Instructor's Solutions Manual

Sal Sciandra
Niagara County Community College

Mathematics with Applications and Finite Mathematics with Applications in the Management, Natural, and Social Sciences

Twelfth Edition

Margaret L. Lial
American River College

Thomas Hungerford
Saint Louis University

John Holcomb
Cleveland State University

Bernadette Mullins
Birmingham-Southern College

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Contents

Chapter 1: Algebra and Equations

Section 1.1 The Real Numbers

1. True. This statement is true, since every integer can be written as the ratio of the integer and 1. For example, $5 = \frac{5}{1}$.

2. False. For example, 5 is a real number, and $5 = \frac{10}{2}$ which is not an irrational number.

3. Answers vary with the calculator, but $\frac{2,508,429,787}{798,458,000}$ is the best.

4. $0 + (-7) = (-7) + 0$
   This illustrates the commutative property of addition.

5. $6(t + 4) = 6t + 6 \cdot 4$
   This illustrates the distributive property.

6. $3 + (-3) = (-3) + 3$
   This illustrates the commutative property of addition.

7. $(-5) + 0 = -5$
   This illustrates the identity property of addition.

8. $(-4) \bullet (\frac{-1}{4}) = 1$
   This illustrates the multiplicative inverse property.

9. $8 + (12 + 6) = (8 + 12) + 6$
   This illustrates the associative property of addition.

10. $1 \bullet (-20) = -20$
    This illustrates the identity property of multiplication.

11. Answers vary. One possible answer: The sum of a number and its additive inverse is the additive identity. The product of a number and its multiplicative inverse is the multiplicative identity.

12. Answers vary. One possible answer: When using the commutative property, the order of the addends or multipliers are changed, while the grouping of the addends or multipliers is changed when using the associative property.

For Exercises 13–16, let $p = -2$, $q = 3$ and $r = -5$.

13. $-3(p + 5q) = -3[-2 + 5(3)] = -3[-2 + 15]$
    $= -3(13) = -39$

14. $2(q - r) = 2(3 + 5) = 2(8) = 16$

15. $\frac{q + r}{q + p} = \frac{3 + (-5)}{3 + (-2)} = \frac{-2}{1} = -2$

16. $\frac{3q}{3p - 2r} = \frac{3(3)}{3(-2) - 2(-5)} = \frac{9}{-6 + 10} = \frac{9}{4}$

17. Let $r = 3.8$.
    $\text{APR} = 12r = 12(3.8) = 45.6\%$

18. Let $r = 0.8$.
    $\text{APR} = 12r = 12(0.8) = 9.6\%$

19. Let $APR = 11$.
    $APR = 12r$
    $11 = 12r$
    $\frac{11}{12} = r$
    $r \approx .9167\%$

20. Let $APR = 13.2$.
    $APR = 12r$
    $13.2 = 12r$
    $\frac{13.2}{12} = r$
    $r = 1.1\%$

21. $3 - 4 \cdot 5 + 5 = 3 - 20 + 5 = -17 + 5 = -12$

22. $8 - (-4)^2 - (-12)$
    Take powers first.
    $8 - 16 - (-12)$
    Then add and subtract in order from left to right.
    $8 - 16 + 12 = -8 + 12 = 4$

23. $(4 - 5) \cdot 6 + 6 = -1 \cdot 6 + 6 = -6 + 6 = 0$

24. $\dfrac{2(3-7)+4(8)}{4(-3)+(-3)(-2)}$

Work above and below fraction bar. Do multiplications and work inside parentheses.

$=\dfrac{2(-4)+32}{-12+6}=\dfrac{-8+32}{-12+6}=\dfrac{24}{-6}=-4$

25. $8-4^2-(-12)$

Take powers first.

8 – 16 – (–12)

Then add and subtract in order from left to right.

8 – 16 + 12 = –8 + 12 = 4

26. $-(3-5)-\left[2-\left(3^2-13\right)\right]$

Take powers first.

–(3 – 5) – [2 – (9 – 13)]

Work inside brackets and parentheses.

– (–2) – [2 – (–4)] = 2 – [2 + 4]

= 2 – 6 = –4

27. $\dfrac{2(-3)+\frac{3}{(-2)}-\frac{2}{\left(-\sqrt{16}\right)}}{\sqrt{64}-1}$

Work above and below fraction bar. Take roots.

$\dfrac{2(-3)+\frac{3}{(-2)}-\frac{2}{(-4)}}{8-1}$

Do multiplications and divisions.

$\dfrac{-6-\frac{3}{2}+\frac{1}{2}}{8-1}$

Add and subtract.

$\dfrac{-\frac{12}{2}-\frac{3}{2}+\frac{1}{2}}{7}=\dfrac{-\frac{14}{2}}{7}=\dfrac{-7}{7}=-1$

28. $\dfrac{6^2-3\sqrt{25}}{\sqrt{6^2+13}}$

Take powers and roots.

$\dfrac{36-3(5)}{\sqrt{36+13}}=\dfrac{36-15}{\sqrt{49}}=\dfrac{21}{7}=3$

29. $\dfrac{2040}{523}, \dfrac{189}{37}, \sqrt{27}, \dfrac{4587}{691}, 6.735, \sqrt{47}$

30. $\dfrac{187}{63}, 2.9884, \sqrt{\sqrt{85}}, \pi, \sqrt{10}, \dfrac{385}{117}$

31. 12 is less than 18.5.

$12 < 18.5$

32. –2 is greater than –20.

$-2 > -20$

33. x is greater than or equal to 5.7.

$x \geq 5.7$

34. y is less than or equal to –5.

$y \leq -5$

35. z is at most 7.5.

$z \leq 7.5$

36. w is negative.

$w < 0$

37. $-6 < -2$

38. $3/4 = .75$

39. $3.14 < \pi$

40. $1/3 > .33$

41. a lies to the right of b or is equal to b.

42. $b + c = a$

43. $c < a < b$

44. a lies to the right of 0

45. (–8, –1)

This represents all real numbers between –8 and –1, not including –8 and –1. Draw parentheses at –8 and –1 and a heavy line between them. The parentheses at –8 and –1 show that neither of these points belongs to the graph.

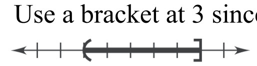
46. [–1, 10]

This represents all real numbers between –1 and 10, including –1 and 10. Draw brackets at –1 and 10 and a heavy line between them.

47. $(-2, 3]$

All real numbers x such that $-2 < x \leq 3$

Draw a heavy line from –2 to 3. Use a parenthesis at –2 since it is not part of the graph. Use a bracket at 3 since it is part of the graph.

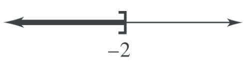


48. $[-2, 2)$
This represents all real numbers between –2 and 2, including –2, but not including 2.
Draw a bracket at –2, a parenthesis at 2, and a heavy line between them.

49. $(-2, \infty)$

All real numbers x such that $x > -2$
Start at –2 and draw a heavy line to the right. Use a parenthesis at –2 since it is not part of the graph.

50. $(-\infty, -2]$
This represents all real numbers less than or equal to –2. Draw a bracket at –2 and a heavy line to the left.



51. $|-9| - |-12| = 9 - (12) = -3$

52. $|8| - |-4| = 8 - (4) = 4$

53. $-|-4| - |-1-14| = -(4) - |-15|$
$= -(4) - 15 = -19$

54. $-|6| - |-12-4| = -(6) - |-16| = -6 - (16) = -22$

55. $|5| \_\_\_ |-5|$
$5 \_\_\_ 5$
$5 = 5$

56. $-|-4| \_\_\_ |4|$
$-4 \_\_\_ 4$
$-4 < 4$

57. $|10-3| \_\_\_ |3-10|$
$|7| \_\_\_ |-7|$
$7 \_\_\_ 7$
$7 = 7$

58. $|6-(-4)| \_\_\_ |-4-6|$
$|10| \_\_\_ |-10|$
$10 \_\_\_ 10$
$10 = 10$

59. $|-2+8| \_\_\_ |2-8|$
$|6| \_\_\_ |-6|$
$6 \_\_\_ 6$
$6 = 6$

60. $|3| \cdot |-5| \_\_\_ |3(-5)|$
$|3| \cdot |-5| \_\_\_ |-15|$
$3 \cdot 5 \_\_\_ 15$
$15 = 15$

61. $|3-5| \_\_\_ |3| - |5|$
$|-2| \_\_\_ 3-5$
$2 \_\_\_ -2$
$2 > -2$

62. $|-5+1| \_\_\_ |-5| + |1|$
$|-4| \_\_\_ 5+1$
$4 \_\_\_ 6$
$4 < 6$

63. When $a < 7$, $a - 7$ is negative.
So $|a-7| = -(a-7) = 7 - a$.

64. When $b \geq c$, $b - c$ is positive.
So $|b-c| = b - c$.

Answers will vary for exercises 65–67. Sample answers are given.

65. No, it is not always true that $|a+b| = |a| + |b|$. For example, let $a = 1$ and $b = -1$. Then,
$|a+b| = |1+(-1)| = |0| = 0$, but
$|a| + |b| = |1| + |(-1)| = 1 + 1 = 2$.

66. Yes, if a and b are any two real numbers, it is always true that $|a-b| = |b-a|$. In general, $a - b = -(b - a)$. When we take the absolute value of each side, we get
$|a-b| = |-(b-a)| = |b-a|$.

67. $|2-b| = |2+b|$ only when $b = 0$. Then each side of the equation is equal to 2. If b is any other value, subtracting it from 2 and adding it to 2 will produce two different values.

68. For females: $|x - 63.5| \leq 8.4$; for males:
$|x - 68.9| \leq 9.3$

69. 1; 30062007

70. 8; 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015

71. 9; 2006, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015

72. 4; 2008, 2010, 2013, 2015

73. 4; 2006, 2007, 2009, 2011

74. 8; 2006, 2007, 2009, 2010, 2011, 2012, 2013, 2014, 2015

75. $|10.6-14.9|=|-4.3|=4.3$

76. $|63.1-(-8.0)|=|71.1|=71.1$

77. $|-1.0-63.1|=|-64.1|=64.1$

78. $|10.6-(-5.7)|=|16.3|=16.3$

79. $|-5.7-(-8.0)|=|2.3|=2.3$

80. $|-1.0-(-5.7)|=|4.7|=4.7$

81. 3; 30062010, 2015, 2016

82. 7; 2010, 2011, 2012, 2013, 2014, 2015, 2016

83. 6; 2010, 2011, 2012, 2013, 2014, 2015

84. 3; 2014, 2015, 2016

Section 1.2 Polynomials

1. $11.2^6 \approx 1{,}973{,}822.685$

2. $(-6.54)^{11} \approx -936{,}171{,}103.1$

3. $\left(-\frac{18}{7}\right)^6 \approx 289.0991339$

4. $\left(\frac{5}{9}\right)^7 \approx .0163339967$

5. -3^2 is negative, whereas $(-3)^2$ is positive. Both -3^3 and $(-3)^3$ are negative.

6. To multiply 4^3 and 4^5, add the exponents since the bases are the same. The product of 4^3 and 3^4 cannot be found in the same way since the bases are different. To evaluate the product, first do the powers, and then multiply the results.

7. $4^2 \cdot 4^3 = 4^{2+3} = 4^5$

8. $(-4)^4 \cdot (-4)^6 = (-4)^{4+6} = (-4)^{10}$

9. $(-6)^2 \cdot (-6)^5 = (-6)^{2+5} = (-6)^7$

10. $(2z)^5 \cdot (2z)^6 = (2z)^{5+6} = (2z)^{11}$

11. $\left[(5u)^4\right]^7 = (5u)^{4\cdot 7} = (5u)^{28}$

12. $(6y)^3 \cdot \left[(6y)^5\right]^4 = (6y)^3 \cdot (6y)^{20}$
$= (6y)^{23}$

13. degree 4; coefficients: 6.2, –5, 4, –3, 3.7; constant term 3.7.

14. degree 7; coefficients: 6, 4, 0, 0, –1, 0, 1, 0; constant term 0.

15. Since the highest power of x is 3, the degree is 3.

16. Since the highest power of x is 5, the degree is 5.

17. $\left(3x^3+2x^2-5x\right)+\left(-4x^3-x^2-8x\right)$
$=\left(3x^3-4x^3\right)+\left(2x^2-x^2\right)+(-5x-8x)$
$=-x^3+x^2-13x$

18. $\left(-2p^3-5p+7\right)+\left(-4p^2+8p+2\right)$
$=-2p^3-4p^2+(-5p+8p)+(7+2)$
$=-2p^3-4p^2+3p+9$

19. $\left(-4y^2-3y+8\right)-\left(2y^2-6y+2\right)$
$=\left(-4y^2-3y+8\right)+\left(-2y^2+6y-2\right)$
$=-4y^2-3y+8-2y^2+6y-2$
$=\left(-4y^2-2y^2\right)+(-3y+6y)+(8-2)$
$=-6y^2+3y+6$

20. $\left(7b^2+2b-5\right)-\left(3b^2+2b-6\right)$
$=\left(7b^2+2b-5\right)+\left(-3b^2-2b+6\right)$
$=\left(7b^2-3b^2\right)+(2b-2b)+(-5+6)$
$=4b^2+1$

21. $\left(2x^3+2x^2+4x-3\right)-\left(2x^3+8x^2+1\right)$
$=\left(2x^3+2x^2+4x-3\right)+\left(-2x^3-8x^2-1\right)$
$=2x^3+2x^2+4x-3-2x^3-8x^2-1$
$=\left(2x^3-2x^3\right)+\left(2x^2-8x^2\right)+(4x)+(-3-1)$
$=-6x^2+4x-4$

22. $\left(3y^3+9y^2-11y+8\right)-\left(-4y^2+10y-6\right)$
$=\left(3y^3+9y^2-11y+8\right)+\left(4y^2-10y+6\right)$
$=3y^3+\left(9y^2+4y^2\right)+(-11y-10y)+(8+6)$
$=3y^3+13y^2-21y+14$

23. $-9m\left(2m^2+6m-1\right)$
$=(-9m)\left(2m^2\right)+(-9m)(6m)+(-9m)(-1)$
$=-18m^3-54m^2+9m$

24. $2a\left(4a^2-6a+8\right)$
$=2a\left(4a^2\right)+2a(-6a)+2a(8)$
$=8a^3-12a^2+16a$

25. $(3z+5)\left(4z^2-2z+1\right)$
$=(3z)\left(4z^2-2z+1\right)+(5)\left(4z^2-2z+1\right)$
$=12z^3-6z^2+3z+20z^2-10z+5$
$=12z^3+14z^2-7z+5$
$=12z^3+20z^2-6z^2-10z+3z+5$
$=12z^3+14z^2-7z+5$

26. $(2k+3)\left(4k^3-3k^2+k\right)$
$=2k\left(4k^3-3k^2+k\right)+3\left(4k^3-3k^2+k\right)$
$=8k^4-6k^3+2k^2+12k^3-9k^2+3k$
$=8k^4+6k^3-7k^2+3k$

27. $(6k-1)(2k+3)$
$=(6k)(2k+3)+(-1)(2k+3)$
$=12k^2+18k-2k-3$
$=12k^2+16k-3$

28. $(8r+3)(r-1)$
Use FOIL.
$=8r^2-8r+3r-3$
$=8r^2-5r-3$

29. $(3y+5)(2y+1)$
Use FOIL.
$=6y^2+3y+10y+5$
$=6y^2+13y+5$

30. $(5r-3s)(5r-4s)$
$=25r^2-20rs-15rs+12s^2$
$=25r^2-35rs+12s^2$

31. $(9k+q)(2k-q)$
$=18k^2-9kq+2kq-q^2$
$=18k^2-7kq-q^2$

32. $(.012x-.17)(.3x+.54)$
$=(.012x)(.3x)+(.012x)(.54)$
$+(-.17)(.3x)+(-.17)(.54)$
$=.0036x^2+.00648x-.051x-.0918$
$=.0036x^2-.04452x-.0918$

33. $(6.2m-3.4)(.7m+1.3)$
$=4.34m^2+8.06m-2.38m-4.42$
$=4.34m^2+5.68m-4.42$

34. $2p-3[4p-(8p+1)]$
$=2p-3(4p-8p-1)$
$=2p-3(-4p-1)$
$=2p+12p+3$
$=14p+3$

35. $5k-[k+(-3+5k)]$
$=5k-[6k-3]$
$=5k-6k+3$
$=-k+3$

36. $(3x-1)(x+2)-(2x+5)^2$

$=\left(3x^2+5x-2\right)-\left(4x^2+20x+25\right)$

$=3x^2+5x-2-4x^2-20x-25$

$=\left(3x^2-4x^2\right)+(5x-20x)+(-2-25)$

$=-x^2-15x-27$

37. $R = 5\,(1000x) = 5000x$
$C = 200{,}000 + 1800x$
$P = (5000x) - (200{,}000 + 1800x)$
$= 3200x - 200{,}000$

38. $R = 8.50(1000x) = 8500x$
$C = 225{,}000 + 4200x$
$P = (8500x) - (225{,}000 + 4200x)$
$= 4300x - 225{,}000$

39. $R = 9.75(1000x) = 9750x$

$C = 260{,}000+(-3x^2+3480x-325)$

$=-3x^2+3480x+259{,}675$

$P=(9750x)-(-3x^2+3480x+259{,}675)$

$=3x^2+6270x-259{,}675$

40. $R = 23.50(1000x) = 23{,}500x$

$C=145{,}000+(-4.2x^2+3220x-425)$

$=-4.2x^2+3220x+144{,}575$

$P=(23{,}500x)-(-4.2x^2+3220x+144{,}575)$

$=4.2x^2+20{,}280x-144{,}575.$

41. **a.** According to the bar graph, the net earnings in 2007 were \$673 million.

b. Let $x = 7$.

$4.79x^3-122.5x^2+1104x-2863$

$=4.79(7)^3-122.5(7)^2+1104(7)-2863$

$=505.47$

According to the polynomial, the net earnings in 2007 were approximately \$505 million.

42. **a.** According to the bar graph, the net earnings in 2015 were \$2757 million.

b. Let $x = 15$.

$4.79x^3-122.5x^2+1104x-2863$

$=4.79(15)^3-122.5(15)^2+1104(15)-2863$

$=2300.75$

According to the polynomial, the net earnings in 2015 were approximately \$2301 million.

43. **a.** According to the bar graph, the net earnings in 2012 were \$1384 million.

b. Let $x = 12$.

$4.79x^3-122.5x^2+1104x-2863$

$=4.79(12)^3-122.5(12)^2+1104(12)-2863$

$=1022.12$

According to the polynomial, the net earnings in 2012 were approximately \$1022 million.

44. **a.** According to the bar graph, the net earnings in 2013 were \$8 million.

b. Let $x = 13$.

$4.79x^3-122.5x^2+1104x-2863$

$=4.79(13)^3-122.5(13)^2+1104(13)-2863$

$=1310.13$

According to the polynomial, the net earnings in 2013 were approximately \$1,310 million.

45. Let $x = 17$.

$4.79x^3-122.5x^2+1104x-2863$

$=4.79(17)^3-122.5(17)^2+1104(17)-2863$

$=4035.77$

According to the polynomial, the net earnings in 2017 will be approximately \$4036 million.

46. Let $x = 18$.

$4.79x^3 - 122.5x^2 + 1104x - 2863$

$= 4.79(18)^3 - 122.5(18)^2 + 1104(18) - 2863$

$= 5254.28$

According to the polynomial, the net earnings in 2018 will be approximately \$5254 million.

47. Let $x = 19$.

$4.79x^3 - 122.5x^2 + 1104x - 2863$

$= 4.79(19)^3 - 122.5(19)^2 + 1104(19) - 2863$

$= 6745.11$

According to the polynomial, the net earnings in 2019 will be approximately \$6745 million.

48. The figures for 2013 – 2015 seem high, but plausible. To see how accurate these conclusions are, search Starbucks.com for later annual reports.

For exercises 49–52, we use the polynomial $9.5x^3 - 401.6x^2 + 6122x - 25,598$.

49. Let $x = 10$.

$9.5(10)^3 - 401.6(10)^2 + 6122(10) - 25,598$

$= 4962$

Thus, the costs were approximately \$4962 million in 2010. The statement is false.

50. Let $x = 15$.

$9.5(15)^3 - 401.6(15)^2 + 6122(15) - 25,598$

$= 7934.5$

Thus, the costs were approximately \$7934.5 million in 2015. The statement is true.

51. Let $x = 12$.

$9.5(12)^3 - 401.6(12)^2 + 6122(12) - 25,598$

$= 6451.6$

Let $x = 15$.

$9.5(15)^3 - 401.6(15)^2 + 6122(15) - 25,598$

$= 7934.5$

Thus, the costs were \$6451.6 million in 2012 and \$7934.5 million in 2015. The statement is false.

52. Let $x = 11$.

$9.5(11)^3 - 401.6(11)^2 + 6122(11) - 25,598$

$= 5794.9$

Let $x = 16$.

$9.5(16)^3 - 401.6(16)^2 + 6122(16) - 25,598$

$= 8456.4$

Thus, the costs were \$5794.9 million in 2011 and \$8456.4 million in 2016. The statement is true.

For exercises 53–58, we use the polynomial $-72.85x^3 + 2082x^2 - 16,532x + 59,357$.

53. Let $x = 7$.

$-72.85x^3 + 2082x^2 - 16,532x + 59,357$

$= -72.85(7)^3 + 2082(7)^2 - 16,532(7) + 59,357$

$= 20,663.45$

Thus, the profit for PepsiCo Inc in 2007 was approximately \$20,663 million.

54. Let $x = 10$.

$-72.85x^3 + 2082x^2 - 16,532x + 59,357$

$= -72.85(10)^3 + 2082(10)^2 - 16,532(10) + 59,357$

$= 29,387$

Thus, the profit for PepsiCo Inc in 2010 was \$29,387 million.

55. Let $x = 12$.

$-72.85x^3 + 2082x^2 - 16,532x + 59,357$

$= -72.85(12)^3 + 2082(12)^2 - 16,532(12) + 59,357$

$= 34,896.2$

Thus, the profit for PepsiCo Inc in 2012 was approximately \$34,896 million.

56. Let $x = 15$.

$-72.85x^3 + 2082x^2 - 16,532x + 59,357$

$= -72.85(15)^3 + 2082(15)^2 - 16,532(15) + 59,357$

$= 33,958.25$

Thus, the profit for PepsiCo Inc in 2015 was approximately \$33,958 million.

57. Let $x = 13$.

$$-72.85x^3 + 2082x^2 - 16{,}532x + 59{,}357$$
$$= -72.85(13)^3 + 2082(13)^2 - 16{,}532(13) + 59{,}357$$
$$= 36{,}247.55$$

Thus, the profit for PepsiCo Inc in 2013 was approximately \$36,248 million.

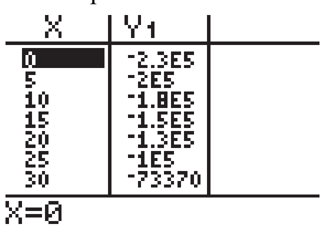
Let $x = 9$.

$$-72.85x^3 + 2082x^2 - 16{,}532x + 59{,}357$$
$$= -72.85(9)^3 + 2082(9)^2 - 16{,}532(9) + 59{,}357$$
$$= 26{,}103.35$$

Thus, the profit for PepsiCo Inc in 2009 was approximately \$26,104 million. Therefore, the profit was higher in 2013.

58. By comparing the answers to problems 55 and 56, the profit was higher in 2012.

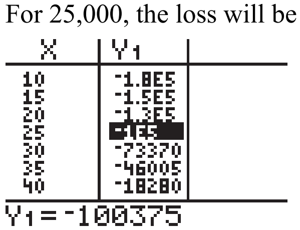
59. $P = 7.2x^2 + 5005x - 230{,}000$. Here is part of the screen capture.



X	Y1
0	-2.3E5
5	-2E5
10	-1.8E5
15	-1.5E5
20	-1.3E5
25	-1E5
30	-73370

X=0

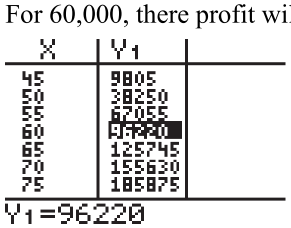
For 25,000, the loss will be \$100,375;



X	Y1
10	-1.8E5
15	-1.5E5
20	-1.3E5
25	-1E5
30	-73370
35	-46005
40	-18280

Y1=-100375

For 60,000, there profit will be \$96,220.



X	Y1
45	9805
50	38250
55	67055
60	96220
65	125745
70	155630
75	185875

Y1=96220

There is a loss at the beginning because of large fixed costs. When more items are made, these costs become a smaller portion of the total costs.

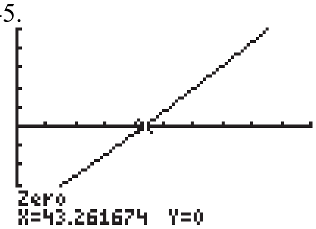
60. In order for the company to make a profit,

$$P = 7.2x^2 + 5005x - 230{,}000 > 0$$

Set the profit function equal to 0 and solve for x.

$$8x^2 + 4450x - 215{,}000 = 0$$

By the quadratic formula, $x \approx 45$ or $x \approx -601$.
Since x represents a positive number, $x = 45$.



Therefore, between 40,000 and 45,000 calculators must be sold for the company to make a profit.

61. Let $x = 100$ (in thousands)

$$7.2(100)^2 + 5005(100) - 230{,}000 = 342{,}500$$

d. The profit for selling 100,000 calculators is \$342,500.

62. Let $x = 150$ (in thousands)

$$7.2(150)^2 + 5005(150) - 230{,}000 = 682{,}750$$

d. The profit for selling 150,000 calculators is \$682,750.

Section 1.3 Factoring

1. $12x^2 - 24x = 12x \cdot x - 12x \cdot 2 = 12x(x-2)$

2. $5y - 65xy = 5y(1) - 5y(13x) = 5y(1-13x)$

3. $r^3 - 5r^2 + r = r\left(r^2\right) - r(5r) + r(1)$
$$= r\left(r^2 - 5r + 1\right)$$

4. $t^3 + 3t^2 + 8t = t\left(t^2\right) + t(3t) + t(8)$
$$= t\left(t^2 + 3t + 8\right)$$

5. $6z^3 - 12z^2 + 18z$
$= 6z\left(z^2\right) - 6z(2z) + 6z(3)$
$= 6z\left(z^2 - 2z + 3\right)$

6. $5x^3 + 55x^2 + 10x$
$= 5x\left(x^2\right) + 5x(11x) + 5x(2)$
$= 5x\left(x^2 + 11x + 2\right)$

7. $3(2y-1)^2 + 7(2y-1)^3$
$= (2y-1)^2(3) + (2y-1)^2 \cdot 7(2y-1)$
$= (2y-1)^2[3 + 7(2y-1)]$
$= (2y-1)^2(3 + 14y - 7)$
$= (2y-1)^2(14y - 4)$
$= 2(2y-1)^2(7y-2)$

8. $(3x+7)^5 - 4(3x+7)^3$
$= (3x+7)^3(3x+7)^2 - (3x+7)^3(4)$
$= (3x+7)^3\left[(3x+7)^2 - 4\right]$
$= (3x+7)^3\left(9x^2 + 42x + 49 - 4\right)$
$= (3x+7)^3\left(9x^2 + 42x + 45\right)$

9. $3(x+5)^4 + (x+5)^6$
$= (x+5)^4 \cdot 3 + (x+5)^4(x+5)^2$
$= (x+5)^4\left[3 + (x+5)^2\right]$
$= (x+5)^4\left(3 + x^2 + 10x + 25\right)$
$= (x+5)^4\left(x^2 + 10x + 28\right)$

10. $3(x+6)^2 + 6(x+6)^4$
$= 3(x+6)^2(1) + 3(x+6)^2\left[2(x+6)^2\right]$
$= 3(x+6)^2\left[1 + 2(x+6)^2\right]$
$= 3(x+6)^2\left[1 + 2\left(x^2 + 12x + 36\right)\right]$
$= 3(x+6)^2\left(1 + 2x^2 + 24x + 72\right)$
$= 3(x+6)^2\left(2x^2 + 24x + 73\right)$

11. $x^2 + 5x + 4 = (x+1)(x+4)$

12. $u^2 + 7u + 6 = (u+1)(u+6)$

13. $x^2 + 7x + 12 = (x+3)(x+4)$

14. $y^2 + 8y + 12 = (y+2)(y+6)$

15. $x^2 + x - 6 = (x+3)(x-2)$

16. $x^2 + 4x - 5 = (x+5)(x-1)$

17. $x^2 + 2x - 3 = (x+3)(x-1)$

18. $y^2 + y - 12 = (y+4)(y-3)$

19. $x^2 - 3x - 4 = (x+1)(x-4)$

20. $u^2 - 2u - 8 = (u+2)(u-4)$

21. $z^2 - 9z + 14 = (z-2)(z-7)$

22. $w^2 - 6w - 16 = (w+2)(w-8)$

23. $z^2 + 10z + 24 = (z+4)(z+6)$

24. $r^2 + 16r + 60 = (r+6)(r+10)$

25. $2x^2 - 9x + 4 = (2x-1)(x-4)$

26. $3w^2 - 8w + 4 = (3w-2)(w-2)$

27. $15p^2 - 23p + 4 = (3p-4)(5p-1)$

28. $8x^2 - 14x + 3 = (4x-1)(2x-3)$

29. $4z^2 - 16z + 15 = (2z-5)(2z-3)$

30. $12y^2 - 29y + 15 = (3y-5)(4y-3)$

31. $6x^2 - 5x - 4 = (2x+1)(3x-4)$

32. $12z^2 + z - 1 = (4z-1)(3z+1)$

33. $10y^2 + 21y - 10 = (5y-2)(2y+5)$

34. $15u^2 + 4u - 4 = (5u-2)(3u+2)$

35. $6x^2 + 5x - 4 = (2x-1)(3x+4)$

36. $12y^2 + 7y - 10 = (3y-2)(4y+5)$

37. $3a^2 + 2a - 5 = (3a+5)(a-1)$

38. $6a^2 - 48a - 120 = 6(a^2 - 8a - 20)$
$= 6(a-10)(a+2)$

39. $x^2 - 81 = x^2 - (9)^2 = (x+9)(x-9)$

40. $x^2 + 17xy + 72y^2 = (x+8y)(x+9y)$.

41. $9p^2 - 12p + 4 = (3p)^2 - 2(3p)(2) + 2^2$
$= (3p-2)^2$

42. $3r^2 - r - 2 = (3r+2)(r-1)$.

43. $r^2 + 3rt - 10t^2 = (r-2t)(r+5t)$.

44. $2a^2 + ab - 6b^2 = (2a-3b)(a+2b)$.

45. $m^2 - 8mn + 16n^2 = (m)^2 - 2(m)(4n) + (4n)^2$
$= (m-4n)^2$

46. $8k^2 - 16k - 10 = 2(4k^2 - 8k - 5)$
$= 2(2k+1)(2k-5)$

47. $4u^2 + 12u + 9 = (2u+3)^2$

48. $9p^2 - 16 = (3p)^2 - 4^2 = (3p-4)(3p+4)$

49. $25p^2 - 10p + 4$
This polynomial cannot be factored

50. $10x^2 - 17x + 3 = (5x-1)(2x-3)$

51. $4r^2 - 9v^2 = (2r+3v)(2r-3v)$

52. $x^2 + 3xy - 28y^2 = (x+7y)(x-4y)$

53. $x^2 + 4xy + 4y^2 = (x+2y)^2$

54. $16u^2 + 12u - 18 = 2(8u^2 + 6u - 9)$
$= 2(4u-3)(2u+3)$

55. $3a^2 - 13a - 30 = (3a+5)(a-6)$.

56. $3k^2 + 2k - 8 = (3k-4)(k+2)$

57. $21m^2 + 13mn + 2n^2 = (7m+2n)(3m+n)$

58. $81y^2 - 100 = (9y+10)(9y-10)$

59. $y^2 - 4yz - 21z^2 = (y-7z)(y+3z)$

60. $49a^2 + 9$
This polynomial cannot be factored.

61. $121x^2 - 64 = (11x+8)(11x-8)$

62. $4z^2 + 56zy + 196y^2$
$= 4(z^2 + 14zy + 49y^2)$
$= 4\left[z^2 + 2(z)(7y) + (7y)^2\right] = 4(z+7y)^2$

63. $a^3 - 64 = a^3 - (4)^3 = (a-4)(a^2 + 4a + 16)$

64. $b^3 + 216 = b^3 + 6^3 = (b+6)(b^2 - 6b + 36)$

65. $8r^3 - 27s^3$
$= (2r)^3 - (3s)^3$
$= (2r - 3s)\left[(2r)^2 + (2r)(3s) + (3s)^2\right]$
$= (2r - 3s)\left(4r^2 + 6rs + 9s^2\right)$

66. $1000p^3 + 27q^3$
$= (10p)^3 + (3q)^3$
$= (10p + 3q)\left(100p^2 - 30pq + 9q^2\right)$

67. $64m^3 + 125$
$= (4m)^3 + (5)^3$
$= (4m + 5)\left[(4m)^2 - (4m)(5) + (5)^2\right]$
$= (4m + 5)\left(16m^2 - 20m + 25\right)$

68. $216y^3 - 343$
$= (6y)^3 - (7)^3$
$= (6y - 7)\left(36y^2 + 42y + 49\right)$

69. $1000y^3 - z^3$
$= (10y)^3 - (z)^3$
$= (10y - z)\left[(10y)^2 + (10y)(z) + (z)^2\right]$
$= (10y - z)\left(100y^2 + 10yz + z^2\right)$

70. $125p^3 + 8q^3 = (5p)^3 + (2q)^3$
$= (5p + 2q)\left(25p^2 - 10pq + 4q^2\right)$

71. $x^4 + 5x^2 + 6 = \left(x^2 + 2\right)\left(x^2 + 3\right)$

72. $y^4 + 7y^2 + 10 = \left(y^2 + 2\right)\left(y^2 + 5\right)$

73. $b^4 - b^2 = b^2\left(b^2 - 1\right) = b^2(b + 1)(b - 1)$

74. $z^4 - 3z^2 - 4 = \left(z^2 - 4\right)\left(z^2 + 1\right)$
$= (z + 2)(z - 2)\left(z^2 + 1\right)$

75. $x^4 - x^2 - 12 = \left(x^2 - 4\right)\left(x^2 + 3\right)$
$= (x + 2)(x - 2)\left(x^2 + 3\right)$

76. $4x^4 + 27x^2 - 81 = \left(4x^2 - 9\right)\left(x^2 + 9\right)$
$= (2x + 3)(2x - 3)\left(x^2 + 9\right)$

77. $16a^4 - 81b^4 = \left(4a^2 - 9b^2\right)\left(4a^2 + 9b^2\right)$
$= (2a + 3b)(2a - 3b)\left(4a^2 + 9b^2\right)$

78. $x^6 - y^6 = \left(x^2\right)^3 - \left(y^2\right)^3$
$= \left(x^2 - y^2\right)\left(x^4 + x^2y^2 + y^4\right)$
$= (x + y)(x - y)\left(x^2 + xy + y^2\right)\cdot$
$\left(x^2 - xy + y^2\right)$

79. $x^8 + 8x^2 = x^2\left(x^6 + 8\right) = x^2\left(\left(x^2\right)^3 + 2^3\right)$
$= x^2\left(x^2 + 2\right)\left(x^4 - 2x^2 + 4\right)$

80. $x^9 - 64x^3 = x^3\left(x^6 - 64\right) = x^3\left(x^6 - 2^6\right)$
$= x^3\left[\left(x^3\right)^2 - \left(2^3\right)^2\right]$
$= x^3\left(x^3 - 2^3\right)\left(x^3 + 2^3\right)$
$= x^3(x - 2)\left(x^2 + 2x + 4\right)\cdot$
$(x + 2)\left(x^2 - 2x + 4\right)$

81. $6x^4 - 3x^2 - 3 = \left(2x^2 + 1\right)\left(3x^2 - 3\right)$ is not the correct complete factorization because $3x^2 - 3$ contains a common factor of 3. This common factor should be factored out as the first step. This will reveal a difference of two squares, which requires further factorization. The correct factorization is
$6x^4 - 3x^2 - 3 = 3\left(2x^4 - x^2 - 1\right)$
$= 3\left(2x^2 + 1\right)\left(x^2 - 1\right)$
$= 3\left(2x^2 + 1\right)(x + 1)(x - 1)$

82. The sum of two squares can be factored when the terms have a common factor. An example is
$(3x)^2 + 3^2 = 9x^2 + 9 = 9(x^2 + 1)$

83. $(x+2)^3 = (x+2)(x+2)^2$
$= (x+2)(x^2+4x+4)$
$= x^3+4x^2+2x^2+8x+4x+8$
$= x^3+6x^2+12x+8,$

which is not equal to x^3+8. The correct factorization is $x^3+8=(x+2)(x^2-2x+4)$.

84. Factoring and multiplication are inverse operations. If we factor a polynomial and then multiply the factors, we get the original polynomial back. For example, we can factor x^2-x-6 to get $(x-3)(x+2)$. Then if we multiply the factors, we get $(x-3)(x+2)=x^2+2x-3x-6=x^2-x-6$

Section 1.4 Rational Expressions

1. $\frac{8x^2}{56x}=\frac{x\cdot 8x}{7\cdot 8x}=\frac{x}{7}$

2. $\frac{27m}{81m^3}=\frac{27m}{27m\cdot 3m^2}=\frac{1}{3m^2}$

3. $\frac{25p^2}{35p^3}=\frac{5\cdot 5p^2}{7p\cdot 5p^2}=\frac{5}{7p}$

4. $\frac{18y^4}{24y^2}=\frac{6y^2\cdot 3y^2}{6y^2\cdot 4}=\frac{3y^2}{4}$

5. $\frac{5m+15}{4m+12}=\frac{5(m+3)}{4(m+3)}=\frac{5}{4}$

6. $\frac{10z+5}{20z+10}=\frac{5(2x+1)}{5(2x+1)\cdot 2}=\frac{1}{2}$

7. $\frac{4(w-3)}{(w-3)(w+6)}=\frac{4}{w+6}$

8. $\frac{-6(x+2)}{(x+4)(x+2)}=\frac{-6}{x+4}$ or $-\frac{6}{x+4}$

9. $\frac{3y^2-12y}{9y^3}=\frac{3y(y-4)}{3y\left(3y^2\right)}$
$=\frac{y-4}{3y^2}$

10. $\frac{15k^2+45k}{9k^2}=\frac{15k(k+3)}{3k\cdot 3k}$
$=\frac{3k\cdot 5(k+3)}{3k\cdot 3k}$
$=\frac{5(k+3)}{3k}$

11. $\frac{m^2-4m+4}{m^2+m-6}=\frac{(m-2)(m-2)}{(m+3)(m-2)}=\frac{m-2}{m+3}$

12. $\frac{r^2-r-6}{r^2+r-12}=\frac{(r-3)(r+2)}{(r+4)(r-3)}=\frac{r+2}{r+4}$

13. $\frac{x^2+2x-3}{x^2-1}=\frac{(x+3)(x-1)}{(x+1)(x-1)}=\frac{x+3}{x+1}$

14. $\frac{z^2+4z+4}{z^2-4}=\frac{(z+2)^2}{(z+2)(z-2)}=\frac{z+2}{z-2}$

15. $\frac{3a^2}{64}\cdot\frac{8}{2a^3}=\frac{3a^2\cdot 8}{64\cdot 2a^3}=\frac{3}{16a}$

16. $\frac{2u^2}{8u^4}\cdot\frac{10u^3}{9u}=\frac{2u^2\cdot 10u^3}{8u^4\cdot 9u}=\frac{5}{18}$

17. $\frac{7x}{11}\div\frac{14x^3}{66y}=\frac{7x}{11}\cdot\frac{66y}{14x^3}=\frac{7x\cdot 66y}{11\cdot 14x^3}=\frac{3y}{x^2}$

18. $\frac{6x^2y}{2x}\div\frac{21xy}{y}=\frac{6x^2y}{2x}\cdot\frac{y}{21xy}=\frac{y}{7}$

19. $\frac{2a+b}{3c}\cdot\frac{15}{4(2a+b)}=\frac{(2a+b)\cdot 15}{(2a+b)\cdot 12c}=\frac{15}{12c}=\frac{5}{4c}$

20. $\frac{4(x+2)}{w}\cdot\frac{3w^2}{8(x+2)}=\frac{4w^2(x+2)\cdot 3}{4w(x+2)\cdot 2}=\frac{3w}{2}$

21. $\frac{15p-3}{6} \div \frac{10p-2}{3} = \frac{15p-3}{6} \cdot \frac{3}{10p-2}$
$= \frac{3(5p-1)\cdot 3}{3\cdot 2\cdot 2\cdot(5p-1)}$
$= \frac{3(5p-1)\cdot 3}{3(5p-1)\cdot 2\cdot 2} = \frac{3}{4}$

22. $\frac{2k+8}{6} \div \frac{3k+12}{3} = \frac{2k+8}{6} \cdot \frac{3}{3k+12}$
$= \frac{2(k+4)}{6} \cdot \frac{3}{3(k+4)}$
$= \frac{6(k+4)}{18(k+4)} = \frac{6}{18} = \frac{1}{3}$

23. $\frac{9y-18}{6y+12} \cdot \frac{3y+6}{15y-30} = \frac{9(y-2)}{6(y+2)} \cdot \frac{3(y+2)}{15(y-2)}$
$= \frac{27(y-2)(y+2)}{90(y+2)(y-2)} = \frac{27}{90} = \frac{3}{10}$

24. $\frac{12r+24}{36r-36} \div \frac{6r+12}{8r-8} = \frac{12(r+2)}{36(r-1)} \div \frac{6(r+2)}{8(r-1)}$
$= \frac{r+2}{3(r-1)} \div \frac{3(r+2)}{4(r-1)}$
$= \frac{r+2}{3(r-1)} \cdot \frac{4(r-1)}{3(r+2)} = \frac{4}{9}$

25. $\frac{4a+12}{2a-10} \div \frac{a^2-9}{a^2-a-20} = \frac{4a+12}{2a-10} \cdot \frac{a^2-a-20}{a^2-9}$
$= \frac{4(a+3)}{2(a-5)} \cdot \frac{(a-5)(a+4)}{(a+3)(a-3)}$
$= \frac{4(a+3)(a-5)(a+4)}{2(a-5)(a+3)(a-3)}$
$= \frac{2(a+4)}{a-3}$

26. $\frac{6r-18}{9r^2+6r-24} \cdot \frac{12r-16}{4r-12}$
$= \frac{6(r-3)}{3\left(3r^2+2r-8\right)} \cdot \frac{4(3r-4)}{4(r-3)} = \frac{2(3r-4)}{\left(3r^2+2r-8\right)}$
$= \frac{2(3r-4)}{(3r-4)(r+2)} = \frac{2}{r+2}$

27. $\frac{k^2-k-6}{k^2+k-12} \cdot \frac{k^2+3k-4}{k^2+2k-3}$
$= \frac{(k-3)(k+2)}{(k+4)(k-3)} \cdot \frac{(k+4)(k-1)}{(k+3)(k-1)}$
$= \frac{(k-3)(k+2)(k+4)(k-1)}{(k+4)(k-3)(k+3)(k-1)} = \frac{k+2}{k+3}$

28. $\frac{n^2-n-6}{n^2-2n-8} \div \frac{n^2-9}{n^2+7n+12}$
$= \frac{(n-3)(n+2)}{(n-4)(n+2)} \div \frac{(n-3)(n+3)}{(n+3)(n+4)}$
$= \frac{n-3}{n-4} \div \frac{n-3}{n+4} = \frac{n-3}{n-4} \cdot \frac{n+4}{n-3} = \frac{n+4}{n-4}$

Answers will vary for exercises 29 and 30. Sample answers are given.

29. To find the least common denominator for two fractions, factor each denominator into prime factors, multiply all unique prime factors raising each factor to the highest frequency it occurred.

30. To add three rational expressions, first factor each denominator completely. Then, find the lowest common denominator and rewrite each expression with that denominator. Next, add the numerators and place over the common denominator. Finally, simplify the resulting expression and write it in lowest terms.

31. The common denominator is $35z$.
$\frac{2}{7z} - \frac{1}{5z} = \frac{2\cdot 5}{7z\cdot 5} - \frac{1\cdot 7}{5z\cdot 7} = \frac{10}{35z} - \frac{7}{35z} = \frac{3}{35z}$

32. The common denominator is $12z$.
$\frac{4}{3z} - \frac{5}{4z} = \frac{4\cdot 4}{3z\cdot 4} - \frac{5\cdot 3}{4z\cdot 3} = \frac{16}{12z} - \frac{15}{12z} = \frac{1}{12z}$

33. $\frac{r+2}{3} - \frac{r-2}{3} = \frac{(r+2)-(r-2)}{3}$
$= \frac{r+2-r+2}{3} = \frac{4}{3}$

34. $\frac{3y-1}{8} - \frac{3y+1}{8} = \frac{(3y-1)-(3y+1)}{8} = \frac{-2}{8} = -\frac{1}{4}$

35. The common denominator is $5x$.
$\frac{4}{x} + \frac{1}{5} = \frac{4\cdot 5}{x\cdot 5} + \frac{1\cdot x}{5\cdot x} = \frac{20}{5x} + \frac{x}{5x} = \frac{20+x}{5x}$

36. The common denominator is $4r$.

$$\frac{6}{r}-\frac{3}{4}=\frac{6\cdot 4}{r\cdot 4}-\frac{3\cdot r}{4\cdot r}=\frac{24}{4r}-\frac{3r}{4r}$$
$$=\frac{24-3r}{4r}=\frac{3(8-r)}{4r}$$

37. The common denominator is $m(m-1)$.

$$\frac{1}{m-1}+\frac{2}{m}=\frac{m\cdot 1}{m\cdot(m-1)}+\frac{(m-1)\cdot 2}{(m-1)\cdot m}$$
$$=\frac{m}{m(m-1)}+\frac{2(m-1)}{m(m-1)}$$
$$=\frac{m+2(m-1)}{m(m-1)}=\frac{m+2m-2}{m(m-1)}$$
$$=\frac{3m-2}{m(m-1)}$$

38. The common denominator is $(y+2)y$.

$$\frac{8}{y+2}-\frac{3}{y}=\frac{8y}{(y+2)y}-\frac{3(y+2)}{y(y+2)}=\frac{8y-3(y+2)}{(y+2)y}$$
$$=\frac{8y-3y-6}{(y+2)y}=\frac{5y-6}{(y+2)y} \text{ or } \frac{5y-6}{y(y+2)}$$

39. The common denominator is $5(b+2)$.

$$\frac{7}{b+2}+\frac{2}{5(b+2)}=\frac{7\cdot 5}{(b+2)\cdot 5}+\frac{2}{5(b+2)}$$
$$=\frac{35+2}{5(b+2)}=\frac{37}{5(b+2)}$$

40. The common denominator is $3(k+1)$.

$$\frac{4}{3(k+1)}+\frac{3}{k+1}=\frac{4}{3(k+1)}+\frac{3\cdot 3}{3(k+1)}$$
$$=\frac{4+9}{3(k+1)}=\frac{13}{3(k+1)}$$

41. The common denominator is $20(k-2)$.

$$\frac{2}{5(k-2)}+\frac{5}{4(k-2)}=\frac{8}{20(k-2)}+\frac{25}{20(k-2)}$$
$$=\frac{8+25}{20(k-2)}=\frac{33}{20(k-2)}$$

42. The common denominator is $6(p+4)$.

$$\frac{11}{3(p+4)}-\frac{5}{6(p+4)}=\frac{22}{6(p+4)}-\frac{5}{6(p+4)}$$
$$=\frac{22-5}{6(p+4)}=\frac{17}{6(p+4)}$$

43. First factor the denominators in order to find the common denominator.

$$x^2-4x+3=(x-3)(x-1)$$
$$x^2-x-6=(x-3)(x+2)$$

The common denominator is $(x-3)(x-1)(x+2)$.

$$\frac{2}{x^2-4x+3}+\frac{5}{x^2-x-6}$$
$$=\frac{2}{(x-3)(x-1)}+\frac{5}{(x-3)(x+2)}$$
$$=\frac{2(x+2)}{(x-3)(x-1)(x+2)}+\frac{5(x-1)}{(x-3)(x+2)(x-1)}$$
$$=\frac{2(x+2)+5(x-1)}{(x-3)(x+2)(x-1)}=\frac{2x+4+5x-5}{(x-3)(x-1)(x+2)}$$
$$=\frac{7x-1}{(x-3)(x-1)(x+2)}$$

44. First factor the denominators in order to find the common denominator.

$$m^2-3m-10=(m-5)(m+2)$$
$$m^2-m-20=(m-5)(m+4)$$

The common denominator is $(m-5)(m+2)(m+4)$.

$$\frac{3}{m^2-3m-10}+\frac{7}{m^2-m-20}$$
$$=\frac{3}{(m-5)(m+2)}+\frac{7}{(m-5)(m+4)}$$
$$=\frac{3(m+4)}{(m-5)(m+2)(m+4)}+\frac{7(m+2)}{(m-5)(m+4)(m+2)}$$
$$=\frac{3m+12+7m+14}{(m-5)(m+2)(m+4)}=\frac{10m+26}{(m-5)(m+2)(m+4)}$$

45. First factor the denominators in order to find the common denominator.

$$y^2+7y+12=(y+3)(y+4)$$
$$y^2+5y+6=(y+3)(y+2)$$

The common denominator is $(y+4)(y+3)(y+2)$.

$$\frac{2y}{y^2+7y+12}-\frac{y}{y^2+5y+6}$$
$$=\frac{2y}{(y+4)(y+3)}-\frac{y}{(y+3)(y+2)}$$
$$=\frac{2y(y+2)}{(y+4)(y+3)(y+2)}-\frac{y(y+4)}{(y+4)(y+3)(y+2)}$$
$$=\frac{2y(y+2)-y(y+4)}{(y+4)(y+3)(y+2)}=\frac{2y^2+4y-y^2-4y}{(y+4)(y+3)(y+2)}$$
$$=\frac{y^2}{(y+4)(y+3)(y+2)}$$

46. First factor the denominators in order to find the common denominator.

$r^2-10r+16=(r-8)(r-2)$

$r^2+2r-8=(r+4)(r-2)$

The common denominator is $(r-8)(r-2)(r+4)$.

$$\frac{-r}{r^2-10r+16}-\frac{3r}{r^2+2r-8}$$
$$=\frac{-r}{(r-8)(r-2)}-\frac{3r}{(r+4)(r-2)}$$
$$=\frac{-r(r+4)}{(r-8)(r-2)(r+4)}-\frac{3r(r-8)}{(r+4)(r-2)(r-8)}$$
$$=\frac{-r^2-4r-\left(3r^2-24r\right)}{(r-8)(r-2)(r+4)}=\frac{-r^2-4r-3r^2+24r}{(r-8)(r-2)(r+4)}$$
$$=\frac{-4r^2+20r}{(r-8)(r-2)(r+4)}$$

47. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$

Multiply both numerator and denominator of this complex fraction by the common denominator, x.

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}=\frac{x\left(1+\frac{1}{x}\right)}{x\left(1-\frac{1}{x}\right)}=\frac{x\cdot 1+x\left(\frac{1}{x}\right)}{x\cdot 1-x\left(\frac{1}{x}\right)}=\frac{x+1}{x-1}$$

48. $\dfrac{2-\frac{2}{y}}{2+\frac{2}{y}}$

Multiply both numerator and denominator by the common denominator, y.

$$\frac{2-\frac{2}{y}}{2+\frac{2}{y}}=\frac{y\left(2-\frac{2}{y}\right)}{y\left(2+\frac{2}{y}\right)}=\frac{2y-2}{2y+2}=\frac{2(y-1)}{2(y+1)}=\frac{y-1}{y+1}$$

49. $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

The common denominator in the numerator is $x(x+h)$

$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\frac{\frac{x-x-h}{x(x+h)}}{h}=\frac{\frac{-h}{x(x+h)}}{h}$$
$$=\frac{-h}{x(x+h)}\div h=\frac{-h}{x(x+h)}\cdot\frac{1}{h}$$
$$=\frac{-1}{x(x+h)}\text{ or }-\frac{1}{x(x+h)}$$

50. $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

The common denominator of the numerator is $(x+h)^2x^2$.

$$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2}{(x+h)^2x^2}-\frac{(x+h)^2}{(x+h)^2x^2}}{\frac{h}{1}}$$
$$=\frac{x^2-(x+h)^2}{(x+h)^2x^2}\cdot\frac{1}{h}$$
$$=\frac{x^2-\left(x^2+2xh+h^2\right)}{(x+h)^2x^2}\cdot\frac{1}{h}$$
$$=\frac{-2xh-h^2}{(x+h)^2x^2h}=\frac{h(-2x-h)}{(x+h)^2x^2h}$$
$$=\frac{-2x-h}{(x+h)^2x^2}$$

51. The length of each side of the dartboard is $2x$, so the area of the dartboard is $4x^2$. The area of the shaded region is πx^2.

a. The probability that a dart will land in the shaded region is $\dfrac{\pi x^2}{4x^2}$.

b. $\dfrac{\pi x^2}{4x^2}=\dfrac{\pi}{4}$

52. The radius of the dartboard is $x+2x+3x=6x$, so the area of the dartboard is $\pi(6x)^2=36\pi x^2$. The area of the shaded region is πx^2.

a. The probability that a dart will land in the shaded region is $\dfrac{\pi x^2}{36\pi x^2}$.

b. $\dfrac{\pi x^2}{36\pi x^2}=\dfrac{1}{36}$

53. The length of each side of the dartboard is $5x$, so the area of the dartboard is $25x^2$. The area of the shaded region is x^2.

a. The probability that a dart will land in the shaded region is $\frac{x^2}{25x^2}$.

b. $\frac{x^2}{25x^2} = \frac{1}{25}$

54. The length of each side of the dartboard is $3x$, so the area of the dartboard is $9x^2$. The area of the shaded region is $\frac{1}{2}x^2$.

a. The probability that a dart will land in the shaded region is $\frac{\frac{1}{2}x^2}{9x^2} = \frac{x^2}{18x^2}$.

b. $\frac{x^2}{18x^2} = \frac{1}{18}$

55. Average cost = total cost C divided by the number of calculators produced.

$$\frac{-7.2x^2 + 6995x + 230,000}{1000x}$$

56. Let $x = 20$ (in thousands).

$$\frac{-7.2(20)^2 + 6995(20) + 230,000}{1000(20)} = \$18.35$$

Let $x = 50$ (in thousands).

$$\frac{-7.2(50)^2 + 6995(50) + 230,000}{1000(50)} = \$11.24$$

Let $x = 125$ (in thousands).

$$\frac{-7.2(125)^2 + 6995(125) + 230,000}{1000(125)} = \$7.94$$

57. Let $x = 13$. Then

$$\frac{.505(13)^2 - 4.587(13) + 27.6}{13+1} \approx 3.8$$

The ad cost approximately \$3.8 million in 2013

58. Let $x = 16$. Then

$$\frac{.505(16)^2 - 4.587(16) + 27.6}{16+1} \approx 4.9$$

The ad cost approximately \$4.9 million in 2016.

59. Let $x = 45$. Then

$$\frac{.072(45)^2 + .744(45) + 1.2}{45+2} = 3.84$$

Let $x = 20$. Then

$$\frac{.505(20)^2 - 4.587(20) + 27.6}{20+1} \approx 6.6$$

The cost of an ad will not reach \$7 million in 2020.

60. Let $x = 45$. Then

$$\frac{.072(45)^2 + .744(45) + 1.2}{45+2} = 3.84$$

Let $x = 22$. Then

$$\frac{.505(22)^2 - 4.587(22) + 27.6}{22+1} \approx 7.4$$

The cost of an ad will not reach \$8 million in 2022.

61. Let $x = 11$. Then

$$\frac{.049(11)^2 + 2.40(11) + .83}{11+2} \approx 2.55$$

The hourly insurance cost in 2011 was \$2.55.

62. Let $x = 17$. Then

$$\frac{.049(17)^2 + 2.40(17) + .83}{17+2} \approx 2.94$$

The hourly insurance cost in 2017 is \$2.94.

63. Let $x = 20$. Then

$$\frac{.049(20)^2 + 2.40(20) + .83}{20+2} \approx 3.11$$

The hourly insurance cost in 2020 will be \$3.11. The annual cost will be 3.11(2100) = \$6531.

64. Let $x = 23$. Then

$$\frac{.049(23)^2 + 2.40(23) + .83}{23+2} \approx 3.28$$

The hourly insurance cost in 2023 will be \$3.28. The annual cost will be 3.28(2100) = \$6888.

No; the annual cost will not reach \$10,000 by 2023.

Section 1.5 Exponents and Radicals

1. $\dfrac{7^5}{7^3} = 7^{5-3} = 7^2 = 49$

2. $\dfrac{(-6)^{14}}{(-6)^6} = (-6)^8 = 1,679,616$

3. $(4c)^2 = 4^2c^2 = 16c^2$

4. $(-2x)^4 = (-2)^4x^4 = 16x^4$

5. $\left(\dfrac{2}{x}\right)^5 = \dfrac{2^5}{x^5} = \dfrac{32}{x^5}$

6. $\left(\dfrac{5}{xy}\right)^3 = \dfrac{5^3}{x^3y^3} = \dfrac{125}{x^3y^3}$

7. $\left(3u^2\right)^3\left(2u^3\right)^2 = \left(27u^6\right)\left(4u^6\right) = 108u^{12}$

8. $\dfrac{\left(5v^2\right)^3}{(2v)^4} = \dfrac{125v^6}{16v^4} = \dfrac{125v^2}{16}$

9. $7^{-1} = \dfrac{1}{7^1} = \dfrac{1}{7}$

10. $10^{-3} = \dfrac{1}{10^3} = \dfrac{1}{1000}$

11. $-6^{-5} = -\dfrac{1}{6^5} = -\dfrac{1}{7776}$

12. $(-x)^{-4} = \dfrac{1}{(-x)^4} = \dfrac{1}{x^4}$

13. $(-y)^{-3} = \dfrac{1}{(-y)^3} = -\dfrac{1}{y^3}$

14. $\left(\dfrac{1}{6}\right)^{-2} = \left(\dfrac{6}{1}\right)^2 = 6^2 = 36$

15. $\left(\dfrac{4}{3}\right)^{-2} = \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

16. $\left(\dfrac{x}{y^2}\right)^{-2} = \left(\dfrac{y^2}{x}\right)^2 = \dfrac{y^4}{x^2}$

17. $\left(\dfrac{a}{b^3}\right)^{-1} = \left(\dfrac{b^3}{a}\right)^1 = \dfrac{b^3}{a}$

18. $-2^{-4} = -\dfrac{1}{2^4} = -\dfrac{1}{16}$, but

$(-2)^{-4} = \dfrac{1}{(-2)^4} = \dfrac{1}{16}$

19. $49^{1/2} = 7$ because $7^2 = 49$.

20. $8^{1/3} = 2$ because $2^3 = 8$.

21. $(5.71)^{1/4} = (5.71)^{.25} \approx 1.55$ Use a calculator.

22. $12^{5/2} = \left(12^{1/2}\right)^5 \approx 498.83$

23. $-64^{2/3} = -\left(64^{1/3}\right)^2 = -(4)^2 = -16$

24. $-64^{3/2} = -\left[\left(64^{1/2}\right)^3\right] = -\left(8^3\right) = -512$

25. $\left(\dfrac{8}{27}\right)^{-4/3} = \left(\dfrac{27^{1/3}}{8^{1/3}}\right)^4 = \left(\dfrac{3}{2}\right)^4 = \dfrac{3^4}{2^4} = \dfrac{81}{16}$

26. $\left(\dfrac{27}{64}\right)^{-1/3} = \left(\dfrac{64}{27}\right)^{1/3} = \dfrac{4}{3}$

27. $\dfrac{5^{-3}}{4^{-2}} = \dfrac{4^2}{5^3}$

28. $\dfrac{7^{-4}}{7^{-3}} = 7^{-4}\cdot 7^3 = 7^{-1} = \dfrac{1}{7}$

29. $4^{-3}\cdot 4^6 = 4^3$

30. $9^{-9}\cdot 9^{10} = 9^1 = 9$

31. $\dfrac{4^{10}\cdot 4^{-6}}{4^{-4}} = 4^{10}\cdot 4^{-6}\cdot 4^4 = 4^8$

32. $\dfrac{5^{-4}\cdot 5^{6}}{5^{-1}} = 5^{-4}\cdot 5^{6}\cdot 5^{1} = 5^{3}$

33. $\dfrac{z^{6}\cdot z^{2}}{z^{5}} = \dfrac{z^{8}}{z^{5}} = z^{8-5} = z^{3}$

34. $\dfrac{k^{6}\cdot k^{9}}{k^{12}} = \dfrac{k^{15}}{k^{12}} = k^{15-12} = k^{3}$

35. $\dfrac{3^{-1}\left(p^{-2}\right)^{3}}{3p^{-7}} = \dfrac{3^{-1}p^{-6}}{3^{1}p^{-7}} = 3^{-1-1}p^{-6-(-7)}$

$= 3^{-2}p^{1} = \dfrac{1}{3^{2}}\cdot p = \dfrac{p}{9}$

36. $\dfrac{\left(5x^{3}\right)^{-2}}{x^{4}} = \dfrac{5^{-2}\left(x^{3}\right)^{-2}}{x^{4}} = \dfrac{5^{-2}x^{-6}}{x^{4}}$

$= 5^{-2}x^{-10} = \dfrac{1}{25x^{10}}$

37. $\left(q^{-5}r^{3}\right)^{-1} = q^{5}r^{-3} = q^{5}\cdot\dfrac{1}{r^{3}} = \dfrac{q^{5}}{r^{3}}$

38. $\left(2y^{2}z^{-2}\right)^{-3} = 2^{-3}\left(y^{2}\right)^{-3}\left(z^{-2}\right)^{-3}$

$= 2^{-3}y^{-6}z^{6} = \dfrac{z^{6}}{2^{3}y^{6}} = \dfrac{z^{6}}{8y^{6}}$

39. $\left(2p^{-1}\right)^{3}\cdot\left(5p^{2}\right)^{-2} = 2^{3}\left(p^{-1}\right)^{3}(5)^{-2}\left(p^{2}\right)^{-2}$

$= 2^{3}\left(p^{-3}\right)\left(\dfrac{1}{5^{2}}\right)\left(p^{-4}\right)$

$= 2^{3}\left(\dfrac{1}{p^{3}}\right)\left(\dfrac{1}{5^{2}}\right)\left(\dfrac{1}{p^{4}}\right)$

$= \dfrac{8}{25p^{7}}$

40. $\left(4^{-1}x^{3}\right)^{-2}\cdot\left(3x^{-3}\right)^{4}$

$= \left(4^{-1}\right)^{-2}\cdot\left(x^{3}\right)^{-2}\cdot(3)^{4}\cdot\left(x^{-3}\right)^{4}$

$= 4^{2}\cdot x^{-6}\cdot 3^{4}\cdot x^{-12} = 1296x^{-18} = \dfrac{1296}{x^{18}}$

41. $(2p)^{1/2}\cdot\left(2p^{3}\right)^{1/3} = 2^{1/2}p^{1/2}\cdot 2^{1/3}\cdot\left(p^{3}\right)^{1/3}$

$= 2^{1/2}p^{1/2}\cdot 2^{1/3}\cdot p^{1}$

$= 2^{5/6}p^{3/2}$

42. $\left(5k^{2}\right)^{3/2}\cdot\left(5k^{1/3}\right)^{3/4} = 5^{3/2}\left(k^{2}\right)^{3/2}\cdot 5^{3/4}\cdot\left(k^{1/3}\right)^{3/4}$

$= 5^{\frac{3}{2}+\frac{3}{4}}k^{3}k^{1/4} = 5^{9/4}k^{13/4}$

43. $p^{2/3}\left(2p^{1/3}+5p\right) = p^{2/3}\left(2p^{1/3}\right) + p^{2/3}(5p)$

$= 2p + 5p^{5/3}$

44. $3x^{3/2}\left(2x^{-3/2}+x^{3/2}\right) = 3x^{3/2}\cdot 2x^{-3/2} + 3x^{3/2}\cdot x^{3/2}$

$= 6x^{0} + 3x^{6/2} = 6 + 3x^{3}$

45. $\dfrac{\left(x^{2}\right)^{1/3}\left(y^{2}\right)^{2/3}}{3x^{2/3}y^{2}} = \dfrac{(x)^{2/3}(y)^{4/3}}{3x^{2/3}y^{2}}$

$= \dfrac{1}{3y^{2-4/3}} = \dfrac{1}{3y^{2/3}}$

46. $\dfrac{\left(c^{1/2}\right)^{3}\left(d^{3}\right)^{1/2}}{\left(c^{3}\right)^{1/4}\left(d^{1/4}\right)^{3}} = \dfrac{\left(c^{3/2}\right)\left(d^{3/2}\right)}{\left(c^{3/4}\right)\left(d^{3/4}\right)}$

$= c^{(3/2)-(3/4)}d^{(3/2)-(3/4)}$

$= c^{3/4}d^{3/4}$

47. $\dfrac{(7a)^{2}(5b)^{3/2}}{(5a)^{3/2}(7b)^{4}} = \dfrac{7^{2}a^{2}5^{3/2}b^{3/2}}{5^{3/2}a^{3/2}7^{4}b^{4}} = \dfrac{a^{2-\frac{3}{2}}}{7^{2}b^{4-\frac{3}{2}}}$

$= \dfrac{a^{1/2}}{49b^{5/2}}$

48. $\dfrac{(4x)^{1/2}\sqrt{xy}}{x^{3/2}y^{2}} = \dfrac{(4x)^{1/2}(xy)^{1/2}}{x^{3/2}y^{2}} = \dfrac{4^{1/2}x^{1/2}x^{1/2}y^{1/2}}{x^{3/2}y^{2}}$

$= 2xy^{1/2}x^{-3/2}y^{-2} = 2x^{-1/2}y^{-3/2}$

$= \dfrac{2}{x^{1/2}y^{3/2}}$

49. $x^{1/2}\left(x^{2/3}-x^{4/3}\right) = x^{1/2}x^{2/3} - x^{1/2}x^{4/3}$

$= x^{7/6} - x^{11/6}$

50. $x^{1/2}\left(3x^{3/2}+2x^{-1/2}\right) = 3x^{1/2}x^{3/2} + 2x^{1/2}x^{-1/2}$

$= 3x^{2} + 2$

51. This is a difference of two squares.

$$\left(x^{1/2}+y^{1/2}\right)\left(x^{1/2}-y^{1/2}\right)=\left(x^{1/2}\right)^2-\left(y^{1/2}\right)^2 = x-y$$

52. Use FOIL.

$$\left(x^{1/3}+y^{1/2}\right)\left(2x^{1/3}-y^{3/2}\right) = x^{1/3}\cdot 2x^{1/3}-x^{1/3}y^{3/2}+2x^{1/3}y^{1/2} - y^{1/2}y^{3/2} = 2x^{2/3}+2x^{1/3}y^{1/2}-x^{1/3}y^{3/2}-y^2$$

53. $(-3x)^{1/3}=\sqrt[3]{-3x}$, (f)

54. $-3x^{1/3}=-3\sqrt[3]{x}$, (b)

55. $(-3x)^{-1/3}=\dfrac{1}{(-3x)^{1/3}}=\dfrac{1}{\sqrt[3]{-3x}}$, (h)

56. $-3x^{-1/3}=\dfrac{-3}{x^{1/3}}=\dfrac{-3}{\sqrt[3]{x}}$, (d)

57. $(3x)^{1/3}=\sqrt[3]{3x}$, (g)

58. $3x^{-1/3}=\dfrac{3}{x^{1/3}}=\dfrac{3}{\sqrt[3]{x}}$, (a)

59. $(3x)^{-1/3}=\dfrac{1}{(3x)^{1/3}}=\dfrac{1}{\sqrt[3]{3x}}$, (c)

60. $3x^{1/3}=3\sqrt[3]{x}$, (e)

61. $\sqrt[3]{125}=125^{1/3}=5$

62. $\sqrt[6]{64}=64^{1/6}=2$

63. $\sqrt[4]{625}=625^{1/4}=5$

64. $\sqrt[7]{-128}=(-128)^{1/7}=-2$

65. $\sqrt{63}\cdot\sqrt{7}=3\sqrt{7}\cdot\sqrt{7}=3\cdot 7=21$

66. $\sqrt[3]{81}\cdot\sqrt[3]{9}=\sqrt[3]{729}=9$

67. $\sqrt{81-4}=\sqrt{77}$

68. $\sqrt{49-16}=\sqrt{33}$

69. $\sqrt{5}\sqrt{15}=\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\sqrt{3}=5\sqrt{3}$

70. $\sqrt{8}\sqrt{96}=\sqrt{8}\sqrt{8\cdot 12}=\sqrt{8}\sqrt{8}\sqrt{12}=8\sqrt{4\cdot 3} = 8\sqrt{4}\sqrt{3}=8\cdot 2\sqrt{3}=16\sqrt{3}$

71. $\sqrt{50}-\sqrt{72}=5\sqrt{2}-6\sqrt{2}=-\sqrt{2}$

72 $\sqrt{75}+\sqrt{192}=5\sqrt{3}+8\sqrt{3}=13\sqrt{3}$

73. $5\sqrt{20}-\sqrt{45}+2\sqrt{80} = 5\cdot 2\sqrt{5}-3\sqrt{5}+2\cdot 4\sqrt{5} = 10\sqrt{5}-3\sqrt{5}+8\sqrt{5}=15\sqrt{5}$

74. $\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right)=\left(\sqrt{3}\right)^2-2^2=3-4=-1$

75. $\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2 = 5-2=3$

76. $\sqrt[3]{4}\cdot\sqrt[3]{4}=4$. A correct statement would be $\sqrt[3]{4}\cdot\sqrt[3]{4}=\sqrt[3]{16}$.

77.
$$\frac{3}{1-\sqrt{2}}=\frac{3}{1-\sqrt{2}}\cdot\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{3\left(1+\sqrt{2}\right)}{(1)^2-\left(\sqrt{2}\right)^2} = \frac{3\left(1+\sqrt{2}\right)}{1-2}=\frac{3\left(1+\sqrt{2}\right)}{-1} = -3\left(1+\sqrt{2}\right)=-3-3\sqrt{2}$$

78.
$$\frac{2}{1+\sqrt{5}}=\frac{2}{1+\sqrt{5}}\cdot\frac{1-\sqrt{5}}{1-\sqrt{5}}=\frac{2\left(1-\sqrt{5}\right)}{1-5} = \frac{2\left(1-\sqrt{5}\right)}{-4}=\frac{1-\sqrt{5}}{-2}\cdot\frac{-1}{-1}=\frac{\sqrt{5}-1}{2}$$

79.
$$\frac{9-\sqrt{3}}{3-\sqrt{3}}=\frac{9-\sqrt{3}}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}}=\frac{27+9\sqrt{3}-3\sqrt{3}-3}{3^2-\left(\sqrt{3}\right)^2} = \frac{24+6\sqrt{3}}{9-3}=\frac{24+6\sqrt{3}}{6}=4+\sqrt{3}$$

80.
$$\frac{\sqrt{3}-1}{\sqrt{3}-2}=\frac{\sqrt{3}-1}{\sqrt{3}-2}\cdot\frac{\sqrt{3}+2}{\sqrt{3}+2}=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}+2\right)}{3-4} = \frac{3+2\sqrt{3}-\sqrt{3}-2}{-1}=\frac{1+\sqrt{3}}{-1}=-1-\sqrt{3}$$

81. $\frac{3-\sqrt{2}}{3+\sqrt{2}} = \frac{3-\sqrt{2}}{3+\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}}$

$= \frac{9-2}{9+6\sqrt{2}+2} = \frac{7}{11+6\sqrt{2}}$

82. $\frac{1+\sqrt{7}}{2-\sqrt{3}} = \frac{1+\sqrt{7}}{2-\sqrt{3}} \cdot \frac{1-\sqrt{7}}{1-\sqrt{7}}$

$= \frac{1-7}{2-2\sqrt{7}-\sqrt{3}+\sqrt{3}\sqrt{7}}$

$= \frac{-6}{2-2\sqrt{7}-\sqrt{3}+\sqrt{21}}$

83. $x = \sqrt{\frac{kM}{f}}$

Note that because x represents the number of units to order, the value of x should be rounded to the nearest integer.

a. $k = \$1, f = \$500, M = 100{,}000$

$x = \sqrt{\frac{1 \cdot 100{,}000}{500}} = \sqrt{200} \approx 14.1$

The number of units to order is 14.

b. $k = \$3, f = \$7, M = 16{,}700$

$x = \sqrt{\frac{3 \cdot 16{,}700}{7}} \approx 84.6$

The number of units to order is 85.

c. $k = \$1, f = \$5, M = 16{,}800$

$x = \sqrt{\frac{1 \cdot 16{,}800}{5}} = \sqrt{3360} \approx 58.0$

The number of units to order is 58.

84. $h = 12.3T^{1/3}$

If $T = 216$, find h.

$h = 12.3(216)^{1/3} = 73.8$

A height of 73.8 in. corresponds to a threshold weight of 216 lb.

For exercises 85–88, we use the model $\text{revenue} = 6.67x^{.188}$, $x \ge 5$, $x = 5$ corresponds to 2005.

85. Let $x = 15$. Then $6.67(15)^{.188} \approx 11.1$

The domestic revenue for 2015 were about \$11.1 billion.

86 Let $x = 18$. Then $6.67(18)^{.188} \approx 11.5$

The domestic revenue for 2018 will be about \$11.5 billion.

87. Let $x = 20$. Then $6.67(20)^{.188} \approx 11.7$

The domestic revenue for 2020 will be about \$11.7 billion.

88. Let $x = 23$. Then $6.67(23)^{.188} \approx 12.0$

The domestic revenue for 2023 will be about \$12.0 billion.

For exercises 89–92, we use the model $\text{revenue} = 1.08x^{.527}$, $x \ge 1$, $x = 1$ corresponds to the first quarter of the year 2013.

89. Let $x = 13$. Then $1.08(13)^{.527} \approx 4.2$

The advertising revenue in the first quarter of 2016 was approximately \$4.2 billion.

90. Let $x = 18$. Then $1.08(18)^{.527} \approx 5.0$

The advertising revenue in the second quarter of 2017 was approximately \$5.0 billion.

91. Let $x = 24$. Then $1.08(24)^{.527} \approx 5.8$

The advertising revenue in the fourth quarter of 2018 will be approximately \$5.8 billion.

92. Let $x = 27$. Then $1.08(27)^{.527} \approx 6.1$

The advertising revenue in the third quarter of 2019 will be approximately \$6.1 billion.

For exercises 93–96, we use the model $\text{Part-time Students} = 2.72x^{.238}; x \ge 10, x = 10$ corresponds to 1990.

93. Let $x = 29$. Then $2.72(29)^{.238} \approx 6.1$

According to the model, there were approximately 6.1 million part-time students attending college or university in 2009.

94. Let $x = 34$. Then $2.72(34)^{.238} \approx 6.3$

According to the model, there were approximately 6.3 million part-time students attending college or university in 2014.

95. Let $x = 38$. Then $2.72(38)^{.238} \approx 6.5$

According to the model, there will be approximately 6.5 million part-time students attending college or university in 2018

96. Let $x = 43$. Then $2.72(43)^{.238} \approx 6.7$

According to the model, there will be approximately 6.7 million part-time students attending college or university in 2023

Section 1.6 First-Degree Equations

1.
$$\begin{aligned} 3x+8&=20\\ 3x+8-8&=20-8\\ 3x&=12\\ \frac{1}{3}(3x)&=\frac{1}{3}(12)\\ x&=4 \end{aligned}$$

2. $4 - 5y = 19$

Add –4 to both sides.

$$\begin{aligned} 4-5y+(-4)&=19+(-4)\\ -5y&=15 \end{aligned}$$

Multiply both sides by $-\frac{1}{5}$.

$$\begin{aligned} -\frac{1}{5}(-5y)&=-\frac{1}{5}(15)\\ y&=-3 \end{aligned}$$

3.
$$\begin{aligned} .6k-.3&=.5k+.4\\ .6k-.5k-.3&=.5k-.5k+.4\\ .1k-.3&=.4\\ .1k-.3+.3&=.4+.3\\ .1k&=.7\\ \frac{.1k}{.1}&=\frac{.7}{.1}\Rightarrow k=7 \end{aligned}$$

4.
$$\begin{aligned} 2.5+5.04m&=8.5-.06m\\ 2.5+5.04m+.06m&=8.5-.06m+.06m\\ 2.5+5.1m&=8.5\\ 2.5+5.1m+(-2.5)&=8.5+(-2.5)\\ 5.1m&=6.0\\ \frac{5.1m}{5.1}&=\frac{6.0}{5.1}\\ m&=\frac{6.0}{5.1}\Rightarrow m\approx 1.18 \end{aligned}$$

5.
$$\begin{aligned} 2a-1&=4(a+1)+7a+5\\ 2a-1&=4a+4+7a+5\\ 2a-1&=11a+9\\ 2a-2a-1&=11a-2a+9\\ -1&=9a+9\\ -1-9&=9a+9-9\\ -10&=9a\\ \frac{-10}{9}&=\frac{9a}{9}\Rightarrow -\frac{10}{9}=a \end{aligned}$$

6.
$$\begin{aligned} 3(k-2)-6&=4k-(3k-1)\\ 3k-6-6&=4k-3k+1\\ 3k-12&=k+1\\ 3k-12+(-k)&=k+1+(-k)\\ 2k-12&=1\\ 2k-12+12&=1+12\\ 2k&=13\\ \frac{2k}{2}&=\frac{13}{2}\Rightarrow k=\frac{13}{2} \end{aligned}$$

7.
$$\begin{aligned} 2[x-(3+2x)+9]&=3x-8\\ 2(x-3-2x+9)&=3x-8\\ 2(-x+6)&=3x-8\\ -2x+12&=3x-8\\ 12&=5x-8\\ 20&=5x\Rightarrow 4=x \end{aligned}$$

8.
$$\begin{aligned} -2\left[4(k+2)-3(k+1)\right]&=14+2k\\ -2(4k+8-3k-3)&=14+2k\\ -2(k+5)&=14+2k\\ -2k-10&=14+2k\\ -2k-10+2k&=14+2k+2k\\ -10&=14+4k\\ -10-14&=14+4k-14\\ -24&=4k\\ \frac{-24}{4}&=\frac{4k}{4}\Rightarrow -6=k \end{aligned}$$

9. $\frac{3x}{5}-\frac{4}{5}(x+1)=2-\frac{3}{10}(3x-4)$

Multiply both sides by the common denominator, 10.

$$10\left(\frac{3x}{5}\right)-10\left(\frac{4}{5}\right)(x+1)$$
$$=(10)(2)-(10)\left(\frac{3}{10}\right)(3x-4)$$
$$2(3x)-8(x+1)=20-3(3x-4)$$
$$6x-8x-8=20-9x+12$$
$$-2x-8=32-9x$$
$$-2x+9x=32+8$$
$$7x=40$$
$$\frac{1}{7}(7x)=\frac{1}{7}(40)\Rightarrow x=\frac{40}{7}$$

10. $\frac{4}{3}(x-2)-\frac{1}{2}=2\left(\frac{3}{4}x-1\right)$

$$\frac{4}{3}x-\frac{8}{3}-\frac{1}{2}=\frac{3}{2}x-2$$
$$\frac{4}{3}x-\frac{19}{6}=\frac{3}{2}x-2$$
$$\frac{4}{3}x-\frac{19}{6}-\frac{4}{3}x=\frac{3}{2}x-2-\frac{4}{3}x$$
$$-\frac{19}{6}=\frac{1}{6}x-2$$
$$-\frac{19}{6}+2=\frac{1}{6}x-2+2$$
$$-\frac{7}{6}=\frac{1}{6}x$$
$$6\left(-\frac{7}{6}\right)=6\left(\frac{1}{6}\right)x\Rightarrow -7=x$$

11. $\frac{5y}{6}-8=5-\frac{2y}{3}$

$$6\left(\frac{5y}{6}-8\right)=6\left(5-\frac{2y}{3}\right)$$
$$6\left(\frac{5y}{6}\right)-6(8)=6(5)-6\left(\frac{2y}{3}\right)$$
$$5y-48=30-4y$$
$$9y-48=30$$
$$9y=78$$
$$y=\frac{78}{9}=\frac{26}{3}$$

12. $\frac{x}{2}-3=\frac{3x}{5}+1$

Multiply both sides by the common denominator, 10, to eliminate the fractions.

$$10\left(\frac{x}{2}-3\right)=10\left(\frac{3x}{5}+1\right)$$
$$5x-30=6x+10$$
$$5x-30-5x=6x+10-5x$$
$$-30=x+10$$
$$-30-10=x+10-10$$
$$-40=x$$

13. $\frac{m}{2}-\frac{1}{m}=\frac{6m+5}{12}$

$$12m\left(\frac{m}{2}-\frac{1}{m}\right)=12m\left(\frac{6m+5}{12}\right)$$
$$(12m)\left(\frac{m}{2}\right)-(12m)\left(\frac{1}{m}\right)=m(6m)+m(5)$$
$$6m^2-12=6m^2+5m$$
$$-12=5m$$
$$\frac{1}{5}(-12)=\frac{1}{5}(5m)\Rightarrow -\frac{12}{5}=m$$

14. $-\frac{3k}{2}+\frac{9k-5}{6}=\frac{11k+8}{k}$

Multiply both sides by the common denominator, $6k$ to eliminate the fractions.

$$6k\left(-\frac{3k}{2}+\frac{9k-5}{6}\right)=6k\left(\frac{11k+8}{k}\right)$$
$$6k\left(-\frac{3k}{2}\right)+6k\left(\frac{9k-5}{6}\right)=6k\left(\frac{11k}{k}\right)+6k\left(\frac{8}{k}\right)$$
$$-9k^2+k(9k-5)=6(11k)+6(8)$$
$$-9k^2+9k^2-5k=66k+48$$
$$-5k=66k+48$$
$$-5k-66k=66k+48-66k$$
$$-71k=48$$
$$\frac{-71k}{-71}=\frac{48}{-71}\Rightarrow k=-\frac{48}{71}$$

15. $\frac{4}{x-3}-\frac{8}{2x+5}+\frac{3}{x-3}=0$

$\frac{4}{x-3}+\frac{3}{x-3}-\frac{8}{2x+5}=0$

$\frac{7}{x-3}-\frac{8}{2x+5}=0$

Multiply each side by the common denominator, $(x-3)(2x+5)$.

$$(x-3)(2x+5)\left(\frac{7}{x-3}\right)-(x-3)(2x+5)\left(\frac{8}{2x+5}\right)=(x-3)(2x+5)(0)$$

$$7(2x+5)-8(x-3)=0$$
$$14x+35-8x+24=0$$
$$6x+59=0$$
$$6x=-59\Rightarrow x=-\frac{59}{6}$$

16. $\frac{5}{2p+3}-\frac{3}{p-2}=\frac{4}{2p+3}$

$\frac{5}{2p+3}-\frac{3}{p-2}-\frac{5}{2p+3}=\frac{4}{2p+3}-\frac{5}{2p+3}$

$-\frac{3}{p-2}=-\frac{1}{2p+3}$

Multiply both sides by the common denominator, $(2p+3)(p-2)$.

$$(2p+3)(p-2)\left(\frac{5}{2p+3}-\frac{3}{p-2}\right)$$

$$\left(-\frac{3}{p-2}\right)(p-2)(2p+3)=\left(-\frac{1}{2p+3}\right)(p-2)(2p+3)$$

$$-3(2p+3)=-1(p-2)$$
$$-6p-9=-p+2$$
$$-5p=11\Rightarrow p=-\frac{11}{5}$$

$$(p-2)5+(2p+3)(-3)=(p-2)4$$
$$5p-10-6p-9=4p-8$$
$$-p-19=4p-8$$
$$-11=5p\Rightarrow -\frac{11}{5}=p$$

17. $\frac{3}{2m+4}=\frac{1}{m+2}-2$

$\frac{3}{2(m+2)}=\frac{1}{m+2}-2$

$$2(m+2)\left(\frac{3}{2(m+2)}\right)=2(m+2)\left(\frac{1}{m+2}\right)-2(m+2)(2)$$

$$=2(m+2)\left(\frac{1}{m+2}\right)-2(m+2)(2)$$

$$3=2-4(m+2)$$
$$3=2-4m-8$$
$$3=-6-4m\Rightarrow 9=-4m\Rightarrow m=-\frac{9}{4}$$

18. $\frac{8}{3k-9}-\frac{5}{k-3}=4$

Multiply both sides by the common denominator, $3k-9$.

$$(3k-9)\left[\frac{8}{3k-9}-\frac{5}{k-3}\right]=(3k-9)4$$

$$(3k-9)\left(\frac{8}{3k-9}\right)+3(k-3)\left(-\frac{5}{k-3}\right)=12k-36$$

$$8-15=12k-36$$
$$-7=12k-36$$
$$29=12k\Rightarrow \frac{29}{12}=k$$

19. $9.06x+3.59(8x-5)=12.07x+.5612$

$$9.06x+28.72x-17.95=12.07x+.5612$$
$$9.06x+28.72x-12.07x=17.95+.5612$$
$$25.71x=18.5112$$
$$x=\frac{18.5112}{25.71}=.72$$

20. $-5.74(3.1-2.7p)=1.09p+5.2588$

$$-17.794+15.498p=1.09p+5.2588$$
$$15.498p-1.09p=5.2588+17.794$$
$$14.408p=23.0528$$
$$p=\frac{23.0528}{14.408}=1.6$$

21. $\frac{2.63r-8.99}{1.25}-\frac{3.90r-1.77}{2.45}=r$

Multiply by the common denominator (1.25)(2.45) to eliminate the fractions.

$$(2.45)(2.63r-8.99)-(1.25)(3.90r-1.77)=(2.45)(1.25)r$$
$$6.4435r-22.0255-4.875r+2.2125=3.0625r$$
$$1.5685r-19.813=3.0625r$$
$$-19.813=1.494r$$
$$-\frac{19.813}{1.494}=\frac{1.494r}{1.494}$$
$$r\approx -13.26$$

22. $\frac{8.19m+2.55}{4.34}-\frac{8.17m-9.94}{1.04}=4m$

$$(1.04)(8.19m+2.55)-(4.34)(8.17m-9.94)=4m(1.04)(4.34)$$
$$8.5176m+2.652-35.4578m+43.1396=18.0544m$$
$$-26.9402m+45.7916=18.0544m$$
$$45.7916=44.9946m$$
$$m=\frac{45.7916}{44.9946}\Rightarrow m\approx 1.02$$

23. $4(a+x)=b-a+2x$

$$4a+4x=b-a+2x$$
$$4a=b-a-2x$$
$$5a-b=-2x$$
$$\frac{5a-b}{-2}=\frac{-2x}{-2}$$
$$-\frac{5a-b}{2}=x \text{ or } x=\frac{b-5a}{2}$$

24. $(3a-b)-bx=a(x-2)$

$$3a-b-bx=ax-2a$$

Isolate terms with x on the right.

$$3a-b=ax-2a+bx$$
$$5a-b=ax+bx$$
$$5a-b=(a+b)x\Rightarrow\frac{5a-b}{a+b}=x$$

25. $5(b-x)=2b+ax$

First, use the distributive property.

$$5b-5x=2b+ax$$
$$5b=2b+ax+5x$$
$$3b=ax+5x$$
$$3b=(a+5)x$$
$$\frac{3b}{a+5}=\frac{(a+5)x}{a+5}\Rightarrow\frac{3b}{a+5}=x$$

Now use the distributive property on the right.

$$3b=(a+5)x$$
$$\frac{3b}{a+5}=\frac{(a+5)x}{a+5}$$
$$\frac{3b}{a+5}=x$$

26. $bx-2b=2a-ax$

Isolate terms with x on the left.

$$bx+ax=2a+2b$$
$$ax+bx=2a+2b$$
$$(a+b)x=2(a+b)$$
$$x=\frac{2(a+b)}{a+b}\Rightarrow x=2$$

27. $PV=k$ for V

$$\frac{1}{P}(PV)=\frac{1}{P}(k)$$
$$V=\frac{k}{P}$$

28. $i=prt$ for p

$$\frac{i}{rt}=p$$

29. $V=V_0+gt$ for g

$$V-V_0=gt$$
$$\frac{V-V_0}{t}=\frac{gt}{t}$$
$$\frac{V-V_0}{t}=g$$

30. $S=S_0+gt^2+k$

$$S-S_0-k=gt^2$$
$$\frac{S-S_0-k}{t^2}=\frac{gt^2}{t^2}\Rightarrow\frac{S-S_0-k}{t^2}=g$$

31. $A = \frac{1}{2}(B+b)h$ for B

$A = \frac{1}{2}Bh + \frac{1}{2}bh$

$2A = Bh + bh$ Multiply by 2.

$2A - bh = Bh$

$\frac{2A - bh}{h} = \frac{Bh}{h}$ Multiply by $\frac{1}{h}$.

$\frac{2A - bh}{h} = \frac{2A}{h} - b = B$

32. $C = \frac{5}{9}(F - 32)$ for F

$\frac{9}{5}C = F - 32 \Rightarrow \frac{9}{5}C + 32 = F$

33. $|2h - 1| = 5$

$2h - 1 = 5$ or $2h - 1 = -5$

$2h = 6$ or $2h = -4$

$h = 3$ or $h = -2$

34. $|4m - 3| = 12$

$4m - 3 = 12$ or $4m - 3 = -12$

$4m = 15$ or $4m = -9$

$m = \frac{15}{4}$ or $m = -\frac{9}{4}$

35. $|6 + 2p| = 10$

$6 + 2p = 10$ or $6 + 2p = -10$

$2p = 4$ or $2p = -16$

$p = 2$ or $p = -8$

36. $|-5x + 7| = 15$

$-5x + 7 = 15$ or $-5x + 7 = -15$

$-5x = 8$ or $-5x = -22$

$x = -\frac{8}{5}$ or $x = \frac{22}{5}$

37. $\left|\frac{5}{r-3}\right| = 10$

$\frac{5}{r-3} = 10$ or $\frac{5}{r-3} = -10$

$5 = 10(r-3)$ or $5 = -10(r-3)$

$5 = 10r - 30$ or $5 = -10r + 30$

$35 = 10r$ or $-25 = -10r$

$\frac{35}{10} = \frac{7}{2} = r$ or $\frac{-25}{-10} = \frac{5}{2} = r$

38. $\left|\frac{3}{2h-1}\right| = 4$

$\frac{3}{2h-1} = 4$ or $\frac{3}{2h-1} = -4$

$3 = 4(2h-1)$ or $3 = -4(2h-1)$

$3 = 8h - 4$ or $3 = -8h + 4$

$7 = 8h$ or $-1 = -8h$

$\frac{7}{8} = h$ or $\frac{1}{8} = h$

39. $-5 = \frac{5}{9}(F - 32)$

$-5\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$-9 = F - 32 \Rightarrow 23 = F$

The temperature –5°C = 23°F.

40. $-15 = \frac{5}{9}(F - 32)$

$-15\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$-27 = F - 32 \Rightarrow 5 = F$

The temperature –15°C = 5°F.

41. $22 = \frac{5}{9}(F - 32)$

$22\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$39.6 = F - 32 \Rightarrow 71.6 = F$

The temperature 22°C = 71.6°F.

42. $36 = \frac{5}{9}(F - 32)$

$36\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$64.8 = F - 32 \Rightarrow 96.8 = F$

The temperature 36°C = 96.8°F.

43. Let $x = 10$.

$$y = 1.15x + 1.62$$
$$y = 1.15(10) + 1.62$$
$$y = 13.12$$

Therefore, the gross federal debt in 2010 was $13.12 trillion.

44. Let $x = 15$.

$$y = 1.15x + 1.62$$
$$y = 1.15(15) + 1.62$$
$$y = 18.87$$

Therefore, the gross federal debt in 2015 was $18.87 trillion.

45. $y = 1.15x + 1.62$

Substitute 22.32 for y.

$$22.32 = 1.15x + 1.62$$
$$20.7 = 1.15x \Rightarrow 18 = x$$

Therefore, the federal deficit will be $22.32 trillion in 2018.

46. $y = 1.15x + 1.62$

Substitute 24.62 for y.

$$24.62 = 1.15x + 1.62$$
$$23 = 1.15x \Rightarrow 20 = x$$

Therefore, the federal deficit will be $24.628.83 trillion in 2020.

47. $y = 1.15x + 1.62$

Substitute 25.77 for y.

$$25.77 = 1.15x + 1.62$$
$$24.15 = 1.15x \Rightarrow 21 = x$$

Therefore, the federal deficit will be $825.77 trillion in 2021.

48. $y = 1.15x + 1.62$

Substitute 30.37 for y.

$$30.37 = 1.15x + 1.62$$
$$28.75 = 1.15x \Rightarrow 25 = x$$

Therefore, the federal deficit will be $30.378 trillion in 2025.

49. $E = .108x + 1.517$

Substitute $2.7052422 in for E.

$$2.705 = .108x + 1.517$$
$$1.188 = .108x \Rightarrow 11 = x$$

The health care expenditures were at $2.7052250 trillion in 2011.

50. $E = .108x + 1.517$

Substitute $3.029 in for E.

$$3.029 = .108x + 1.517$$
$$1.512 = .108x \Rightarrow 14 = x$$

The health care expenditures were at $3.029 trillion in 2014.

51. $E = .108x + 1.517$

Substitute $3.461 in for E.

$$3.461 = .108x + 1.517$$
$$1.944 = .108x \Rightarrow 18 = x$$

The health care expenditures will be $3.4612422 trillion in 2018.

52. $E = .108x + 1.517$

Substitute $3.893 in for E.

$$3.893 = .108x + 1.517$$
$$2.376 = .108x \Rightarrow 22 = x$$

The health care expenditures will be $3.893 trillion in 2022.

53. $114.8(x - 2010) = 5y - 3390.5$

Substitute 746.98 for y and solve for x.

$$114.8(x - 2010) = 5(746.98) - 3390.5$$
$$114.8x - 230,748 = 344.4$$
$$114.8x = 231,092.4$$
$$x = 2013$$

The amount of income was $746.98 billion in 2013.

54. $114.8(x - 2010) = 5y - 3390.5$

Substitute 815.86 for y and solve for x.

$$114.8(x - 2010) = 5(815.86) - 3390.5$$
$$114.8x - 230,748 = 688.8$$
$$114.8x = 231,436.8$$
$$x = 2016$$

The amount of income was $815.86 billion in 2016.

55. $114.8(x - 2010) = 5y - 3390.5$

Substitute 907.7 for y and solve for x.

$$114.8(x - 2010) = 5(907.7) - 3390.5$$
$$114.8x - 230,748 = 1148$$
$$114.8x = 231,896$$
$$x = 2020$$

The amount of income will be $907.7 billion in 2020.

56. $114.8(x-2010)=5y-3390.5$

Substitute 1022.5 for y and solve for x.

$$114.8(x-2010)=5(1022.5)-3390.5$$
$$114.8x-230,748=1722$$
$$114.8x=232,470$$
$$x=2025$$

The amount of income will be \$1022.5 billion in 2025.

57. $f = 800$, $n = 18$, $q = 36$

$$u=f\cdot\frac{n(n+1)}{q(q+1)}=800\cdot\frac{18(19)}{36(37)}$$
$$=800\cdot\frac{342}{1332}\approx 205.41$$

The amount of unearned interest is \$205.41.

58. $f = 1400$, $q = 48$, $n = 12$

$$u=f\cdot\frac{n(n+1)}{q(q+1)}=1400\cdot\frac{12(12+1)}{48(48+1)}\approx \$92.86$$

The amount of unearned interest is \$92.86.

59. Let x = the number invested at 5%.

$$.05x+.04(52,000-x)=2290$$
$$.05x+2080-.04x=2290$$
$$.01x+2080=2290$$
$$.01x+2080-2080=2290-2080$$
$$.01x=210$$
$$\frac{.01x}{.01x}=\frac{210}{.01}$$
$$x=21,000$$

Joe invested \$21,000 at 5%.

60. Let x represent the amount invested at 4%. Then 20,000 – x is the amount invested at 6%. Since the total interest is \$1040,

$$.04x+.06(20,000-x)=1040$$
$$.04x+1200-.06x=1040$$
$$-.02x+1200=1040$$
$$-.02x=-160$$
$$x=8000$$

She invested \$8000 at 4%.

61. Let x = price of first plot.
Then 120,000 – x = price of second plot.
$.15x$ = profit from first plot
$-.10(120,000 - x)$ = loss from second plot.

$$.15x-.10(120,000-x)=5500$$
$$.15x-12,000+.10x=5500$$
$$.25x=17,500$$
$$x=70,000$$

She paid \$70,000 for the first plot and 120,000 – 70,000, or \$50,000 for the second plot.

62. Let x represent the amount invested at 4%. \$20,000 invested at 5% (or .05) plus x dollars invested at 4% (or .04) must equal 4.8% (or .048) of the total investment (or 20,000 + x). Solve this equation.

$$.05(20,000)+.04x=.048(20,000+x)$$
$$1000+.04x=960+.048x$$
$$1000=960+.008x$$
$$40=.008x\Rightarrow x=5000$$

\$5000 should be invested at 4%.

63. Let x = average rate of growth of Twitter.
Then 5.7 + x = average rate of growth of Instagram.
$35x$ = visitors to Twitter
$21(5.7 + x)$ = visitors to Instagram.

$$35x=21(5.7+x)$$
$$35x=119.7+21x$$
$$14x=119.7$$
$$x\approx 8.6$$

Since x represents the average rate of growth of Twitter, there was an average growth of approximately 8.6 million visitorsShe .

64. The average rate of growth of Instagram was 5.7 + x or approximately 14.3 million visitors.

65. The number of visitors to Twitter was $35x = 35(8.6) = 301$ million.

66. The number of visitors to Instagram was $21(5.7 + x) = 21(5.7 + 8.6) = 300.3$ million (Note: Due to rounding error, the values in 65 and 66 are different)

67. Let x = the number of liters of 94 octane gas;
200 = the number of liters of 99 octane gas;
200 + x = the number of liters of 97 octane gas.

$$94x + 99(200) = 97(200 + x)$$
$$94x + 19{,}800 = 19{,}400 + 97x$$
$$400 = 3x$$
$$\frac{400}{3} = x$$

Thus, $\frac{400}{3}$ liters of 94 octane gas are needed.

68. Let x be the amount of 92 octane gasoline. Then $12 - x$ is the amount of 98 octane gasoline. A mixture of the two must yield 12 L of 96 octane gasoline, so

$$92x + 98(12 - x) = 96(12)$$
$$92x + 1176 - 98x = 1152$$
$$-6x = -24$$
$$x = 4$$

Then $12 - x = 12 - 4 = 8$.
Mix 4 L of 92 octane gasoline with 8 L of 98 octane gasoline.

Section 1.7 Quadratic Equations

1. $(x + 4)(x - 14) = 0$

$x + 4 = 0$ or $x - 14 = 0$

$x = -4$ or $x = 14$

The solutions are –4 and 14.

2. $(p - 16)(p - 5) = 0$

$p - 16 = 0$ or $p - 5 = 0$

$p = 16$ or $p = 5$

The solutions are 16 and 5.

3. $x(x + 6) = 0$

$x = 0$ or $x + 6 = 0$

$x = -6$

The solutions are 0 and –6.

4. $x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ or $x - 2 = 0$

$x = 2$

The solutions are 0 and 2.

5. $2z^2 = 4z$

$2z^2 - 4z = 0$

$2z(z - 2) = 0$

$2z = 0$ or $z - 2 = 0$

$z = 0$ or $z = 2$

The solutions are 0 and 2.

6. $x^2 - 64 = 0$

$(x - 8)(x + 8) = 0$

$x - 8 = 0$ or $x + 8 = 0$

$x = 8$ or $x = -8$

The solutions are 8 and –8.

7. $y^2 + 15y + 56 = 0$

$(y + 7)(y + 8) = 0$

$y + 7 = 0$ or $y + 8 = 0$

$y = -7$ or $y = -8$

The solutions are –7 and –8.

8. $k^2 - 4k - 5 = 0$

$(k + 1)(k - 5) = 0$

$k + 1 = 0$ or $k - 5 = 0$

$k = -1$ or $k = 5$

The solutions are –1 and 5.

9. $2x^2 = 7x - 3$

$2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$

$2x - 1 = 0$ or $x - 3 = 0$

$x = \frac{1}{2}$ or $x = 3$

The solutions are $\frac{1}{2}$ and 3.

10. $2 = 15z^2 + z$

$0 = 15z^2 + z - 2$

$0 = (3z - 1)(5z + 2)$

$3z - 1 = 0$ or $5z + 2 = 0$

$z = \frac{1}{3}$ or $z = -\frac{2}{5}$

The solutions are $\frac{1}{3}$ and $-\frac{2}{5}$.

11. $6r^2 + r = 1$

$6r^2 + r - 1 = 0$

$(3r-1)(2r+1) = 0$

$3r - 1 = 0$ or $2r + 1 = 0$

$r = \frac{1}{3}$ or $r = -\frac{1}{2}$

The solutions are $\frac{1}{3}$ and $-\frac{1}{2}$.

12. $3y^2 = 16y - 5$

$3y^2 - 16y + 5 = 0$

$(3y-1)(y-5) = 0$

$3y - 1 = 0$ or $y - 5 = 0$

$y = \frac{1}{3}$ or $y = 5$

The solutions are $\frac{1}{3}$ and 5.

13. $2m^2 + 20 = 13m$

$2m^2 - 13m + 20 = 0$

$(2m-5)(m-4) = 0$

$2m - 5 = 0$ or $m - 4 = 0$

$m = \frac{5}{2}$ or $m = 4$

The solutions are $\frac{5}{2}$ and 4.

14. $6a^2 + 17a + 12 = 0$

$(2a+3)(3a+4) = 0$

$2a + 3 = 0$ or $3a + 4 = 0$

$a = -\frac{3}{2}$ or $a = -\frac{4}{3}$

The solutions are $-\frac{3}{2}$ and $-\frac{4}{3}$.

15. $m(m+7) = -10$

$m^2 + 7m + 10 = 0$

$(m+5)(m+2) = 0$

$m + 5 = 0$ or $m + 2 = 0$

$m = -5$ or $m = -2$

The solutions are –5 and –2.

16. $z(2z+7) = 4$

$2z^2 + 7z - 4 = 0$

$(2z-1)(z+4) = 0$

$2z - 1 = 0$ or $z + 4 = 0$

$z = \frac{1}{2}$ or $z = -4$

The solutions are $\frac{1}{2}$ and –4.

17. $9x^2 - 16 = 0$

$(3x+4)(3x-4) = 0$

$3x + 4 = 0$ or $3x - 4 = 0$

$3x = -4$ $\quad$ $3x = 4$

$x - \frac{4}{3}$ or $x = \frac{4}{3}$

The solutions are $-\frac{4}{3}$ and $\frac{4}{3}$.

18. $36y^2 - 49 = 0$

$(6y-7)(6y+7) = 0$

$6y - 7 = 0$ or $6y + 7 = 0$

$y = \frac{7}{6}$ or $y = -\frac{7}{6}$

The solutions are $\frac{7}{6}$ and $-\frac{7}{6}$.

19. $16x^2 - 16x = 0$

$16x(x-1) = 0$

$16x = 0$ or $x - 1 = 0$

$x = 0$ or $x = 1$

The solutions are 0 and 1.

20. $12y^2 - 48y = 0$

$12y(y-4) = 0$

$y = 0$ or $y - 4 = 0$

$y = 0$ or $y = 4$

The solutions are 0 and 4.

21. $(r-2)^2 = 7$

$r - 2 = \sqrt{7}$ or $r - 2 = -\sqrt{7}$

$r = 2 + \sqrt{7}$ or $r = 2 - \sqrt{7}$

We abbreviate the solutions as $2 \pm \sqrt{7}$.

22. $(b+4)^2 = 27$

$b+4=\sqrt{27}$ or $b+4=-\sqrt{27}$

$b+4=3\sqrt{3}$ or $b+4=-3\sqrt{3}$

$b=-4+3\sqrt{3}$ or $b=-4-3\sqrt{3}$

We abbreviate the solutions as $-4\pm 3\sqrt{3}$.

23. $(4x-1)^2 = 20$

Use the square root property.

$4x-1=\sqrt{20}$ or $4x-1=-\sqrt{20}$

$4x-1=2\sqrt{5}$ or $4x-1=-2\sqrt{5}$

$4x=1+2\sqrt{5}$ or $4x=1-2\sqrt{5}$

The solutions are $\frac{1\pm 2\sqrt{5}}{4}$.

24. $(3t+5)^2 = 11$

$3t+5=\sqrt{11}$ or $3t+5=-\sqrt{11}$

$3t=-5+\sqrt{11}$ or $3t=-5-\sqrt{11}$

$t=\frac{-5+\sqrt{11}}{3}$ or $t=\frac{-5\pm\sqrt{11}}{3}$

The solutions are $\frac{-5\pm\sqrt{11}}{3}$.

25. $2x^2+7x+1=0$

Use the quadratic formula with $a = 2$, $b = 7$, and $c = 1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-7\pm\sqrt{7^2-4(2)(1)}}{2(2)}$$

$$=\frac{-7\pm\sqrt{49-8}}{4}=\frac{-7\pm\sqrt{41}}{4}$$

The solutions are $\frac{-7+\sqrt{41}}{4}$ and $\frac{-7-\sqrt{41}}{4}$, which are approximately –.1492 and –3.3508.

26. $3x^2-x-7=0$

Use the quadratic formula with $a = 3$, $b = -1$, and $c = -7$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-7)}}{2(3)}$$

$$=\frac{1\pm\sqrt{1+84}}{6}=\frac{1\pm\sqrt{85}}{6}$$

The solutions $\frac{1\pm\sqrt{85}}{6}$ are approximately 1.7033 and –1.3699.

27. $4k^2+2k=1$

Rewrite the equation in standard form.

$4k^2+2k-1=0$

Use the quadratic formula with $a = 4$, $b = 2$, and $c = -1$.

$$k=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}$$

$$=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm 2\sqrt{5}}{8}=\frac{2\left(-1\pm\sqrt{5}\right)}{2\cdot 4}$$

$$k=\frac{-1\pm\sqrt{5}}{4}$$

The solutions are $\frac{-1+\sqrt{5}}{4}$ and $\frac{-1-\sqrt{5}}{4}$, which are approximately .3090 and –.8090.

28. $r^2=3r+5$

$r^2-3r-5=0$

Use the quadratic formula with $a = 1$, $b = -3$, and $c = -5$.

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$r=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-5)}}{2(1)}$$

$$=\frac{3\pm\sqrt{9+20}}{2}=\frac{3\pm\sqrt{29}}{2}$$

The solutions $\frac{3\pm\sqrt{29}}{2}$ are approximately 4.1926 and –1.1926.

29. $5y^2 + 5y = 2$

$5y^2 + 5y - 2 = 0$

$a = 5,\ b = 5,\ c = -2$

$$y = \frac{-5 \pm \sqrt{5^2 - 4(5)(-2)}}{2(5)} = \frac{-5 \pm \sqrt{25 + 40}}{10}$$
$$= \frac{-5 \pm \sqrt{65}}{10} = \frac{-5 \pm \sqrt{65}}{10}$$

The solutions are $\frac{-5 + \sqrt{65}}{10}$ and $\frac{-5 - \sqrt{65}}{10}$, which are approximately .3062 and –1.3062.

30. $2z^2 + 3 = 8z$

$2z^2 - 8z + 3 = 0$

$a = 2,\ b = -8$, and $c = 3$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{64 - 24}}{4} = \frac{8 \pm \sqrt{40}}{4} = \frac{8 \pm 2\sqrt{10}}{4}$$
$$= \frac{2\left(4 \pm \sqrt{10}\right)}{4} = \frac{4 \pm \sqrt{10}}{2}$$

The solutions $\frac{4 \pm \sqrt{10}}{2}$, are approximately 3.5811 and .4189.

31. $6x^2 + 6x + 4 = 0$

$a = 6,\ b = 6,\ c = 4$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(6)(4)}}{2(6)} = \frac{-6 \pm \sqrt{36 - 96}}{12}$$
$$= \frac{-6 \pm \sqrt{-60}}{12}$$

Because $\sqrt{-60}$ is not a real number, the given equation has no real number solutions.

32. $3a^2 - 2a + 2 = 0$

$a = 3,\ b = -2$, and $c = 2$.

$$a = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(2)}}{2(3)}$$
$$= \frac{2 \pm \sqrt{4 - 24}}{6} = \frac{2 \pm \sqrt{-20}}{6}$$

Since $\sqrt{-20}$ is not a real number, the given equation has no real number solutions.

33. $2r^2 + 3r - 5 = 0$

$a = 2,\ b = 3,\ c = -5$

$$r = \frac{-(3) \pm \sqrt{9 - 4(2)(-5)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 40}}{4}$$
$$= \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4}$$
$$r = \frac{-3 + 7}{4} = \frac{4}{4} = 1 \text{ or } r = \frac{-3 - 7}{4} = \frac{-10}{4} = \frac{-5}{2}$$

The solutions are $-\frac{5}{2}$ and 1.

34. $8x^2 = 8x - 3$

$8x^2 - 8x + 3 = 0$

$a = 8,\ b = -8$, and $c = 3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(8)(3)}}{2(8)}$$
$$= \frac{8 \pm \sqrt{64 - 96}}{16} = \frac{8 \pm \sqrt{-32}}{16}$$

Because $\sqrt{-32}$ is not a real number, there are no real number solutions.

35. $2x^2 - 7x + 30 = 0$

$a = 2,\ b = -7,\ c = 30$

$$x = \frac{-(-7) \pm \sqrt{49 - 4(2)(30)}}{2(2)} = \frac{7 \pm \sqrt{-191}}{4}$$

Since $\sqrt{-191}$ is not a real number, there are no real solutions.

36. $3k^2 + k = 6$

$3k^2 + k - 6 = 0$

$a = 3,\ b = 1$, and $c = -6$.

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$k = \frac{-1 \pm \sqrt{1^2 - 4(3)(-6)}}{2(3)} = \frac{-1 \pm \sqrt{1 + 72}}{6}$$
$$k = \frac{-1 \pm \sqrt{73}}{6}$$

The solutions $\frac{-1 \pm \sqrt{73}}{6}$ are approximately 1.2573 and –1.5907.

37. $1+\frac{7}{2a}=\frac{15}{2a^2}$

To eliminate fractions, multiply both sides by the common denominator, $2a^2$.

$2a^2+7a=15 \Rightarrow 2a^2+7a-15=0$

$a = 2, b = 7, c = -15$

$$a=\frac{-7\pm\sqrt{7^2-4(2)(-15)}}{2(2)}=\frac{-7\pm\sqrt{49+120}}{4}$$

$$=\frac{-7\pm\sqrt{169}}{4}\Rightarrow a=\frac{-7+13}{4}=\frac{6}{4}=\frac{3}{2} \text{ or}$$

$$a=\frac{-7-13}{4}=\frac{-20}{4}=-5$$

The solutions are $\frac{3}{2}$ and -5.

38. $5-\frac{4}{k}-\frac{1}{k^2}=0$

Multiply both sides by k^2.

$5k^2-4k-1=0$

$a = 5, b = -4, c = -1$

$$k=\frac{-(-4)\pm\sqrt{(-4)^2-4(5)(-1)}}{2(5)}$$

$$=\frac{4\pm\sqrt{16+20}}{10}=\frac{4\pm\sqrt{36}}{10}=\frac{4\pm 6}{10}$$

$$k=\frac{4+6}{10}=\frac{10}{10}=1 \text{ or } k=\frac{4-6}{10}=\frac{-2}{10}=-\frac{1}{5}$$

The solutions are $-\frac{1}{5}$ and 1.

39. $25t^2+49=70t$

$25t^2-70t+49=0$

$b^2-4ac=(-70)^2-4(25)(49)$

$=4900-4900$

$=0$

The discriminant is 0.
There is one real solution to the equation.

40. $9z^2-12z=1$

$9z^2-12z-1=0$

$b^2-4ac=(-12)^2-4(9)(-1)$

$=144+36$

$=180$

The discriminant is positive.
There are two real solutions to the equation.

41. $13x^2+24x-5=0$

$b^2-4ac=(24)^2-4(13)(-5)$

$=576+260=836$

The discriminant is positive.
There are two real solutions to the equation.

42. $20x^2+19x+5=0$

$b^2-4ac=(19)^2-4(20)(5)$

$=361-400=-39$

The discriminant is negative.
There are no real solutions to the equation.

For Exercises 43–46 use the quadratic formula:
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

43. $4.42x^2-10.14x+3.79=0$

$$x=\frac{-(-10.14)\pm\sqrt{(-10.14)^2-4(4.42)(3.79)}}{2(4.42)}$$

$$\approx\frac{10.14\pm 5.9843}{8.84}\approx .4701 \text{ or } 1.8240$$

44. $3x^2-82.74x+570.4923=0$

$$x=\frac{-(-82.74)\pm\sqrt{(-82.74)^2-4(3)(570.4923)}}{2(3)}$$

$$=\frac{82.74}{6}=13.79$$

45. $7.63x^2+2.79x=5.32$

$7.63x^2+2.79x-5.32=0$

$$x=\frac{-2.79\pm\sqrt{(2.79)^2-4(7.63)(-5.32)}}{2(7.63)}$$

$$\approx\frac{-2.79\pm 13.0442}{15.26}\approx -1.0376 \text{ or } .6720$$

46. $8.06x^2+25.8726x=25.047256$

$8.06x^2+25.8726x-25.047256=0$

$$x=\frac{-25.8726\pm\sqrt{(25.8726)^2-4(8.06)(-25.047256)}}{2(8.06)}$$

$$\approx\frac{-25.8726\pm 38.4307}{16.12}\approx -3.9890 \text{ or } .7790$$

47. Let $x = 10$.

$P = .396x^2 - 7.09x + 74.9$

$P = .396(10)^2 - 7.09(10) + 74.9$

$P = 43.6$

The average annual pay for truckers in 2010 was \$43.6 thousand.

48. Let $x = 15$.

$P = .396x^2 - 7.09x + 74.9$

$P = .396(15)^2 - 7.09(15) + 74.9$

$P = 57.65$

The average annual pay for truckers in 2015 was \$57.65 thousand.

49. Let $P = 48$.

$P = .396x^2 - 7.09x + 74.9$

$48 = .396x^2 - 7.09x + 74.9$

$0 = .396x^2 - 7.09x + 26.9$

$$x = \frac{7.09 \pm \sqrt{(-7.09)^2 - 4(.396)(26.9)}}{2(.396)}$$

≈ 5.46 or 12.45

The first full year the average annual pay for truckers was below \$48,000 was 2006.

50. Using the answer from 49, the first full year the average pay for truckers recovered to be above \$48,000 was 2013.

51. Let $P = 55$.

$P = .396x^2 - 7.09x + 74.9$

$55 = .396x^2 - 7.09x + 74.9$

$0 = .396x^2 - 7.09x + 19.9$

$$x = \frac{7.09 \pm \sqrt{(-7.09)^2 - 4(.396)(19.9)}}{2(.396)}$$

≈ 3.49 or 14.42

The first full year the average annual pay for truckers recovered to be above \$55,000 was 2015.

52. Let $P = 60$.

$P = .396x^2 - 7.09x + 74.9$

$60 = .396x^2 - 7.09x + 74.9$

$0 = .396x^2 - 7.09x + 14.9$

$$x = \frac{7.09 \pm \sqrt{(-7.09)^2 - 4(.396)(14.9)}}{2(.396)}$$

≈ 2.43 or 15.47

The first full year the average annual pay for truckers recovered to be above \$60,000 was 2016.

53. Let $x = 3$.

$O = .867x^2 - 9.31x + 63.32$

$O = .867(3)^2 - 9.31(3) + 63.32$

$O = 43.19$

The price of crude oil for the third quarter of 2015 was \$43.19.

54. Let $x = 6$.

$O = .867x^2 - 9.31x + 63.32$

$O = .867(6)^2 - 9.31(6) + 63.32$

$O = 38.67$

The price of crude oil for the second quarter of 2016 was \$38.67.

55. Let $O = 45$.

$O = .867x^2 - 9.31x + 63.32$

$45 = .867x^2 - 9.31x + 63.32$

$0 = .867x^2 - 9.31x + 18.32$

$$x = \frac{9.31 \pm \sqrt{(-9.31)^2 - 4(.867)(18.32)}}{2(.867)}$$

≈ 2.59 or 8.14

The most recent quarter where the price of crude oil was at least \$45 was the first quarter of 2017.

56. Let $O = 55$.

$O = .867x^2 - 9.31x + 63.32$

$55 = .867x^2 - 9.31x + 63.32$

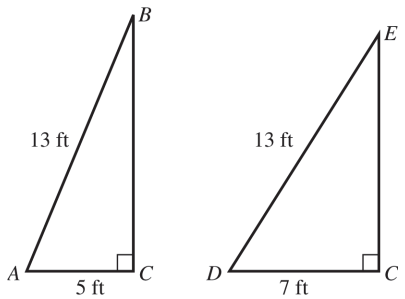
$0 = .867x^2 - 9.31x + 8.32$

$$x = \frac{9.31 \pm \sqrt{(-9.31)^2 - 4(.867)(8.32)}}{2(.867)}$$

≈ 0.98 or 9.75

The most recent quarter where the price of crude oil was at least \$55 was the second quarter of 2017.

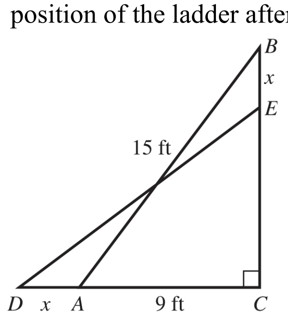
57. Triangle ABC represents the original position of the ladder, while triangle DEC represents the position of the ladder after it was moved.



Use the Pythagorean theorem to find the distance from the top of the ladder to the ground.
In triangle ABC,
$13^2 = 5^2 + BC^2 \Rightarrow 169 - 25 = BC^2 \Rightarrow$
$144 = BC^2 \Rightarrow 12 = BC$
Thus, the top of the ladder was originally 12 feet from the ground.
In triangle DEC,
$13^2 = 7^2 + EC^2 \Rightarrow 169 - 49 = EC^2 \Rightarrow$
$120 = EC^2 \Rightarrow EC = \sqrt{120}$

The top of the ladder was $\sqrt{120}$ feet from the ground after the ladder was moved. Therefore, the ladder moved down $12 - \sqrt{120} \approx 1.046$ feet.

58. Triangle ABC shows the original position of the ladder against the wall, and triangle DEC is the position of the ladder after it was moved.

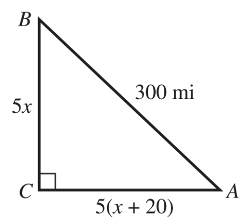


In triangle ABC,
$15^2 = BC^2 + 9^2 \Rightarrow BC^2 = 15^2 - 9^2 = 144 \Rightarrow$
$BC = 12$
In triangle DEC
$15^2 = (12 - x)^2 + (9 + x)^2 \Rightarrow$
$225 = 144 - 24x + x^2 + 81 + 18x + x^2 \Rightarrow$
$0 = -6x + 2x^2 \Rightarrow 2x(x - 3) = 0 \Rightarrow$
$2x = 0 \Rightarrow x = 0$ or $x - 3 = 0 \Rightarrow x = 3$
The bottom of the ladder should be pulled 3 feet away from the wall.

59. a. The eastbound train travels at a speed of $x + 20$.

b. The northbound train travels a distance of $5x$ in 5 hours.
The eastbound train travels a distance of $5(x + 20) = 5x + 100$ in 5 hours.

c.



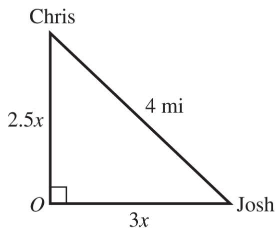
By the Pythagorean theorem,
$(5x)^2 + (5x + 100)^2 = 300^2$

d. Expand and combine like terms.
$25x^2 + 25x^2 + 1000x + 10,000 = 90,000$
$50x^2 + 1000x - 80,000 = 0$
Factor out the common factor, 50, and divide both sides by 50.
$50(x^2 + 20x - 1600) = 0$
$x^2 + 20x - 1600 = 0$
Now use the quadratic formula to solve for x.

$$x = \frac{-20 \pm \sqrt{20^2 - 4(1)(-1600)}}{2(1)}$$
≈ -51.23 or 31.23

Since x cannot be negative, the speed of the northbound train is $x \approx 31.23$ mph, and the speed of the eastbound train is $x + 20 \approx 51.23$ mph.

60. Let x represent the length of time they will be able to talk to each other.
Since $d = rt$, Tyrone' distance is $2.5x$ and Miguel's distance is $3x$.



This is a right triangle, so

$$(2.5x)^2 + (3x)^2 = 4^2 \Rightarrow 6.25x^2 + 9x^2 = 16 \Rightarrow$$
$$15.25x^2 = 16 \Rightarrow x^2 = \frac{16}{15.25} \Rightarrow$$
$$x = \pm\sqrt{\frac{16}{15.25}} \approx \pm 1.024$$

Since time must be nonnegative, they will be able to talk to each other for about 1.024 hr or approximately 61 min.

61. **a.** Let x represent the length. Then, $\frac{300-2x}{2}$ or $150 - x$ represents the width.

b. Use the formula for the area of a rectangle.
$LW = A \Rightarrow x(150 - x) = 5000$

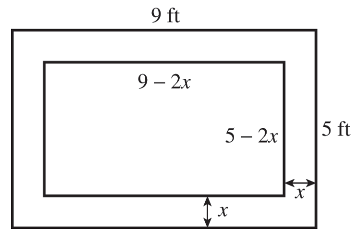
c. $150x - x^2 = 5000$
Write this quadratic equation in standard form and solve by factoring.

$$0 = x^2 - 150x + 5000$$

$$x^2 - 150x + 5000 = 0$$
$$(x - 50)(x - 100) = 0$$
$$x - 50 = 0 \quad \text{or} \quad x - 100 = 0$$
$$x = 50 \quad \text{or} \quad x = 100$$

Choose $x = 100$ because the length is the larger dimension. The length is 100 m and the width is $150 - 100 = 50$ m.

62. Let x represent the width of the border.

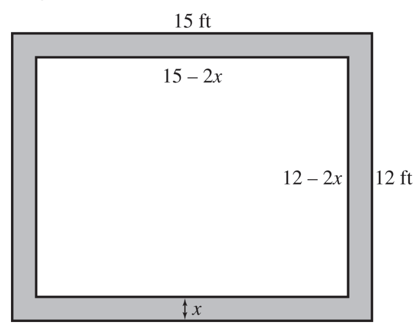


The area of the flower bed is $9 \cdot 5 = 45$. The area of the center is $(9 - 2x)(5 - 2x)$. Therefore,

$$45 - (9 - 2x)(5 - 2x) = 24$$
$$45 - \left(45 - 28x + 4x^2\right) = 24$$
$$45 - 45 + 28x - 4x^2 = 24$$
$$0 = 4x^2 - 28x + 24$$
$$0 = 4\left(x^2 - 7x + 6\right)$$
$$0 = 4(x - 1)(x - 6)$$
$$x - 1 = 0 \quad \text{or} \quad x - 6 = 0$$
$$x = 1 \quad \text{or} \quad x = 6$$

The solution $x = 6$ is impossible since both $9 - 2x$ and $5 - 2x$ would be negative. Therefore, the width of the border is 1 ft.

63. Let x = the width of the uniform strip around the rug.



The dimensions of the rug are $15 - 2x$ and $12 - 2x$. The area, 108, is the length times the width.

Solve the equation.

$(15-2x)(12-2x)=108$

$180-54x+4x^2=108$

$4x^2-54x+72=0$

$2x^2-27x+36=0$

$(x-12)(2x-3)=0$

$x-12=0$ or $2x-3=0$

$x=12$ or $x=\frac{3}{2}$

Discard $x = 12$ since both $12 - 2x$ and $15 - 2x$ would be negative.

If $x=\frac{3}{2}$, then

$15-2x=15-2\left(\frac{3}{2}\right)=12$

and $12-2x=12-2\left(\frac{3}{2}\right)=9$.

The dimensions of the rug should be 9 ft by 12 ft.

64. Let x = Haroud's speed. Then $x + 92$ = Franchitti's speed. From the formula $d = rt$, we have Haroud's time = $\frac{500}{x}$ and Franchitti's time = $\frac{500}{x+92}$. Then, we have

$\frac{500}{x+92}=\frac{500}{x}-3.72 \Rightarrow$

$500x=500(x+92)-3.72x(x+92) \Rightarrow$

$500x=500x+46{,}000-3.72x^2-342.24x \Rightarrow$

$0=-3.72x^2-342.24x+46{,}000$

Use the quadratic formula to solve for x.

$$x=\frac{-(-342.24)\pm\sqrt{(-342.24)^2-4(-3.72)(46{,}000)}}{2(-3.72)}$$

≈ -166.3 or 74.3

The negative value is not applicable. Haroud's speed was 74.3 mph and Franchitti's speed was 74.3 + 92 = 166.3 mph.

For exercises 65–70, use the formula $h=-16t^2+v_0t+h_0$, where h_0 is the height of the object when $t = 0$, and v_0 is the initial velocity at time $t = 0$.

65. $v_0=0,\ h_0=625,\ h=0$

$0=-16t^2+(0)t+625 \Rightarrow 16t^2-625=0 \Rightarrow$

$(4t-25)(4t+25)=0 \Rightarrow t=\frac{25}{4}=6.25$ or

$t=-\frac{25}{4}=-6.25$

The negative solution is not applicable. It takes 6.25 seconds for the baseball to reach the ground.

66. When the ball has fallen 196 feet, it is 625 – 196 = 429 feet above the ground.

$v_0=0,\ h_0=625,\ h=429$

$429=-16t^2+(0)t+625 \Rightarrow 16t^2-196=0 \Rightarrow$

$(4t-14)(4t+14)=0 \Rightarrow t=\frac{14}{4}=3.5$ or

$t=-\frac{14}{4}=-3.5$

The negative solution is not applicable. It takes 3.5 seconds for the ball to fall 196 feet.

67. **a.** $v_0=0,\ h_0=200,\ h=0$

$0=-16t^2-(0)t+200 \Rightarrow 16t^2=200 \Rightarrow$

$t^2=\frac{200}{16} \Rightarrow t=\pm\frac{\sqrt{200}}{4}\approx \pm 3.54$

The negative solution is not applicable. It will take about 3.54 seconds for the rock to reach the ground if it is dropped.

b. $v_0=-40,\ h_0=200,\ h=0$

$0=-16t^2-40t+200$

Using the quadratic formula, we have

$$t=\frac{-(-40)\pm\sqrt{(-40)^2-4(-16)(200)}}{2(-16)}$$

$=-5$ or 2.5

The negative solution is not applicable. It will take about 2.5 seconds for the rock to reach the ground if it is thrown with an initial velocity of 40 ft/sec.

c. $v_0=-40,\ h_0=200,\ t=2$

$h=-16(2)^2-40(2)+200=56$

After 2 seconds, the rock is 56 feet above the ground. This means it has fallen 200 – 56 = 144 feet.

68. a. $v_0 = 800,\ h_0 = 0,\ h = 3200$

$3200 = -16t^2 + 800t + 0 \Rightarrow$
$16t^2 - 800t + 3200 = 0 \Rightarrow$
$t^2 - 50t + 200 = 0$

Using the quadratic formula, we have

$$t = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(1)(200)}}{2(1)} \Rightarrow$$

$t \approx 4.38$ or $t \approx 45.62$

The rocket rises to 3200 feet after about 4.38 seconds. It also reaches 3200 feet as it falls back to the ground after about 45.62 seconds. Because we are asked how long to rise to 3200 feet, the answer will be about 4.38 sec.

b. $v_0 = 800,\ h_0 = 0,\ h = 0$

$0 = -16t^2 + 800t + 0 \Rightarrow$
$16t^2 - 800t = 0 \Rightarrow 16t(t - 50) = 0 \Rightarrow$
$t = 0$ or $t = 50$

The rocket hits the ground after 50 seconds.

69. a. $v_0 = 64,\ h_0 = 0,\ h = 64$

$64 = -16t^2 + 64t + 0 \Rightarrow$
$16t^2 - 64t + 64 = 0 \Rightarrow$
$t^2 - 4t + 4 = 0 \Rightarrow (t-2)^2 = 0 \Rightarrow t = 2$

The ball will reach 64 feet after 2 seconds.

b. $v_0 = 64,\ h_0 = 0,\ h = 39$

$39 = -16t^2 + 64t + 0 \Rightarrow$
$16t^2 - 64t + 39 = 0 \Rightarrow (4t - 13)(4t - 3) \Rightarrow$
$t = \frac{13}{4} = 3.25$ or $t = \frac{3}{4} = .75$

The ball will reach 39 feet after .75 seconds and after 3.25 seconds.

c. Two answers are possible because the ball reaches the given height twice, once on the way up and once on the way down.

70. a. $v_0 = 100,\ h_0 = 0,\ h = 50$

$50 = -16t^2 + 100t + 0 \Rightarrow$
$16t^2 - 100t + 50 = 0 \Rightarrow$
$8t^2 - 50t + 25 = 0 \Rightarrow t \approx .55$ or 5.7

The ball will reach 50 feet on the way up after approximately 0.55 seconds.

b. $v_0 = 100,\ h_0 = 0,\ h = 35$

$35 = -16t^2 + 100t + 0 \Rightarrow$
$16t^2 - 100t + 35 = 0 \Rightarrow$
$t \approx .37$ or 5.88

The ball will reach 35 feet on the way up after approximately 0.37 seconds.

In exercises 71–76, we discard negative roots since all variables represent positive real numbers.

71. $S = \frac{1}{2}gt^2$ for t

$$2S = gt^2$$
$$\frac{2S}{g} = t^2$$
$$\sqrt{\frac{2S}{g}} \cdot \frac{\sqrt{g}}{\sqrt{g}} = t$$
$$\frac{\sqrt{2Sg}}{g} = t$$

72. Solve for r.

$$a = \pi r^2$$
$$\frac{a}{\pi} = r^2$$
$$\sqrt{\frac{a}{\pi}} \cdot \frac{\sqrt{\pi}}{\sqrt{\pi}} = r \quad (r > 0)$$
$$r = \sqrt{\frac{a}{\pi}} = \frac{\sqrt{\pi a}}{\pi}$$

73. $L = \frac{d^4k}{h^2}$ for h

$$Lh^2 = d^4k$$
$$h^2 = \frac{d^4k}{L}$$
$$h = \sqrt{\frac{d^4k}{L}} \cdot \frac{\sqrt{L}}{\sqrt{L}} = \frac{\sqrt{d^4kL}}{L}$$
$$h = \frac{d^2\sqrt{kL}}{L}$$

74. Solve for v.

$$F = \frac{kMv^2}{r}$$

$$\frac{Fr}{kM} = v^2$$

$$\sqrt{\frac{Fr}{kM}} \cdot \frac{\sqrt{kM}}{\sqrt{kM}} = v \ (v > 0)$$

$$\sqrt{\frac{Fr}{kM}} = \frac{\sqrt{FrkM}}{kM} = v$$

75. $$P = \frac{E^2 R}{(r+R)^2} \text{ for } R$$

$$P(r+R)^2 = E^2 R$$

$$P\left(r^2 + 2rR + R^2\right) = E^2 R$$

$$Pr^2 + 2PrR + PR^2 = E^2 R$$

$$PR^2 + \left(2Pr - E^2\right)R + Pr^2 = 0$$

Solve for R by using the quadratic formula with $a = P,\ b = 2\text{Pr} - E^2$, and $c = \text{Pr}^2$.

$$R = \frac{-\left(2\text{Pr} - E^2\right) \pm \sqrt{\left(2\text{Pr} - E^2\right)^2 - 4P \cdot \text{Pr}^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{4P^2r^2 - 4\,\text{Pr}\,E^2 + E^4 - 4P^2r^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{E^4 - 4\,\text{Pr}\,E^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{E^2\left(E^2 - 4\text{Pr}\right)}}{2P}$$

$$R = \frac{-2\text{Pr} + E^2 \pm E\sqrt{E^2 - 4\text{Pr}}}{2P}$$

76. Solve for r.

$S = 2\pi rh + 2\pi r^2$

Write as a quadratic equation in r.

$(2\pi)r^2 + (2\pi h)r - S = 0$

Solve for r using the quadratic formula with $a = 2\pi$, $b = 2\pi h$, and $c = -\text{S}$.

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-S)}}{2(2\pi)}$$

$$= \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi S}}{4\pi}$$

$$= \frac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + 2\pi S}}{4\pi}$$

$$= \frac{2\left(-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S}\right)}{4\pi}$$

$$r = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S}}{2\pi}$$

Chapter 1 Review Exercises

1. 0 and 6 are whole numbers.

2. –12, –6, $-\sqrt{4}$, 0, and 6 are integers.

3. –12, –6, $-\frac{9}{10}$, $-\sqrt{4}$, 0, $\frac{1}{8}$, and 6 are rational numbers.

4. $-\sqrt{7}, \frac{\pi}{4}, \sqrt{11}$ are irrational numbers.

5. 9[(–3)4] = 9[4(–3)]
Commutative property of multiplication

6. 7(4 + 5) = (4 + 5)7
Commutative property of multiplication

7. $6(x + y - 3) = 6x + 6y + 6(-3)$
Distributive property

8. 11 + (5 + 3) = (11 + 5) + 3
Associative property of addition

9. x is at least 9.
$x \geq 9$

10. x is negative.
$x < 0$

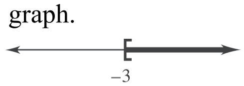
11. $|6-4| = 2,\ -|-2| = -2,\ |8+1| = |9| = 9,$
$-|3-(-2)| = -|3+2| = -|5| = -5$
Since –5, –2, 2, 9 are in order, then
$-|3-(-2)|, -|-2|, |6-4|, |8+1|$ are in order.

12. $-\left|\sqrt{16}\right| = -4,\ -\sqrt{8},\ \sqrt{7},\ \left|-\sqrt{12}\right| = \sqrt{12}$

13. $7 - |-8| = 7 - 8 = -1$

14. $|-3| - |-9+6| = 3 - |-3| = 3 - 3 = 0$

15. $x \geq -3$
Start at –3 and draw a ray to the right. Use a bracket at –3 to show that –3 is a part of the graph.



16. $-4 < x \leq 6$
Put a parenthesis at –4 and a bracket at 6. Draw a line segment between these two endpoints.

17. $\dfrac{-9+(-6)(-3)\div 9}{6-(-3)} = \dfrac{-9+18\div 9}{6+3} = \dfrac{-9+2}{9} = -\dfrac{7}{9}$

18. $\dfrac{20\div 4\cdot 2\div 5-1}{-9-(-3)-12\div 3} = \dfrac{5\cdot 2\div 5-1}{-9-(-3)-4}$
$= \dfrac{10\div 5-1}{-9+3-4} = \dfrac{2-1}{-6-4} = -\dfrac{1}{10}$

19. $\left(3x^4 - x^2 + 5x\right) - \left(-x^4 + 3x^2 - 6x\right)$
$= 3x^4 - x^2 + 5x + x^4 - 3x^2 + 6x$
$= \left(3x^4 + x^4\right) + \left(-x^2 - 3x^2\right) + (5x + 6x)$
$= 4x^4 - 4x^2 + 11x$

20. $\left(-8y^3 + 8y^2 - 5y\right) - \left(2y^3 + 4y^2 - 10\right)$
$= -8y^3 + 8y^2 - 5y - 2y^3 - 4y^2 + 10$
$= -8y^3 - 2y^3 + 8y^2 - 4y^2 - 5y + 10$
$= -10y^3 + 4y^2 - 5y + 10$

21. $(5k-2h)(5k+2h) = (5k)^2 - (2h)^2 = 25k^2 - 4h^2$

22. $(2r-5y)(2r+5y) = (2r)^2 - (5y)^2$
$= 4r^2 - 25y^2$

23. $(3x+4y)^2 = (3x)^2 + 2(3x)(4y) + (4y)^2$
$= 9x^2 + 24xy + 16y^2$

24. $(2a-5b)^2 = (2a)^2 - 2(2a)(5b) + (5b)^2$
$= 4a^2 - 20ab + 25b^2$

25. $2kh^2 - 4kh + 5k = k\left(2h^2 - 4h + 5\right)$

26. $2m^2n^2 + 6mn^2 + 16n^2 = 2n^2\left(m^2 + 3m + 8\right)$

27. $5a^4 + 12a^3 + 4a^2 = a^2\left(5a^2 + 12a + 4\right)$
$= a^2(5a+2)(a+2)$

28. $24x^3 + 4x^2 - 4x = 4x\left(6x^2 + x - 1\right)$
$= 4x(3x-1)(2x+1)$

29. $$144p^2 - 169q^2 = (12p)^2 - (13q)^2 = (12p - 13q)(12p + 13q)$$

30. $$81z^2 - 25x^2 = (9z)^2 - (5x)^2 = (9z + 5x)(9z - 5x)$$

31. $$27y^3 - 1 = (3y)^3 - 1^3 = (3y - 1)\left[(3y)^2 + (3y)(1) + 1^2\right] = (3y - 1)\left(9y^2 + 3y + 1\right)$$

32. $$125a^3 + 216 = (5a)^3 + (6)^3 = (5a + 6)\left[(5a)^2 - 5a(6) + 6^2\right] = (5a + 6)\left(25a^2 - 30a + 36\right)$$

33. $$\frac{3x}{5} \cdot \frac{45x}{12} = \frac{3x \cdot 45x}{5 \cdot 12} = \frac{3 \cdot 5 \cdot 3 \cdot 3x^2}{4 \cdot 5 \cdot 3} = \frac{9x^2}{4}$$

34. $$\frac{5k^2}{24} - \frac{70k}{36} = \frac{5k^2 \cdot 3}{24 \cdot 3} - \frac{70k \cdot 2}{36 \cdot 2} = \frac{15k^2}{72} - \frac{140k}{72} = \frac{15k^2 - 140k}{72} = \frac{5k(3k - 28)}{72}$$

35. $$\frac{c^2 - 3c + 2}{2c(c - 1)} \div \frac{c - 2}{8c} = \frac{(c - 1)(c - 2)}{2c(c - 1)} \cdot \frac{8c}{(c - 2)} = \frac{8c(c - 1)(c - 2)}{2c(c - 1)(c - 2)} = \frac{8}{2} = 4$$

36. $$\frac{p^3 - 2p^2 - 8p}{3p\left(p^2 - 16\right)} \div \frac{p^2 + 4p + 4}{9p^2}$$
$$= \frac{p\left(p^2 - 2p - 8\right)}{3p(p + 4)(p - 4)} \cdot \frac{9p^2}{(p + 2)(p + 2)}$$
$$= \frac{p(p - 4)(p + 2) \cdot 9p^2}{3p(p + 4)(p - 4)(p + 2)(p + 2)}$$
$$= \frac{3p(p - 4)(p + 2) \cdot 3p^2}{3p(p - 4)(p + 2) \cdot (p + 4)(p + 2)}$$
$$= \frac{3p^2}{(p + 4)(p + 2)}$$

37. $$\frac{2m^2 - 4m + 2}{m^2 - 1} \div \frac{6m + 18}{m^2 + 2m - 3}$$
$$= \frac{2\left(m^2 - 2m + 1\right)}{(m + 1)(m - 1)} \cdot \frac{m^2 + 2m - 3}{6m + 18}$$
$$= \frac{2(m - 1)^2}{(m + 1)(m - 1)} \cdot \frac{(m + 3)(m - 1)}{6(m + 3)}$$
$$= \frac{2(m - 1)(m + 3) \cdot (m - 1)^2}{2(m - 1)(m + 3) \cdot 3(m + 1)} = \frac{(m - 1)^2}{3(m + 1)}$$

38. $$\frac{x^2 + 6x + 5}{4\left(x^2 + 1\right)} \cdot \frac{2x(x + 1)}{x^2 - 25}$$
$$= \frac{(x + 5)(x + 1) \cdot 2x(x + 1)}{4\left(x^2 + 1\right) \cdot (x + 5)(x - 5)}$$
$$= \frac{2(x + 5) \cdot x(x + 1)^2}{2(x + 5) \cdot 2\left(x^2 + 1\right)(x - 5)}$$
$$= \frac{x(x + 1)^2}{2\left(x^2 + 1\right)(x - 5)}$$

39. $$5^{-3} = \frac{1}{5^3} \text{ or } \frac{1}{125}$$

40. $$10^{-2} = \frac{1}{10^2} \text{ or } \frac{1}{100}$$

41. $$-8^0 = -\left(8^0\right) = -1$$

42. $$\left(-\frac{5}{6}\right)^{-2} = \left(-\frac{6}{5}\right)^2 = \frac{36}{25}$$

43. $$4^6 \cdot 4^{-3} = 4^{6+(-3)} = 4^3$$

44. $$7^{-5} \cdot 7^{-2} = 7^{-5+(-2)} = 7^{-7} = \frac{1}{7^7}$$

45. $$\frac{8^{-5}}{8^{-4}} = 8^{-5-(-4)} = 8^{-5+4} = 8^{-1} = \frac{1}{8}$$

46. $$\frac{6^{-3}}{6^4} = 6^{-3-4} = 6^{-7} = \frac{1}{6^7}$$

47. $$5^{-1} + 2^{-1} = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$$

48. $5^{-2}+5^{-1}=\frac{1}{5^2}+\frac{1}{5}=\frac{1}{25}+\frac{1}{5}=\frac{6}{25}$

49. $\frac{5^{1/3}5^{1/2}}{5^{3/2}}=5^{1/3+1/2-3/2}=5^{-2/3}=\frac{1}{5^{2/3}}$

50. $\frac{2^{3/4}\cdot 2^{-1/2}}{2^{1/4}}=\frac{2^{1/4}}{2^{1/4}}=1$

51. $\left(3a^2\right)^{1/2}\cdot\left(3^2a\right)^{3/2}=3^{1/2}a\cdot 3^3a^{3/2}=3^{7/2}a^{5/2}$

52.
$$\begin{aligned}(4p)^{2/3}\cdot\left(2p^3\right)^{3/2}&=4^{2/3}p^{2/3}\cdot 2^{3/2}\cdot p^{9/2}\\&=\left(2^2\right)^{2/3}p^{2/3}\cdot 2^{3/2}p^{9/2}\\&=2^{4/3}\cdot 2^{3/2}p^{2/3}p^{9/2}\\&=2^{17/6}p^{31/6}\end{aligned}$$

53. $\sqrt[3]{27}=3$

54. $\sqrt[6]{-64}$ is not a real number.

55.
$$\begin{aligned}\sqrt[3]{54p^3q^5}&=\sqrt[3]{27\cdot 2p^3q^3q^2}\\&=\sqrt[3]{27p^3q^3}\cdot\sqrt[3]{2q^2}=3pq\sqrt[3]{2q^2}\end{aligned}$$

56. $\sqrt[4]{64a^5b^3}=\sqrt[4]{16a^4}\cdot\sqrt[4]{4ab^3}=2a\sqrt[4]{4ab^3}$

57.
$$\begin{aligned}3\sqrt{3}-12\sqrt{12}&=3\sqrt{3}-12\sqrt{4\cdot 3}=3\sqrt{3}-12\cdot 2\sqrt{3}\\&=3\sqrt{3}-24\sqrt{3}=-21\sqrt{3}\end{aligned}$$

58.
$$\begin{aligned}8\sqrt{7}+2\sqrt{63}&=8\sqrt{7}+2\sqrt{9\cdot 7}\\&=8\sqrt{7}+6\sqrt{7}=14\sqrt{7}\end{aligned}$$

59.
$$\begin{aligned}\frac{\sqrt{3}}{1+\sqrt{2}}&=\frac{\sqrt{3}\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)}=\frac{\sqrt{3}-\sqrt{6}}{1-2}\\&=\frac{\sqrt{3}-\sqrt{6}}{-1}=\sqrt{6}-\sqrt{3}\end{aligned}$$

60.
$$\begin{aligned}\frac{4+\sqrt{2}}{4-\sqrt{5}}&=\frac{\left(4+\sqrt{2}\right)}{\left(4-\sqrt{5}\right)}\cdot\frac{\left(4+\sqrt{5}\right)}{\left(4+\sqrt{5}\right)}\\&=\frac{16+4\sqrt{2}+4\sqrt{5}+\sqrt{10}}{16-\left(\sqrt{5}\right)^2}\\&=\frac{16+4\sqrt{2}+4\sqrt{5}+\sqrt{10}}{11}\end{aligned}$$

61.
$$\begin{aligned}3x-4(x-2)&=2x+9\\3x-4x+8&=2x+9\\-x+8&=2x+9\\-1&=3x\Rightarrow x=-\frac{1}{3}\end{aligned}$$

62.
$$\begin{aligned}4y+9&=-3(1-2y)+5\\4y+9&=-3+6y+5\\4y+9&=2+6y\\-2y&=-7\Rightarrow y=\frac{7}{2}\end{aligned}$$

63.
$$\begin{aligned}\frac{2m}{m-3}&=\frac{6}{m-3}+4\\2m&=6+4(m-3)\\2m&=6+4m-12\\2m&=4m-6\\6&=2m\Rightarrow 3=m\end{aligned}$$

Because $m = 3$ would make the denominators of the fractions equal to 0, making the fractions undefined, the given equation has no solution.

64. $\frac{15}{k+5}=4-\frac{3k}{k+5}$

Multiply both sides of the equation by the common denominator $k + 5$.

$15=4(k+5)-3k$

$15=4k+20-3k\Rightarrow k=-5$

If $k = -5$, the fractions would be undefined, so the given equation has no solution.

65.
$$\begin{aligned}8ax-3&=2x\\8ax-2x&=3\\x(8a-2)&=3\\x&=\frac{3}{8a-2}\end{aligned}$$

66.
$$\begin{aligned}b^2x-2x&=4b^2\\\left(b^2-2\right)x&=4b^2\\x&=\frac{4b^2}{b^2-2}\end{aligned}$$

67. $\left|\frac{2-y}{5}\right| = 8$

$\frac{2-y}{5} = 8$ or $\frac{2-y}{5} = -8$

$5\left(\frac{2-y}{5}\right) = 5(8)$ or $5\left(\frac{2-y}{5}\right) = 5(-8)$

$2-y = 40$ or $2-y = -40$

$-y = 38$ or $-y = -42$

$y = -38$ or $y = 42$

The solutions are –38 and 42.

68. $|4k+1| = |6k-3|$

$4k+1 = 6k-3$ or $4k+1 = -(6k-3)$

$4k+1 = 6k-3$ or $4k+1 = -6k+3$

$-2k = -4$ or $10k = 2$

$k = 2$ or $k = \frac{1}{5}$

The solutions are 2 and $\frac{1}{5}$.

69. $(b+7)^2 = 5$

Use the square root property to solve this quadratic equation.

$b+7 = \sqrt{5}$ or $b+7 = -\sqrt{5}$

$b = -7+\sqrt{5}$ or $b = -7-\sqrt{5}$

The solutions are $-7+\sqrt{5}$ and $-7-\sqrt{5}$, which we abbreviate as $-7 \pm \sqrt{5}$.

70. $(2p+1)^2 = 7$

Solve by the square root property.

$2p+1 = \sqrt{7}$ or $2p+1 = -\sqrt{7}$

$2p = -1+\sqrt{7}$ or $2p = -1-\sqrt{7}$

$p = \frac{-1+\sqrt{7}}{2}$ or $p = \frac{-1-\sqrt{7}}{2}$

The solutions are $\frac{-1 \pm \sqrt{7}}{2}$.

71. $2p^2 + 3p = 2$

Write the equation in standard form and solve by factoring.

$2p^2 + 3p - 2 = 0$

$(2p-1)(p+2) = 0$

$2p-1 = 0$ or $p+2 = 0$

$p = \frac{1}{2}$ or $p = -2$

The solutions are $\frac{1}{2}$ and –2.

72. $2y^2 = 15 + y$

Write the equation in standard form and solve by factoring.

$2y^2 - y - 15 = 0$

$(y-3)(2y+5) = 0$

$y = 3$ or $y = -\frac{5}{2}$

The solutions are 3 and $-\frac{5}{2}$.

73. $2q^2 - 11q = 21 \Rightarrow 2q^2 - 11q - 21 = 0 \Rightarrow$

$(2q+3)(q-7) = 0$

$2q+3 = 0$ or $q-7 = 0$

$q = -\frac{3}{2}$ or $q = 7$

The solutions are $-\frac{3}{2}$ and 7.

74. $3x^2 + 2x = 16 \Rightarrow 3x^2 + 2x - 16 = 0 \Rightarrow$

$(3x+8)(x-2) = 0$

$3x+8 = 0$ or $x-2 = 0$

$x = -\frac{8}{3}$ or $x = 2$

The solutions are $-\frac{8}{3}$ and 2.

75. $6k^4 + k^2 = 1 \Rightarrow 6k^4 + k^2 - 1 = 0$

Let $p = k^2$, so $p^2 = k^4$.

$6p^2 + p - 1 = 0$

$(3p-1)(2p+1) = 0$

$3p-1 = 0$ or $2p+1 = 0$

$p = \frac{1}{3}$ or $p = -\frac{1}{2}$

If $p = \frac{1}{3}$, $k^2 = \frac{1}{3} \Rightarrow k = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

If $p = -\frac{1}{2}$, $k^2 = -\frac{1}{2}$ has no real number solution.

The solutions are $\pm\frac{\sqrt{3}}{3}$.

76. $21p^4 = 2 + p^2 \Rightarrow 21p^4 - p^2 - 2 = 0$

Let $u = p^2$; then $u^2 = p^4$.

$$21u^2 - u - 2 = 0$$
$$(3u-1)(7u+2) = 0$$
$$3u - 1 = 0 \quad \text{or} \quad 7u + 2 = 0$$
$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{2}{7}$$
$$p^2 = \frac{1}{3} \quad \text{or} \quad p^2 = -\frac{2}{7}$$

If $x = -\frac{2}{7}$, $p^2 = -\frac{2}{7}$ has no real number solution.

$$p = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$$

The solutions are $\pm\frac{\sqrt{3}}{3}$.

77. $p = \frac{E^2R}{(r+R)^2}$ for r.

$$p(r+R)^2 = E^2R$$
$$p\left(r^2 + 2rR + R^2\right) = E^2R$$
$$pr^2 + 2rpR + R^2p = E^2R$$
$$pr^2 + 2rpR + R^2p - E^2R = 0$$

Use the quadratic formula to solve for r.

$$r = \frac{-2pR \pm \sqrt{4p^2R^2 - 4p\left(R^2p - E^2R\right)}}{2p}$$
$$r = \frac{-2pR \pm \sqrt{4pE^2R}}{2p} = \frac{-pR \pm E\sqrt{pR}}{p}$$

78. $p = \frac{E^2R}{(r+R)^2}$ for E.

$$p(r+R)^2 = E^2R \Rightarrow E^2 = \frac{p(r+R)^2}{R} \Rightarrow$$
$$E = \pm\sqrt{\frac{p(r+R)^2}{R}} = \frac{\pm(r+R)\sqrt{pR}}{R}$$

79. $K = s(s-a)$ for s.

$K = s^2 - as \Rightarrow s^2 - as - K = 0$

Use the quadratic formula.

$$s = \frac{a \pm \sqrt{a^2 - 4(-K)}}{2} = \frac{a \pm \sqrt{a^2 + 4K}}{2}$$

80. $kz^2 - hz - t = 0$ for z.

Use the quadratic formula with $a = k$, $b = -h$, and $c = -t$.

$$z = \frac{-(-h) \pm \sqrt{(-h)^2 - 4(k)(-t)}}{2k}$$
$$= \frac{h \pm \sqrt{h^2 + 4kt}}{2k}$$

81. $|27 - (-55)| = |82| = 82\%$

82. $|-22.5 - (-2.5)| = |-20| = 20\%$

83. Let x = the original price

$$x - .2x = 1229$$
$$.8x = 1229$$
$$x = 1536.25$$

The original price of the laptop was \$1536.25.

84. Let x = the original price

$$x + .273x = 29.83$$
$$1.273x = 29.83$$
$$x = 23.43$$

The original price of the stock was \$23.43.

85. Let $O = 3.63$

$$3.63 = .11x + 2.2$$
$$1.43 = .11x$$
$$13 = x$$

The amount of outlays reached \$3.63 trillion in the year 2013.

86. Let $O = 3.96$

$$3.96 = .11x + 2.2$$
$$1.76 = .11x$$
$$16 = x$$

The amount of outlays reached \$3.96 trillion in the year 2016.

87. Let $O = 10.8$

$$10.8 = .2x + 8$$
$$2.8 = .2x$$
$$14 = x$$

The crude oil output reached 10.8 million barrels per day in the year 2014.

88. Let $O = 11.4$

$$11.4 = .2x + 8$$
$$3.4 = .2x$$
$$17 = x$$

The crude oil output reached 11.4 million barrels per day in the year 2017.

89. Let $x = 7$

$$V = .04x^2 - .47x + 18.38$$
$$V = .04(7)^2 - .47(7) + 18.38$$
$$V = 17.05$$

The number of vehicles sold in July 2016 was 17.05 million.

90. Let $x = 10$

$$V = .04x^2 - .47x + 18.38$$
$$V = .04(10)^2 - .47(10) + 18.38$$
$$V = 17.68$$

The number of vehicles sold in October 2016 was 17.68 million.

91. Let $V = 17.5$.

$$V = .04x^2 - .47x + 18.38$$
$$17.5 = .04x^2 - .47x + 18.38$$
$$0 = .04x^2 - .47x + .88$$
$$x = \frac{.47 \pm \sqrt{(-.47)^2 - 4(.04)(.88)}}{2(.04)}$$
$$\approx 2.33 \text{ or } 9.41$$

The first full month in which sales were below 17.5 million vehicles was March 2016.

92. Using the solution to 91, the second month of the year in which sales was 17.5 million was October 2016.

93. Let $x = 13$

$$F = \frac{.061(13)^2 + 6.09(13) + 38.8}{13 + 1} \approx 9.16$$

The total number of flights that departed in 2013 was approximately 9.16 million.

94. Let $x = 20$

$$F = \frac{.061(20)^2 + 6.09(20) + 38.8}{20 + 1} \approx 8.81$$

The total number of flights that will depart in 2020 will be approximately 8.81 million.

95. Let $F = 9.2$.

$$F = \frac{.061x^2 + 6.09x + 38.8}{x + 1}$$
$$9.2 = \frac{.061x^2 + 6.09x + 38.8}{x + 1}$$
$$9.2(x + 1) = .061x^2 + 6.09x + 38.8$$
$$0 = .061x^2 - 3.11x + 29.6$$
$$x = \frac{3.11 \pm \sqrt{(-3.11)^2 - 4(.061)(29.6)}}{2(.061)}$$
$$\approx 12.67 \text{ or } 38.32$$

The number of flight departures fell below 9.2 million in 2013.

96. Let $F = 8.9$.

$$F = \frac{.061x^2 + 6.09x + 38.8}{x + 1}$$
$$8.9 = \frac{.061x^2 + 6.09x + 38.8}{x + 1}$$
$$8.9(x + 1) = .061x^2 + 6.09x + 38.8$$
$$0 = .061x^2 - 2.81x + 29.9$$
$$x = \frac{2.81 \pm \sqrt{(-2.81)^2 - 4(.061)(29.9)}}{2(.061)}$$
$$\approx 16.70 \text{ or } 29.38$$

The number of flight departures fell below 8.9 million in 2017.

97. Let $x = 12$

$$R = 16.2(12)^{.26} \approx 30.91$$

The total amount spent on basic research by the U.S. government in 2012 was approximately $30.91 billion.

98. Let $x = 17$

$$R = 16.2(17)^{.26} \approx 33.84$$

The total amount spent on basic research by the U.S. government in 2017 was approximately $33.84 billion.

99. Let x = the single interest rate

$$2000(.12) + 500(.07) = 2500x$$
$$275 = 2500x$$
$$.11 = x$$

Therefore, a single interest rate of 11% will yield the same results.

100. Let x = the amount of beef. Then $30 - x$ = the amount of pork.

$2.8x + 3.25(30 - x) = 3.10(30)$
$2.8x + 97.5 - 3.25x = 93$
$-.45x = -4.5$
$x = 10$

Therefore the butcher should use 10 pounds of beef and 30 – 10 = 20 pounds of pork.

101. Let x = 112

$D = .047x^2 - 7.69x + 341.6$
$D = .047(112)^2 - 7.69(112) + 341.6$
$D = 69.9$

The ratio of federal debt to GDP in 2012 was approximately 69.9%.

102. Let D = 55%.

$D = .047x^2 - 7.69x + 341.6$
$55 = .047x^2 - 7.69x + 341.6$
$0 = .047x^2 - 7.69x + 286.6$

$$x = \frac{7.69 \pm \sqrt{(-7.69)^2 - 4(.047)(286.6)}}{2(.047)}$$
$$\approx 57.42 \text{ or } 106.20$$

The most recent time the ratio of federal debt to GDP was 55% was in 2006.

103. Let x = the width of the walk.

$$\text{The area } = (10 + 2x)(15 + 2x) - 10(15)$$
$$= 150 + 20x + 30x + 4x^2 - 150$$
$$= 4x^2 + 50x$$

To use all of the cement, solve

$200 = 4x^2 + 50x$
$0 = 4x^2 + 50x - 200$

Using the quadratic formula, we have

$$x = \frac{-50 \pm \sqrt{(50)^2 - 4(4)(-200)}}{2(4)}$$
$$= \frac{-50 \pm \sqrt{5700}}{2(4)} \approx -15.6875 \text{ or } 3.1875$$

The width cannot be negative, so the solution is approximately 3.2 feet.

104. Let x = the length of the yard. Then $160 - x$ = the width of the yard. Area is equal to length times width.

The area $= lw$

$4000 = (x)(160 - x)$
$0 = x^2 - 160x + 4000$

Using the quadratic formula, we have

$$x = \frac{160 \pm \sqrt{(-160)^2 - 4(1)(4000)}}{2(1)}$$
$$= \frac{160 \pm \sqrt{9600}}{2(1)} \approx 31.01 \text{ or } 128.99$$

The length is longer than the width, so the length is approximately 129 feet and the width is approximately 160 – 129 = 31 feet.

105. $v_0 = 150,\ h_0 = 0,\ h = 200$

$200 = -16t^2 + 150t + 0 \Rightarrow$
$16t^2 - 150t + 200 = 0 \Rightarrow$
$8t^2 - 75t + 100 = 0 \Rightarrow t \approx 1.61 \text{ or } 7.77$

The ball will reach 200 feet on its downward trip after approximately 7.77 seconds.

106. $v_0 = 55,\ h_0 = 700,\ h = 0$

$0 = -16t^2 - 55t + 700 \Rightarrow$
$16t^2 + 55t - 700 = 0 \Rightarrow t \approx -8.55 \text{ or } 5.12$

The ball will reach the ground after approximately 5.12 seconds

Case 1 Consumers Often Defy Common Sense

1. The total cost to buy and run this electric hot water tank for x years is $E = 218 + 508x$.

2. The total cost to buy and run this gas hot water tank for x years is $G = 328 + 309x$.

3. Over 10 years, the electric hot water tank costs 218 + 508(10) = 5298 or $5298, and the gas hot water tank costs 328 + 309(10) = 3418 or $3418. The gas hot water tank costs $1880 less over 10 years.

4. The total costs for the two hot water tanks will be equal when $218 + 508x = 328 + 309x \Rightarrow$ $199x = 110 \Rightarrow x \approx 0.55$.
   The costs will be equal within the first year.

5. The total cost to buy and run this Maytag refrigerator for x years is $M = 1529.10 + 50x$.

6. The total cost to buy and run this LG refrigerator for x years is $L = 1618.20 + 44x$.

7. Over 10 years, the Maytag refrigerator costs 1529.10 + 50(10) = 2029.10 or $2029.10, and the LG refrigerator costs 1618.20 + 44(10) = 2058.20 or $2058.20. The LG refrigerator costs $29.10 more over 10 years.