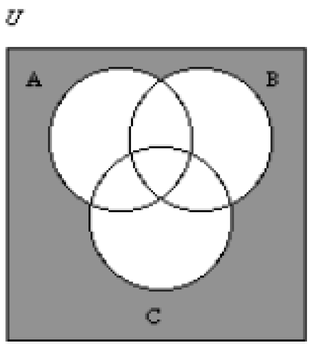
$(A' \cap B') \cup C'$

$((A \cup B) \cap C)' = (A' \cap B') \cup C'$

46.



$(A \cap B) \cap C$



$(A \cup B \cup C)'$

$(A \cap B) \cap C \neq (A \cup B \cup C)'$

47. $R \cap G \cap B'$
$B' = \{R, Y, G\}$
$R = \{R, M, W, Y\}$
$G = \{G, Y, W, C\}$
$R \cap G \cap B' = Y$ (yellow)

48. $R \cap G' \cup B$
$G' = \{R, M, B\}$
$R = \{R, M, W, Y\}$
$B = \{B, M, W, C\}$
$R \cap G' \cup B = M$(magenta)

49. $R' \cap G \cap B$
$R' = \{B, C, G\}$
$G = \{G, Y, W, C\}$
$B = \{B, M, W, C\}$
$R \cap G \cap B = C$ (cyan)

50. $Y' = \{C, B, M\}$
$C = \{C, G, K, B\}$
$M = \{M, B, K, R\}$
$C \cap M \cap Y' = B$ (blue)

51. $C' = \{Y, R, M\}$
$M = \{M, B, K, R\}$
$Y = \{Y, G, K, R\}$
$C' \cup M \cap Y =$ R (red)

52. $C = \{C, G, K, B\}$
$M' = \{C, G, Y\}$
$Y = \{Y, G, K, R\}$
$C \cap M' \cap Y = G$ (green)

In Exercises 53–62, one possible answer is given. Your answers may vary from the given answers.

53. $A \cap B'$

54. $(A \cup B) \cap (A \cap B)'$

55. $(A \cup B)'$

56. $A \cap (B \cup C)$

57. $B \cup C$

58. $C \cup (A \cap B)$

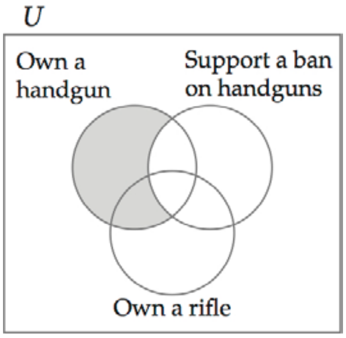
59. $C \cap (A \cup B)'$

60. $(A \cup B)'$

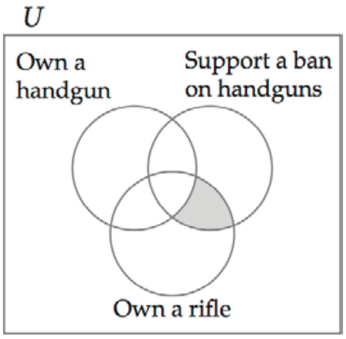
61. $(A \cup B)' \cup (A \cap B \cap C)$

62. $C \cup (A \cap B')$

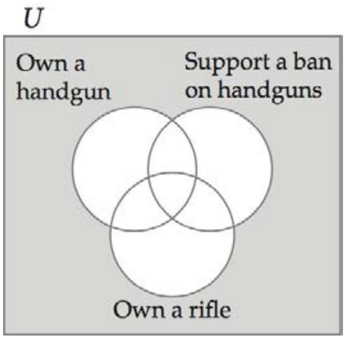
63. a.



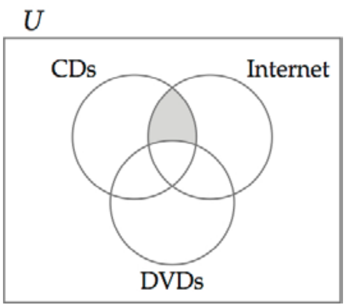
b.



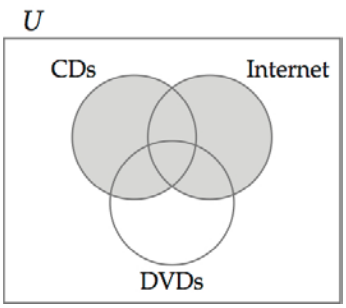
c.



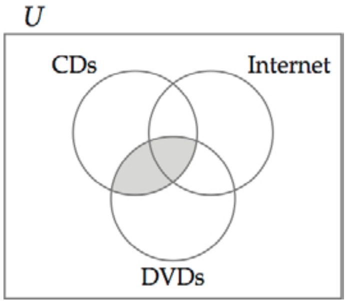
64. a.



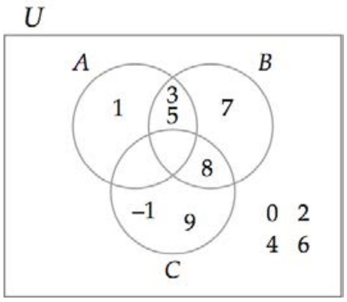
b.



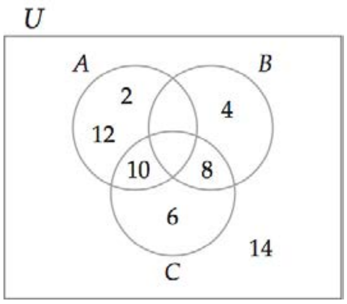
c.



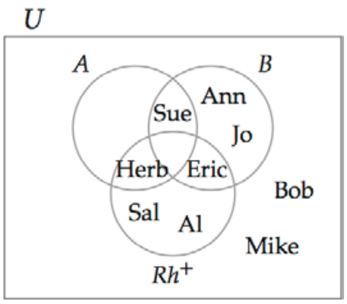
65.



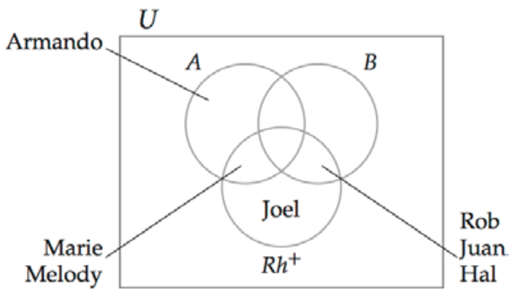
66.



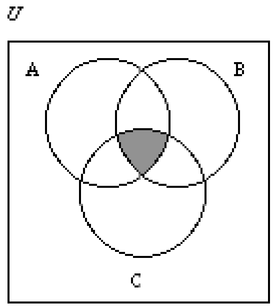
67.



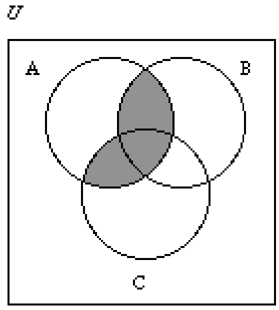
68.



69.



70.



71. $B - A = \{2, 3, 8, 9\} - \{2, 4, 6, 8\}$
$= \{3, 9\}$

72. $A - B = \{2, 4, 6, 8\} - \{2, 3, 8, 9\}$
$= \{4, 6\}$

73. $A - B' = \{2, 4, 6, 8\} - \{1, 4, 5, 6, 7\}$
$= \{2, 8\}$

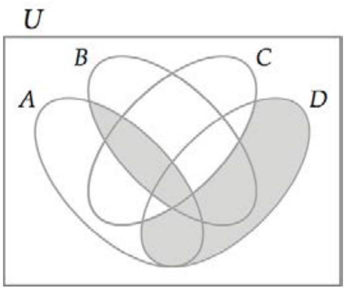
74. $B' - A = \{1, 4, 5, 6, 7\} - \{2, 4, 6, 8\}$
$= \{1, 5, 7\}$

75. $A' - B' = \{1, 3, 5, 7, 9\} - \{1, 4, 5, 6, 7\}$
$= \{3, 9\}$

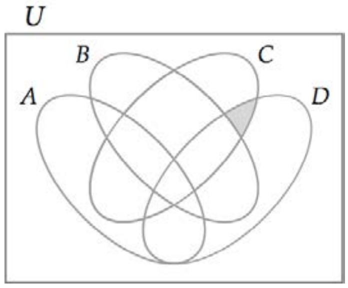
76. $A' - B = \{1, 3, 5, 7, 9\} - \{2, 3, 8, 9\}$
$= \{1, 5, 7\}$

77. Responses will vary.

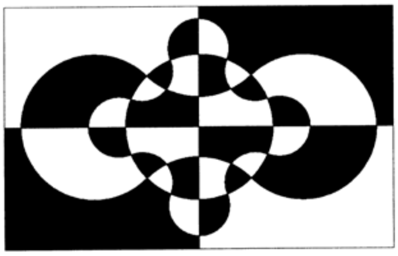
78.



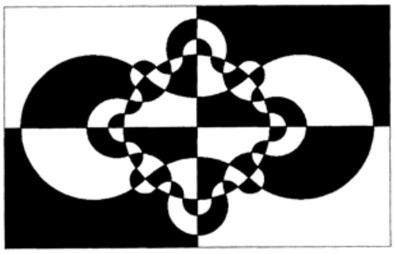
79.



80. A Venn diagram for five sets.



A Venn diagram for six sets.



EXCURSION EXERCISES 2.4

1. a. {Ryan, Susan}, {Ryan, Trevor}, {Susan, Trevor}, {Ryan, Susan, Trevor}

   b. Ø, {Ryan}, {Susan}, {Trevor}

2. {M, N}, {M, P}, {M, N, P}

3. In $\{M, N\}$, if either voter leaves the coalition, the coalition becomes a losing coalition. The same is true for $\{M, P\}$. However, in $\{M, N, P\}$, if N or P leaves, the coalition still wins. The minimal winning coalition is $\{M, N\}$ and $\{M, P\}$.

EXERCISE SET 2.4

1. $B \cup C$ = {Math, Physics, Chemistry, Psychology, Drama, French, History} so $n(B \cup C) = 7$

2. $A \cup B$ = {English, History, Psychology, Drama, Math, Physics, Chemistry} so $n(A \cup B) = 7$

3. $n(B) + n(C) = 5 + 3 = 8$

4. $n(A) + n(B) = 4 + 5 = 9$

5. $(A \cup B) \cup C$ = {English, History, Psychology, Drama, Math, Physics, Chemistry, French} so $n[(A \cup B) \cup C] = 8$

6. $A \cap B$ = {Psychology, Drama} so $n(A \cap B) = 2$

7. $n(A) + n(B) + n(C) = 4 + 5 + 3 = 12$

8. $A \cap B \cap C = \varnothing$ so $n(A \cap B \cap C) = 0$

9. $B \cap C$ = {Chemistry}
$A \cup (B \cap C)$ = {English, History, Psychology, Drama, Chemistry}
so $n[A \cup (B \cap C)] = 5$

10. $B \cup C$ = {Math, Physics, Chemistry, Psychology, Drama, French, History}
$A \cap (B \cup C)$ = {History, Psychology, Drama}
so $n[A \cap (B \cup C)] = 3$

11. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 4 + 5 - 2 = 7$

12. $n(A \cup C) = n(A) + n(C) - n(A \cap C)$
$= 4 + 3 - 1 = 6$

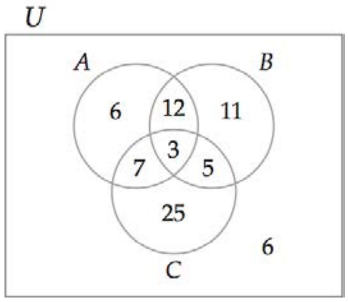
13. Using the formula:
$$n(J \cup K) = n(J) + n(K) - n(J \cap K)$$
$$310 = 245 + 178 - n(J \cap K)$$
$$310 = 423 - n(J \cap K)$$
$$-113 = -n(J \cap K)$$
$$n(J \cap K) = 113$$

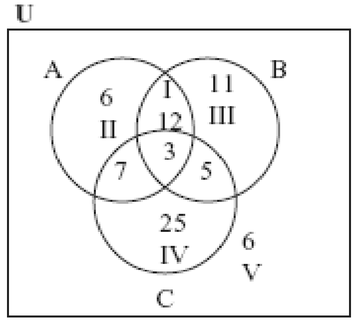
14. $$n(L \cup M) = n(L) + n(M) - n(L \cap M)$$
$$n(L \cup M) = 780 + 240 - 50 = 970$$

15. Using the formula:
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$2250 = 1500 + n(B) - 310$$
$$2250 = 1190 + n(B)$$
$$1060 = n(B)$$

16. Using the formula:
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$765 = 640 + 280 - n(A \cap B)$$
$$765 = 920 - n(A \cap B)$$
$$-155 = -n(A \cap B)$$
$$n(A \cap B) = 155$$

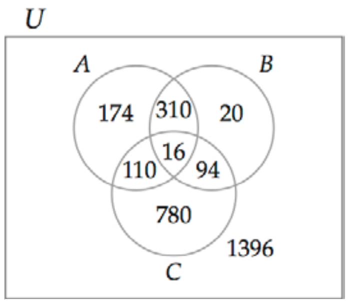
17.

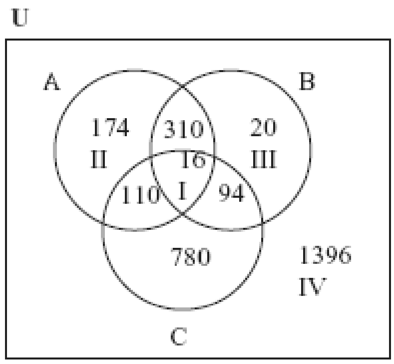




To find n(I): Since $n(A \cap B) = 15$ and 3 has been accounted for, $15 - 3 = 12$ for n(I).
To find n(II): $n(A) = 28$. $7 + 3 + 12 = 22$ is accounted for so $28 - 22 = 6$ for n(II).
To find n(III): $n(B) = 31$. $5 + 3 + 12 = 20$. $31 - 20 = 11$.
To find n(IV): $n(C) = 40$. $40 - (7 + 3 + 5) = 25$.
To find n(V): $n(U) = 75$ so
$n(\text{V}) = 75 - (6 + 12 + 11 + 7 + 3 + 5 + 25) = 6$.

18.





To find n(I): $n(C) = 1000$.
$1000 - (110 + 94 + 780) = 16$.
To find n(II):
$n(A) = 610$. $610 - (110 + 310 + 16) = 174$.
To find n(III): $n(B) = 440$.
$440 - (310 + 16 + 94) = 20$.

To find $n(\text{IV})$: $n(U) = 2900$.
$2900 - (174 + 310 + 20 + 110 + 16 + 94 + 780) = 1396$.

19. a. S = {investors in stocks} and let B = {investors in bonds}. Since 75 had not invested in either stocks or bonds, $n(S \cup B) = 600 - 75 = 525$.

$$\begin{aligned} n(S \cup B) &= n(S) + n(B) - n(S \cap B) \\ 525 &= 380 + 325 - n(S \cap B) \\ 525 &= 705 - n(S \cap B) \\ -180 &= -n(S \cap B) \\ n(S \cap B) &= 180 \end{aligned}$$

$n(S \cap B) = 180$ represents the number of investors in both stocks and bonds.

b. $n(S \text{ only}) = 380 - 180 = 200$

20. a. Let S = {commuters taking the subway} and let B = {commuters taking the bus}. Since 120 commuters do not take either the subway or the bus, $n(S \cup B) = 1500 - 120 = 1380$.

$$\begin{aligned} n(S \cup B) &= n(S) + n(B) - n(S \cap B) \\ 1380 &= 1140 + 680 - n(S \cap B) \\ 1380 &= 1820 - n(S \cap B) \\ -440 &= -n(S \cap B) \\ n(S \cap B) &= 440 \end{aligned}$$

$n(S \cap B) = 440$ commuters take both the subway and the bus.

b. Using the formula:

$$\begin{aligned} n(S \text{ only}) &= n(S) - n(S \cap B) \\ &= 1140 - 440 = 700 \end{aligned}$$

21. Draw a Venn diagram to represent the data: Since 44% responded favorably to both forms, place 44% in the intersection of the two sets. A total of 72% responded favorably to an analgesic, so 72% – 44% = 28% who responded favorably to only the analgesic. Similarly, 59% – 44% = 15% who responded favorably only to the muscle relaxant.

a. 15% responded favorably to the muscle relaxant but not the analgesic.

b. Since the universe must contain 100%, subtract the known values from 100%: 100% – (28% + 44% + 15%) =13%. This represents the percent of athletes who were treated who did not respond favorably to either form of treatment.

22. Draw a Venn diagram to represent the data. Fill in the diagram starting from the innermost region.

i: 245 represents guests who tip all three services.
ii: 755 tip only wait staff and luggage handlers, but this includes the 245 so 755 – 245 = 510.
iii: 700 tip only wait staff and the maids, but this includes the 245 so 700 – 245 = 455
iv: 275 – 245 = 30
v: 831 – (455 + 245 + 30) = 101
vi: 1219 – (510 + 245 + 30) = 434
vii: 1785 – (455 + 245 + 510) = 575
viii: 210 do not tip these services.

a. Exactly two of the three services are represented by regions ii, iii, and iv: 510 + 455 + 30 = 995

b. Only the wait staff is region vii: 575

c. Only one of the three services is represented by regions v, vi, and vii: 101 + 434 + 575 = 1110

23. Draw a Venn diagram to represent the data. Fill in the diagram starting from the innermost region.

i: 85
ii: 150 – 85 = 65
iii: 135 – 85 = 50
iv: 110 – 85 = 25
v: 390 – (65 + 85 + 50) = 190
vi: 290 – (25 + 85 + 50) = 130
vii: 305 – (65 + 85 + 25) = 130
viii: 770 – (130+25+130+65+85+50 +190) = 95

a. exactly one of these forms of advertising is represented by v, vi, and vii: 190 + 130 + 130 = 450

b. exactly two of these forms are represented by ii, iii, and iv: 65 + 50 + 25 = 140

c. PC World and neither of the other two forms is represented by vii: 130.

24. a. We are given $p(\text{A}) = 45.8\%$, $p(\text{B}) = 14.2\%$, and $p(\text{A} \cap \text{B}) = 4.1\%$. Substituting in the percent inclusion-exclusion formula gives:

$$\begin{aligned} P(\text{A} \cup \text{B}) &= P(\text{A}) + p(\text{B}) - p(\text{A} \cap \text{B}) \\ &= 45.8\% + 14.2\% - 4.1\% \\ &= 55.9\% \end{aligned}$$

55.9% of donors has the A antigen or B antigen

b.
$$\begin{aligned} p(\text{B} \cup \text{Rh+}) &= p(\text{B}) + p(\text{Rh+}) - p(\text{B} \cap \text{Rh+}) \\ 87.2\% &= 14.2\% + 84.7\% - p(\text{B} \cap \text{Rh+}) \\ 87.2\% &= 98.9\% - p(\text{B} \cap \text{Rh+}) \\ 11.7\% &= p(\text{B} \cap \text{Rh+}) \end{aligned}$$

11.7% of donors have B antigen and are Rh+

25. Draw a Venn diagram to represent the data. Fill in the diagram starting from the innermost region.

    i: 52
    ii: 10 – 52 = 88
    iii: 437 – (52 + 88 + 202) = 95
    iv: 74 – 52 = 22
    v: 271 – (22 + 88 + 52) = 109
    vi: 202
    vii: 497 – (22 + 52 + 95) = 328
    viii: 1000 – (109+88+52+22+202+95+328) = 104

    a. this group is represented by v: 109
    b. this group is represented by vii: 328
    c. this group is represented by viii: 104

26. Draw a Venn diagram to represent the data. Fill in the diagram starting from the innermost region.

    i: 98
    ii: 155 – 98 = 57
    iii: 268 – (36 + 98 + 114) = 20
    iv: 212 – 98 = 114
    v: 365 – (57 + 98 + 114) = 96
    vi: 298 – (20 + 98 + 57) = 123
    vii: 36

    a. this group is represented by v: 96
    b. this group is represented by vi: 123
    c. this group is represented by i, ii, iii, iv, v, and vi:
       98 + 57 + 20 + 114 + 96 + 123 = 508
    d. this group is represented by i and iii:
       98 + 20 = 118

27. a. 101
    b. 124 + 82 + 65 + 51 + 48 = 370
    c. 124+82+133 + 41 = 380
    d. 124+82+101 + 66 = 373
    e. 124 +101 = 225
    f. 124 + 82 + 65 + 101 + 66 + 51 + 41 = 530

28. a. 175
    b. 180+162+190+110 + 86 = 728
    c. 114 +175 +162 +126 +190 +110 + 86 = 963
    d. 210 +175 = 385
    e. 110
    f. 210 + 175 + 180 + 162 = 727

29. Given $n(A) = 47$ and $n(B) = 25$.

    a. If A and B are disjoint sets,
       $n(A \cup B) = n(A) + n(B) = 47 + 25 = 72$.
    b. If $B \subset$ A, then $A \cup B = A$ and $n(A) = 47$.
    c. If $B \subset A$, then $A \cap B = B$ and $n(B) = 25$.
    d. If A and B are disjoint sets, then $A \cap B = \varnothing$ and $n(A \cap B) = 0$.

30. Given $n(A) = 16$, $n(B) = 12$, $n(C) = 7$.

    a. If A, B, and C are disjoint sets,
       $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
       $= 16 + 12 + 7 = 35$
    b. If $C \subset B \subset A$, then $A \cup B \cup C = A$ and $n(A) = 16$
    c. If $B \cup C = A$, then $A \cap (B \cup C) = A$ and $n(A) = 16$
    d. If A, B, and C are disjoint sets, then $A \cap (B \cup C) = \varnothing$ and $n(A \cap (B \cup C)) = 0$.
       If $C \subset B$ and $A \cap B = \varnothing$, then $n(A \cap (B \cup C)) = 0$.

31. Complete the Venn diagram. Since there are 450 users of Webcrawler,
    450 – (45 + 41 + 30 + 50 + 60 + 80 + 45) = 99 gives the total who use only Webcrawler.
    Similarly, 585 – (55 + 50 + 60 + 41 + 34 + 100 + 45) = 200 gives the total who use only Bing.
    To find Yahoo only users:
    620 – (55 + 50 + 100 + 60 +80 +41 +30) = 204.
    To find Ask only users:
    560 – (50 +80 +50 +60 + 34 + 100 + 45) = 141.

    a. only Google: 200
    b. To find the number who use exactly three search engines, add the numbers given for people who use only 3 search engines:
       100 + 41 + 50 + 80 = 271
    c. Total number in the regions: 204 + 55+ 200 + 141 + 50 + 50 + 34 + 45 + 80 + 60 + 100 + 99 + 30 + 41 + 45 = 1234.
       Since 1250 people were surveyed, this means 1250 – 1234 = 16 people do not use any of the search engines.

32. Using the data from Example 2 and letting A = Rock, B = Rap, C = Heavy Metal,
    $n(A \cup B \cup C)$
    $= n(A) + n(B) + n(C) - n(A \cap B)$
    $-n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
    455 = 395 + 320 + 295 – 280 – 245 –190 +160
    Equation d is the correct choice.

EXCURSION EXERCISES 2.5

1. Let $C = \{3, 4, 5, 6, \ldots, n + 2, \ldots\}$. C and N have the same cardinality because the elements of C can be paired with the elements of N using the general correspondence $(n+2) \leftrightarrow n$.

   Let $D = \{1, 2\}$. Because $C \cup D = N$, we can establish the following equations.
   $$\begin{aligned} n(C) + n(D) &= n(N) \\ \aleph_0 + 2 &= \aleph_0 \end{aligned}$$

2. Let W be the set of whole numbers and let B be the set of negative integers. Then
   $$\begin{aligned} n(W) + n(B) &= n(W \cup B) \\ \aleph_0 + \aleph_0 &= n(I) \\ \aleph_0 + \aleph_0 &= \aleph_0 \end{aligned}$$

3. Let $C = \{1, 2, 3, 4, 5, 6\}$. Then
   $$\begin{aligned} n(N) - n(C) &= n(N \cap C') \\ \aleph_0 - 6 &= n(\{7, 8, 9, 10, \ldots\}) \\ \aleph_0 - 6 &= \aleph_0 \end{aligned}$$

EXERCISE SET 2.5

1. a. Comparing:
   $$\begin{array}{l} V = \{a, e, i\} \\ \quad\ \ \updownarrow\updownarrow\updownarrow \\ M = \{3, 6, 9\} \end{array}$$

   b. The possible one-to-one correspondences (listed as ordered pairs) are: $\{(a, 6), (i, 3), (e, 9)\}$, $\{(a, 9), (i, 3), (e, 6)\}$, $\{(a, 3), (i, 9), (e, 6)\}$, $\{(a, 6), (i, 9), (e, 3)\}$, $\{(a, 9), (i, 6), (e, 3)\}$ plus the pairing shown in part a. produce 6 one-to-one correspondences.

2. Write the sets so that one is aligned below the other. One possible pairing is shown below.
   $$\begin{array}{l} N = \{1, 2, 3, 4, \ldots, n, \ldots\} \\ \quad\ \ \updownarrow\ \updownarrow\ \ \updownarrow\ \updownarrow\quad\ \ \updownarrow \\ F = \{5, 10, 15, 20, \ldots 5n, \ldots\} \end{array}$$
   By pairing n of N with $5n$ of F a one-to-one correspondence is established.

3. Write the sets so that one is aligned below the other. One possible pairing is shown below.
   $$\begin{array}{l} D = \{1, 3, 5, \ldots, 2n-1, \ldots\} \\ \quad\ \ \updownarrow\updownarrow\updownarrow\quad\ \ \updownarrow \\ M = \{3, 6, 9, \ldots 3n, \ldots\} \end{array}$$
   Pair $(2n - 1)$ of D with $(3n)$ of M to establish a one-to-one correspondence.

4. 4. The cardinality of a finite set is the number of elements in the set.

5. The general correspondence $(n) \leftrightarrow (7n - 5)$ establishes a one-to-one correspondence between the elements of N and the elements of the given set. Thus the cardinality is $\aleph_0$.

6. $\aleph_0$. The natural numbers, integers and rational numbers all have cardinality $\aleph_0$.

7. c

8. c

9. c. Any set of the form $\{x \mid a \le x \le b\}$ where a and b are real numbers and $a \ne b$ has cardinality c.

10. The set of subsets of a set with n elements is 2^n. The given set has 4 elements and $2^4 = 16$ subsets. Therefore the cardinality is 16.

11. Sets with equal cardinality are equivalent. The cardinality of N = cardinality of $I = \aleph_0$ therefore the sets are equivalent.

12. The sets are not equivalent since the cardinality of W is $\aleph_0$ and the cardinality of R is c.

13. The sets are equivalent since the set of rational numbers and the set of integers have cardinality $\aleph_0$.

14. The sets are not equivalent. The cardinality of $Q = \aleph_0$ and the cardinality of $R = c$.

15. Let $S = \{10, 20, 30, \ldots, 10n, \ldots\}$. Then S is a proper subset of A. A rule for a one-to-one correspondence between A and S is $(5n) \leftrightarrow (10n)$. Because A can be placed in a one-to-one correspondence with a proper subset of itself, A is an infinite set.

16. Let $F = \{15, 19, 23, 27, \ldots, 4n + 11, \ldots\}$. Then F is a proper subset of B. A rule for a one-to-one correspondence between B and F is $(4n + 7) \leftrightarrow (4n + 11)$. Because B can be placed in a one-to-one correspondence with a proper subset of itself, B is an infinite set.

17. Let $R = \left\{\frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots, \frac{2n+1}{2n+2}, \ldots\right\}$. Then R is a proper subset of C. A rule for a one-to-one correspondence between C and R is $\left(\frac{2n-1}{2n}\right) \leftrightarrow \left(\frac{2n+1}{2n+2}\right)$. Because C can be placed in a one-to-one correspondence with a proper subset of itself, C is an infinite set.

18. Let $H = \left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, ..., \frac{1}{n+2}, ...\right\}$. Then H is a proper subset of D. A rule for a one-to-one correspondence between D and H is $\left(\frac{1}{n+1}\right) \leftrightarrow \left(\frac{1}{n+2}\right)$. Because D can be placed in a one-to-one correspondence with a proper subset of itself, D is an infinite set.

In Exercises 19-26, let N = {1, 2, 3, 4, .., n, . .}. Then a one-to-one correspondence between the given sets and the set of natural numbers N is given by the following general correspondences.

19. $(n + 49) \leftrightarrow (n)$

20. $(-5n + 15) \leftrightarrow (n)$

21. $\left(\frac{1}{3^{n-1}}\right) \leftrightarrow (n)$

22. $(-6n - 6) \leftrightarrow (n)$

23. $(10^n) \leftrightarrow (n)$

24. $\left(\frac{1}{2^{n-1}}\right) \leftrightarrow (n)$

25. $(n^3) \leftrightarrow (n)$

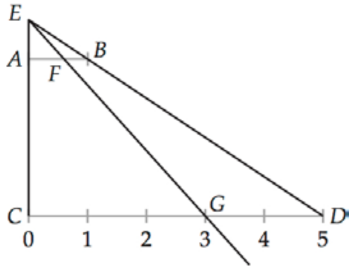
26. $(10^{-n}) \leftrightarrow (n)$

27. a. For any natural number n, the two natural numbers preceding $3n$ are not multiples of 3. Pair these two numbers, $3n - 2$ and $3n - 1$, with the multiples of 3 given by $6n - 3$ and $6n$, respectively.
Using the two general correspondences $(6n - 3) \leftrightarrow (3n - 2)$ and $(6\ n) \leftrightarrow (3n - 1)$, we can establish a one-to- one correspondence between the multiples of 3 (set M) and the set K of all natural numbers that are not multiples of 3.

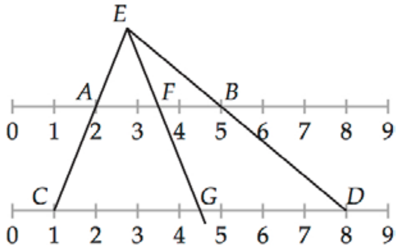
    b. First find n. $6n - 606$, $n = 101$.
Then $(6n) \leftrightarrow (3n - 1)$ means
$(6\ (101)) \leftrightarrow (3\ (101) - 1) = 302$.

    c. Solve.
$$3n - 1 = 899$$
$$3n = 900$$
$$n = 300$$
Then $(6n) \leftrightarrow (3n - 1)$ means
$(6(300)) \leftrightarrow (3(300) - 1)$ and
$(1800) \leftrightarrow (899)$.

28. a. In the following figure, the line from E that passes through $\overline{AB}$ and $\overline{CD}$ illustrates a method of establishing a one-to-one correspondence between the sets $\{x \mid 0 \le x \le 1\}$ and $\{x \mid 0 \le x \le 5\}$.



    b. In the following figure, the line from E that passes through the intervals $\overline{AB}$ and $\overline{CD}$ illustrates a method of establishing a one-to-one correspondence between the sets $\{x \mid 2 \le x \le 5\}$ and $\{x \mid 1 \le x \le 8\}$.



29. The set of real numbers x such that $0 < x < 1$ is equivalent to the set of all real numbers.

30. Written responses will vary.
In the Hilbert Hotel there is always room for one more guest, even when the hotel is full. For example, if every room is occupied, a new guest can be accommodated by having each of the current guests move to the room with the next higher natural number. This will allow the new guest to occupy room 1. Even if a bus with $\aleph_0$ new guests arrives, the manager of the hotel can make room for the new guests by having each of the current guests move to the room that has a number twice as large as the guest's current room number. Now the new guests can be assigned to the empty rooms in the following manner. The first new arrival will get room 1, the second will get room 3, and, in general, the nth new arrival will get room $2n - 1$.

CHAPTER 2 REVIEW EXERCISES

1. {January, June, July}

2. {Alaska, Hawaii}

3. {0, 1, 2, 3, 4, 5, 6, 7}

4. {–8, 8}. Since the set of integers includes positives and negatives, –8 and 8 satisfy $x^2 = 64$.

5. Solving:
   x + 3 £ 7
   x £ 4
   {1, 2, 3, 4}.

6. {1, 2, 3, 4, 5, 6}. Counting numbers begin at 1.

7. $\{x \mid x \in I \text{ and } x > -6\}$

8. {*x* | *x* is the name of a month with exactly 30 days}

9. {*x* | *x* is the name of a U.S. state that begins with the letter *K*}

10. $\{x^3 \mid x = 1, 2, 3, 4, 5\}$

11. The sets are equivalent, since each set has exactly four elements and are not equal.

12. The sets are both equal and equivalent.

13. False. The set contains numbers, not sets.
    $\{3\} \notin \{1, 2, 3, 4\}$.

14. True. The set of integers includes positive and negative integers.

15. True. The symbol ~ means equivalent and sets with the same number of elements are equivalent.

16. False. The word small is not precise.

17. $A \cap B = \{2, 6, 10\} \cap \{6, 10, 16, 18\} = \{6, 10\}$

18. $A \cup B = \{2, 6, 10\} \cup \{6, 10, 16, 18\}$
    $= \{2, 6, 10, 16, 18\}$

19. $A' \cap C = \{8, 12, 14, 16, 18\} \cap \{14, 16\}$
    $= \{14, 16\} = C$

20. $B \cup C' = \{6, 10, 16, 18\} \cup \{2, 6, 8, 10, 12, 18\}$
    $= \{2, 6, 8, 10, 12, 16, 18\}$

21. $B \cap C = \{6, 10, 16, 18\} \cap \{14, 16\} = \{16\}$
    $A \cup \{16\} = \{2, 6, 10\} \cup \{16\} = \{2, 6, 10, 16\}$

22. $A \cup C = \{2, 6, 10\} \cup \{14, 16\}$
    $= \{2, 6, 10, 14, 16\}$
    $(A \cup C)' = \{8, 12, 18\}$
    $\{8, 12, 18\} \cap \{2, 8, 12, 14\} = \{8, 12\}$

23. $A \cap B' = \{2, 6, 10\} \cap \{2, 8, 12, 14\} = \{2\}$
    $(\{2\})' = \{6, 8, 10, 12, 14, 16, 18\}$

24. $A \cup B \cup C = \{2, 6, 10, 14, 16, 18\}$
    $(A \cup B \cup C)' = \{8, 12\}$

25. No, the first set is not a subset of the second set because 0 is not in the set of natural numbers.

26. No, the first set is not a subset of the second set because 9.5 is not in the set of integers.

27. Proper subset. All natural numbers are whole numbers, but 0 is not a natural number so $N \subset W$.

28. Proper subset. All integers are real numbers, but $\frac{1}{2}$ is a real number that is not an integer so $I \subset R$.

29. Counting numbers and natural numbers represent the same set of numbers. The set of counting numbers is not a proper subset of the set of natural numbers.

30. The set of real numbers is not a proper subset of the set of rational numbers.

31. Ø, {I}, {II}, {I, II}

32. Ø, {*s*} {*u*}, {*n*}, {*s*, *u*}, {*s*, *n*}, {*u*, *n*}, { *s*, *u*, *n*}

33. Ø, {penny}, {nickel}, {dime}, {quarter}, {penny, nickel}, {penny, dime}, {penny, quarter}, {nickel, dime}, {nickel, quarter}, {dime, quarter}, {penny, nickel, dime}, {penny, nickel, quarter}, {penny, dime, quarter}, {nickel, dime, quarter}, {penny, nickel, dime, quarter}

34. Ø, {A}, {B}, {C}, {D}, {E}, {A, B}, {A, C}, {A, D}, {A, E}, {B, C}, {B, D}, {B, E}, {C, D}, {C, E}, {D, E}, {A, B, C}, {A, B, D}, {A, B, E}, {A, C, D}, {A, C, E},{A, D, E}, {B, C, D}, {B, C, E}, {B, D, E}, {C, D, E}, {A, B, C, D}, {A, B, C, E}, {A, B, D, E}, {A, C, D, E},{B, C, D, E}, {A, B, C, D, E}

35. The number of subsets of a set with *n* elements is 2^n. The set of four musketeers has 4 elements and $2^4 = 16$ subsets.

36. $n = 26$.
    $2^{26} = 67{,}108{,}864$ subsets

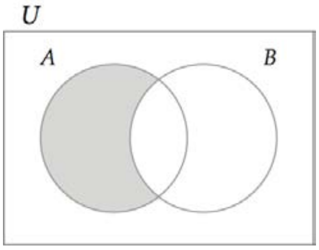
37. The number of letters is 15.
    $2^{15} = 32, 768$ subsets

38. $n = 7$. $2^7 = 128$ subsets

39. True, by De Morgan's Law: $(A \cup B')' = A' \cap B$

40. True, by De Morgan's Law: $(A' \cap B')' = A \cup B$

41.



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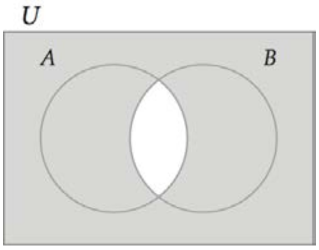

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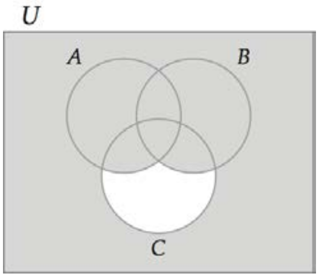

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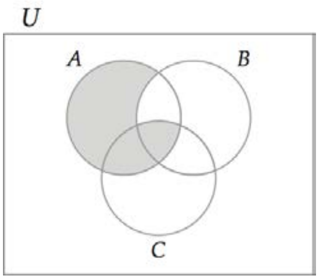
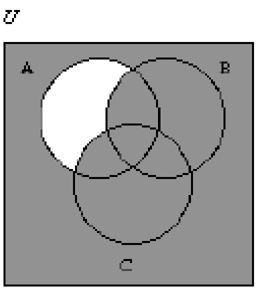
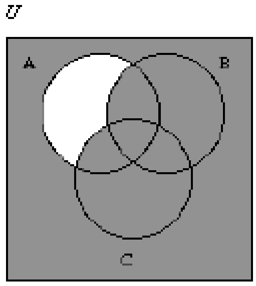


45.



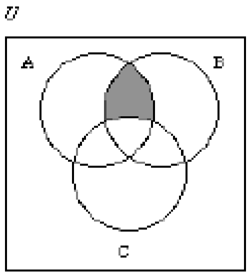
$A' \cup (B \cup C)$



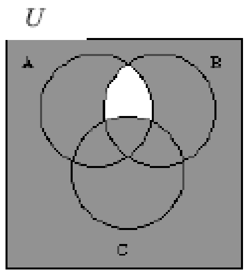
$(A' \cup B) \cup (A' \cup C)$

$A' \cup B \cup C = (A' \cup B) \cup (B' \cup C)$

46.



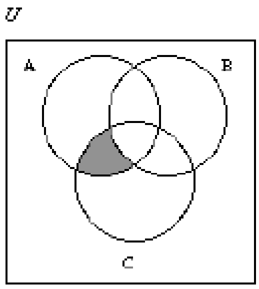
$(A \cap B) \cap C'$



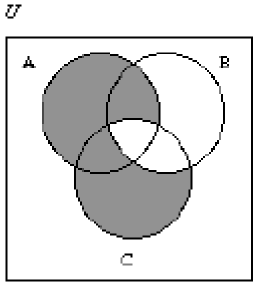
$(A' \cup B') \cup C$

$(A \cap B) \cap C' \neq (A' \cup B') \cup C$

47.



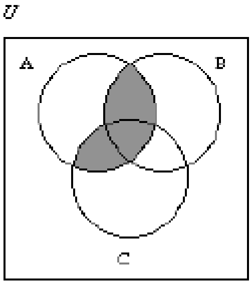
$A \cap (B' \cap C)$



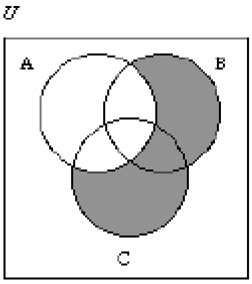
$(A \cup B') \cap (A \cup C)$

$A \cap (B' \cap C) \neq (A \cup B') \cap (A \cup C)$

48.



$A \cap (B \cup C)$



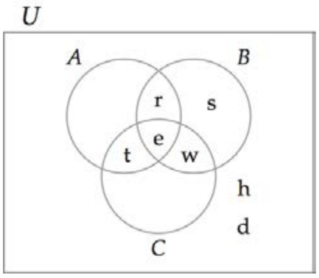
$A' \cap (B \cup C)$

$A \cap (B \cup C) \neq A' \cap (B \cup C)$

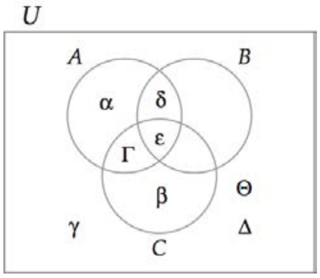
49. $(A \cup B)' \cap C$ or $C \cap (A' \cap B')$

50. $(A \cap B) \cup (B \cap C')$

51.



52.



53. Use a Venn diagram to represent the survey results. Total the numbers from each region to find the number of members surveyed:
111 + 97 + 48 + 135 = 391

54. Use a Venn diagram to represent the survey results: After placing 96 in the intersection of all three types, use the information on customers who like two types of coffee:
116 espresso and cappuccino – 96 like all three = 20
136 espresso and chocolate-flavored – 96 = 40
127 cappuccino and chocolate-flavored – 96=31
221 espresso – (20 + 96 + 40) = 65 who like only espresso.
182 cappuccino – (20 + 96 + 31) = 35 like only cappuccino.
209 chocolate-flavored coffee – (40 + 96 + 31) = 42 who like only chocolate- flavored coffee.

a. 42 customers

b. 31 customers

c. 20 customers

d. 65 + 35 + 42 = 142 customers

55. Let O = (athletes playing offense} and D = {athletes playing defense}.

$$\begin{aligned} n(O \cup D) &= n(O) + n(D) - n(O \cap D) \\ 43 &= 27 + 22 - n(O \cap D) \\ 43 &= 49 - n(O \cap D) \\ 6 &= n(O \cap D) \end{aligned}$$

Therefore, 6 athletes play both offense and defense.

56. Let B = {students registered in biology} and let P = {students registered in psychology}.

$$\begin{aligned} n(B \cup P) &= n(B) + n(P) - n(B \cap P) \\ &= 625 + 433 - 184 \\ &= 874 \end{aligned}$$

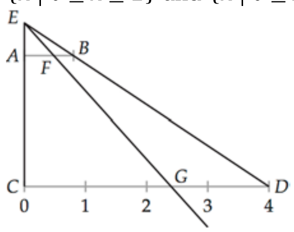
Therefore, 874 students are registered in biology or psychology.

57. One possible one-to-one correspondence between {1, 3, 6, 10} and {1, 2, 3, 4} is given by
$$\begin{array}{c}\{1,3,6,10\}\\ \updownarrow\updownarrow\updownarrow\ \updownarrow\\ \{1,2,3,4\}\end{array}$$

58. $\{x \mid x > 10 \text{ and } x \in N\} = \{11, 12, 13, 14, ..., n + 10, ...\}$
Thus a one-to-one correspondence between the sets is given by
$$\begin{array}{c}\{11,12,13,14,...n+10,...\}\\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow\qquad\ \updownarrow\\ \{2,\ 4,\ 6,\ 8,\ ...,2n,...\}\end{array}$$

59. One possible one-to-one correspondence between the set is given by
$$\begin{array}{c}\{3,\quad 6,\quad 9,\ ...\quad 3n,...\}\\ \updownarrow\ \ \updownarrow\ \ \updownarrow\qquad\ \updownarrow\\ \{10,100,1000,\ ...,10^n,...\}\end{array}$$

60. In the following figure, the line from E that passes through $\overline{AB}$ and $\overline{CD}$ illustrates a method of establishing a one-to-one correspondence between the sets $\{x \mid 0 \le x \le 1\}$ and $\{x \mid 0 \le x \le 4\}$.



61. A proper subset of A is $S = \{10, 14, 18,\ldots,4n + 6, ...\}$. A one-to-one correspondence between A and S is given by
$$\begin{array}{l}A = \{6,10,14,18,...,4n+2,...\}\\ \qquad\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow\qquad\ \updownarrow\\ S = \{10,14,18,22,...,4n+6,...\}\end{array}$$
Because A can be placed in a one-to-one correspondence with a proper subset of itself, A is an infinite set.

62. A proper subset of B is $T = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16},...,\frac{1}{2^n},...\right\}$. A one-to-one correspondence between B and T is given by:
$$\begin{array}{l}B = \left\{1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},...,\frac{1}{2^{n+1}},...\right\}\\ \qquad\ \updownarrow\ \ \updownarrow\ \ \updownarrow\ \ \updownarrow\qquad\ \updownarrow\\ T = \left\{\frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{16},...,\frac{1}{2^n},...\right\}\end{array}$$
Because B can be placed in a one-to-one correspondence with a proper subset of itself, B is an infinite set.

63. 5

64. 10

65. 2

66. 5

67. $\aleph_0$

68. $\aleph_0$

69. c

70. c

71. $\aleph_0$

72. c

CHAPTER 2 TEST

1. $(A \cap B)' = (\{3, 5, 7, 8\} \cap \{2, 3, 8, 9, 10\})'$
$= \{3, 8\}' = \{1, 2, 4, 5, 6, 7, 9, 10\}$

2. $A' \cap B = \{1, 2, 4, 6, 9, 10\} \cap \{2, 3, 8, 9, 10\}$
$= \{2, 9, 10\}$

3. $A' \cup (B \cap C')$
$= \{1, 2, 4, 6, 9, 10\} \cup (\{2, 3, 8, 9\ 10\}$
$\cap \{2, 3, 5, 6, 9, 10\})$
$= \{1, 2, 4, 6, 9, 10\} \cup \{2, 3, 9, 10\}$
$= \{1, 2, 3, 4, 6, 9, 10\}$

4. $A \cap (B' \cup C)$
$= \{3, 5, 7, 8\} \cap (\{1, 4, 5, 6, 7\} \cup \{1, 4, 7, 8\})$
$= \{3, 5, 7, 8\} \cap \{1, 4, 5, 6, 7, 8\}$
$= \{5, 7, 8\}$

5. $\{x \mid x \in W \text{ and } x < 7\}$

6. $\{x \mid x \in I \text{ and } -3 \le x \le 2\}$

7. a. The set of whole numbers less than 4 $= \{0, 1, 2, 3\}$.
$n(\{0, 1, 2, 3\}) = 4$

   b. $\aleph_0$

8. a. Neither, the sets do not have the same number of elements and are not equal.

   b. Equivalent, the sets have the same number of elements but are not equal.

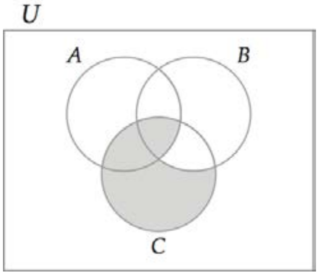
9. a. Equivalent. The set of natural numbers and the set of integers have cardinality $\aleph_0$. The sets are not equal since integers such as –3 and 0 are not natural numbers.

b. Equivalent. Both sets have cardinality $\aleph_0$. The sets are not equal because $0 \in W$ but 0 is not a positive integer.

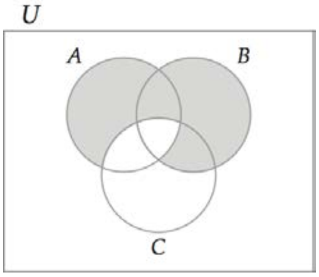
10. Ø, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c,}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}

11. The number of subsets of a set of n elements is 2^n.
2^{21} = 2,097,152 subsets

12.



13.



14. $(A \cup B)' = A \cap B'$ by De Morgan's Laws

15. a. 2^9 = 512 different versions of this sedan

b. Use the method of guess and check to find the smallest natural numbers n for which $2^n > 2500$.

$$2^n > 2500$$
$$2^{10} = 1024$$
$$2^{11} = 2048$$
$$2^{12} = 4096$$

The company must provide a minimum of <u>12</u> upgrade options if it wishes to offer at least 2500 versions of this sedan.

16. Let F = {students receiving financial aid} and let B = {students who are business majors}

$$\begin{aligned} n(F \cup B) &= n(F) + n(B) - n(F \cap B) \\ &= 841 + 525 - 202 \\ &= 1164 \end{aligned}$$

Therefore, 1164 students are receiving financial aid or are business majors.

17. a. {2007, 2008, 2014}

b. {2009, 2010, 2013, 2014}

c. Æ

18. Draw a Venn diagram to represent the data. Fill in the diagram starting from the innermost region.

i:	105
ii:	412 – 105 = 307
iii:	280 – (80 + 105 + 64) = 232
iv:	185 – 105 = 80
v:	724 – (105 + 80 + 307) = 190
vi:	545 – (31 + 105 + 307) = 102
vii:	64
viii:	1000 – (105 + 307 + 31 + 80 + 232 + 102 + 64) = 79

a. this group is represented by v: 232 households

b. this group is represented by vi: 102 households

c. this group is represented by i, ii, iii, iv, v, and vi:
105 + 307 + 31 + 80 + 232 + 102
= 857 households

d. this group is represented by viii: 79 households

19. A possible correspondence:

$$\begin{array}{l} \{5, 10, 15, 20, 25, \ldots, 5n, \ldots\} \\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow \qquad \updownarrow \\ \{0,\ 1,\ 2,\ 3,\ 4, \ldots, n-1, \ldots\} \\ (5n) \leftrightarrow (n-1) \end{array}$$

20. A possible correspondence:

$$\begin{array}{l} \{3,\ 6,\ 9,\ 12, \ldots, 3n, \ldots\} \\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow \qquad \updownarrow \\ \{6, 12, 18, 24, \ldots, n-1, \ldots\} \\ (3n) \leftrightarrow (6n) \end{array}$$

Problem Solving

Section 1.2 Problem Solving with Patterns

Terms of a Sequence

Terms of a Sequence

An ordered list of numbers such as 5, 14, 27, 44, 65, ... is called a **sequence**. The numbers in a sequence that are separated by commas are the **terms** of the sequence.

In the above sequence, 5 is the first term, 14 is the second term, 27 is the third term, 44 is the fourth term, and 65 is the fifth term.

The three dots "..." indicate that the sequence continues beyond 65, which is the last written term. It is customary to use the subscript notation a_n to designate the ***n*th term of a sequence**.

Terms of a Sequence

We often construct a **difference table**, which shows the differences between successive terms of the sequence. The following table is a difference table for the sequence 2, 5, 8, 11, 14, ...

sequence:	2		5		8		11		14		…	
first differences:		3		3		3		3		…		(1)

Each of the numbers in row (1) of the table is the difference between the two closest numbers just above it (upper right number minus upper left number). The differences in row (1) are called the **first differences** of the sequence.

Terms of a Sequence

In this case, the first differences are all the same.

Thus, if we use the above difference table to predict the next number in the sequence, we predict that 14 + 3 = 17 is the next term of the sequence.

This prediction might be wrong; however, the pattern shown by the first differences seems to indicate that each successive term is 3 larger than the preceding term.

Terms of a Sequence

The following table is a difference table for the sequence 5, 14, 27, 44, 65, ...

sequence:	5		14		27		44		65		…	
first differences:		9		13		17		21		…		(1)
second differences:			4		4		4		…			(2)

In this table, the first differences are *not* all the same. In such a situation it is often helpful to compute the successive differences of the first differences. These are shown in row (2).

Terms of a Sequence

These differences of the first differences are called the **second differences**.

The differences of the second differences are called the **third differences**.

To predict the next term of a sequence, we often look for a pattern in a row of differences.

Example 1 – *Predict the Next Term of a Sequence*

Use a difference table to predict the next term in the sequence.

2, 7, 24, 59, 118, 207, ...

Solution:

Construct a difference table as shown below.

sequence:	2		7		24		59		118		207		332		…	
first differences:		5		17		35		59		89		125		…		(1)
second differences:			12		18		24		30		36		…			(2)
third differences:				6		6		6		6		…				(3)

Example 1 – *Solution*

cont'd

The third differences, shown in blue in row (3), are all the same constant, 6. Extending row (3) so that it includes an additional 6 enables us to predict that the next second difference will be 36.

Adding 36 to the first difference 89 gives us the next first difference, 125. Adding 125 to the sixth term 207 yields 332.

Using the method of extending the difference table, we predict that 332 is the next term in the sequence.

*n*th-Term Formula for a Sequence

*n*th-Term Formula for a Sequence

In Example 1 we used a difference table to predict the next term of a sequence.

In some cases we can use patterns to predict a formula, called an ***n*th-term formula**, that generates the terms of a sequence.

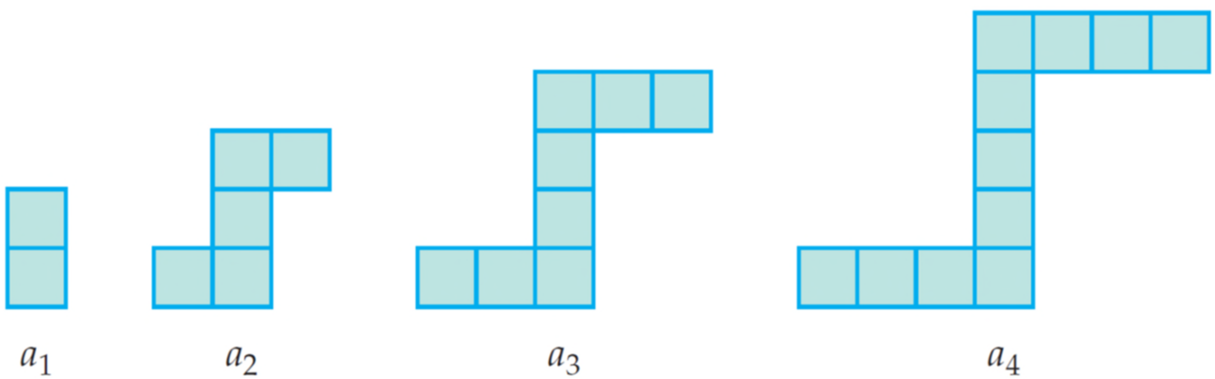
Example 2 – *Find an nth-Term Formula*

Assume the pattern shown by the square tiles in the following figures continues.

a. What is the *n*th-term formula for the number of tiles in the *n*th figure of the sequence?

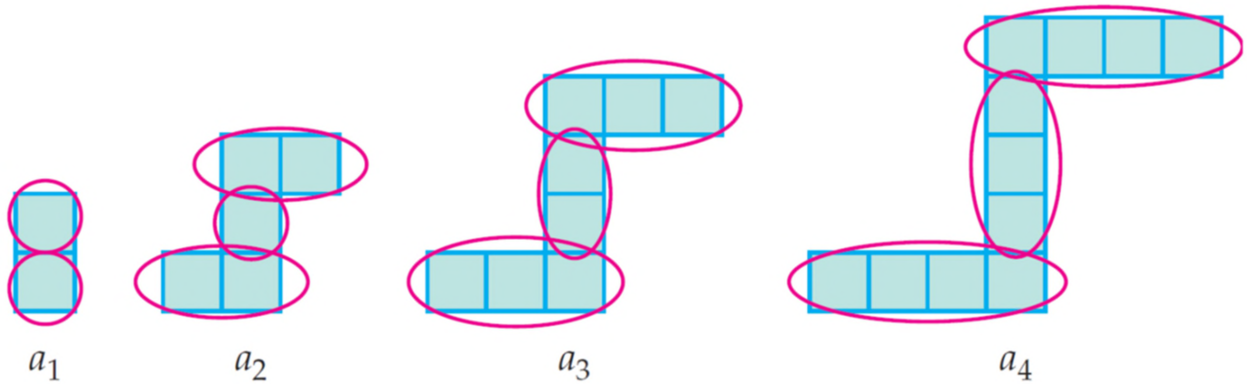
b. How many tiles are in the eighth figure of the sequence?

c. Which figure will consist of exactly 320 tiles?



Example 2 – *Solution*

a. Examine the figures for patterns.



Note that the second figure has two tiles on each of the horizontal sections and one tile between the horizontal sections.

The third figure has three tiles on each horizontal section and two tiles between the horizontal sections.

The fourth figure has four tiles on each horizontal section and three tiles between the horizontal sections.

Example 2 – *Solution*

cont'd

Thus the number of tiles in the nth figure is given by two groups of n plus a group of n less one.

That is,

$$a_n = 2n + (n - 1)$$

$$a_n = 3n - 1$$

b. The number of tiles in the eighth figure of the sequence is $3(8) - 1 = 23$.

c. To determine which figure in the sequence will have 320 tiles, we solve the equation $3n - 1 = 320$.

Example 2 – *Solution*

cont'd

$$3n - 1 = 320$$

$$3n = 321 \qquad \text{Add 1 to each side.}$$

$$n = 107 \qquad \text{Divide each side by 3.}$$

The 107th figure is composed of 320 tiles.

The Fibonacci Sequence

The Fibonacci Sequence

Here is a statement of Fibonacci’s rabbit problem.

At the beginning of a month, you are given a pair of newborn rabbits. After a month the rabbits have produced no offspring; however, every month thereafter, the pair of rabbits produces another pair of rabbits.

The offspring reproduce in exactly the same manner. If none of the rabbits dies, how many pairs of rabbits will there be at the start of each succeeding month?

The Fibonacci Sequence

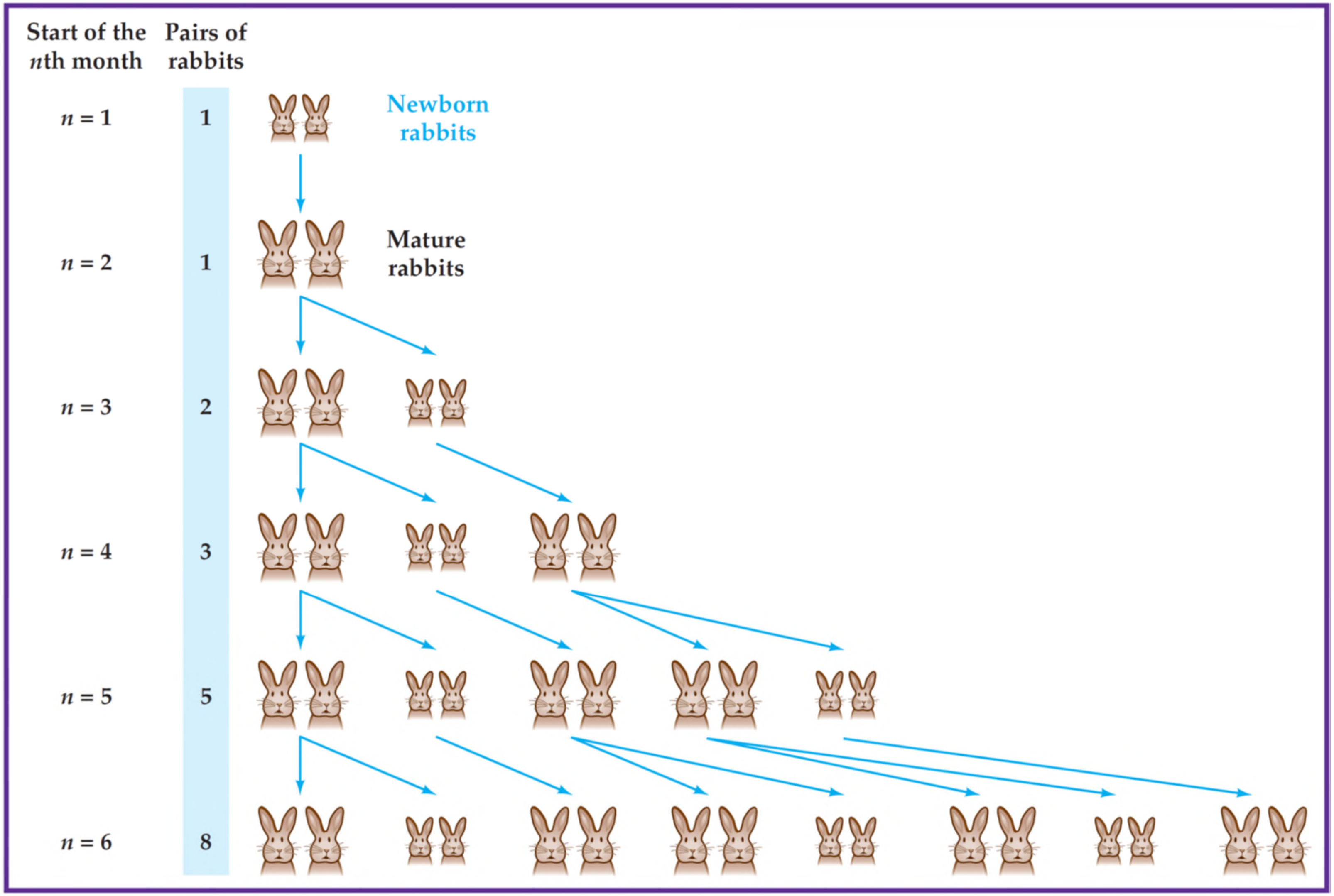
The solution of this problem is a sequence of numbers that we now call the **Fibonacci sequence.**

The following figure shows the numbers of pairs of rabbits on the first day of each of the first six months.

The larger rabbits represent mature rabbits that produce another pair of rabbits each month.

The numbers in the blue region—1, 1, 2, 3, 5, 8—are the first six terms of the Fibonacci sequence.

The Fibonacci Sequence



The Fibonacci Sequence

Fibonacci discovered that the number of pairs of rabbits for any month after the first two months can be determined by adding the numbers of pairs of rabbits in each of the *two previous* months.

For instance, the number of pairs of rabbits at the start of the sixth month is $3 + 5 = 8$.

A **recursive definition** for a sequence is one in which each successive term of the sequence is defined by using some of the preceding terms.

The Fibonacci Sequence

If we use the mathematical notation F_n to represent the *n*th Fibonacci number, then the numbers in the Fibonacci sequence are given by the following recursive definition.

The Fibonacci Numbers

$F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$.

Example 3 – *Find a Fibonacci Number*

Use the definition of Fibonacci numbers to find the seventh and eighth Fibonacci numbers.

Solution:

The first six Fibonacci numbers are 1, 1, 2, 3, 5, and 8. The seventh Fibonacci number is the sum of the two previous Fibonacci numbers.

Thus,

$$F_7 = F_6 + F_5$$

$$= 8 + 5$$

$$= 13$$

Example 3 – *Solution*

cont'd

The eighth Fibonacci number is

$$F_8 = F_7 + F_6$$

$$= 13 + 8$$

$$= 21$$