Solutions to Problems

Getting Started

1 **(a)** This is shown in Figure SI.1.

	A	B	C	D
1	Economics Examination Marks			
2				
3	Candidate	Section A	Section B	
4	Fofaria	20	17	
5	Bull	38	12	
6	Eoin	34	38	
7	Arefin	40	52	
8	Cantor	29	34	
9	Devaux	30	49	
10				

Figure SI.1

(b) Type the heading Total Mark in cell D3. Type `=B4+C4` into cell D4. Click and drag down to D9.

(c) Type the heading Average: in cell C10. Type `=(SUM 4:D9))/6` in cell D10 and press Enter. [Note: Excel has lots of built-in functions for performing standard calculations such as this. To find the average you could just type `=AVERAGE(D4:D9)` in cell D10.]

(d) This is shown in Figure SI.2.

Economics Examination Marks

Candidate	Section A Mark	Section B Mark	Total Mark
Arefin	40	52	92
Bull	38	12	50
Cantor	29	34	63
Devaux	30	49	79
Eoin	34	38	72
Fofaria	20	17	37
		Average:	65.5

Figure SI.2

(e) Just put the cursor over cell C5 and type in the new mark of 42. Pressing the Enter key causes cells D5 and D10 to be automatically updated. The new spreadsheet is shown in Figure SI.3.

Economics Examination Marks

Candidate	Section A Mark	Section B Mark	Total Mark
Arefin	40	52	92
Bull	38	42	80
Cantor	29	34	63
Devaux	30	49	79
Eoin	34	38	72
Fofaria	20	17	37
		Average:	70.5

Figure SI.3

2 **(a)** 14

(b) 11

(c) 5

3 **(a)** 4; is the solution of the equation $2x - 8 = 0$.

(b) Figure SI.4 shows the graph of $2x - 8$ plotted between $x = 0$ and 10.

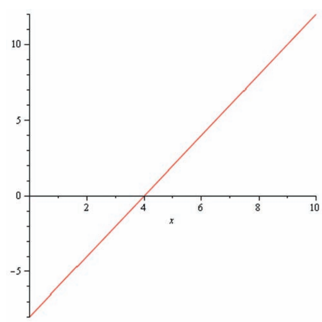


Figure SI.4

(c) $x^2 + 4x + 4$; the brackets have been 'multiplied out' in the expression $(x + 2)^2$.

(d) $7x + 4$; like terms in the expression $2x + 6 + 5x - 2$ have been collected together.

(e) Figure SI.5 shows the three-dimensional graph of the surface $x^3 - 3x + xy$ plotted between -2 and 2 in both the x and y directions.

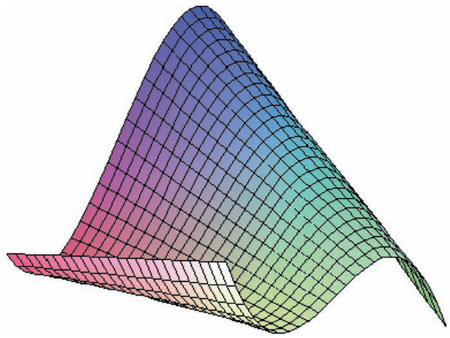


Figure SI.5

Chapter 1

Section 1.1

Practice Problems

1 (a) -30 (b) 2 (c) -5
(d) 5 (e) 36 (f) -1

2 (a) -1 (b) -7 (c) 5
(d) 0 (e) -91 (f) -5

3 (a) 19 (b) 1500 (c) 32 (d) 35

4 (a) $x + 9y$ (b) $2y + 4z$ (c) Not possible.
(d) $8r^2 + s + rs - 3s^2$ (e) $-4f$
(f) Not possible. (g) 0

5 (a) $5z - 2z^2$
(b) $6x - 6y + 3y - 6x = -3y$
(c) $x - y + z - x^2 - x + y = z - x^2$

6 (a) $7(d + 3)$ (b) $4(4w - 5q)$
(c) $3(3x - y + 3z)$ (d) $5Q(1 - 2Q)$

7 (a) $x^2 - 2x + 3x - 6 = x^2 + x - 6$
(b) $x^2 - xy + yx - y^2 = x^2 - y^2$
(c) $x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$
(d) $5x^2 - 5xy + 5x + 2yx - 2y^2 + 2y$
$= 5x^2 - 3xy + 5x - 2y^2 + 2y$

8 (a) $(x + 8)(x - 8)$
(b) $(2x + 9)(2x - 9)$

Exercise 1.1 (p. 30)

1 (a) -20 (b) 3 (c) -4 (d) 1
(e) -12 (f) 50 (g) -5 (h) 3
(i) 30 (j) 4

2 (a) -1 (b) -3 (c) -11 (d) 16
(e) -1 (f) -13 (g) 11 (h) 0
(i) -31 (j) -2

3 (a) -3 (b) 2 (c) 18 (d) -15
(e) -41 (f) -3 (g) 18 (h) -6
(i) -25 (j) -6

4 (a) $2PQ$ (b) $8I$ (c) $3xy$
(d) $4qwz$ (e) b^2 (f) $3k^2$

5 (a) $19w$ (b) $4x - 7y$ (c) $9a + 2b - 2c$
(d) $x^2 + 2x$ (e) $4c - 3cd$ (f) $2st + s^2 + t^2 + 9$

6 (a) 10 (b) 18 (c) 2000
(d) 96 (e) 70

7 (a) 1 (b) 5 (c) -6
(d) -6 (e) -30 (f) 44

8 (a) 16
(b) Presented with the calculation, -4^2, your calculator uses BIDMAS, so squares first to get 16 and then subtracts from zero to give a final answer, -16. To obtain the correct answer you need to use brackets:

[( ] [−] [4] [ )] [x^2] [=]

9 (a) 9 (b) 21; no

10 (a) 43.96 (b) 1.13 (c) 10.34 (d) 0.17
(e) 27.38 (f) 3.72 (g) 62.70 (h) 2.39

11 (a) $7x - 7y$ (b) $15x - 6y$ (c) $4x + 12$
(d) $21x - 7$ (e) $3x + 3y + 3z$ (f) $3x^2 - 4x$
(g) $-2x - 5y + 4z$

12 (a) $5(5c + 6)$ (b) $9(x - 2)$ (c) $x(x + 2)$
(d) $4(4x - 3y)$ (e) $2x(2x - 3y)$ (f) $5(2d - 3e + 10)$

13 (a) $x^2 + 7x + 10$ (b) $a^2 + 3a - 4$ (c) $d^2 - 5d - 24$
(d) $6s^2 + 23s + 21$ (e) $2y^2 + 5y + 3$ (f) $10t^2 - 11t - 14$
(g) $9n^2 - 4$ (h) $a^2 - 2ab + b^2$

14 (a) $6x + 2y$ (b) $11x^2 - 3x - 3$ (c) $14xy + 2x$;
(d) $6xyz + 2xy$ (e) $10a - 2b$ (f) $17x + 22y$
(g) $11 - 3p$ (h) $x^2 + 10x$

15 (a) $(x + 2)(x - 2)$ (b) $(Q + 7)(Q - 7)$
(c) $(x + y)(x - y)$ (d) $(3x + 10y)(3x - 10y)$

16 (a) $4x^2 + 8x - 2$ (b) $-13x$

Exercise 1.1* (p. 32)

1 (a) 3 (b) 5 (c) -7

2 (a) $2 - 7 - (9 + 3) = -17$ (b) $8 - (2 + 3) - 4 = -1$
(c) $7 - (2 - 6 + 10) = 1$

3 (a) -6 (b) 6 (c) -5 (d) -96
(e) -1 (f) 6 (g) $\frac{5}{4}$ (h) 63

4 (a) 6 (b) 2 (c) 5

5 $-y^2 + xy - 5x + 2y - 6$

6 (a) $2x - 2y$ (b) $2x$ (c) $-2x + 3y$

7 (a) $x^2 - 2x - 24$ (b) $6x^2 - 29x + 35$
(c) $6x^2 + 2xy - 4x$ (d) $12 - 2g + 3h - 2g^2 + gh$
(e) $2x - 2x^2 - 3xy + y - y^2$ (f) $a^2 - b^2 - c^2 - 2bc$

8 (a) $3(3x - 4y)$ (b) $x(x - 6)$
(c) $5x(2y + 3x)$ (d) $3xy(y - 2x + 4)$
(e) $x^2(x - 2)$ (f) $5xy^3(12x^3y^3 - 3xy + 4)$

9 (a) $(p + 5)(p - 5)$ (b) $(3c + 8)(3c - 8)$
(c) $2(4v + 5d)(4v - 5d)$ (d) $(4x^2 + y^2)(2x + y)(2x - y)$

10 (a) 112 600 000 (b) 1.7999
(c) 283 400 (d) 246 913 577

Section 1.2

Practice Problems

1 (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2y}$ (d) $\frac{1}{2+3x}$ (e) $\frac{1}{x-4}$

2 (a) $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$

(b) $\cancel{7} \times \frac{1}{\cancel{14}_2} = \frac{1}{2}$

(c) $\frac{2}{3} \div \frac{8}{9} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{9}^3}{\cancel{8}_4} = \frac{3}{4}$

(d) $\frac{8}{9} \div 16 = \frac{^1\cancel{8}}{9} \times \frac{1}{\cancel{16}_2} = \frac{1}{18}$

3 (a) $\frac{3}{7} - \frac{1}{7} = \frac{2}{7}$

(b) $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

(c) $\frac{7}{18} - \frac{1}{4} = \frac{14}{36} - \frac{9}{36} = \frac{5}{36}$

4 (a) $\frac{5}{\cancel{x-1}} \times \frac{\cancel{x-1}}{x+2} = \frac{5}{x+2}$

(b) $\frac{x^2}{x+10} \div \frac{x}{x+1} = \frac{x^{\cancel{2}}}{x+10} \times \frac{x+1}{\cancel{x}} = \frac{x(x+1)}{x+10}$

(c) $\frac{4}{x+1} + \frac{1}{x+1} = \frac{4+1}{x+1} = \frac{5}{x+1}$

(d) $\frac{2}{x+1} - \frac{1}{x+2}$

$= \frac{2(x+2)}{(x+1)(x+2)} - \frac{(1)(x+1)}{(x+1)(x+2)}$

$= \frac{(2x+4) - (x+1)}{(x+1)(x+2)} = \frac{(x+3)}{(x+1)(x+2)}$

5 (a) $4x + 5 = 5x - 7$
$5 = x - 7$ (subtract $4x$ from both sides)
$12 = x$ (add 7 to both sides)

(b) $3(3 - 2x) + 2(x - 1) = 10$
$9 - 6x + 2x - 2 = 10$ (multiply out brackets)
$7 - 4x = 10$ (collect like terms)
$-4x = 3$ (subtract 7 from both sides)
$x = -\frac{3}{4}$ (divide both sides by -4)

(c) $\frac{4}{x-1} = 5$
$4 = 5(x - 1)$ (multiply both sides by $x - 1$)
$4 = 5x - 5$ (multiply out brackets)
$9 = 5x$ (add 5 to both sides)
$\frac{9}{5} = x$ (divide both sides by 5)

(d) $\frac{3}{x} = \frac{5}{x-1}$
$3(x - 1) = 5x$ (cross-multiplication)
$3x - 3 = 5x$ (multiply out brackets)
$-3 = 2x$ (subtract $3x$ from both sides)
$-\frac{3}{2} = x$ (divide both sides by 2)

6 (a) $12 > 9$ (true) (b) $12 > 6$ (true)
(c) $3 > 0$ (true) (d) same as (c)
(e) $2 > 1$ (true) (f) $-24 > -12$ (false)
(g) $-6 > -3$ (false) (h) $2 > -1$ (false)
(i) $-4 > -7$ (true)

7 (a) $2x < 3x + 7$
$-x < 7$ (subtract $3x$ from both sides)
$x > -7$ (divide both sides by -1 changing sense because $-1 < 0$)

(b) $21x - 19 \geq 4x + 15$
$17x - 19 \geq 15$ (subtract $4x$ from both sides)
$17x \geq 34$ (add 19 to both sides)
$x \geq 2$ (divide both sides by 17, leaving inequality unchanged because $17 > 0$)

Exercise 1.2 (p. 47)

1 (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{1}{3}$ (e) $\frac{4}{3} = 1\frac{1}{3}$

2 (a) $\frac{2x}{3}$ (b) $\frac{1}{2x}$ (c) $\frac{1}{ac}$ (d) $\frac{2}{3xy}$ (e) $\frac{3a}{4b}$

3 (a) $\frac{p}{2q+3r}$ (b) $\frac{1}{x-4}$ (c) $\frac{b}{2a+1}$ (d) $\frac{2}{3-e}$

(e) $\frac{1}{x-2}$ (because $x^2 - 4 = (x + 2)(x - 2)$)

4 $\frac{x-1}{2x-2} = \frac{x-1}{2(x-1)} = \frac{1}{2}$; other two have no common factors on top and bottom.

5 (a) $\frac{3}{7}$ (b) $-\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{7}{20}$

(e) $\frac{7}{18}$ (f) $\frac{5}{6}$ (g) $\frac{5}{8}$ (h) $\frac{2}{5}$

(i) $\frac{7}{12}$ (j) $\frac{1}{30}$ (k) $\frac{2}{27}$ (l) $\frac{21}{2} = 10\frac{1}{2}$

6 (a) $\frac{1}{x}$ (b) $\frac{2}{5}$ (c) $\frac{3x-2}{x^2}$ (d) $\frac{7y+2x}{xy}$

(e) 3 (f) $\frac{15c+10d}{36}$ (g) $\frac{x+2}{x+3}$ (h) $\frac{18h^2}{7}$

(i) $\frac{t}{20}$ (j) 1

7 (a) 5 (b) 6 (c) 18 (d) 2

(e) 10 (f) −1 (g) 60 (h) $1\frac{2}{3}$

(i) −5 (j) −3 (k) −2 (l) $-3\frac{2}{3}$

(m) $3\frac{1}{4}$ (n) 3 (o) $\frac{1}{4}$

8 (a), (d), (e), (f)

9 (a) $x > 1$ (b) $x \geq 3$ (c) $x \geq -3$ (d) $x > 2$

10 $\frac{2}{x^3}$

11 (a) $-\frac{7}{26}$ (b) $x \leq 10$

Exercise 1.2* (p. 48)

1 (a) $\frac{x-3}{2}$ (b) $\frac{3}{2x-1}$ (c) 4 (d) −1

(e) $\frac{1}{x-6}$ (f) $\frac{x+3}{x+4}$ (g) $\frac{1}{2x^2-5x+3}$

(h) $\frac{2x+5y}{3}$

2 (a) $\frac{5}{7}$ (b) $\frac{1}{10}$ (c) $\frac{3}{2}$ (d) $\frac{5}{48}$

(e) $\frac{8}{13}$ (f) $\frac{11}{9}$ (g) $\frac{141}{35}$ (h) $\frac{34}{5}$

(i) 6 (j) $\frac{7}{10}$ (k) $\frac{7}{9}$ (l) 4

3 (a) $x + 6$ (b) $\frac{x+1}{x}$ or equivalently $1 + \frac{1}{x}$

(c) $\frac{5}{xy}$ (d) $\frac{5x+2}{6}$ (e) $\frac{7x+3}{x(x+1)}$ (f) $\frac{3x+5}{x^2}$

(g) $\frac{x^2+x-2}{x+1}$ (h) $\frac{x+3}{x(x+1)}$

4 (a) $-\frac{11}{7}$ (b) 1 (c) $-\frac{35}{9}$ (d) 8

(e) $\frac{4}{5}$ (f) $\frac{1}{4}$ (g) $-\frac{11}{7}$ (h) 8

(i) 9 (j) $\frac{71}{21}$ (k) 7 (l) −9

(m) 1 (n) −5 (o) 3 (p) 5

5 $1.6 + \frac{5x}{7} = 6.75$; \$7.21

6 (a) \$3221.02 (b) \$60 000 (c) 10

7 (a) $x < -8\frac{3}{5}$ (b) $x > -\frac{12}{13}$ (c) $x \leq 13$

(d) $x > -3$ (e) $-1 < x \leq 3$

8 −3, −2, −1, 0.

9 (a) $\frac{3}{2x-1}$ (b) −1 (c) $x \geq \frac{11}{5}$

10 $\frac{x(x-1)}{2}$

Section 1.3

Practice Problems

1 From Figure S1.1 note that all five points lie on a straight line.

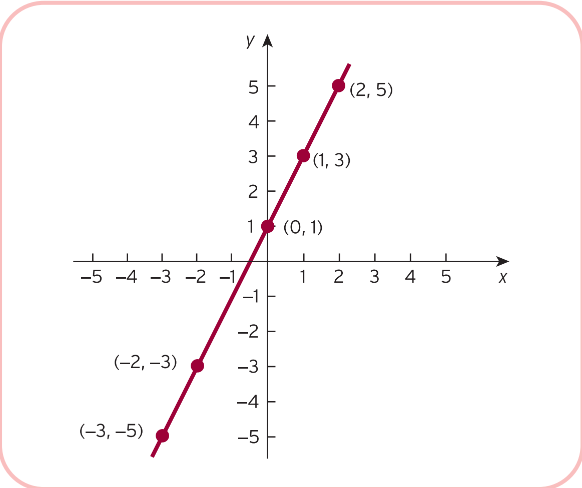


Figure S1.1

2

Point	Check	
(−1, 2)	2(−1) + 3(2) = −2 + 6 = 4	✓
(−4, 4)	2(−4) + 3(4) = −8 + 12 = 4	✓
(5, −2)	2(5) + 3(−2) = 10 − 6 = 4	✓
(2, 0)	2(2) + 3(0) = 4 + 0 = 4	✓

The graph is sketched in Figure S1.2.

The graph shows that (3, −1) does not lie on the line. This can be verified algebraically:

$$2(3) + 3(-1) = 6 - 3 = 3 \neq 4$$

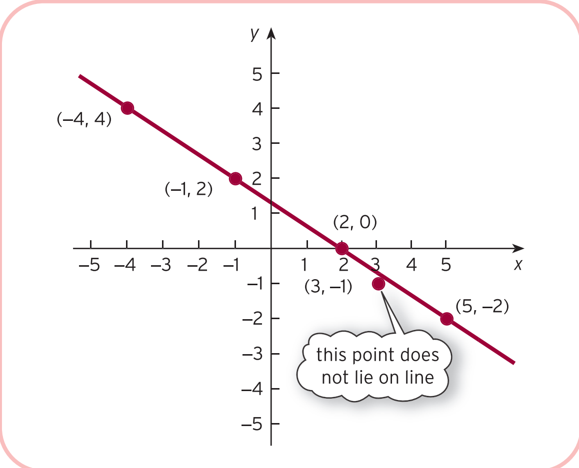


Figure S1.2

3 $3x - 2y = 4$

$3(2) - 2y = 4$ (substitute $x = -2$)

$6 - 2y = 4$

$-2y = -2$ (subtract 6 from both sides)

$y = 1$ (divide both sides by −2)

Hence (2, 1) lies on the line.

$3x - 2y = 4$

$3(-2) - 2y = 4$

$-6 - 2y = 4$ (substitute $x = 2$)

$-2y = 10$ (add 6 to both sides)

$y = -5$ (divide both sides by −2)

Hence (−2, −5) lies on the line.

The line is sketched in Figure S1.3.

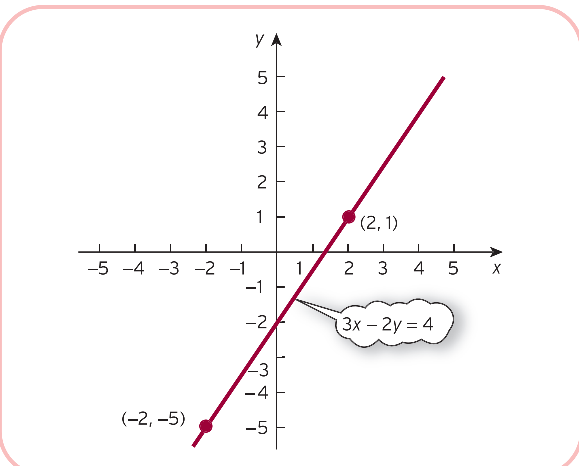


Figure S1.3

4 $x - 2y = 2$

$0 - 2y = 2$ (substitute $x = 0$)

$-2y = 2$

$y = -1$ (divide both sides by −2)

Hence (0, −1) lies on the line.

$x - 2y = 2$

$x - 2(0) = 2$ (substitute $y = 0$)

$x - 0 = 2$

$x = 2$

Hence (2, 0) lies on the line.

The graph is sketched in Figure S1.4.

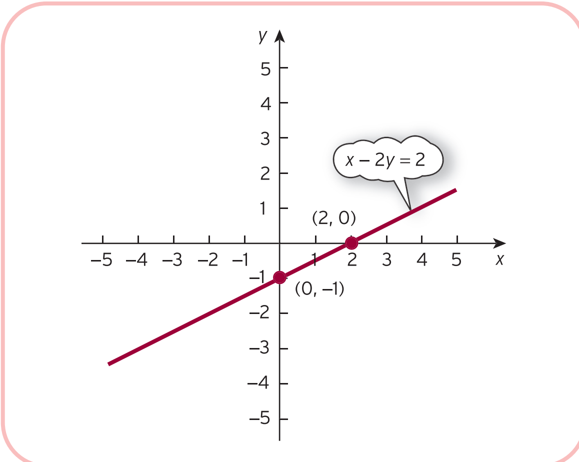


Figure S1.4

5 From Figure S1.5 the point of intersection is $(1, -\frac{1}{2})$.

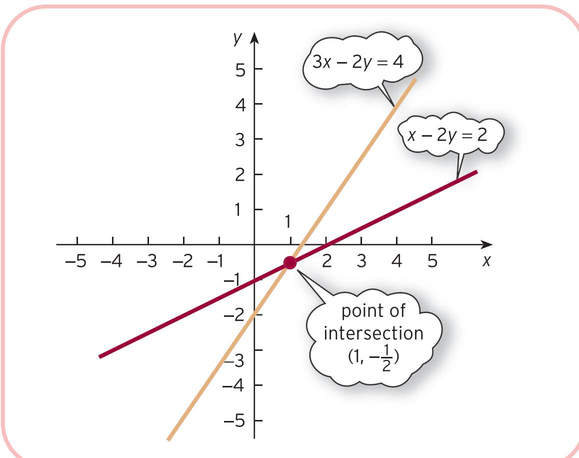


Figure S1.5

6 **(a)** $a = 1$, $b = 2$. The graph is sketched in Figure S1.6.

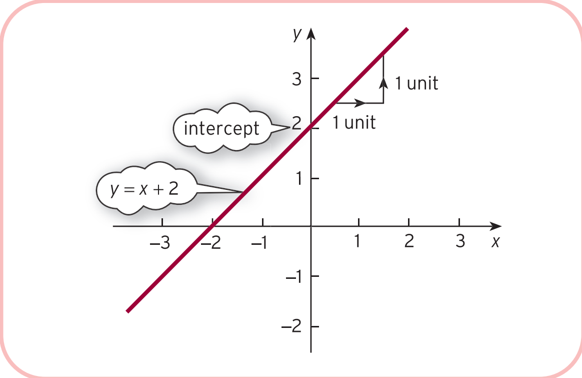


Figure S1.6

(b) $4x + 2y = 1$

$2y = 1 - 4x$ (subtract $4x$ from both sides)

$y = {}^1/_2 - 2x$ (divide both sides by 2)

so $a = -2$, $b = {}^1/_2$. The graph is sketched in Figure S1.7.

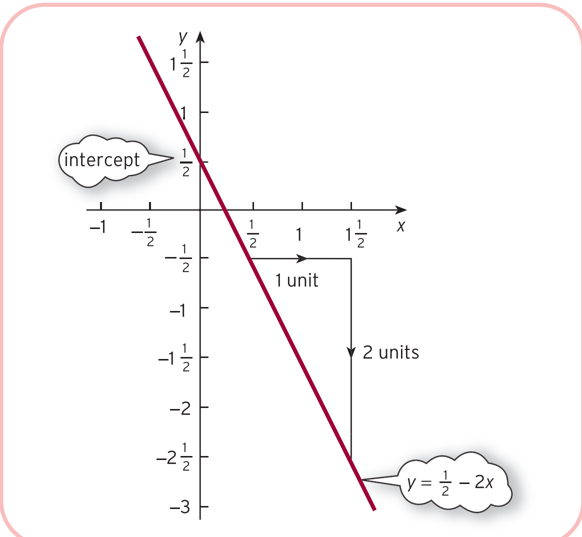


Figure S1.7

Exercise 1.3 (p. 64)

1 From Figure S1.8 the point of intersection is (2, 3).

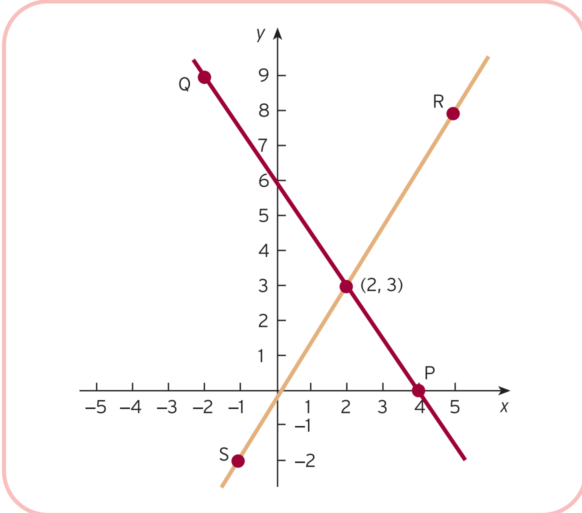


Figure S1.8

2 A, C, D, E

3 **(a)** 6 **(b)** -1; $(6, 2)$, $(1, -1)$

4

x	y
0	8
6	0
3	4

The graph is sketched in Figure S1.9.

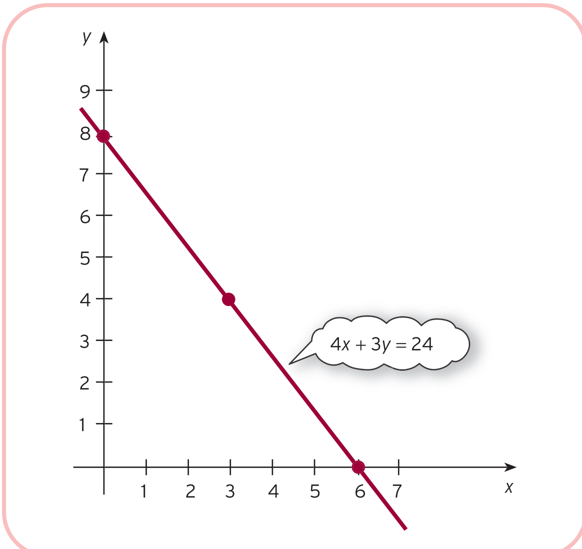


Figure S1.9

5 **(a)** $(-2, -2)$ **(b)** $(2, 1^1/_2)$ **(c)** $(1^1/_2, 1)$ **(d)** $(10, -9)$

6 **(a)** 5, 9 **(b)** 3, -1 **(c)** -1, 13 **(d)** 1, 4

(e) $-2, \dfrac{5}{2}$ **(f)** 5, -6

7 **(a)** The graph is sketched in Figure S1.10.

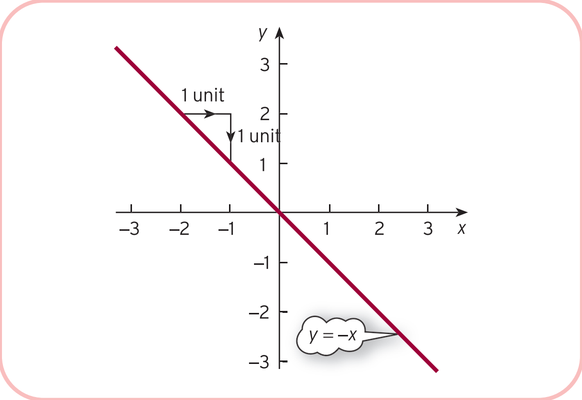


Figure S1.10

(b) The graph is sketched in Figure S1.11.

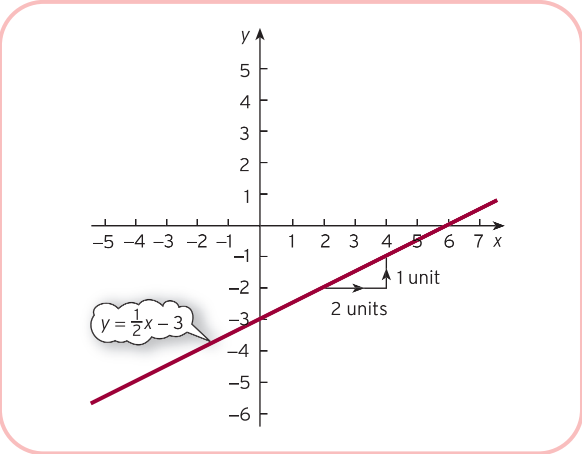


Figure S1.11

Exercise 1.3* (p. 65)

1 (5, –2), (10, 1), (0, –5)

2 **(a)** (2, 5) **(b)** (1, 4) **(c)** (–2, 3) **(d)** (–8, 3)

3 **(a)** 7, –34 **(b)** –1, 1 **(c)** $\frac{3}{2}$, –3

(d) 2, $\frac{5}{2}$ **(e)** $\frac{1}{5}$, 0 **(f)** 0, 2

(g) The vertical line, $x = 4$, has no gradient and does not intercept the y axis.

4 **(b)** and **(d)**

5 **(a)** $0.5x + 70$ **(b)** $x + 20$ **(c)** 100

6 **(1)** Gradients are $-\frac{a}{b}$ and $-\frac{d}{e}$ respectively so the lines are parallel when $\frac{a}{b} = \frac{d}{e}$ which gives $ae = bd$.

(2) Parallel lines so no solution.

7 $\left(0, \frac{c}{b}\right), \left(\frac{c}{a}, 0\right)$

Section 1.4

Practice Problems

1 **(a)** Step 1

It is probably easiest to eliminate y. This can be done by subtracting the second equation from the first:

$$\begin{aligned} 3x - 2y &= 4 \\ x - 2y &= 2\ - \\ \hline 2x \qquad &= 2 \end{aligned}$$

Step 2

The equation $2x = 2$ has solution $x = 2/2 = 1$.

Step 3

If this is substituted into the first equation then

$$\begin{aligned} 3(1) - 2y &= 4 \\ 3 - 2y &= 4 \\ -2y &= 1 \qquad \text{(subtract 3 from both sides)} \\ y &= -^1/_2 \qquad \text{(divide both sides by } -2\text{)} \end{aligned}$$

Step 4

As a check the second equation gives

$$\begin{aligned} x - 2y &= 1 - 2(-1/2) \\ &= 1 - (-1) = 2 \ \checkmark \end{aligned}$$

Hence the solution is $x = 1$, $y = -^1/_2$.

If you decide to eliminate x then the corresponding steps are as follows:

Step 1

Triple the second equation and subtract from the first:

$$\begin{aligned} 3x - 2y &= 4 \\ 3x - 6y &= 6\ - \\ \hline 4y &= -2 \end{aligned}$$

Step 2

The equation $4y = -2$ has solution $y = -2/4 = -^1/_2$.

Step 3

If this is substituted into the first equation then

$$\begin{aligned} 3x - 2(-^1/_2) &= 4 \\ 3x + 1 &= 4 \\ 3x &= 3 \end{aligned}$$

(subtract 1 from both sides)

$$x = 1$$

(divide both sides by 3)

(b) **Step 1**

It is immaterial which variable is eliminated. To eliminate x multiply the first equation by 5, multiply the second by 3 and add:

$$15x + 25y = 95$$
$$\underline{-15x + 6y = -33} +$$
$$31y = 62$$

Step 2

The equation $31y = 62$ has solution $y = 62/31 = 2$.

Step 3

If this is substituted into the first equation then

$$3x + 5(2) = 19$$
$$3x + 10 = 19$$
$$3x = 9$$

(subtract 10 from both sides)

$$x = 3$$

(divide both sides by 3)

Step 4

As a check the second equation gives

$$-5x + 2y = -5(3) + 2(2)$$
$$= -15 + 4 = -11 \quad ✓$$

Hence the solution is $x = 3$, $y = 2$.

2 (a) **Step 1**

To eliminate x multiply the first equation by 4, multiply the second equation by 3 and add:

$$12x + 24y = -8$$
$$\underline{-12x + 24y = -3} +$$
$$0y = -11$$

Step 2

This is impossible, so there are no solutions.

(b) **Step 1**

To eliminate x multiply the first equation by 2 and add to the second:

$$-10x + 2y = 8$$
$$\underline{10x - 2y = -8} +$$
$$0y = 0$$

Step 2

This is true for any value of y, so there are infinitely many solutions.

3 **Step 1**

To eliminate x from the second equation multiply equation (2) by 2 and subtract from equation (1):

$$2x + 2y - 5z = -5$$
$$\underline{2x - 2y + 2z = 6} -$$
$$4y - 7z = -11 \qquad (4)$$

To eliminate x from the third equation multiply equation (1) by 3, multiply equation (3) by 2 and add:

$$6x + 6y - 15z = -15$$
$$\underline{-6x + 2y + 4z = -4} +$$
$$8y - 11z = -19 \qquad (5)$$

The new system is

$$2x + 2y - 5z = -5 \qquad (1)$$
$$4y - 7z = -11 \qquad (4)$$
$$8y - 11z = -19 \qquad (5)$$

Step 2

To eliminate y from the third equation multiply equation (4) by 2 and subtract equation (5):

$$8y - 14z = -22$$
$$\underline{8y - 11z = -19}$$
$$-3z = -3 \qquad (6)$$

The new system is

$$2x + 2y - 5z = -5 \qquad (1)$$
$$4y - 7z = -11 \qquad (4)$$
$$-3z = -3 \qquad (6)$$

Step 3

Equation (6) gives $z = -3/-3 = 1$. If this is substituted into equation (4) then

$$4y - 7(1) = -11$$
$$4y - 7 = -11$$
$$4y = -4 \quad \text{(add 7 to both sides)}$$
$$y = -1 \quad \text{(divide both sides by 4)}$$

Finally, substituting $y = -1$ and $z = 1$ into equation (1) produces

$$2x + 2(-1) - 5(1) = -5$$
$$2x - 7 = -5$$
$$2x = 2$$

(add 7 to both sides)

$$x = 1$$

(divide both sides by 2)

Step 4

As a check the original equations (1), (2) and (3) give

$$2(1) + 2(-1) - 5(1) = -5 \quad ✓$$
$$1 - (-1) + 1 = 3 \quad ✓$$
$$-3(1) + (-1) + 2(1) = -2 \quad ✓$$

Hence the solution is $x = 1$, $y = -1$, $z = 1$.

Exercise 1.4 (p. 77)

1 (a) $x = -2, y = -2$

(b) $x = 2, y = 3/2$

(c) $x = 3/2, y = 1$

(d) $x = 10, y = -9$

2 The lines are sketched in Figure S1.12.

(a) Infinitely many. (b) No solution.

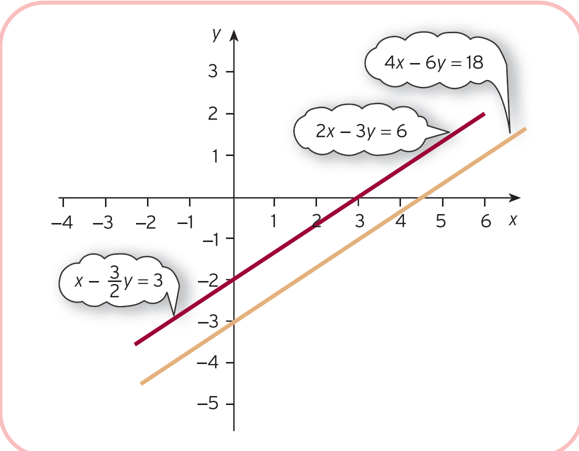


Figure S1.12

3 (a) Infinitely many. (b) No solution.

4 $k = -1$

Exercise 1.4* (p. 78)

1 (a) $x = 2, y = 5$ (b) $x = 1, y = 4$
(c) $x = -2, y = 3$ (d) $x = -8, y = 3$

2 (a) $a = 4, b = 8$ (b) $a = -3, b \neq \frac{1}{2}$

3 (a) $x = 3, y = -2, z = -1$

(b) $x = -1, y = 3, z = 4$

4 (a) No solution.

(b) Infinitely many solutions.

5 $k = 6$; no solutions otherwise.

Section 1.5

Practice Problems

1 (a) 0 (b) 48 (c) 16
(d) 25 (e) 1 (f) 17

The function g reverses the effect of f and takes you back to where you started. For example, if 25 is put into the function f, the outgoing number is 0; and when 0 is put into g, the original number, 25, is produced. We describe this by saying that g is the inverse of f (and vice versa).

2 The demand curve that passes through (0, 75) and (25, 0) is sketched in Figure S1.13. From this diagram we see that

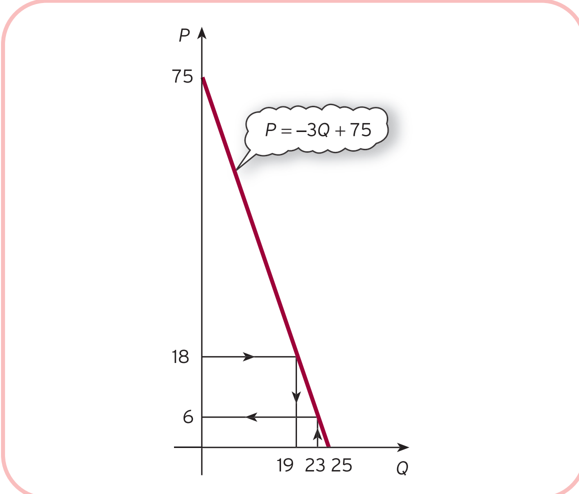


Figure S1.13

(a) $P = 6$ when $Q = 23$

(b) $Q = 19$ when $P = 18$

Alternatively, using algebra:

(a) Substituting $Q = 23$ gives

$$P = -3(23) + 75 = 6$$

(b) Substituting $P = 18$ gives $18 = -3Q + 75$ with solution $Q = 19$

3 (a) In equilibrium, $Q_S = Q_D = Q$, so

$$P = -4Q + 120$$

$$P = \tfrac{1}{3}Q + 29$$

Hence

$$-4Q + 120 = \tfrac{1}{3}Q + 29$$

(since both sides equal P)

$$-4\tfrac{1}{3}Q + 120 = 29$$

(subtract $\tfrac{1}{3}Q$ from both sides)

$$-4\tfrac{1}{3}Q = -91$$

(subtract 120 from both sides)

$$Q = 21$$

(divide both sides by $-4\tfrac{1}{3}$)

Substituting this value into either the demand or supply equations gives $P = 36$.

(b) After the imposition of a \$13 tax the supply equation becomes

$$P - 13 = \tfrac{1}{3}Q_S + 29$$

$$P = \tfrac{1}{3}Q_S + 42$$

(add 13 to both sides)

The demand equation remains unchanged, so, in equilibrium,

$$P = -4Q + 120$$

$$P = {}^1/_3Q + 42$$

Hence

$$-4Q + 120 = {}^1/_3Q + 42$$

This equation can now be solved as before to get $Q = 18$ and the corresponding price is $P = 48$. The equilibrium price rises from \$36 to \$48, so the consumer pays an additional \$12. The remaining \$1 of the tax is paid by the firm.

4 For good 1, $Q_{D_1} = Q_{S_1} = Q_1$ in equilibrium, so the demand and supply equations become

$$Q_1 = 40 - 5P_1 + P_2$$

$$Q_1 = -3 + 4P_1$$

Hence

$$40 - 5P_1 - P_2 = -3 + 4P_1$$

(since both sides equal Q_1)

$$40 - 9P_1 - P_2 = -3$$

(subtract $4P_1$ from both sides)

$$-9P_1 - P_2 = -43$$

(subtract 40 from both sides)

For good 2, $Q_{D_2} = Q_{S_2} = Q_2$ in equilibrium, so the demand and supply equations become

$$Q_2 = 50 - 2P_1 + 4P_2$$

$$Q_2 = -7 + 3P_2$$

Hence

$$50 - 2P_1 - 4P_2 = -7 + 3P_2$$

(since both sides equal Q_2)

$$50 - 2P_1 - 7P_2 = -7$$

(subtract $3P_2$ from both sides)

$$-2P_1 - 7P_2 = -57$$

(subtract 50 from both sides)

The equilibrium prices therefore satisfy the simultaneous equations

$$-9P_1 - P_2 = -43 \quad (1)$$

$$-2P_1 - 7P_2 = -57 \quad (2)$$

Step 1

Multiply equation (1) by 2 and (2) by 9 and subtract to get

$$61P_2 = 427 \quad (3)$$

Step 2

Divide both sides of equation (3) by 61 to get $P_2 = 7$.

Step 3

Substitute P_2 into equation (1) to get $P_1 = 4$.

If these equilibrium prices are substituted into either the demand or the supply equations then $Q_1 = 13$ and $Q_2 = 14$.

The goods are complementary because the coefficient of P_2 in the demand equation for good 1 is negative, and likewise for the coefficient of P_1 in the demand equation for good 2.

Exercise 1.5 (p. 94)

1 **(a)** 21 **(b)** 45 **(c)** 15 **(d)** 2
(e) 10 **(f)** 0; inverse

2 The supply curve is sketched in Figure S1.14.

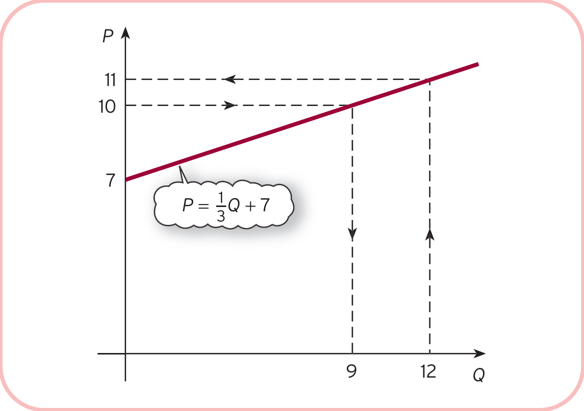


Figure S1.14

(a) 11 **(b)** 9

(c) 0; once the price falls below 7 the firm does not plan to produce any goods.

3 **(a)** Demand is 173. Additional advertising expenditure is 12.

(b) Superior.

4 **(a)** 53

(b) Substitutable; a rise in P_A leads to an increase in Q.

(c) 6

5 **(a)** 20, 10, 45; line passes through these three points.

(b) Line passing through (50, 0) and (0, 50)

$Q = 20$, $P = 30$

(c) Price increases; quantity increases.

6 6

7 $P_1 = 40$, $P_2 = 10$; $Q_1 = 30$, $Q_2 = 55$

8 **(a)** $Q = 30$

(b) Substitutable; e.g. since coefficient of P_r is positive.

(c) $P = 14$

(d) **(i)** slope = –20, intercept = 135

(ii) slope = $-\frac{1}{20}$, intercept = 6.75

9 Superior; graph of **(b)** lies above that of **(a)**.

Substitutable; graph of **(c)** lies below that of **(a)**.

Exercise 1.5* (p. 96)

1 **(a)** As P_S rises, consumers are likely to switch to the good under consideration, so demand for this good also rises: that is, the graph shifts to the right.

(b) As P_C rises, demand for the bundle of goods as a whole is likely to fall, so the graph shifts to the left.

(c) Assuming that advertising promotes the good and is successful, demand rises and the graph shifts to the right. For some goods, such as drugs, advertising campaigns are intended to discourage consumption, so the graph shifts to the left.

2 $m = -\frac{3}{2}$, $c = 9$

3 0 and 30

4 **(1)** $P = 30$, $Q = 10$

(2) New supply equation is $0.85P = 2Q_S + 10$; $P = 33.6$, $Q = 9.28$.

5 **(a)** 17, 9 **(b)** \$324

6 $P_1 = 20, P_2 = 5, P_3 = 8; Q_1 = 13, Q_2 = 16, Q_3 = 11$

7 Change supply equation to $P = 2Q_S + 40 + t$.

In equilibrium

$$-3Q + 60 = 2Q + 40 + t$$

$$-5Q = -20 + t$$

$$Q = 4 - \frac{t}{5}$$

Substitute to get $P = 48 + \frac{3}{5}t$.

(a) $t = 5$ firm pays \$2

(b) $P = 45$, $Q = 5$

8 \$180, \$200, $\frac{1}{3}$.

(a) $\frac{1}{2}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{2}{3}$; fraction is $\frac{k}{k+2}$

When $k = 6$, the fraction is $^3/_4$ so the consumer pays \$45.

[In general, if the supply and demand equations are

$$P = -aQ_D + b$$

$$P = cQ_S + d$$

the fraction of tax paid by the consumer is

$\frac{a}{a+d}$.]

Section 1.6

Practice Problems

1 **(a)** $\frac{1}{2}Q = 4$ (subtract 13 from both sides)

$Q = 8$ (multiply both sides by 2)

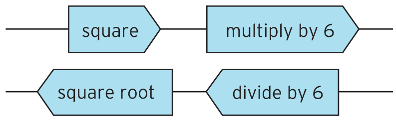
(b) $\frac{1}{2}Q = P - 13$ (subtract 13 from both sides)

$Q = 2(P - 13)$ (multiply both sides by 2)

$Q = 2P - 26$ (multiply out brackets)

(c) $Q = 2 \times 17 - 26 = 8$

2 **(a)**

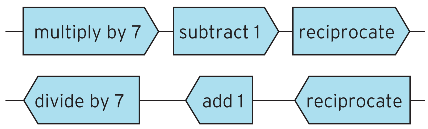


$6x^2 = y$

$x^2 = \frac{y}{6}$ (divide both sides by 6)

$x = \sqrt{\frac{y}{6}}$ (square root both sides)

(b)



$\frac{1}{7x-1} = y$

$7x - 1 = \frac{1}{y}$ (reciprocate both sides)

$7x = \frac{1}{y} + 1$ (add 1 to both sides)

$x = \frac{1}{7}\left(\frac{1}{y} + 1\right)$ (divide both sides by 7)

3 **(a)** $x - ay = cx + y$

$x = cx + y + ay$

(add ay to both sides)

$x - cx = y + ay$

(subtract cx from both sides)

$(1 - c)x = (1 + a)y$

(factorize both sides)

$x = \left(\frac{1+a}{1-c}\right)y$

(divide both sides by $1 - c$)

(b) $$y = \frac{x-2}{x+4}$$

$(x+4)y = x-2$

(multiply both sides by $x+4$)

$xy + 4y = x - 2$

(multiply out the brackets)

$xy = x - 2 - 4y$

(subtract $4y$ from both sides)

$xy - x = -2 - 4y$

(subtract x from both sides)

$(y-1)x = -2 - 4y$

(factorize left-hand side)

$$x = \frac{-2-4y}{y-1}$$

(divide both sides by $y-1$)

Exercise 1.6 (p. 106)

1 $Q = \frac{1}{2}P - 4$; 22

2 (a) $y = 2x + 5$ (b) $y = 2(x+5)$

(c) $y = \frac{5}{x^2}$ (d) $y = 2(x+4)^2 - 3$

3 (a) multiply by 5 → add 3

(b) add 3 → multiply by 5

(c) multiply by 6 → subtract 9

(d) square → multiply by 4 → subtract 6

(e) divide by 2 → add 7

(f) reciprocate → multiply by 2

(g) add 3 → reciprocate

4 (a) $x = \frac{1}{9}(y+6)$ (b) $x = 3y - 4$ (c) $x = 2y$

(d) $x = 5(y-8)$ (e) $x = \frac{1}{y} - 2$ (f) $x = \frac{1}{3}\left(\frac{4}{y} + 7\right)$

5 (a) $P = \frac{Q}{a} - \frac{b}{a}$ (b) $Y = \frac{b+1}{1-a}$ (c) $P = \frac{1}{aQ} - \frac{b}{a}$

6 $x = \frac{3}{y+2}$

Exercise 1.6* (p. 107)

1 (1)

(a) multiply by 9 → add 1

(b) multiply by −1 → add 3

(c) square → multiply by 5 → subtract 8

(d) multiply by 3 → add 5 → square root

(e) square → add 8 → reciprocate → multiply by 4

(2) (a) $x = \frac{y-1}{9}$ (b) $x = 3 - y$ (c) $x = \pm\sqrt{\frac{y+8}{5}}$

(d) $x = \frac{y^2-5}{3}$ (e) $x = \pm\sqrt{\frac{4}{y} - 8}$

2 (a) $x = \frac{c-a}{b}$ (b) $x = \frac{a^2-b}{a+1}$ (c) $x = (g-e)^2 - f$

(d) $x = \frac{ma^2}{b^2} + n$ (e) $x = \frac{n^2}{m^2} + m$ (f) $x = \left(\frac{a^2+b^2}{b-a}\right)^2$

3 $t = \frac{V+1}{V-5}$; 11

4 $r = 100\left(\sqrt[n]{\frac{S}{P}} - 1\right)$

5 (a) $G = Y(1 - a + at) + aT - b - I$

(b) $T = \frac{G + b + I - Y(1 - a + at)}{a}$

(c) $t = \frac{G + b + I - Y + aY - aT}{aY}$

(d) $a = \frac{G + b + I - Y}{T - Y + tY}$

Section 1.7

Practice Problems

1 $S = Y - C$

$= Y - (0.8Y + 25)$ (substitute expression for C)

$= Y - 0.8Y - 25$ (multiply out brackets)

$= 0.2Y - 25$ (collect terms)

2 $Y = C + I$ (from theory)

$C = 0.8Y + 25$ (given in question)

$I = 17$ (given in question)

Substituting the given value of I into the first equation gives

$Y = C + 17$

and if the expression for C is substituted into this then

$Y = 0.8Y + 42$

$0.2Y = 42$ (subtract $0.8Y$ from both sides)

$Y = 210$ (divide both sides by 0.2)

Repeating the calculations with $I = 18$ gives $Y = 215$, so a 1 unit increase in investment leads to a 5 unit increase in income. The scale factor, 5, is called the investment multiplier. In general, the investment multiplier is given by $1/(1 - a)$, where a is the marginal propensity to consume. The foregoing is a special case of this with $a = 0.8$.

3 $Y = C + I + G$ (1)

$G = 40$ (2)

$I = 55$ (3)

$C = 0.8Y_d + 25$ (4)

$T = 0.1Y + 10$ (5)

$Y_d = Y - T$ (6)

Substituting equations (2) and (3) into equation (1) gives

$Y = C + 95$ (7)

Substituting equation (5) into (6) gives

$Y_d = Y - (0.1Y + 10)$

$= 0.9Y - 10$

so, from equation (4),

$C = 0.8(0.9Y - 10) + 25$

$= 0.72Y + 17$ (8)

Finally, substituting equation (8) into (7) gives

$Y = 0.72Y + 112$

which has solution $Y = 400$.

4 The commodity market is in equilibrium when

$Y = C + I$

so we can substitute the given expressions for consumption ($C = 0.7Y + 85$) and investment ($I = 50r + 1200$) to deduce that

$Y = 0.7Y - 50r + 1285$

which rearranges to give the IS schedule,

$0.3Y + 50r = 1285$ (1)

The money market is in equilibrium when

$M_S = M_D$

Now we are given that $M_S = 500$ and that total demand,

$M_D = L_1 + L_2 = 0.2Y - 40r + 230$

so that

$500 = 0.2Y - 40r + 230$

which rearranges to give the LM schedule,

$0.2Y - 40r = 270$ (2)

We now solve equations (1) and (2) as a pair of simultaneous equations.

Step 1

Multiply equation (1) by 0.2 and (2) by 0.3 and subtract to get

$22r = 176$

Step 2

Divide through by 22 to get $r = 8$.

Step 3

Substitute $r = 8$ into equation (1) to give $Y = 2950$.

The IS and LM curves shown in Figure S1.15 confirm this, since the point of intersection has coordinates (8, 2950). A change in I does not affect the LM schedule. However, if the autonomous level of investment increases from its current level of 1200 then the right-hand side of the IS schedule (1) will rise. The IS curve moves upwards, causing both r and Y to increase.

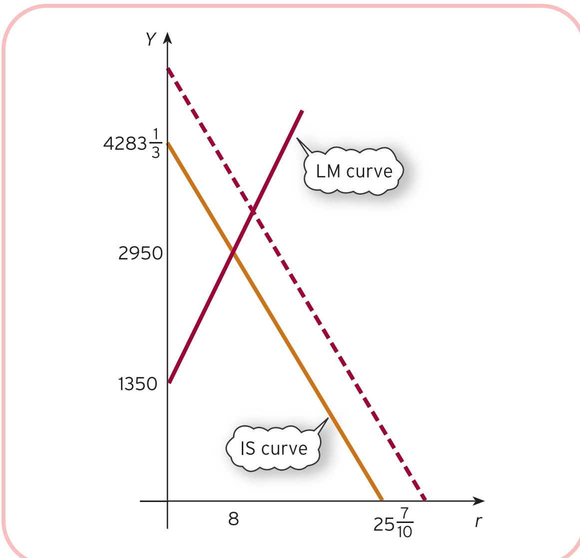


Figure S1.15

Exercise 1.7 (p. 123)

1 (a) 40 (b) 0.7; $Y = \frac{10}{7}(C - 40)$; 100

2 (a) $S = 0.1Y - 72$ (b) $S = 0.2Y - 100$

3 (a) 325 (b) 225 (c) 100

4 $10a + b = 28$

$30a + b = 44$

$a = 0.8, b = 20; \quad Y = 165$

5 187.5

Exercise 1.7* (p. 124)

1 **(a)** $S = 0.3Y - 30$

(b) $S = \dfrac{10Y - 500}{Y + 10}$

2 $C = \dfrac{aI^* + b}{1 - a}$

3 $a = \dfrac{Y - b - I^*}{Y}$

4 825

5 $Y = 2500, r = 10$

6 $C = a(Y - 20) + 50 = aY - 20a + 50$

$Y = aY - 20a + 74$

$(1 - a)Y = 74 - 20a \quad \Rightarrow \quad Y = \dfrac{74 - 20a}{1 - a}$

$a = \dfrac{Y - 74}{Y - 20}; \quad a = 0.6, C = 131$

7 **(a)** $C = 120 + 0.8Y$

(b) $C = 40 + 0.8Y$

(c) $C = 120 + 0.6Y$

With a lump sum tax, the graph has the same slope but has been shifted downwards.

With a proportional tax, the graph has the same intercept but is less steep.

(a) 600 **(b)** 200 **(c)** 300

8 $0.9Y + 30$; 300

(a) Slope decreases; 150.

(b) Shifts up 5 units; 350.

Chapter 2

Section 2.1

Practice Problems

1 **(a)**
$$x^2 - 100 = 0$$
$$x^2 = 100$$
$$x = \pm\sqrt{100}$$
$$x = \pm 10$$

(b)
$$2x^2 - 8 = 0$$
$$2x^2 = 8$$
$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

(c)
$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$
$$x = \pm 1.73 \quad \text{(to 2 decimal places)}$$

(d)
$$x^2 - 5.72 = 0$$
$$x^2 = 5.72$$
$$x = \pm\sqrt{5.72}$$
$$x = \pm 2.39 \quad \text{(to 2 decimal places)}$$

(e)
$$x^2 + 1 = 0$$
$$x^2 = -1$$

This equation does not have a solution, because the square of a number is always positive. Try using your calculator to find $\sqrt{(-1)}$. An error message should be displayed.

(f)
$$3x^2 + 6.21 = 0$$
$$3x^2 = -6.21$$
$$x^2 = -2.07$$

This equation does not have a solution, because it is impossible to find the square root of a negative number.

(g) $x^2 = 0$

This equation has exactly one solution, $x = 0$.

2 **(a)** $a = 2, b = -19, c = -10$

$$x = \frac{-(-19) \pm \sqrt{((-19)^2 - 4(2)(-10))}}{2(2)}$$

$$= \frac{19 \pm \sqrt{(361 + 80)}}{4}$$

$$= \frac{19 \pm \sqrt{441}}{4} = \frac{19 \pm 21}{4}$$

This equation has two solutions:

$$x = \frac{19 + 21}{4} = 10$$

$$x = \frac{19 - 21}{4} = -\frac{1}{2}$$

(b) $a = 4, b = 12, c = 9$

$$x = \frac{-12 \pm \sqrt{((12)^2 - 4(4)(9))}}{2(4)}$$

$$= \frac{-12 \pm \sqrt{(144 - 144)}}{8}$$

$$= \frac{-12 \pm 0}{8}$$

This equation has one solution, $x = -^3/_2$.