CHAPTER 2

Fluid Statics

FE-type Exam Review Problems: Problems 2.1 to 2.9

2.1 (**C**) $p = \gamma_{Hg} h = (13.6 \times 9810) \times (28.5 \times 0.0254) = 96\ 600 \text{ Pa}$

2.2 (**D**) $p = p_0 - \rho g h = 84\ 000 - 1.00 \times 9.81 \times 4000 = 44\ 760 \text{ Pa}$

2.3 (**C**) $p_w = p_{atm} + \gamma_x h_x - \gamma_{water} h_w = 0 + 30\ 000 \times 0.3 - 9810 \times 0.1 = 8020 \text{ Pa}$

2.4 (**A**) $p_a = -\gamma H = -(13.6 \times 9810) \times 0.16 = -21\,350 \text{ Pa}$

$p_{a,after} = -21\,350 + 10\ 000 = -11\,350 = 13.6 \times 9810 H_{after}$. $\therefore H_{after} = 0.0851 \text{ m}$

2.5 (**B**) The force acts 1/3 the distance from the hinge to the water line:

$$(2 \times \frac{5}{3}) \times P = \frac{1}{3} \times (2 \times \frac{5}{3}) \times [9800 \times 1 \times 3 \times (2 \times \frac{5}{3})]. \quad \therefore P = 32\ 670 \text{ N}$$

2.6 (**A**) The gate opens when the center of pressure in at the hinge:

$$\bar{y} = \frac{1.2 + h}{2} + 5. \quad y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = \frac{11.2 + h}{2} + \frac{b(1.2 + h)^3 / 12}{(1.2 + h) b (11.2 + h) / 2} = 5 + 1.2.$$

This can be solved by trial-and –error, or we can simply substitute one of the answers into the equation and check to see if it is correct. This yields $h = 1.08$ m.

2.7 (**D**) Place the force $\vec{F}_H + \vec{F}_V$ at the center of the circular arc. F_H passes through the hinge:

$$\therefore P = F_V = 4 \times 1.2 w \times 9800 + (\pi \times 1.2^2 / 4) w \times 9800 = 300\ 000. \quad \therefore w = 5.16 \text{ m.}$$

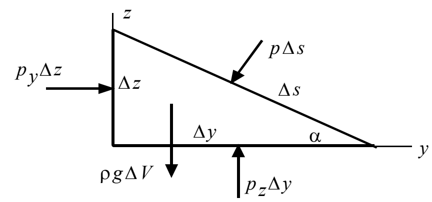
2.8 (**A**) $W = \gamma \forall$

$900 \times 9.81 = 9810 \times 0.01 \times 15 w. \quad \therefore w = 6 \text{ m}$

2.9 (**A**) $p_{plug} = 20\ 000 + \gamma h = 20\ 000 + 6660 \times (1.2 \times \frac{5}{9.81}) = 24\ 070 \text{ Pa}$

$F_{plug} = p_{plug} A = 24\ 070 \times \pi \times 0.02^2 = 30.25 \text{ N}$.

Chapter 2 Problems: Pressure



2.10 $\Sigma F_y = ma_y$: $p_y \Delta z - p\Delta s \sin\alpha = \rho \dfrac{\Delta y \Delta z}{2} a_y$

$\Sigma F_z = ma_z$: $p_z \Delta y - p\Delta s \cos\alpha = \rho \dfrac{\Delta y \Delta z}{2} a_z + \rho g \dfrac{\Delta y \Delta z}{2}$

Since $\Delta s \cos\alpha = \Delta y$ and $\Delta s \sin\alpha = \Delta z$, we have

$$p_y - p = \rho \frac{\Delta y}{2} a_y \quad \text{and} \quad p_z - p = \rho \frac{\Delta z}{2}(a_z + g)$$

Let $\Delta y \to 0$ and $\Delta z \to 0$: then $\left. \begin{array}{l} p_y - p = 0 \\ p_z - p = 0 \end{array} \right\}$ $\quad \therefore p_y = p_z = p.$

2.11 $p = \gamma h$.
a) 9810 × 10 = 98 100 Pa or <u>98.1 kPa</u>
b) (0.8 × 9810) × 10 = 78 480 Pa or <u>78.5 kPa</u>
c) (13.6 × 9810) × 10 = 1 334 000 Pa or <u>1334 kPa</u>
d) (1.59 × 9810) × 10 = 155 980 Pa or <u>156.0 kPa</u>
e) (0.68 × 9810) × 10 = 66 710 Pa or <u>66.7 kPa</u>

2.12 $h = p/\gamma$.
a) h = 250 000/9810 = <u>25.5 m</u>
b) h = 250 000/(0.8 × 9810) = <u>31.9 m</u>
c) h = 250 000/(13.6 × 9810) = <u>1.874 m</u>
d) h = 250 000/(1.59 × 9810) = <u>16.0 m</u>
e) h = 250 000/(0.68 × 9810) = <u>37.5 m</u>

2.13 $S = \dfrac{p}{\gamma h} = \dfrac{20 \times 144}{62.4 \times 20} = \underline{2.31}$. $\quad \rho = 1.94 \times 2.31 = \underline{4.48\ \text{slug/ft}^3}$.

2.14 a) $p = \gamma h = 0.76 \times (13.6 \times 9810) = 9810\, h$. $\quad \therefore h = \underline{10.34\ \text{m}}$.
b) $(13.6 \times 9810) \times 0.75 = 9810\, h$. $\quad \therefore h = \underline{10.2\ \text{m}}$.
c) $(13.6 \times 9810) \times 0.01 = 9810\, h$. $\quad \therefore h = 0.136$ m or <u>13.6 cm</u>.

2.15 a) $p = \gamma_1 h_1 + \gamma_2 h_2 = 9810 \times 0.2 + (13.6 \times 9810) \times 0.02 = 4630$ Pa or <u>4.63 kPa</u>.
b) 9810 × 0.052 + 15 630 × 0.026 = 916 Pa or <u>0.916 kPa</u>.
c) 9016 × 3 + 9810 × 2 + (13.6 × 9810) × 0.1 = 60 010 Pa or <u>60.0 kPa</u>.

2.16 $\Delta p = \rho g h = 0.0024 \times 32.2\,(-10{,}000) = -773$ psf or <u>–5.37 psi</u>.

2.17
$$\left.\begin{aligned}\Delta p_{outside} &= \rho_o g\Delta h = \frac{pg}{RT_o}\Delta h = \frac{100\times 9.81}{0.287\times 253}\times 3 = 13.51 \text{ Pa}\\ \Delta p_{\text{inside}} &= \rho_i g\Delta h = \frac{pg}{RT_i}\Delta h = \frac{100\times 9.81}{0.287\times 293}\times 3 = 11.67 \text{ Pa}\end{aligned}\right\} \quad \therefore \Delta p_{base} = \underline{1.84 \text{ Pa}}$$

If no wind is present this Δp_{base} would produce a small infiltration since the higher pressure outside would force outside air into the bottom region (through cracks).

2.18 $p = \rho g dh$ where $h = -z$. From the given information $S = 1.0 + h/100$ since $S(0) = 1$ and $S(10) = 1.1$. By definition $\rho = 1000\ S$, where $\rho_{water} = 1000$ kg/m^3. Then $dp = 1000\ (1 + h/100)\ gdh$. Integrate:

$$\int_0^p dp = \int_0^{10} 1000(1 + h/100)gdh$$

$$p = 1000\times 9.81(10 + \frac{10^2}{2\times 100}) = 103\ 000 \text{ Pa or } \underline{103 \text{ kPa}}$$

Note: we could have used an average S: $S_{\text{avg}} = 1.05$, so that $\rho_{avg} = 1050$ kg/m^3.

2.19 $$\nabla p = \frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}$$

$$= -\rho a_x\mathbf{i} - \rho \alpha_y\mathbf{j} - \rho\alpha_z\mathbf{k} - \rho g\mathbf{k} = -\rho\left(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}\right) - \rho g\mathbf{k} = -\rho\mathbf{a} - \rho\mathbf{g}$$

$$\underline{\therefore \nabla p = -\rho(\mathbf{a} + \mathbf{g})}$$

2.20 $p = p_{atm}[(T_0 - \alpha z)/T_0]^{g/\alpha R}$

$= 100\ [(288 - 0.0065 \times 300)/288]^{9.81/0.0065 \times 287} = \underline{96.49 \text{ kPa}}$

$$p = p_{atm} - \rho gh = 100 - \frac{100}{0.287\times 288}\times 9.81\times 300/1000 = \underline{96.44 \text{ kPa}}$$

$$\%\ \text{error} = \frac{96.44 - 96.49}{96.49}\times 100 = \underline{-0.052\%}$$

The density variation can be ignored over heights of 300 m or less.

2.21 $$\Delta p = p - p_0 = p_{atm}\left(\frac{T_0 - \alpha z}{T_0}\right)^{g/\alpha R} - p_{atm}$$

$$= 100\left[\left(\frac{288 - 0.0065\times 20}{288}\right)^{9.81/0.0065\times 287} - 1\right] = -0.237 \text{ Pa} \quad \text{or} \quad \underline{-0.000237 \text{ kPa}}$$

This change is very small and can most often be ignored.

2.22 Eq. 1.5.11 gives $310,000\times144=\rho\dfrac{dp}{d\rho}$. But, $dp = \rho g dh$. Therefore,

$$\rho g dh=\frac{4.464\times10^{7}}{\rho}d\rho \quad\text{or}\quad \frac{d\rho}{\rho^{2}}=\frac{32.2}{4.464\times10^{7}}dh$$

Integrate, using ρ_0 = 2.00 slug/ft^3:

$$\int_2^{\rho}\frac{d\rho}{\rho^2}=\frac{32.2}{4.464\times10^7}\int_0^h dh. \quad \therefore -\left(\frac{1}{\rho}-\frac{1}{2}\right)=7.21\times10^{-7}h \quad\text{or}\quad \rho=\frac{2}{1-14.42\times10^{-7}h}$$

Now,

$$p=\int_0^h \rho g dh=\int_0^h\frac{2g}{1-14.42\times10^{-7}h}dh=\frac{2g}{-14.42\times10^{-7}}\ln(1-14.42\times10^{-7}h)$$

Assume ρ = const:

$$p=\rho g h=2.0\times32.2\times h=64.4h$$

a) For h = 1500 ft: p_{accurate} = $\underline{96{,}700\text{ psf}}$ and p_{estimate} = $\underline{96{,}600\text{ psf}}$.

$$\%\text{ error }=\frac{96{,}600-96{,}700}{96{,}700}\times100=\underline{-0.103\ \%}$$

b) For h = 5000 ft: p_{accurate} = $\underline{323{,}200\text{ psf}}$ and p_{estimate} = $\underline{322{,}000\text{ psf.}}$

$$\%\text{ error }=\frac{322{,}000-323{,}200}{323{,}200}\times100=\underline{-0.371\ \%}$$

c) For h = 15,000 ft: p_{accurate} = $\underline{976{,}600\text{ psf}}$ and p_{estimate} = $\underline{966{,}000\text{ psf.}}$

$$\%\text{ error }=\frac{966{,}000-976{,}600}{976{,}600}\times100=\underline{-1.085\ \%}$$

2.23 Use the result of Example 2.2: $p = 101\,e^{-gz/RT}$.

a) $p = 101\,e^{-9.81\times10\,000/287\times273}$ = $\underline{28.9\text{ kPa}}$.

b) $p = 101\,e^{-9.81\times10\,000/287\times288}$ = $\underline{30.8\text{ kPa}}$.

c) $p = 101\,e^{-9.81\times10\,000/287\times258}$ = $\underline{26.9\text{ kPa}}$.

2.24 Use Eq. 2.4.8: $p=101(1-0.0065z/288)^{\frac{9.81}{0.0065\times287}}$.

a) z = 3000. $\therefore p$ = $\underline{69.9\text{ kPa}}$. b) z = 6000. $\therefore p$ = $\underline{47.0\text{ kPa}}$.

c) z = 9000. $\therefore p$ = $\underline{30.6\text{ kPa}}$. d) z = 11 000. $\therefore p$ = $\underline{22.5\text{ kPa}}$.

2.25 Use the result of Example 2.2: $\frac{p}{p_0} = e^{-gz/RT}$.

$$\ln\frac{p}{p_0} = -\frac{gz}{RT}. \qquad \ln\frac{0.001}{14.7} = -\frac{32.2z}{1716\times 455}. \qquad \therefore z = \underline{232{,}700 \text{ ft}}.$$

Manometers

2.26 $p = \gamma h = (13.6 \times 9810) \times 0.25 = 33\ 350$ Pa or $\underline{33.35 \text{ kPa}}$.

2.27 a) $p = \gamma h$. $450\ 000 = (13.6 \times 9810)\, h$. $\therefore h = \underline{3.373 \text{ m}}$

b) $p + 11.78 \times 1.5 = (13.6 \times 9810)\, h$. Use $p = 450\ 000$, then $h = \underline{3.373 \text{ m}}$

The % error is $\underline{0.000\ \%}$.

2.28 Referring to Fig. 2.6a, the pressure in the pipe is $p = \rho g h$. If $p = 2400$ Pa, then

$$2400 = \rho g h = \rho \times 9.81h \quad \text{or} \quad \rho = \frac{2400}{9.81h}.$$

a) $\rho = \frac{2400}{9.81\times 0.36} = 680 \text{ kg/m}^3$. $\therefore$ $\underline{\text{gasoline}}$

b) $\rho = \frac{2400}{9.81\times 0.272} = 899 \text{ kg/m}^3$. $\therefore$ $\underline{\text{benzene}}$

c) $\rho = \frac{2400}{9.81\times 0.245} = 999 \text{ kg/m}^3$. $\therefore$ $\underline{\text{water}}$

d) $\rho = \frac{2400}{9.81\times 0.154} = 1589 \text{ kg/m}^3$. $\therefore$ $\underline{\text{carbon tetrachloride}}$

2.29 Referring to Fig. 2.6a, the pressure is $p = \rho_w g h = \frac{1}{2}\rho_a V^2$. Then $V^2 = \frac{2\rho_w g h}{\rho_a}$.

a) $V^2 = \frac{2\times 1000\times 9.81\times 0.06}{1.23} = 957.$ $\therefore V = \underline{30.9 \text{ m/s}}$

b) $V^2 = \frac{2\times 1.94\times 32.2\times 3/12}{0.00238} = 13{,}124.$ $\therefore V = \underline{115 \text{ ft/sec}}$

c) $V^2 = \frac{2\times 1000\times 9.81\times 0.1}{1.23} = 1595.$ $\therefore V = \underline{39.9 \text{ m/s}}$

d) $V^2 = \frac{2\times 1.94\times 32.2\times 5/12}{0.00238} = 21{,}870.$ $\therefore V = \underline{148 \text{ ft/sec}}$

2.30 See Fig. 2.6b: $p_1 = -\gamma_1 h + \gamma_2 H.$

$$p_1 = -0.86 \times 62.4 \times \frac{5}{12} + 13.6 \times 62.4 \times \frac{9.5}{12} = 649.5 \text{ psf or } \underline{4.51 \text{ psi}}.$$

2.31 $p = p_0 + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 + \rho_4 g h_4$

$= 3200 + 917\times9.81\times0.2 + 1000\times9.81\times0.1 + 1258\times9.81\times0.15 + 1593\times9.81\times0.18$

$= 10\,640$ Pa or $\underline{10.64 \text{ kPa}}$

2.32 $p_1 - p_4 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$ (Use $\Delta p = \rho g \Delta h$)

$40\,000 - 16\,000 = 1000\times9.81(-0.2) + 13\,600\times9.81\times H + 920\times9.81\times0.3.$

$\therefore H = 0.1743$ m or $\underline{17.43 \text{ cm}}$

2.33 $p_1 - p_4 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$ (Use $\Delta p = \rho g \Delta h$)

$p_o - p_w = 900\times9.81(-0.2) + 13\,600\times9.81(-0.1) + 1000\times9.81\times0.15$

$= -12\,300$Pa or $\underline{-12.3 \text{ kPa}}$

2.34 $p_1 - p_5 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4) + (p_4 - p_5)$

$p_1 = 9810(-0.02) + 13\,600\times9.81(-0.04) + 9810(-0.02) + 13\,600\times9.81\times0.16$

$= 15\,620$ Pa or $\underline{15.62 \text{ kPa}}$

2.35 $p_w + 9810 \times 0.15 - 13.6 \times 9810 \times 0.1 - 0.68 \times 9810 \times 0.2 + 0.86 \times 9810 \times 0.15 = p_o.$

$\therefore p_w - p_o = 11\,940$ Pa or $\underline{11.94 \text{ kPa}}$.

2.36 $p_w - 9810 \times 0.12 - 0.68 \times 9810 \times 0.1 + 0.86 \times 9810 \times 0.1 = p_o.$

With $p_w = 15\,000$, $p_o = 14\,000$ Pa or $\underline{14.0 \text{ kPa}}$.

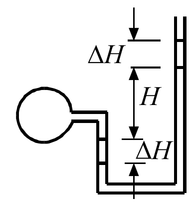
2.37 a) $p + 9810 \times 2 = 13.6 \times 9810 \times 0.1.$ $\therefore p = -6278$ Pa or $\underline{-6.28 \text{ kPa}}$.

b) $p + 9810 \times 0.8 = 13.6 \times 9810 \times 0.2.$ $\therefore p = 18\,835$ Pa or $\underline{18.84 \text{ kPa}}$.

c) $p + 62.4 \times 6 = 13.6 \times 62.4 \times 4/12.$ $\therefore p = -91.5$ psf or $\underline{-0.635 \text{ psi}}$.

d) $p + 62.4 \times 2 = 13.6 \times 62.4 \times 8/12.$ $\therefore p = 441$ psf or $\underline{3.06 \text{ psi}}$.

2.38 $p - 9810 \times 4 + 13.6 \times 9810 \times 0.16 = 0.$ $\therefore p = 17\,890$ Pa or $\underline{17.89 \text{ kPa}}$.



2.39 $8200 + 9810 \times 0.25 = 1.59 \times 9810 \times H.$ $\therefore H = 0.683$ m

$H_{new} = 0.683 + 0.273 = 0.956$ m. $\Delta H = \frac{0.273}{2} = 0.1365.$

$p + 9810\,(0.25 + 0.1365) = 1.59 \times 9810 \times 0.956.$

$\therefore p = 11\,120$ Pa or 11.12 kPa.

2.40 $p + 9810 \times 0.05 + 1.59 \times 9810 \times 0.07 - 0.8 \times 9810 \times 0.1 = 13.6 \times 9810 \times 0.05.$

$\therefore p = 5873$ Pa or 5.87 kPa.

Note: In our solutions we usually retain 3 significant digits in the answers (if a number starts with "1" then 4 digits are retained). In most problems a material property is used, i.e., $S = 1.59$. This is only 3 significant digits! $\therefore$ only 3 are usually retained in the answer!

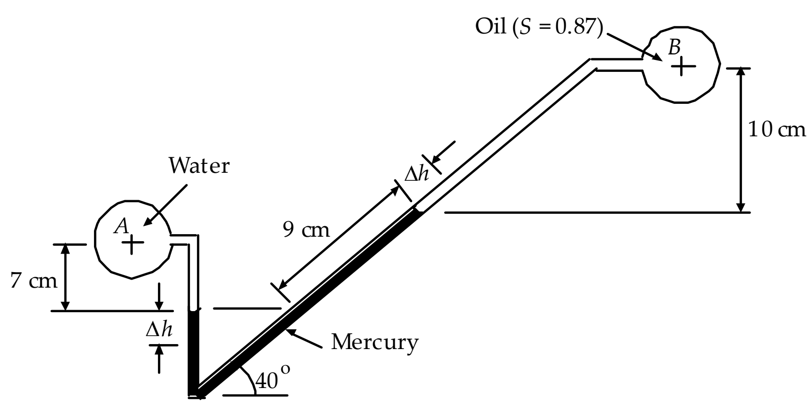
2.41 The equation for the manometer is

$$p_A + \gamma_{\text{water}} \times 0.07 = p_B + \gamma_{\text{oil}} \times 0.1 + \gamma_{\text{HG}} \times 0.09 \sin 40$$

Solve for p_B:

$$\begin{aligned} p_B &= p_A + \gamma_{\text{water}} \times 0.07 - \gamma_{\text{HG}} \times 0.09 \sin 40 - \gamma_{\text{oil}} \times 0.1 \\ &= p_A + \gamma_{\text{water}} \times 0.07 - 13.6\gamma_{\text{water}} \times 0.09 \sin 40 - 0.87\gamma_{\text{water}} \times 0.1 \\ &= p_A + (0.07 - 13.6 \times 0.09 \sin 40 - 0.87 \times 0.1) \times \gamma_{\text{water}} \\ &= 10 \text{ kPa} + (0.07 - 13.6 \times 0.09 \sin 40 - 0.87 \times 0.1) \times 9.81 \text{ kN/m}^3 = 2.11 \text{ kPa} \end{aligned}$$

2.42 The distance the mercury drops on the left equals the distance along the tube that the mercury rises on the right. This is shown in the sketch.



From the previous problem we have

$$(p_B)_1 = p_A + \gamma_{\text{water}} \times 0.07 - \gamma_{\text{HG}} \times 0.09 \sin 40 - \gamma_{\text{oil}} \times 0.1 \qquad (1)$$

For the new condition

$$(p_B)_2 = p_A + \gamma_{\text{water}} \times (0.07 + \Delta h) - \gamma_{\text{HG}} \times 0.11 \sin 40 - \gamma_{\text{oil}} \times (0.1 - \Delta h \sin 40) \qquad (2)$$

where Δh in this case is calculated from the new manometer reading as

$$\Delta h + \Delta h / \sin 40 = 11 - 9 \quad \Rightarrow \quad \Delta h = 0.783 \text{ cm}$$

Subtracting Eq.(1) from Eq.(2) yields

$$\left(p_B\right)_2-\left(p_B\right)_1=\gamma_{water}\times\left(\Delta h\right)-\gamma_{HG}\times 0.02\sin 40-\gamma_{oil}\times\left(-\Delta h\sin 40\right)$$

Substituting the values of Δh and $\left(p_B\right)_1$ gives $\left(p_B\right)_2$

$$\left(p_B\right)_2=2.11+\left[\left(0.00783\right)-13.6\times 0.02\sin 40-0.87\times\left(-0.00783\sin 40\right)\right]\times 9.81$$
$$=\underline{0.52\text{ kPa}}$$

2.43 Before pressure is applied the air column on the right is 48" high. After pressure is applied, it is (4 – H/2) ft high. For an isothermal process $p_1 V_1 = p_2 V_2$ using absolute pressures. Thus,

$$14.7\times 144\times 4A=p_2(4-H/2)A \qquad \text{or} \qquad p_2=p_2=8467/(4-H/2)$$

From a pressure balance on the manometer (pressures in psf):

$$30\times 144+14.7\times 144=13.6\times 62.4\,H+\frac{8467}{4-H/2},$$

or $H^2 - 15.59\,H + 40.73 = 0.$ $\quad \therefore H = 12.27$ or $\underline{3.32\text{ ft}}$.

2.44 a) $p_1-p_5=\left(p_1-p_2\right)+\left(p_2-p_3\right)+\left(p_3-p_4\right)+\left(p_4-p_5\right)$

$$4000 = 9800(0.16–0.22) + 15\ 600(0.10–0.16) + 133\ 400H + 15\ 600(0.07–H).$$

$$\therefore H = 0.0376\text{ m or }\underline{3.76\text{ cm}}$$

b) $0.6\times 144 = 62.4(–2/12) + 99.5(–2/12) + 849H + 99.5(2.5/12 – H).$

$\therefore H = 0.1236$ ft or $\underline{1.483\text{ in}}$.

2.45 a) $$\frac{\Delta H}{\Delta p_1}=\frac{2D^2/d^2}{-\gamma_1+2\gamma_2+2(\gamma_3-\gamma_2)D^2/d^2}$$

$$=\frac{2(0.1/0.005)^2}{-9800+2\times 15\ 600+2(133\ 400-15\ 600)(0.1/0.005)^2}=8.487\times 10H^{-6}$$

$\therefore\ \Delta H = 8.487\times 10^{-6}\times 400 = 0.0034$ m or 3.4 mm

b) $$\Delta H=\frac{2(4/0.2)^2}{-62.4+2\times 99.5+2(849-99.5)(4/0.2)^2}0.06\times 144=0.01153\text{ ft} \quad\text{or}\quad \underline{0.138\text{ in}}.$$

2.46 $p_1-p_4=\left(p_1-p_2\right)+\left(p_2-p_3\right)+\left(p_3-p_4\right)$ (p_{oil} = 14.0 kPa from No. 2.30)

$15\ 500 – 14\ 000 = 9800(0.12 + \Delta z) + 680(0.1 – 2\Delta z) + 860(–0.1 – \Delta z).$

$\therefore \Delta z = 0.0451$ m or $\underline{4.51\text{ cm}}$

2.47 a) $p_{air} = –6250 + 625 = –5620$ Pa.

$–5620 + 9800(2 + \Delta z) – 13\ 600 \times 9.81(0.1 + 2\Delta z) = 0.$ $\quad \therefore \Delta z = 0.0025.$

$\therefore h = 0.1 + 2\Delta z = 0.15$ m or $\underline{15\text{ cm}}$

b) $p_{air} = 18\ 800 + 1880 = 20\ 680$ Pa.

$20\ 680 + 9800(0.8 + \Delta z) - 13\ 600 \times 9.81(0.2 + 2\Delta z) = 0. \quad \therefore \Delta z = 0.00715$ m

$\therefore h = 0.2 + 2\Delta z = 0.214$ or $\underline{21.4 \text{ cm}}$

c) $p_{air} = -91.5 + 9.15 = -82.4$ psf.

$-82.4 + 62.4(6 + \Delta z) - 13.6 \times 62.4(4/12 + 2\Delta z) = 0. \quad \therefore \Delta z = 0.00558$ ft.

$\therefore h = 4/12 + 2\ (0.00558) = 0.3445$ ft or $\underline{4.13 \text{ in.}}$

d) $p_{air} = 441 + 44.1 = 485$ psf

$485 + 62.4(2 + \Delta z) - 13.6 \times 62.4(8/12 + 2\Delta z) = 0. \quad \therefore \Delta z = 0.0267$ ft.

$\therefore h = 8/12 + 2\ (0.0267) = 0.7205$ ft or $\underline{8.65 \text{ in.}}$

Forces on Plane Areas

2.48 $F = \gamma \bar{h} A = 9810 \times 10 \times \pi \times 0.3^2/4 = \underline{6934 \text{ N}}$.

2.49 $\left(2\times\frac{5}{3}\right)\times P = \frac{1}{3}\times\left(2\times\frac{5}{3}\right)\times\left[9800\times1\times3\times\left(2\times\frac{5}{3}\right)\right]. \quad \therefore P = 32\ 670 \text{ N}$

a) $F = p_c A = 9800 \times 2 \times 4^2 = 313\ 600$ N or $\underline{313.6 \text{ kN}}$

b) $F = p_c A = 9800\times1\times(2\times4) + 9800\times\frac{2}{3}\times2 + 9800\times\frac{2}{3}\times1 = 98\ 000$ N or $\underline{98.0 \text{ kN}}$

c) $F = p_c A = 9800 \times 1 \times 2 \times 4 \times \sqrt{2} = 110\ 900$ N or $\underline{110.9 \text{ kN}}$

d) $F = p_c A = 9800 \times 1 \times 2 \times 4/0.866 = 90\ 500$ N or $\underline{90.5 \text{ kN}}$

2.50 For saturated ground, the force on the bottom tending to lift the vault is

$F = p_c A = 9800 \times 1.5 \times (2 \times 1) = 29\ 400$ N

The weight of the vault is approximately

$W = \rho g \forall_{walls} = 2400\times9.81\,[2(2\times1.5\times0.1) + 2(2\times1\times0.1) + 20(.8\times1.3\times0.1)] = 28\ 400$ N.

The vault will tend to rise out of the ground.

2.51 $F = p_c A = 6660 \times 2 \times \pi \times 2^2 = 167\ 400$ N or $\underline{167.4 \text{ kN}}$

Find γ in Table B.5 in the Appendix.

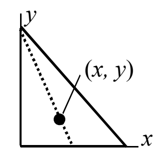
2.52 a) $F = p_c A = 9800\ (10 - 2.828/3)\ (2.828 \times 2/2) = 251\ 000$ N or $\underline{251 \text{ kN}}$

where the height of the triangle is $(3^2 - 1^2)^{1/2} = 2.828$ m.

b) $F = p_c A = 9800 \times 10\ (2.828 \times 2/2) = 277\ 100$ N or $\underline{277.1 \text{ kN}}$

c) $F = p_c A = 9800\ (10 - 2.828 \times 0.866/3)\ (2.828 \times 2/2) = 254\ 500$ N or $\underline{254.5 \text{ kN}}$

2.53 a) $F = \gamma \bar{h} A = 62.4 \times 27.33 \times 24 = \underline{40{,}930 \text{ lb}}$.



$$y_p = 27.33 + \frac{6 \times 8^3/36}{27.33 \times 24} = 27.46 \text{ ft.} \quad \therefore y = 30 - 27.46 = 2.54 \text{ ft.}$$

$8/5.46 = 3/x$. $\quad \therefore x = 2.05'$. $\quad \underline{(2.05, 2.54) \text{ ft}}$.

b) $F = 62.4 \times 30 \times 24 = \underline{44{,}930 \text{ lb}}$. The centroid is the center of pressure.

$$y = 2.667 \text{ ft.} \qquad \frac{8}{5.333} = \frac{3}{x}. \qquad \therefore x = 2.000 \text{ ft} \qquad \underline{(2.000, 2.667) \text{ ft}}.$$

c) $F = 62.4\,(30 - 2.667 \times 0.707) \times 24 = \underline{42{,}100 \text{ lb}}$.

$$y_p = 39.77 + \frac{6 \times 8^3 / 36}{39.77 \times 24} = 39.86 \text{ ft.} \qquad y = 42.43 - 39.86 = 2.57 \text{ ft}$$

$$\frac{8}{5.46} = \frac{3}{x}\, 8/5.43 = 3/x. \qquad \therefore x = 2.04 \text{ ft.} \qquad \underline{(2.04, 2.57) \text{ ft}}.$$

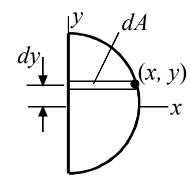
2.54 a) $F = \gamma \bar{h} A = 9810 \times 6 \times \pi 2^2 = 739\ 700 \text{ N}$ or $\underline{739.7 \text{ kN}}$.

$$y_p = \bar{y} + \bar{I}/A\bar{y} = 6 + \pi \times 2^4/4(4\pi \times 6) = 6.167 \text{ m.} \qquad \therefore (x, y)_p = \underline{(0, -0.167) \text{ m}}$$

b) $F = \gamma \bar{h} A = 9810 \times 6 \times 2\pi = 369\ 800 \text{ N}$ or $\underline{369.8 \text{ kN}}$.

$$y_p = 6 + \frac{\pi \times 2^4 / 8}{2\pi \times 6} = 6.167 \text{ m.} \ x^2 + y^2 = 4$$

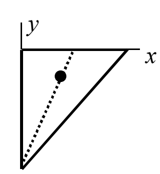
$$x_p F = \int \frac{x}{2} p\,dA = \frac{\gamma}{2} \int_{-2}^{2} x(6-y)x\,dy = \frac{\gamma}{2} \int_{-2}^{2} (4-y^2)(6-y)\,dy.$$



$$\therefore x_p \gamma 6 \times 2\pi = \frac{\gamma}{2} \int_{-2}^{2} (24 - 4y - 6y^2 + y^3)\,dy = 32\gamma. \ \therefore x_p = 0.8488 \text{ m}$$

$$\therefore (x, y)_p = \underline{(0.8488, -0.167) \text{ m}}$$

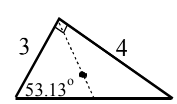
c) $F = 9810 \times (4 + 4/3) \times 6 = 313\ 900 \text{ N}$ or $\underline{313.9 \text{ kN}}$.



$$y_p = 5.333 + \frac{3 \times 4^3 / 36}{5.333 \times 6} = 5.500 \text{ m.} \qquad \therefore y = -1.5$$

$$4/2.5 = \frac{1.5}{x}. \quad \therefore x = 0.9375. \qquad \therefore (x, y)_p = \underline{(0.9375, -1.5) \text{ m}}$$

d) $F = 9810 \times (4 + \frac{2}{3} \times 4 \sin 36.9°) \times 6 = \underline{330\ 000 \text{ N}}$

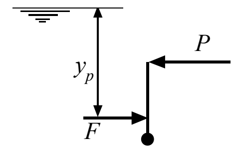


$$y_p = 5.6 + 5 \times 2.4^3/36(6 \times 5.6) = 5.657 \text{ m.} \ \therefore y = 0.343 \text{ m}$$

$3 \cos 53.13° = 1.8$, $\quad 2.5 - 1.8 = 0.7$, $\quad 2.4/2.057 = .7/x_1$. $\quad \therefore x_1 = 0.6$.

$x = 1.8 + 0.6 = 2.4$. $\qquad \therefore (x, y)_p = \underline{(2.4, 0.343) \text{ m}}$.

2.55 $F = \gamma \bar{h} A = 62.4 \times 11 \times (6 \times 10) = 41{,}180$ lb.



$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 11 + \frac{6 \times 10^3 / 12}{11 \times 60} = 11.758 \text{ ft.}$$

$(16 - 11.758)\ 41{,}180 = 10P. \quad \therefore P = \underline{17{,}470 \text{ lb}}.$

2.56 $F = \gamma \bar{h} A = 9810 \times 6 \times 20 = 1.777 \times 10^6$ N, or 1177 kN.

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 7.5 + \frac{4 \times 5^3 / 12}{7.5 \times 20} = 7.778 \text{ m.}$$

$(10 - 7.778)\ 1177 = 5\,P. \quad \therefore P = \underline{523 \text{ kN}}.$

2.57 a) $F = \gamma \bar{h} A = 9810 \times 8 \times 20 = 1.70 \times 10^6$ N, or 1700 kN.

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 10 + \frac{4 \times 5^3 / 12}{20 \times 10} = 10.21 \text{ m.}$$

$(12.5 - 10.21)\ 1700 = 5\,P. \quad \therefore P = \underline{779 \text{ kN}}.$

b) $F = \gamma \bar{h} A = 9810 \times 10 \times 20 = 1.962 \times 10^6$ N, or 1962 kN.

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 12.5 + \frac{4 \times 5^3 / 12}{20 \times 12.5} = 12.667 \text{ m.}$$

$(15 - 12.667)\ 1962 = 5\,P. \quad \therefore P = \underline{915 \text{ kN}}.$

c) $F = \gamma \bar{h} A = 9810 \times 12 \times 20 = 2.354 \times 10^6$ N, or 2354 kN.

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 15 + \frac{4 \times 5^3 / 12}{20 \times 15} = 15.14 \text{ m.}$$

$(17.5 - 15.14)\ 2354 = 5\,P. \therefore P = \underline{1112 \text{ kN}}.$

2.58 $y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = \frac{H}{2} + \frac{bH^3 / 12}{bH \times H / 2} = \frac{H}{2} + \frac{H}{6} = \frac{2}{3}H.$ y_p is measured from the surface.

$\therefore$ From the bottom, $H - y_p = H - \frac{2}{3}H = \frac{1}{3}H.$

Note: This result is independent of the angle α, so it is true for a vertical area or a sloped area.

2.59 $F = \gamma \frac{1}{2} l \sin 40^\circ \times 3l. \quad F \times \frac{l}{3} = (l + 2)P \sin 40^\circ. \quad \therefore \gamma l^3 = 2(l + 2)P.$

a) $9810 \times 2^3 = 2(2 + 2)P. \quad \therefore P = \underline{9810 \text{ N}}$

b) $9810 \times 4^3 = 2(4 + 2)P. \quad \therefore P = \underline{52\,300 \text{ N}}$

c) $9810 \times 5^3 = 2(5 + 2)P. \quad \therefore P = \underline{87\,600 \text{ N}}$

2.60 $h = \sqrt{1.2^2 - 0.4^2}$ = 1.1314 m. A = 1.2 × 1.1314 + 0.4 × 1.1314 = 1.8102 m^2

Use 2 forces: $F_1 = \gamma h_c A_1 = 9800 \times 0.5657 \times (1.2 \times 1.1314) = 7527$ N

$$F_2 = \gamma h_c A_2 = 9800 \times \frac{1.1314}{3} \times (0.4 \times 1.1314) = 1673 \text{ N}$$

$$y_{p1} = \frac{2}{3}(1.1314). \qquad y_{p2} = \bar{y} + \frac{\bar{I}_2}{A_2 \bar{y}} = \frac{1.1314}{3} + \frac{0.4 \times 1.1314^3 / 36}{0.4 \times (1.1314/2) \times (1.1314/3)} = 0.5657 \text{ m}$$

$\Sigma M_{hinge} = 0$: $7527 \times 1.1314/3 + 1673 \times (1.1314 - 0.5657) - 1.1314P = 0$. $\therefore$ $\underline{P = 3346 \text{ N}}$.

2.61 To open, the resultant force must be just above the hinge, i.e., y_p must be just less than h. Let $y_p = h$, the condition when the gate is about to open:

$$\bar{y} = (h+H)/3,\ A = (h+H)^2,\ \bar{I} = [2(h+H)](h+H)^3/36$$

$$\therefore y_p = \frac{h+H}{3} + \frac{2(h+H)^4/36}{(h+H)^2(h+H)/3} = \frac{h+H}{3} + \frac{h+H}{6} = \frac{h+H}{2}$$

a) $h = \dfrac{h+H}{2}$. $\quad\therefore h = H = \underline{0.9 \text{ m}}$

b) $h = H = \underline{1.2 \text{ m}}$

c) $h = H = \underline{1.5 \text{ m}}$

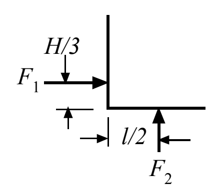
2.62 The gate is about to open when the center of pressure is at the hinge.

a) $y_p = 1.2 + H = (1.8/2 + H) + \dfrac{b \times 1.8^3/12}{(0.9+H)1.8b}$. $\quad\therefore H = \underline{0}$.

b) $y_p = 1.2 + H = (2.0/2 + H) + \dfrac{b \times 2^3/12}{(1+H)2b}$. $\quad\therefore H = \underline{0.6667 \text{ m}}$.

c) $y_p = 1.2 + H = (2.2/2 + H) + \dfrac{b \times 2.2^3/12}{(1.1+H)2.2b}$. $\quad\therefore H = \underline{2.933 \text{ m}}$.

2.63 $F_1 = \gamma \dfrac{H}{2} \times bH = \dfrac{1}{2}\gamma b H^2$

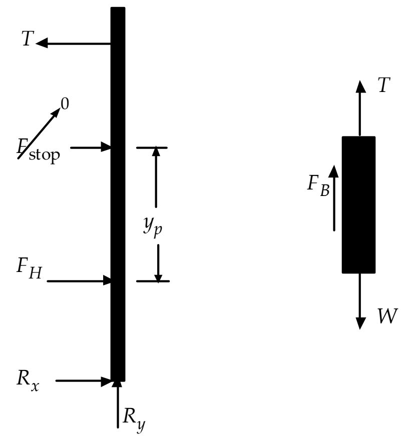


$F_2 = \gamma H \times \ell b = \gamma b \ell H$

$\dfrac{1}{2}\gamma b H^2 \times \dfrac{H}{3} = \gamma b \ell H \times \dfrac{\ell}{2}$. $\quad\therefore H = \sqrt{3}\ell$

a) $H = \sqrt{3} \times 2 = \underline{3.464 \text{ m}}$ b) $H = \underline{1.732}$ m c) $H = \underline{10.39'}$ d) $H = \underline{5.196'}$

2.64 A free-body-diagram of the gate and block is sketched.



Sum forces on the block:

$$\Sigma F_y = 0 \qquad \therefore W = T + F_B$$

where F_B is the buoyancy force which is given by

$$F_B = \gamma\left[\pi R^2(3-H)\right]$$

Take moments about the hinge:

$$T \times 3.5 = F_H \times (3 - y_p)$$

where F_H is the hydrostatic force acting on the gate. It is, using $\bar{h} = 1.5$ m and $A = 2\times 3 = 6\text{ m}^2$,

$$F_H = \gamma \bar{h} A = \left(9.81\text{ kN/m}^3\right)\left(1.5\text{ m}\times 6\text{ m}^2\right) = 88.29\text{ kN}$$

From the given information,

$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 1.5 + \frac{2\left(3^3\right)/12}{1.5\times 6} = 2\text{ m}$$

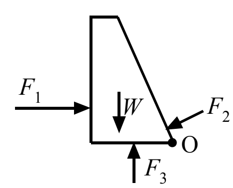
$$\therefore T = \frac{88.29\times(3-2)}{3.5} = 25.23\text{ kN}$$

$$F_B = W - T = 70 - 25.23 = 44.77\text{ kN.} \qquad \therefore \gamma\pi R^2(3-H) = 44.77$$

$$H = 3\text{ m} - \frac{44.77\text{ kN}}{\left(9.81\text{ kN/m}^3\right)\pi(1\text{ m})^2} = \underline{1.55\text{ m}}$$

2.65 The dam will topple if the moment about "O" of F_1 and F_3 exceeds the restoring moment of W and F_2.

Assume 1 m deep



a) $W = (2.4 \times 9810)(6 \times 50 + 24 \times 50 / 2) = 21.19 \times 10^6$ N

$$d_w = \frac{300\times 27 + 600\times 16}{300+600} = 19.67\text{ m.}$$ (d_w is from O to W.)

$F_2 = 9810 \times 5 \times 11.09 = 0.544 \times 10^6$ N. $\quad d_2 = \dfrac{11.09}{3} = 3.697$ m.

$F_1 = 9810 \times \dfrac{45}{2} \times 45 = 9.933 \times 10^6$ N. $\qquad d_1 = 15$ m. (d_1 is from O to F_1.)

$F_3 = 9810 \times \dfrac{45+10}{2} \times 30 = 8.093 \times 10^6$ N. $\quad d_3 = \dfrac{2.943\times 15 + 5.150\times 20}{2.943 + 5.150} = 18.18$ m.

$$\left.\begin{aligned} Wd_w + F_2 d_2 &= 418.8\times 10^6\,\text{N}\cdot\text{m} \\ F_1 d_1 + F_3 d_3 &= 296.1\times 10^6\,\text{N}\cdot\text{m} \end{aligned}\right\} \therefore \underline{\text{will not topple}}.$$

b) $W = (2.4 \times 9810)(6 \times 65 + 65 \times 12) = 27.55 \times 10^6$ N.

$$d_w = \frac{390 \times 27 + 780 \times 16}{390 + 780} = 19.67 \text{ m.}$$

$F_2 \cong 0.54 \times 10^6$ N. $\quad d_2 \cong 3.70$ m.

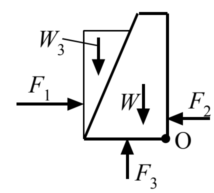
$F_1 = 9810 \times 30 \times 60 = 17.66 \times 10^6$ N. $\quad d_1 = 20$ m.

$$F_3 = 9810 \times \frac{60+10}{2} \times 30 = 10.3 \times 10^6 \text{ N.} \qquad d_3 = \frac{2.943 \times 15 + 7.358 \times 20}{2.943 + 7.358} = 18.57 \text{ m.}$$

$$\left.\begin{aligned} Wd_w + F_2 d_2 &= 543.9 \times 10^6 \text{ N}\cdot\text{m} \\ F_1 d_1 + F_3 d_3 &= 544.5 \times 10^6 \text{ N}\cdot\text{m} \end{aligned}\right\} \quad \therefore \underline{\text{it will topple}}.$$

c) Since it will topple for $H = 60$, it certainly will topple if $H = 75$ m.

2.66 The dam will topple if there is a net clockwise moment about "O."



a) $W = W_1 + W_2$. $\quad W_1 = (6 \times 43 \times 1) \times 62.4 \times 2.4 = 38{,}640$ lb.

$W_2 = (24 \times 43 / 2) \times 62.4 \times 2.4 = 77{,}280$ lb.

$W_3 = (40 \times 22.33 / 2) \times 62.4 = 27{,}870$ lb @ 20.89 ft.

$F_1 = 62.4 \times 20 \times (40 \times 1) = 49{,}920$ lb @ 40/3 ft.

$F_2 = 62.4 \times 5 \times (10 \times 1) = 3120$ lb @ 3.33 ft

$$F_3 = \begin{cases} F_{p1} = 18{,}720 \text{ lb @ 15 ft} \\ F_{p2} = 28{,}080 \text{ lb @ 20 ft} \end{cases}$$

ΣM_O: $(49{,}920)(40/3) + (18{,}720)(15) + (28{,}080)(20) - (38{,}640)(3)$
$- (77{,}280)(14) - (27{,}870)(20.89) - (3120)(3.33) < 0$. $\therefore$ <u>won't tip</u>.

b) $W_1 = 6 \times 63 \times 62.4 \times 2.4 = 56{,}610$ lb. $\quad W_2 = (24 \times 63/2) \times 62.4 \times 2.4 = 113{,}220$ lb.

$F_1 = 62.4 \times 30 \times 60 = 112{,}300$ lb. $\quad W_3 = (60 \times 22.86/2) \times 62.4 = 42{,}790$ lb.

$F_2 = 62.4 \times 5 \times 10 = 3120$ lb

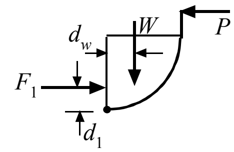
$F_{p1} = 62.4 \times 10 \times 30 = 18{,}720$ lb. $\quad F_{p2} = 62.4 \times 50 \times 30 / 2 = 46{,}800$ lb.

ΣM_O: $(112{,}300)(20) + (18{,}720)(15) + (46{,}800)(20) - (56{,}610)(3)$
$- (113{,}220)(14) - 42{,}790(21.24) = 799{,}000 > 0$. $\therefore$ <u>will tip.</u>

c) Since it will topple for $H = 60$ ft., it will also topple for $H = 80$ ft.

Forces on Curved Surfaces

2.67 $\Sigma M_{hinge} = 0. \quad 2.5P - d_w \times W - d_1 \times F_1 = 0.$



$$\therefore P = \frac{1}{2.5}\left[\frac{2}{3}\times 9800\times 1\times 8 + \frac{4\times 2}{3\pi}\times 9800\times\frac{\pi\times 2^2}{4}\times 4\right] = \underline{62\ 700\ \text{N}}$$

Note: This calculation is simpler than that of Example 2.7. Actually, We could have moved the horizontal force F_H and a vertical force F_V (equal to W) simultaneously to the center of the circle and then $2.5P = 2F_H = 2F_1$. This was outlined at the end of Example 2.7.

2.68 Since all infinitesimal pressure forces pass through the center, we can place the resultant forces at the center. Since the vertical components pass through the bottom point, they produce no moment about that point. Hence, consider only horizontal forces:

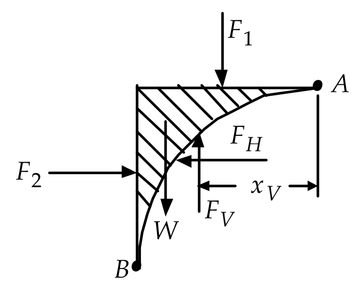
$$(F_H)_{water} = 9810\times 2\times(4\times 10) = 784\ 800\text{N}$$
$$(F_H)_{oil} = 0.86\times 9810\times 1\times 20 = 168\ 700\text{N}$$
$$\Sigma M:\ 2P = 784.8\times 2 - 168.7\times 2. \qquad \therefore \underline{P = 616.1\ \text{kN}}.$$

2.69 Place the resultant force $\vec{F}_H + \vec{F}_V$ at the center of the circular arc. $\vec{F}_H$ passes through the hinge showing that $P = F_V$.

a) $P = F_V = 9810(6\times 2\times 4 + \pi\times 4) = 594\ 200$ N or $\underline{594.2\ \text{kN}}$.

b) $P = F_V = 62.4\,(20\times 6\times 12 + 9\pi\times 12) = \underline{111{,}000\ \text{lb}}$.

2.70 a) A free-body-diagram of the volume of water in the vicinity of the surface is shown. Force balances in the horizontal and vertical directions give:



$$F_H = F_2$$
$$F_V = W + F_1$$

where F_H and F_V are the horizontal and vertical components of the force acting on the water by the surface AB. Hence,

$$F_H = F_2 = \left(9.81\ \text{kN/m}^3\right)(8+1)(2\times 4) = 706.3\ \text{kN}$$

The line of action of F_H is the same as that of F_2. Its distance from the surface is

$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 9 + \frac{4\left(2^3\right)/12}{9\times 8} = 9.037\ \text{m}$$

To find F_V we find W and F_1:

$$W = \gamma V\!\!\!/ = \left(9.81\ \text{kN/m}^3\right)\left[2\times 2 - \frac{\pi}{4}\left(2^2\right)\right]\times 4 = 33.7\ \text{kN}$$

$$F_1 = 9.81\ \text{kN/m}^3\left(8\times 2\times 4\right) = 628\ \text{kN}$$

$$\therefore F_V = F_1 + W = 33.7 + 628 = 662 \text{ kN}$$

To find the line of action of F_V, we take moments at point A:

$$F_V \times x_V = F_1 \times d_1 + W \times d_2$$

where $d_1 = 1$ m, and $d_2 = \dfrac{2R}{3(4-\pi)} = \dfrac{2\times 2}{3(4-\pi)} = 1.553$ m:

$$\therefore x_V = \frac{F_1 \times d_1 + W \times d_2}{F_V} = \frac{628\times 1 + 33.7\times 1.553}{662} = 1.028 \text{ m}$$

Finally, the forces F_H and F_V that act on the surface AB are equal and opposite to those calculated above. So, on the surface, F_H acts to the right and F_V acts downward.

b) If the water exists on the opposite side of the surface AB, the pressure distribution would be identical to that of Part (a). Consequently, the forces due to that pressure distribution would have the same magnitudes. The vertical force F_V = 662 N would act upward and the horizontal force F_H = 706.3 N would act to the left.

2.71 Place the resultant $\mathbf{F}_H + \mathbf{F}_V$ at the circular arc center. $\mathbf{F}_H$ passes through the hinge so that $P = F_V$. Use the water that could be contained above the gate; it produces the same pressure distribution and hence the same F_V. We have

$$P = F_V = 9810\ (6 \times 3 \times 4 + 9\pi) = 983\ 700 \text{ N or } \underline{983.7 \text{ kN}}.$$

2.72 Place the resultant $\mathbf{F}_H + \mathbf{F}_V$ at the center. $\mathbf{F}_V$ passes through the hinge

$$2 \times (9810 \times 1 \times 10) = 2.8 \text{ P}. \qquad \therefore P = 70\ 070 \text{ N or } \underline{70.07 \text{ kN}}.$$

2.73 The incremental pressure forces on the circular quarter arc pass through the hinge so that no moment is produced by such forces. Moments about the hinge gives:

$$3\ P = 0.9\ W = 0.9 \times 400. \qquad \therefore \underline{P = 120 \text{ N}}.$$

2.74 The resultant $\mathbf{F}_H + \mathbf{F}_V$ of the unknown liquid acts through the center of the circular arc. $\mathbf{F}_V$ passes through the hinge. Thus, we use only $(F_H)_{oil}$. Assume 1 m wide:

a) ΣM: $\dfrac{R}{3}\left(9810\dfrac{R}{2}R\right) + \dfrac{4R}{3\pi}\left(9800S\dfrac{\pi R^2}{4}\right) = R\left(\gamma_x \dfrac{R}{2}R\right)$. $\qquad \therefore \gamma_x = \underline{4580 \text{ N/m}^3}$

b) ΣM: $\dfrac{R}{3}\left(62.4\dfrac{R}{2}R\right) + \dfrac{4R}{3\pi}\left(62.4S\dfrac{\pi R^2}{4}\right) = R\left(\gamma_x \dfrac{R}{2}R\right)$. $\qquad \therefore \gamma_x = \underline{29.1 \text{ lb/ft}^3}$

2.75 The force of the water is only vertical $(F_V)_w$, acting through the center. The force of the oil can also be positioned at the center:

a) $P = (F_H)_o = (0.8 \times 9810) \times 0.3 \times 3.6 = \underline{8476 \text{ N}}$.

$$\Sigma F_y = 0 = W + (F_V)_o - (F_V)_w$$

$$0 = S \times 9810\pi \times 0.6^2 \times 6 + \left(0.36 - \frac{0.36\pi}{4}\right) \times 6 \times (0.8 \times 9810) - 9810\pi \times 0.18 \times 6$$

$$-9810 \times 0.8 \times 2 \times 0.6^2 - 6 \qquad \therefore S = \underline{0.955}.$$

b) $\rho g V\!\!\!/ = W. = \underline{1996 \text{ lb}}$.

$$\Sigma F_y = 0 = W + (F_V)_o - (F_V)_w$$

$$0 = S \times 62.4 \times \pi \times 2^2 \times 20 + \left(4 - \frac{4\pi}{4}\right) \times 20 \times 0.8 \times 62.4 - 62.4 \times \pi \times 2 \times 20$$

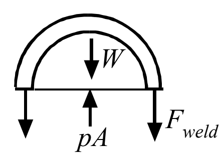
$$-62.4 \times .8 \times 2 \times 2^2 \times 20. \qquad \therefore S = \underline{0.955}.$$

2.76 The pressure in the dome is

a) $p = 60\,000 - 9810 \times 3 - 0.8 \times 9810 \times 2 = 14\,870$ Pa or 14.87 kPa.

The force is $F = pA_{projected} = (\pi \times 3^2) \times 14.87 = \underline{420.4 \text{ kN}}$.

b) From a free-body diagram of the dome filled with oil:



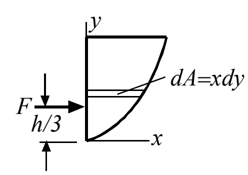
$$F_{weld} + W = pA$$

Using the pressure from part (a):

$$F_{weld} = 14\,870 \times \pi \times 3^2 - (0.8 \times 9810) \times \frac{1}{2}\left(\frac{4}{3}\pi \times 3^3\right) = -23\,400 \text{ N}$$

or $\underline{-23.4 \text{ kN}}$

2.77 A free-body diagram of the gate and water is shown.



$$\frac{H}{3}F + d_w W = H \times P.$$

a) $H = 2$ m. $F = 9810 \times 1 \times 4 = 39\,240$ N.

$$W = 9810\int_0^2 2x\,dy = 9810\int_0^2 2\frac{y^{1/2}}{\sqrt{2}}dy = \frac{2 \times 9810}{\sqrt{2}}\frac{2^{3/2}}{3/2} = 26\,160 \text{ N}.$$

$$d_w = \bar{x} = \frac{\int \frac{x}{2} x\,dy}{\int x\,dy} = \frac{\frac{1}{2}\int_0^1 4x^3 dx}{\int_0^1 4x^2 dx} = \frac{1}{2}\left(\frac{1/4}{1/3}\right) = 0.375 \text{ m}.$$

$$\therefore P = \frac{1}{3} \times 39\,240 + \frac{0.375}{2} \times 26\,160 = 17\,980 \text{ N} \quad \text{or} \quad \underline{17.98 \text{ kN}}.$$

b) $H = 8$ ft. $F = 62.4 \times 4 \times 32 = 7987$ lb

$$W = 62.4\int 4xdy = 62.4 \times 4\int_0^2 4x^2 dx = 62.4 \times 16 \times 2^3 / 3 = 2662 \text{ lb.}$$

$$d_w = \bar{x} = \frac{\frac{1}{2}\int_0^2 4x^3 dx}{\int_0^2 4x^2 dx} = \frac{1}{2}\left(\frac{16/4}{8/3}\right) = 0.75 \text{ ft.} \quad \therefore P = \frac{1}{8}\left(\frac{8}{3}\times 7987 + 0.75\times 2662\right) = \underline{2910 \text{ lb}}$$

Buoyancy

2.78 W = weight of displaced water.

a) $20\,000 + 250\,000 = 9810 \times 3\,(6d + d^2/2)$. $\therefore d^2 + 12d - 18.35 = 0$. $\therefore d = \underline{1.372 \text{ m}}$.

b) $270\,000 = 1.03 \times 9810 \times 3\,(6d + d^2/2)$. $d^2 + 12d - 17.81 = 0$. $\therefore d = \underline{1.336 \text{ m}}$.

2.79 $25 + F_B = 100$. $\therefore F_B = 75 = 9810\, V\!\!\!/$. $\therefore V\!\!\!/ = 7.645 \times 10^{-3}$ m^3

$\gamma \times 7.645 \times 10^{-3} = 100$. or $\underline{7645 \text{ cm}^3}$

$\therefore \gamma = \underline{13\,080 \text{ N/m}^3}$.

2.80 $3000 \times 60 = 25 \times 300\, \Delta d \times 62.4$. $\therefore \Delta d = 0.3846'$ or $4\underline{.62''}$.

2.81 $100\,000 \times 9.81 + 6\,000\,000 = (12 \times 30 + 8h \times 30)\, 9810$

$\therefore h = 1.465$ m. $\therefore$ distance from top $= 2 - 1.465 = \underline{0.535 \text{ m}}$

2.82 $T + F_B = W$. (See Fig. 2.11 c.)

$T = 40\,000 - 1.59 \times 9810 \times 2 = 8804$ N or $\underline{8.804 \text{ kN}}$.

2.83 The forces acting on the balloon are its weight W, the buoyant force F_B, and the weight of the air in the balloon F_a. Sum forces:

$$F_B = W + F_a \quad \text{or} \quad \frac{4}{3}\pi R^3 \rho g = 1000 + \frac{4}{3}\pi R^3 \rho_a g$$

$$\frac{4}{3}\pi \times 5^3 \frac{100\times 9.81}{0.287\times 293} = 1000 + \frac{4}{3}\pi \times 5^3 \frac{100\times 9.81}{0.287 T_a}. \quad \therefore T_a = 350.4 \text{ K or } \underline{77.4°\text{C}}$$

2.84 The forces acting on the blimp are the payload F_p, the weight of the blimp W, the buoyant force F_B, and the weight of the helium F_h:

$$F_B = F_p + W + F_h$$

$$1500\pi\times150^2\times\frac{100\times9.81}{0.287\times288} = F_p + 0.1\,F_p + 1500\,\pi\times150^2\times\frac{100\times9.81}{2.077\times288}$$

$$\therefore F_P = 9.86\times10^8 \quad \text{and} \quad N_{people} = \frac{9.86\times10^8}{800} = \underline{1.23\times10^6}$$

Of course equipment and other niceties such as gyms, pools, restaurants, etc., would add significant weight.

2.85 Neglect the buoyant force of air. A force balance yields

$$F_B = W + F$$
$$= 50 + 10 = 60 = 9800\,V\!\!\!\!/. \qquad \therefore V\!\!\!\!/ = 0.006122\text{ m}^3$$

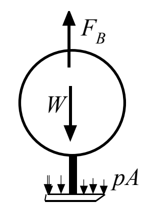
Density: $\rho g V\!\!\!\!/ = W$.

$$\rho\times9.81\times0.006122 = 50. \qquad \therefore \rho = \underline{832.5\text{ kg/m}^3}$$

Specific wt: $\gamma = \rho g = 832.5\times9.81 = \underline{8167\text{ N/m}^3}$

Specific gravity: $S = \dfrac{\rho}{\rho_{\text{water}}} = \dfrac{832.5}{1000}\,S = \underline{0.8325}$

2.86 From a force balance $F_B = W + pA$.

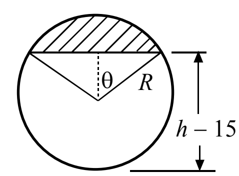


a) The buoyant force is found as follows ($h > 16'$):

$$\cos\theta = \frac{h-15-R}{R}, \qquad \text{Area} = \theta R^2 - (h-15-R)\,R\sin\theta$$

$$\therefore F_B = 10\times62.4[\pi R^2 - \theta R^2 + (h-15-R)\,R\sin\theta].$$

$$F_B = 1500 + \gamma hA.$$



The h that makes the above 2 F_B's equal is found by trial-and-error:

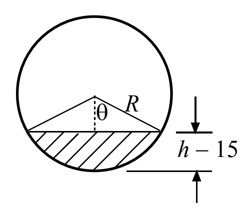
$h = 16.5$: 1859 $\overset{?}{=}$ 1577 $h = 16.8$: 1866 $\overset{?}{=}$ 1858

$h = 17.0$: 1870 $\overset{?}{=}$ 1960 $\therefore h = \underline{16.8\text{ ft.}}$

b) Assume $h > 16.333$ ft and use the above equations with $R = 1.333$ ft:

$h = 16.4$: 1857 $\overset{?}{=}$ 1853 $\therefore h = \underline{16.4\text{ ft}}$.

c) Assume $h < 16.667$ft. With $R = 1.667$ ft,



$$F_B = 10\times62.4[\theta R^2 - (R-h+15)\,R\sin\theta]$$

$$F_B = 1500 + \gamma hA. \qquad \cos\theta = \frac{R-h+15}{R}$$

Trial-and-error for h:

$h = 16$: 1849 $\overset{?}{=}$ 1374 $h = 16.2$: 1853 $\overset{?}{=}$ 1765

$h = 16.4$: 1857 $\overset{?}{=}$ 2170 $\therefore h = \underline{16.25\text{ ft.}}$

2.87 a) $W = F_B$. $\left[0.01 + 13.6 \times 1000 \times h\pi \times .015^2 / 4\right] \times 9.81 = 9810 \forall$.

$$\forall = \frac{\pi \times .015^2}{4} \times .15 + \frac{\pi \times .005^2}{4} \times .06 = 2.769 \times 10^{-5}\,\text{m}^3. \qquad \therefore h = 7.361 \times 10^{-3}\text{ m}$$

$$\therefore m_{Hg} = 13.6 \times 1000 \times h\pi \times .015^2 / 4 = \underline{0.01769\text{ kg}}$$

b) $(0.01 + 0.01769)\, 9.81 = 9810 \left[\frac{\pi \times 0.015^2}{4} \times 0.15 + \frac{\pi \times 0.005^2}{4} \times 0.12\right] S_x$.

$\therefore S_x = \underline{0.959}$.

c) $(0.01 + 0.01769)\, 9.81 = 9810 \, \frac{\pi \times 0.015^2}{4} \times 0.15 \;\; S_x$. $\qquad \therefore S_x = \underline{1.045}$.

2.88 $(0.01 + m_{Hg}) 9.81 = 9810 \left[\frac{\pi \times 0.015^2}{4} \times 0.15 + \frac{\pi \times 0.005^2}{4} \times 0.12\right]$. $\qquad \therefore m_{Hg} = 0.01886$.

a) $(0.01 + 0.01886)\, 9.81 = 9810 \, \frac{\pi \times 0.015^2}{4} \times 0.15\, S_x$. $\qquad \therefore S_x = \underline{1.089}$.

b) $m_{Hg} = \underline{0.01886\text{ kg}}$.

Stability

2.89 a) $I_o = \frac{\pi d^4}{64} = \frac{\pi \times (10/12)^4}{64} = 0.02367\text{ ft}^4$.

$$\forall = \frac{W}{r_{H_2O}} = \frac{.8 \times 62.4 \times \pi \times (5/12)^2 \times 12/12}{62.4} = 0.4363. \text{ depth} = \frac{0.4363}{\pi (5/12)^2} = 0.8\text{ ft}$$

$\therefore \overline{GM} = 0.02367 / 0.4363 - (0.5 - 0.4) = -0.0457'$. $\therefore$ <u>It will not float with ends horizontal</u>.

b) $I_o = 0.02367\text{ ft}^4$, $\forall = 0.3636\text{ ft}^3$, depth $= 0.6667$ ft

$\overline{GM} = 0.02367 / 0.3636 - (5-4)/12 = -0.01823$ ft. $\qquad \therefore$ <u>It will not float as given</u>.

c) $\forall = 0.2909$, depth = 6.4", GM $= \frac{0.02367}{0.2909} - \frac{4 - 3.2}{12} = 0.0147$ ft. $\therefore$ <u>It will float</u>.

2.90 With ends horizontal $I_o = \pi d^4 / 64$. The displaced volume is

$\forall = \gamma_x \pi d^2 h / 4 \times 9800 = 8.014 \times 10^{-5} \gamma_x d^3$ since $h = d$. The depth the cylinder will sink is

$$\text{depth} = \frac{\forall}{A} = 8.014 \times 10^{-5} \gamma_x d^3 \, / \, \pi d^2 \, / \, 4 = 10.20 \times 10^{-5} \gamma_x d$$

The distance $\overline{CG}$ is $\overline{CG} = \frac{h}{2} - 10.2 \times 10^{-5} \gamma_x d / 2$. Then

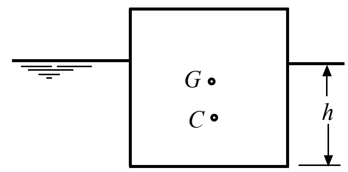
$$\overline{GM} = \frac{\pi d^4 \ / \ 64}{8.014\times10^{-5}\gamma_x d^3} - \frac{d}{2} + 10.2\times10^{-5}\gamma_x d/2 > 0.$$

This gives (divide by d and multiply by γ_x): $612.5 - 0.5\,\gamma_x + 5.1\times10^{-5}\,\gamma_x^2 > 0$.

Consequently, $\gamma_x >$ <u>8369 N/m^3</u> or $\gamma_x <$ <u>1435 N/m^3</u>

2.91 $\forall = \dfrac{W}{\gamma_{water}} = \dfrac{S\gamma_{water}d^3}{\gamma_{water}} = S\,d^3$. $\forall = \dfrac{W}{\gamma_{water}} = \dfrac{S\gamma_{water}d^3}{\gamma_{water}} = S\,d^3$. $\therefore h = Sd$.

$$\overline{GM} = \frac{d^4/12}{Sd^3} - (d/2 - Sd/2) = d\left(\frac{1}{12S} - \frac{1}{2} + \frac{S}{2}\right).$$



If $\overline{GM} = 0$ the cube is neutral and $6S^2 - 6S + 1 = 0$.

$$\therefore S = \frac{6 \pm \sqrt{36-24}}{12} = 0.7887,\ 0.2113.$$

The cube is unstable if <u>$0.2113 < S < 0.7887$</u>.

Note: Try $S = 0.8$ and $S = 0.1$ to see if $\overline{GM} > 0$. This indicates stability.

2.92 As shown, $\bar{y} = 16\times9 + 16\times4/(16+16) = 6.5$ cm above the bottom edge.

$$G = \frac{4\gamma\times9.5 + 16\gamma\times8.5 + 16S_A\gamma\times4}{0.5\gamma\times8 + 2\gamma\times8 + S_A\gamma\times16} = 6.5 \text{ cm}.$$

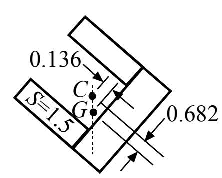
$\therefore 130 + 104\,S_A = 174 + 64\,S_A$. $\therefore S_A =$ <u>1.1.</u>

2.93 a) $\bar{y} = \dfrac{16\times4 + 8\times1 + 8\times7}{16+8+8} = 4.$ $\bar{x} = \dfrac{16\times1 + 8\times4 + 8\times4}{16+8+8} = 2.5.$

For G: $y = \dfrac{1.2\times16\times4 + 0.5\times8\times1 + 1.5\times8\times7}{1.2\times16 + 0.5\times8 + 1.5\times8} = 4.682.$

$$x = \frac{1.2\times16 + 0.5\times8\times4 + 1.5\times8\times4}{1.2\times16 + 0.5\times8 + 1.5\times8} = 2.364.$$

G must be directly under C.



$\tan\theta = \dfrac{0.136}{0.682}$. $\therefore$ <u>$\theta = 11.3°$</u>.

b) $\bar{y} = \dfrac{4\times2 + 2\times\frac{1}{2} + 2\times3.5}{4+2+2} = 2.$ $\bar{x} = \dfrac{4\times\frac{1}{2} + 2\times2 + 2\times2}{4+2+2} = 1.25$

For G: $y = \dfrac{1.2\times4\times2 + 0.5\times1 + 1.5\times7}{1.2\times4 + 0.5\times2 + 1.5\times2} = 2.34.$ $x = \dfrac{1.2\times2 + 0.5\times4 + 1.5\times4}{1.2\times4 + 0.5\times2 + 1.5\times2} = 1.182$

$\Delta y = 0.34$, $\Delta x = 0.068$. $\tan\theta = \dfrac{0.068}{0.34}$. $\therefore$ <u>$\theta = 11.3°$</u>.

2.94 The centroid C is 1.5 m below the water surface. $\therefore \overline{CG} = 1.5$ m.

Using Eq. 2.4.47: $\overline{GM} = \dfrac{\ell \times 8^3 / 12}{\ell \times 8 \times 3} - 1.5 = 1.777 - 1.5 = 0.277 > 0.$

$\therefore$ <u>The barge is stable</u>.

2.95 $\bar{y} = \dfrac{8.485 \times 3.414 + 16.97 \times 1}{8.485 + 16.97} = 1.8$ m. $\therefore \overline{CG} = 1.8 - 1.5 = 0.3$ m.

Using Eq. 2.4.47: $\overline{GM} = \dfrac{\ell \times 8.485^3 / 12}{34.97\ell} - 0.3 = 1.46 - 0.3 = 1.16.$ $\therefore$ <u>Stable</u>.

Linearly Accelerating Containers

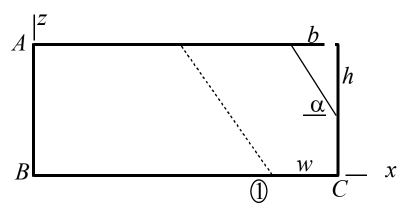
2.96 a) $\tan\alpha = \dfrac{20}{9.81} = \dfrac{H}{4}$. $\therefore H = 8.155$ m. $p_{max} = 9810\,(8.155 + 2) =$ <u>99 620 Pa</u>

b) $p_{max} = \rho(g + a_z)\, h = 1000\,(9.81 + 20) \times 2 =$ <u>59 620 Pa</u>

c) $p_{max} = 1.94 \times 60\,(-12) - 1.94\,(32.2 + 60)\,(-6) = 2470$ psf or <u>17.15 psi</u>

d) $p_{max} = 1.94\,(32.2 + 60)\,(-6) = 1073$ psf or <u>7.45 psi</u>

2.97 The air volume is the same before and after.



$\therefore\ 0.5 \times 8 = hb/2.\ \ \tan\alpha = \dfrac{10}{9.81} = \dfrac{h}{b}.$

$4 = \dfrac{h}{2}\dfrac{9.81}{10}h.\ \ \therefore h = 2.856.$ $\therefore$ Use dotted line.

$2.5w + \dfrac{1}{2} \times 2.5 \times 2.452 = 4.\ \ \therefore w = 0.374$ m.

a) $p_A = -1000 \times 10\,(0 - 7.626) - 1000 \times 9.81 \times 2.5 = 51\,740$ Pa or <u>51.74 kPa</u>

b) $p_B = -1000 \times 10\,(0 - 7.626) = 76\,260$ Pa or <u>76.26 kPa</u>

c) $p_C =$ <u>0</u>. Air fills the space to the dotted line.

2.98 Use Eq. 2.5.2: Assume an air-water surface as shown in the above figure.

a) $60\,000 = -1000\, a_x\,(0-8) - 1000 \times 9.81 \left[0 - \left(2.5 - \sqrt{\dfrac{8a_x}{9.81}}\right)\right]$

$4 = \dfrac{h^2 \times 9.81}{2a_x}$ $60 = 8\,a_x + 24.52 - 9.81\sqrt{\dfrac{8a_x}{9.81}}$ or $a_x - 4.435 = 1.1074\sqrt{a_x}$.

$a_x^2 - 10.1\,a_x + 19.67 = 0$ $\therefore a_x = 2.64$, <u>7.46 m/s²</u>

b) $60\,000 = -1000\,a_x\,(-8) - 1000\,(9.81 + 10)\left(-2.5 + \sqrt{\dfrac{8a_x}{9.81}}\right)$.

$$60 = 8\,a_x + 49.52 - 19.81\sqrt{\frac{8a_x}{19.81}} \quad \text{or} \quad a_x - 1.31 = 1.574\sqrt{a_x}\,.$$

$$a_x^2 - 5.1\,a_x + 1.44 = 0 \qquad \therefore a_x = 0.25,\ \underline{4.8\text{ m/s}^2}$$

c) $$60\,000 = -1000\,a_x\,(-8) - 1000\,(9.81 + 5)\left(-2.5 + \sqrt{\frac{8a_x}{14.81}}\right).$$

$$60 = 8\,a_x + 37.0 - 14.81\sqrt{\frac{8a_x}{14.81}} \quad \text{or} \quad a_x - 2.875 = 1.361\sqrt{a_x}\,.$$

$$a_x^2 - 7.6\,a_x + 8.266 = 0 \qquad \therefore a_x = 1.32,\ \underline{6.28\text{ m/s}^2}$$

2.99 a) $a_x = 20 \times 0.866 = 17.32$ m/s^2, $a_z = 10$ m/s^2. Use Eq. 2.5.2 with the peep hole as position 1. The x-axis is horizontal passing through A. We have

$p_A = -1000 \times 17.32\,(0 - 1.232) - 1000\,(9.81 + 10)\,(0 - 1.866) = \underline{58\,290\text{ Pa}}$

b) $p_A = -1000 \times 8.66\,(0 - 1.848) - 1000\,(9.81 + 5)\,(0 - 2.799) = \underline{57\,460\text{ Pa}}$

c) The peep hole is located at (3.696, 5.598). Use Eq. 2.5.2:

$p_A = -1.94 \times 51.96\,(0 - 3.696) - 1.94\,(32.2 + 30)\,(0 - 5.598) = \underline{1048\text{ psf}}$

d) The peep hole is located at (4.928, 7.464). Use Eq. 2.5.2:

$p_A = -1.94 \times 25.98\,(-4.928) - 1.94\,(32.2 + 15)\,(-7.464) = \underline{932\text{ psf}}$

2.100 a) The pressure on the end AB (z is zero at B) is, using Eq. 2.5.2,

$p(z) = -1000 \times 10\,(-7.626) - 1000 \times 9.81(z) = 76\,260 - 9810\,z$

$$\therefore F_{AB} = \int_0^{2.5}(76\,260 - 9810z)4dz = 640\,000\text{ N} \quad \text{or} \quad \underline{640\text{ kN}}$$

b) The pressure on the bottom BC is

$p(x) = -1000 \times 10\,(x - 7.626) = 76\,260 - 10\,000\,x.$

$$\therefore F_{BC} = \int_0^{7.626}(76\,260 - 10\,000x)4dx = 1.163 \times 10^6\text{ N} \quad \text{or} \quad \underline{1163\text{ kN}}$$

c) On the top $p(x) = -1000 \times 10\,(x - 5.174)$ where position 1 is on the top surface:

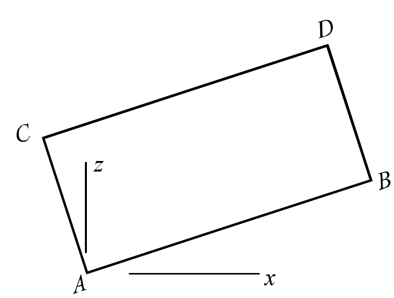
$$\therefore F_{top} = \int_0^{5.174}(51\,740 - 10\,000x)4dx = 5.35 \times 10^5\text{ N} \quad \text{or} \quad \underline{535\text{ kN}}$$

2.101 a) The pressure at A is 58.29 kPa. At B it is

$p_B = -1000 \times 17.32\,(1.732 - 1.232)$
$\quad - 1000\,(19.81)\,(1 - 1.866) = 8495$ Pa.

Since the pressure varies linearly over AB, we can use an average pressure times the area:

$$F_{AB} = \frac{58\,290 + 8495}{2} \times 1.5 \times 2 = 100\,200\text{ N or } \underline{100.2\text{ kN}}$$

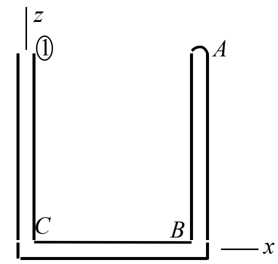


b) $p_D = 0$. $p_C = -1000 \times 17.32\ (-0.5-1.232) - 1000 \times 19.81(0.866-1.866) = 49\ 810$ Pa.

$F_{CD} = \frac{1}{2} \times 49\ 810 \times 1.5 \times 2 = 74\ 720$ N or $\underline{74.72 \text{ kN}}$.

c) $p_A = 58\ 290$ Pa. $p_C = 49\ 810$ Pa. $\therefore F_{AC} = \frac{58.29 + 49.81}{2} \times 1.5 = \underline{81.08 \text{ kN}}$.

2.102 Use Eq. 2.5.2 with position 1 at the open end:



a) $p_A = \underline{0}$ since $z_2 = z_1$.

$p_B = 1000 \times 19.81 \times 0.6 = \underline{11\ 890 \text{ Pa.}}$

$p_C = \underline{11\ 890 \text{ Pa}}$.

b) $p_A = -1000 \times 10\ (0.9-0) = \underline{-9000 \text{ Pa}}$.

$p_B = -000 \times 10\ (0.9) - 1000 \times 9.81(-0.6) = \underline{-3114 \text{ Pa}}$

$p_C = -1000 \times 9.81 \times (-0.6) = \underline{5886 \text{ Pa}}$.

c) $p_A = -1000 \times 20\ (0.9) = \underline{-18\ 000 \text{ Pa}}$.

$p_B = -1000 \times 20 \times 0.9 - 1000 \times 19.81(-0.6) = \underline{-6110 \text{ Pa.}}$ $p_C = \underline{11\ 890 \text{ Pa}}$

d) $p_A = \underline{0}$. $p_B = 1.94 \times (32.2\text{-}60) \left(\frac{25}{12}\right) = \underline{-112 \text{ psf}}$. $p_C = \underline{-112 \text{ psf}}$.

e) $p_A = 1.94 \times 60 \left(-\frac{37.5}{12}\right) = \underline{-364 \text{ psf}}$.

$p_B = 1.94 \times 60 \left(-\frac{37.5}{12}\right) - 1.94 \times 32.2 \left(-\frac{25}{12}\right) = \underline{-234 \text{ psf}}$.

$p_C = -1.94 \times 32.2 \left(-\frac{25}{12}\right) = \underline{130 \text{ psf}}$.

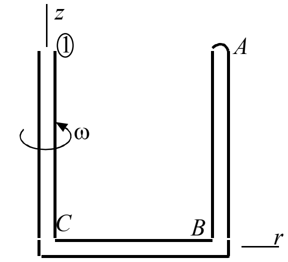
f) $p_A = 1.94 \times 30 \left(\frac{37.5}{12}\right) = \underline{182 \text{ psf}}$.

$p_B = -1.94(-30) \left(\frac{37.5}{12}\right) - 1.94 \times 62.2 \left(-\frac{25}{12}\right) = \underline{433 \text{ psf}}$.

$p_C = -1.94 \times 62.2 \times \left(-\frac{25}{12}\right) = \underline{251 \text{ psf}}$.

Rotating Containers

2.103 Use Eq. 2.6.4 with position 1 at the open end:



$$\omega = \frac{50 \times 2\pi}{60} = 5.236 \text{ rad/s.}$$

a) $p_A = (1000 \times 5.236^2/2) \times (0.6 \times 1.5)^2 = \underline{11\ 100 \text{ Pa}}$.

$p_B = \frac{1}{2} \times 1000 \times 5.236^2 \times 0.9^2 + 9810 \times 0.6 = \underline{16\ 990 \text{ Pa}}$.

$p_C = 9810 \times 0.6 = \underline{5886 \text{ Pa}}$.

b) $p_A = \frac{1}{2} \times 1000 \times 5.236^2 \times 0.6^2 = \underline{4935 \text{ Pa.}}$

$p_B = \frac{1}{2} \times 1000 \times 5.236^2 \times 0.6^2 + 9810 \times 0.4 = \underline{8859 \text{ Pa}}.$

$p_C = 9810 \times 0.4 = \underline{3924 \text{ Pa}}.$

c) $p_A = \frac{1}{2} \times 1.94 \times 5.236^2 \times \left(\frac{37.5}{12}\right)^2 = \underline{259.7 \text{ psf}}.$

$p_B = \frac{1}{2} \times 1.94 \times 5.236^2 \times \left(\frac{37.5}{12}\right)^2 + 62.4 \times \frac{25}{12} = \underline{389.7 \text{ psf}}.$

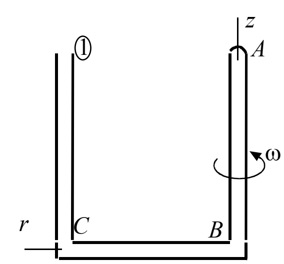
$p_C = 62.4 \times 25/12 = \underline{130 \text{ psf}}.$

d) $p_A = \frac{1}{2} \times 1.94 \times 5.236^2 \times \left(\frac{22.5}{12}\right)^2 = \underline{93.5 \text{ psf}}.$

$p_B = \frac{1}{2} \times 1.94 \times 5.236^2 \times \left(\frac{22.5}{12}\right)^2 + 62.4 \times \frac{15}{12} = \underline{171.5 \text{ psf}}.$

$p_C = 62.4 \times 15/12 = \underline{78 \text{ psf}}.$

2.104 Use Eq. 2.6.4 with position 1 at the open end.



a) $p_A = \frac{1}{2} \times 1000 \times 10^2 (0 - 0.9^2) = \underline{-40\,500 \text{ Pa}}.$

$p_B = -40\,500 + 9810 \times 0.6 = \underline{-34\,600 \text{ Pa}}.$

$p_C = 9810 \times 0.6 = \underline{5886 \text{ Pa}}.$

b) $p_A = \frac{1}{2} \times 1000 \times 10^2 (0 - 0.6^2) = \underline{-18\,000 \text{ Pa.}}$

$p_B = -18\,000 + 9810 \times 0.4 = \underline{-14\,080 \text{ Pa}}.$

$p_C = 9810 \times 0.4 = \underline{3924 \text{ Pa}}.$

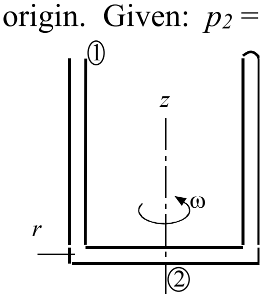
c) $p_A = \frac{1}{2} \times 1.94 \times 10^2 \left(0 - \frac{37.5^2}{144}\right) = \underline{-947 \text{ psf}}.$

$p_B = -947 + 62.4 \times 25/12 = \underline{-817 \text{ psf}}.$ $p_C = 62.4 \times 25/12 = \underline{130 \text{ psf.}}$

d) $p_A = \frac{1}{2} \times 1.94 \times 10^2 \left(-\frac{22.5^2}{12^2}\right) = \underline{-341 \text{ psf}}.$

$p_B = -341 + 62.4 \times 15/12 = \underline{-263 \text{ psf}}.$ $p_C = 62.4 \times 15/12 = \underline{78 \text{ psf.}}$

2.105 Use Eq. 2.6.4 with position 1 at the open end and position 2 at the origin. Given: $p_2 = 0$.



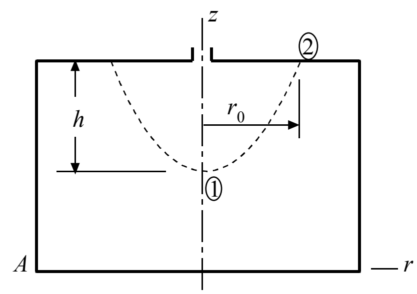
a) $0 = \frac{1}{2} \times 1000\ \omega^2 (0 - 0.45^2) - 9810 (0 - 0.6)$. $\therefore \omega = \underline{7.62 \text{ rad/s}}.$

b) $0 = \frac{1}{2} \times 1000\ \omega^2 (0 - 0.3^2) - 9810 (0 - 0.4)$. $\therefore \omega = \underline{9.34 \text{ rad/s}}.$

c) $0 = \frac{1}{2} \times 1.94\ \omega^2 \left(0 - \frac{18.75^2}{12^2}\right) - 62.4\left(-\frac{25}{12}\right)$. $\therefore \omega = \underline{7.41 \text{ rad/s}}.$

d) $0 = \frac{1}{2} \times 1.94\ \omega^2 \left(-\frac{11.25^2}{12^2}\right) - 62.4\left(-\frac{15}{12}\right)$. $\therefore \omega =$ 9.57 rad/s

2.106 The air volume before and after is equal.



$\therefore \frac{1}{2}\pi r_0^2 h = \pi \times .6^2 \times .2.$ $\therefore r_0^2 h = 0.144.$

a) Using Eq. 2.6.5: $r_0^2 \times 5^2\ /\ 2 = 9.81\ h$

$\therefore h = 0.428$ m

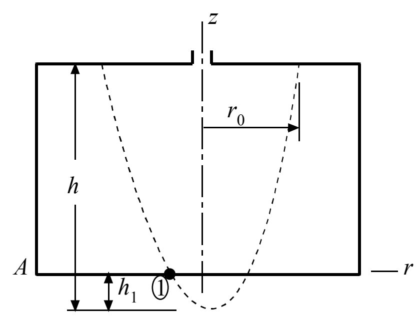
$\therefore p_A = \frac{1}{2} \times 1000 \times 5^2 \times 0.6^2 - 9810\ (-0.372)$

$=$ 8149 Pa.

b) $r_0^2 \times 7^2\ /\ 2 = 9.81\ h$. $\therefore h = 0.6$ m.

$\therefore p_A = \frac{1000}{2} \times 7^2 \times 0.6^2 + 9810 \times 0.2 =$ 10 780 Pa.

c) For $\omega = 10$, part of the bottom is bared.



$$\pi \times .6^2 \times .2 = \frac{1}{2}\pi r_0^2 h - \frac{1}{2}\pi r_1^2 h_1.$$

Using Eq. 2.6.5:

$$\frac{\omega^2 r_0^2}{2g} = h, \qquad \frac{\omega^2 r_1^2}{2g} = h_1.$$

$$\therefore 0.144 = \frac{2g}{\omega^2}h^2 - \frac{2g}{\omega^2}h_1^2 \qquad \text{or}$$

$$h^2 - h_1^2 = \frac{0.144 \times 10^2}{2 \times 9.81}.$$

Also, $h - h_1 = 0.8$. $1.6h - 0.64 = 0.7339$. $\therefore h = 0.859$ m, $r_1 = 0.108$ m.

$\therefore p_A = 1000 \times 10^2\ (0.6^2 - 0.108^2)/2 =$ 17 400 Pa.

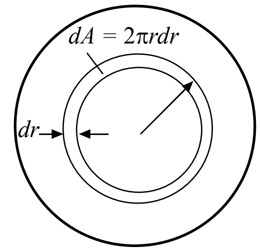
d) Following part (c): $h^2 - h_1^2 = \frac{0.144 \times 20^2}{2 \times 9.81}$. $1.6h - 0.64 = 2.936.\therefore\ h = 2.235$ m.

$\therefore p_A = 1000 \times 20^2\ (0.6^2 - 0.265^2)/2 =$ 57 900 Pa $\qquad r_1 = 0.265$ m

2.107 The answers to Problem 2.105 are increased by 25 000 Pa.

a) 33 150 Pa b) 35 780 Pa c) 42 400 Pa d) 82 900 Pa

2.108 $p(r) = \frac{1}{2}\rho\omega^2 r^2 - \rho g[0-(0.8-h)]$.



$$p(r) = 500\omega^2 r^2 + 9810(0.8-h) \qquad \text{if } h < 0.8.$$

$$p(r) = 500\omega^2 (r^2 - r_1^2) \qquad \text{if } h > 0.8.$$

a) $F = \int p2\pi r dr = 2\pi \int_0^{0.6} (12\,500r^3 + 3650r)dr = \underline{6670 \text{ N}}$.

(We used h = 0.428 m)

b) $F = \int p2\pi r dr = 2\pi \int_0^{0.6} (24\,500r^3 + 1962r)dr = \underline{7210 \text{ N}}$. (We used h = 0.6 m)

c) $F = \int p2\pi r dr = 2\pi \int_{-0.108}^{0.6} (50\,000(r^3 - 0.108^2 r)dr = \underline{9920 \text{ N}}$. (We used r_1 = 0.108 m)

d) $F = \int p2\pi r dr = 2\pi \int_{0.265}^{0.6} (200\,000(r^3 - 0.265^2 r)dr = \underline{26\,400 \text{ N}}$. (We used r_1 = 0.265 m)