Chapter 2

2.1 Exercises

1. {1, 3, 5, 7, 9} matches F, the set of odd positive integer less than 10.

2. {x| x is an even integer greater than 4 and less than 6} matches G, the empty set, since there is no even integer larger than 4 and smaller than 6.

3. {..., –4, –3, –2, –1} matches E, the set of all negative integers.

4. {..., –5, –3, –1, 1, 3, 5, ...} matches D, the set of all odd integers.

5. {2, 4, 8, 16, 32} matches B, the set of the five least positive integer powers of 2, since each element represents a successive power of 2 beginning with 2^1.

6. {..., –4, –2, 0, 2, 4, ...} matches A, the set of all even integers.

7. {2, 4, 6, 8, 10} matches H, the set of the five least positive integer multiples of 2, since this set represents the first five positive even integers. Remember that all even numbers are multiples of 2.

8. {2, 4, 6, 8} matches C, the set of even positive integers less than 10.

9. The set of all counting numbers less than or equal to 6 can be expressed as {1, 2, 3, 4, 5, 6}.

10. The set of all whole numbers greater than 8 and less than 18 can be expressed as {9, 10, 11, 12, 13, 14, 15, 16, 17}.

11. The set of all whole numbers not greater than 4 can be expressed as {0, 1, 2, 3, 4}.

12. The set of all natural numbers between 4 and 14 can be expressed as {5, 6, 7, 8, 9, 10, 11, 12, 13}.

13. In the set {6, 7, 8, ..., 14}, the ellipsis (three dots) indicates a continuation of the pattern. A complete listing of this set is {6, 7, 8, 9, 10, 11, 12, 13, 14}.

14. The set {3, 6, 9, 12, ..., 30} contains all multiples of 3 from 3 to 30 inclusive. A complete listing of this set is {3, 6, 9, 12, 15, 18, 21, 24, 27, 30}.

15. The set {2, 4, 8, ..., 256} contains all powers of two from 2 to 256 inclusive. A complete listing of this set is {2, 4, 8, 16, 32, 64, 128, 256}.

16. In the set {90, 87, 84, ..., 69}, each member after the first is found by subtracting 3 from the previous member. The set contains multiples of 3 in decreasing order, from 90 to 69 inclusive. A complete listing is {90, 87, 84, 81, 78, 75, 72, 69}.

17. A complete listing of the set {x| x is an even whole number less than 11} is {0, 2, 4, 6, 8, 10}. Remember that 0 is the first whole number.

18. The set {x| x is an odd integer between –8 and 7} can be represented as {–7, –5, –3, –1, 1, 3, 5}.

19. The set of all multiples of 20 that are greater than 200 is represented by the listing {220, 240, 260, ...}.

20. The set {x| x is a negative multiple of 6} is represented by the listing {–6, –12, –18, –24, –30, ...}.

21. The set of U. S. Great Lakes is represented by {Lake Erie, Lake Huron, Lake Michigan, Lake Ontario, Lake Superior}.

22. The set of United States presidents who served after Richard Nixon and before Barack Obama is represented by {George W. Bush, William Clinton, George H.W. Bush, Ronald Reagan, Jimmy Carter, Gerald Ford}.

23. The set {x| x is the reciprocal of a natural number} is represented by the listing $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...\right\}$.

24. The set {x| x is a positive integer power of 4} is represented by the listing {4, 16, 64, 256, 1024, ...} since $4^1 = 4,$ $4^2 = 16,$ and so forth.

25. The set $\{(x, y) | x$ and y are whole numbers and $x^2 + y^2 = 25\}$ can be represented as $\{(0, 5), (3, 4), (4, 3), (5, 0)\}$.

26. The set $\{(x, y) | x$ and y are integers, and $x^2 = 9y^2 + 16\}$ can be represented as $\{(5,1),\ (5,-1),\ (-5,1),\ (-5,-1),\ (4,0),\ (-4,0)\}$.

Note that in Exercises 27–34, there are other ways to describe the sets.

27. The set of all rational numbers may be represented using set-builder notation as $\{x | x$ is a rational number$\}$.

28. The set of all even natural numbers may be represented using set-builder notation as $\{x | x$ is an even natural number$\}$.

29. The set $\{1, 3, 5, ..., 75\}$ may be represented using set-builder notation as $\{x | x$ is an odd natural number less than $76\}$.

30. The set $\{35, 40, 45, ..., 95\}$ may be represented using set-builder notation as $\{x | x$ is a multiple of 5 between 30 and $100\}$.

31. $\{-9, -8, -7, ..., 7, 8, 9\}$ is the set of single-digit integers.

32. $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ...\right\}$ is the set of positive rational numbers with denominator = numerator + 1.

33. {Alabama, Alaska, Arizona, ..., Wisconsin, Wyoming} is the set of states of the United States.

34. {Alaska, California, Hawaii, Oregon, Washington} is the set of U.S. states that touch the Pacific Ocean.

35. The set $\{2, 4, 6, ..., 932\}$ is finite since the cardinal number associated with this set is a whole number.

36. The set $\{6, 12, 18\}$ is finite since the cardinal number is a whole number.

37. The set $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$ is infinite since there is no last element, and therefore its cardinal number is not a whole number.

38. The set $\{3, 6, 9, \ldots\}$ is infinite, since there is no last element, and therefore its cardinal number is not a whole number.

39. The set $\{x | x$ is a natural number greater than $50\}$ is infinite since there is no last element, and therefore its cardinal number is not a whole number.

40. The set $\{x | x$ is a natural number less than $50\}$ is finite since its cardinal number (49) is a whole number.

41. The set $\{x | x$ is a rational number$\}$ is infinite since there is no last element, and therefore its cardinal number is not a whole number.

42. The set $\{x | x$ is a rational number between 0 and $1\}$ is an infinite set. One could never finish counting the elements of this set because between every two rational numbers, we can always find another rational number.

43. For any set A, $n(A)$ represents the cardinal number of the set, that is, the number of elements in the set. The set $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ contains 8 elements. Thus, $n(A) = 8$.

44. The set $A = \{-3, -1, 1, 3, 5, 7, 9\}$ contains 7 elements. Thus, $n(A) = 7$.

45. The set $A = \{2, 4, 6, ..., 1000)$ contains 500 elements. Thus, $n(A) = 500$.

46. The set $A = \{0, 1, 2, 3, ..., 2000\}$ contains 2001 elements. Thus, $n(A) = 2001$.

47. The set $A = \{$a, b, c, ..., z$\}$ has 26 elements (letters of the alphabet). Thus, $n(A) = 26$.

48. The set $\{x | x$ is a vowel in the English alphabet$\}$ has 5 members since there are 5 vowels, a, e, i, o, and u. Thus, $n(A) = 5$.

49. The set A = the set of integers between -20 and 20 has 39 members. The set can be indicated as $\{-19, -18, ..., 18, 19\}$, or 19 negative integers, 19 positive integers, and 0. Thus, $n(A) = 39$.

50. The set A = set of current US senators has 100 members (two from each state). Thus, $n(A) = 100$.

51. The set $A = \left\{\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \ldots, \frac{27}{29}, \frac{28}{30}\right\}$ has 28 elements. Thus, $n(A) = 28$.

52. The set $A = \left\{\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, \ldots, \frac{1}{10}, -\frac{1}{10}\right\}$ has 18 (nine negative and nine positive) elements. Thus, $n(A) = 18$.

53. Writing exercise; answers will vary.

54. Writing exercise; answers will vary.

55. The set $\{x | x$ is a real number$\}$ is well defined since we can always tell if a number is real and belongs to this set.

56. The set $\{x | x$ is a good athlete$\}$ is not well defined since set membership, in this case, is a value judgment, and there is no clear-cut way to determine whether a particular athlete is "good."

57. The set $\{x | x$ is a difficult course$\}$ is not well defined since membership is a value judgment, and there is no clear-cut way to determine whether a particular course is "difficult."

58. The set $\{x | x$ is a counting number less than 2$\}$ is well defined since we can always tell if a number satisfies the conditions and, hence, belongs to this set. Note that there is only one element, 1, in the set.

59. $3 \boxed{\notin} \{2, 4, 5, 7\}$ since 3 is not a member of the set.

60. $-4 \boxed{\notin} \{4, 7, 8, 12\}$ since -4 is not contained in the set.

61. $8 \boxed{\in} \{3, 8, 12, 18\}$ because 8 is a member of the set.

62. $0 \boxed{\in} \{-2, 0, 5, 9\}$ since 0 is a member of the set.

63. $8 \boxed{\in} \{10 - 2, 10\}$ since $8 = 10 - 2$.

64. $\{6\} \boxed{\notin} \{6, 7\}$ (which is equivalent to the original set) because the set $\{6\}$ is not a member of the second set even though the number 6 is.

65. False; the set $\{0\}$ is nonempty.

66. $9 - 6 = 3$, so 3 is an element of the set.

67. The statement $3 \in \{2, 5, 6, 8\}$ is false since the element 3 is not a member of the set.

68. The statement m $\in$ {l, m, n, o, p} is true since m is contained in the set.

69. The statement c $\in$ {c, d, a, b} is true since c is contained in the set.

70. The statement $2 \in \{-2, 5, 8, 9\}$ is false since the number 2 is not a member of the set.

71. The statement {k, c, r, a} = {k, c, a, r} is true since both sets contain exactly the same elements.

72. The statement {e, h, a, n} = {a, h, e, n} is true since both sets contain exactly the same elements.

73. The statement $\{5, 8, 9\} = \{5, 8, 9, 0\}$ is false because the second set contains a different element from the first set, 0.

74. The statement $\{3, 7\} = \{3, 7, 0\}$ is false because the second set contains a different element from the first set, 0.

75. The statement $\{4\} \in \{\{3\}, \{4\}, \{5\}\}$ is true since the element, $\{4\}$, is a member of the set.

76. The statement $4 \in \{\{3\}, \{4\}, \{5\}\}$ is false since the element, 4, is not a member of the set. Rather $\{4\}$ is a member.

77. The statement $\{x | x$ is a natural number less than 3$\}$ $= \{1, 2\}$ is true since both represent sets with exactly the same elements.

78. The statement $\{x | x$ is a natural number greater than 10$\} = \{11, 12, 13, \ldots\}$ is true since both represent sets with exactly the same elements.

79. The statement $4 \in A$ is true since 4 is a member of set A.

80. The statement $10 \in B$ is true since 10 is a member of set B.

81. The statement $4 \notin C$ is false since 4 is a member of the set C.

82. The statement $10 \notin A$ is false since 10 is a member of the set A.

83. Every element of C is also an element of A is true since the members, 4, 10, and 12 of set C, are also members of set A.

84. The statement, every element of C is also an element of B, is false since the element, 12, of set C is not also an element of set B.

85. Writing exercise; answers will vary.

86. Writing exercise; answers will vary.

87. An example of two sets that are not equivalent and not equal would be {2} and {3, 4}. Other examples are possible.

88. An example of two sets that are equal but not equivalent is impossible. If they are equal, they have the same number of elements and must be equivalent.

89. An example of two sets that are equivalent but not equal would be {a, b} and {a, c}. Other examples are possible.

90. Two sets that are both equal and equivalent would be {5} and {4 + 1}. Other examples are possible.

91. **(a)** All sets of hires that would include two RNs and one LVN are
$\{\text{Bernice, Heather, Marcy}\}$,
$\{\text{Bernice, Heather, Natalie}\}$,
$\{\text{Bernice, Susan, Marcy}\}$,
$\{\text{Bernice, Susan, Natalie}\}$,
$\{\text{Heather, Susan, Marcy}\}$, and
$\{\text{Heather, Susan, Natalie}\}$.

(b) All sets of hires that would include one RN and two LVNs are
$\{\text{Bernice, Marcy, Natalie}\}$,
$\{\text{Heather, Marcy, Natalie}\}$, and
$\{\text{Susan, Marcy, Natalie}\}$.

(c) The only set of hires that includes no LVNs is $\{\text{Bernice, Heather, Susan}\}$.

92. **(a)** Since there are 220 calories in the cotton candies that Alexis likes, she must burn off $3 \times 220 = 660$ calories. The following sets indicate activities that will burn off the required number of calories and will take no more than two hours: {r}, {g, s}, {c, s}, {v, r}, {g, r}, {c, r}, and {s, r}.

(b) The required number of calories to burn off is $5 \times 220 = 1100$ calories. The following sets indicated activities taking less than or equal to three hours and burning off the required number of calories: {v, g, r}, {v, c, r}, {v, s, r}, {g, c, r}, {g, s, r}, and {c, s, r}.

(c) The required number of calories to burn off is $7 \times 220 = 1540$ calories. The following sets indicate activities taking less than or equal to four hours and burning off the required number of calories: $\{v, c, s, r\}$ and $\{g, c, s, r\}$.

2.2 Exercises

1. {p}, {q}, {p, q}, ∅ matches D, the subsets of {p, q}.

2. {p}, {q}, ∅ matches A, the proper subsets of {p, q}. Note that the set {p, q}, itself, though a subset, is not a proper subset.

3. {a, b} matches B, the complement of {c, d}, if U = {a, b, c, d}.

4. ∅ matches C, the complement of U.

5. $\{-2, 0, 2\} \not\subseteq \{-2, -1, 1, 2\}$

6. $\{M, W, F\} \not\subseteq \{S, M, T, W, Th\}$ since the element "F" is not a member of the second set.

7. $\{2, 5\} \subseteq \{0, 1, 5, 3, 7, 2\}$

8. $\{a, n, d\} \subseteq \{r, a, n, d, y\}$

9. $\emptyset \subseteq \{a, b, c, d, e\}$, since the empty set is considered a subset of any given set.

10. $\emptyset \subseteq \emptyset$, since the empty set is considered a subset of every set including itself.

11. $\{-5, 2, 9\} \not\subseteq \{x \mid x \text{ is an odd integer}\}$ since the element "2" is not an element of the second set.

12. $\{1, 2, \frac{9}{3}\} \subseteq$ the set of rational numbers since 1, 2, and $\frac{9}{3}$ are rational numbers.

13. $\{P, Q, R\} \subset \{P, Q, R, S\}$ and $\{P, Q, R\} \subseteq \{P, Q, R, S\}$, i.e. both.

14. {red, blue, yellow} $\subseteq$ {yellow, blue, red}.

15. $\{9, 1, 7, 3, 5\} \subseteq \{1, 3, 5, 7, 9\}$

16. {S, M, T, W, Th} $\not\subseteq$ {W, E, E, K}; therefore, neither.

17. $\varnothing \subset \{0\}$ and $\varnothing \subseteq \{0\}$, i.e., both.

18. $\varnothing \subseteq \varnothing$ only.

19. $\{0, 1, 2, 3\} \not\subseteq \{1, 2, 3, 4\}$; therefore, neither. Note that if a set is not a subset of another set, it cannot be a proper subset either.

20. $\left\{\frac{5}{6}, \frac{9}{8}\right\} \not\subseteq \left\{\frac{6}{5}, \frac{8}{9}\right\}$ since neither member of the first set is a member of the second. Thus, neither is the correct answer.

21. $A \subset U$ is true since all sets must be subsets of the universal set by definition, and U contains at least one more element than A.

22. $C \not\subset U$ is false since all sets must be subsets of the universal set by definition.

23. $D \subseteq B$ is false since the element "d" in set D is not also a member of set B.

24. $D \not\subseteq A$ is true since D contains an element not contained in A.

25. $A \subset B$ is true. All members of A are also members of B, and there are elements in set B not contained in set A.

26. $B \subseteq C$ is false since the elements "a" and "e" in set B are not also members of set C.

27. $\varnothing \not\subset A$ is false since $\varnothing$ is a subset of all sets.

28. $\varnothing \subseteq D$ is true since the empty set, $\varnothing$, is considered a subset of all sets.

29. $D \not\subseteq B$ is true. Set D is not a subset of B because the element "d," though a member of set D, is not also a member of set B.

30. $A \not\subseteq B$ is false. Set A is a subset of set B since all of the elements in set A are also in set B.

31. There are exactly 6 subsets of C is false. Since there are 3 elements in set C, there are $2^3 = 8$ subsets.

32. There are exactly 31 subsets of B is false. Since there are 5 elements in set B, there are $2^5 = 32$ subsets.

33. There are exactly 3 proper subsets of A is true. Since there are 2 elements in set A, there are $2^2 = 4$ subsets, and one of those is the set A itself, so there are 3 proper subsets of A.

34. There are exactly 4 subsets of D is true. Since there are 2 elements in set D, there are $2^2 = 4$ subsets.

35. The Venn diagram does not represent the correct relationships among the sets since D is not a subset of A. Thus, the answer is false.

36. The Venn diagram shows that C is a subset of B and that B is a subset of U. This is a correct relationship since all members of set C are also members of set B, and B is a subset of U. Thus, the answer is true.

37. Since the given set has 6 elements, there are
(a) $2^6 = 64$ subsets, and
(b) $2^6 - 1 = 63$ proper subsets.

38. Since the given set has 7 elements, which are the days of the week, there are
(a) $2^7 = 128$ subsets and
(b) $2^7 - 1 = 127$ proper subsets.

39. The set $\{x \mid x$ is an odd integer between -4 and $6\}$ $= \{-3, -1, 1, 3, 5\}$. Since the set contains 5 elements, there are (a) $2^5 = 32$ subsets and (b) $2^5 - 1 = 32 - 1 = 31$ proper subsets.

40. The set $\{x| \, x \text{ is an even whole number less than } 4\} = \{0, 2\}$. Since the set contains 2 elements, there are (a) $2^2 = 4$ subsets and (b) $2^2 - 1 = 3$ proper subsets.

41. The complement of the universal set, U, is the empty set, $\emptyset$.

42. The complement of $\emptyset$, the empty set, is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, the universal set.

43. The complement of $\{1, 2, 3, 4, 6, 8\}$ is $\{5, 7, 9, 10\}$, that is, all of the elements in U not also in the given set.

44. The complement of $\{2, 5, 9, 10\}$ is $\{1, 3, 4, 6, 7, 8\}$.

45. In order to contain all of the indicated considerations, the universal set U = {Higher cost, Lower cost, Educational, More time to see the sights in California, Less time to see the sights in California, Cannot visit friends along the way, Can visit friends along the way}.

46. Since F contains the considerations of the flying option, F' = {Lower cost, Less time to see the sights in California, Can visit friends along the way}.

47. Since D contains the set of considerations of the driving option, D' = {Higher cost, More time to see the sights in California, Cannot visit friends along the way}.

48. The set of element(s) common to set F and D is {Educational}.

49. The set of element(s) common to F' and D' is $\emptyset$, the empty set, since there are no common elements.

50. The set of element(s) common to F and D' is {Higher cost, More time to see the sights in California, Cannot visit friends along the way}.

51. The only possible set is $\{A, B, C, D, E\}$. (All are present.)

52. The possible subsets of four people would include $\{A, B, C, D\}$, $\{A, B, C, E\}$, $\{A, B, D, E\}$, $\{A, C, D, E\}$, and $\{B, C, D, E\}$.

53. The possible subsets of three people would include $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, E\}$, $\{A, C, D\}$, $\{A, C, E\}$, $\{A, D, E\}$, $\{B, C, D\}$, $\{B, C, E\}$, $\{B, D, E\}$, and $\{C, D, E\}$.

54. The possible subsets of two people would include $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, E\}$, $\{B, C\}$, $\{B, D\}$, $\{B, E\}$, $\{C, D\}$, $\{C, E\}$, and $\{D, E\}$.

55. The possible subsets consisting of one person would include $\{A\}$, $\{B\}$, $\{C\}$, $\{D\}$, and $\{E\}$.

56. The set indicating that no people get together (no one shows up) is $\emptyset$.

57. Adding the number of subsets in Exercises 51−56, we have $1 + 5 + 10 + 10 + 5 + 1 = 32$ ways that the group can gather.

58. They are the same: $32 = 2^5$. The number of ways that people, from a group of five, can gather is the same as the number of subsets there are of a set of five elements.

59. Because at least one member must attend, sending no members (the empty set) is not a possible subset of the members that can be sent. So the total number of different delegations that can possibly be sent is $2^{25} - 1 = 33{,}554{,}431$.

60. If ten of the club members do not want to be part of the delegation, then only 15 members can be considered. At least one member must attend, so the total number of possible delegations is $2^{15} - 1 = 32{,}767$.

61. **(a)** Consider all possible subsets of a set with four elements (the number of bills). The number of subsets would be $2^4 = 16$. Since 16 includes also the empty set (and we must choose one bill), we will subtract one from this or $16 - 1 = 15$ possible sums of money.

(b) Removing the condition says, in effect, that we may also choose no bills. Thus, there are $2^4 = 16$ subsets or possible sums of money; it is now possible to select no bills.

62. (a) Consider all possible subsets of a set with five elements (the number of coins). The number of subsets would be $2^5 = 32$. But the 32 includes also the empty set, and we must select at least one coin. Thus, there are $32 - 1 = 31$ sums of coins.

(b) Removing the condition says, in effect, that we may also choose no coins. Thus, there are $2^5 = 32$ subsets or possible sums of coins including no coins.

63. (a) There are s subsets of B that do not contain e. These are the subsets of the original set A.

(b) There is one subset of B for each of the original subsets of set A, which is formed by including e as the element of that subset of A. Thus, B has s subsets which do contain e.

(c) The total number of subsets of B is the sum of the numbers of subsets containing e and of those not containing e. This number is $s + s$ or $2s$.

(d) Adding one more element will always double the number of subsets, so we conclude that the formula 2^n is true in general.

64. Writing exercise; answers will vary.

2.3 Exercises

1. The intersection of A and B, $A \cap B$, matches B, the set of elements common to both A and B.

2. The union of A and B, $A \cup B$, matches F, the set of elements that are in A or in B or in both A and B.

3. The difference of A and B, $A - B$, matches A, the set of elements in A that are not in B.

4. The complement of A, A', matches C, the set of elements in the universal set that are not in A.

5. The Cartesian product of A and B, $A \times B$, matches E, the set of ordered pairs such that each first element is from A and each second element is from B, with every element of A paired with every element of B.

6. The difference of B and A, $B - A$, matches D, the set of elements of B that are not in A.

7. $X \cap Y = \{a, c\}$ since these are the elements that are common to both X and Y.

8. $X \cup Y = \{a, b, c, e, g\}$ since these are the elements that are contained in X or Y (or both).

9. $Y \cup Z = \{a, b, c, d, e, f\}$ since these are the elements that are contained in Y or Z (or both).

10. $Y \cap Z = \{b, c\}$ since these are the elements that are common to both Y and Z.

11. $X' = \{b, d, f\}$ since these are the only elements in U not contained in X.

12. $Y' = \{d, e, f, g\}$ since these are the only elements in U not contained in Y.

13. $X' \cap Y' = \{b, d, f\} \cap \{d, e, f, g\} = \{d, f\}$

14. $X' \cap Z = \{b, d, f\} \cap \{b, c, d, e, f\} = \{b, d, f\}$

15. $X \cup (Y \cap Z) = \{a, c, e, g\} \cup \{b, c\}$
$= \{a, b, c, e, g\}$
Observe that the intersection must be done first.

16. $Y \cap (X \cup Z) = \{a, b, c\} \cap \{a, b, c, d, e, f, g\}$
$= \{a, b, c\}$
Observe that the union must be done first.

17. $X - Y = \{e, g\}$
Since these are the only two elements that belong to X and not to Y.

18. $Y - X = \{b\}$
Since this is the only element that belongs to Y and not to X.

19. $(Z \cup X')' \cap Y$
$= (\{b, c, d, e, f\} \cup \{b, d, f\})' \cap \{a, b, c\}$
$= \{b, c, d, e, f\}' \cap \{a, b, c\}$
$= \{a, g\} \cap \{a, b, c\}$
$= \{a\}$

20. $(Y \cap X')' \cup Z'$
$= (\{a, b, c\} \cap \{b, d, f\})' \cup \{a, g\}$
$= \{b\}' \cup \{a, g\}$
$= \{a, c, d, e, f, g\} \cup \{a, g\}$
$= \{a, c, d, e, f, g\}$

21. $X \cap (X - Y) = \{a, c, e, g\} \cap \{e, g\} = \{e, g\}$
Observe that we must find $X - Y$ first.

22. $Y \cup (Y - X) = \{a, b, c\} \cup \{b\} = \{a, b, c\}$

23. $X' - Y = \{b, d, f\} - \{a, b, c\} = \{d, f\}$
Observe that we must find X' first.

24. $Y' - (X \cap Z) = \{d, e, f, g\} - \{c, e\} = \{d, f, g\}$
Observe that we must find Y' first.

25. $A \cup (B' \cap C')$ is the set of all elements that are in A, or are not in B and not in C.

26. $(A \cap B') \cup (B \cap A')$ is the set of all elements that are in A but not in B, or in B but not in A.

27. $(C - B) \cup A$ is the set of all elements that are in C but not in B, or they are in A.

28. $(A' \cap B') \cup C'$ is the set of all elements that are not in A and not in B, or are not in C.

29. The smallest set representing the universal set U is $\{e, h, c, l, b\}$.

30. $T \cap A$ is the set of adverse effects of both tobacco and alcohol use: $T \cap A = \{h\}$.

31. $T \cup A$ is the set of all adverse effects that are either tobacco related or alcohol related: $T \cup A = \{e, h, c, l, b\} = U$.

32. $T \cap A'$ is the set of adverse tobacco related effects that are not alcohol related: $T \cap A' = \{e, c\}$.

33. $C - A$ is the set of all tax returns filed in 2014 without itemized deductions.

34. $D \cup A'$ is the set of all tax returns selected for audit or without itemized deductions.

35. $(A \cup B) - D$ is the set of all tax returns with itemized deductions or showing business income, but not selected for audit.

36. $(C \cap A) \cap B'$ is the set of all tax returns filed in 2014 with itemized deductions but not showing business income.

37. $(A \cap B) \subseteq A$ is always true since the elements of $A \cap B$ must be in A.

38. $A \subseteq (A \cap B)$ is not always true since $A \cap B$ may not contain all the elements of A.

39. $n(A \cup B) = n(A) + n(B)$ is not always true. If there are any common elements to A and B, they will be counted twice.

40. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ is always true, since any elements common to sets A and B which are counted twice by $n(A) + n(B)$ are returned to a single count by the subtraction of $n(A \cap B)$.

41. **(a)** $X \cup Y = \{1, 2, 3, 5\}$

(b) $Y \cup X = \{1, 2, 3, 5\}$

(c) For any sets X and Y, $X \cup Y = Y \cup X$. This conjecture indicates that set union is a commutative operation.

42. **(a)** $X \cap Y = \{1, 3\}$

(b) $Y \cap X = \{1, 3\}$

(c) For any sets X and Y, $X \cap Y = Y \cap X$. This conjecture indicates that set intersection is a commutative operation.

43. **(a)** $X \cup (Y \cup Z)$
$= \{1, 3, 5\} \cup (\{1, 2, 3\} \cup \{3, 4, 5\})$
$= \{1, 3, 5\} \cup \{1, 2, 3, 4, 5\}$
$= \{1, 3, 5, 2, 4\}$

(b) $(X \cup Y) \cup Z$
$= (\{1, 3, 5\} \cup \{1, 2, 3\}) \cup \{3, 4, 5\}$
$= \{1, 3, 5, 2\} \cup \{3, 4, 5\}$
$= \{1, 3, 5, 2, 4\}$

(c) For any sets X, Y, and Z, $X \cup (Y \cup Z) = (X \cup Y) \cup Z$. This conjecture indicates that set union is an associative operation.

44. **(a)** $X \cap (Y \cap Z)$
$= \{1, 3, 5\} \cup (\{1, 2, 3\} \cap \{3, 4, 5\})$
$= \{1, 3, 5\} \cap \{3\}$
$= \{3\}$

(b) $(X \cap Y) \cap Z$
$= (\{1, 3, 5\} \cap \{1, 2, 3\}) \cap \{3, 4, 5\}$
$= \{1, 3\} \cap \{3, 4, 5\}$
$= \{3\}$

(c) For any sets X, Y, and Z,
$X \cap (Y \cap Z) = (X \cap Y) \cap Z$.
This conjecture indicates that set intersection is an associative operation.

45. The statement $(3, 2) = (5 - 2, 1 + 1)$ is true.

46. The statement $(2, 13) = (13, 2)$ is false. Corresponding elements in ordered pairs must be equal.

47. The statement $\{6, 3\} = \{3, 6\}$ is true since order is not important when listing elements in sets.

48. The statement $\{(5, 9), (4, 8), (4, 2)\} = \{(4, 8), (5, 9), (2, 4)\}$ false, because the ordered pair (4, 2) is different from the ordered pair (2, 4). Thus the two sets do not contain the same elements.

49. For $A = \{d, o, g\}$ and $B = \{p, i, g\}$,
$A \times B = \{(d, p), (d, i), (d, g), (o, p), (o, i), (o, g), (g, p), (g, i), (g, g)\}$;
$B \times A = \{(p, d), (p, o), (p, g), (i, d), (i, o), (i, g), (g, d), (g, o), (g, g)\}$.

50. For $A = \{3, 6, 9, 12\}$ and $B = \{6, 8\}$,
$A \times B = \{(3, 6), (3, 8), (6, 6), (6, 8), (9, 6), (9, 8), (12, 6), (12, 8)\}$;
$B \times A = \{(6, 3), (6, 6), (6, 9), (6, 12), (8, 3), (8, 6), (8, 9), (8, 12)\}$.

51. For $n(A) = 35$ and $n(B) = 6$,
$n(A \times B) = n(A) \times n(B) = 35 \times 6 = 210$
$n(B \times A = n(B) \times n(A) = 6 \times 35 = 210$

52. For $n(A) = 13$ and $n(B) = 5$,
$n(A \times B) = n(A) \times n(B) = 13 \times 5 = 65$
$n(B \times A) = n(B) \times n(A) = 5 \times 13 = 65$

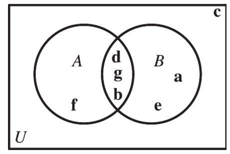
53. To find $n(B)$ when $n(A \times B) = 72$ and $n(A) = 12$, we have:

$$n(A \times B) = n(A) \times n(B)$$
$$72 = 12 \times n(B)$$
$$6 = n(B)$$

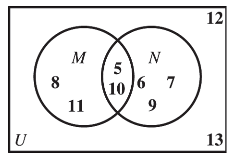
54. To find $n(A)$ when $n(A \times B) = 300$ and $n(B) = 30$, we have:

$$n(A \times B) = n(A) \times n(B)$$
$$300 = n(A) \times 30$$
$$10 = n(A)$$

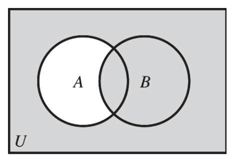
55. Let $U = \{a, b, c, d, e, f, g\}$, $A = \{b, d, f, g\}$, and $B = \{a, b, d, e, g\}$.



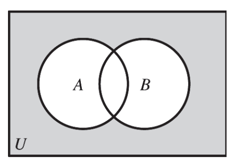
56. Let $U = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$, $M = \{5, 8, 10, 11\}$, and $N = \{5, 6, 7, 9, 10\}$.



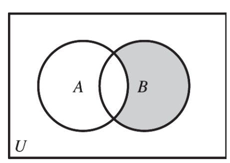
57. The set operations for $A' \cup B$ indicate those elements not in A or in B.



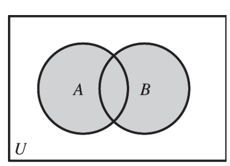
58. The set operations for $A' \cap B'$ indicate those elements not in A and, at the same time, not in B.



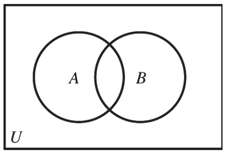
59. The set operations for $B \cap A'$ indicate those elements in B and not in A.



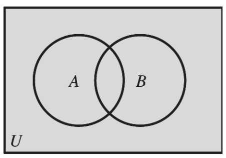
60. The set operation for $A \cup B$ indicates those elements in A or in B.



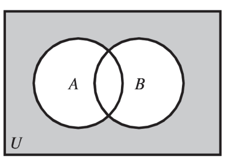
61. The set operations $B' \cap B$ indicate those elements not in B and in B at the same time, and since there are no elements that can satisfy both conditions, we get the null set (empty set), $\varnothing$.



62. The set operations for $A' \cup A$ indicate all elements not in A along with those in A, that is, the universal set, U.

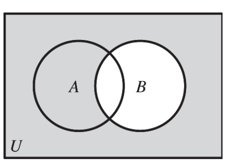


63. The indicated set operations mean those elements not in B or those not in A as long as they are also not in B. It is a help to shade the region representing "not in A" first, then that region representing "not in B." Identify the intersection of these regions (covered by both shadings). As in algebra, the general strategy when deciding which order to do operations is to begin inside parentheses and work out.

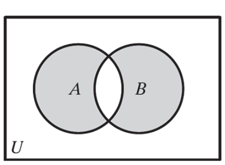


$A' \cap B'$

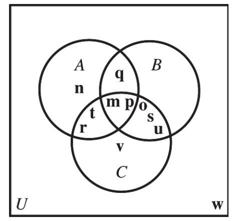
Finally, the region of interest will be that "not in B" along with (union of) the above intersection—$(A' \cap B')$. That is, the final region of interest is given by



64. The set operations $(A - B) \cup (B - A)$ indicate all those elements in A but not in B, along with those elements in B but not in A.

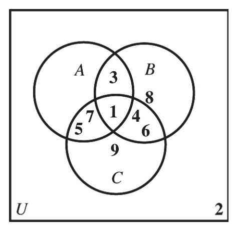


65. Let U = {m, n, o, p, q, r, s, t, u, v, w}, A = {m, n, p, q, r, t}, B = {m, o, p, q, s, u}, and C = {m, o, p, r, s, t, u, v}.
Placing the elements of these sets in the proper location on a Venn diagram will yield the following diagram.

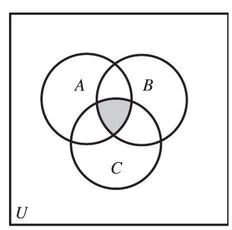


It helps to identify those elements in the intersection of A, B, and C first, then those elements not in this intersection but in each of the two set intersections (e.g., $A \cap B$, etc.), next, followed by elements that lie in only one set, etc.

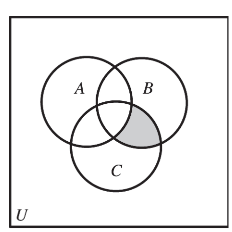
66. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 3, 5, 7}, B = {1, 3, 4, 6, 8}, and C = {1, 4, 5, 6, 7, 9}.
Placing the elements of these sets in the proper location on a Venn diagram will yield the following diagram.



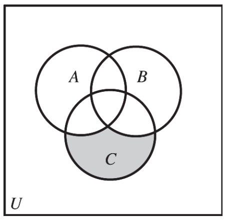
67. The set operations $(A \cap B) \cap C$ indicate those elements common to all three sets.



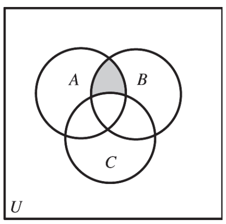
68. The set operations $(A' \cap B) \cap C$ indicate those elements not in A but which are in B and also in C.



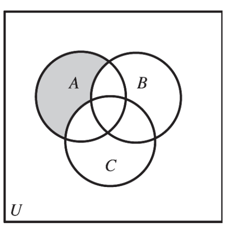
69. The set operations $(A' \cap B') \cap C$ indicate those elements that are in C while simultaneously outside of both A and B.



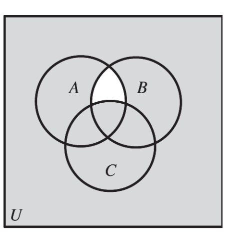
70. The set operations $(A \cap C') \cap B$ indicate those elements which are in A and outside of C while at the same time, inside B.



71. The set operations $(A \cap B') \cap C'$ indicate the region in A and outside B and at the same time outside C.



72. The set operations $(A \cap B)' \cup C$ indicate those elements not common to A and B, along with all elements of C.



73. The shaded area indicates the region $(A \cup B)'$ or $A' \cap B'$.

74. Since only $A - B$ is unshaded, we get the shaded region by $(A - B)'$. This may also be indicated by the set $(A \cap B')'$ or, using one form of De Morgan's law, as $(A' \cup B)$.

75. Since this is the region in A or in B but, at the same time, outside of A and B, we have the set $(A \cup B) \cap (A \cap B)'$ or $(A \cup B) - (A \cap B)$.

76. Since this region represents all elements in A which are outside of B, the area can be indicated by the set $A \cap B'$ or $A - B$.

77. The shaded area may be represented by the set $(A \cap B) \cup (A \cap C)$; that is, the region in the intersection of A and B along with the region in the intersection of A and C or, by the distributive property, $A \cap (B \cup C)$.

78. The shaded area is in A and, at the same time, outside the union of B with C. This region can be represented by the set $A \cap (B \cup C)'$ or $A \cap (B' \cap C')$.

79. If $A = A - B$, then A and B must not have any common elements, or $A \cap B = \emptyset$.

80. $A = B - A$ is true only if $A = B = \emptyset$.

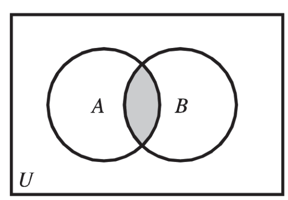
81. $A = A - \emptyset$ is true for any set A.

82. $A \cap \emptyset = \emptyset$ is true for any set A.

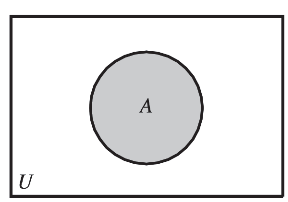
83. $A \cup B = A$ only if B is a subset of A, or $B \subseteq A$.

84. $A \cap B = B$ only if B is a subset of A, or $B \subseteq A$.

85. $(A \cap B) \subseteq A$



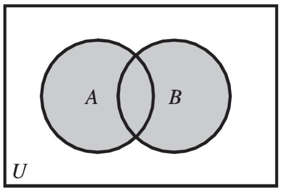
$A \cap B$



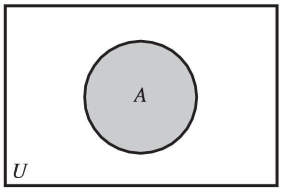
A

Thus, by the Venn diagrams, the shaded region is in A; therefore, the statement is always true.

86. $(A \cup B) \subseteq A$



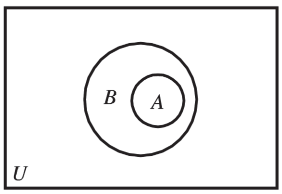
$(A \cup B)$



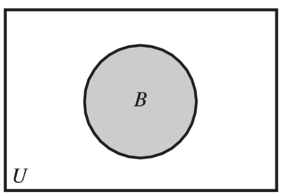
A

Thus, the statement is not always true.

87. If $A \subseteq B$, then $A \cup B = A$.



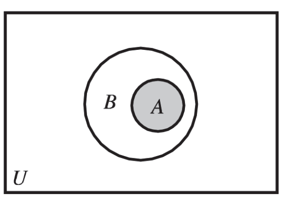
$A \subseteq B$



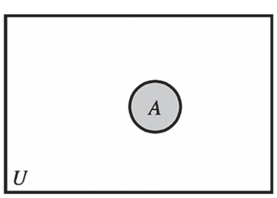
$A \cup B$

Thus, the statement is not always true.

88. If $A \subseteq B$, then $A \cap B = A$.



$A \cap B$



A

Thus, the statement is always true.

89. Writing exercise; answers may vary.

90. Writing exercise; answers may vary.

2.4 Exercises

1. **(a)** $n(A \cap B) = 5$ since A and B have 5 elements in common.

(b) $n(A \cup B) = 7$ since there are a total of 7 elements in A or in B.

(c) $n(A \cap B') = 0$ since there are 0 elements which are in A and, at the same time, outside B.

(d) $n(A' \cap B) = 2$ since there are 2 elements which are in B and, at the same time, outside A.

(e) $n(A' \cap B') = 8$ since there are 8 elements which are outside of A and, at the same time, outside of B.

2. **(a)** $n(A \cap B) = 1$ since A and B share only one element.

(b) $n(A \cup B) = 11$, since there are a total of 11 elements in A or in B.

(c) $n(A \cap B') = 3$ since there are 3 elements which are in A and, at the same time, outside B.

(d) $n(A' \cap B) = 7$ since there are 7 elements which are in B and, at the same time, outside A.

(e) $n(A' \cap B') = 5$ since there are 5 elements which are outside of A and, at the same time, outside of B.

3. **(a)** $n(A \cap B \cap C) = 1$ since there is only one element shared by all three sets.

(b) $n(A \cap B \cap C') = 3$ since there are 3 elements in A and B while, at the same time, outside of C.

(c) $n(A \cap B' \cap C) = 4$ since there are 4 elements in A and C while, at the same time, outside of B.

(d) $n(A' \cap B \cap C) = 0$ since there are 0 elements which are outside of A while, at the same time, in B and C.

(e) $n(A' \cap B' \cap C) = 2$ since there are 2 elements outside of A and outside of B while, at the same time, in C.

(f) $n(A \cap B' \cap C') = 8$ since there are 8 elements in A which at the same time, are outside of B and outside of C.

(g) $n(A' \cap B \cap C') = 2$ since there are 2 elements outside of A and, at the same time, outside of C but inside of B.

(h) $n(A' \cap B' \cap C') = 6$ since there are 6 elements which are outside all three sets at the same time.

4. **(a)** $n(A \cap B \cap C) = 1$ since there is only one element shared by all three sets.

(b) $n(A \cap B \cap C') = 2$ since there are 2 elements in A and B while, at the same time, outside of C.

(c) $n(A \cap B' \cap C) = 4$ since there are 4 elements in A and C while, at the same time, outside of B.

(d) $n(A' \cap B \cap C) = 7$ since there are 7 elements which are outside of A while, at the same time, in B and C.

(e) $n(A' \cap B' \cap C) = 5$ since there are 5 elements outside of A and outside of B while, at the same time, inside of C.

(f) $n(A \cap B' \cap C') = 3$ since there are 3 elements in A which, at the same time, are outside of B and outside of C.

(g) $n(A' \cap B \cap C') = 6$ since there are 6 elements outside of A and, at the same time, outside of C but inside of B.

(h) $n(A' \cap B' \cap C') = 10$ since there are 10 elements which are outside all three sets at the same time.

5. Using the Cardinal Number Formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we have $n(A \cup B) = 12 + 14 - 5 = 21$.

6. Using the Cardinal Number Formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we have $n(A \cup B) = 16 + 28 - 5 = 39$.

7. Using the Cardinal Number Formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we have $25 = 20 + 12 - n(A \cap B)$. Solving for $n(A \cap B)$, we get $n(A \cap B) = 7$.

8. Using the Cardinal Number Formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we have $30 = 20 + 24 - n(A \cap B)$. Solving for $n(A \cap B)$, we get $n(A \cap B) = 14$.

9. Using the Cardinal Number Formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we have $55 = n(A) + 35 - 15$. Solving for $n(A)$, we get $n(A) = 35$.

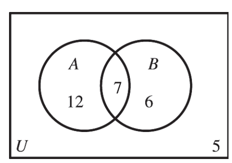
10. Using the Cardinal Number Formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we have $30 = 20 + n(B) - 6$. Solving for $n(B)$, we get $n(B) = 16$.

11. Using the Cardinal Number Formula, we find that

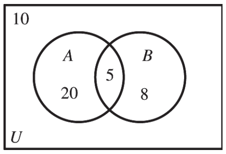
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 25 &= 19 + 13 - n(A \cap B) \\ n(A \cap B) &= 19 + 13 - 25 \\ &= 7. \end{aligned}$$

Then $n(A \cap B') = 19 - 17 = 12$ and $n(B \cap A') = 13 - 7 = 6$.

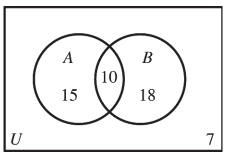
Since $n(A') = 11$ and $n(B \cap A') = 6$, we have $n(A \cup B)' = n(A' \cap B') = 11 - 6 = 5$. Completing the cardinalities for each region, we arrive at the following Venn diagram.



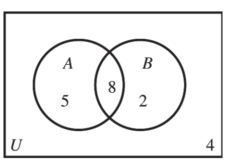
12. Use deduction to complete the cardinalities of the unknown regions. For example since $n(B') = 30$, there are 13 elements in B $[n(U) - n(B') = 43 - 30]$; therefore, 8 elements are in B that are not in A $[n(B) - n(A \cap B) = 13 - 5]$. Since there is a total of 25 elements in A and 5 are accounted for in $A \cap B$, there must be 20 elements in A that are not in B. This leaves a total of 33 elements in the regions formed by A along with B. Thus, there are 10 elements left in U that are not in A or in B. Completing the cardinalities for each region, we arrive at the following Venn diagram.



13. Since $n(B) = 28$ and $n(A \cap B) = 10$, we have $n(B \cap A') = 28 - 10 = 18$. Since $n(A') = 25$ and $n(B \cap A') = 18$, it follows that $n(A' \cap B') = 7$.
By De Morgan's laws, $A' \cup B' = (A \cap B)'$.
So, $n[(A \cap B)'] = 40$.
Thus, $n(A \cap B') = 40 - (18 + 7) = 15$.
Completing the cardinalities for each region, we arrive at the following Venn diagram.



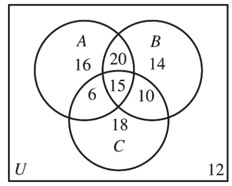
14. Use deduction to complete the cardinalities of each region outside of $A \cap B$. Since there is a total of 13 elements in A and 8 accounted for in the intersection of A and B, there must be 5 elements in A outside of B. Since there is a total of 15 elements in the union of A and B with 13 accounted for in A, there must be 2 elements in B which are not in A. The region $A' \cup B'$ is equivalent (by De Morgan's law) to $(A \cap B)'$, or the elements outside the intersection. Since this totals 11, we must have 4[11 − 7] elements outside the union but inside U.
Completing the cardinalities for each region, we arrive at the following Venn diagram.



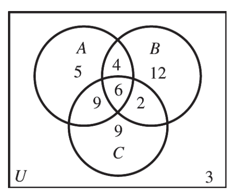
15. Fill in the cardinal numbers of the regions, beginning with $(A \cap B \cap C)$. Since $n(A \cap B \cap C) = 15$ and $n(A \cap B) = 35$, we have $n(A \cap B \cap C') = 35 - 15 = 20$. Since $n(A \cap C) = 21$, we have $n(A \cap C \cap B') = 21 - 15 = 6$. Since $n(B \cap C) = 25$, we have $n(B \cap C \cap A') = 25 - 15 = 10$. Since $n(C) = 49$, we have $n(C \cap A' \cap B') = 49 - (6 + 15 + 10) = 18$.
Since $n(A) = 57$, we have
$n(A \cap B' \cap C') = 57 - (20 + 15 + 6) = 16$.
Since $n(B') = 52$, we have

$$\begin{aligned} n(A \cup B \cup C)' &= n(A' \cap B' \cap C') \\ &= 52 - (16 + 6 + 18) \\ &= 12. \end{aligned}$$

Completing the cardinalities for each region, we arrive at the following Venn diagram.



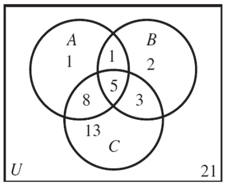
16. Fill in the cardinal numbers of the regions, beginning with the $(A \cap B \cap C)$. Since there is a total of 15 elements in $A \cap C$ of which 6 are accounted for in $A \cap B \cap C$, we conclude that there are 9 elements in $A \cap C$ but outside of B. Similarly, there must be 2 elements in $B \cap C$ but outside A and 4 elements in $A \cap B$ but outside of C. Since there are 26 elements in C of which we have accounted for 17[9 + 6 + 2], there must be 9 elements in C but outside of A or B. Similarly, there are 12 elements in B outside of A or C and 5 elements in A outside of B or C. And finally, adding the elements in the regions of A, B, and C gives a total of 47. Thus, the number of elements outside of A, B, and C is $n(U) - 47$ or $50 - 47 = 3$.
Completing the cardinalities for each region, we arrive at the following Venn diagram.



17. Fill in the cardinal numbers of the regions, beginning with the $(A \cap B \cap C)$.
$n(A \cap B \cap C) = 5$ and $n(A \cap C) = 13$, so $n(A \cap C \cap B') = 13 - 5 = 8$. $n(B \cap C) = 8$, so $n(B \cap C \cap A') = 8 - 5 = 3$.
$n(A \cap B') = 9$, so
$n(A \cap B' \cap C') = 9 - 8 = 1$. $n(A) = 15$, so
$n(A \cap B \cap C') = 15 - (1 + 8 + 5) = 1$.
$n(B \cap C') = 3$, so
$n(B \cap A' \cap C') = 3 - 1 = 2$.
$n(A' \cap B' \cap C') = n(A \cup B \cup C)' = 21$.
$n(B \cup C) = 32$, so

$n(A' \cap B' \cap C) = 32 - (1+2+5+3) = 21$

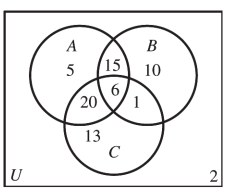
Completing the cardinalities for each region, we arrive at the following Venn diagram.



18. Fill in the cardinal numbers of the regions, beginning with $n(A \cap B \cap C) = 6$. Since there are 21 elements in A and B and 6 elements in all three sets, the number of elements in A and B but not in C is given by $n(A \cap B \cap C') = 21 - 6 = 15$. Since $n(A \cap C) = 26$, we have those elements in A and C but not in B as $n(A \cap C \cap B') = 26 - 6 = 20$.

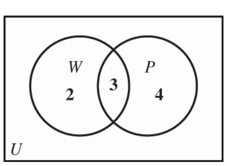
Since $n(B \cap C) = 7$, $n(B \cap C \cap A') = 7 - 6 = 1$. Since $n(A \cap C') = 20$, we have those elements in A but not in either B or C as $n(A \cap (B \cup C)') = 20 - 15 = 5$. Since $n(B \cap C') = 25$, we have those elements in B but not in A or C as $n(B \cap (A \cup C)') = 25 - 15 = 10$. Since $n(C) = 40$, we have those elements in C but not in A or B as $n(C \cap (A \cup B)') = 40 - (20 + 6 + 1) = 13$.

Observe that $n(A' \cap B' \cap C') = n(A \cup B \cup C)'$ (by De Morgan's). That is, there are 2 elements outside the union of the three sets.



19. Complete a Venn diagram showing the cardinality for each region. Let W = set of projects Joe Long writes. Let P = set of projects Joe long produces.

Begin with $(W \cap P)$. $n(W \cap P) = 3$. Since $n(W) = 5$, $n(W \cap P') = 5 - 3 = 2$. Since $n(P) = 7$, $n(P \cap W') = 7 - 3 = 4$.



Interpreting the resulting cardinalities we see that:

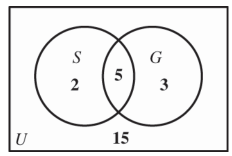
(a) He wrote but did not produce $n(W \cap P') = 2$ projects.

(b) He produced but did not write $n(P \cap W') = 4$ projects.

20. Complete a Venn diagram showing the cardinality for each region. Let S = the set of CDs featuring Paul Simon, G = the set of CDs featuring Art Garfunkel.

Beginning with $n(S \cap G) = 5$ and $n(S) = 7$, we conclude that $n(S \cap G') = 7 - 5 = 2$.

Since $n(G) = 8$, we conclude that $n(S' \cap G) = 8 - 5 = 3$. There are 15 CDs on which neither sing, so $n(S \cup G)' = 15$.



Interpreting the resulting cardinalities we see that:

(a) There are 2 CDs which feature only Paul Simon.

(b) There are 3 CDs which feature only Art Garfunkel.

(c) There are 10 CDs which feature at least one of these two artists.

(d) There are 20 CDs which feature at most one of these two artists.

21. Construct a Venn diagram and label the number of elements in each region. Let M = set of students who like Mozart, B = set of students who like Beethoven, and H = set of students who like Haydn.

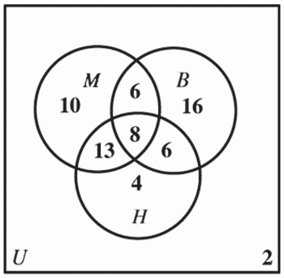
Begin with the region indicating the intersection of all three sets, $n(M \cap B \cap H) = 8$.

Since $n(M \cap B) = 14$, $n(M \cap B \cap H') = 14 - 8 = 6$.

Since $n(B \cap H) = 14$, $n(B \cap H \cap M') = 14 - 8 = 6$.

Since $n(M \cap H) = 21$, $n(M \cap H \cap B') = 21 - 8 = 13$.

Since $n(M) = 37$, the number of elements inside M and not in B or H is $37 - (13 + 8 + 6) = 10$. Since $n(B) = 36$, the number of elements inside B and not in M or H is $36 - (6 + 8 + 6) = 16$. Since $n(H) = 31$, the number of elements inside H and not in M or B is $31 - (13 + 8 + 6) = 4$. Since $n(U) = 65$, there are $65 - (10 + 6 + 16 + 13 + 8 + 6 + 4)$ $= 2$ elements outside the three sets. That is, $n(M \cup B \cup H)' = 2$. The completed Venn diagram is as follows:



(a) There are $6 + 13 + 6 = 25$ students that like exactly two of these composers.

(b) There are $10 + 16 + 4 = 30$ students that like exactly one of these composers.

(c) There are 2 students that like none of these composers.

(d) There are 10 students that like Mozart, but neither Beethoven nor Haydn.

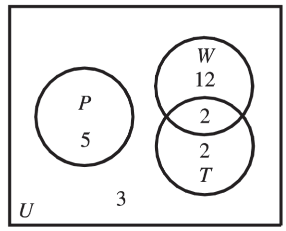
(e) There are $13 + 6 = 19$ students that like Haydn and exactly one of the other two.

(f) There are $65 - 8 = 57$ students like no more than two of these composers.

22. Let U be the set of mathematics majors receiving federal aid. Since half of the 48 mathematics majors receive federal aid,
$n(U) = \frac{1}{2}(48) = 24.$
Let P, W, and T be the sets of students receiving Pell Grants, participating in Work Study, and receiving TOPS scholarships, respectively.
Construct a Venn diagram and label the number of elements in each region. Since the 5 with Pell Grants had no other federal aid, 5 goes in set P, which does not intersect the other regions. The most manageable data remaining are the 2 who had TOPS scholarships and participated in Work Study. Place 2 in the intersection of W and T. Then since 14 altogether participated in Work Study, $14 - 2 = 12$ is the number who participated in Work Study but did not have TOPS scholarships or Pell Grants; in symbols, $n(W \cap T' \cap P')$. Also, $4 - 2 = 2$ is the number who had TOPS scholarships, but did not participate in Work Study nor had Pell Grants, $n(T \cap W' \cap P')$. Finally,

$$\begin{aligned} n(P' \cap W' \cap T') &= n(P \cup W \cup T)' \\ &= 24 - (5 + 12 + 2 + 2) \\ &= 3 \end{aligned}$$

is the number in the region not included in the sets P, W, and T. The completed Venn diagram is as follows.



(a) Since half of the 48 mathematics majors received federal aid, the other half, 24 math majors received no federal aid.

(b) There were only 2 students who received more than one of these three forms of aid. This is shown by the number in the region $T \cap W$.

(c) Since 24 students received federal aid, but only 21 are accounted for in the given information (the sum of the numbers in the three circles), there were 3 math majors who received other kinds of federal aid.

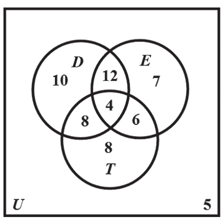
(d) The number of students receiving a TOPS scholarship or participating in Work Study is $12 + 2 + 2 = 16$.

(e) The number of students receiving exactly one of these three forms of aid is $5 + 12 + 2 = 19$.

(f) The number of students receiving no more than one of these three forms of aid is $48 - 2 = 46$. (Note that the 48 includes the 24 math majors who did not receive any form of federal aid.

23. Let U be the set of students interviewed, and let D, E, and T represent the sets of students who had seen *Despicable Me, Epic,* and *Turbo,* respectively.
Construct a Venn diagram and label the cardinal number of each region, beginning with the region $(D \cap E \cap T)$.

$n(D \cap E \cap T) = 4$. Since $n(D \cap E) = 16$,
$n(D \cap E \cap T') = 16 - 4 = 12$. Since
$n(D \cap T) = 12$, $n(D \cap T \cap E') = 12 - 4 = 8$.
Since $n(T \cap E) = 10$,
$n(D \cap E \cap T') = 16 - 4 = 12$.
Since $n(D) = 34$,
$$\begin{aligned} n(D \cap T' \cap E') &= n(D \cap (T \cup E)') \\ &= 34 - (8 + 4 + 12) \\ &= 10. \end{aligned}$$
Since $n(E) = 29$,
$n(E \cap D' \cap T') = 29 - (12 + 4 + 6) = 7$.
Since $n(T) = 26$,
$$\begin{aligned} n(T \cap D' \cap E') &= n(T \cap (D \cup E)') \\ &= 26 - (8 + 4 + 6) \\ &= 8. \end{aligned}$$
The completed Venn diagram is as follows.

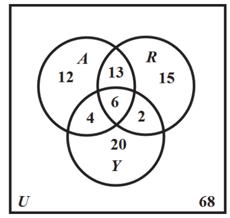


(a) 8 students had seen *Turbo* only.

(b) The number of students who had seen exactly two of the films is $12 + 8 + 6 = 26$.

(c) Adding the cardinalities of all regions in the Venn diagram results in 60, the number of students who were surveyed.

24. Construct a Venn diagram and label the cardinal number of each region beginning with the intersection of all three sets,
$n(A \cap R \cap Y) = 6$.
Since $n(A \cap R) = 19$,
$n(A \cap R \cap Y') = 19 - 6 = 13$.
Since $n(R \cap Y) = 8$,
$n(R \cap Y \cap A') = 8 - 6 = 2$.
Since $n(A \cap Y) = 10$,
$n(A \cap Y \cap R') = 10 - 6 = 4$.
Since $n(A) = 35$,
$n(A \cap Y' \cap R') = 35 - (13 + 6 + 4) = 12$.
Since $n(R) = 36$,
$n(R \cap A' \cap Y') = 36 - (13 + 6 + 2) = 15$.
Since $n(Y) = 32$,
$n(Y \cap A' \cap R') = 32 - (4 + 6 + 2) = 20$.
Since $n(U) = 140$, there are
$140 - (12 + 13 + 15 + 4 + 6 + 2 + 20)$
$= 68$ elements outside the three sets. That is,
$n(A \cup R \cup Y)' = 68$. The completed Venn diagram is as follows.



(a) There are 12 + 4 = 16 respondents who believe in astrology but not reincarnation.

(b) There are 140 − 68 = 72 respondents who believe in at least one of these three things.

(c) There are 15 respondents who believe in reincarnation but neither of the others.

(d) There are 13 + 2 + 4 = 19 respondents who believe in exactly two of these three things.

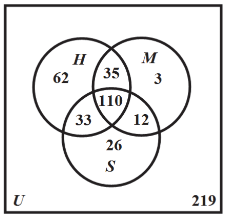
(e) There are 68 respondents who believe in none of the three things.

25. Construct a Venn diagram and label the cardinal number of each region.
Let H = the set of respondents who think Hollywood is unfriendly toward religion,
M = the set of respondents who think the media are unfriendly toward religion,
S = the set of respondents who think scientists are unfriendly toward religion.
Then we are given the following information.

$n(H) = 240$	$n(H \cap M) = 145$
$n(M) = 160$	$n(S \cap M) = 122$
$n(S) = 181$	$n(H \cap M \cap S) = 110$
	$n(H \cup M \cup S)' = 219$

Since $n(H \cap M) = 145$,
$n(H \cap M \cap S') = 145 - 110 = 35$.
Since $n(S \cap M) = 122$,
$n(S \cap M \cap H') = 122 - 110 = 12$.
The total number of respondents who think exactly two of these groups are unfriendly toward religion is 80, so
$n(H \cap S \cap M') = 80 - (35 + 12) = 33$.
Since $n(H) = 240$,
$$\begin{aligned} n(H \cap S' \cap M') &= 240 - (33 + 110 + 35) \\ &= 62. \end{aligned}$$
Since $n(M) = 160$,
$n(M \cap H' \cap S') = 160 - (35 + 110 + 12) = 3$.

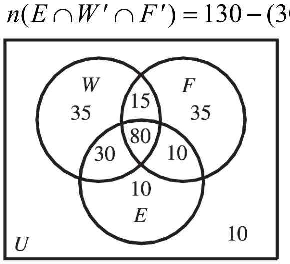
Since $n(S) = 181$,
$n(S \cap H' \cap M') = 181 - (33 + 110 + 12) = 26.$



(a) The total number of respondents surveyed is
62 + 33 + 110 + 35 + 3 + 12 + 26 + 219
= 500.

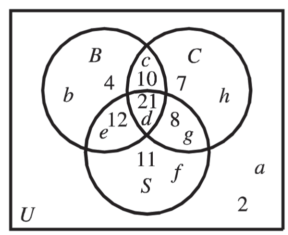
(b) The number of respondents who think exactly one of these three groups is unfriendly toward religion is
62 + 3 + 26 = 91.

26. Construct a Venn diagram and label the cardinal number of each region beginning with the intersection of all three sets, $n(W \cap F \cap E) = 80$. Since $n(E \cap F) = 90$,
$n(E \cap F \cap W') = 90 - 80 = 10.$
Since $n(W \cap F) = 95$,
$n(W \cap F \cap E') = 95 - 80 = 15.$
Since $n(F) = 140$,
$n(F \cap W' \cap E') = 140 - (15 + 10 + 80) = 35.$
$n(W' \cap F' \cap E') = n(W \cup F \cup E)' = 10.$
Since $n(W \cap E)$ is not given, it is not obvious how to label the three remaining regions. We need to use the information that $n(E') = 95$. The only region not yet labeled that is outside of E is $(W \cap E' \cap F')$.
Since $n(E') = 95$,
$n(W \cap E' \cap F') = 95 - (10 + 35 + 15) = 35.$
Since $n(W) = 160$,
$n(W \cap E \cap F') = 160 - (35 + 15 + 80) = 30.$
Since $n(E) = 130$,
$n(E \cap W' \cap F') = 130 - (30 + 10 + 80) = 10.$



Add the cardinal numbers of all the regions to find that the total number of students interviewed was 225.

27. Construct a Venn diagram to represent the survey data beginning with the region representing the intersection of S, B, and C. Rather than representing each region as a combination of sets and set operations, we will label the regions a–h. There are 21 patients in Nadine's survey that are in the intersection of all three sets, i.e., in region d. Since there are 31 patients in $B \cap C$, we can deduce that there must be 10 patients in region c. Similarly since there are 33 patients in $B \cap S$, there must be 12 patients in region e. From the given information, there is a total of 51 patients in regions c, d, e, and g. Thus, there are 51 – (10 + 21 + 12) = 8 patients in region g. Since there are 52 patients in S, we can deduce that there are 52 – (12 + 21 + 8) = 11 patients in region f. Similarly, there are 4 patients in region b, and 7 patients in region h. There is a total of 73 patients found in regions b–h. Thus, there must be 2 patients in region a.



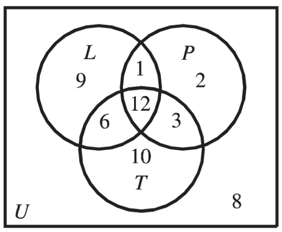
(a) The number of these patients who had either high blood pressure or high cholesterol levels, but not both is represented by regions b, e, g, and h for a total of 31 patients.

(b) The number of these patients who had fewer than two of the indications listed are found in regions a, b, f, and h for a total of 24 patients.

(c) The number of these patients who were smokers but had neither high blood pressure nor high cholesterol levels are found in region f, which has 11 members.

(d) The number of these patients who did not have exactly two of the indications listed would be those excluded from regions c, e, and g (representing patients with exactly two of the indications). We arrive at a total of 45 patients.

28. Let L, P, and T represent the sets of songs about love, prison, and trucks, respectively. Construct a Venn diagram and label the cardinal number of each region.
$n(T \cap L \cap P) = 12$
Since $n(P \cap L) = 13$,
$n(P \cap L \cap T') = 13 - 12 = 1.$ Since
$n(T \cap L) = 18,\ n(T \cap L \cap P') = 18 - 12 = 6.$
We have $n(T \cap P \cap L') = 3,$
$n(P \cap L' \cap T') = 2,$ and
$n(P' \cap L' \cap T') = 8.$
Since $n(L) = 28,$
$n(L \cap P' \cap T') = 28 - (1 + 12 + 6) = 9.$
Since $n(T \cap P') = 16,$
$n(T \cap P' \cap L') = 16 - 6 = 10.$



(a) The total in all eight regions is 51, the total number of songs.

(b) $n(T) = 6 + 12 + 3 + 10 = 31$

(c) $n(P) = 1 + 2 + 12 + 3 = 18$

(d) $n(T \cap P) = 12 + 3 = 15$

(e) $n(P') = n(U) - n(P) = 51 - 18 = 33$

(f) $n(L') = n(U) - n(L) = 51 - 28 = 23$

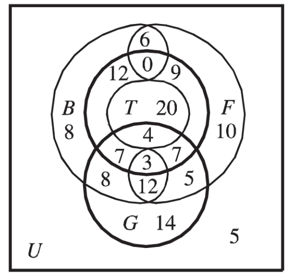
29. (a) The set $A \cap B \cap C \cap D$ is region 1 (in text).

(b) The set $A \cup B \cup C \cup D$ includes the regions 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15.

(c) The set $(A \cap B) \cup (C \cap D)$ includes the set of regions
$\{1, 3, 9, 11\} \cup \{1, 2, 4, 5\}$ or the regions 1, 2, 3, 4, 5, 9, and 11.

(d) The set $(A' \cap B') \cap (C \cup D)$ includes the set of regions
$\{5, 13, 8, 16\} \cap \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13\}$, which is represented by regions 5, 8, and 13.

30. Let F, B, T, and G represent the sets of those who watch football, basketball, tennis, and golf, respectively. Construct a Venn diagram, similar to the figure in Exercise 29 in the text, with the four sets. Be careful to indicate the number of elements in each region. Begin at the bottom of the list and work upwards.



(a) The region which represents those who watch football, basketball, and tennis, but not golf, is in the north central portion of the diagram and has 0 elements. Thus, the answer is none.

(b) Adding cardinalities associated with the regions representing exactly one of these four sports, we get
8 + 10 + 14 + 20 = 52 viewers.

(c) Adding cardinalities associated with the regions representing exactly two of these four sports, we get
6 + 9 + 5 + 8 + 12 + 4 = 44 viewers.

31. (a) $n(J \cap G) = 9$, coming from the intersection of the first row with the first column.

(b) $n(S \cap N) = 9$, coming from the intersection of second row and the third column.

(c) $n(N \cup (S \cap F)) = 20$ since there are 20 players who are in either N(total of 15) or in S and F (just 5), at the same time.

(d) $n(S' \cap (G \cup N)) = 20$ since there are
9 + 4 + 5 + 2 = 20 players who are not in S, but are in G or in N.

(e) $n((S \cap N') \cup (C \cap G')) = 27$
There are 27 players who are in S but not in N(12 + 5), or who are in C but not in G (8 + 2).

(f) $n(N' \cap (S' \cap C')) = 15$
There are 15 (9 + 6) players who are not in N and at the same time are not in S and not in C.

32. (a) $n(W \cap O) = 6$, coming from the intersection of second row with the third column.

(b) $n(C \cup B) = 473$, coming from the union of the first row along with the first column and is given by 95 + 390 − 12. Observe that the 12 are counted twice when adding the totals for the respective row and column and hence must be subtracted (once).

(c) $n(R' \cup W') = n(R \cap W)' = 835,$ which is the number of army personnel outside of the intersection of the second column (R) with the second row (W) or 840 − 5.

(d) $n((C \cup W) \cap (B \cup R)) = 12 + 29 + 4 + 5$
$= 50$
This comes from adding the numbers in the first two rows (representing $C \cup W$) that are also in the first two columns (representing $B \cup R$).

(e) Using the cardinal number formula.
$$\begin{aligned} &n((C \cap B) \cup (E \cap O)) \\ &= n(C \cap B) + n(E \cap O) \\ &\qquad - n(C \cap B \cap E \cap O) \\ &= 12 + 285 - 0 \\ &= 297 \end{aligned}$$

(f)
$$\begin{aligned} n(B \cap (W \cup R)') &= n(B \cap (W' \cap R')) \\ &= n(B \cap W' \cap R') \\ &= n(B \cap W') \\ &= n(B) - n(B \cap W) \\ &= 390 - 4 \\ &= 386 \end{aligned}$$

33. Writing exercise; answers will vary

34. Writing exercise; answers will vary.

Chapter 2 Test

1.
$$\begin{aligned} A \cup C &= \{a, b, c, d\} \cup \{e, a\} \\ &= \{a, b, c, d, e\} \end{aligned}$$

2.
$$\begin{aligned} B \cap A &= \{b, e, a, d\} \cap \{a, b, c, d\} \\ &= \{a, b, d\} \end{aligned}$$

3. $B' = \{b, e, a, d\}' = \{c, f, g, h\}$

4.
$$\begin{aligned} &A - (B \cap C') \\ &= A - (\{b, e, a, d\} \cap \{b, c, d, f, g, h\}) \\ &= \{a, b, c, d\} - \{b, d\} \\ &= \{a, c\} \end{aligned}$$

5. $e \in A$ is false since e is not a member of set A.

6. $C \subseteq B$ is true since each member of set C is also a member of set B.

7. $B \subset (A \cup C)$ is true since all members of set B are also members of $A \cup C$.

8. $c \notin C$ is true because c is not a member of set C.

9. $n[(A \cup B) - C] = 4$ is false. Because,
$$\begin{aligned} n[(A \cup B) - C] &= n[\{a, b, c, d, e\} - \{a, e\}] \\ &= n(\{b, c, d\}) \\ &= 3 \end{aligned}$$

10. $\emptyset \not\subset C$ is false. The empty set is considered a subset of any set. Since C is not empty, $\emptyset$ is a proper subset of C.

11. $(A \cap B')$ is equivalent to $(B \cap A')$ is true. Because, $n(A \cap B') = n(\{c\}) = 1,$
$n(B \cap A') = n(\{e\}) = 1.$

12. $(A \cup B)' = A' \cap B'$ is true by one of De Morgan's laws.

13. $n(B \times A) = n(B) \times n(A) = 4 \times 4 = 16$

14. The number of proper subsets of B is $2^4 - 1 = 16 - 1 = 15.$

Answers may vary for Exercises 15–18.

15. A word description for $\{-3, -1, 1, 3, 5, 7, 9\}$ is the set of all odd integers between −4 and 10.

16. A word description for {Sun, Mon, Tues, ..., Sat} is the set of days of the week.

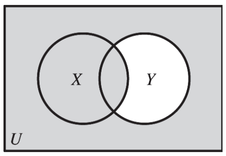
17. Set-builder notation for $\{-1, -2, -3, -4, ...\}$ would be $\{x | x \text{ is a negative integer}\}$.

18. Set-builder notation for $\{24, 32, 40, 48, ..., 88\}$ would be $\{x| \ x$ is a multiple of 8 between 20 and 90$\}$.

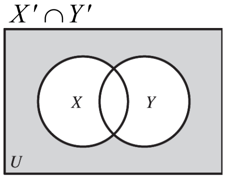
19. $\varnothing \subseteq \{x| \ x$ is a counting number between 20 and 21$\}$ since the empty set is a subset of any set.

20. $\{3, 5, 7\}$ neither $\{4, 5, 6, 7, 8, 9, 10\}$ since the element 3 is not a member of the second set.

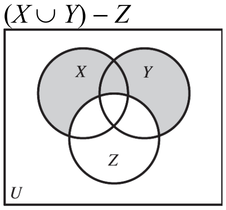
21. $X \cup Y'$



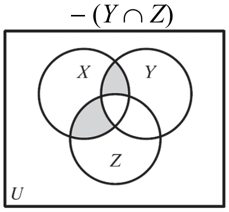
22. $X' \cap Y'$



23. $(X \cup Y) - Z$



24. $[(X \cap Y) \cup (X \cap Z)] - (Y \cap Z)$



25. Writing exercise; answers will vary.

26. $A \cap T$ = {Electric razor, Telegraph, Zipper} $\cap$ {Electric razor, Fiber optics, Geiger counter, Radar} = {Electric Razor}

27. $(A \cup T)'$
= ({Electric razor, Telegraph, Zipper} $\cup$ {Electric razor, Fiber optics, Geiger counter, Radar})$'$
= {Electric razor, Fiber optics, Geiger counter, Radar, Telegraph, Zipper}$'$
= {Adding machine, Baking powder, Pendulum clock, Thermometer}

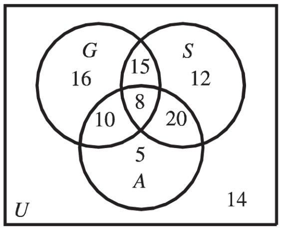
28. $T' - A'$
= {Electric razor, Fiber optics, Geiger counter, Radar}$'$ − {Electric razor, Telegraph, Zipper}$'$
= {Adding machine, Baking powder, Pendulum clock, Telegraph, Thermometer, Zipper} − {Adding machine, Baking powder, Fiber optics, Geiger counter, Pendulum clock, Radar, Thermometer}
= {Telegraph, Zipper}

29. **(a)** $n(A \cup B) = 12 + 3 + 7 = 22$

(b) $n(A \cap B') = n(A - B) = 12$
These are the elements in A but outside of B.

(c) $n\left((A \cap B)'\right) = n\left(U - (A \cap B)\right)$
$= 31 - 3 = 28$
These are the elements outside the intersection of A and B.

30. Let G = set of students who are receiving government grants. Let S = set of students who are receiving private scholarships. Let A = set of students who are receiving aid from the college.
Complete a Venn diagram by inserting the appropriate cardinal number for each region in the diagram. Begin with the intersection of all three sets: $n(G \cap S \cap A) = 8$. Since $n(S \cap A) = 28$, $n(S \cap A \cap G') = 28 - 8 = 20$.
Since $n(G \cap A) = 18$, $n(G \cap A \cap S') = 18 - 8 = 10$. Since $n(G \cap S) = 23$, $n(G \cap S \cap A') = 23 - 8 = 15$.
Since $n(A) = 43$,
$$\begin{aligned} n(A \cap (G \cup S)') &= 43 - (10 + 8 + 20) \\ &= 43 - 38 \\ &= 5. \end{aligned}$$
Since $n(S) = 55$,
$$\begin{aligned} n(S \cap (G \cup A)') &= 55 - (15 + 8 + 20) \\ &= 55 - 43 \\ &= 12. \end{aligned}$$
Since $n(G) = 49$,
$$\begin{aligned} n(G \cap (S \cup A)') &= 49 - (10 + 8 + 15) \\ &= 49 - 33 \\ &= 16. \end{aligned}$$



Thus,

(a) $n(G \cap (S \cup A)') = 16$ have a government grant only.

(b) $n(S \cap G') = 32$ have a private scholarship but not a government grant.

(c) 16 + 12 + 5 = 33 receive financial aid from only one of these sources.

(d) 10 + 15 + 20 = 45 receive aid from exactly two of these sources.

(e) $n(G \cup S \cup A)' = 14$ receive no financial aid from any of these sources.

(f) $n(S \cap (A \cup G)') + n(A \cup G \cup S)'$
$= 12 + 14$
$= 26$
received private scholarships or no aid at all.