Chapter 2 Solutions

Problem 2.2-1

$$L = 10\text{ft} \qquad q_0 = 2 \cdot \frac{\text{kip}}{\text{ft}} \qquad k = 4 \cdot \frac{\text{kip}}{\text{in}}$$

$$\Sigma M_B = 0 \qquad A_y = \frac{1}{L} \cdot \left(\frac{1}{2} \cdot q_0 \cdot L \cdot \frac{L}{3} \right) = 3.333 \cdot \text{kip} \qquad \delta_A = \frac{A_y}{k} = 0.833 \cdot \text{in} \qquad \frac{\delta_A}{L} = 6.944 \times 10^{-3}$$

Problem 2.2-2

$E = 200\text{GPa}$ $d_r = 25\text{mm}$ $q = 5\dfrac{\text{kN}}{\text{m}}$ $L_r = 0.75\text{m}$ $P = 10\text{kN}$ $a = 2.5\text{m}$ $b = 0.75\text{m}$

$$A_r = \frac{\pi}{4}\cdot d_r^2 = 0.761\cdot\text{in}^2$$

Force in rod $\Sigma M_A = 0$ $F_r = \dfrac{1}{a}\cdot\left[q\cdot a\cdot\dfrac{a}{2} + P\cdot(a + b)\right] = 19.25\cdot\text{kN}$

Change in length of rod $\delta_{\text{rod}} = \dfrac{F_r\cdot L_r}{E\cdot A_r} = 0.1471\cdot\text{mm}$
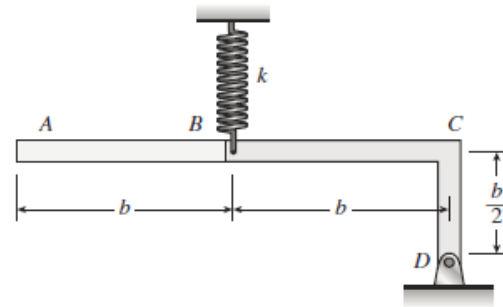
Displacement at B using similar triangles $\delta_B = \dfrac{a + b}{a}\cdot\delta_{\text{rod}} = 0.1912\cdot\text{mm}$

Problem 2.2-3

(a) SUM MOMENTS ABOUT A

$$\Sigma M_A = 0 \qquad \frac{2b}{\frac{5}{2}b}Wb + \frac{\frac{b}{2}}{\frac{5}{2}b}W(2b) = k\delta b$$

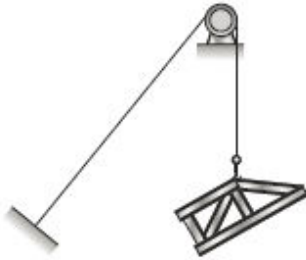$$\delta = \frac{\dfrac{2b}{\frac{5}{2}b}Wb + \dfrac{\frac{b}{2}}{\frac{5}{2}b}W(2b)}{kb} = \frac{6W}{5k}$$

(b) $\Sigma M_D = 0 \quad kb\delta = \dfrac{2b}{\frac{5}{2}b}Wb = \dfrac{4Wb}{5}$ so $\delta = \dfrac{\dfrac{2b}{\frac{5}{2}b}Wb}{kb} = \dfrac{4W}{5k}$

(b)

Problem 2.2-4

$A = 304 \text{ mm}^2$ (from Table 2-1)

$W = 38 \text{ kN}$

$E = 140 \text{ GPa}$

$L = 14 \text{ m}$

(b) FACTOR OF SAFETY

$P_{ULT} = 406 \text{ kN}$ (from Table 2-1)

$P_{max} = 70 \text{ kN}$

$$n = \frac{P_{ULT}}{P_{max}} = \frac{406 \text{ kN}}{70 \text{ kN}} = 5.8 \quad \leftarrow$$

(a) STRETCH OF CABLE

$$\delta = \frac{WL}{EA} = \frac{(38 \text{ kN})(14 \text{ m})}{(140 \text{ GPa})(304 \text{ mm}^2)}$$

$$= 12.5 \text{ mm} \quad \leftarrow$$

Problem 2.2-5

(a) $\dfrac{\delta_a}{\delta_s} = \dfrac{\dfrac{PL}{E_a A}}{\left(\dfrac{PL}{E_s A}\right)} \rightarrow \dfrac{E_s}{E_a}$

$E_s = 30{,}000\ \text{ksi} \quad E_a = 11{,}000\ \text{ksi}$

$\boxed{\dfrac{E_s}{E_a} = 2.727} \quad \dfrac{30}{11} = 2.727$

(b) $\delta_a = \delta_s$ so $\dfrac{PL}{E_a A_a} = \dfrac{PL}{E_s A_s}$ so $\dfrac{A_a}{A_s} = \dfrac{E_s}{E_a}$ and $\boxed{\dfrac{d_a}{d_s} = \sqrt{\dfrac{E_s}{E_a}} = 1.651}$

(c) SAME DIAMETER, SAME LOAD, FIND RATIO OF LENGTH OF ALUMINUM TO STEEL WIRE IF ELONGATION OF ALUMINUM IS 1.5 TIMES THAT OF STEEL WIRE

$\dfrac{\delta_a}{\delta_s} = \dfrac{\dfrac{PL_a}{E_a A}}{\left(\dfrac{PL_s}{E_s A}\right)} \qquad \dfrac{\dfrac{PL_a}{E_a A}}{\left(\dfrac{PL_s}{E_s A}\right)} = 1.5 \qquad \boxed{\dfrac{L_a}{L_s} = 1.5\dfrac{E_a}{E_s} = 0.55}$

(d) SAME DIAMETER, SAME LENGTH, SAME LOAD—BUT WIRE 1 ELONGATION 1.7 TIMES THE STEEL WIRE $>$ WHAT IS WIRE 1 MATERIAL?

$\dfrac{\delta_1}{\delta_s} = \dfrac{\dfrac{PL}{E_1 A}}{\left(\dfrac{PL}{E_s A}\right)} \qquad \dfrac{\dfrac{PL}{E_1 A}}{\left(\dfrac{PL}{E_s A}\right)} = 1.7 \qquad E_1 = \dfrac{E_s}{1.7} = 17{,}647\ \text{ksi} \qquad \boxed{<\text{cast iron or copper alloy (see App. I)}}$

Problem 2.2-6
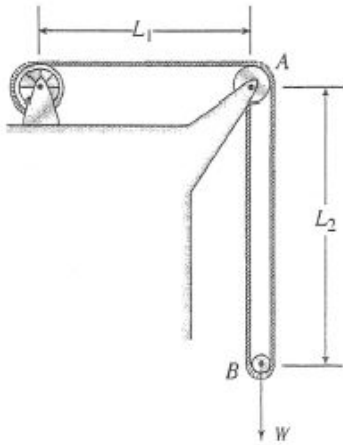
$d_A = 300$ mm

$d_B = 150$ mm

$L_1 = 4.6$ m

$L_2 = 10.5$ m

$EA = 10{,}700$ kN

$W = 22$ kN

LENGTH OF CABLE

$$L = L_1 + 2L_2 + \frac{1}{4}(\pi d_A) + \frac{1}{2}(\pi d_B)$$

$$= 4{,}600 \text{ mm} + 21{,}000 \text{ mm} + 236 \text{ mm} + 236 \text{ mm}$$

$$= 26{,}072 \text{ mm}$$

ELONGATION OF CABLE

$$\delta = \frac{TL}{EA} = \frac{(11 \text{ kN})(26{,}072 \text{ mm})}{(10{,}700 \text{ kN})} = 26.8 \text{ mm}$$

LOWERING OF THE CAGE

$h = $ distance the cage moves downward

$$h = \frac{1}{2}\delta = 13.4 \text{ mm} \quad \leftarrow$$

TENSILE FORCE IN CABLE

$$T = \frac{W}{2} = 11 \text{ kN}$$

Problem 2.2-7

$d_o = 15\text{in}$ $d_i = 14.4\text{in}$ $E = 29000\text{ksi}$ $P = 5\text{kip}$

$L_{DC} = \sqrt{(3\text{ft})^2 + (4\text{ft})^2} = 5\cdot\text{ft}$ $A_{DC} = \dfrac{\pi}{4}\cdot\left(d_o^2 - d_i^2\right) = 13.854\cdot\text{in}^2$

Find force in DC - use FBD of ACB

$\Sigma M_A = 0$ $\dfrac{3}{5}F_{DC}\cdot(4\text{ft}) = P\cdot(9\text{ft})$ so $F_{DC} = \dfrac{5}{3}\cdot P\cdot\left(\dfrac{9}{4}\right) = 18.75\cdot\text{kip}$ compression

Change in length of strut

$\Delta_{DC} = \dfrac{F_{DC}\cdot L_{DC}}{E\cdot A_{DC}} = 2.8\times 10^{-3}\cdot\text{in}$ shortening

Vertical displacement at C (see Example 2-7) and at B

$\delta_C = \dfrac{\Delta_{DC}}{\sin(ACD)}$ $\delta_C = \dfrac{\Delta_{DC}}{\dfrac{3}{5}} = 4.667\times 10^{-3}\cdot\text{in}$ $\delta_B = \dfrac{9}{4}\cdot\delta_C = 1.05\times 10^{-2}\cdot\text{in}$ downward

Problem 2.2-8

$L_{BD} = 350\text{mm}$ $L_{CE} = 450\text{mm}$ $A = 720\text{mm}^2$ $E = 200\text{GPa}$ $P = 20\text{kN}$

Statics - find axial forces in BD and CE - remove pins at B and E, use FBD of beam ABC - assume beam is rigid

$$\Sigma M_B = 0 \qquad CE = \frac{1}{350\text{mm}} \cdot [P \cdot (600\text{mm})] = 34.286 \cdot \text{kN} \quad \text{CE is in tension; force CE acts downward on ABC}$$

$$\Sigma F_y = 0 \qquad BD = P + CE = 54.286 \cdot \text{kN} \qquad \text{BD is in compression; force BD acts upward on ABC}$$

Use force-displacement relation to find <u>change in lengths of CE and BD and vertical displacements at B and C</u>

$$\delta_{BD} = \frac{BD \cdot L_{BD}}{E \cdot A} = 0.13194 \cdot \text{mm} \quad \text{shortening} \qquad\qquad \delta_{CE} = \frac{CE \cdot L_{CE}}{E \cdot A} = 0.10714 \cdot \text{mm elongation}$$
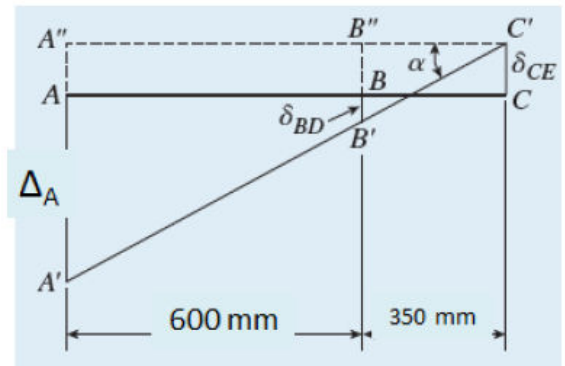
Use geometry to find downward displacement at A

$$\alpha = \text{atan}\left(\frac{\left|\delta_{BD}\right| + \delta_{CE}}{350\text{mm}}\right) = 0.03914 \cdot \text{deg}$$

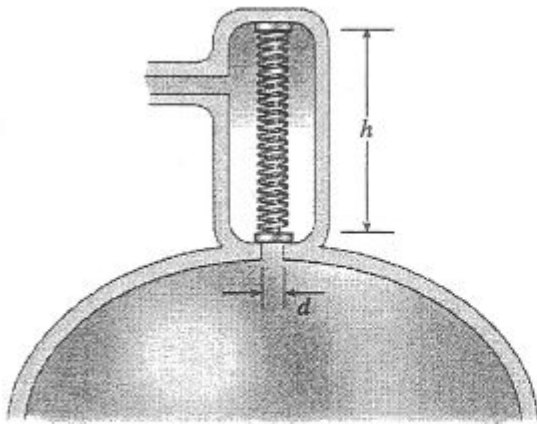$$\Delta_A = 950\text{mm} \cdot \tan(\alpha) - \delta_{CE} = 0.542 \cdot \text{mm} \quad \text{downward}$$

or similar triangles (see figure)
$$\frac{\Delta_A + \delta_{CE}}{600 + 350} = \frac{\left|\delta_{BD}\right| + \delta_{CE}}{350}$$

$$\Delta_A = \left(\left|\delta_{BD}\right| + \delta_{CE}\right) \cdot \left(\frac{950}{350}\right) - \delta_{CE} = 0.542 \cdot \text{mm}$$
downward

$$\frac{\delta_A + \delta_{CE}}{600 + 350} = \frac{\delta_{BD} + \delta_{CE}}{350}$$

Problem 2.2-9

h = height of valve (compressed length of the spring)

d = diameter of discharge hole

p = pressure in tank

p_{max} = pressure when valve opens

L = natural length of spring $(L > h)$

k = stiffness of spring

FORCE IN COMPRESSED SPRING

$F = k(L - h)$ (From Eq. 2-1a)

PRESSURE FORCE ON SPRING

$$P = p_{max}\left(\frac{\pi d^2}{4}\right)$$

EQUATE FORCES AND SOLVE FOR h:

$$F = P \quad k(L - h) = \frac{\pi p_{max} d^2}{4}$$

$$h = L - \frac{\pi p_{max} d^2}{4k} \quad \leftarrow$$
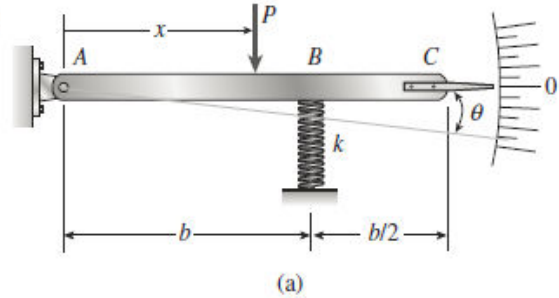
Problem 2.2-10

   $k = 950$ N/m $b = 165$ mm $P = 11$ N $\theta = 2.5°$ $\theta_{max} = 2°$

$W_p = 3$ N $W_s = 2.75$ N

(a) If the load $P = 11$ N, at what distance x should the load be placed so that the pointer will read $\theta = 2.5°$ on the scale (see Fig. a)?
Sum moments about A, then solve for x:

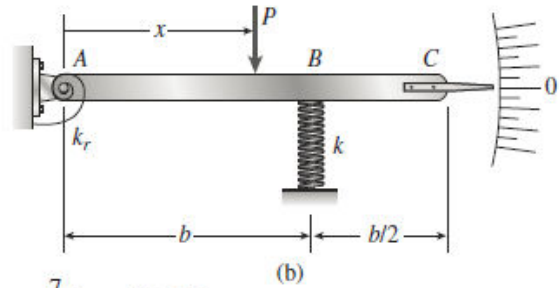$$x = \frac{k\theta b^2}{P} = 102.6 \text{ mm} \quad \boxed{x = 102.6 \text{ mm}}$$

(a)

(b) Repeat (a) if a rotational spring $k_r = kb^2$ is added at A (see Fig. b).

$k_r = k b^2 = 25864$ N·mm

Sum moments about A, then solve for x:

$$x = \frac{k\theta b^2 + k_r\theta}{P} = 205 \text{ mm} \quad \frac{x}{b} = 1.244 \quad \boxed{x = 205 \text{ mm}}$$

(b)

(c) Now if $x = 7b/8$, what is P_{max} (N) if θ cannot exceed $2°$? $x = \frac{7}{8} b = 144.375$ mm

Sum moments about A, then solve for P: $P_{max} = \dfrac{k\theta_{max} b^2 + k_r\theta_{max}}{\frac{7}{8}b} = 12.51$ N $\boxed{P_{max} = 12.51 \text{ N}}$

(d) Now, if the weight of the pointer ABC is known to be $W_p = 3$ N and the weight of the spring is $W_s = 2.75$ N, what initial angular position (i.e., θ in degrees) of the pointer will result in a zero reading on the angular scale once the pointer is released from rest? Assume $P = k_r = 0$.

Deflection at spring due to W_p: Deflection at B due to self weight of spring:

$$\delta_{Bp} = \frac{W_p\left(\frac{3}{4}b\right)}{kb} = 2.368 \text{ mm} \qquad \delta_{Bk} = \frac{W_s}{2k} = 1.447 \text{ mm}$$

$$\delta_B = \delta_{Bp} + \delta_{Bk} = 3.816 \text{ mm} \qquad \theta_{init} = \frac{\delta_B}{b} = 1.325°$$

$$\text{OR} \quad \theta_{init} = \arctan\left(\frac{\delta_B}{b}\right) = 1.325° \quad \boxed{\theta_{init} = 1.325°}$$
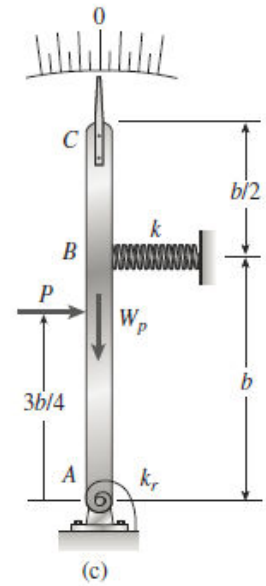
(e) If the pointer is rotated to a vertical position (figure part c), find the required load P, applied at mid-height of the pointer that will result in a pointer reading of $\theta = 2.5°$ on the scale. Consider the weight of the pointer, W_p, in your analysis.

$$k = 950 \text{ N/m} \quad b = 165 \text{ mm} \quad W_p = 3 \text{ N}$$

$$k_r = kb^2 = 25.864 \text{ N·m} \quad \theta = 2.5°$$

Sum moments about A to get P:

$$P = \frac{\theta}{\left(\dfrac{3b}{4}\right)}\left[k_r + k\left(\frac{5}{4}b^2\right) - W_p\left(\frac{3b}{4}\right)\right] = 20.388 \text{ N} \qquad \boxed{P = 20.4 \text{ N}}$$
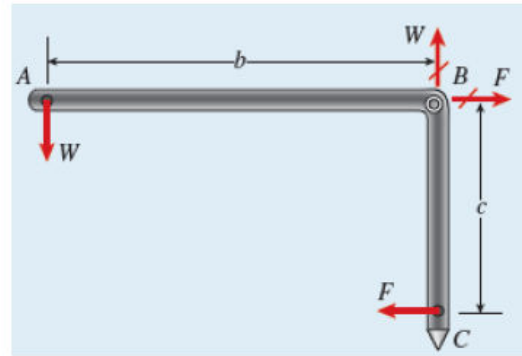

(c)

Problem 2.2-11

$b = 10\text{in}$ $c = 7\text{in}$ $k = 5\dfrac{\text{lbf}}{\text{in}}$ $p = \dfrac{1}{16}\text{in}$ $n = 12$

Use FBD of ABC (pin forces $B_x = F$ and $B_y = W$ at B; see fig.); sum moments about B s.t. Wb = Fc, F = force in spring

$\Sigma M_B = 0$ $W = F \cdot \dfrac{c}{b}$

Force in spring is $F = k \cdot (n \cdot p) = 3.75 \cdot \text{lbf}$ so $W = F \cdot \dfrac{c}{b} = 2.625 \cdot \text{lbf}$

Problem 2.2-12

$b = 30\text{cm}$ $c = 20\text{cm}$ $k = 3650\dfrac{\text{N}}{\text{m}}$ $p = 1.5\text{mm}$ $W = 65\text{N}$
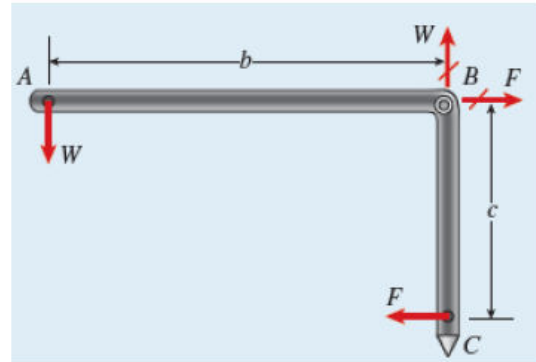
Force in parallel springs is $F = 2 \cdot k \cdot (n \cdot p)$

Sum moments about B (see FBD) to find F in terms of weight W

$Wb = Fc$ so $F = W \cdot \dfrac{b}{c}$

Substitute expression for F and solve for n

$$n = \frac{W \cdot \dfrac{b}{c}}{2 \cdot k \cdot p} = 8.904$$

Problem 2.2-13

(a) Derive a formula for the displacement δ_4 at point 4 when the load P is applied at joint 3 and moment PL is applied at joint 1, as shown.

Cut horizontally through both springs to create upper and lower FBD's. Sum moments about joint 1 for upper FBD and also sum moments about joint 6 for lower FBD to get two equations of equilibrium; assume both springs are in tension.

Note that $\quad \delta_2 = \dfrac{2}{3}\delta_3 \quad$ and $\quad \delta_5 = \dfrac{3}{4}\delta_4$

Force in left spring: $\quad k\left(\delta_4 - \dfrac{2}{3}\delta_3\right)$

Force in right spring: $\quad 2k\left(\dfrac{3}{4}\delta_4 - \delta_3\right)$

Summing moments about joint 1 (upper FBD) and about joint 6 (lower FBD) then dividing through by k gives

$$\begin{pmatrix} \dfrac{-22}{9} & \dfrac{13}{6} \\[2mm] \dfrac{-26}{9} & \dfrac{17}{6} \end{pmatrix}\begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \dfrac{-2P}{k} \\ 0 \end{pmatrix} \quad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \dfrac{-22}{9} & \dfrac{13}{6} \\[2mm] \dfrac{-26}{9} & \dfrac{17}{6} \end{pmatrix}^{-1}\begin{pmatrix} \dfrac{-2P}{k} \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{17P}{2k} \\[2mm] \dfrac{26P}{3k} \end{pmatrix} \quad \begin{matrix} \dfrac{17}{2} = 8.5 \\[3mm] \dfrac{26}{3} = 8.667 \end{matrix} \quad \boxed{\delta_4 = \dfrac{26P}{3k}}$$

^ deltas are **positive downward**

(b) Repeat part (a) if a rotational spring $k_r = kL^2$ is now added at joint 6. What is the ratio of the deflection $\delta4$ in part (a) to that in (b)?

Upper FBD—sum moments about joint 1:

$$k\left(\delta_4 - \dfrac{2}{3}\delta_3\right)\dfrac{2L}{3} + 2k\left(\dfrac{3}{4}\delta_4 - \delta_3\right)L = -2PL \quad \text{OR} \quad \left(\dfrac{22Lk}{9}\right)\delta_3 + \dfrac{13Lk}{6}\delta_4 = -2PL$$

Lower FBD—sum moments about joint 6:

$$k\left(\delta_4 - \dfrac{2}{3}\delta_3\right)\dfrac{4L}{3} + 2k\left(\dfrac{3}{4}\delta_4 - \delta_3\right)L - k_r\theta_6 = 0$$

$$\left[k\left(\delta_4 - \dfrac{2}{3}\delta_3\right)\dfrac{4L}{3} + 2k\left(\dfrac{3}{4}\delta_4 - \delta_3\right)L\right] + (kL^2)\left(\dfrac{\delta_4}{\dfrac{4}{3}L}\right) = 0 \quad \text{OR} \quad \left(\dfrac{26Lk}{9}\right)\delta_3 + \dfrac{43Lk}{12}\delta_4 = 0$$

Divide matrix equilibrium equations through by k to get the following displacement equations:

$$\begin{pmatrix} \dfrac{-22}{9} & \dfrac{13}{6} \\[2mm] \dfrac{-26}{9} & \dfrac{43}{6} \end{pmatrix}\begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \dfrac{-2P}{k} \\ 0 \end{pmatrix} \quad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \dfrac{-22}{9} & \dfrac{13}{6} \\[2mm] \dfrac{-26}{9} & \dfrac{43}{12} \end{pmatrix}^{-1}\begin{pmatrix} \dfrac{-2P}{k} \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{43P}{15k} \\[2mm] \dfrac{104P}{45k} \end{pmatrix} \quad \begin{matrix} \dfrac{43}{15} = 2.867 \\[3mm] \dfrac{104}{45} = 2.311 \end{matrix} \quad \boxed{\delta_4 = \dfrac{104P}{45k}}$$

^ deltas are positive downward

Ratio of the deflection δ_4 in part (a) to that in (b): $\quad \dfrac{\dfrac{26}{3}}{\dfrac{104}{45}} = \dfrac{15}{4} \quad \boxed{\text{Ratio} = \dfrac{15}{4} = 3.75}$

Problem 2.2-14

(a)

NUMERICAL DATA

$A = 3900 \text{ mm}^2 \quad E = 200 \text{ GPa}$

$P = 475 \text{ kN} \quad L = 3000 \text{ mm}$

$\delta_{Bmax} = 1.5 \text{ mm}$

(a) FIND HORIZONTAL DISPLACEMENT OF JOINT B
    STATICS TO FIND SUPPORT REACTIONS AND THEN MEMBER FORCES:

$$\Sigma M_A = 0 \quad B_y = \frac{1}{L}\left(2P\frac{L}{2}\right)$$
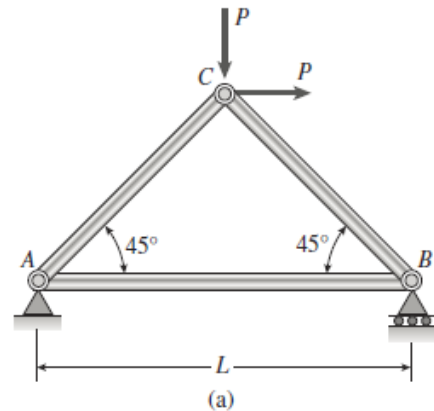
$$B_y = P$$

$$\Sigma F_H = 0 \quad A_x = -P$$

$$\Sigma F_V = 0 \quad A_y = P - B_y \quad A_y = 0$$

METHOD OF JOINTS: $AC_V = A_Y \quad AC_V = 0 \quad$ Force in $AC = 0$

$AB = A_X$

Force in AB is P (tension) so elongation of AB is the horizontal displacement of joint B.
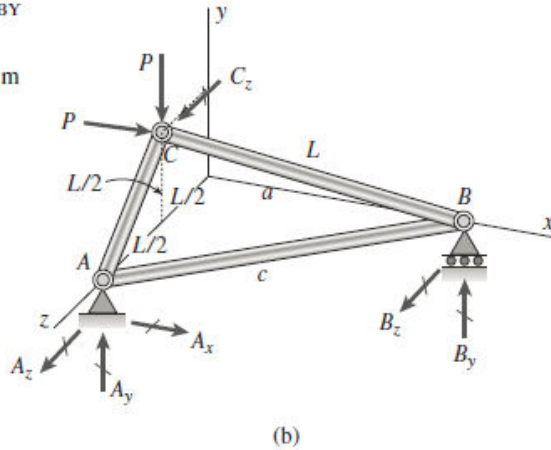
$$\delta_B = \frac{F_{AB}L}{EA} \quad \delta_B = \frac{PL}{EA} \quad \delta_B = 1.82692 \text{ mm} \quad \boxed{\delta_B = 1.827 \text{ mm}}$$

(b) FIND P_{max} IF DISPLACEMENT OF JOINT $B = \delta_{Bmax} = 1.5 \text{ mm} \quad P_{max} = \frac{EA}{L}\delta_{Bmax} \quad \boxed{P_{max} = 390 \text{ kN}}$

(c) REPEAT PARTS (a) AND (b) IF THE PLANE TRUSS IS REPLACED BY
    A SPACE TRUSS (SEE FIGURE PART b).

    FIND MISSING DIMENSIONS a AND c: $\quad P = 475 \text{ kN} \quad L = 3 \text{ m}$

(b)

$$a = \sqrt{L^2 - 2\left(\frac{L}{2}\right)^2} = 2.12132 \text{ m} \qquad \frac{a}{L} = 0.707 \qquad a = \frac{L}{\sqrt{2}} = 2.12132 \text{ m}$$

$$c = \sqrt{L^2 + a^2} = 3.67423 \text{ m} \qquad c = \sqrt{L^2 + \left(\frac{L}{\sqrt{2}}\right)^2} = 3.67423 \text{ m} \qquad c = L\sqrt{\frac{3}{2}} = 3.67423 \text{ m}$$

(1) SUM MOMENTS ABOUT A LINE THRU A WHICH IS PARALLEL TO THE y-AXIS

$$B_z = -P\frac{L}{a} = -671.751 \text{ kN}$$

(2) Sum moments about the z-axis

$$B_y = \frac{P\left(\dfrac{L}{2}\right)}{a} = 335.876 \text{ kN} \qquad \text{SO} \qquad A_y = P - B_y = 139.124 \text{ kN}$$

(3) Sum moments about the x-axis

$$C_z = \frac{A_y L - P \dfrac{L}{2}}{\dfrac{L}{2}} = -196.751 \text{ kN}$$

(4) Sum forces in the x- and z-directions $\qquad A_x = -P = -475 \text{ kN} \qquad A_z = -C_z - B_z = 868.503 \text{ kN}$

(5) Use method of joints to find member forces

Sum forces in x-direction at joint A: $\quad \dfrac{a}{c}F_{AB} + A_x = 0 \qquad F_{AB} = \dfrac{-c}{a}A_x = 823 \text{ kN}$

Sum forces in y-direction at joint A: $\quad \dfrac{\dfrac{L}{2}}{\sqrt{2}\dfrac{L}{2}}F_{AC} + A_y = 0 \qquad F_{AC} = \sqrt{2}(-A_y) = -196.8 \text{ kN}$

Sum forces in y-direction at joint B: $\quad \dfrac{\dfrac{L}{2}}{L}F_{BC} + B_y = 0 \qquad F_{BC} = -2B_y = -672 \text{ kN}$

(6) Find displacement along x-axis at joint B

Find change in length of member AB then find its projection along x axis:

$$\delta_{AB} = \frac{F_{AB}c}{EA} = 3.875 \text{ mm} \quad \beta = \arctan\left(\frac{L}{a}\right) = 54.736° \quad \delta_{Bx} = \frac{\delta_{AB}}{\cos(\beta)} = 6.713 \text{ mm} \quad \boxed{\delta_{Bx} = 6.71 \text{ mm}}$$

(7) Find P_{max} for space truss if δ_{Bx} must be limited to 1.5 mm

Displacements are linearly related to the loads for this linear elastic small displacement problem, so reduce load variable P from 475 kN to

$$\frac{1.5}{6.71254}\, 475 = 106.145 \text{ kN} \qquad \boxed{P_{max} = 106.1 \text{ kN}}$$

Repeat space truss analysis using vector operations $\quad a = 2.121 \text{ m} \quad L = 3 \text{ m} \quad P = 475 \text{ kN}$

Position and unit vectors:

$$r_{AB} = \begin{pmatrix} a \\ 0 \\ -L \end{pmatrix} \quad e_{AB} = \frac{r_{AB}}{|r_{AB}|} = \begin{pmatrix} 0.577 \\ 0 \\ -0.816 \end{pmatrix} \quad r_{AC} = \begin{pmatrix} 0 \\ \dfrac{L}{2} \\ \dfrac{-L}{2} \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} 0 \\ 0.707 \\ -0.707 \end{pmatrix}$$

Find moment at A:

$$M_A = r_{AB} \times R_B + r_{AC} \times R_C$$

$$M_A = r_{AB} \times \begin{pmatrix} 0 \\ RB_y \\ RB_z \end{pmatrix} + r_{AC} \times \begin{pmatrix} 2.P \\ -P \\ RC_z \end{pmatrix} = \begin{pmatrix} 3.0 \text{ m } RB_y + 1.5 \text{ m } RC_z - 712.5 \text{ kN·m} \\ -2.1213 \text{ m } RB_Z - 1425.0 \text{ kN·m} \\ 2.1213 \text{ m } RB_y - 1425.0 \text{ kN·m} \end{pmatrix}$$

FIND MOMENTS ABOUT LINES OR AXES:

$M_A e_{AB} = -1.732 \text{ m } RB_y + 1.7321 \text{ m } RB_y + 0.86603 \text{ m } RC_z + 752.15 \text{ kN·m}$

$$RC_z = \frac{-244.12}{0.72169} = -338.262 \qquad C_z = -196.751 \text{ kN}$$

$M_A e_{AC} = -1.5 \text{ m } RB_y + -1.5 \text{ m } RB_z \qquad \text{So} \qquad RB_y = -RB_z$

$$M_A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -2.1213 \text{ m } RB_z + -1425.0 \text{ kN·m} \quad \text{So} \quad RB_z = \frac{462.5}{-1.7678} = -261.625 \quad B_z = -671.75 \text{ kN}$$

$$M_A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2.1213 \text{ m } RB_y + -1425.0 \text{ kN·m} \quad \text{So} \quad RB_y = -RB_z = 261.625 \quad B_y = -335.876 \text{ kN}$$

$\Sigma F_y = 0 \qquad A_y = P - B_y = 139.124 \text{ kN}$

Reactions obtained using vector operations agree with those based on scalar operations.

Problem 2.2-15

$d = \dfrac{1}{10}$ in. $L = 12(12)$ in. $E = 10{,}600 \times (10^3)$ psi

$\delta_a = \dfrac{1}{8}$ in. $\sigma_a = 10 \times (10^3)$ psi

$A = \dfrac{\pi d^2}{4}$ $A = 7.854 \times 10^{-3}$ in.^{2}

$EA = 8.325 \times 10^4$ lb

Maximum load based on elongation:

$$P_{max1} = \dfrac{EA}{L}\delta_a \quad P_{max1} = 72.3 \text{ lb} \quad \leftarrow \text{ controls}$$

Maximum load based on stress:

$$P_{max2} = \sigma_a A \quad P_{max2} = 78.5 \text{ lb}$$

Problem 2.2-16

$W = 25\,\text{N}$ $k_1 = 0.300\,\text{N/mm}$ $L_1 = 250\,\text{mm}$

$k_2 = 0.400\,\text{N/mm}$ $L_2 = 200\,\text{mm}$

$L = 350\,\text{mm}$ $h = 80\,\text{mm}$ $P = 18\,\text{N}$

(a) LOCATION OF LOAD P TO BRING BAR TO HORIZONTAL POSITION

Use statics to get forces in both springs:

$$\sum M_A = 0 \qquad F_2 = \frac{1}{L}\left(W\frac{L}{2} + Px\right)$$

$$F_2 = \frac{W}{2} + P\frac{x}{L}$$

$$\sum F_V = 0 \qquad F_1 = W + P - F_2$$

$$F_1 = \frac{W}{2} + P\left(1 - \frac{x}{L}\right)$$

Use constraint equation to define horizontal position, then solve for location x:

$$L_1 + \frac{F_1}{k_1} = L_2 + h + \frac{F_2}{k_2}$$

Substitute expressions for F_1 and F_2 above into constraint equilibrium and solve for x:

$$x = \frac{-2L_1\,L\,k_1\,k_2 - k_2 WL - 2k_2\,P\,L + 2L_2\,L\,k_1\,k_2 + 2\,h\,L\,k_1\,k_2 + k_1 WL}{-2P(k_1 + k_2)}$$

$$x = 134.7\,\text{mm} \quad \leftarrow$$

(b) NEXT REMOVE P AND FIND NEW VALUE OF SPRING CONSTANT k_1 SO THAT BAR IS HORIZONTAL UNDER WEIGHT W

Now, $F_1 = \dfrac{W}{2}$ $F_2 = \dfrac{W}{2}$ since $P = 0$
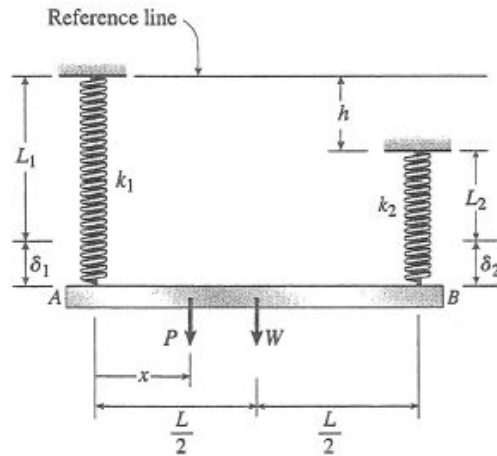
Same constraint equation as above but now $P = 0$:

$$L_1 + \frac{\frac{W}{2}}{k_1} - (L_2 + h) - \frac{\left(\frac{W}{2}\right)}{k_2} = 0$$

Solve for k_1:

$$k_1 = \frac{-Wk_2}{[2k_2[L_1 - (L_2 + h)]] - W}$$

$$k_1 = 0.204\,\text{N/mm} \quad \leftarrow$$

PART (c)—CONTINUED (from page below)

STATICS

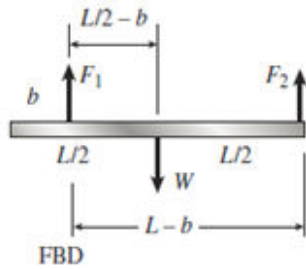$$\sum M_{k_1} = 0 \qquad F_2 = \frac{w\left(\frac{L}{2} - b\right)}{L - b}$$

$$\sum F_V = 0$$

$$F_1 = W - F_2$$

$$F_1 = W - \frac{w\left(\frac{L}{2} - b\right)}{L - b}$$

$$F_1 = \frac{WL}{2(L - b)}$$

Part (c) continued in right column below

(c) USE $k_1 = 0.300$ N/mm BUT RELOCATE SPRING k_1 $(x = b)$ SO THAT BAR ENDS UP IN HORIZONTAL POSITION UNDER WEIGHT W

FBD

$$b = \frac{2L_1k_1k_2L + WLk_2 - 2L_2k_1k_2L - 2hk_1k_2L - Wk_1L}{(2L_1k_1k_2) - 2L_2k_1k_2 - 2hk_1k_2 - 2Wk_1}$$

(d) REPLACE SPRING k_1 WITH SPRINGS IN SERIES: $k_1 = 0.3$ N/mm, $L_1/2$, AND k_3, $L_1/2$. FIND k_3 SO THAT BAR HANGS IN HORIZONTAL POSITION

STATICS $F_1 = \dfrac{W}{2}$ $F_2 = \dfrac{W}{2}$

$$k_3 = \frac{Wk_1k_2}{-2L_1k_1k_2 - Wk_2 + 2L_2k_1k_2 + 2hk_1k_2 + Wk_1}$$

NOTE—equivalent spring constant for series springs:
$$k_e = \frac{k_1k_3}{k_1 + k_3}$$

Constraint equation—substitute above expressions for F_1 and F_2 and solve for b:

$$L_1 + \frac{F_1}{k_1} - (L_2 + h) - \frac{F_2}{k_2} = 0$$

Use the following data:

$k_1 = 0.300$ N/mm $k_2 = 0.4$ N/mm $L_1 = 250$ mm

$L_2 = 200$ mm $L = 350$ mm

$b = 74.1$ mm   ←

New constraint equation; solve for k_3:

$$L_1 + \frac{F_1}{k_1} + \frac{F_1}{k_3} - (L_2 + h) - \frac{F_2}{k_2} = 0$$

$$L_1 + \frac{W/2}{k_1} + \frac{W/2}{k_3} - (L_2 + h) - \frac{W/2}{k_2} = 0$$

$k_3 = 0.638$ N/mm   ←

$k_e = 0.204$ N/mm   ←   checks—same as (b) above

Problem 2.2-17

The figure shows a section cut through the pipe, cap, and rod.

NUMERICAL DATA

$E_c = 12000$ ksi $E_b = 14,000$ ksi

$W = 2$ k $d_c = 6$ in. $d_r = \dfrac{1}{2}$ in.

$\sigma_a = 5$ ksi $\delta_a = 0.02$ in.

Unit weights (see Table I-1): $\gamma_s = 2.836 \times 10^{-4}$ k/in.^{3}

$\gamma_b = 3.009 \times 10^{-4}$ k/in.^{3}

$L_c = 48$ in. $L_r = 42$ in.

$t_s = 1$ in.

(a) MINIMUM REQUIRED WALL THICKNESS OF CAST IRON PIPE, t_{cmin}

First check allowable stress then allowable shortening:

$$W_{cap} = \gamma_s \left(\frac{\pi}{4} d_c^2 t_s \right)$$

$$W_{cap} = 8.018 \times 10^{-3} \text{ k}$$

$$W_{rod} = \gamma_b \left(\frac{\pi}{4} d_r^2 L_r \right)$$

$$W_{rod} = 2.482 \times 10^{-3} \text{ k}$$

$W_t = W + W_{cap} + W_{rod}$ $W_t = 2.01$ k

$$A_{min} = \frac{W_t}{\sigma_a} \qquad A_{min} = 0.402 \text{ in.}^2$$

$$A_{pipe} = \frac{\pi}{4}[d_c^2 - (d_c - 2t_c)^2]$$

$A_{pipe} = \pi t_c(d_c - t_c)$

$$t_c(d_c - t_c) = \frac{W_t}{\pi \sigma_a}$$

Let $\alpha = \dfrac{W_t}{\pi \sigma_a}$ $\alpha = 0.128$:

$t_c^2 - d_c t_c + \alpha = 0$

$$t_c = \frac{d_c - \sqrt{d_c^2 - 4\alpha}}{2} \qquad t_c = 0.021 \text{ in.}$$

^ minimum based on σ_a

Now check allowable shortening requirement:

$$\delta_{pipe} = \frac{W_t L_c}{E_c A_{min}} \qquad A_{min} = \frac{W_t L_c}{E_c \delta_a}$$

$A_{min} = 0.447$ in.^{2} < larger than value based on

σ_a above

$$\pi t_c(d_c - t_c) = \frac{W_t L_c}{E_c \delta_a}$$

$$t_c^2 - d_c t_c + \beta = 0 \qquad \beta = \frac{W_t L_c}{\pi E_c \delta_a}$$

$$\beta = 0.142$$

$$t_c = \frac{d_c - \sqrt{d_c^2 - 4\beta}}{2}$$

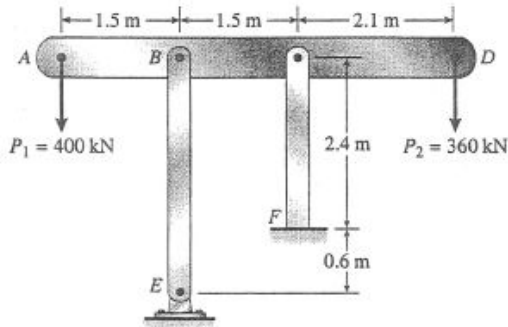$t_c = 0.021$ in.   ← minimum based on δ_a and σ_a

controls

(b) ELONGATION OF ROD DUE TO SELF WEIGHT AND ALSO WEIGHT W

$$\delta_r = \frac{\left(W + \dfrac{W_{rod}}{2} \right) L_r}{E_b \left(\dfrac{\pi}{4} d_r^2 \right)} \qquad \delta_r = 0.031 \text{ in.} \quad \leftarrow$$

(c) MINIMUM CLEARANCE h

$h_{min} = \delta_a + \delta_r$ $h_{min} = 0.051$ in.   ←

Problem 2.2-18

$A_{BE} = 11{,}100$ mm^2

$A_{CF} = 9{,}280$ mm^2

$E = 200$ GPa

$L_{BE} = 3.0$ m

$L_{CF} = 2.4$ m

$P_1 = 400$ kN; $P_2 = 360$ kN

FREE-BODY DIAGRAM OF BAR *ABCD*
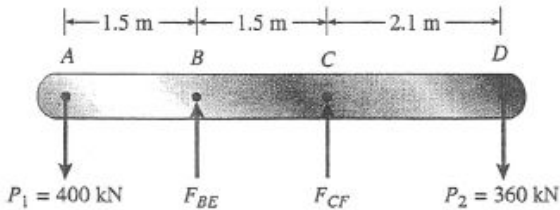
$\Sigma M_B = 0 \;\curvearrowright$

$(400 \text{ kN})(1.5 \text{ m}) + F_{CF}(1.5 \text{ m}) - (360 \text{ kN})(3.6 \text{ m}) = 0$

$F_{CF} = 464$ kN

$\Sigma M_C = 0$

$(400 \text{ kN})(3.0 \text{ m}) - F_{BE}(1.5 \text{ m}) - (360 \text{ kN})(2.1 \text{ m}) = 0$
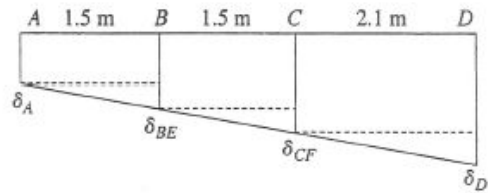
$F_{BE} = 296$ kN

SHORTENING OF BAR *BE*

$$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA_{BE}} = \frac{(296 \text{ kN})(3.0 \text{ m})}{(200 \text{ GPa})(11{,}100 \text{ mm}^2)}$$
$$= 0.400 \text{ mm}$$

SHORTENING OF BAR *CF*

$$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA_{CF}} = \frac{(464 \text{ kN})(2.4 \text{ m})}{(200 \text{ GPa})(9{,}280 \text{ mm}^2)}$$
$$= 0.600 \text{ mm}$$

DISPLACEMENT DIAGRAM

$\delta_{BE} - \delta_A = \delta_{CF} - \delta_{BE}$ or $\delta_A = 2\delta_{BE} - \delta_{CF}$

$\delta_A = 2(0.400 \text{ mm}) - 0.600 \text{ m}$

$\quad = 0.200 \text{ mm} \quad \leftarrow$

$\quad$ (Downward)

$\delta_D - \delta_{CF} = \dfrac{2.1}{1.5}(\delta_{CF} - \delta_{BE})$

or $\quad \delta_D = \dfrac{12}{5}\delta_{CF} - \dfrac{7}{5}\delta_{BE}$

$\quad\quad = \dfrac{12}{5}(0.600 \text{ mm}) - \dfrac{7}{5}(0.400 \text{ mm})$

$\quad\quad = 0.880 \text{ mm} \quad \leftarrow$

$\quad\quad$ (Downward)

Problem 2.2-19

(a) DISPLACEMENT δ_D

Use *FBD* of beam *BCD* $\Sigma M_B = 0$ $R_C = \dfrac{1}{L}\left[\left(2\dfrac{P}{L}\right)\left(\dfrac{3}{4}L\right)\left(\dfrac{3}{8}L\right) + \dfrac{P}{4}\left(L + \dfrac{3}{4}L\right)\right] = P$ <compression force in column *CF*

$$\Sigma F_V = 0 \quad R_B = \left(2\dfrac{P}{L}\right)\left(\dfrac{3}{4}L\right) + \dfrac{P}{4} - R_C = \dfrac{3P}{4} \quad \text{<compression force in column } BA$$

Downward displacements at *B* and *C*: $\delta_B = R_B f_1 = \dfrac{3Pf_1}{4}$ $\delta_C = R_C f_2 = Pf_2$

Geometry: $\delta_D = \delta_B + (\delta_C - \delta_B)\left(\dfrac{L + \dfrac{3}{4}L}{L}\right) = \dfrac{7Pf_2}{4} - \dfrac{9Pf_1}{16}$ $\boxed{\delta_D = \dfrac{7Pf_2}{4} - \dfrac{9Pf_1}{16} = \dfrac{P}{16}(28f_2 - 9f_1)}$

(b) DISPLACEMENT TO HORIZONTAL POSITION, SO $\delta_C = \delta_B$ and $\dfrac{3Pf_1}{4} = Pf_2$ or $\dfrac{f_1}{f_2} = \dfrac{4}{3}$

$\dfrac{\dfrac{L_1}{EA_1}}{\dfrac{L_2}{EA_2}} = \dfrac{4}{3}$ or $\dfrac{L_1}{L_2} = \dfrac{4}{3}\left(\dfrac{A_1}{A_2}\right)$ $\dfrac{L_1}{L_2} = \dfrac{4}{3}\left(\dfrac{\dfrac{\pi}{4}d_1^2}{\dfrac{\pi}{4}d_2^2}\right) = \dfrac{4d_1^2}{3d_2^2}$ $\dfrac{L_1}{L_2} = \dfrac{4}{3}\left(\dfrac{d_1}{d_2}\right)^2$ with $\dfrac{d_1}{d_2} = \dfrac{9}{8}$

$\dfrac{L_1}{L_2} = \dfrac{4}{3}\left(\dfrac{9}{8}\right)^2 = \dfrac{27}{16}$ $\boxed{\dfrac{L_1}{L_2} = \dfrac{27}{16}}$

(c) IF $L_1 = 2L_2$, FIND THE d_1/d_2 RATIO SO THAT BEAM *BCD* DISPLACES DOWNWARD TO A HORIZONTAL POSITION

$\dfrac{L_1}{L_2} = 2$ and $\delta_C = \delta_B$ from part (b). $\left(\dfrac{d_1}{d_2}\right)^2 = \dfrac{3}{4}\left(\dfrac{L_1}{L_2}\right)$ so $\boxed{\dfrac{d_1}{d_2} = \sqrt{\dfrac{3}{4}(2)} = 1.225}$

(d) IF $d_1 = (9/8)\,d_2$ AND $L_1/L_2 = 1.5$, AT WHAT HORIZONTAL DISTANCE *x* FROM *B* SHOULD LOAD *P/4* AT *D* BE PLACED?

Given $\dfrac{d_1}{d_2} = \dfrac{9}{8}$ and $\dfrac{L_1}{L_2} = 1.5$ or $\dfrac{f_1}{f_2} = \dfrac{L_1}{L_2}\left(\dfrac{A_2}{A_1}\right)$ $\dfrac{f_1}{f_2} = \dfrac{L_1}{L_2}\left(\dfrac{d_2}{d_1}\right)^2 = \dfrac{3}{2}\left(\dfrac{8}{9}\right)^2 = \dfrac{32}{27}$

Recompute column forces R_B and R_C but now with load *P/4* positioned at distance *x* from *B*.

Use *FBD* of beam *BCD*: $\Sigma M_B = 0$ $R_C = \dfrac{1}{L}\left[\left(2\dfrac{P}{L}\right)\left(\dfrac{3}{4}L\right)\left(\dfrac{3}{8}L\right) + \dfrac{P}{4}(x)\right] = \dfrac{\dfrac{9LP}{16} + \dfrac{Px}{4}}{L}$

$$\Sigma F_V = 0 \quad R_B = \left(2\dfrac{P}{L}\right)\left(\dfrac{3}{4}L\right) + \dfrac{P}{4} - R_C = \dfrac{7P}{4} - \dfrac{\dfrac{9LP}{16} + \dfrac{Px}{4}}{L}$$

Horizontal displaced position under load *q* and load *P/4* so $\delta_C = \delta_B$ or $R_C f_2 = R_B f_1$.

$$\left(\frac{\dfrac{9LP}{16} + \dfrac{Px}{4}}{L}\right) f_2 = \left(\frac{7P}{4} - \frac{\dfrac{9LP}{16} + \dfrac{Px}{4}}{L}\right) f_1 \text{ solve, } x = -\frac{9Lf_2 - 19Lf_1}{4f_1 + 4f_2} = -\frac{L(9f_2 - 19f_1)}{4(f_1 + f_2)}$$

$$x = -\frac{L(9f_2 - 19f_1)}{4(f_1 + f_2)} \text{ or } x = L\left[\frac{19\dfrac{f_1}{f_2} - 9}{4\left(\dfrac{f_1}{f_2} + 1\right)}\right]$$

Now substitute f_1/f_2 ratio from above:
$$x = L\left[\frac{19\dfrac{32}{27} - 9}{4\left(\dfrac{32}{27} + 1\right)}\right] = \frac{365L}{236} \qquad \frac{365}{236} = 1.547$$

Problem 2.2-20

Apply the laws of statics to the structure in its displaced position; also use FBD's of the left and right bars alone (referred to as LHFB and RHFB below).

OVERALL FBD: $\quad \sum F_H = 0 \qquad H_A - k_1\delta = 0 \qquad$ so $\qquad H_A = k_1\delta$

$\qquad\qquad\qquad \sum F_V = 0 \qquad R_A + R_C = P$

$\qquad\qquad\qquad \sum M_A = 0 \qquad k_r(\alpha - \theta) - P\dfrac{L_2}{2} + R_C L_2 = 0 \qquad R_C = \dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right]$

LHFB: $\qquad\qquad \sum M_B = 0 \qquad H_A h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) - R_A\left(\dfrac{L_2}{2}\right) + k_r(\alpha - \theta) = 0$

$\qquad\qquad\qquad\qquad\qquad R_A = \dfrac{2}{L_2}\left[k_1\delta h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_r(\alpha - \theta)\right]$

RHFB: $\qquad\qquad \sum M_B = 0 \qquad -k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) - k_1\delta h + R_C\dfrac{L_2}{2} = 0 \qquad R_C = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right]$

Equate the two expressions for R_C then substitute expressions for L_2, k_r, k_1, h and δ

$\dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right] \quad$ OR

$\dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\dfrac{2}{L_2}\left[k\dfrac{2b(\cos(\theta) - \cos(\alpha))\,b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE δ

$\quad b = 200$ mm $\quad k = 3.2$ kN/m $\quad \alpha = 45° \quad P = 50$ N $\quad k_1 = 0 \quad k_r = 0$

$\quad L_2 = 2b\cos(\theta) \quad L_1 = 2b\cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b\sin(\theta)$

$\dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\dfrac{1}{L_2}\left[k\dfrac{2b(\cos(\theta) - \cos(\alpha))\,b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$

Solving above equation numerically gives $\boxed{\theta = 35.1°}$ $\boxed{\delta = 44.6 \text{ mm}}$

COMPUTE REACTIONS

$R_C = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right] = 25$ N $\qquad R_C = \dfrac{1}{L_C}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] = 25$ N

$R_A = \dfrac{2}{L_2}\left[k_1\delta h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_r(\alpha - \theta)\right] = 25$ N $\qquad M_A = k_r(\alpha - \theta) = 0$

$R_A + R_C = 50$ N $\quad<$check $\qquad\qquad\qquad\qquad\qquad \boxed{R_A = 25 \text{ N}} \qquad \boxed{R_C = 25 \text{ N}}$

(b) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE δ

$\quad b = 200$ mm $\qquad k = 3.2$ kN/m $\qquad \alpha = 45° \qquad P = 50$ N $\qquad k_1 = \dfrac{k}{2} \qquad k_r = \dfrac{k}{2}b^2$

$\quad L_2 = 2b\cos(\theta) \qquad L_1 = 2b\cos(\alpha) \qquad \delta = L_2 - L_1 \qquad \delta = 2b(\cos(\theta) - \cos(\alpha)) \qquad h = b\sin(\theta)$

$\dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\dfrac{2}{L_2}\left[k\dfrac{2b(\cos(\theta) - \cos(\alpha))\,b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$

Solving above equation numerically gives $\boxed{\theta = 43.3°}$ $\boxed{\delta = 8.19 \text{ mm}}$

COMPUTE REACTIONS

$R_C = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right] = 18.5$ N $\quad R_2 = \dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] = 18.5$ N

$R_A = \dfrac{2}{L_2}\left[k_1\delta h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_r(\alpha - \theta)\right] = 31.5$ N $\quad M_A = k_r(\alpha - \theta) = 1.882$ N·m

$R_A + R_C = 50$ N $\quad<$ check $\qquad \boxed{R_A = 31.5 \text{ N}} \qquad \boxed{R_C = 18.5 \text{ N}} \qquad \boxed{M_A = 1.882 \text{ N·m}}$

Problem 2.2-21

Apply the laws of statics to the structure in its displaced position; also use FBD's of the left and right bars alone (referred to as LHFB and RHFB below)

OVERALL FBD $\quad \Sigma F_H = 0 \qquad H_A - k_1\delta = 0 \qquad$ so $\qquad H_A = k_1\delta$

$\qquad\qquad\qquad \Sigma F_V = 0 \qquad R_A + R_C = P$

$\qquad\qquad\qquad \Sigma M_A = 0 \qquad k_r(\alpha - \theta) - P\dfrac{L_2}{2} + R_C L_2 = 0 \qquad R_C = \dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right]$

LHFB $\qquad\quad \Sigma M_B = 0 \qquad H_A h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) - R_A\dfrac{L_2}{2} + k_r(\alpha - \theta) = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad R_A = \dfrac{2}{L_2}\left[k_1\delta h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_r(\alpha - \theta)\right]$

RHFB $\qquad\quad \Sigma M_B = 0 \qquad -k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) - k_1\delta h + R_C\dfrac{L_2}{2} = 0 \qquad R_C = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right]$

Equate the two expressions above for R_C then substitute expressions for L_2, k_r, k_1, h, and δ

$\dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right] \qquad$ OR

$\dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] - \dfrac{2}{L_2}\left[k\dfrac{2b(\cos(\theta) - \cos(\alpha))}{2}\dfrac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right] = 0$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE δ

$\quad b = 8$ in. $\qquad k = 16$ lb/in. $\qquad \alpha = 45° \qquad P = 10 lb \qquad k_1 = 0 \qquad k_r = 0$

$\quad L_2 = 2b\cos(\theta) \qquad L_1 = 2b\cos(\alpha) \qquad \delta = L_2 - L_1 \qquad \delta = 2b(\cos(\theta) - \cos(\alpha)) \qquad h = b\sin(\theta)$

$\quad \dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] - \dfrac{2}{L_2}\left[k\dfrac{2b(\cos(\theta) - \cos(\alpha))b\sin(\theta)}{2}\dfrac{}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right] = 0$

Solving above equation numerically gives $\quad \boxed{\theta = 35.1°} \quad \boxed{\delta = 1.782 \text{ in.}}$

COMPUTE REACTIONS

$R_C = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right] = 5$ lb $\qquad R_C = \dfrac{1}{L_C}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] = 5$ lb

$R_A = \dfrac{2}{L_2}\left[k_1\delta h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1(\alpha - \theta)\right] = 5$ lb $\qquad M_A = k_r(\alpha - \theta) = 0$

$R_A + R_C = 10$ lb $\qquad <$ check $\qquad \boxed{R_A = 5 lb} \quad \boxed{R_C = 5 lb}$

(b) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE δ

$b = 8$ in. $k = 16$ lb/in. $\alpha = 45°$ $P = 10 lb$ $k_1 = \dfrac{k}{2}$ $k_r = \dfrac{k}{2} b^2$

$L_2 = 2b\cos(\theta)$ $L_1 = 2b\cos(\alpha)$ $\delta = L_2 - L_1$ $\delta = 2b(\cos(\theta) - \cos(\alpha))$ $h = b\sin(\theta)$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta) \right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2} \frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta)) \right] \right] = 0$$

Solving above equation numerically gives $\boxed{\theta = 43.3°}$ $\boxed{\delta = 0.327 \text{ in.}}$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2}\left[k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_1\delta h \right] = 3.71 \text{ lb} \qquad R_C = \frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta) \right] = 3.71 \text{ lb}$$

$$R_A = \frac{2}{L_2}\left[k_1\delta h + k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_r(\alpha - \theta) \right] = 6.3 \text{ lb} \qquad M_A = k_r(\alpha - \theta) = 1.252 \text{ ft·lb}$$

$R_A + R_C = 10.01 \text{ lb}$ < check $\boxed{R_A = 6.3 \text{ lb}}$ $\boxed{R_C = 3.71 \text{ lb}}$ $\boxed{M_A = 1.252 \text{ lb·ft}}$

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Problem 2.3-1

NUMERICAL DATA

$P = 3$ k $L_1 = 20$ in. $L_2 = 50$ in. $d_A = 0.5$ in. $d_B = 1$ in. $E = 18000$ ksi

(a) TOTAL ELONGATION

$$\delta_1 = \frac{4PL_1}{\pi E d_A d_B} = 0.00849 \text{ in.} \qquad \delta_2 = \frac{PL_2}{E\frac{\pi}{4}d_B{}^2} = 0.01061 \text{ in.}$$

$\delta = 2\delta_1 + \delta_2 = 0.0276$ in. $\boxed{\delta = 0.0276 \text{ in.}}$

(b) FIND NEW DIAMETERS AT B AND C IF TOTAL ELONGATION CANNOT EXCEED 0.025 in.

$$2\left(\frac{4PL_1}{\pi E d_A d_B}\right) + \frac{PL_2}{E\frac{\pi}{4}d_B^2} = 0.025 \text{ in.} \qquad \text{Solving for } d_B\text{:} \qquad \boxed{d_B = 1.074 \text{ in.}}$$

Problem 2.3-2

Steel
post

Copper
bar

L_s

L_c

P
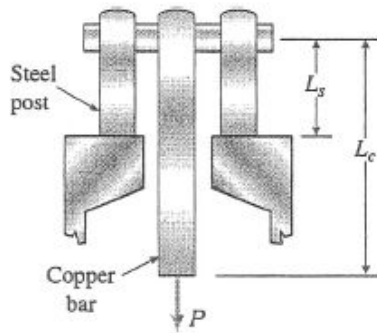
$L_c = 2.0$ m

$A_c = 4800$ mm^2

$E_c = 120$ GPa

$L_s = 0.5$ m

$A_s = 4500$ mm^2

$E_s = 200$ GPa

(a) DOWNWARD DISPLACEMENT δ ($P = 180$ kN)

$$\delta_c = \frac{PL_c}{E_c A_c} = \frac{(180\ \text{kN})(2.0\ \text{m})}{(120\ \text{GPa})(4800\ \text{mm}^2)}$$

$$= 0.625\ \text{mm}$$

$$\delta_s = \frac{(P/2)L_s}{E_s A_s} = \frac{(90\ \text{kN})(0.5\ \text{m})}{(200\ \text{GPa})(4500\ \text{mm}^2)}$$

$$= 0.050\ \text{mm}$$

$$\delta = \delta_c + \delta_s = 0.625\ \text{mm} + 0.050\ \text{mm}$$

$$= 0.675\ \text{mm}\ \ \leftarrow$$

(b) MAXIMUM LOAD P_{max} ($\delta_{max} = 1.0$ mm)

$$\frac{P_{max}}{P} = \frac{\delta_{max}}{\delta} \qquad P_{max} = P\left(\frac{\delta_{max}}{\delta}\right)$$

$$P_{max} = (180\ \text{kN})\left(\frac{1.0\ \text{mm}}{0.675\ \text{mm}}\right) = 267\ \text{kN}\ \ \leftarrow$$

Problem 2.3-3

Numerical data

$A = 0.40$ in.^{2} $P_1 = 1700$ lb

$P_2 = 1200$ lb $P_3 = 1300$ lb

$E = 10.4 (10^6)$ psi

$a = 60$ in. $b = 24$ in. $c = 36$ in.

(a) Total elongation

$$\delta = \frac{1}{EA} \left[(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c \right] = 0.01125 \text{ in.} \quad \boxed{\delta = 0.01125 \text{ in.}} \quad \text{(elongation)}$$

(b) Increase P_3 so that bar does not change length

$\dfrac{1}{EA} \left[(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c \right] = 0$ solve, $P_3 = 1690$ lb $\boxed{\text{So new value of } P_3 \text{ is 1690 lb, an increase of 390 lb.}}$

(c) Now change cross-sectional area of AB so that bar does not change length $P_3 = 1300$ lb

$$\frac{1}{E} \left[(P_1 + P_2 - P_3)\frac{a}{A_{AB}} + (P_2 - P_3)\frac{b}{A} + (-P_3)\frac{c}{A} \right] = 0$$

Solving for A_{AB}: $\boxed{A_{AB} = 0.78 \text{ in.}^2}$ $\dfrac{A_{AB}}{A} = 1.951$

Problem 2.3-4

$E = 200\text{GPa}$

$A_1 = 6000\text{mm}^2 \qquad A_2 = 5000\text{mm}^2 \qquad A_3 = 4000\text{mm}^2 \qquad L_1 = 500\text{mm} \qquad L_2 = L_1 \qquad L_3 = L_1$

$P_B = 50\text{N} \qquad P_C = 250\text{N} \qquad P_E = 350\text{N}$

Internal forces in each segment (tension +) - cut bar and use lower FBD

$N_{AB} = -P_B + P_C + P_E = 550\,\text{N} \qquad N_{BC} = P_C + P_E = 600\,\text{N} \qquad N_{CD} = P_E = 350\,\text{N} \qquad N_{DE} = P_E = 350\,\text{N}$

Use force-displacement relation to find segment elongations then sum elongations to find downward displacements

$$\delta_B = \frac{N_{AB} \cdot L_1}{E \cdot A_1} = 2.292 \times 10^{-4}\,\text{mm} \quad \text{downward} \qquad\qquad \delta_C = \delta_B + \frac{N_{BC} \cdot \frac{L_2}{2}}{E \cdot A_2} = 3.792 \times 10^{-4}\,\text{mm}$$

$$\delta_D = \delta_C + \frac{N_{CD} \cdot \frac{L_2}{2}}{E \cdot A_2} = 4.667 \times 10^{-4}\,\text{mm} \qquad\qquad \delta_E = \delta_D + \frac{N_{DE} \cdot L_3}{E \cdot A_3} = 6.854 \times 10^{-4}\,\text{mm}$$

Axial displacement diagram - x origin at A, positive downward

$$\delta(x) = \left|\begin{array}{l} \delta_B \cdot \dfrac{x}{L_1} \quad \text{if } x \leq L_1 \\[3mm] \delta_B + (\delta_C - \delta_B) \cdot \left(\dfrac{x - L_1}{\frac{L_2}{2}}\right) \quad \text{if } L_1 \leq x \leq L_1 + \dfrac{L_2}{2} \\[5mm] \delta_C + (\delta_D - \delta_C) \cdot \left[\dfrac{x - \left(L_1 + \frac{L_2}{2}\right)}{\frac{L_2}{2}}\right] \quad \text{if } L_1 + \dfrac{L_2}{2} \leq x \leq L_1 + L_2 \\[5mm] \delta_D + (\delta_E - \delta_D) \cdot \left[\dfrac{x - (L_1 + L_2)}{L_3}\right] \quad \text{otherwise} \end{array}\right.$$

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Downward axial displacement (meters)

Problem 2.3-5

$E = 29000\text{ksi}$ $A = 8.24\text{in}^2$ $L_1 = 20\text{in}$ $L_2 = 20\text{in}$ $L_3 = 40\text{in}$

$P_B = 50\text{lbf}$ $P_C = 100\text{lbf}$ $P_D = 200\text{lbf}$

Internal forces in each segment (tension +) - cut bar and use lower FBD

$$N_{AB} = -P_B + P_C - P_D = -150 \cdot \text{lbf} \qquad N_{BC} = P_C - P_D = -100 \cdot \text{lbf} \qquad N_{CD} = -P_D = -200 \cdot \text{lbf}$$

Use force-displacement relation to find segment elongations then sum elongations to find downward displacements

$$\delta_B = \frac{N_{AB} \cdot L_1}{E \cdot A} = -1.255 \times 10^{-5} \cdot \text{in} \qquad \delta_C = \delta_B + \frac{N_{BC} \cdot L_2}{E \cdot A} = -2.092 \times 10^{-5} \cdot \text{in} \qquad \delta_D = \delta_C + \frac{N_{CD} \cdot L_3}{E \cdot A} = -5.44 \times 10^{-5} \cdot \text{in}$$

upward   upward   upward

Problem 2.3-6

$\gamma = 77.0 \dfrac{kN}{m^3}$ from Table I-1

$E = 200 GPa$

$A_1 = 6000 mm^2$ $A_2 = 5000 mm^2$ $A_3 = 4000 mm^2$ $L_1 = 500 mm$ $L_2 = L_1$ $L_3 = L_1$

$P_B = 50 N$ $P_C = 250 N$ $P_E = 350 N$

Internal forces in each segment (tension +) - cut bar and use lower FBD - weight per unit length = γA_i

$P_{AB} = -P_B + P_C + P_E = 550\,N$ $P_{BC} = P_C + P_E = 600\,N$ $P_{CD} = P_E = 350\,N$ $P_{DE} = P_E = 350\,N$

Now add weight per unit length - x origin at A, positive downward

$N_{AB}(x) = P_{AB} + \gamma \cdot A_1 \cdot (L_1 - x) + \gamma \cdot A_2 \cdot L_2 + \gamma \cdot A_3 \cdot L_3$ $N_{BC}(x) = P_{BC} + \gamma \cdot A_2 \cdot \left(L_1 + \dfrac{L_2}{2} - x \right) + \gamma \cdot A_2 \cdot \dfrac{L_2}{2} + \gamma \cdot A_3 \cdot L_3$

$N_{CD}(x) = P_{CD} + \gamma \cdot A_2 \cdot (L_1 + L_2 - x) + \gamma \cdot A_3 \cdot L_3$ $N_{DE}(x) = P_{DE} + \gamma \cdot A_3 \cdot (L_1 + L_2 + L_3 - x)$

Note that total bar weight is not small compared to applied loads $W = \gamma \cdot (A_1 \cdot L_1 + A_2 \cdot L_2 + A_3 \cdot L_3) = 577.5\,N$

Use force-displacement relation to find segment elongations then sum elongations to find displacements

$\Delta_B = \displaystyle\int_0^{L_1} \dfrac{N_{AB}(x)}{E \cdot A_1}\,dx = 4.217 \times 10^{-4}\,mm$ $\Delta_C = \Delta_B + \displaystyle\int_{L_1}^{L_1 + \frac{L_2}{2}} \dfrac{N_{BC}(x)}{E \cdot A_2}\,dx = 6.463 \times 10^{-4}\,mm$

$\Delta_D = \Delta_C + \displaystyle\int_{L_1 + \frac{L_2}{2}}^{L_1 + L_2} \dfrac{N_{CD}(x)}{E \cdot A_2}\,dx = 7.843 \times 10^{-4}\,mm$ $\Delta_E = \Delta_D + \displaystyle\int_{L_1 + L_2}^{L_1 + L_2 + L_3} \dfrac{N_{DE}(x)}{E \cdot A_3}\,dx = 1.051 \times 10^{-3}\,mm$

Compare to
Prob. 2.3-4

$\dfrac{\Delta_B}{2.292 \cdot \left(10^{-4}\right) \cdot mm} = 1.84$ $\dfrac{\Delta_C}{3.792 \cdot \left(10^{-4}\right) \cdot mm} = 1.704$ $\dfrac{\Delta_D}{4.667 \cdot \left(10^{-4}\right) \cdot mm} = 1.681$ $\dfrac{\Delta_E}{6.854 \cdot \left(10^{-4}\right) \cdot mm} = 1.534$

Axial displacement diagram including weight of bar - x origin at A, positive downward

$$\Delta(x) = \begin{cases} \displaystyle\int_0^x \frac{N_{AB}(x)}{E \cdot A_1}\, dx & \text{if } x \le L_1 \\[2em] \Delta_B + \displaystyle\int_{L_1}^x \frac{N_{BC}(x)}{E \cdot A_2}\, dx & \text{if } L_1 \le x \le L_1 + \dfrac{L_2}{2} \\[2em] \Delta_C + \displaystyle\int_{L_1+\frac{L_2}{2}}^x \frac{N_{CD}(x)}{E \cdot A_2}\, dx & \text{if } L_1 + \dfrac{L_2}{2} \le x \le L_1 + L_2 \\[2em] \Delta_D + \displaystyle\int_{L_1+L_2}^x \frac{N_{DE}(x)}{E \cdot A_3}\, dx & \text{if } x \ge L_1 + L_2 \end{cases}$$

Downward axial displacement (meters)
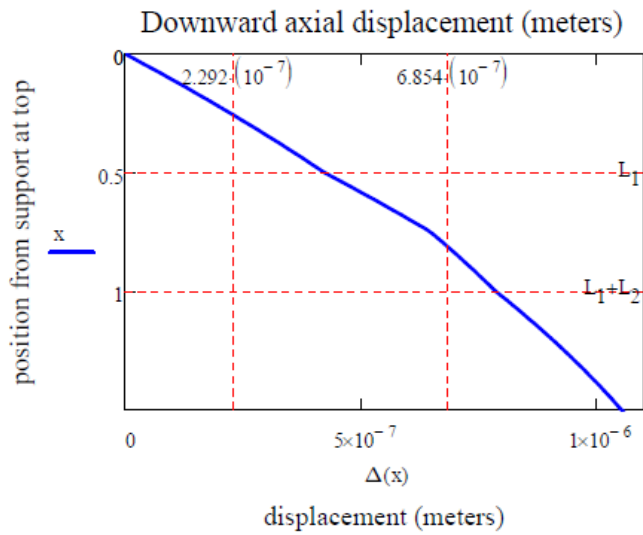
Compare ADD's without (δ) & with (Δ) weight of bar (plotted horizontally)
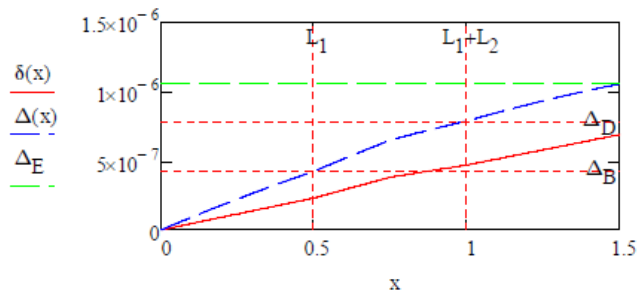
Problem 2.3-7

$$E = 29000 \text{ksi} \qquad A = 8.24 \text{in}^2 \qquad \gamma = 490 \frac{\text{lbf}}{\text{ft}^3}$$

$$L_1 = 20 \text{in} \qquad L_2 = 20 \text{in} \qquad L_3 = 40 \text{in}$$

$$P_B = 50 \text{lbf} \qquad P_C = 100 \text{lbf} \qquad P_D = 200 \text{lbf}$$

Internal forces in each segment (tension +) - cut bar and use lower FBD
x origin at A, positive downward

$$P_{AB} = -P_B + P_C - P_D = -150 \cdot \text{lbf} \qquad P_{BC} = P_C - P_D = -100 \cdot \text{lbf} \qquad P_{CD} = -P_D = -200 \cdot \text{lbf}$$

$$N_{AB}(x) = P_{AB} + \gamma \cdot A \cdot (L_1 - x) + \gamma \cdot A \cdot (L_2 + L_3) \qquad N_{BC}(x) = P_{BC} + \gamma \cdot A \cdot (L_1 + L_2 - x) + \gamma \cdot A \cdot L_3$$

$$N_{CD}(x) = P_{CD} + \gamma \cdot A \cdot (L_1 + L_2 + L_3 - x)$$

Note that total bar weight is not small compared to applied loads $\qquad W = \gamma \cdot A \cdot (L_1 + L_2 + L_3) = 186.926 \cdot \text{lbf}$

Use force-displacement relation to find segment elongations then sum elongations to find downward displacements

$$\delta_B = \int_0^{L_1} \frac{N_{AB}(x)}{E \cdot A} dx = 1.135 \times 10^{-6} \cdot \text{in} \qquad \delta_C = \delta_B + \int_{L_1}^{L_1+L_2} \frac{N_{BC}(x)}{E \cdot A} dx = 2.543 \times 10^{-6} \cdot \text{in}$$
$$\text{downward} \qquad\qquad\qquad \text{downward}$$

$$\delta_D = \delta_C + \int_{L_1+L_2}^{L_1+L_2+L_3} \frac{N_{CD}(x)}{E \cdot A} dx = -2.311 \times 10^{-5} \cdot \text{in}$$
$$\text{upward}$$

Problem 2.3-8

(a) $\delta = \dfrac{P}{E}\left(\dfrac{2\dfrac{L}{4}}{bt} + \dfrac{\dfrac{L}{2}}{\dfrac{3}{4}bt}\right) = \dfrac{7LP}{6\,Ebt}$ $\boxed{\delta = \dfrac{7PL}{6\,Ebt}}$

(b) Numerical data $E = 210\ \text{GPa}$ $L = 750\ \text{mm}$ $\sigma_{mid} = 160\ \text{MPa}$

so $\sigma_{mid} = \dfrac{P}{\dfrac{3}{4}bt}$ and $\dfrac{P}{bt} = \dfrac{3}{4}\sigma_{mid}$

$\delta = \dfrac{7LP}{6\,Ebt}$ or $\delta = \dfrac{7L}{6E}\left(\dfrac{3}{4}\sigma_{mid}\right) = 0.5\ \text{mm}$ $\boxed{\delta = 0.5\ \text{mm}}$

(c) $\delta_{max} = \dfrac{P}{E}\left(\dfrac{L - L_{slot}}{bt} + \dfrac{L_{slot}}{\dfrac{3}{4}bt}\right)$ or $\delta_{max} = \left(\dfrac{P}{bt}\right)\left(\dfrac{1}{E}\right)\left(L - L_{slot} + \dfrac{4}{3}L_{slot}\right)$

or $\delta_{max} = \left(\dfrac{3}{4}\sigma_{mid}\right)\left(\dfrac{1}{E}\right)\left(L + \dfrac{L_{slot}}{3}\right)$ Solving for L_{slot} with $\delta_{max} = 0.475\ \text{mm}$

$L_{slot} = \dfrac{4E\delta_{max} - 3L\sigma_{mid}}{\sigma_{mid}} = 244\ \text{mm}$ $\boxed{L_{slot} = 244\ \text{mm}}$ $\dfrac{L_{slot}}{L} = 0.325$

Problem 2.3-9

(a)
$$\delta = \frac{P}{E}\left(\frac{2\frac{L}{4}}{bt} + \frac{\frac{L}{2}}{\frac{3}{4}bt}\right) = \frac{7LP}{6Ebt}$$

(b) $E = 30{,}000$ ksi $\qquad L = 30$ in. $\qquad \sigma_{mid} = 24$ ksi

So $\qquad \sigma_{mid} = \dfrac{P}{\frac{3}{4}bt}$ $\qquad$ and $\qquad \dfrac{P}{bt} = \dfrac{3}{4}\sigma_{mid}$

$$\delta = \frac{7LP}{6Ebt} \qquad \text{or} \qquad \delta = \frac{7L}{6E}\left(\frac{3}{4}\sigma_{mid}\right) = 0.021 \text{ in.} \qquad \boxed{\delta = 0.021 \text{ in.}}$$

(c) $\delta_{max} = \dfrac{P}{E}\left(\dfrac{L - L_{slot}}{bt} + \dfrac{L_{slot}}{\frac{3}{4}bt}\right)$ $\qquad$ or $\qquad \delta_{max} = \left(\dfrac{P}{bt}\right)\left(\dfrac{1}{E}\right)\left(L - L_{slot} + \dfrac{4}{3}L_{slot}\right)$

or $\qquad \delta_{max} = \left(\dfrac{3}{4}\sigma_{mid}\right)\left(\dfrac{1}{E}\right)\left(L + \dfrac{L_{slot}}{3}\right)$ $\qquad$ Solving for L_{slot} with $\delta_{max} = 0.02$ in.:

$$L_{slot} = \frac{4E\delta_{max} - 3L\sigma_{mid}}{\sigma_{mid}} = 10 \text{ in.} \qquad \boxed{L_{slot} = 10 \text{ in.}} \qquad \frac{L_{slot}}{L} = 0.333$$

Problem 2.3-10

$P_1 = 400$ kN

C

L

B | $P_2 = 720$ kN

L

A

L = length of each column

= 3.75 m

$E = 206$ GPa

$A_{AB} = 11,000$ mm^2

$A_{BC} = 3,900$ mm^2
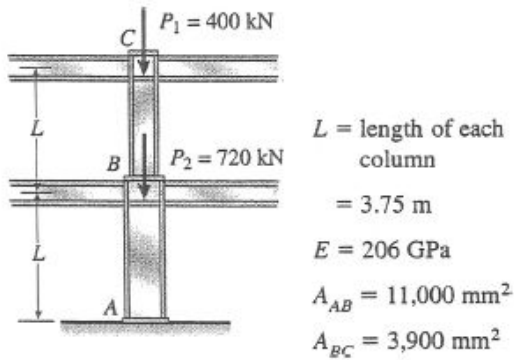
(a) Shortening δ_{AC} of the two columns

$$\delta_{AC} = \Sigma \frac{N_i L_i}{E_i A_i} = \frac{N_{AB} L}{EA_{AB}} + \frac{N_{BC} L}{EA_{BC}}$$

$$= \frac{(1120 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(11,000 \text{ mm}^2)}$$

$$+ \frac{(400 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(3,900 \text{ mm}^2)}$$

$$= 1.8535 \text{ mm} + 1.8671 \text{ mm} = 3.7206 \text{ mm}$$

$\delta_{AC} = 3.72$ mm ←

(b) Additional load P_0 at point C

$$(\delta_{AC})_{max} = 4.0 \text{ mm}$$

δ_0 = additional shortening of the two columns due to the load P_0

$$\delta_0 = (\delta_{AC})_{max} - \delta_{AC} = 4.0 \text{ mm} - 3.7206 \text{ mm}$$

$$= 0.2794 \text{ mm}$$

Also, $\delta_0 = \dfrac{P_0 L}{EA_{AB}} + \dfrac{P_0 L}{EA_{BC}} = \dfrac{P_0 L}{E}\left(\dfrac{1}{A_{AB}} + \dfrac{1}{A_{BC}}\right)$

Solve for P_0:

$$P_0 = \frac{E\delta_0}{L}\left(\frac{A_{AB} A_{BC}}{A_{AB} + A_{BC}}\right)$$

Substitute numerical values:

$E = 206 \times 10^9$ N/m^2 $\delta_0 = 0.2794 \times 10^{-3}$ m

$L = 3.75$ m $A_{AB} = 11,000 \times 10^{-6}$ m^2

$A_{BC} = 3,900 \times 10^{-6}$ m^2

$P_0 = 44,200$ N $= 44.2$ kN   ←

Problem 2.3-11

Numerical data $E = 30 \cdot (10^6) \, \text{psi}$ $P = 5000 \, \text{lb}$ $L = 4 \text{ft}$ $d_1 = 0.75 \text{in}$ $d_2 = 0.5 \text{in}$

Part (a)

$$\delta_a = \frac{P \cdot L}{E} \cdot \left(\frac{1}{\frac{\pi}{4} \cdot d_1^2} + \frac{1}{\frac{\pi}{4} \cdot d_2^2} \right) = 0.0589 \cdot \text{in} \qquad \boxed{\delta_a = 0.0589 \cdot \text{in}}$$

Part (b)

$$\text{Vol}_a = \left(\frac{\pi}{4} \cdot d_1^2 + \frac{\pi}{4} \cdot d_2^2 \right) \cdot L = 30.631 \cdot \text{in}^3 \qquad d = \sqrt{\frac{\text{Vol}_a}{\frac{\pi}{4} \cdot (2 \cdot L)}} = 0.637 \cdot \text{in} \qquad A = \frac{\pi}{4} \cdot d^2 = 0.31907 \cdot \text{in}^2$$

$$\delta_b = \frac{P \cdot (2 \cdot L)}{E \cdot A} = 0.0501 \cdot \text{in} \qquad \boxed{\delta_b = 0.0501 \cdot \text{in}}$$

Part (c) $q = 1250 \, \frac{\text{lb}}{\text{ft}} \qquad L = 4 \, \text{ft}$

$$\delta_c = \frac{q \cdot L^2}{2 \cdot E \cdot \left(\frac{\pi}{4} \cdot d_1^2 \right)} + \frac{P \cdot L}{E \cdot \left(\frac{\pi}{4} \cdot d_2^2 \right)} = 0.0498 \cdot \text{in} \qquad \boxed{\frac{\delta_c}{\delta_a} = 0.846} \qquad \boxed{\frac{\delta_c}{\delta_b} = 0.993}$$

Problem 2.3-12

NUMERICAL DATA

$d_1 = 100$ mm $d_2 = 60$ mm

$L = 1200$ mm $E = 4.0$ GPa $P = 110$ kN

$\delta_a = 8.0$ mm

(a) FIND d_{max} IF SHORTENING IS LIMITED TO δ_a

$$A_1 = \frac{\pi}{4}d_1^2 \quad A_2 = \frac{\pi}{4}d_2^2$$

$$\delta = \frac{P}{E}\left[\frac{\frac{L}{4}}{\frac{\pi}{4}(d_1^2 - d_{max}^2)} + \frac{\frac{L}{4}}{A_1} + \frac{\frac{L}{2}}{A_2}\right]$$

Set δ to δ_a, and solve for d_{max}:

$$d_{max} = d_1\sqrt{\frac{E\delta_a\pi d_1^2 d_2^2 - 2PLd_2^2 - 2PLd_1^2}{E\delta_a\pi d_1^2 d_2^2 - PLd_2^2 - 2PLd_1^2}}$$

$d_{max} = 23.9$ mm   ←

(b) Now, IF d_{max} IS INSTEAD SET AT $d_2/2$, AT WHAT DISTANCE b FROM END C SHOULD LOAD P BE APPLIED TO LIMIT THE BAR SHORTENING TO $\delta_a = 8.0$ mm?

$$A_0 = \frac{\pi}{4}\left[d_1^2 - \left(\frac{d_2}{2}\right)^2\right]$$

$$A_1 = \frac{\pi}{4}d_1^2 \quad A_2 = \frac{\pi}{4}d_2^2$$

$$\delta = \frac{P}{E}\left[\frac{L}{4A_0} + \frac{L}{4A_1} + \frac{\left(\frac{L}{2} - b\right)}{A_2}\right]$$

No axial force in segment at end of length b; set $\delta = \delta_a$ and solve for b:

$$b = \left[\frac{L}{2} - A_2\left[\frac{E\delta_a}{P} - \left(\frac{L}{4A_0} + \frac{L}{4A_1}\right)\right]\right]$$

$b = 4.16$ mm   ←

(c) FINALLY IF LOADS P ARE APPLIED AT THE ENDS AND $d_{max} = d_2/2$, WHAT IS THE PERMISSIBLE LENGTH x OF THE HOLE IF SHORTENING IS TO BE LIMITED TO $\delta_a = 8.0$ mm?

$$\delta = \frac{P}{E}\left[\frac{x}{A_0} + \frac{\left(\frac{L}{2} - x\right)}{A_1} + \frac{\left(\frac{L}{2}\right)}{A_2}\right]$$

Set $\delta = \delta_a$ and solve for x:

$$x = \frac{\left[A_0A_1\left(\frac{E\delta_a}{P} - \frac{L}{2A_2}\right)\right] - \frac{1}{2}A_0L}{A_1 - A_0}$$

$x = 183.3$ mm   ←

Problem 2.3-13

AFD LINEAR

(a) $N(y) = fy$
$$\delta = \int_0^L \frac{(fy)}{EA} dy = \frac{L^2 f}{2AE} \qquad \boxed{\delta = \frac{PL}{2EA}}$$

(b) $\sigma(y) = \dfrac{N(y)}{A}$ $\qquad \sigma(y) = \dfrac{fy}{A} \qquad \sigma(L) = \dfrac{fL}{A} = \dfrac{P}{A}$

$\sigma(0) = 0$ $\qquad$ So linear variation, zero at bottom, P/A at top (i.e., at ground surface)

$N(L) = f$ $\qquad \boxed{\sigma(y) = \dfrac{P}{A}\left(\dfrac{y}{L}\right)}$
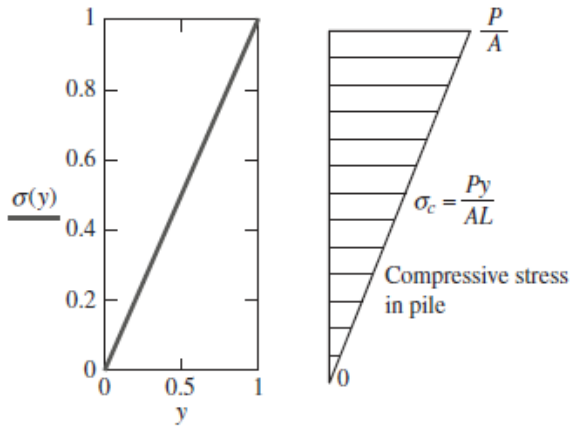
$f(y)$ is constant
and AFD is linear
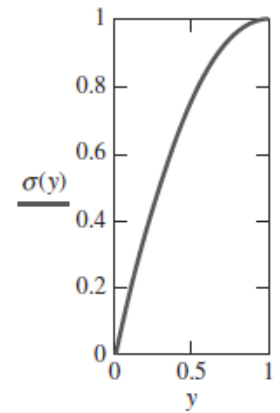
$f(y)$ is linear and
AFD quadratic

(c) $N(y) = f(y)\,y$

$$N(y) = \int_0^y f_0\left(1 - \frac{\zeta}{L}\right) d\zeta = \frac{f_0 y(y-2)}{2} \qquad N(L) = \frac{f_0}{2} \qquad N(0) = 0$$

$$\delta = \frac{\left(\dfrac{f_0 L}{2}\right)}{\dfrac{3}{2}EA} \qquad P = \frac{1}{2}f_0 L \qquad \boxed{\delta = \frac{PL}{EA}\left(\frac{2}{3}\right)} \qquad \boxed{\sigma(y) = \frac{P}{A}\left[\frac{y}{L}\left(2 - \frac{y}{L}\right)\right]} \qquad \sigma(0) = 0 \qquad \sigma(L) = \frac{f_0}{2} = P/A$$

Problem 2.3-14

NUMERICAL DATA

$P = 5$ kN $\qquad E_c = 120$ GPa

$L_2 = 18$ mm $\qquad L_4 = L_2$

$L_3 = 40$ mm

$d_{o3} = 22.2$ mm $\quad t_3 = 1.65$ mm

$d_{o5} = 18.9$ mm $\quad t_5 = 1.25$ mm

$\tau_Y = 30$ MPa $\quad \sigma_Y = 200$ MPa

$FS_\tau = 2$ $\qquad FS_\sigma = 1.7$

$\tau_a = \dfrac{\tau_Y}{FS_\tau}$ $\quad \tau_a = 15$ MPa

$\sigma_a = \dfrac{\sigma_Y}{FS_\sigma}$ $\quad \sigma_a = 117.6$ MPa

(a) ELONGATION OF SEGMENT 2-3-4

$A_2 = \dfrac{\pi}{4}[d_{o3}^2 - (d_{o5} - 2t_5)^2]$

$A_3 = \dfrac{\pi}{4}[d_{o3}^2 - (d_{o3} - 2t_3)^2]$

$A_2 = 175.835$ mm^2 $\quad A_3 = 106.524$ mm^2

$\delta_{24} = \dfrac{P}{E_c}\left(\dfrac{L_2 + L_4}{A_2} + \dfrac{L_3}{A_3}\right)$

$\delta_{24} = 0.024$ mm $\quad \leftarrow$

(b) MAXIMUM LOAD P_{max} THAT CAN BE APPLIED TO THE JOINT

First check normal stress:

$A_1 = \dfrac{\pi}{4}[d_{o5}^2 - (d_{o5} - 2t_5)^2]$

$A_1 = 69.311$ mm^2 $\quad <$ smallest cross-sectional area controls normal stress

$P_{max\sigma} = \sigma_a A_1$ $\quad P_{max\sigma} = 8.15$ kN $\quad \leftarrow$ smaller than P_{max} based on shear below so normal stress controls

Next check shear stress in solder joint:

$A_{sh} = \pi d_{o5} L_2$ $\qquad A_{sh} = 1.069 \times 10^3$ mm^2

$P_{max\tau} = \tau_a A_{sh}$ $\quad P_{max\tau} = 16.03$ kN

(c) FIND THE VALUE OF L_2 AT WHICH TUBE AND SOLDER CAPACITIES ARE EQUAL

Set P_{max} based on shear strength equal to P_{max} based on tensile strength and solve for L_2:

$L_2 = \dfrac{\sigma_a A_1}{\tau_a(\pi d_{o5})}$ $\quad L_2 = 9.16$ mm $\quad \leftarrow$

Problem 2.3-15

(a) Statics $\sum F_H = 0$ $R_1 = -P - \dfrac{P}{2}$

$$R_1 = \dfrac{-3}{2}P \quad \leftarrow$$

(b) Draw FBD's cutting through segment 1 and again through segment 2

$$N_1 = \dfrac{3P}{2} \quad < \text{tension} \quad N_2 = \dfrac{P}{2} \quad < \text{tension}$$

(c) Find x required to obtain axial displacement at joint 3 of $\delta_3 = PL/EA$

Add axial deformations of segments 1 and 2, then set to δ_3; solve for x:

$$\dfrac{N_1 x}{E\frac{3}{4}A} + \dfrac{N_2(L - x)}{EA} = \dfrac{PL}{EA}$$

$$\dfrac{\frac{3P}{2}x}{E\frac{3}{4}A} + \dfrac{\frac{P}{2}(L - x)}{EA} = \dfrac{PL}{EA}$$

$$\dfrac{3}{2}x = \dfrac{L}{2} \quad x = \dfrac{L}{3} \quad \leftarrow$$

(d) What is the displacement at joint 2, δ_2?

$$\delta_2 = \dfrac{N_1 x}{E\frac{3}{4}A} \quad \delta_2 = \dfrac{\left(\frac{3P}{2}\right)\frac{L}{3}}{E\frac{3}{4}A}$$

$$\delta_2 = \dfrac{2}{3}\dfrac{PL}{EA}$$

(e) If $x = 2L/3$ and $P/2$ at joint 3 is replaced by βP, find β so that $\delta_3 = PL/EA$

$$N_1 = (1 + \beta)P \quad N_2 = \beta P \quad x = \dfrac{2L}{3}$$

substitute in axial deformation expression above and solve for β

$$\dfrac{[(1 + \beta)P]\frac{2L}{3}}{E\frac{3}{4}A} + \dfrac{\beta P\left(L - \frac{2L}{3}\right)}{EA} = \dfrac{PL}{EA}$$

$$\dfrac{1}{9}PL\dfrac{8 + 11\beta}{EA} = \dfrac{PL}{EA}$$

$$(8 + 11\beta) = 9$$

$$\beta = \dfrac{1}{11} \quad \leftarrow$$

$$\beta = 0.091$$

(f) Draw AFD, ADD—see plots for $x = \dfrac{L}{3}$

No plots provided here

Problem 2.3-16
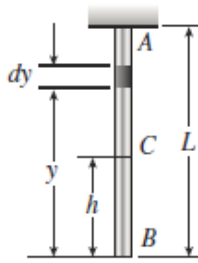
W = Weight of bar

(a) Downward displacement δ_C
Consider an element at distance y from the lower end.

$$N(y) = \frac{Wy}{L} \qquad d\delta = \frac{N(y)\,dy}{EA} = \frac{Wy\,dy}{EAL}$$

$$\delta_C = \int_h^L d\delta = \int_h^L \frac{Wy\,dy}{EAL} = \frac{W}{2EAL}(L^2 - h^2)$$

$$\delta_C = \frac{W}{2EAL}(L^2 - h^2) \quad \leftarrow$$

(b) Elongation of bar ($h = 0$)

$$\delta_B = \frac{WL}{2EA} \quad \leftarrow$$

(c) Ratio of elongations

Elongation of upper half of bar $\left(h = \dfrac{L}{2}\right)$:

$$\delta_{upper} = \frac{3WL}{8EA}$$

Elongation of lower half of bar:

$$\delta_{lower} = \delta_B - \delta_{upper} = \frac{WL}{2EA} - \frac{3WL}{8EA} = \frac{WL}{8EA}$$

$$\beta = \frac{\delta_{upper}}{\delta_{lower}} = \frac{3/8}{1/8} = 3 \quad \leftarrow$$

(d) Numerical data

$$\gamma_s = 77 \text{ kN/m}^3 \qquad \gamma_w = 10 \text{ kN/m}^3 \qquad L = 1500 \text{ m} \qquad A = 0.0157 \text{ m}^2 \qquad E = 210 \text{ GPa}$$

In sea water:

$$W = (\gamma_s - \gamma_w)AL = 1577.85 \text{ kN} \qquad \boxed{\delta = \frac{WL}{2EA} = 359 \text{ mm}} \qquad \frac{\delta}{L} = 2.393 \times 10^{-4}$$

In air:

$$W = (\gamma_s)AL = 1813.35 \text{ kN} \qquad \boxed{\delta = \frac{WL}{2EA} = 412 \text{ mm}} \qquad \frac{\delta}{L} = 2.75 \times 10^{-4}$$

Problem 2.3-17
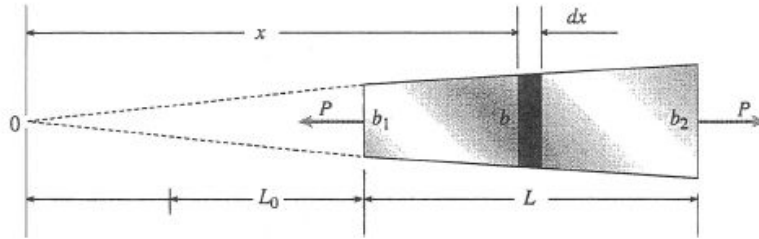
t = thickness (constant)

$b = b_1\left(\dfrac{x}{L_0}\right)$ $b_2 = b_1\left(\dfrac{L_0 + L}{L_0}\right)$ (Eq. 1)

$A(x) = bt = b_1 t\left(\dfrac{x}{L_0}\right)$

(a) Elongation of the bar

$d\delta = \dfrac{P dx}{EA(x)} = \dfrac{PL_0\, dx}{Eb_1\, tx}$

$\delta = \displaystyle\int_{L_0}^{L_0+L} d\delta = \dfrac{PL_0}{Eb_1\, t}\int_{L_0}^{L_0+L} \dfrac{dx}{x}$

$= \dfrac{PL_0}{Eb_1\, t}\, \ln x \Big|_{L_0}^{L_0+L} = \dfrac{PL_0}{Eb_1\, t}\, \ln\dfrac{L_0 + L}{L_0}$ (Eq. 2)

From Eq. (1): $\dfrac{L_0 + L}{L_0} = \dfrac{b_2}{b_1}$ (Eq. 3)

Solve Eq. (3) for L_0: $L_0 = L\left(\dfrac{b_1}{b_2 - b_1}\right)$ (Eq. 4)

Substitute Eqs. (3) and (4) into Eq. (2):

$\delta = \dfrac{PL}{Et\,(b_2 - b_1)}\, \ln\dfrac{b_2}{b_1}$ (Eq. 5)

(b) Substitute numerical values:

$L = 5$ ft $= 60$ in. $t = 10$ in.

$P = 25$ k $b_1 = 4.0$ in.

$b_2 = 6.0$ in. $E = 30 \times 10^6$ psi

From Eq. (5): $\delta = 0.010$ in.   ←

Problem 2.3-18

$P = 200\text{kN}$ $L = 2\text{m}$ $t = 20\text{mm}$ $b_1 = 100\text{mm}$ $b_2 = 115\text{mm}$ $E = 96\text{GPa}$

Bar width at B at L/2 $b_B = \dfrac{b_1 + b_2}{2} = 107.5 \cdot \text{mm}$

Axial forces in bar segments (use RHFB) $N_{AB} = 2 \cdot P - P = 200 \cdot \text{kN}$ $N_{BC} = 2 \cdot P = 400 \cdot \text{kN}$

Axial displacement at B $\delta_B = \dfrac{N_{AB} \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_2 - b_B)} \cdot \ln\left(\dfrac{b_2}{b_B}\right) = 0.937 \cdot \text{mm}$

Axial displacement at C $\delta_C = \delta_B + \dfrac{N_{BC} \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_B - b_1)} \cdot \ln\left(\dfrac{b_B}{b_1}\right) = 2.946 \cdot \text{mm}$

Problem 2.3-19

$P = 50\text{kip}$ $L = 5\text{ft}$ $t = \dfrac{3}{8} \cdot \text{in}$ $b_1 = 3\text{in}$ $b_2 = 2.75\text{in}$ $E = 16000\text{ksi}$

$N_{AB} = 2 \cdot P - P = 50 \cdot \text{kip}$ $N_{BC} = 2 \cdot P = 100 \cdot \text{kip}$

Axial displacement at B $\delta_B = \dfrac{N_{AB} \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) = 0.087 \cdot \text{in}$

Axial displacement at C $\delta_C = \delta_B + \dfrac{N_{BC} \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) = 0.261 \cdot \text{in}$

Problem 2.3-20

$E = 72 \text{GPa}$ $P_2 = 200 \text{kN}$ $L = 2\text{m}$ $t = 20\text{mm}$ $b_1 = 100\text{mm}$ $b_2 = 115\text{mm}$

$A_{BC} = b_1 \cdot t = 2 \times 10^3 \cdot \text{mm}^2$

If only load P_2 is applied at C $\delta_B = \dfrac{P_2 \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) = 1.294 \cdot \text{mm}$ $\delta_C = \delta_B + \dfrac{P_2 \cdot \dfrac{L}{2}}{E \cdot A_{BC}} = 2.683 \cdot \text{mm}$

Now apply both P_1 (to the left) and P_2 at C and solve for P_1 s.t. axial displacement at C = 0

Given

$$\dfrac{(P_2 - P_1) \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) + \dfrac{P_2 \cdot \dfrac{L}{2}}{E \cdot A_{BC}} = 0$$ $\text{Find}(P_1) = 414.651 \cdot \text{kN}$

Axial displacement at B with both loads applied as shown

Let $P_1 = 414.651 \text{kN}$ Check

$$\delta_B = \dfrac{(P_2 - P_1) \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) = -1.389 \cdot \text{mm}$$ $$\dfrac{(P_2 - P_1) \cdot \dfrac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) + \dfrac{P_2 \cdot \dfrac{L}{2}}{E \cdot A_{BC}} = -0 \, \text{m}$$

leftward

Problem 2.3-21

$$d_A = 4in \qquad d_B = 8in \qquad P = 45kip \qquad \delta_A = 0.02in \qquad d(x) = d_A + \left(\frac{d_B - d_A}{L}\right) \cdot x$$

$$A(x) = \frac{\pi}{4} \cdot d(x)^2 \qquad E = 10400ksi$$

$$\int_0^L \frac{P}{E \cdot A(x)} dx = \delta_A \qquad \text{expand integral to obtain following expression} \qquad \frac{4 \cdot P \cdot L}{\pi \cdot E \cdot d_A \cdot d_B} = \delta_A$$

$$\text{Solving for L} \qquad L = \frac{\pi \cdot E \cdot d_A \cdot d_B}{4 \cdot P} \cdot \delta_A = 9.681 \cdot ft$$

Problem 2.3-22

$L = 1.8\,\text{m}$ $r = 36\,\text{mm}$ $E = 72\,\text{GPa}$ $a = \dfrac{r}{8} = 4.5\,\text{mm}$ $\sigma_2 = 180\,\text{MPa}$

$A_1 = \pi \cdot r^2 = 4071.504\,\text{mm}^2$ Use formulas in *Appendix E, Case 15* for area of slotted segment

$\alpha = \text{acos}\!\left(\dfrac{a}{r}\right) = 1.445$ $b = \sqrt{r^2 - a^2} = 35.718\,\text{mm}$ $A_2 = 2 \cdot r^2 \!\left(\alpha - \dfrac{a \cdot b}{r^2}\right) = 3425.196\,\text{mm}^2$ $\dfrac{A_2}{A_1} = 0.841$

Stress in middle half is known so use to find force P $P = \sigma_2 \cdot A_2 = 616.535\,\text{kN}$

Compute bar elongation now that P is known $\delta = 2 \cdot \dfrac{P \cdot \dfrac{L}{4}}{E \cdot A_1} + \dfrac{P \cdot \dfrac{L}{2}}{E \cdot A_2} = 4.143\,\text{mm}$

Problem 2.3-23
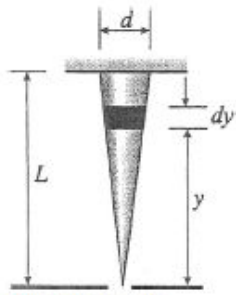
TERMINOLOGY

N_y = axial force acting on element dy

A_y = cross-sectional area at element dy

A_B = cross-sectional area at base of cone

$= \dfrac{\pi d^2}{4}$ V = volume of cone

$= \dfrac{1}{3} A_B L$ V_y = volume of cone below element dy

$= \dfrac{1}{3} A_y y$ W_y = weight of cone below element dy

$= \dfrac{V_y}{V}(W) = \dfrac{A_y y W}{A_B L}$ $N_y = W_y$

ELEMENT OF BAR
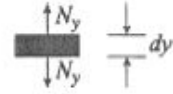
W = weight of cone

ELONGATION OF ELEMENT dy

$$d\delta = \frac{N_y\, dy}{E A_y} = \frac{W y\, dy}{E A_B L} = \frac{4W}{\pi d^2\, EL} y\, dy$$

ELONGATION OF CONICAL BAR

$$\delta = \int d\delta = \frac{4W}{\pi d^2\, EL} \int_0^L y\, dy = \frac{2WL}{\pi d^2\, E} \quad \leftarrow$$

Problem 2.3-24
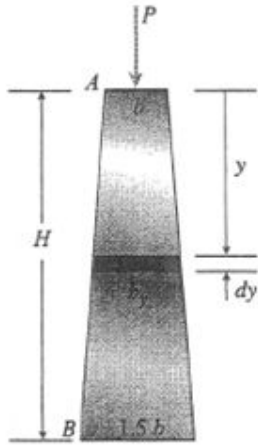
Square cross sections:

b = width at A

$1.5b$ = width at B

b_y = width at distance y

$$= b + (1.5b - b)\frac{y}{H}$$

$$= \frac{b}{H}(H + 0.5y)$$

A_y = cross-sectional area at distance y

$$= (b_y)^2 = \frac{b^2}{H^2}(H + 0.5y)^2$$

SHORTENING OF ELEMENT dy

$$d\delta = \frac{Pdy}{EA_y} = \frac{Pdy}{E\left(\frac{b^2}{H^2}\right)(H + 0.5y)^2}$$

SHORTENING OF ENTIRE POST

$$\delta = \int d\delta = \frac{PH^2}{Eb^2}\int_0^H \frac{dy}{(H + 0.5y)^2}$$

From Appendix C : $\int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$

$$\delta = \frac{PH^2}{Eb^2}\left[-\frac{1}{(0.5)(H + 0.5y)}\right]_0^H$$

$$= \frac{PH^2}{Eb^2}\left[-\frac{1}{(0.5)(1.5H)} + \frac{1}{0.5H}\right]$$

$$= \frac{2PH}{3Eb^2} \quad \leftarrow$$

Problem 2.3-25

$$\frac{dy}{dx} = \frac{8hx}{L^2}$$

FREE-BODY DIAGRAM OF HALF OF CABLE

$$\Sigma M_B = 0 \quad \curvearrowright$$

$$-Hh + \frac{qL}{2}\left(\frac{L}{4}\right) = 0$$

$$H = \frac{qL^2}{8h}$$

$$\Sigma F_{horizontal} = 0$$

$$H_B = H = \frac{qL^2}{8h} \qquad \text{(Eq. 1)}$$

$$\Sigma F_{vertical} = 0$$
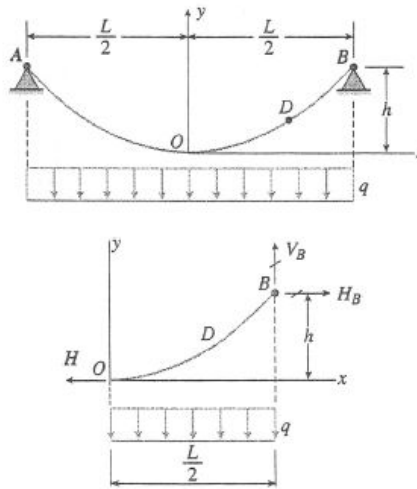
$$V_B = \frac{qL}{2} \qquad \text{(Eq. 2)}$$

Equation of parabolic curve:

$$y = \frac{4hx^2}{L^2}$$

FREE-BODY DIAGRAM OF SEGMENT DB OF CABLE
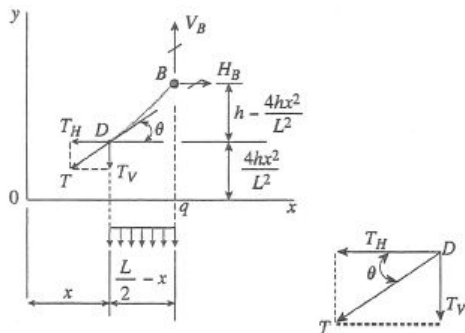
$$\Sigma F_{horiz} = 0 \quad T_H = H_B = \frac{qL^2}{8h} \qquad \text{(Eq. 3)}$$

$$\Sigma F_{vert} = 0 \quad V_B - T_v - q\left(\frac{L}{2} - x\right) = 0$$

$$T_v = V_B - q\left(\frac{L}{2} - x\right) = \frac{qL}{2} - \frac{qL}{2} + qx$$

$$= qx \qquad \text{(Eq. 4)}$$

TENSILE FORCE T IN CABLE

$$T = \sqrt{T_H^2 + T_v^2} = \sqrt{\left(\frac{qL^2}{8h}\right)^2 + (qx)^2}$$

$$= \frac{qL^2}{8h}\sqrt{1 + \frac{64h^2x^2}{L^4}} \qquad \text{(Eq. 5)}$$

ELONGATION $d\delta$ OF AN ELEMENT OF LENGTH ds
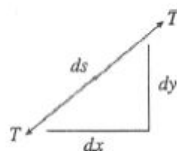
$$d\delta = \frac{T ds}{EA}$$

$$ds = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$= dx\sqrt{1 + \left(\frac{8hx}{L^2}\right)^2}$$

$$= dx\sqrt{1 + \frac{64h^2x^2}{L^4}} \qquad \text{(Eq. 6)}$$

(a) ELONGATION δ OF CABLE AOB

$$\delta = \int d\delta = \int \frac{T ds}{EA}$$

Substitute for T from Eq. (5) and for ds from Eq. (6):

$$\delta = \frac{1}{EA}\int \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$$

For both halves of cable:

$$\delta = \frac{2}{EA}\int_0^{L/2} \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$$

$$\delta = \frac{qL^3}{8hEA}\left(1 + \frac{16h^2}{3L^4}\right) \quad \leftarrow \qquad \text{(Eq. 7)}$$

(b) GOLDEN GATE BRIDGE CABLE

$L = 4200$ ft $\qquad h = 470$ ft

$q = 12{,}700$ lb/ft $\quad E = 28{,}800{,}000$ psi

27,572 wires of diameter $d = 0.196$ in.

$$A = (27{,}572)\left(\frac{\pi}{4}\right)(0.196 \text{ in.})^2 = 831.90 \text{ in.}^2$$

Substitute into Eq. (7):

$\delta = 133.7$ in $= 11.14$ ft $\quad \leftarrow$

196

Problem 2.3-26

(a) ELONGATION δ FOR CASE OF CONSTANT DIAMETER HOLE

$$d(\zeta) = d_A\left(1 + \frac{\zeta}{L}\right) \qquad A(\zeta) = \frac{\pi}{4}d(\zeta)^2 \qquad < \text{solid portion of length } L - x$$

$$A(\zeta) = \frac{\pi}{4}(d(\zeta)^2 - d_A{}^2) \qquad < \text{hollow portion of length } x$$

$$\delta = \frac{P}{E}\left(\int \frac{1}{A(\zeta)}d\zeta\right) \qquad \delta = \frac{P}{E}\left[\int_0^{L-x}\frac{4}{\pi d(\zeta)^2}d\zeta + \int_{L-x}^L \frac{4}{\pi(d(\zeta)^2 - d_A{}^2)}d\zeta\right]$$

$$\delta = \frac{P}{E}\left[\int_0^{L-x}\frac{1}{\left[\frac{\pi}{4}\left[d_A\left(1 + \frac{\zeta}{L}\right)\right]^2\right]}d\zeta + \int_{L-x}^L \frac{1}{\left[\frac{\pi}{4}\left[\left[d_A\left(1 + \frac{\zeta}{L}\right)\right]^2 - d_A{}^2\right]\right]}d\zeta\right]$$

$$\delta = \frac{P}{E}\left[4\frac{L^2}{(-2+x)\pi d_A{}^2} + \left[\left[4\frac{L}{\pi d_A{}^2} + \int_{L-x}^L \frac{1}{\left[\frac{\pi}{4}\left[\left[d_A\left(1 + \frac{\zeta}{L}\right)\right]^2 - d_A{}^2\right]\right]}d\zeta\right]\right]\right]$$

$$\delta = \frac{P}{E}\left[4\frac{L^2}{(-2+x)\pi d_A{}^2} + \left(4\frac{L}{\pi d_A{}^2} - 2L\frac{\ln(3)}{\pi d_A{}^2} + 2L\frac{-\ln(L-x) + \ln(3L-x)}{\pi d_A{}^2}\right)\right]$$

$$\text{if } x = L/2 \quad \delta = \frac{P}{E}\left(\frac{4}{3}\frac{L}{\pi d_A^2} - 2L\frac{\ln(3)}{\pi d_A^2} + 2L\frac{-\ln\left(\frac{1}{2}L\right) + \ln\left(\frac{5}{2}L\right)}{\pi d_A^2}\right)$$

Substitute numerical data:

$\delta = 2.18$ mm $\quad \leftarrow$

(b) ELONGATION δ FOR CASE OF VARIABLE DIAMETER HOLE BUT CONSTANT WALL THICKNESS $t = d_A/20$ OVER SEGMENT x

$$d(\zeta) = d_A\left(1 + \frac{\zeta}{L}\right) \qquad\qquad A(\zeta) = \frac{\pi}{4}d(\zeta)^2 \quad < \text{solid portion of length } L - x$$

$$A(\zeta) = \frac{\pi}{4}\left[d(\zeta)^2 - \left(d(\zeta) - 2\frac{d_A}{20}\right)^2\right] \quad < \text{hollow portion of length } x$$

$$\delta = \frac{P}{E}\left(\int \frac{1}{A(\zeta)}d\zeta\right) \qquad\qquad \delta = \frac{P}{E}\left[\int_0^{L-x}\frac{4}{\pi d(\zeta)^2}d\zeta + \int_{L-x}^L \frac{4}{\pi\left[d(\zeta)^2 - \left(d(\zeta) - 2\frac{d_A}{20}\right)^2\right]}d\zeta\right]$$

$$\delta = \frac{P}{E}\left[\int_0^{L-x} \frac{4}{\pi\left[d_A\left(1+\frac{\zeta}{L}\right)\right]^4}\,d\zeta + \int_{L-x}^{L} \frac{4}{\pi\left[\left[d_A\left(1+\frac{\zeta}{L}\right)\right]^2 - \left[d_A\left(1+\frac{\zeta}{L}\right) - 2\frac{d_A}{20}\right]^2\right]^2}\,d\zeta \right]$$

$$\delta = \frac{P}{E}\left[4\frac{L^2}{(-2L+x)\pi d_A^2} + 4\frac{L}{\pi d_A^2} + 20L\frac{\ln(3)+\ln(13)+2\ln(d_A)+\ln(L)}{\pi d_A^2} \right.$$
$$\left. - 20L\frac{2\ln(d_A)+\ln(39L-20x)}{\pi d_A^2} \right]$$

if $x = L/2$

$$\delta = \frac{P}{E}\left(\frac{4}{3}\frac{L}{\pi d_A^2} + 20L\frac{\ln(3)+\ln(13)+2\ln(d_A)+\ln(L)}{\pi d_A^2} - 20L\frac{2\ln(d_A)+\ln(29L)}{\pi d_A^2} \right)$$

Substitute numerical data:

$\delta = 6.74$ mm $\quad\leftarrow$

Problem 2.3-27

$P_1 = 2.5\text{kip}$ $P_2 = 1\text{kip}$ $M = 25\text{kip·in}$ $E = 29000\text{ksi}$ $A_1 = 0.25\text{in}^2$ $A_2 = 0.15\text{in}^2$

Find pin force at B - use FBD of bar BDE $\Sigma M_D = 0$ $B_y = \dfrac{1}{25\text{in}} \cdot \left[P_2 \cdot (25\text{in}) - M \right] = 0 \cdot \text{kip}$

No pin force at B so bar ABC is subjected force P_1 at C only $\delta_C = \dfrac{P_1}{E} \cdot \left(\dfrac{20\text{in}}{A_1} + \dfrac{35\text{in}}{A_2} \right) = 0.027 \cdot \text{in}$

     downward

Problem 2.3-28

Find pin force at B - use FBD of bar ABC $\qquad$ $\Sigma M_A = 0$ $\qquad$ $B_y = \dfrac{1}{\dfrac{L}{3}} \cdot (3 \cdot P \cdot L)$ $\qquad$ $B_y \to 9 \, P$ $\qquad$ acts upward on ABC so downward on DBF

Vertical displacements at B and F

$N_{BD} = P - 9 \cdot P \to -8 \, P$ $\qquad$ $\delta_B = \dfrac{N_{BD} \cdot L}{2 \cdot EA}$ $\qquad$ $\delta_B \to -4 \dfrac{PL}{EA}$ $\qquad$ downward

$N_{BF} = P$ $\qquad$ $\delta_C = \delta_B + \dfrac{N_{BF} \cdot \dfrac{L}{3}}{EA}$ $\qquad$ $\delta_C \to -\dfrac{11}{3} \dfrac{PL}{EA}$ $\qquad$ downward

Axial force (N(y)) and displacement (δ(y)) diagrams - origin of y at D, positive upward (rotated CW to horiz, position below)

$N(y) = \begin{Vmatrix} N_{BD} & \text{if } y \le L \\ N_{BF} & \text{otherwise} \end{Vmatrix}$ $\qquad$ $\delta(y) = \begin{Vmatrix} \left[N_{BD} \cdot y \cdot \left(\dfrac{L}{2 \cdot EA} \right) \right] & \text{if } y \le L \\ \left[\delta_B + N_{BF} \cdot (y - L) \cdot \left(\dfrac{L}{3 \cdot EA} \right) \right] & \text{otherwise} \end{Vmatrix}$ $\qquad$ $\delta(0) \to 0$ $\qquad$ $\delta(L) \to -4$

$\delta\left(\dfrac{4 \cdot L}{3} \right) \to -\dfrac{35}{9} = -3.889$

times PL/EA
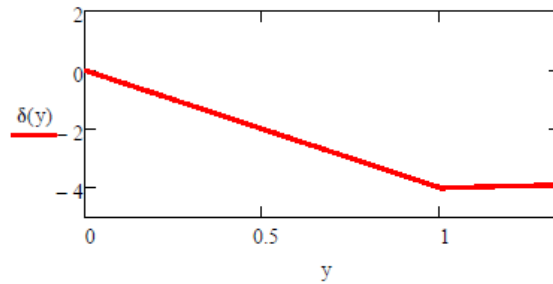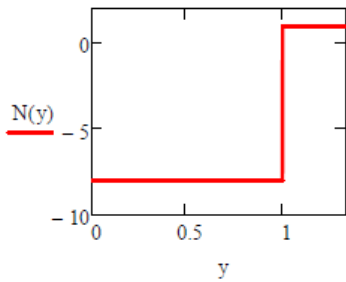
Problem 2.3-29

Find pin force at B - use FBD of bar ABC $\Sigma F_y = 0$ $B_y = 2P$ upward at B on ABC so downward on DBF

Axial forces in column segments (tension is positive)

$$N_{DB} = -P \qquad N_{BF} = -P - B_y \rightarrow -3 \cdot P \qquad$$ so AFD is constant and compressive over each column segment

Vertical displacements at B and D (positive upward)

$$\delta_B = \frac{N_{BF} \cdot \frac{L}{2}}{2 \cdot EA} \rightarrow -\frac{3 \cdot L \cdot P}{4 \cdot EA} \qquad \delta_D = \delta_B + \frac{N_{DB} \cdot \frac{L}{2}}{EA} \rightarrow -\frac{5 \cdot L \cdot P}{4 \cdot EA} \qquad$$ so ADD is linear and downward over each column segment

Problem 2.3-30

Use FBD of beam ABC - find pin force at B $\Sigma F_y = 0$ $B_y = 2 \cdot P$ upward on ABC so downward on DBF

Axial forces in column segments (tension is positive)

$N_{DB} = 0$ $N_{BF} = -B_y \rightarrow -2 \cdot P$ so AFD is 0 over DB and constant and compressive over column segment BF

Vertical displacements at B and D (positive upward)

$$\delta_B = \frac{N_{BF} \cdot \frac{L}{2}}{2 \cdot EA} \rightarrow -\frac{L \cdot P}{2 \cdot EA}$$ $$\delta_D = \delta_B \rightarrow -\frac{L \cdot P}{2 \cdot EA}$$ so ADD is linear over BF and constant over column segment DB, both downward

Problem 2.3-31
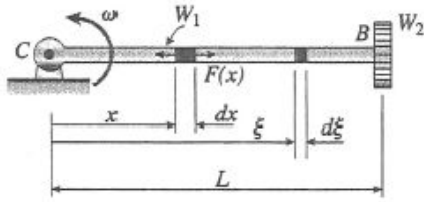
ω = angular speed

A = cross-sectional area

E = modulus of elasticity

g = acceleration of gravity

$F(x)$ = axial force in bar at distance x from point C

Consider an element of length dx at distance x from point C.

To find the force $F(x)$ acting on this element, we must find the inertia force of the part of the bar from distance x to distance L, plus the inertia force of the weight W_2.

Since the inertia force varies with distance from point C, we now must consider an element of length $d\xi$ at distance ξ, where ξ varies from x to L.

Mass of element $d = \dfrac{d}{L}\left(\dfrac{W_1}{g}\right)$

Acceleration of element $= \xi\omega^2$

Centrifugal force produced by element

$= (\text{mass})(\text{acceleration}) = \dfrac{W_1\omega^2}{gL}d$

Centrifugal force produced by weight W_2

$= \left(\dfrac{W_2}{g}\right)(L\omega^2)$

AXIAL FORCE $F(x)$

$$F(x) = \int_{=x}^{=L}\frac{W_1\omega^2}{gL}d + \frac{W_2L\omega^2}{g}$$

$$= \frac{W_1\omega^2}{2gL}(L^2 - x^2) + \frac{W_2L\omega^2}{g}$$

ELONGATION OF BAR BC

$$\delta = \int_0^L \frac{F(x)\,dx}{EA}$$

$$= \int_0^L \frac{W_1\omega^2}{2gL}(L^2 - x^2)dx + \int_0^L \frac{W_2L\omega^2 dx}{gEA}$$

$$= \frac{W1L\omega^2}{2gLEA}\left[\int_0^L L^2\,dx - \int_0^L x^2\,dx\right] + \frac{W_2L\omega^2 dx}{gEA}\int_0^L dx$$

$$= \frac{W_1L^2\omega^2}{3gEA} + \frac{W_2L^2\omega^2}{gEA}$$

$$= \frac{L^2\omega^2}{3gEA} + (W_1 + 3W_2) \quad\leftarrow$$

Problem 2.4-1

Steel shell

Brass core
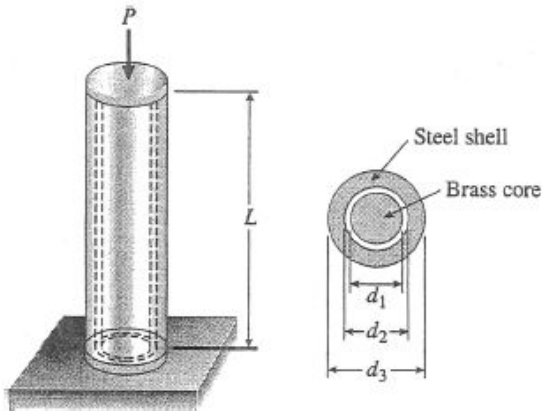
$d_1 = 0.25$ in. $E_b = 15 \times 10^6$ psi

$d_2 = 0.28$ in. $E_s = 30 \times 10^6$ psi

$d_3 = 0.35$ in. $A_s = \frac{\pi}{4}(d_3^2 - d_2^2) = 0.03464$ in.^{2}

$L = 4.0$ in. $A_b = \frac{\pi}{4}d_1^2 = 0.04909$ in.^{2}

(a) DECREASE IN LENGTH ($\delta = 0.003$ in.)

Use Eq. (2-18) of Example 2-6.

$$\delta = \frac{PL}{E_s A_s + E_b A_b} \quad \text{or}$$

$$P = (E_s A_s + E_s A_b)\left(\frac{\delta}{L}\right)$$

Substitute numerical values:

$$E_s A_s + E_b A_b = (30 \times 10^6 \text{ psi})(0.03464 \text{ in.}^2)$$

$$+ (15 \times 10^6 \text{ psi})(0.04909 \text{ in.}^2)$$

$$= 1.776 \times 10^6 \text{ lb}$$

$$P = (1.776 \times 10^6 \text{ lb})\left(\frac{0.003 \text{ in.}}{4.0 \text{ in.}}\right)$$

$$= 1330 \text{ lb} \quad \leftarrow$$

(b) ALLOWABLE LOAD

$\sigma_s = 22$ ksi $\sigma_b = 16$ ksi

Use Eqs. (2-17a and b) of Example 2-6.

For steel:

$$\sigma_s = \frac{PE_s}{E_s A_s + E_b A_b} \quad P_s = (E_s A_s + E_b A_b)\frac{\sigma_s}{E_s}$$

$$P_s = (1.776 \times 10^6 \text{ lb})\left(\frac{22 \text{ ksi}}{30 \times 10^6 \text{ psi}}\right) = 1300 \text{ lb}$$

For brass:

$$\sigma_b = \frac{PE_b}{E_s A_s + E_b A_b} \quad P_s = (E_s A_s + E_b A_b)\frac{\sigma_b}{E_b}$$

$$P_s = (1.776 \times 10^6 \text{ lb})\left(\frac{16 \text{ ksi}}{15 \times 10^6 \text{ psi}}\right) = 1890 \text{ lb}$$

Steel governs. $P_{allow} = 1300$ lb $\leftarrow$

Problem 2.4-2
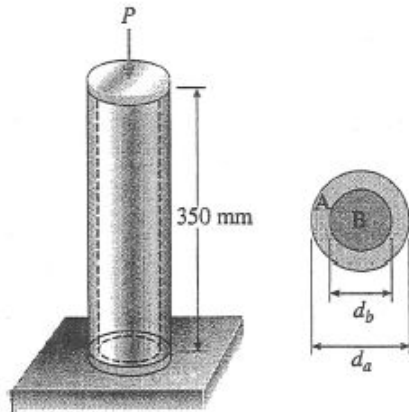
A = aluminum

B = brass

L = 350 mm

d_a = 40 mm

d_b = 25 mm

$A_a = \dfrac{\pi}{4}(d_a^2 - d_b^2)$

$\quad = 765.8 \text{ mm}^2$

$E_a = 72 \text{ GPa} \quad E_b = 100 \text{ GPa} \quad A_b = \dfrac{\pi}{4}d_b^2$

$\quad = 490.9 \text{ mm}^2$

(a) DECREASE IN LENGTH

$\quad (\delta = 0.1\% \text{ of } L = 0.350 \text{ mm})$

Use Eq. (2-18) of Example 2-6.

$$\delta = \frac{PL}{E_a A_a + E_b A_b} \quad \text{or}$$

$$P = (E_a A_a + E_b A_b)\left(\frac{\delta}{L}\right)$$

Substitute numerical values:

$E_a A_a + E_b A_b = (72 \text{ GPa})(765.8 \text{ mm}^2)$
$\qquad\qquad\qquad +(100 \text{ GPa})(490.9 \text{ mm}^2)$
$\qquad\qquad\quad = 55.135 \text{ MN} + 49.090 \text{ MN}$
$\qquad\qquad\quad = 104.23 \text{ MN}$

$P = (104.23 \text{ MN})\left(\dfrac{0.350 \text{ mm}}{350 \text{ mm}}\right)$

$\quad = 104.2 \text{ kN} \quad \leftarrow$

(b) ALLOWABLE LOAD

$\sigma_a = 80 \text{ MPa} \quad \sigma_b = 120 \text{ MPa}$

Use Eqs. (2-17a and b) of Example 2-6.

For aluminum:

$\sigma_a = \dfrac{PE_a}{E_a A_a + E_b A_b} \quad P_a = (E_a A_a + E_b A_b)\left(\dfrac{\sigma_a}{E_a}\right)$

$P_a = (104.23 \text{ MN})\left(\dfrac{80 \text{ MPa}}{72 \text{ GPa}}\right) = 115.8 \text{ kN}$

For brass:

$\sigma_b = \dfrac{PE_b}{E_a A_a + E_b A_b} \quad P_b = (E_a A_a + E_b A_b)\left(\dfrac{\sigma_b}{E_b}\right)$

$P_b = (104.23 \text{ MN})\left(\dfrac{120 \text{ MPa}}{100 \text{ GPa}}\right) = 125.1 \text{ kN}$

Aluminum governs. $P_{max} = 116 \text{ kN} \quad \leftarrow$

Problem 2.4-3

$E = 29000 \text{ksi}$ $A = 8 \text{in}^2$

Use superposition - select A_y as the redundant

Released structure with actual load P at C $\delta_{A1} = \dfrac{2.5\text{kip} \cdot (6\text{ft})}{E \cdot A} = 7.759 \times 10^{-4} \cdot \text{in}$ upward

Released structure with redundant A_y applied at A $\delta_{A2} = A_y \cdot \left(\dfrac{3\text{ft} + 6\text{ft}}{E \cdot A} \right)$ $\dfrac{3\text{ft} + 6\text{ft}}{E \cdot A} = 4.655 \times 10^{-4} \cdot \dfrac{\text{in}}{\text{kip}}$

Compatibility equation $\delta_{A1} + \delta_{A2} = 0$ solve for redundant A_y $A_y = \dfrac{-\delta_{A1}}{\frac{3\text{ft}+6\text{ft}}{E \cdot A}} = -1.667 \cdot \text{kip}$

Statics $B_y = -\left(A_y + 2.5\text{kip} \right) = -0.833 \cdot \text{kip}$

Axial displacement at C $\dfrac{-B_y \cdot (6\text{ft})}{E \cdot A} = 2.586 \times 10^{-4} \cdot \text{in}$ upward ... or $\dfrac{-A_y \cdot (3\text{ft})}{E \cdot A} = 2.586 \times 10^{-4} \cdot \text{in}$

use either extension of segment BC or compression of AC to find upward displ. δ_C

Problem 2.4-4

$P = 10\text{kN}$ $E = 200\text{GPa}$ $\sigma_Y = 400\text{MPa}$ $FS_Y = 2$ $\sigma_a = \dfrac{\sigma_Y}{FS_Y} = 200\cdot\text{MPa}$

Static equilibrium - cut through cables, use lower FBD (see fig.)

$a = 1.5\text{m}$ $b = 1.5\text{m}$ $\alpha_B = \text{atan}\left(\dfrac{a}{b}\right) = 45\cdot\text{deg}$

$\alpha_C = \text{atan}\left(\dfrac{a}{2\cdot b}\right) = 26.565\cdot\text{deg}$

$\Sigma M_A = 0$

$T_1\cdot\sin(\alpha_B) + 2\cdot T_2\cdot\sin(\alpha_C) = P\cdot(2)$
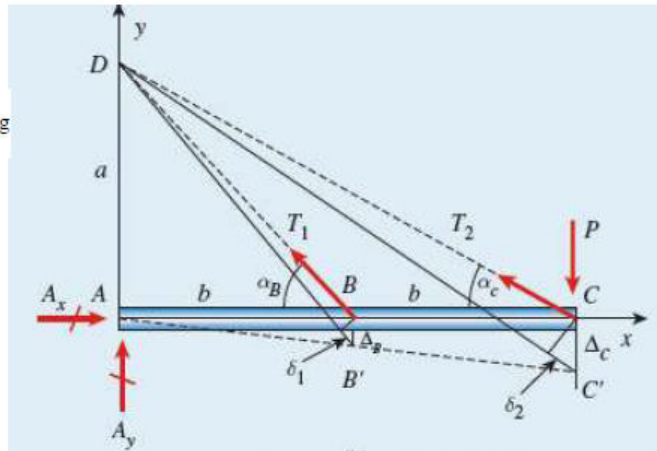
Compatibility - from figure, see that $\Delta_C = 2\cdot\Delta_B$

Cable elongations

$\delta_1 = \Delta_B\cdot\sin(\alpha_B)$ $\delta_2 = \Delta_C\cdot\sin(\alpha_C)$

so $\delta_2 = 2\cdot\left(\dfrac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot\delta_1$ $2\cdot\left(\dfrac{\sin(\alpha_C)}{\sin(\alpha_B)}\right) = 1.26491$

Force-displacement relations for cables

$L_1 = \sqrt{a^2 + b^2} = 2.121\,\text{m}$ $L_2 = \sqrt{a^2 + (2\cdot b)^2} = 3.354\,\text{m}$

$\delta_1 = T_1\cdot f_1$ $f_1 = \dfrac{L_1}{E\cdot A_1}$ $\delta_2 = T_2\cdot f_2$ $f_2 = \dfrac{L_2}{E\cdot A_2}$
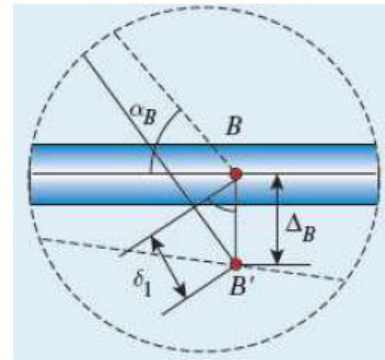
where $T_2\cdot f_2 = 2\cdot\left(\dfrac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot T_1\cdot f_1$ or $T_2 = 2\cdot\left(\dfrac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot\left(\dfrac{f_1}{f_2}\right)\cdot T_1$ and $A_1 = A_2$ so $\dfrac{f_1}{f_2} = \dfrac{L_1}{L_2}$

Substitute T_2 expression into equilibrium equation and solve for T_1 then solve for T_2

$T_1 = \left(\dfrac{2\cdot f_2\cdot\sin(\alpha_B)}{f_2\cdot\sin(\alpha_B)^2 + 4\cdot f_1\cdot\sin(\alpha_C)^2}\right)\cdot P$ or $T_1 = \left[\dfrac{2\cdot\sin(\alpha_B)}{\sin(\alpha_B)^2 + 4\cdot\left(\dfrac{L_1}{L_2}\right)\cdot\sin(\alpha_C)^2}\right]\cdot P = 14.058\cdot\text{kN}$

and $T_2 = 2\cdot\left(\dfrac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot\left(\dfrac{L_1}{L_2}\right)\cdot T_1 = 11.247\cdot\text{kN}$

Use allowable stress σ_a to find minimum required cross sectional area of each cable

$A_1 = \dfrac{T_1}{\sigma_a} = 70.291\cdot\text{mm}^2$ $A_2 = \dfrac{T_2}{\sigma_a} = 56.233\cdot\text{mm}^2$ so $A_{reqd} = 70.3\text{mm}^2$

Problem 2.4-5

$$\sigma_{ys} = 50ksi \qquad \sigma_{yA} = 60ksi \qquad A_s = 12in^2 \qquad A_A = 6in^2 \qquad L = 20in \qquad E_s = 29000ksi \quad E_A = 10600ksi$$

Axial stiffnesses of cylinder and tube - treat as springs in parallel

$$k_s = \frac{E_s \cdot A_s}{L} = 1.74 \times 10^4 \cdot \frac{kip}{in} \qquad\qquad k_A = \frac{E_A \cdot A_A}{L} = 3.18 \times 10^3 \cdot \frac{kip}{in} \qquad\qquad k_T = k_s + k_A = 2.058 \times 10^4 \cdot \frac{kip}{in}$$

Each "spring" carries a force in proportion to its stiffness

$$P_s(P) = \frac{k_s}{k_T} \cdot P \text{ float}, 5 \rightarrow 0.84548 \cdot P \qquad\qquad\qquad P_A(P) = \frac{k_A}{k_T} \cdot P \text{ float}, 5 \rightarrow 0.15452 \cdot P$$

Maximum force in each component is governed by its yield stress

$$\sigma_{ys}(P) = \frac{P_s(P)}{A_s} \text{ float}, 5 \rightarrow \frac{0.070457 \cdot P}{in^2} \qquad\qquad \sigma_{ys}(P) - 50ksi \begin{vmatrix} solve, P \\ float, 5 \end{vmatrix} \rightarrow 709.65 \cdot in^2 \cdot ksi = 709.65 \cdot kip$$

$$\sigma_{yA}(P) = \frac{P_A(P)}{A_A} \text{ float}, 5 \rightarrow \frac{0.025753 \cdot P}{in^2} \qquad\qquad \sigma_{yA}(P) - 60ksi \begin{vmatrix} solve, P \\ float, 5 \end{vmatrix} \rightarrow 2329.8 \cdot in^2 \cdot ksi = 2329.8 \cdot kip$$

So the allowable load P is limited by yield stress in steel cylinder $P_{all} = 709.655kip$

Problem 2.4-6
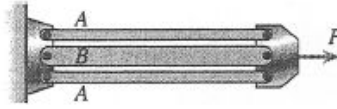
FREE-BODY DIAGRAM OF END PLATE
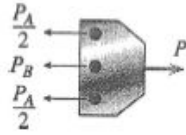
EQUATION OF EQUILIBRIUM

$$\Sigma F_{\text{horiz}} = 0 \quad P_A + P_B - P = 0 \tag{1}$$

EQUATION OF COMPATIBILITY

$$\delta_A = \delta_B \tag{2}$$

FORCE-DISPLACEMENT RELATIONS

A_A = total area of both outer bars

$$\delta_A = \frac{P_A L}{E_A A_k} \quad \delta_B = \frac{P_B L}{E_B A_B} \tag{3}$$

Substitute into Eq. (2):

$$\frac{P_A L}{E_A A_A} = \frac{P_B L}{E_B A_B} \tag{4}$$

SOLUTION OF THE EQUATIONS

Solve simultaneously Eqs. (1) and (4):

$$P_A = \frac{E_A A_A P}{E_A A_A + E_B A_B} \quad P_B = \frac{E_B A_B P}{E_A A_A + E_B A_B} \tag{5}$$

Substitute into Eq. (3):

$$\delta = \delta_A = \delta_B = \frac{PL}{E_A A_A + E_B A_B} \tag{6}$$

STRESSES:

$$\sigma_A = \frac{P_A}{A_A} = \frac{E_A P}{E_A A_A + E_B A_B}$$

$$\sigma_B = \frac{P_B}{A_B} = \frac{E_B P}{E_A A_A + E_B A_B} \tag{7}$$

(a) LOAD IN MIDDLE BAR

$$\frac{P_B}{P} = \frac{E_B A_B}{E_A A_A + E_B A_B} = \frac{1}{\dfrac{E_A A_A}{E_B A_B} + 1}$$

$$\text{Given:} \frac{E_A}{E_B} = 2 \quad \frac{A_A}{A_B} = \frac{1+1}{1.5} = \frac{4}{3}$$

$$\therefore \frac{P_B}{P} = \frac{1}{\left(\dfrac{E_A}{E_B}\right)\left(\dfrac{A_A}{A_B}\right) + 1} = \frac{1}{\dfrac{8}{3} + 1} = \frac{3}{11} \quad \leftarrow$$

(b) RATIO OF STRESSES

$$\frac{\sigma_B}{\sigma_A} = \frac{E_B}{E_A} = \frac{1}{2} \quad \leftarrow$$

(c) RATIO OF STRAINS

All bars have the same strain

Ratio = 1 $\quad \leftarrow$

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Problem 2.4-7

(a) REACTIONS AT A AND B DUE TO LOAD P AT $L/2$

$$A_{AC} = \frac{\pi}{4}\left[d^2 - \left(\frac{d}{2}\right)^2\right] \qquad A_{AC} = \frac{3}{16}\pi d^2$$

$$A_{CB} = \frac{\pi}{4}d^2$$

Select R_B as the redundant; use superposition and a compatibility equation at B:

$$\text{if } x \leq L/2 \qquad \delta_{B1a} = \frac{Px}{EA_{AC}} + \frac{P\left(\frac{L}{2} - x\right)}{EA_{CB}} \qquad \delta_{B1a} = \frac{P}{E}\left(\frac{x}{\frac{3}{16}\pi d^2} + \frac{\frac{L}{2} - x}{\frac{\pi}{4}d^2}\right)$$

$$\delta_{B1a} = \frac{2}{3}P\frac{2x + 3L}{E\pi d^2}$$

$$\text{if } x \geq L/2 \qquad \delta_{B1b} = \frac{P\frac{L}{2}}{EA_{AC}} \qquad \delta_{B1b} = \frac{P\frac{L}{2}}{E\left(\frac{3}{16}\pi d^2\right)} \qquad \delta_{B1b} = \frac{8}{3}\frac{PL}{E\pi d^2}$$

The following expression for δ_{B2} is good for all x:

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{x}{A_{AC}} + \frac{L - x}{A_{CB}}\right) \qquad \delta_{B2} = \frac{R_B}{E}\left(\frac{x}{\frac{3}{16}\pi d^2} + \frac{L - x}{\frac{\pi}{4}d^2}\right)$$

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{16}{3}\frac{x}{\pi d^2} + 4\frac{L - x}{\pi d^2}\right)$$

Solve for R_B and R_A assuming that $x \leq L/2$:

Compatibility: $\quad \delta_{B1a} + \delta_{B2} = 0 \qquad R_{Ba} = \dfrac{-\left(\frac{2}{3}P\frac{2x + 3L}{\pi d^2}\right)}{\left(\frac{16}{3}\frac{x}{\pi d^2} + 4\frac{L - x}{\pi d^2}\right)} \qquad R_{Ba} = \frac{-1}{2}P\frac{2x + 3L}{x + 3L} \quad \leftarrow$

$^\wedge$ check—if $x = 0$, $R_B = -P/2$

Statics: $\quad R_{Aa} = -P - R_{Ba} \qquad R_{Aa} = -P - \frac{-1}{2}P\frac{2x + 3L}{x + 3L} \qquad R_{Aa} = \frac{-3}{2}P\frac{L}{x + 3L} \quad \leftarrow$

$^\wedge$ check—if $x = 0$, $R_{Aa} = -P/2$

Solve for R_B and R_A assuming that $x \geq L/2$:

Compatibility: $\quad \delta_{B1b} + \delta_{B2} = 0 \qquad R_{Bb} = \dfrac{\dfrac{-8}{3}\dfrac{PL}{\pi d^2}}{\left(\dfrac{16}{3}\dfrac{x}{\pi d^2} + 4\dfrac{L-x}{\pi d^2}\right)} \qquad R_{Bb} = \dfrac{-2PL}{x+3L} \quad \leftarrow$

$\wedge$ check—if $x = L$, $R_B = -P/2$

Statics: $\quad R_{Ab} = -P - R_{Bb} \qquad R_{Ab} = -P - \left(\dfrac{-2PL}{x+3L}\right) \qquad R_{Ab} = -P\dfrac{x+L}{x+3L} \quad \leftarrow$

(b) FIND δ AT POINT OF LOAD APPLICATION; AXIAL FORCE FOR SEGMENT 0 TO $L/2 = -R_A$ AND δ = ELONGATION OF THIS SEGMENT

Assume that $x \leq L/2$:

$$\delta_a = \dfrac{-R_{Aa}}{E}\left(\dfrac{x}{A_{AC}} + \dfrac{\dfrac{L}{2}-x}{A_{CB}}\right) \qquad \delta_a = \dfrac{-\left(\dfrac{-3}{2}P\dfrac{L}{x+3L}\right)}{E}\left(\dfrac{x}{\dfrac{3}{16}\pi d^2} + \dfrac{\dfrac{L}{2}-x}{\dfrac{\pi}{4}d^2}\right)$$

$$\delta_a = PL\dfrac{2x+3L}{(x+3L)E\pi d^2}$$

For $x = L/2$, $\qquad \delta_a = \dfrac{8}{7}L\dfrac{P}{E\pi d^2} \quad \leftarrow$

ASSUME THAT $x \geq L/2$:

$$\delta_b = \dfrac{(-R_{Ab})\dfrac{L}{2}}{EA_{AC}} \qquad \delta_b = \dfrac{\left(P\dfrac{x+L}{x+3L}\right)\dfrac{L}{2}}{E\left(\dfrac{3}{16}\pi d^2\right)} \qquad \delta_b = \dfrac{8}{3}P\left(\dfrac{x+L}{x+3L}\right)\dfrac{L}{E\pi d^2} \quad \leftarrow$$

for $x = L/2$ $\qquad \delta_b = \dfrac{8}{7}P\dfrac{L}{E\pi d^2} \qquad <$ same as δ_a above (OK)

(c) FOR WHAT VALUE OF x IS $R_B = (6/5) R_A$?
Guess that $x < L/2$ here and use R_{Ba} expression above to find x:

$$\dfrac{-1}{2}P\dfrac{2x+3L}{x+3L} - \dfrac{6}{5}\left(\dfrac{-3}{2}P\dfrac{L}{x+3L}\right) = 0 \qquad \dfrac{-1}{10}P\dfrac{10x-3L}{x+3L} = 0 \qquad x = \dfrac{3L}{10} \quad \leftarrow$$

Now try $R_{Bb} = (6/5)R_{Ab}$, assuming that $x > L/2$

$$\dfrac{-2PL}{x+3L} - \dfrac{6}{5}\left(-P\dfrac{x+L}{x+3L}\right) = 0 \qquad \dfrac{2}{5}P\dfrac{-2L+3x}{x+3L} = 0 \qquad x = \dfrac{2}{3}L \quad \leftarrow$$

So, there are two solutions for x.

(d) FIND REACTIONS IF THE BAR IS NOW ROTATED TO A VERTICAL POSITION, LOAD P IS REMOVED, AND THE BAR IS HANGING UNDER ITS OWN WEIGHT (ASSUME MASS DENSITY = ρ). ASSUME THAT $x = L/2$.

$$A_{AC} = \frac{3}{16}\pi d^2 \qquad A_{CB} = \frac{\pi}{4}d^2$$

Select R_B as the redundant; use superposition and a compatibility equation at B from (a) above.   compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{x}{A_{AC}} + \frac{L-x}{A_{CB}}\right) \qquad \text{For } x = L/2, \; \delta_{B2} = \frac{R_B}{E}\left(\frac{14}{3}\frac{L}{\pi d^2}\right)$$

$$\delta_{B1} = \int_0^{\frac{L}{2}} \frac{N_{AC}}{EA_{AC}}d\zeta + \int_{\frac{L}{2}}^{L} \frac{N_{CB}}{EA_{CB}}d\zeta$$

Where axial forces in bar due to self weight are $W_{AC} = \rho g A_{AC}\dfrac{L}{2}$ $W_{CB} = \rho g A_{CB}\dfrac{L}{2}$ (assume ζ is measured upward from A):

$$N_{AC} = -\left[\rho g A_{CB}\frac{L}{2} + \rho g A_{AC}\left(\frac{L}{2} - \zeta\right)\right] \qquad A_{AC} = \frac{3}{16}\pi d^2 \qquad A_{CB} = \frac{\pi}{4}d^2$$

$$N_{CB} = -[\rho\, g A_{CB}(L - \zeta)]$$

$$N_{AC} = \frac{-1}{8}\rho g \pi d^2 L - \frac{3}{16}\rho g \pi d^2\left(\frac{1}{2}L - \zeta\right) \qquad N_{CB} = -\left[\frac{1}{4}\rho g \pi d^2(L - \zeta)\right]$$

$$\delta_{B1} = \int_0^{\frac{L}{2}} \frac{\dfrac{-1}{8}\rho g \pi d^2 L - \dfrac{3}{16}\rho g \pi d^2\left(\dfrac{1}{2}L - \zeta\right)}{E\left(\dfrac{3}{16}\pi d^2\right)}d\zeta + \int_{\frac{L}{2}}^{L} \frac{-\left[\dfrac{1}{4}\rho g \pi d^2(L-\zeta)\right]}{E\left(\dfrac{\pi}{4}d^2\right)}d\zeta$$

$$\delta_{B1} = \left(\frac{-11}{24}\rho g\frac{L^2}{E} + \frac{-1}{8}\rho g\frac{L^2}{E}\right) \qquad \delta_{B1} = \frac{-7}{12}\rho g\frac{L^2}{E} \qquad \frac{7}{12} = 0.583$$

Compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{-\left(\dfrac{-7}{12}\rho g\dfrac{L^2}{E}\right)}{\left(\dfrac{14}{3}\dfrac{L}{E\pi d^2}\right)} \qquad R_B = \frac{1}{8}\rho g \pi d^2 L \qquad \leftarrow$$

Statics: $R_A = (W_{AC} + W_{CB}) - R_B$

$$R_A = \left[\left[\rho g\left(\frac{3}{16}\pi d^2\right)\frac{L}{2} + \rho g\left(\frac{\pi}{4}d^2\right)\frac{L}{2}\right] - \frac{1}{8}\rho g \pi d^2 L\right]$$

$$R_A = \frac{3}{32}\rho g \pi d^2 L \qquad \leftarrow$$

Problem 2.4-8

$P = 200\text{kN}$ $L = 2\text{m}$ $t = 20\text{mm}$ $b_1 = 100\text{mm}$ $b_2 = 115\text{mm}$ $E = 96\text{GPa}$

Select reaction R_C as the redundant; use superposition

axial displacement at C due to actual load P at B $\delta_{C1} = \dfrac{P\cdot\left(\dfrac{3\cdot L}{5}\right)}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right) = 1.165\cdot\text{mm}$

axial displacement at C due to redundant R_C $\delta_{C2} = R_C\cdot\left[\dfrac{\dfrac{3\cdot L}{5} + \dfrac{2\cdot L}{5}}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right)\right]$

$$\dfrac{\dfrac{3\cdot L}{5} + \dfrac{2\cdot L}{5}}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right) = 9.706 \times 10^{-3}\cdot\dfrac{\text{mm}}{\text{kN}}$$

Compatibility equation $\delta_{C1} + \delta_{C2} = 0$ solve for R_C $R_C = \dfrac{-\delta_{C1}}{\left[\dfrac{\dfrac{3\cdot L}{5} + \dfrac{2\cdot L}{5}}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right)\right]} = -120\cdot\text{kN}$

Statics $\Sigma F = 0$ $R_A = -(P + R_C) = -80\cdot\text{kN}$

Negative reactions so both act to left

Compute extension of AB or compression of BC to find displ. δ_B (to the right)

$-R_A\cdot\left[\dfrac{\left(\dfrac{3\cdot L}{5}\right)}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right)\right] = 0.466\cdot\text{mm}$ or $-R_C\cdot\left[\dfrac{\left(\dfrac{2\cdot L}{5}\right)}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right)\right] = 0.466\cdot\text{mm}$

Problem 2.4-9

$P = 20\text{kip}$ $\qquad$ $L = 3\text{ft}$ $\qquad$ $t = \dfrac{1}{4}\text{in}$ $\qquad$ $b_1 = 2\text{in}$ $\qquad$ $b_2 = 2.5\text{in}$ $\qquad$ $E = 10400\text{ksi}$

$A_{BC} = b_1 \cdot t = 0.5\cdot\text{in}^2$ $\qquad$ $b_{ave} = \dfrac{b_1 + b_2}{2} = 2.25\cdot\text{in}$

Select reaction R_C as the redundant; use superposition

axial displacement at C due to actual load P at middle of AB $\qquad$ $\delta_{C1} = \dfrac{P\cdot\left(\dfrac{L}{4}\right)}{E\cdot t\cdot(b_2 - b_{ave})}\cdot\ln\left(\dfrac{b_2}{b_{ave}}\right) = 0.029\cdot\text{in}$

axial displacement at C due to redundant R_C $\qquad$ $\delta_{C2} = R_C\cdot\left[\dfrac{\dfrac{L}{2}}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right) + \dfrac{\dfrac{L}{2}}{E\cdot A_{BC}}\right]$

flexibility constant for bar $\qquad$ $\dfrac{\dfrac{L}{2}}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right) + \dfrac{\dfrac{L}{2}}{E\cdot A_{BC}} = 6.551 \times 10^{-3}\cdot\dfrac{\text{in}}{\text{kip}}$

Compatibility equation $\qquad$ $\delta_{C1} + \delta_{C2} = 0$ $\qquad$ solve for R_C $\qquad$ $R_C = \dfrac{-\delta_{C1}}{\left[\dfrac{\dfrac{L}{2}}{E\cdot t\cdot(b_2 - b_1)}\cdot\ln\left(\dfrac{b_2}{b_1}\right) + \dfrac{\dfrac{L}{2}}{E\cdot A_{BC}}\right]} = -4.454\cdot\text{kip}$

Statics $\qquad$ $\Sigma F = 0$ $\qquad$ $R_A = -(P + R_C) = -15.546\cdot\text{kip}$

Negative reactions so both act to left

Compute deformations of AB (two terms, more difficult) or deformation of BC (easier) to find displ. δ_B (to the right)

$$-R_A\cdot\left[\dfrac{\left(\dfrac{L}{4}\right)}{E\cdot t\cdot(b_2 - b_{ave})}\cdot\ln\left(\dfrac{b_2}{b_{ave}}\right)\right] + \dfrac{(-R_A - P)\cdot\dfrac{L}{4}}{E\cdot t\cdot(b_{ave} - b_1)}\cdot\ln\left(\dfrac{b_{ave}}{b_1}\right) = 1.542 \times 10^{-2}\cdot\text{in}$$

or $\qquad$ $-R_C\cdot\left(\dfrac{\dfrac{L}{2}}{E\cdot A_{BC}}\right) = 1.542 \times 10^{-2}\cdot\text{in}$

Problem 2.4-10
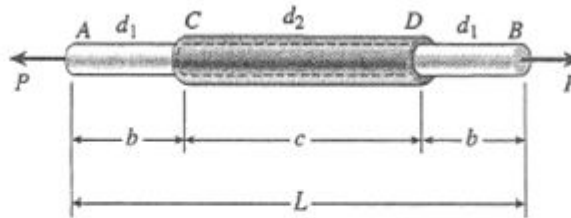
$P = 12$ kN $d_1 = 30$ mm $b = 100$ mm

$L = 500$ mm $d_2 = 45$ mm $c = 300$ mm

Rod: $E_1 = 3.1$ GPa

Sleeve: $E_2 = 2.5$ GPa

Rod: $A_1 = \dfrac{\pi d_1^2}{4} = 706.86$ mm^2

Sleeve: $A_2 = \dfrac{\pi}{4}(d_2^2 - d_1^2) = 883.57$ mm^2

$E_1 A_1 + E_2 A_2 = 4.400$ MN

(a) ELONGATION OF ROD

Part AC: $\delta_{AC} = \dfrac{Pb}{E_1 A_1} = 0.5476$ mm

Part CD: $\delta_{CD} = \dfrac{Pc}{E_1 A_1 + E_2 A_2}$

$= 0.81815$ mm

(From Eq. 2-16 of Example 2-8)

$\delta = 2\delta_{AC} + \delta_{CD} = 1.91$ mm   ←

(b) SLEEVE AT FULL LENGTH

$\delta = \delta_{CD}\left(\dfrac{L}{c}\right) = (0.81815 \text{ mm})\left(\dfrac{500 \text{ mm}}{300 \text{ mm}}\right)$

$= 1.36$ mm   ←

(c) SLEEVE REMOVED

$\delta = \dfrac{PL}{E_1 A_1} = 2.74$ mm   ←

Problem 2.4-11
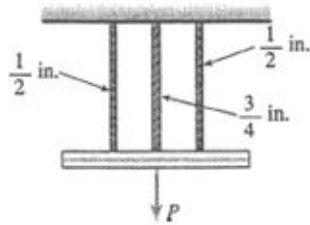
AREAS OF CABLES (from Table 2-1)

Middle cable: $A_M = 0.268$ in.^{2}

Outer cables: $A_O = 0.119$ in.^{2}

(for each cable)

FIRST LOADING

$P_1 = 12$ k $\left(\text{Each cable carries } \dfrac{P_1}{3} \text{ or } 4 \text{ k.}\right)$

SECOND LOADING

$P_2 = 9$ k (additional load)
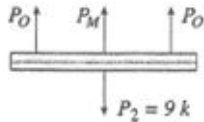
EQUATION OF EQUILIBRIUM

$\Sigma F_{\text{vert}} = 0 \qquad 2P_O + P_M - P_2 = 0$ (1)

EQUATION OF COMPATIBILITY

$\delta_M = \delta_O$ (2)

FORCE-DISPLACEMENT RELATIONS

$\delta_M = \dfrac{P_M L}{EA_M} \qquad \delta_O = \dfrac{P_o L}{EA_o}$ (3, 4)

SUBSTITUTE INTO COMPATIBILITY EQUATION:

$\dfrac{P_M L}{EA_M} = \dfrac{P_O L}{EA_O} \qquad \dfrac{P_M}{A_M} = \dfrac{P_O}{A_O}$ (5)

SOLVE SIMULTANEOUSLY EQS. (1) AND (5):

$P_M = P_2\left(\dfrac{A_M}{A_M + 2A_O}\right) = (9 \text{ k})\left(\dfrac{0.268 \text{ in.}^2}{0.506 \text{ in.}^2}\right)$

    $= 4.767$ k

$P_o = P_2\left(\dfrac{A_o}{A_M + 2A_O}\right) = (9 \text{ k})\left(\dfrac{0.119 \text{ in.}^2}{0.506 \text{ in.}^2}\right)$

    $= 2.117$ k

FORCES IN CABLES

Middle cable: Force $= 4$ k $+ 4.767$ k $= 8.767$ k

Outer cables: Force $= 4$ k $+ 2.117$ k $= 6.117$ k

(for each cable)

(a) PERCENT OF TOTAL LOAD CARRIED BY MIDDLE CABLE

$\text{Percent} = \dfrac{8.767 \text{ k}}{21 \text{ k}}(100\%) = 41.7\%$ ←

(b) STRESSES IN CABLES ($\sigma = P/A$)

Middle cable: $\sigma_M = \dfrac{8.767 \text{ k}}{0.268 \text{ in.}^2} = 32.7$ ksi   ←

Outer cables: $\sigma_O = \dfrac{6.117 \text{ k}}{0.119 \text{ in.}^2} = 51.4$ ksi   ←

Problem 2.4-12
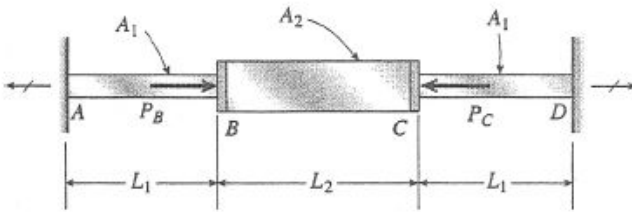
$P_B = 25.5$ kN $\qquad P_C = 17.0$ kN

$L_1 = 200$ mm $\qquad L_2 = 250$ mm

$A_1 = 840$ mm^2 $\qquad A_2 = 1260$ mm^2

$m = $ meter

FREE-BODY DIAGRAM

SOLUTION OF EQUATIONS

Substitute Eqs. (3), (4), and (5) into Eq. (2):

$$\frac{R_A}{E}\left(238.095\frac{1}{m}\right) + \frac{R_A}{E}\left(198.413\frac{1}{m}\right)$$

$$-\frac{P_B}{E}\left(198.413\frac{1}{m}\right) + \frac{R_D}{E}\left(238.095\frac{1}{m}\right) = 0$$

Simplify and substitute $P_B = 25.5$ kN:

$$R_A\left(436.508\frac{1}{m}\right) + R_D\left(238.095\frac{1}{m}\right)$$

$$= 5,059.53 \text{ kN/m} \qquad \text{(Eq. 6)}$$
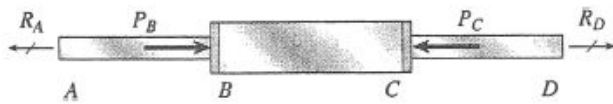
EQUATION OF EQUILIBRIUM

$\Sigma F_{\text{horiz}} = 0 \;\rightarrow\; \leftarrow$

$P_B + R_D - P_C - R_A = 0$ or

$R_A - R_D = P_B - P_C = 8.5$ kN $\qquad$ (Eq. 1)

EQUATION OF COMPATIBILITY

$\delta_{AD} = $ elongation of entire bar

$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 0$ $\qquad$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$\delta_{AB} = \frac{R_A L_1}{EA_1} = \frac{R_A}{E}\left(238.05\frac{1}{m}\right) \qquad \text{(Eq. 3)}$$

$$\delta_{BC} = \frac{(R_A - P_B)L_2}{EA_2}$$

$$= \frac{R_A}{E}\left(198.413\frac{1}{m}\right) - \frac{P_B}{E}\left(198.413\frac{1}{m}\right) \qquad \text{(Eq. 4)}$$

$$\delta_{CD} = \frac{R_D L_1}{EA_1} = \frac{R_D}{E}\left(238.095\frac{1}{m}\right) \qquad \text{(Eq. 5)}$$

(a) REACTIONS R_A AND R_D

Solve simultaneously Eqs. (1) and (6).

From (1): $R_D = R_A - 8.5$ kN

Substitute into (6) and solve for R_A:

$$R_A\left(674.603\frac{1}{m}\right) = 7083.34 \text{ kN/m}$$

$R_A = 10.5$ kN $\quad \leftarrow$

$R_D = R_A - 8.5$ kN $= 2.0$ kN $\quad \leftarrow$

(b) COMPRESSIVE AXIAL FORCE F_{BC}

$F_{BC} = P_B - R_A = P_C - R_D = 15.0$ kN $\quad \leftarrow$

Problem 2.4-13

$n = 6$ $d_b = 0.5$ in. $\sigma_a = 14$ ksi $A_b = \dfrac{\pi}{4} d_b^2 = 0.196$ in.^{2}

(a) FORMULAS FOR REACTIONS F

Segment ABC flexibility: $f_1 = \dfrac{2\left(\dfrac{L}{4}\right)}{EA} = \dfrac{L}{2EA}$

Segment CDE flexibility: $f_2 = \dfrac{2\left(\dfrac{L}{4}\right)}{\dfrac{1}{2}EA} = \dfrac{L}{EA}$

Loads at points B and D:

$P_B = -2P$ $P_D = 3P$

(1) Select R_E as the redundant; find axial displacement δ_1 = displacement at E due to loads P_B and P_D:

$$\delta_1 = \dfrac{(P_B + P_D)\dfrac{L}{4}}{EA} + \dfrac{P_D\dfrac{L}{4}}{EA} + \dfrac{P_D\dfrac{L}{4}}{\dfrac{1}{2}EA} = \dfrac{5LP}{2EA}$$

(2) Next apply redundant R_E and find axial displacement δ_2 = displacement at E due to redundant R_E:

$$\delta_2 = R_E(f_1 + f_2) = \dfrac{3LR_E}{2EA}$$

(3) Use compatibility equation to find redundant R_E then use statics to find R_A:

$\delta_1 + \delta_2 = 0$ solve, $R_E = -\dfrac{5P}{3}$ $R_E = \dfrac{-5}{3}P$

$R_A = -R_E - P_B - P_D = \dfrac{2P}{3}$ $R_A = \dfrac{2P}{3}$ $\boxed{R_A = \dfrac{2P}{3}}$ $\boxed{R_E = \dfrac{-5P}{3}}$

(b) DETERMINE THE AXIAL DISPLACEMENTS δ_B, δ_C, AND δ_D AT POINTS B, C, AND D, RESPECTIVELY.

$\delta_B \dfrac{\left(\dfrac{-2P}{3}\right)\left(\dfrac{L}{4}\right)}{EA} = \dfrac{LP}{6EA}$ leftward

$\delta_C = \delta_B + \dfrac{\left(2P - \dfrac{2P}{3}\right)\left(\dfrac{L}{4}\right)}{EA} = \dfrac{LP}{6EA}$ to the right

$\delta_D = \dfrac{\left(\dfrac{5P}{3}\right)\left(\dfrac{L}{4}\right)}{EA} = \dfrac{5LP}{6EA}$ to the right

(c) DRAW AN AXIAL-DISPLACEMENT DIAGRAM (ADD) IN WHICH THE ABSCISSA IS THE DISTANCE x FROM SUPPORT A TO ANY POINT ON THE BAR AND THE ORDINATE IS THE HORIZONTAL DISPLACEMENT δ AT THAT POINT.

AFD for use below in Part (d)

AFD is composed of 4 constant segments, so ADD is linear with zero displacements at supports A and E.

Axial Force Diagram (AFD)

Axial force (times P) $\dfrac{N(x)}{}$

R_A

R_E

Distance x (times L)
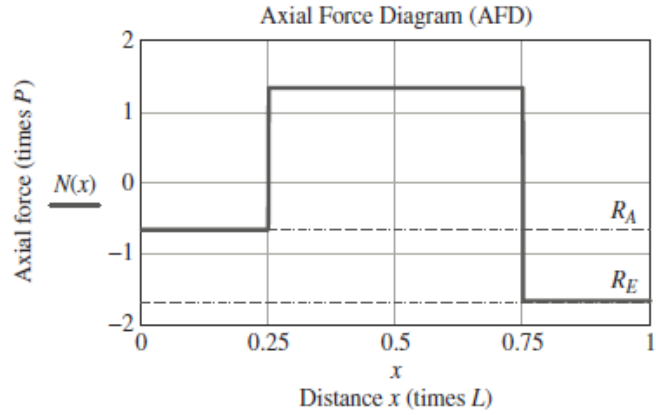
Plot displacements δ_B, δ_C, and δ_D from part (b) above, then connect points using straight lines showing linear variation of axial displacement Between points

$$\delta_{max} = \delta_D \qquad \delta_{max} = \frac{5LP}{6EA} \quad \text{to the right}$$

Boundary conditions at supports:

$$\delta_A = \delta_E = 0$$

Axial Displacement Diagram (ADD)

Axial displ (times L/EA) $\dfrac{\delta(x)}{}$

$\dfrac{5}{6}$

$-\dfrac{1}{6}$

Distance (times L)
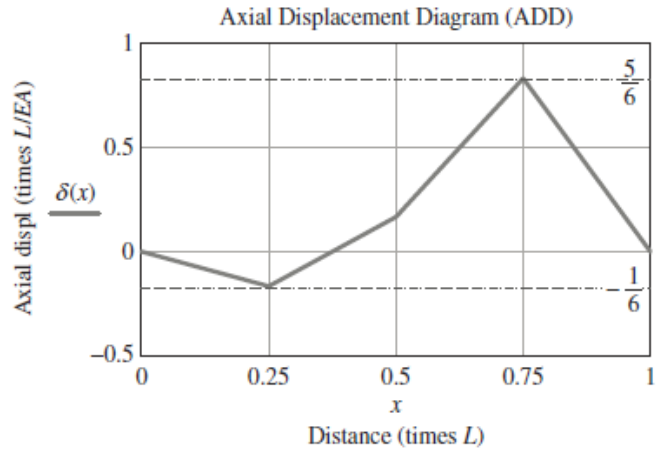
(d) MAXIMUM PERMISSIBLE VALUE OF LOAD VARIABLE P BASED ON ALLOWABLE NORMAL STRESS IN FLANGE BOLTS
FROM AFD, FORCE AT $L/2$:

$$F_{max} = \frac{4}{3}P \qquad \text{and} \qquad F_{max} = n\sigma_a A_b = 16.493 \text{ k}$$

$$P_{max} = \frac{3}{4}F_{max} = 12.37 \text{ k} \qquad \boxed{P_{max} = 12.37 \text{ k}}$$

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Problem 2.4-14

(a) STRESSES AND REACTIONS: SELECT R_1 AS REDUNDANT AND DO SUPERPOSITION ANALYSIS (HERE $q = 0$; DEFLECTION POSITIVE UPWARD)

$d_1 = 50$ mm $\quad d_2 = 60$ mm $\quad d_3 = 57$ mm $\quad d_4 = 64$ mm $\quad A_1 = \dfrac{\pi}{4}(d_2{}^2 - d_1{}^2) = 863.938$ mm^2

$E = 110$ MPa

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad A_2 = \dfrac{\pi}{4}(d_4{}^2 - d_3{}^2) = 665.232$ mm^2

SEGMENT FLEXIBILITIES $\quad L_1 = 2$ m $\quad L_2 = 3$ m

$f_1 = \dfrac{L_1}{EA_1} = 0.02105$ mm/N $\quad f_2 = \dfrac{L_2}{EA_2} = 0.041$ mm/N $\quad \dfrac{f_1}{f_2} = 0.513$

TENSILE stress (σ_1) is known in upper segment so $R_1 = \sigma_1 \times A_1 \qquad \sigma_1 = 10.5$ MPa $\qquad R_1 = \sigma_1 A_1 = 9.07$ kN

$\delta_{1a} = -Pf_2 \quad \delta_{1b} = R_1(f_1 + f_2) \quad$ Compatibility: $\delta_{1a} + \delta_{1b} = 0$

Solve for P: $\quad P = R_1\left(\dfrac{f_1 + f_2}{f_2}\right) = 13.73$ kN

Finally, use statics to find R_2: $\quad R_2 = P - R_1 = 4.66$ kN $\qquad \sigma_2 = \dfrac{R_2}{A_2} = 7$ MPa $\quad <$ **compressive** since R_2 is positive (upward)

$\boxed{P = 13.73 \text{ kN}} \qquad \boxed{R_1 = 9.07 \text{ kN}} \qquad \boxed{R_2 = 4.66 \text{ kN}} \qquad \boxed{\sigma_2 = 7 \text{ MPa}}$

(b) DISPLACEMENT AT CAP PLATE

$\delta_c = R_1 f_1 = 190.909$ mm $\quad <$ downward OR $\delta_c = (R_2)f_2 = 190.909$ mm $\quad <$ downward (neg. x-direction)

$\delta_{cap} = \delta_c = 0.191$ m $\quad \boxed{\delta_{cap} = 190.9 \text{ mm}}$

AFD and ADD: $\quad R_1 = 9.071 \qquad R_2 = 4.657 \qquad L_1 = 2 \qquad A_1 = 863.938 \qquad A_2 = 665.232 \qquad E = 110$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad L_2 = 3$

NOTE: x is measured up from lower support.

Axial Force Diagram (AFD)

Axial Displacement Diagram (ADD)
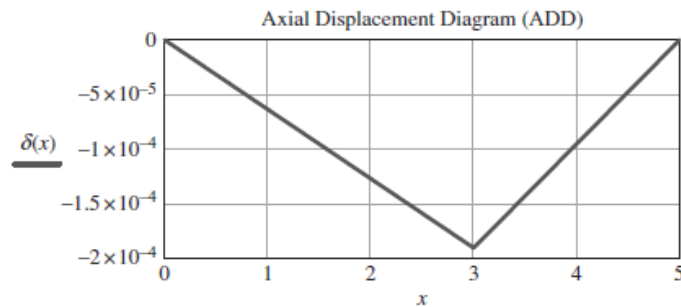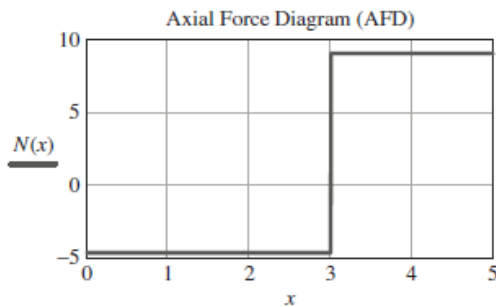
(c) UNIFORM LOAD Q ON SEGMENT 2 SUCH THAT $R_2 = 0$

$P = 13.728$ kN $\qquad R_1 = \sigma_1 A_1 = 9.071$ kN $\qquad L_2 = 3$ m

Equilibrium: $R_1 + R_2 = P - qL_2 \; <$ set $R_2 = 0$, solve for req'd $q \qquad q = \dfrac{P - R_1}{L_2} = 1.552$ kN/m

$\boxed{q = 1.552 \text{ kN/m}}$

Problem 2.4-15
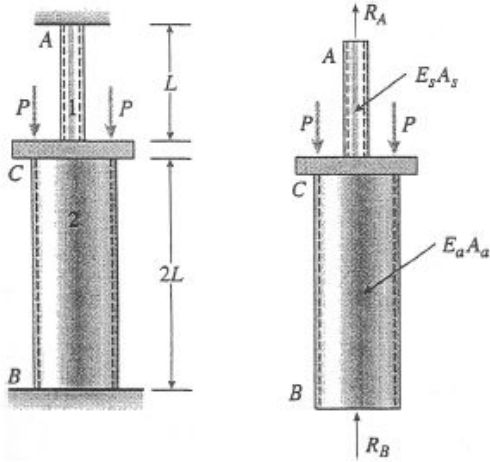
Pipe 1 is steel.
Pipe 2 is aluminum.

EQUATION OF EQUILIBRIUM

$$\Sigma F_{\text{vert}} = 0 \qquad R_A + R_B = 2P \qquad \text{(Eq. 1)}$$

EQUATION OF COMPATIBILITY

$$\delta_{AB} = \delta_{AC} + \delta_{CB} = 0 \qquad \text{(Eq. 2)}$$

(A positive value of δ means elongation.)

FORCE-DISPLACEMENT RELATIONS

$$\delta_{AC} = \frac{R_A L}{E_s A_s} \qquad \delta_{BC} = -\frac{R_B(2L)}{E_a A_a} \qquad \text{(Eqs. 3, 4))}$$

SOLUTION OF EQUATIONS

Substitute Eqs. (3) and (4) into Eq. (2):

$$\frac{R_A L}{E_s A_s} - \frac{R_B(2L)}{E_a A_a} = 0 \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$R_A = \frac{4E_s A_s P}{E_a A_a + 2E_s A_s} \qquad R_B = \frac{2E_a A_a P}{E_a A_a + 2E_s A_s}$$

$$\text{(Eqs. 6, 7)}$$

(a) AXIAL STRESSES

$$\text{Aluminum: } \sigma_a = \frac{R_B}{A_a} = \frac{2E_a P}{E_a A_a + 2E_s A_s} \quad \leftarrow$$

$$\text{(compression)} \qquad \text{(Eq. 8)}$$

$$\text{Steel: } \sigma_s = \frac{R_A}{A_s} = \frac{4E_s P}{E_a A_a + 2E_s A_s} \quad \leftarrow \quad \text{(Eq. 9)}$$

$$\text{(tension)}$$

(b) NUMERICAL RESULTS

$$P = 12 \text{ k} \qquad A_a = 8.92 \text{ in.}^2 \qquad A_s = 1.03 \text{ in.}^2$$

$$E_a = 10 \times 10^6 \text{ psi} \qquad E_s = 29 \times 10^6 \text{ psi}$$

Substitute into Eqs. (8) and (9):

$$\sigma_a = 1,610 \text{ psi (compression)} \quad \leftarrow$$

$$\sigma_s = 9,350 \text{ psi (tension)} \quad \leftarrow$$

Problem 2.4-16

Numerical data:

$W = 800$ N $\qquad L = 150$ mm

$\quad a = 50$ mm $\qquad d_S = 2$ mm

$d_A = 4$ mm $\qquad E_S = 210$ GPa

$E_A = 70$ GPa

$\sigma_{Sa} = 220$ MPa $\qquad \sigma_{Aa} = 80$ MPa

$$A_A = \frac{\pi}{4}d_A^2 \qquad A_S = \frac{\pi}{4}d_S^2$$

$A_A = 13$ mm^2 $\qquad A_S = 3$ mm^2

(a) P_{allow} AT CENTER OF BAR

One-degree statically indeterminate - use reaction (R_A) at top of aluminum bar as the redundant

compatibility: $\delta_1 - \delta_2 = 0$ $\qquad$ Statics: $\qquad 2R_S + R_A = P + W$

$$\delta_1 = \frac{P+W}{2}\left(\frac{L}{E_S A_S}\right) \qquad \begin{array}{l}< \text{downward displacement due to elongation of each steel wire under } P + W \text{ if} \\ \text{aluminum wire is cut at top}\end{array}$$

$$\delta_2 = R_A\left(\frac{L}{2E_S A_S} + \frac{L}{E_A A_A}\right) \qquad \begin{array}{l}< \text{upward displ. due to shortening of steel wires and elongation of aluminum} \\ \text{wire under redundant } R_A\end{array}$$

Enforce compatibility and then solve for R_A:

$$\delta_1 = \delta_2 \quad \text{so} \quad R_A = \frac{\dfrac{P+W}{2}\left(\dfrac{L}{E_S A_S}\right)}{\dfrac{L}{2E_S A_S} + \dfrac{L}{E_A A_A}} \qquad R_A = (P+W)\frac{E_A A_A}{E_A A_A + 2E_S A_S} \qquad \text{and} \qquad \sigma_{Aa} = \frac{R_A}{A_A}$$

Now use statics to find R_S:

$$R_S = \frac{P+W-R_A}{2} \qquad R_S = \frac{P+W-(P+W)\dfrac{E_A A_A}{E_A A_A + 2E_S A_S}}{2} \qquad R_S = (P+W)\frac{E_S A_S}{E_A A_A + 2E_S A_S}$$

$$\text{and} \qquad \sigma_{Sa} = \frac{R_S}{A_S}$$

Compute stresses and apply allowable stress values:

$$\sigma_{Aa} = (P+W)\frac{E_A}{E_A A_A + 2E_S A_S} \qquad \sigma_{Sa} = (P+W)\frac{E_S}{E_A A_A + 2E_S A_S}$$

Solve for allowable load P:

$$P_{Aa} = \sigma_{Aa}\left(\frac{E_A A_A + 2E_S A_S}{E_A}\right) - W \qquad P_{Sa} = \sigma_{Sa}\left(\frac{E_A A_A + 2E_S A_S}{E_S}\right) - W \qquad \text{(lower value of } P \text{ controls)}$$

$P_{Aa} = 1713$ N $\qquad\qquad\qquad\qquad P_{Sa} = 1504$ N $\qquad \leftarrow P_{\text{allow}}$ is controlled by steel wires

(b) P_{allow} IF LOAD P AT $x = a/2$

Again, cut aluminum wire at top, then compute elongations of left and right steel wires:

$$\delta_{1L} = \left(\frac{3P}{4} + \frac{W}{2}\right)\left(\frac{L}{E_SA_S}\right) \qquad \delta_{1R} = \left(\frac{P}{4} + \frac{W}{2}\right)\left(\frac{L}{E_SA_S}\right)$$

$$\delta_1 = \frac{\delta_{1L} + \delta_{1R}}{2} \qquad \delta_1 = \frac{P + W}{2}\left(\frac{L}{E_SA_S}\right) \text{ where } \delta_1 = \text{displacement at } x = a$$

Use δ_2 from part (a):

$$\delta_2 = R_A\left(\frac{L}{2E_SA_S} + \frac{L}{E_AA_A}\right)$$

So equating δ_1 and δ_2, solve for R_A: $\quad R_A = (P + W)\dfrac{E_AA_A}{E_AA_A + 2E_SA_S}$

$\wedge$ same as in part (a)

$$R_{SL} = \frac{3P}{4} + \frac{W}{2} - \frac{R_A}{2} \qquad < \text{stress in left steel wire exceeds that in right steel wire}$$

$$R_{SL} = \frac{3P}{4} + \frac{W}{2} - \frac{(P + W)\dfrac{E_AA_A}{E_AA_A + 2E_SA_S}}{2}$$

$$R_{SL} = \frac{PE_AA_A + 6PE_SA_S + 4WE_SA_S}{4E_AA_A + 8E_SA_S} \qquad \sigma_{Sa} = \frac{PE_AA_A + 6PE_SA_S + 4WE_SA_S}{4E_AA_A + 8E_SA_S}\left(\frac{1}{A_S}\right)$$

Solve for P_{allow} based on allowable stresses in steel and aluminum:

$$P_{Sa} = \frac{\sigma_{Sa}(4A_SE_AA_A + 8E_SA_S{}^2) - (4WE_SA_S)}{E_AA_A + 6E_SA_S} \qquad P_{Aa} = 1713 \text{ N} \qquad < \text{same as in part(a)}$$

$P_{Sa} = 820 \text{ N} \qquad \leftarrow \quad$ steel controls

(c) P_{allow} IF WIRES ARE SWITCHED AS SHOWN AND $x = a/2$

Select R_A as the redundant; statics on the two released structures:

(1) Cut aluminum wire—apply P and W, compute forces in left and right steel wires, then compute displacements at each steel wire:

$$R_{SL} = \frac{P}{2} \qquad R_{SR} = \frac{P}{2} + W$$

$$\delta_{1L} = \frac{P}{2}\left(\frac{L}{E_SA_S}\right) \qquad \delta_{1R} = \left(\frac{P}{2} + W\right)\left(\frac{L}{E_SA_S}\right)$$

By geometry, δ at aluminum wire location at far right is $\qquad \delta_1 = \left(\dfrac{P}{2} + 2W\right)\left(\dfrac{L}{E_S A_S}\right)$

(2) Next apply redundant R_A at right wire, compute wire force and displacement at aluminum wire:

$$R_{SL} = -R_A \qquad R_{SR} = 2R_A \qquad \delta_2 = R_A\left(\dfrac{5L}{E_S A_S} + \dfrac{L}{E_A A_A}\right)$$

(3) Compatibility equate δ_1, δ_2 and solve for R_A, then P_{allow} for aluminum wire:

$$R_A = \dfrac{\left(\dfrac{P}{2} + 2W\right)\left(\dfrac{L}{E_S A_S}\right)}{\dfrac{5L}{E_S A_S} + \dfrac{L}{E_A A_A}} \qquad R_A = \dfrac{E_A A_A P + 4E_A A_A W}{10E_A A_A + 2E_S A_S} \qquad \sigma_{Aa} = \dfrac{R_A}{A_A}$$

$$\sigma_{Aa} = \dfrac{E_A P + 4E_A W}{10E_A A_A + 2E_S A_S}$$

$$P_{Aa} = \dfrac{\sigma_{Aa}(10E_A A_A + 2E_S A_S) - 4E_A W}{E_A} \qquad P_{Aa} = 1713 \text{ N}$$

(4) Statics or superposition—find forces in steel wires, then P_{allow} for steel wires:

$$R_{SL} = \dfrac{P}{2} + R_A \qquad R_{SL} = \dfrac{P}{2} + \dfrac{E_A A_A P + 4E_A A_A W}{10E_A A_A + 2E_S A_S}$$

$$R_{SL} = \dfrac{6E_A A_A P + PE_S A_S + 4E_A A_A W}{10E_A A_A + 2E_S A_S} \qquad \begin{array}{l} < \text{larger than } R_{SR}\text{, so use in allowable stress} \\ \text{calculations} \end{array}$$

$$R_{SR} = \dfrac{P}{2} + W - 2R_A \qquad R_{SR} = \dfrac{P}{2} + W - \dfrac{E_A A_A P + 4E_A A_A W}{5E_A A_A + E_S A_S}$$

$$R_{SR} = \dfrac{3E_A A_A P + PE_S A_S + 2E_A A_A W + 2WE_S A_S}{10E_A A_A + 2E_S A_S}$$

$$\sigma_{Sa} = \dfrac{R_{SL}}{A_S} \qquad P_{Sa} = \sigma_{Sa} A_S\left(\dfrac{10E_A A_A + 2E_S A_S}{6E_A A_A + E_S A_S}\right) - \dfrac{4E_A A_A W}{6E_A A_A + E_S A_S}$$

$$P_{Sa} = \dfrac{10\sigma_{Sa} A_S E_A A_A + 2\sigma_{Sa} A_S^2 E_S - 4E_A A_A W}{6E_A A_A + E_S A_S} \qquad \begin{array}{l} P_{Sa} = 703 \text{ N} \quad \leftarrow \\ \text{^ steel controls} \end{array}$$

Problem 2.4-17
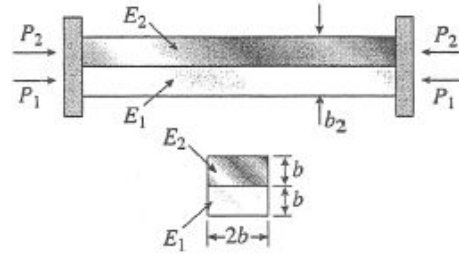
FREE-BODY DIAGRAM

(Plate at right-hand end)
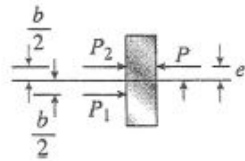
EQUATIONS OF EQUILIBRIUM

$\Sigma F = 0 \quad P_1 + P_2 = P$ (Eq. 1)

$\Sigma M = 0 \quad Pe + P_1\left(\dfrac{b}{2}\right) - P_2\left(\dfrac{b}{2}\right) = 0$ (Eq. 2)

EQUATION OF COMPATIBILITY

$\delta_2 = \delta_1$

$\dfrac{P_2 L}{E_2 A} = \dfrac{P_1 L}{E_1 A}$ or $\dfrac{P_2}{E_2} = \dfrac{P_1}{E_1}$ (Eq. 3)

(a) AXIAL FORCES

Solve simultaneously Eqs. (1) and (3):

$$P_1 = \dfrac{PE_1}{E_1 + E_2} \quad P_2 = \dfrac{PE_2}{E_1 + E_2} \quad \leftarrow$$

(b) ECCENTRICITY OF LOAD P

Substitute P_1 and P_2 into Eq. (2) and solve for e:

$$e = \dfrac{b(E_2 - E_1)}{2(E_2 + E_1)} \quad \leftarrow$$

(c) RATIO OF STRESSES

$$\sigma_1 = \dfrac{P_1}{A} \quad \sigma_2 = \dfrac{P_2}{A} \quad \dfrac{\sigma_1}{\sigma_2} = \dfrac{P_1}{P_2} = \dfrac{E_1}{E_2} \quad \leftarrow$$

Problem 2.4-18

$L = 2.5$ m $b = 0.71$ $L = 1.775$ m $E = 210$ GPa $A = 3500$ mm^2 $P = 185$ kN $\theta_A = 60°$

$\sigma_a = 150$ MPa

FIND MISSING DIMENSIONS AND ANGLES IN PLANE TRUSS FIGURE

$x_c = b\cos(\theta_A) = 0.8875$ m $y_c = b\sin(\theta_A) = 1.5372$ m

$$\frac{b}{\sin(\theta_B)} = \frac{L}{\sin(\theta_A)} \quad \text{so} \quad \theta_B = a\sin\left(\frac{b\sin(\theta_A)}{L}\right) = 37.94306°$$

$\theta_C = 180° - (\theta_A + \theta_B) = 82.05694°$

$$c = \frac{L}{\sin(\theta_A)}\sin(\theta_C) = 2.85906 \text{ m} \quad \text{or} \quad c = \sqrt{b^2 + L^2 - 2bL\cos(\theta_C)} = 2.85906 \text{ m}$$

(a) SELECT B_X AS THE REDUNDANT; PERFORM SUPERPOSITION ANALYSIS TO FIND B_X THEN USE STATICS TO FIND REMAINING REACTIONS. FINALLY USE METHOD OF JOINTS TO FIND MEMBER FORCES (SEE EXAMPLE 1-1)

   δ_{Bx1} = displacement in x-direction in released structure acted upon by loads P and $2P$ at joint C:

   $\delta_{Bx1} = 1.2789911$ mm   < this displacement equals force in AB divided by flexibility of AB

   δ_{Bx2} = displacement in x-direction in released structure acted upon by redundant B_x: $\delta_{BX2} = B_x\dfrac{c}{EA}$

   COMPATIBILITY EQUATION: $\delta_{BX1} + \delta_{BX2} = 0$ so $B_X = \dfrac{-EA}{c}\delta_{BX1} = -328.8$ kN

   STATICS: $\Sigma F_X = 0$ $A_X = -B_X - 2P = -41.2$ kN

   $\Sigma M_A = 0$ $B_y = \dfrac{1}{c}[2P(b\sin(\theta_A)) + P(b\cos(\theta_A))] = 256.361$ kN

   $\Sigma F_y = 0$ $A_y = P - B_y = -71.361$ kN

   REACTIONS:

$A_x = -41.2$ kN	$A_y = -71.4$ kN	$B_x = -329$ kN	$B_y = 256$ kN

(b) FIND MAXIMUM PERMISSIBLE VALUE OF LOAD VARIABLE P IF ALLOWABLE NORMAL STRESS IS 150 MPA

   (1) Use reactions and Method of Joints to find member forces in each member for above loading.

   Results: $F_{AB} = 0$ $F_{BC} = -416.929$ kN $F_{AC} = 82.40$ kN

   (2) Compute member stresses:

   $\sigma_{AB} = 0$ $\sigma_{BC} = \dfrac{-416.93 \text{ kN}}{A} = -119.123$ MPa $\sigma_{AC} = \dfrac{82.4 \text{ kN}}{A} = 23.543$ MPa

   (3) Maximum stress occurs in member BC. For linear analysis, the stress is proportional to the load so

$P_{max} = \left|\dfrac{\sigma_a}{\sigma_{BC}}\right|P = 233$ kN

   So when downward load $P = 233$ kN is applied at C and horizontal load $2P = 466$ kN is applied to the right at C, the stress in BC is 150 MPa

Problem 2.4-19
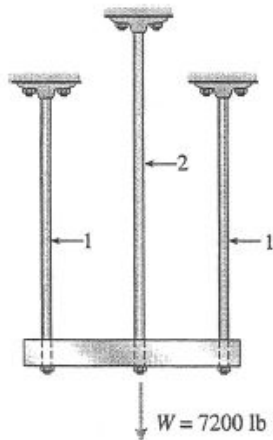
BAR 1 ALUMINUM

$E_1 = 10 \times 10^6$ psi

$d_1 = 0.4$ in.

$L_1 = 40$ in.

$\sigma_1 = 24{,}000$ psi

BAR 2 MAGNESIUM

$E_2 = 6.5 \times 10^6$ psi

$d_2 = ?$ $L_2 = ?$

$\sigma_2 = 13{,}000$ psi

$W = 7200$ lb

FREE-BODY DIAGRAM OF RIGID BAR
EQUATION OF EQUILIBRIUM
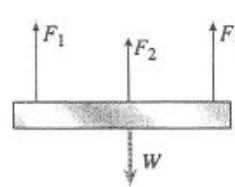
$\Sigma F_{\text{vert}} = 0$

$2F_1 + F_2 - W = 0$ (Eq. 1)

FULLY STRESSED RODS

$F_1 = \sigma_1 A_1$ $F_2 = \sigma_2 A_2$

$$A_1 = \frac{\pi d_1^2}{4} \qquad A_2 = \frac{\pi d_2^2}{4}$$

Substitute into Eq. (1):

$$2\sigma_1\left(\frac{\pi d_1^2}{4}\right) + \sigma_2\left(\frac{\pi d_2^2}{4}\right) = W$$

Diameter d_1 is known; solve for d_2:

$$d_2^2 = \frac{4W}{\pi\sigma_2} - \frac{2\sigma_1 d_1^2}{\sigma_2} \quad \leftarrow \qquad \text{(Eq. 2)}$$

SUBSTITUTE NUMERICAL VALUES:

$$d_2^2 = \frac{4(7200 \text{ lb})}{\pi(13{,}000 \text{ psi})} - \frac{2(24{,}000 \text{ psi})(0.4 \text{ in.})^2}{13{,}000 \text{ psi}}$$

$$= 0.70518 \text{ in.}^2 - 0.59077 \text{ in.}^2 = 0.11441 \text{ in.}^2$$

$d_2 = 0.338$ in. $\leftarrow$

EQUATION OF COMPATIBILITY

$$\delta_1 = \delta_2 \qquad \text{(Eq. 3)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{F_1 L_1}{E_1 A_1} = \sigma_1\left(\frac{L_1}{E_1}\right) \qquad \text{(Eq. 4)}$$

$$\delta_2 = \frac{F_2 L_2}{E_2 A_2} = \sigma_2\left(\frac{L_2}{E_2}\right) \qquad \text{(Eq. 5)}$$

Substitute (4) and (5) into Eq. (3):

$$\sigma_1\left(\frac{L_1}{E_1}\right) = \sigma_2\left(\frac{L_2}{E_2}\right)$$

Length L_1 is known; solve for L_2:

$$L_2 = L_1\left(\frac{\sigma_1 E_2}{\sigma_2 E_1}\right) \quad \leftarrow \qquad \text{(Eq. 6)}$$

SUBSTITUTE NUMERICAL VALUES:

$$L_2 = (40 \text{ in.})\left(\frac{24{,}000 \text{ psi}}{13{,}000 \text{ psi}}\right)\left(\frac{6.5 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}}\right)$$

$$= 48.0 \text{ in.}$$

Problem 2.4-20
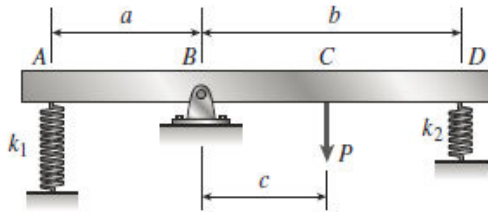
NUMERICAL DATA

$a = 250$ mm

$b = 500$ mm

$c = 200$ mm

$k_1 = 10$ kN/m

$k_2 = 25$ kN/m

$\theta_{max} = 3° = \dfrac{\pi}{60}$ rad

FREE-BODY DIAGRAM AND DISPLACEMENT DIAGRAM
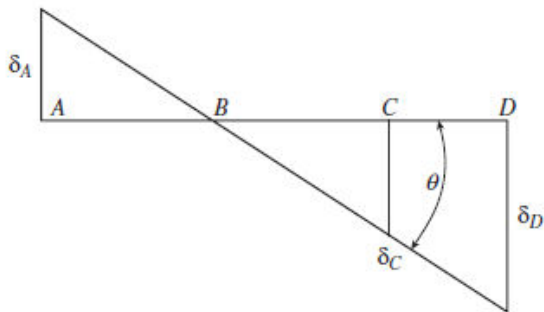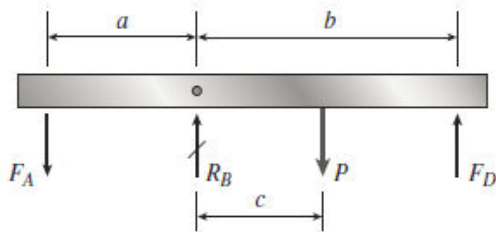
EQUATION OF EQUILIBRIUM

$\Sigma M_B = 0 + - F_A(a) - P(c) + F_D(b) = 0$ 　　(Eq. 1)

EQUATION OF COMPATIBILITY

$\dfrac{\delta_A}{a} = \dfrac{\delta_D}{b}$ 　　(Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$\delta_A = \dfrac{F_A}{k_1} \quad \delta_D = \dfrac{F_D}{k_2}$ 　　(Eqs. 3, 4)

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$\dfrac{F_A}{ak_1} = \dfrac{F_D}{bk_2}$ 　　(Eq. 5)

SOLVE SIMULTANEOUSLY EQS. (1) AND (5):

$F_A = \dfrac{ack_1 P}{a^2 k_1 + b^2 k_2} \qquad F_D = \dfrac{bck_2 P}{a^2 k_1 + b^2 k_2}$

ANGLE OF ROTATION

$\delta_D = \dfrac{F_D}{k_2} = \dfrac{bcP}{a^2 k_1 + b^2 k_2} \qquad \theta = \dfrac{\delta_D}{b} = \dfrac{cP}{a^2 k_1 + b^2 k_2}$

MAXIMUM LOAD

$P = \dfrac{\theta}{c}(a^2 k_1 + b^2 k_2)$

$P_{max} = \dfrac{\theta_{max}}{c}(a^2 k_1 + b^2 k_2)$ 　　←

SUBSTITUTE NUMERICAL VALUES:

$P_{max} = \dfrac{\pi/60 \text{ rad}}{200 \text{ mm}}[(250 \text{ mm})^2(10 \text{ kN/m})$

$+ (500 \text{ mm})^2(25 \text{ kN/m})]$

$= 1800$ N 　　←

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Problem 2.4-21
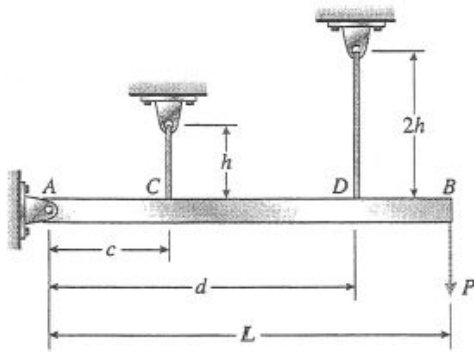
$h = 18$ in.

$2h = 36$ in.

$c = 20$ in.

$d = 50$ in.

$L = 66$ in.

$E = 30 \times 10^6$ psi

$A = 0.0272$ in.^{2}

$P = 340$ lb

FREE-BODY DIAGRAM
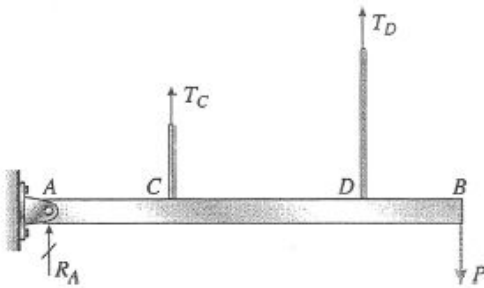
DISPLACEMENT DIAGRAM
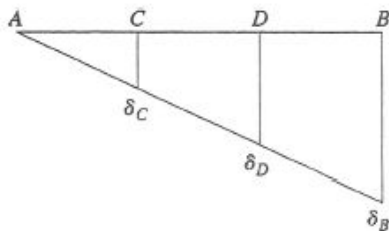
EQUATION OF EQUILIBRIUM

$\Sigma M_A = 0 \quad \curvearrowleft \curvearrowright \quad T_C(c) + T_D(d) = PL$ (Eq. 1)

EQUATION OF COMPATIBILITY

$$\frac{\delta_C}{c} = \frac{\delta_D}{d}$$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$\delta_C = \frac{T_C h}{EA} \quad \delta_D = \frac{T_D(2h)}{EA}$$ (Eqs. 3, 4)

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{T_C h}{cEA} = \frac{T_D(2h)}{dEA} \quad \text{or} \quad \frac{T_C}{c} = \frac{2T_D}{d}$$ (Eq. 5)

TENSILE FORCES IN THE WIRES

Solve simultaneously Eqs. (1) and (5):

$$T_C = \frac{2cPL}{2c^2 + d^2} \quad T_D = \frac{dPL}{2c^2 + d^2}$$ (Eqs. 6, 7)

TENSILE STRESSES IN THE WIRES

$$\sigma_C = \frac{T_C}{A} = \frac{2cPL}{A(2c^2 + d^2)}$$ (Eq. 8)

$$\sigma_D = \frac{T_D}{A} = \frac{dPL}{A(2c^2 + d^2)}$$ (Eq. 9)

DISPLACEMENT AT END OF BAR

$$\delta_B = \delta_D\left(\frac{L}{d}\right) = \frac{2hT_D}{EA}\left(\frac{L}{d}\right) = \frac{2hPL^2}{EA(2c^2 + d^2)}$$ (Eq. 10)

SUBSTITUTE NUMERICAL VALUES

$2c^2 + d^2 = 2(20 \text{ in.})^2 + (50 \text{ in.})^2 = 3300 \text{ in.}^2$

(a) $\sigma_C = \dfrac{2cPL}{A(2c^2 + d^2)} = \dfrac{2(20 \text{ in.})(340 \text{ lb})(66 \text{ in.})}{(0.0272 \text{ in.}^2)(3300 \text{ in.}^2)}$

$\qquad = 10{,}000$ psi $\leftarrow$

$\sigma_D = \dfrac{dPL}{A(2c^2 + d^2)} = \dfrac{(50 \text{ in.})(340 \text{ lb})(66 \text{ in.})}{(0.0272 \text{ in.}^2)(3300 \text{ in.}^2)}$

$\qquad = 12{,}500$ psi $\leftarrow$

(b) $\delta_B = \dfrac{2hPL^2}{EA(2c^2 + d^2)}$

$\qquad = \dfrac{2(18 \text{ in.})(340 \text{ lb})(66 \text{ in.})^2}{(30 \times 10^6 \text{ psi})(0.0272 \text{ in.}^2)(3300 \text{ in.}^2)}$

$\qquad = 0.0198$ in. $\leftarrow$

Problem 2.4-22

Remove pin at B; draw separate FBD's of beam and column. Find selected forces using statics

   From FBD of column DBF

$$\Sigma M_B = D_x \cdot \frac{L}{2} = 0 \qquad D_x = 0$$

$$\Sigma F_x = D_x - B_x = 0 \qquad B_x = D_x$$

   From FBD of beam ABC

$$\Sigma F_x = A_x + B_x = 0 \qquad A_x = 0$$

$$\Sigma M_B = M_A - 2P \cdot \frac{L}{3} = 0 \qquad M_A = 2P \cdot \frac{L}{3}$$

$$\Sigma F_y = B_y - 2P = 0 \qquad B_y = 2P$$
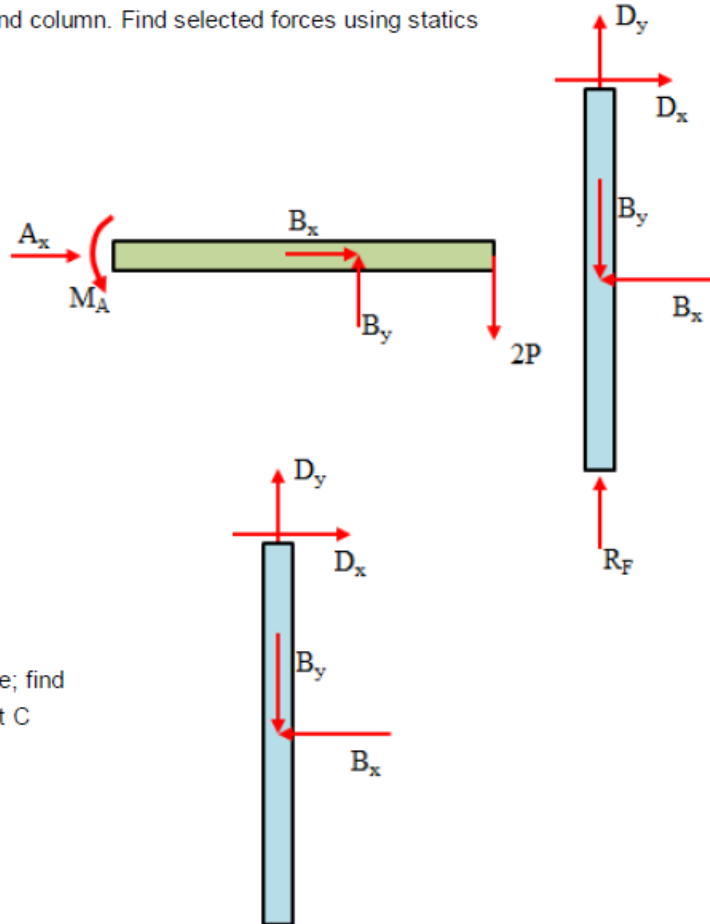
Remove reaction R_F to create the release structure; find vertical displacement at F due to actual load 2P at C

$$\delta_{F1} = \frac{B_y \cdot \frac{L}{2}}{EA} \qquad \delta_{F1} = \frac{P \cdot L}{EA}$$

Apply redundant R_F to released structure; find vertical displacement at F

$$B_y = 0 \qquad \delta'_{F2} = \frac{-R_F \cdot \frac{L}{2}}{2EA} - \frac{R_F \cdot \frac{L}{2}}{EA} \qquad \delta'_{F2} = -R_F \left(\frac{L}{4 \cdot EA} + \frac{L}{2EA} \right) \qquad \delta'_{F2} = -R_F \cdot \frac{3L}{4 \cdot EA}$$

Compatibility equation - solve for R_F

$$\delta_{F1} + \delta'_{F2} = 0 \qquad R_F = \frac{\frac{P \cdot L}{EA}}{\left(\frac{3L}{4 \cdot EA} \right)} \qquad R_F = \frac{4}{3} P$$

Finally solve for reaction D_y using FBD of DBF

$$\Sigma F_y = 0 \qquad D_y = B_y - R_F \qquad D_y = 2P - \frac{4}{3}P \qquad D_y = \frac{2}{3}P$$
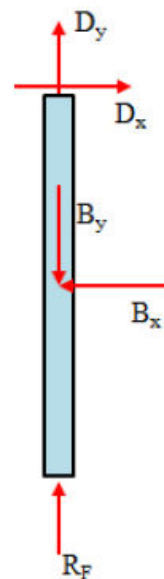
Problem 2.4-23

Numerical properties (kips, inches):

$d_c = 2.25$ in. $d_b = 1.75$ in. $d_s = 1.25$ in.

$E_c = 18,000$ ksi $E_b = 16,000$ ksi

$E_s = 30000$ ksi

$P = 9$ k

$$A_s = \frac{\pi}{4} d_s^2$$

$$A_b = \frac{\pi}{4}(d_b^2 - d_s^2)$$

$$A_c = \frac{\pi}{4}(d_c^2 - d_b^2)$$

EQUATION OF EQUILIBRIUM

$\Sigma F_{\text{vert}} = 0$ $P_s + P_b + P_c = P$ (Eq. 1)

EQUATIONS OF COMPATIBILITY

$\delta_s = \delta_b$ $\delta_c = \delta_s$ (Eqs. 2)

FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_b = \frac{P_b L}{E_b A_b} \quad \delta_c = \frac{P_c L}{E_c A_c}$$ (Eqs. 3, 4, 5)

SOLUTION OF EQUATIONS

Substitute (3), (4), and (5) into Eqs. (2):

$$P_b = P_s \frac{E_b A_b}{E_s A_s} \quad P_c = P_s \frac{E_c A_c}{E_s A_s}$$ (Eqs. 6, 7)

SOLVE SIMULTANEOUSLY EQS. (1), (6), AND (7):

$$P_s = P\frac{E_s A_s}{E_s A_s + E_b A_b + E_c A_c} = 3.95 \text{ k}$$

$$P_b = P\frac{E_b A_b}{E_s A_s + E_b A_b + E_c A_c} = 2.02 \text{ k}$$

$$P_c = P\frac{E_c A_c}{E_s A_s + E_b A_b + E_c A_c} = 3.03 \text{ k}$$

$P_s + P_b + P_c = 9$ statics check

COMPRESSIVE STRESSES

Let $\Sigma EA = E_s A_s + E_b A_b + E_c A_c$

$$\sigma_s = \frac{P_s}{A_s} = \frac{PE_s}{\Sigma EA} \qquad \sigma_s = 3.22 \text{ ksi} \qquad \leftarrow$$

$$\sigma_b = \frac{P_b}{A_b} = \frac{PE_b}{\Sigma EA} \qquad \sigma_b = 1.716 \text{ ksi} \qquad \leftarrow$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{PE_c}{\Sigma EA} \qquad \sigma_c = 1.93 \text{ ksi} \qquad \leftarrow$$

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Problem 2.4-24

Remove R_F to create <u>released structure</u>; use superposition to find redundant R_F = y-dir reaction at F

Released structure under actual load; use FBD of ABC to find pin force B_y

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{\frac{L}{3}} \cdot (3 \cdot P \cdot L) \qquad B_y \rightarrow 9 \cdot P \qquad \text{acts upward on ABC so acts downward on DBF}$$

Find vert. displ. of F in released structure under actual loads $\quad \delta_{F1} = \frac{-B_y \cdot L}{2 \cdot EA} \qquad \delta_{F1} \rightarrow -\frac{9 \cdot L \cdot P}{2 \cdot EA} \quad$ downward

Apply redundant R_F and find vertical displ. at F in released structure $\qquad \delta_{F2} = R_F \left(\frac{\frac{L}{3}}{EA} + \frac{L}{2 \cdot EA} \right) \quad \delta_{F2} \rightarrow \frac{5 \cdot L \cdot R_F}{6 \cdot EA}$

Compatibility equ. $\quad \delta_{F1} + \delta_{F2} = 0 \qquad R_F = \frac{-\delta_{F1}}{\frac{5}{6} \cdot \frac{L}{EA}} \qquad R_F \rightarrow \frac{27 \cdot P}{5} \qquad\qquad$ upward

<u>Now use statics to find all remaining reactions</u> $\qquad$ FBD of DBF $\qquad \Sigma M_B = 0 \quad$ so $\qquad D_x = 0$

Entire structure $\qquad \Sigma F_x = 0 \quad A_x = 0 \qquad\qquad\qquad \Sigma M_B = 0 \quad A_y = \frac{1}{\frac{L}{3}} \left[-3 \cdot P \cdot \left(\frac{2 \cdot L}{3} \right) \right] \qquad A_y \rightarrow -6 \cdot P$

$$\Sigma F_y = 0 \qquad D_y = -R_F + 3 \cdot P - A_y \qquad D_y \rightarrow \frac{18 \cdot P}{5}$$

Problem 2.5-1

The rails are prevented from expanding because of their great length and lack of expansion joints.

Therefore, each rail is in the same condition as a bar with fixed ends (see Example 2-9).

The compressive stress in the rails may be calculated as follows.

$\Delta T = 120°F - 60°F = 60°F$

$\sigma = E\alpha(\Delta T)$

$\quad = (30 \times 10^6 \text{ psi})(6.5 \times 10^{-6}/°F)(60°F)$

$\sigma = 11{,}700 \text{ psi} \quad \leftarrow$

Problem 2.5-2

$L_a = 60$ m $T_0 = 10°C$

$L_s = 60.005$ m $T_0 = 10°C$

$\alpha_a = 23 \times 10^{-6}/°C$ $\alpha_s = 12 \times 10^{-6}/°C$

FINAL CONDITIONS

Aluminum pipe is longer than the steel pipe by the amount $\Delta L = 15$ mm.

ΔT = increase in temperature

$\delta_a = \alpha_a(\Delta T)L_a$ $\delta_s = \alpha_s(\Delta T)L_s$
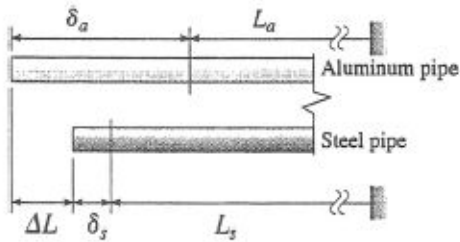
From the figure above:

$\delta_a + L_a = \Delta L + \delta_s + L_s$

or, $\alpha_a(\Delta T)L_a + L_a = \Delta L + \alpha_s(\Delta T)L_s + L_s$

Solve for ΔT:

$$\Delta T = \frac{\Delta L + (L_s - L_a)}{\alpha_a L_a - \alpha_s L_s} \quad \leftarrow$$

Substitute numerical values:

$\alpha_a L_a - \alpha_s L_s = 659.9 \times 10^{-6}$ m/°C

$$\Delta T = \frac{15 \text{ mm} + 5 \text{ mm}}{659.9 \times 10^{-6} \text{ m/°C}} = 30.31°C$$

$T = T_0 + \Delta T = 10°C + 30.31°C$

$\quad = 40.3°C \quad \leftarrow$

Problem 2.5-3
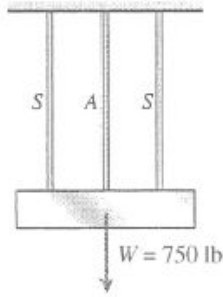
S = steel A = aluminum

$W = 750$ lb

$d = \dfrac{1}{8}$ in.

$A_s = \dfrac{\pi d^2}{4} = 0.012272$ in.^{2}

$E_s = 30 \times 10^6$ psi

$E_s A_s = 368{,}155$ lb

$\alpha_s = 6.5 \times 10^{-6}/°\text{F}$

$\alpha_a = 12 \times 10^{-6}/°\text{F}$

L = Initial length of wires
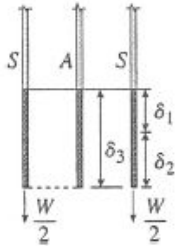
δ_1 = increase in length of a steel wire due to temperature increase ΔT

    $= \alpha_s (\Delta T) L$

δ_2 = increase in length of a steel wire due to load $W/2$

    $= \dfrac{WL}{2 E_s A_s}$

δ_3 = increase in length of aluminum wire due to temperature increase ΔT

    $= \alpha_a (\Delta T) L$

For no load in the aluminum wire:

$\delta_1 + \delta_2 = \delta_3$

$\alpha_s (\Delta T) L + \dfrac{WL}{2 E_s A_s} = \alpha_a (\Delta T) L$

or

$\Delta T = \dfrac{W}{2 E_s A_s (\alpha_a - \alpha_s)}$ ←

Substitute numerical values:

$\Delta T = \dfrac{750 \text{ lb}}{(2)(368{,}155 \text{ lb})(5.5 \times 10^{-6}/°\text{F})}$

    $= 185°\text{F}$ ←

NOTE: If the temperature increase is larger than ΔT, the aluminum wire would be in compression, which is not possible. Therefore, the steel wires continue to carry all of the load. If the temperature increase is less than ΔT, the aluminum wire will be in tension and carry part of the load.

Problem 2.5-4

NUMERICAL PROPERTIES

$$d_r = 15 \text{ mm} \qquad d_b = 12 \text{ mm} \qquad d_w = 20 \text{ mm} \qquad t_c = 10 \text{ mm} \qquad t_{wall} = 18 \text{ mm}$$

$$\tau_b = 45 \text{ MPa} \qquad \alpha = 12(10^{-6}) \qquad E = 200 \text{ GPa}$$

(a) TEMPERATURE DROP RESULTING IN BOLT SHEAR STRESS $\quad \varepsilon = \alpha \Delta T \qquad \sigma = E \alpha \Delta T$

Rod force $= P = (E\alpha \, \Delta T)\dfrac{\pi}{4}d_r^2 \qquad$ and bolt in double shear with shear stress $\quad \tau = \dfrac{\dfrac{P}{2}}{A_s} \qquad \tau = \dfrac{P}{2\dfrac{\pi}{4}d_b^2}$

$$\tau_b = \frac{2}{\pi d_b^2}\left[(E\alpha \, \Delta T)\frac{\pi}{4}d_r^2\right] \qquad \tau_b = \frac{E\alpha \, \Delta T}{2}\left(\frac{d_r}{d_b}\right)^2$$

$\tau_b = 45 \text{ MPa}$

$$\Delta T = \frac{2\tau_b}{E(1000)\alpha}\left(\frac{d_b}{d_r}\right)^2 \qquad \Delta T = 24°C \qquad P = (E\alpha \, \Delta T)\frac{\pi}{4}d_r^2 \qquad P = 10 \text{ kN}$$

$$\sigma_{rod} = \frac{P\,1000}{\dfrac{\pi}{4}d_r^2} \qquad \boxed{\sigma_{rod} = 57.6 \text{ MPa}}$$

(b) BEARING STRESSES

BOLT AND CLEVIS $\qquad \sigma_{bc} = \dfrac{\dfrac{P}{2}}{d_b t_c} \qquad \boxed{\sigma_{bc} = 42.4 \text{ MPa}}$

WASHER AT WALL $\qquad \sigma_{bw} = \dfrac{P}{\dfrac{\pi}{4}(d_w^2 - d_r^2)} \qquad \boxed{\sigma_{bw} = 74.1 \text{ MPa}}$

(c) If the connection to the wall at B is changed to an end plate with two bolts (see Fig. b), what is the required diameter d_b of each bolt if temperature drop $\Delta T = 38°C$ and the allowable bolt stress is 90 MPa?
   Find force in rod due to temperature drop.

$\Delta T = 38°C \qquad\qquad P = (E\alpha \, \Delta T)\dfrac{\pi}{4}d_r^2$

$$P = 200 \text{ GPa}\frac{\pi}{4}(15 \text{ mm})^2\left[12(10^{-6})\right](38) = 16116 \text{ N} \qquad P = 16.12 \text{ kN}$$

Each bolt carries one half of the force P:

$$d_b = \sqrt{\frac{\dfrac{16\,12 \text{ kN}}{2}}{\dfrac{\pi}{4}(90 \text{ MPa})}} = 10.68 \text{ mm} \qquad \boxed{d_b = 10.68 \text{ mm}}$$

Problem 2.5-5

(a) ONE DEGREE STATICALLY INDETERMINATE—USE SUPERPOSITION SELECT REACTION R_B AS THE REDUNDANT; FOLLOW PROCEDURE

Bar with nonuniform temperature change.
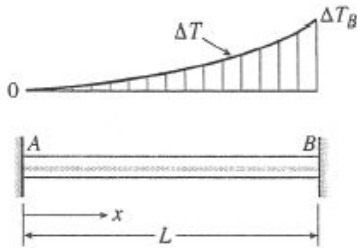
At distance x:

$$\Delta T = \Delta T_B \left(\frac{x^3}{L^3} \right)$$

REMOVE THE SUPPORT AT THE END B OF THE BAR:
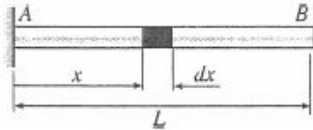
Consider an element dx at a distance x from end A.

$d\delta$ = Elongation of element dx

$$d\delta = \alpha(\Delta T)dx = \alpha(\Delta T_B)\left(\frac{x^3}{L^3} \right)dx$$

$d\delta$ = elongation of bar

$$\delta = \int_0^L d\delta = \int_0^L \alpha(\Delta T_B)\left(\frac{x^3}{L^3} \right)dx = \frac{1}{4}\alpha(\Delta T_B)L$$

COMPRESSIVE FORCE P REQUIRED TO SHORTEN THE BAR BY THE AMOUNT δ

$$P = \frac{EA\delta}{L} = \frac{1}{4}EA\alpha(\Delta T_B)$$

COMPRESSIVE STRESS IN THE BAR

$$\sigma_c = \frac{P}{A} = \frac{E\alpha(\Delta T_B)}{4} \quad \leftarrow$$

(b) ONE DEGREE STATICALLY INDETERMINATE—USE SUPERPOSITION.

Select reaction R_B as the redundant then compute bar elongations due to ΔT and due to R_B

$$\delta_{B1} = \alpha\Delta T_B \frac{L}{4} \quad \text{due to temperature from above}$$

$$\delta_{B2} = R_B\left(\frac{1}{k} + \frac{L}{EA} \right)$$

Compatibility: solve for R_B: $\quad \delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{-\left(\alpha\Delta T_B \dfrac{L}{4} \right)}{\left(\dfrac{1}{k} + \dfrac{L}{EA} \right)}$$

$$R_B = -\alpha\Delta T_B \left[\frac{EA}{4\left(\dfrac{EA}{kL} + 1 \right)} \right]$$

So compressive stress in bar is

$$\sigma_c = \frac{R_B}{A} \qquad \sigma_c = \frac{E\alpha(\Delta T_B)}{4\left(\dfrac{EA}{kL} + 1 \right)} \quad \leftarrow$$

NOTE: σ_c in part (b) is the same as in part (a) if spring constant k goes to infinity.

Problem 2.5-6

$$A = 2 \cdot \left(1090 \text{mm}^2\right) = 2180 \cdot \text{mm}^2 \qquad k = 1750 \frac{\text{kN}}{\text{m}} \qquad \Delta T = 45 \qquad \alpha = 12 \cdot \left(10^{-6}\right) \qquad L = 3\text{m} \qquad E = 205\text{GPa}$$

Assume that beam and spring are stress free at the start, then apply temperature increase ΔT. Select R_c as the redundant to remove to create the released structure

Apply ΔT to beam in released structure $\qquad \delta_{C1} = \alpha \cdot \Delta T \cdot L = 1.62 \cdot \text{mm}$

Apply redundant R_c $\qquad \delta_{C2} = R_C \cdot \left(\dfrac{L}{E \cdot A} + \dfrac{1}{k} \right) \qquad \dfrac{L}{E \cdot A} + \dfrac{1}{k} = 0.578 \cdot \dfrac{\text{mm}}{\text{kN}}$

Compatibility equation and solution for redundant $\qquad \delta_{C1} + \delta_{C2} = 0 \qquad R_C = \dfrac{-(\alpha \cdot \Delta T \cdot L)}{\left(\dfrac{L}{E \cdot A} + \dfrac{1}{k} \right)} = -2.802 \cdot \text{kN}$

Axial normal compressive stress in beam $\qquad \sigma_T = \dfrac{R_C}{A} = -1.285 \cdot \text{MPa}$

Displacement at B using superposition $\qquad \delta_B = \dfrac{R_C \cdot L}{E \cdot A} + \alpha \cdot \Delta T \cdot L = 1.601 \cdot \text{mm} \qquad \dfrac{R_C}{k} = -1.601 \cdot \text{mm}$

elongation of beam is equal to shortening of spring

Problem 2.5-7

$E = 29000\text{ksi}$ $\alpha = 6.5 \cdot 10^{-6}$ $\Delta T = 20$ $A = 8.24\text{in}^2$ $L = 10\text{ft}$

Select reaction R_B as the redundant; remove R_B to create released structure. Use superposition - apply ΔT to released structure, then apply redundant. Solve compatibility equation to find R_B then use statics to get R_A

$$\delta_{B1} = \alpha \cdot \Delta T \cdot L = 0.016 \cdot \text{in} \qquad \delta_{B2} = R_B \cdot \frac{L}{EA}$$

Compatibility $\delta_{B1} + \delta_{B2} = 0$ solve for R_B $R_B = \dfrac{-E \cdot A}{L} \cdot (\alpha \cdot \Delta T \cdot L) = -31.065 \cdot \text{kip}$ negative so R_B acts to left

Statics $R_A + R_B = 0$ so $R_A = -R_B = 31.065 \cdot \text{kip}$

Beam is in uniform axial compression due to temperature change; compressive normal stress is $\sigma_T = \dfrac{R_B}{A} = -3.77 \cdot \text{ksi}$

Problem 2.5-8

$d_1 = 50$ mm $d_2 = 75$ mm

$L_1 = 225$ mm $L_2 = 300$ mm

$E = 6.0$ GPa $\alpha = 100 \times 10^{-6}/°C$

$\Delta T = 30°C$ $k = 50$ MN/m

(a) COMPRESSIVE FORCE N, MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PT. C

$$A_1 = \frac{\pi}{4}d_1^2 \quad A_2 = \frac{\pi}{4}d_2^2$$

One-degree statically indeterminate—use R_B as redundant

$$\delta_{B1} = \alpha\Delta T(L_1 + L_2)$$

$$\delta_{B2} = R_B\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2}\right)$$

Compatibility: $\delta_{B1} = \delta_{B2}$, solve for R_B

$$R_B = \frac{\alpha\Delta T(L_1 + L_2)}{\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2}} \quad N = R_B$$

$N = 51.8$ kN $\leftarrow$

Maximum compressive stress in AC since it has the smaller area ($A_1 < A_2$):

$$\sigma_{cmax} = \frac{N}{A_1} \quad \sigma_{cmax} = 26.4 \text{ MPa}$$

Displacement δ_C of point C = superposition of displacements in two released structures at C:

$$\delta_C = \alpha\Delta T(L_1) - R_B\frac{L_1}{EA_1}$$

$\delta_C = -0.314$ mm $\leftarrow$ ($-$) sign means joint C moves left

(b) COMPRESSIVE FORCE N, MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PART C FOR ELASTIC SUPPORT CASE

Use R_B as redundant as in part (a):

$$\delta_{B1} = \alpha\Delta T(L_1 + L_2)$$

$$\delta_{B_2} = R_B\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k}\right)$$

Now add effect of elastic support; equate δ_{B1} and δ_{B2} then solve for R_B:

$$R_B = \frac{\alpha\Delta T(L_1 + L_2)}{\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2} + \dfrac{1}{k}} \quad N = R_B$$

$N = 31.2$ kN $\leftarrow$

$$\sigma_{cmax} = \frac{N}{A_1} \quad \sigma_{cmax} = 15.91 \text{ MPa} \quad \leftarrow$$

Superposition:

$$\delta_C = \alpha\Delta T(L_1) - R_B\left(\frac{L_1}{EA_1} + \frac{1}{k}\right)$$

$\delta_C = -0.546$ mm $\leftarrow$ ($-$) sign means joint C moves left

Problem 2.5-9

$$\Delta T = 20 \qquad \alpha = 13 \cdot \left(10^{-6}\right) \qquad E = 10400 \text{ksi} \qquad L = 3 \text{ft} \qquad b_1 = 2 \text{in} \qquad b_2 = 2.5 \text{in} \qquad t = \frac{1}{4} \text{in}$$

Select reaction R_C as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha \cdot \Delta T \cdot L = 9.36 \times 10^{-3} \cdot \text{in} \qquad \delta_{C2} = R_C \cdot \left[\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)} \right]$$

$$\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)} = 6.551 \times 10^{-3} \cdot \frac{\text{in}}{\text{kip}}$$

Write compatibility equation then solve for R_C

$$\delta_{C1} + \delta_{C2} = 0 \qquad R_C = \frac{-(\alpha \cdot \Delta T \cdot L)}{\left[\dfrac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \dfrac{\frac{L}{2}}{E \cdot (b_1 \cdot t)} \right]} = -1.429 \cdot \text{kip}$$

Statics $\qquad R_A + R_C = 0 \qquad R_A = -R_C = 1.429 \cdot \text{kip}$

Displacement at B using superposition $\qquad \delta_B = -R_A \cdot \left[\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right] + \alpha \cdot \Delta T \cdot \frac{L}{2} = 2.656 \times 10^{-4} \cdot \text{in}$

joint B moves to right

$$\text{OR} \qquad \frac{R_C \cdot \frac{L}{2}}{E \cdot (b_1 \cdot t)} + \alpha \cdot \Delta T \cdot \frac{L}{2} = -2.656 \times 10^{-4} \cdot \text{in}$$

shortening of BC

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Problem 2.5-10

$\Delta T = 30$ $\alpha = 19 \cdot \left(10^{-6}\right)$ $L = 2m$ $t = 20mm$ $b_1 = 100mm$ $b_2 = 115mm$ $E = 96GPa$

Select reaction R_C as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha \cdot \Delta T \cdot L = 1.14 \cdot mm \qquad \delta_{C2} = R_C \cdot \left[\frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right] \qquad \frac{L}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 9.706 \times 10^{-3} \cdot \frac{mm}{kN}$$

Write compatibility equation then solve for R_C

$$\delta_{C1} + \delta_{C2} = 0 \qquad R_C = \frac{-(\alpha \cdot \Delta T \cdot L)}{\left[\frac{L}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right]} = -117.457 \cdot kN$$

Statics $R_A + R_C = 0$ $R_A = -R_C = 117.457 \cdot kN$

Displacement at B using superposition $\delta_B = -R_A \cdot \left[\frac{\frac{3 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right] + \alpha \cdot \Delta T \cdot \frac{3 \cdot L}{5} = 0 \cdot mm$

no elongation of AB

OR $R_C \cdot \left[\frac{\frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right] + \alpha \cdot \Delta T \cdot \frac{2L}{5} = 0 \cdot mm$

no shortening of BC

Extra - find displ. at x = 2L/5 $b_{2L5} = b_2 - \frac{2}{3} \cdot (b_2 - b_1)$ $b_{2L5} \rightarrow 105 \cdot mm$

$$\delta_{2L5} = -R_A \cdot \left[\frac{\frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_{2L5})} \cdot \ln\left(\frac{b_2}{b_{2L5}}\right) \right] + \alpha \cdot \Delta T \cdot \frac{2 \cdot L}{5} = 0.011 \cdot mm$$

OR $R_C \cdot \left[\frac{\frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{5}}{E \cdot t \cdot (b_{2L5} - b_1)} \cdot \ln\left(\frac{b_{2L5}}{b_1}\right) \right] + \alpha \cdot \Delta T \cdot \frac{3L}{5} = -0.011 \cdot mm$

Problem 2.5-11
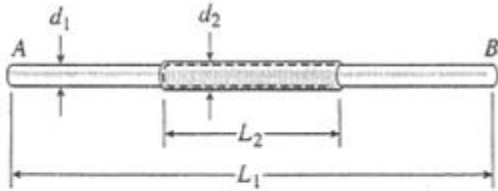
$L_1 = 36$ in. $L_2 = 12$ in.

ELONGATION OF THE TWO OUTER PARTS OF THE BAR

$\delta_1 = \alpha_s(\Delta T)(L_1 - L_2)$

$= (6.5 \times 10^{-6}/°F)(500°F)(36 \text{ in.} - 12 \text{ in.})$

$= 0.07800$ in.

ELONGATION OF THE MIDDLE PART OF THE BAR
The steel rod and bronze sleeve lengthen the same amount, so they are in the same condition as the bolt and sleeve of Example 2-10. Thus, we can calculate the elongation from Eq. (2-21):

$$\delta_2 = \frac{(\alpha_s E_s A_s + \alpha_b E_b A_b)(\Delta T)L_2}{E_s A_s + E_b A_b}$$

SUBSTITUTE NUMERICAL VALUES

$\alpha_s = 6.5 \times 10^{-6}/°F$ $\alpha_b = 11 \times 10^{-6}/°F$

$E_s = 30 \times 10^6$ psi $E_b = 15 \times 10^6$ psi

$d_1 = 1.0$ in.

$A_s = \frac{\pi}{4}d_1^2 = 0.78540 \text{ in.}^2$

$d_2 = 1.25$ in.

$A_b = \frac{\pi}{4}(d_2^2 - d_1^2) = 0.44179 \text{ in.}^2$

$\Delta T = 500°F$ $L_2 = 12.0$ in.

$\delta_2 = 0.04493$ in.

TOTAL ELONGATION

$\delta = \delta_1 + \delta_2 = 0.123$ in. $\leftarrow$

Problem 2.5-12

$\Delta T = 15$ $\alpha_T = 23 \cdot \left(10^{-6}\right)$ $L = 1.8\,m$ $r = 36mm$ $E = 72GPa$ $a = \dfrac{r}{8} = 4.5 \cdot mm$

$A_1 = \pi \cdot r^2 = 4071.504 \cdot mm^2$ Use formulas in **Appendix E, Case 15** for area of slotted segment

$\alpha = acos\left(\dfrac{a}{r}\right) = 1.445$ $b = \sqrt{r^2 - a^2} = 35.718 \cdot mm$ $A_2 = 2 \cdot r^2 \cdot \left(\alpha - \dfrac{a \cdot b}{r^2}\right) = 3425.196 \cdot mm^2$ $\dfrac{A_2}{A_1} = 0.841$

Select reaction R_C as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha_T \cdot \Delta T \cdot L = 0.621 \cdot mm \quad \delta_{C2} = R_C \cdot \left(\dfrac{2 \cdot \dfrac{L}{4}}{E \cdot A_1} + \dfrac{\dfrac{L}{2}}{E \cdot A_2}\right) \qquad \dfrac{2 \cdot \dfrac{L}{4}}{E \cdot A_1} + \dfrac{\dfrac{L}{2}}{E \cdot A_2} = 6.72 \times 10^{-3} \cdot \dfrac{mm}{kN}$$

Write compatibility equation then solve for R_C $\delta_{C1} + \delta_{C2} = 0$ $R_C = \dfrac{-\left(\alpha_T \cdot \Delta T \cdot L\right)}{\left(\dfrac{2 \cdot \dfrac{L}{4}}{E \cdot A_1} + \dfrac{\dfrac{L}{2}}{E \cdot A_2}\right)} = -92.417 \cdot kN$

Statics $R_A + R_C = 0$ $R_A = -R_C = 92.417 \cdot kN$

Thermal compressive stress in solid bar segments $\sigma_{T1} = \dfrac{R_C}{A_1} = -22.698 \cdot MPa$

and in slotted middle segment $\sigma_{T2} = \dfrac{R_C}{A_2} = -26.982 \cdot MPa$

Problem 2.5-13

0.5 in. × 2.0 in.
1.0 in. × 2.0 in.
0.5 in. × 2.0 in.

Pin   Copper   Aluminum
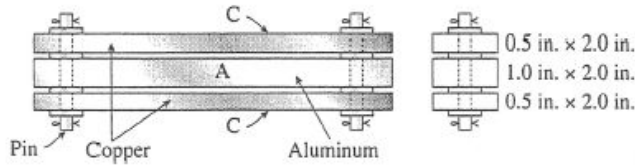
Diameter of pin: $d_P = \dfrac{7}{16}$ in. $= 0.4375$ in.

Area of pin: $A_P = \dfrac{\pi}{4} d_P^2 = 0.15033$ in.^{2}

Area of two copper bars: $A_c = 2.0$ in.^{2}

Area of aluminum bar: $A_a = 2.0$ in.^{2}

$\Delta T = 100°F$

Copper: $E_c = 18,000$ ksi $\alpha_c = 9.5 \times 10^{-6}/°F$

Aluminum: $E_a = 10,000$ ksi

$\alpha_a = 13 \times 10^{-6}/°F$

Use the results of Example 2-10.

Find the forces P_a and P_c in the aluminum bar and copper bar, respectively, from Eq. (2-19).

Replace the subscript "S" in that equation by "a" (for aluminum) and replace the subscript "B" by "c" (for copper), because α for aluminum is larger than α for copper.

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T)E_a A_a E_c A_c}{E_a A_a + E_c A_c}$$

Note that P_a is the compressive force in the aluminum bar and P_c is the combined tensile force in the two copper bars.

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T)E_c A_c}{1 + \dfrac{E_c A_c}{E_a A_a}}$$

SUBSTITUTE NUMERICAL VALUES:

$$P_a = P_c = \frac{(3.5 \times 10^{-6}/°F)(100°F)(18,000 \text{ ksi})(2 \text{ in.}^2)}{1 + \left(\dfrac{18}{10}\right)\left(\dfrac{2.0}{2.0}\right)}$$

$= 4,500$ lb

FREE-BODY DIAGRAM OF PIN AT THE LEFT END

$\dfrac{P_c}{2}$

P_a

$\dfrac{P_c}{2}$
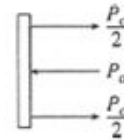
$V =$ shear force in pin

$= P_c/2$

$= 2,250$ lb

$\tau =$ average shear stress on cross section of pin

$$\tau = \frac{V}{A_P} = \frac{2,250 \text{ lb}}{0.15033 \text{ in.}^2}$$

$\tau = 15.0$ ksi   ←

Problem 2.5-14
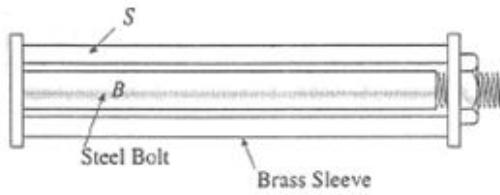
Subscript S means "sleeve".

Subscript B means "bolt".

Use the results of Example 2-10.

σ_S = compressive force in sleeve

EQUATION (2-20a):

$$\sigma_S = \frac{(\alpha_S - \alpha_B)(\Delta T)E_S E_B A_B}{E_S A_S + E_B A_B} \text{ (Compression)}$$

SOLVE FOR ΔT:

$$\Delta T = \frac{\sigma_S(E_S A_S + E_B A_B)}{(\alpha_S - \alpha_B)E_S E_B A_B}$$

or

$$\Delta T = \frac{\sigma_S}{E_S(\alpha_S - \alpha_B)}\left(1 + \frac{E_S A_S}{E_B A_B}\right) \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_S = 25$ MPa

$d_2 = 36$ mm $d_1 = 26$ mm $d_B = 25$ mm

$E_S = 100$ GPa $E_B = 200$ GPa

$\alpha_S = 21 \times 10^{-6}/°C$ $\alpha_B = 10 \times 10^{-6}/°C$

$$A_S = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(620 \text{ mm}^2)$$

$$A_B = \frac{\pi}{4}(d_B)^2 = \frac{\pi}{4}(625 \text{ mm}^2) \; 1 + \frac{E_S A_S}{E_B A_B} = 1.496$$

$$\Delta T = \frac{25 \text{ MPa} (1.496)}{(100 \text{ GPa})(11 \times 10^{-6}/°C)}$$

$\Delta T = 34°C \quad \leftarrow$

(Increase in temperature)

Problem 2.5-15
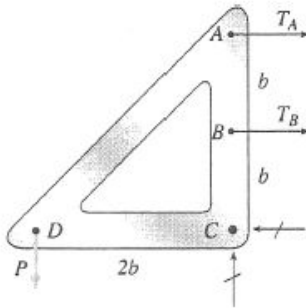
$$\Sigma M_C = 0 \quad \curvearrowright\curvearrowleft$$

$$P(2b) - T_A(2b) - T_B(b) = 0 \quad \text{or} \quad 2T_A + T_B = 2P \quad \text{(Eq. 1)}$$

DISPLACEMENT DIAGRAM
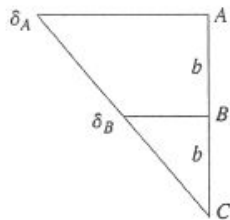
EQUATION OF COMPATIBILITY

$$\delta_A = 2\delta_B \tag{Eq. 2}$$

(a) LOAD P ONLY

Force-displacement relations:

$$\delta_A = \frac{T_A L}{EA} \quad \delta_B = \frac{T_B L}{EA} \tag{Eq. 3, 4}$$

(L = length of wires at A and B.)
Substitute (3) and (4) into Eq. (2):

$$\frac{T_A L}{EA} = \frac{2T_B L}{EA}$$

or $\quad T_A = 2T_B$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$$T_A = \frac{4P}{5} \quad T_B = \frac{2P}{5} \tag{Eqs. 6, 7}$$

Numerical values:
$P = 500$ lb
$\therefore T_A = 400$ lb $\quad T_B = 200$ lb $\quad \leftarrow$

(b) LOAD P AND TEMPERATURE INCREASE ΔT

Force-displacement and temperature-displacement relations:

$$\delta_A = \frac{T_A L}{EA} + \alpha(\Delta T)L \tag{Eq. 8}$$

$$\delta_B = \frac{T_B L}{EA} + \alpha(\Delta T)L \tag{Eq. 9}$$

Substitute (8) and (9) into Eq. (2):

$$\frac{T_A L}{EA} + \alpha(\Delta T)L = \frac{2T_B L}{EA} + 2\alpha(\Delta T)L$$

or $\quad T_A - 2T_B = EA\alpha(\Delta T)$ (Eq. 10)

Solve simultaneously Eqs. (1) and (10):

$$T_A = \frac{1}{5}[4P + EA\alpha(\Delta T)] \tag{Eq. 11}$$

$$T_B = \frac{2}{5}[P - EA\alpha(\Delta T)] \tag{Eq. 12}$$

Substitute numerical values:

$$P = 500 \text{ lb} \quad EA = 120{,}000 \text{ lb}$$

$$\Delta T = 180°F$$

$$\alpha = 12.5 \times 10^{-6}/°F$$

$$T_A = \frac{1}{5}(2000 \text{ lb} + 270 \text{ lb}) = 454 \text{ lb} \quad \leftarrow$$

$$T_B = \frac{2}{5}(500 \text{ lb} - 270 \text{ lb}) = 92 \text{ lb} \quad \leftarrow$$

(c) WIRE B BECOMES SLACK

Set $T_B = 0$ in Eq. (12):

$$P = EA\alpha(\Delta T)$$

or

$$\Delta T = \frac{P}{EA\alpha} = \frac{500 \text{ lb}}{(120{,}000 \text{ lb})(12.5 \times 10^{-6}/°F)}$$

$$= 333.3°F$$

Further increase in temperature:

$$\Delta T = 333.3°F - 180°F$$

$$= 153°F \quad \leftarrow$$

Problem 2.5-16

FREE-BODY DIAGRAM OF BAR $ABCD$
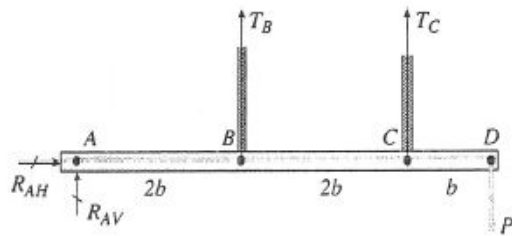
T_B = force in cable B T_C = force in cable C
d_B = 12 mm d_C = 20 mm

From Table 2-1:

$A_B = 76.7 \text{ mm}^2$ $E = 140 \text{ GPa}$
$\Delta T = 60°\text{C}$ $A_C = 173 \text{ mm}^2$
    $\alpha = 12 \times 10^{-6}/°\text{C}$

EQUATION OF EQUILIBRIUM

$\Sigma M_A = 0$ ⟳⟲ $T_B(2b) + T_C(4b) - P(5b) = 0$
or $2T_B + 4T_C = 5P$ (Eq. 1)

DISPLACEMENT DIAGRAM
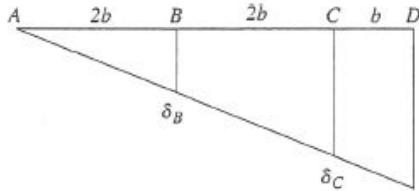
COMPATIBILITY:

$\delta_C = 2\delta_B$ (Eq. 2)

FORCE-DISPLACEMENT AND TEMPERATURE-DISPLACEMENT
RELATIONS

$\delta_B = \dfrac{T_B L}{E A_B} + \alpha(\Delta T)L$ (Eq. 3)

$\delta_C = \dfrac{T_C L}{E A_C} + \alpha(\Delta T)L$ (Eq. 4)

SUBSTITUTE EQS. (3) AND (4) INTO EQ. (2):

$\dfrac{T_C L}{E A_C} + \alpha(\Delta T)L = \dfrac{2T_B L}{E A_B} + 2\alpha(\Delta T)L$

or

$2T_B A_C - T_C A_B = -E\alpha(\Delta T)A_B A_C$ (Eq. 5)

SUBSTITUTE NUMERICAL VALUES INTO EQ. (5):

$T_B(346) - T_C(76.7) = -1{,}338{,}000$ (Eq. 6)
in which T_B and T_C have units of newtons.

SOLVE SIMULTANEOUSLY EQS. (1) AND (6):

$T_B = 0.2494\,P - 3{,}480$ (Eq. 7)
$T_C = 1.1253\,P + 1{,}740$ (Eq. 8)
in which P has units of newtons.

SOLVE EQS. (7) AND (8) FOR THE LOAD P:

$P_B = 4.0096\,T_B + 13{,}953$ (Eq. 9)
$P_C = 0.8887\,T_C - 1{,}546$ (Eq. 10)

ALLOWABLE LOADS

From Table 2-1:

$(T_B)_{\text{ULT}} = 102{,}000 \text{ N}$ $(T_C)_{\text{ULT}} = 231{,}000 \text{ N}$

Factor of safety = 5

$(T_B)_{\text{allow}} = 20{,}400 \text{ N}$ $(T_C)_{\text{allow}} = 46{,}200 \text{ N}$

From Eq. (9): $P_B = (4.0096)(20{,}400 \text{ N}) + 13{,}953 \text{ N}$
                    $= 95{,}700 \text{ N}$

From Eq. (10): $P_C = (0.8887)(46{,}200 \text{ N}) - 1546 \text{ N}$
                    $= 39{,}500 \text{ N}$

Cable C governs.

$P_{\text{allow}} = 39.5 \text{ kN}$ ←

Problem 2.5-17

Numerical data:

$L = 25$ in. $d = 2$ in. $\delta = 0.008$ in.

$k = 1.2 \times (10^6)$ lb/in. $E = 16 \times (10^6)$ psi

$\alpha = 9.6 \times (10^{-6})/°F$ $\Delta T = 50°F$

$A = \dfrac{\pi}{4}d^2$ $A = 3.14159$ in.^{2}

(a) ONE-DEGREE STATICALLY INDETERMINATE IF GAP CLOSES

$\Delta = \alpha \Delta T L$ $\Delta = 0.012$ in. <exceeds gap

Select R_A as redundant and do superposition analysis:

$\delta_{A1} = \Delta$ $\delta_{A2} = R_A\left(\dfrac{L}{EA} + \dfrac{1}{k}\right)$

Compatibility: $\delta_{A1} + \delta_{A2} = \delta$ $\delta_{A2} = \delta - \delta_{A1}$

$R_A = \dfrac{\delta - \Delta}{\dfrac{L}{EA} + \dfrac{1}{k}}$ $R_A = -3006$ lb

Compressive stress in bar:

$\sigma = \dfrac{R_A}{A}$ $\sigma = -957$ psi

(b) FORCE IN SPRING $F_k = R_C$

STATICS

$R_A + R_C = 0$

$R_C = -R_A$

$R_C = 3006$ lb $\leftarrow$

(c) FIND COMPRESSIVE STRESS IN BAR IF k GOES TO INFINITY FROM EXPRESSION FOR R_A ABOVE, $1/k$ GOES TO ZERO

$R_A = \dfrac{\delta - \Delta}{\dfrac{L}{EA}}$ $R_A = -8042$ lb $\sigma = \dfrac{R_A}{A}$

$\sigma = -2560$ psi $\leftarrow$

Problem 2.5-18

Initial prestress: $\sigma_1 = 42$ MPa

Initial temperature: $T_1 = 20°C$

$E = 200$ GPa

$\alpha = 14 \times 10^{-6}/°C$

(a) STRESS σ WHEN TEMPERATURE DROPS TO $0°C$

   $T_2 = 0°C \quad \Delta T = 20°C$

NOTE: *Positive ΔT* means a *decrease* in temperature and an *increase* in the stress in the wire.

Negative ΔT means an *increase* in temperature and a *decrease* in the stress.

Stress σ equals the initial stress σ_1 plus the additional stress σ_2 due to the temperature drop.

$\sigma_2 = E\alpha(\Delta T)$

$\sigma = \sigma_1 + \sigma_2 = \sigma_1 + E\alpha(\Delta T)$

$\quad = 42$ MPa $+ (200$ GPa$)(14 \times 10^{-6}/°C)(20°C)$

$\quad = 42$ MPa $+ 56$ MPa $= 98$ MPa $\quad \leftarrow$

(b) TEMPERATURE WHEN STRESS EQUALS ZERO

$\sigma = \sigma_1 + \sigma_2 = 0 \quad \sigma_1 + E\alpha(\Delta T) = 0$

$\Delta T = -\dfrac{\sigma_1}{E\alpha}$

(Negative means increase in temp.)

$\Delta T = -\dfrac{42 \text{ MPa}}{(200 \text{ GPa})(14 \times 10^{-6}/°C)} = -15°C$

$T = 20°C + 15°C = 35°C \quad \leftarrow$

Problem 2.5-19

$n = 1.5$ $p = \dfrac{1}{16} \text{in}$ $A_s = 0.85\text{in}^2$ $A_A = 4.5\text{in}^2$

$L = 20\text{in}$ $E_s = 29000\text{ksi}$ $E_A = 10600\text{ksi}$

Select force in tube as the redundant. Cut through aluminum tube at right end to expose internal force F_A to create released structure. Apply n turns of turnbuckles to released structure to find relative displacement between ends of cut tube

$\delta_1 = 2 \cdot n \cdot p = 0.187 \cdot \text{in}$ Note that n turns of a turnbuckle moves ends together by factor of two

Now apply pair of internal forces F_T to ends of tube then again find relative displacement. Force F_A shortens both cables and elongates the tube.

$$\delta_2 = F_A \cdot \left(\frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s} \right) \qquad \frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s} = 8.25 \times 10^{-4} \cdot \frac{\text{in}}{\text{kip}}$$

Compatibility equation $\delta_1 + \delta_2 = 0$ solve for F_A $F_A = \dfrac{-2 \cdot n \cdot p}{\dfrac{L}{E_A \cdot A_A} + \dfrac{L}{2 \cdot E_s \cdot A_s}} = -227.282 \cdot \text{kip}$

Statics - force in each cable = F_s $2 \cdot F_s + F_A = 0$ $F_s = \dfrac{-F_A}{2} = 113.641 \cdot \text{kip}$

Shortening of aluminum tube $\delta_A = \dfrac{F_A \cdot L}{E_A \cdot A_A} = -0.0953 \cdot \text{in}$

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Problem 2.5-20
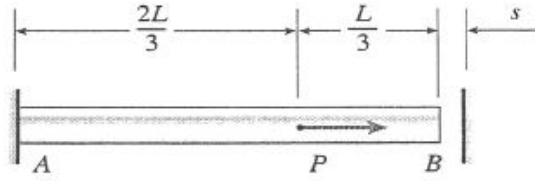
L = length of bar

s = size of gap

EA = axial rigidity

Reactions must be equal; find s.

COMPATIBILITY EQUATION

$\delta_1 - \delta_2 = s$ or

$$\frac{2PL}{3EA} - \frac{R_B L}{EA} = s \qquad \text{(Eq. 1)}$$

EQUILIBRIUM EQUATION

R_A = reaction at end A (to the left)

R_B = reaction at end B (to the left)

$P = R_A + R_B$

FORCE-DISPLACEMENT RELATIONS
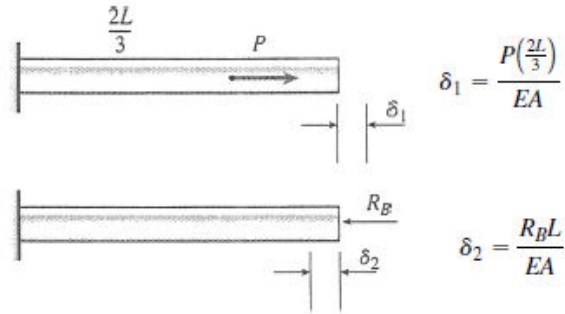
$$\delta_1 = \frac{P\left(\frac{2L}{3}\right)}{EA}$$

$$\delta_2 = \frac{R_B L}{EA}$$

Reactions must be equal.

$$\therefore R_A = R_B \quad P = 2R_B \quad R_B = \frac{P}{2}$$

Substitute for R_B in Eq. (1):

$$\frac{2PL}{3EA} - \frac{PL}{2EA} = s \quad \text{or} \quad s = \frac{PL}{6EA} \; \leftarrow$$

NOTE: The gap closes when the load reaches the value $P/4$. When the load reaches the value P, equal to $6EAs/L$, the reactions are equal ($R_A = R_B = P/2$). When the load is between $P/4$ and P, R_A is greater than R_B. If the load exceeds P, R_B is greater than R_A.

Problem 2.5-21

(a) FIND REACTIONS AT A AND B FOR APPLIED FORCE P_1
First compute P_1, required to close gap:

$$P_1 = \frac{E_1 A_1}{L_1} s \quad P_1 = 231.4 \text{ k} \quad \leftarrow$$

Statically indeterminate analysis with R_B as the redundant:

$$\delta_{B1} = -s \quad \delta_{B2} = R_B \left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right)$$

Compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{s}{\left(\dfrac{L_1}{E_1 A_1} + \dfrac{L_2}{E_2 A_2} \right)} \quad R_B = 55.2 \text{ k} \quad \leftarrow$$

$$R_A = -R_B \quad \leftarrow$$

(b) FIND REACTIONS AT A AND B FOR APPLIED FORCE P_2

$$P_2 = \frac{E_2 A_2}{\dfrac{L_2}{2}} s \quad P_2 = 145.1 \text{ k} \quad \leftarrow$$

Analysis after removing P_2 is same as in part (a), so reaction forces are the same

---

(c) MAXIMUM SHEAR STRESS IN PIPE 1 OR 2 WHEN EITHER P_1 OR P_2

IS APPLIED $\tau_{maxa} = \dfrac{\dfrac{P_1}{A_1}}{2} \quad \tau_{maxa} = 13.39 \text{ ksi} \quad \leftarrow$

$$\tau_{maxb} = \dfrac{\dfrac{P_2}{A_2}}{2} \quad \tau_{maxb} = 19.44 \text{ ksi} \quad \leftarrow$$

(d) REQUIRED ΔT AND REACTIONS AT A AND B

$$\Delta T_{reqd} = \frac{s}{\alpha_1 L_1 + \alpha_2 L_2} \quad \Delta T_{reqd} = 65.8°F \quad \leftarrow$$

If pin is inserted but temperature remains at ΔT above ambient temperature, reactions are zero.

(e) IF TEMPERATURE RETURNS TO ORIGINAL AMBIENT TEMPERATURE, FIND REACTIONS AT A AND B
statically indeterminate analysis with R_B as the redundant Compatibility: $\delta_{B1} + \delta_{B2} = 0$
Analysis is the same as in parts (a) and (b) above since gap s is the same, so reactions are the same.

Problem 2.5-22

With gap s closed due to ΔT, structure is one-degree statically-indeterminate; select internal force (Q) at juncture of bar and spring as the redundant. Use superposition of two released structures in the solution.

δ_{rel1} = relative displacement between end of bar at C and end of spring due to ΔT

$\delta_{rel1} = \alpha\Delta T(L_1 + L_2)$
  δ_{rel1} is greater than gap length s

δ_{rel2} = relative displacement between ends of bar and spring due to pair of forces Q, one on end of bar at C and the other on end of spring

$\delta_{rel2} = Q\left(\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2}\right) + \dfrac{Q}{k_3}$

$\delta_{rel2} = Q\left(\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2} + \dfrac{1}{k_3}\right)$

Compatibility: $\delta_{rel1} + \delta_{rel2} = s$ $\delta_{rel2} = s - \delta_{rel1}$

$\delta_{rel2} = s - \alpha\Delta T(L_1 + L_2)$

$Q = \dfrac{s - \alpha\Delta T(L_1 + L_2)}{\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2} + \dfrac{1}{k_3}}$

$Q = \dfrac{EA_1A_2k_3}{L_1A_2k_3 + L_2A_1k_3 + EA_1A_2}$

$[s - \alpha\Delta T(L_1 + L_2)]$

(a) REACTIONS AT A AND D

Statics: $R_A = -Q$ $R_D = Q$

$R_A = \dfrac{-s + \alpha\Delta T(L_1 + L_2)}{\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2} + \dfrac{1}{k_3}}$ $\leftarrow$

$R_D = -R_A$ $\leftarrow$

(b) DISPLACEMENTS AT B AND C
  Use superposition of displacements in the two released structures:

$\delta_B = \alpha\Delta T(L_1) - R_A\left(\dfrac{L_1}{EA_1}\right)$ $\leftarrow$

$\delta_B = \alpha\Delta T(L_1) -$
  $\dfrac{[-s + \alpha\Delta T(L_1 + L_2)]}{\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2} + \dfrac{1}{k_3}}\left(\dfrac{L_1}{EA_1}\right)$

$\delta_C = \alpha\Delta T(L_1 + L_2) -$
  $R_A\left(\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2}\right)$ $\leftarrow$

$\delta_C = \alpha\Delta T(L_1 + L_2) -$
  $\dfrac{[-s + \alpha\Delta T(L_1 + L_2)]}{\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2} + \dfrac{1}{k_3}}\left(\dfrac{L_1}{EA_1} + \dfrac{L_2}{EA_2}\right)$

Problem 2.5-23
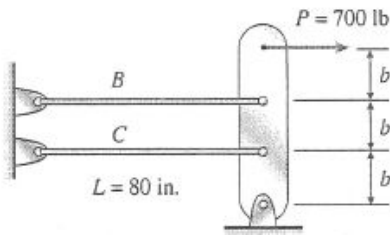
P = 700 lb

$P = 700$ lb

$A = 0.03$ in.^{2}

$E = 30 \times 10^6$ psi

$L_B = 79.98$ in.

$L_C = 79.95$ in.

EQUILIBRIUM EQUATION

$P = 700$ lb
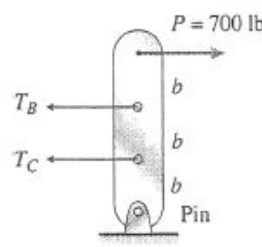
$\Sigma M_{pin} = 0$ ←

$T_C(b) + T_B(2b) = P(3b)$

$2T_B + T_C = 3P$ (Eq. 1)

DISPLACEMENT DIAGRAM

$S_B = 80$ in. $- L_B = 0.02$ in.

$S_C = 80$ in. $- L_C = 0.05$ in.
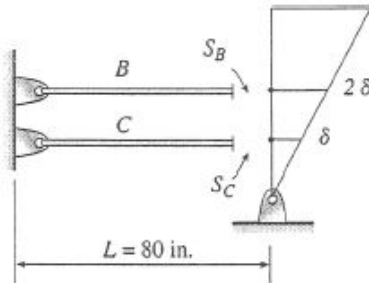
Elongation of wires:

$\delta_B = S_B + 2\delta$ (Eq. 2)

$\delta_C = S_C + \delta$ (Eq. 3)

FORCE-DISPLACEMENT RELATIONS

$\delta_B = \dfrac{T_B L}{EA}$ $\delta_C = \dfrac{T_C L}{EA}$ (Eqs. 4, 5)

SOLUTION OF EQUATIONS

Combine Eqs. (2) and (4):

$\dfrac{T_B L}{EA} = S_B + 2\delta$ (Eq. 6)

Combine Eqs. (3) and (5):

$\dfrac{T_C L}{EA} = S_C + \delta$ (Eq. 7)

Eliminate δ between Eqs. (6) and (7):

$T_B - 2T_C = \dfrac{EAS_B}{L} - \dfrac{2EAS_C}{L}$ (Eq. 8)

Solve simultaneously Eqs. (1) and (8):

$T_B = \dfrac{6P}{5} + \dfrac{EAS_B}{5L} - \dfrac{2EAS_C}{5L}$ ←

$T_C = \dfrac{3P}{5} - \dfrac{2EAS_B}{5L} + \dfrac{4EAS_C}{5L}$ ←

SUBSTITUTE NUMERICAL VALUES:

$\dfrac{EA}{5L} = 2250$ lb/in.

$T_B = 840$ lb $+ 45$ lb $- 225$ lb $= 660$ lb ←

$T_C = 420$ lb $- 90$ lb $+ 450$ lb $= 780$ lb ←

(Both forces are positive, which means tension, as required for wires.)

Problem 2.5-24
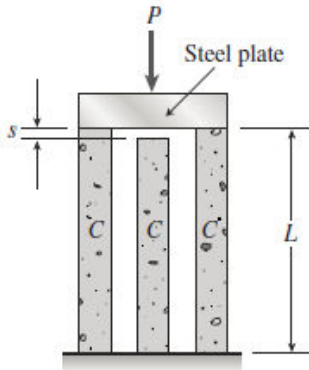
s = size of gap = 1.0 mm

L = length of posts = 2.0 m

A = 40,000 mm²

σ_{allow} = 20 MPa

E = 30 GPa

C = concrete post

DOES THE GAP CLOSE?

Stress in the two outer posts when the gap is just closed:

$$\sigma = E\varepsilon = E\left(\frac{s}{L}\right) = (30 \text{ GPa})\left(\frac{1.0 \text{ mm}}{2.0 \text{ m}}\right)$$

$$= 15 \text{ MPa}$$

Since this stress is less than the allowable stress, the allowable force P will close the gap.

EQUILIBRIUM EQUATION

$$2P_1 + P_2 = P \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION

δ_1 = shortening of outer posts

δ_2 = shortening of inner post

$$\delta_1 = \delta_2 + s \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{P_1 L}{EA} \quad \delta_2 = \frac{P_2 L}{EA} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_1 L}{EA} = \frac{P_2 L}{EA} + s \quad \text{or} \quad P_1 - P_2 = \frac{EAs}{L} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P = 3P_1 - \frac{EAs}{L}$$

By inspection, we know that P_1 is larger than P_2. Therefore, P_1 will control and will be equal to $\sigma_{\text{allow}} A$.

$$P_{\text{allow}} = 3\sigma_{\text{allow}} A - \frac{EAs}{L}$$

$$= 2400 \text{ kN} - 600 \text{ kN} = 1800 \text{ kN}$$

$$= 1.8 \text{ MN} \quad \leftarrow$$

Problem 2.5-25

The figure shows a section through the pipe, cap and rod

NUMERICAL PROPERTIES

$L_{ci} = 48$ in. $E_s = 30000$ ksi $E_b = 14{,}000$ ksi

$E_c = 12{,}000$ ksi $t_c = 1$ in. $p = 52 \times (10^{-3})$ in. $n = \dfrac{1}{4}$

$d_w = \dfrac{3}{4}$ in. $d_r = \dfrac{1}{2}$ in. $d_o = 6$ in. $d_i = 5.625$ in.

(a) FORCES AND STRESSES IN PIPE AND ROD

One degree statically indeterminate—cut rod at cap and use force in rod (Q) as the redundant:

δ_{rel1} = relative displacement between cut ends of rod due to 1/4 turn of nut

$\delta_{rel1} = -np$ Ends of rod move apart, not together, so this is ($-$).

δ_{rel2} = relative displacement between cut ends of rod due pair of forces Q

$$\delta_{rel2} = Q\left(\frac{L + 2t_c}{E_b A_{rod}} + \frac{L_{ci}}{E_c A_{pipe}}\right)$$

$$A_{rod} = \frac{\pi}{4}d_r^2 \qquad A_{pipe} = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$A_{rod} = 0.196$ in.^{2} $A_{pipe} = 3.424$ in.^{2}

Compatibility equation: $\delta_{rel1} + \delta_{rel2} = 0$

$$Q = \frac{np}{\dfrac{L_{ci} + 2t_c}{E_b A_{rod}} + \dfrac{L_{ci}}{E_c A_{pipe}}}$$

$Q = 0.672$ k $F_{rod} = Q$

Statics: $F_{pipe} = -Q$

Stresses: $\sigma_c = \dfrac{F_{pipe}}{A_{pipe}}$ $\sigma_c = -0.196$ ksi $\leftarrow$

$\sigma_b = \dfrac{F_{rod}}{A_{rod}}$ $\sigma_b = 3.42$ ksi $\leftarrow$

(b) BEARING AND SHEAR STRESSES IN STEEL CAP

$\sigma_b = \dfrac{F_{rod}}{\dfrac{\pi}{4}(d_w^2 - d_r^2)}$ $\sigma_b = 2.74$ ksi $\leftarrow$

$\tau_c = \dfrac{F_{rod}}{\pi d_w t_c}$ $\tau_c = 0.285$ ksi $\leftarrow$

Problem 2.5-26
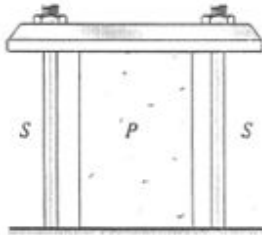
$L = 200$ mm
$P = 1.0$ mm
$E_s = 200$ GPa
$A_s = 36.0$ mm² (for one bolt)
$E_p = 7.5$ GPa
$A_p = 960$ mm²
$n = 1$ (See Eq. 2-22)

EQUILIBRIUM EQUATION

P_s = tensile force in one steel bolt
P_p = compressive force in plastic cylinder
$P_p = 2P_s$ (Eq. 1)

COMPATIBILITY EQUATION

δ_s = elongation of steel bolt
δ_p = shortening of plastic cylinder
$\delta_s + \delta_p = np$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_p = \frac{P_p L}{E_p A_p}$$ (Eq. 3, Eq. 4)

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np$$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2np E_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2np\, E_s\, A_s\, E_p}{L(E_p A_p + 2E_s A_s)}$$
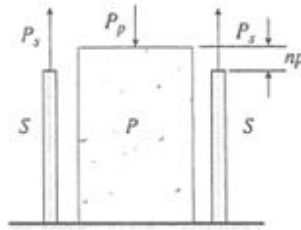
SUBSTITUTE NUMERICAL VALUES:

$N = E_s A_s E_p = 54.0 \times 10^{15}$ N²/m²

$D = E_p A_p + 2E_s A_s = 21.6 \times 10^6$ N

$$\sigma_p = \frac{2np}{L}\left(\frac{N}{D}\right) = \frac{2(1)(1.0 \text{ mm})}{200 \text{ mm}}\left(\frac{N}{D}\right)$$

$= 25.0$ MPa $\leftarrow$

Problem 2.5-27
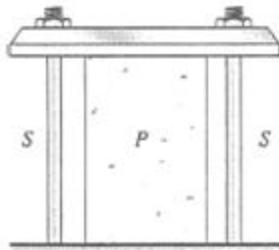
$L = 10$ in.

$p = 0.058$ in.

$E_s = 30 \times 10^6$ psi

$A_s = 0.06$ in.^{2} (for one bolt)

$E_p = 500$ ksi

$A_p = 1.5$ in.^{2}

$n = 1$ (see Eq. 2-22)

EQUILIBRIUM EQUATION

P_s = tensile force in one steel bolt

P_p = compressive force in plastic cylinder

$P_p = 2P_s$ (Eq. 1)

COMPATIBILITY EQUATION

δ_s = elongation of steel bolt

δ_p = shortening of plastic cylinder

$\delta_s + \delta_p = np$ (Eq. 2)
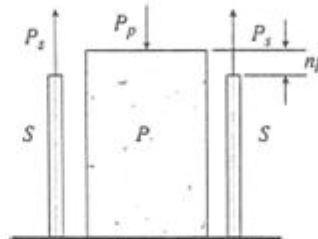
FORCE-DISPLACEMENT RELATIONS

$\delta_s = \dfrac{P_s L}{E_s A_s}$ $\delta_p = \dfrac{P_p L}{E_p A_p}$ (Eq. 3, Eq. 4)

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$\dfrac{P_s L}{E_s A_s} + \dfrac{P_p L}{E_p A_p} = np$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2\, np\, E_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2\, np\, E_s A_s E_p}{L(E_p A_p + 2E_s A_s)} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$N = E_s A_s E_p = 900 \times 10^9$ lb^2/in.^{2}

$D = E_p A_p + 2E_s A_s = 4350 \times 10^3$ lb

$$\sigma_p = \frac{2np}{L}\left(\frac{N}{D}\right) = \frac{2(1)(0.058\text{in.})}{10 \text{ in.}}\left(\frac{N}{D}\right)$$

$= 2400$ psi $\leftarrow$

Problem 2.5-28

The figure shows a section through the sleeve, cap, and bolt.

$$A_b = \frac{\pi}{4} d_b^2 \qquad A_1 = \frac{\pi}{4}[d_1^2 - (d_1 - 2t_1)^2]$$

$$A_b = 19.635 \text{ mm}^2 \qquad A_1 = 263.894 \text{ mm}^2$$

NUMERICAL PROPERTIES

$$n = \frac{1}{2} \qquad p = 1.0 \text{ mm} \qquad \Delta T = 30°C$$

$$A_2 = \frac{\pi}{4}[d_2^2 - (d_2 - 2t_2)^2] \qquad A_2 = 131.947 \text{ mm}^2$$

$$E_c = 120 \text{ GPa} \qquad \alpha_c = 17 \times (10^{-6})/°C$$

$$E_s = 200 \text{ GPa} \qquad \alpha_s = 12 \times (10^{-6})/°C$$

$$\tau_{aj} = 18.5 \text{ MPa} \qquad s = 26 \text{ mm} \qquad d_b = 5 \text{ mm}$$

$$L_1 = 40 \text{ mm} \quad t_1 = 4 \text{ mm} \quad L_2 = 50 \text{ mm} \quad t_2 = 3 \text{ mm}$$

$$d_1 = 25 \text{ mm} \qquad d_1 - 2t_1 = 17 \text{ mm} \qquad d_2 = 17 \text{ mm}$$

(a) FORCES IN SLEEVE AND BOLT
One-degree statically indeterminate—cut bolt and use force in bolt (P_B) as redundant (see sketches):

$$\delta_{B1} = -np + \alpha_s \Delta T(L_1 + L_2 - s)$$

$$\delta_{B2} = P_B\left[\frac{L_1 + L_2 - s}{E_s A_b} + \frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c(A_1 + A_2)}\right]$$

Compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$P_B = \frac{-[-np + \alpha_s \Delta T(L_1 + L_2 - s)]}{\left[\dfrac{L_1 + L_2 - s}{E_s A_b} + \dfrac{L_1 - s}{E_c A_1} + \dfrac{L_2 - s}{E_c A_2} + \dfrac{s}{E_c(A_1 + A_2)}\right]} \qquad P_B = 25.4 \text{ kN} \quad \leftarrow \qquad P_s = -P_B \quad \leftarrow$$

Sketches illustrating superposition procedure for statically-indeterminate analysis
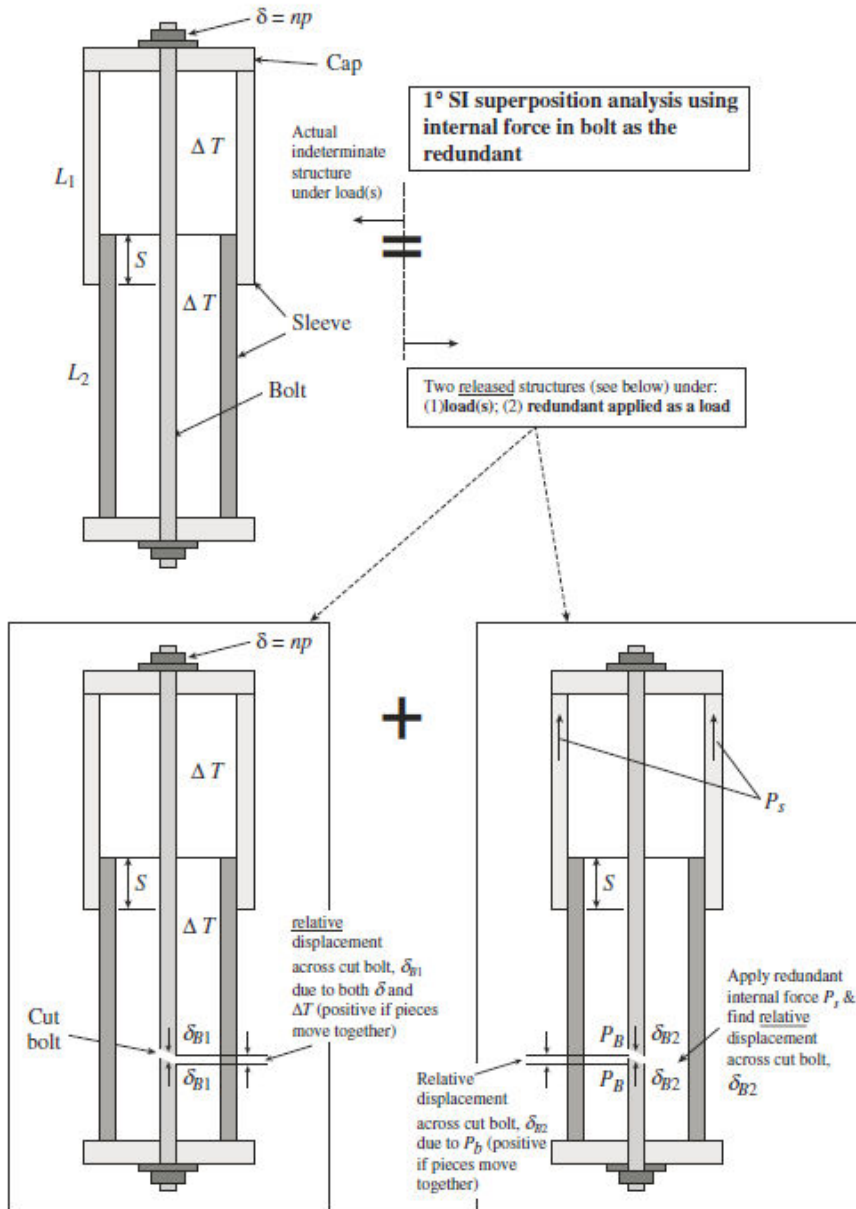
(b) REQUIRED LENGTH OF SOLDER JOINT≈

$$\tau = \frac{P}{A_s} \qquad A_s = \pi d_2 s$$

$$s_{reqd} = \frac{P_B}{\pi d_2 \tau_{aj}} \qquad s_{reqd} = 25.7 \text{ mm}$$

$$\delta_s = P_s \left[\frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c(A_1 + A_2)} \right]$$

$$\delta_s = -0.064 \text{ mm}$$

$$\delta_f = \delta_b + \delta_s \qquad \delta_f = 0.35 \text{ mm} \quad \leftarrow$$

(c) FINAL ELONGATION

δ_f = net of elongation of bolt (δ_b) and shortening of sleeve (δ_s)

$$\delta_b = P_B \left(\frac{L_1 + L_2 - s}{E_s A_b} \right) \qquad \delta_b = 0.413 \text{ mm}$$

Problem 2.5-29

The figure shows a section through the tube, cap, and spring.

Properties and dimensions:

$d_o = 6$ in. $t = \dfrac{1}{8}$ in. $E_t = 100$ ksi

$A_t = \dfrac{\pi}{4}[d_o^2 - (d_o - 2t)^2]$ $A_t = 2.307$ in.^{2}

$L_1 = 12.125$ in. $> L = 12$ in. $k = 1.5$ k/in.

Spring is 1/8 in. longer than tube

$\delta = L_1 - L$ $\delta = 0.125$ in.

$\alpha_k = 6.5(10^{-6})/°F$ $<$ $\alpha_t = 80 \times (10^{-6})/°F$

$\Delta T = 0$ $<$ note that Q result below is for zero temperature (until part(d))

(a) FORCE IN SPRING F_k = REDUNDANT Q

Flexibilities: $f = \dfrac{1}{k}$ $f_t = \dfrac{L}{E_t A_t}$

δ_2 = relative displacement across cut spring due to redundant = $Q(f + f_t)$

δ_1 = relative displacement across cut spring due to precompression and $\Delta T = \delta + \alpha_k \Delta T L_1 - \alpha_t \Delta T L$

Compatibility: $\delta_1 + \delta_2 = 0$

Solve for redundant Q:

$$Q = \dfrac{-\delta + \Delta T(-\alpha_k L_1 + \alpha_t L)}{f + f_t} = F_k$$

$F_k = -0.174$ k $\leftarrow$ compressive force in spring (F_k) and also tensile force in tube

(b) F_t = force in tube = $-Q$ $\leftarrow$

NOTE: If tube is rigid, $F_k = -k\delta = -0.1875$ k

(c) FINAL LENGTH OF TUBE

$L_f = L + \delta_{c1} + \delta_{c2}$ $<$ i.e., add displacements for the two released structures to initial tube length L

$L_f = L - Qf_t + \alpha_t(\Delta T)L$ $L_f = 12.01$ in. $\leftarrow$

(d) SET $Q = 0$ TO FIND ΔT REQUIRED TO REDUCE SPRING FORCE TO ZERO

$$\Delta T_{reqd} = \dfrac{\delta}{(-\alpha_k L_1 + \alpha_t L)}$$

$\Delta T_{reqd} = 141.9°F$

Since $\alpha_t > \alpha_k$, a temp. increase is req'd to expand tube so that spring force goes to zero.

Problem 2.5-30

Steel wires
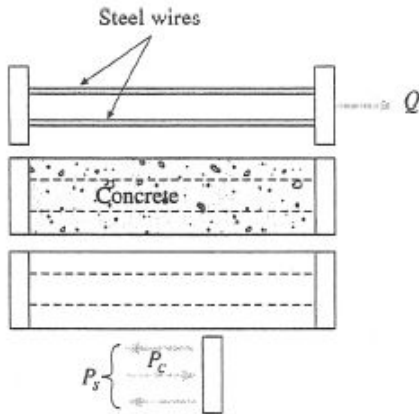
Concrete

P_s { P_c |

EQUILIBRIUM EQUATION

$P_s = P_c$ (Eq. 1)

COMPATIBILITY EQUATION AND FORCE-DISPLACEMENT RELATIONS

δ_1 = initial elongation of steel wires

$$= \frac{QL}{E_s A_s} = \frac{\sigma_0 L}{E_s}$$

δ_2 = final elongation of steel wires

$$= \frac{P_s L}{E_s A_s}$$

δ_3 = shortening of concrete

$$= \frac{P_c L}{E_c A_c}$$

$\delta_1 - \delta_2 = \delta_3$ or

$$\frac{\sigma_0 L}{E_s} - \frac{P_s L}{E_s A_s} = \frac{P_c L}{E_c A_c}$$ (Eq. 2, Eq. 3)

Solve simultaneously Eqs. (1) and (3):

$$P_s = P_c = \frac{\sigma_0 A_s}{1 + \dfrac{E_s A_s}{E_c A_c}}$$

L = length

σ_0 = initial stress in wires

$$= \frac{Q}{A_s} = 620 \text{ MPa}$$

A_s = total area of steel wires

A_c = area of concrete

$$= 50 A_s$$

$E_s = 12 E_c$

P_s = final tensile force in steel wires

P_c = final compressive force in concrete

STRESSES

$$\sigma_s = \frac{P_s}{A_s} = \frac{\sigma_0}{1 + \dfrac{E_s A_s}{E_c A_c}} \quad \leftarrow$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{\sigma_0}{\dfrac{A_c}{A_s} + \dfrac{E_s}{E_c}} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_0 = 620 \text{ MPa} \qquad \frac{E_s}{E_c} = 12 \qquad \frac{A_s}{A_c} = \frac{1}{50}$$

$$\sigma_s = \frac{620 \text{ MPa}}{1 + \dfrac{12}{50}} = 500 \text{ MPa (Tension)} \quad \leftarrow$$

$$\sigma_c = \frac{620 \text{ MPa}}{50 + 12} = 10 \text{ MPa (Compression)} \quad \leftarrow$$

Problem 2.5-31

The figure shows a section through the tube, cap, and spring.

Properties and dimensions:

$d_o = 6$ in. $t = \dfrac{1}{8}$ in. $E_t = 100$ ksi

$L = 12$ in. $> L_1 = 11.875$ in. $k = 1.5$ k/in.

$\alpha_k = 6.5(10^{-6}) < \alpha_t = 80 \times (10^{-6})$

$A_t = \dfrac{\pi}{4}[d_o^2 - (d_o - 2t)^2]$

$A_t = 2.307$ in.^{2}

Pretension and temperature:
Spring is 1/8 in. shorter than tube.

$\delta = L - L_1$ $\delta = 0.125$ in. $\Delta T = 0$

Note that Q result below is for zero temperature (until part (d)).

Flexibilities: $f = \dfrac{1}{k}$ $f_t = \dfrac{L}{E_t A_t}$

(a) FORCE IN SPRING (F_k) = REDUNDANT (Q)

Follow solution procedure outlined in Prob. 2.5-29 solution:

$$Q = \frac{\delta + \Delta T(-\alpha_k L_1 + \alpha_t L)}{f + f_t} = F_k$$

$F_k = 0.174$ k   ← also the compressive force in the tube

(b) FORCE IN TUBE $F_t = -Q = -0.174$ k   ←
(c) FINAL LENGTH OF TUBE AND SPRING $L_f = L + \delta_{c1} + \delta_{c2}$

$L_f = L - Qf_t + \alpha_t(\Delta T)L$ $L_f = 11.99$ in.   ←

(d) SET $Q = 0$ TO FIND ΔT REQUIRED TO REDUCE SPRING FORCE TO ZERO

$$\Delta T_{reqd} = \frac{-\delta}{(-\alpha_k L_1 + \alpha_t L)}$$

$\Delta T_{reqd} = -141.6°F$

Since $\alpha_t > \alpha_k$, a temperature drop is required to shrink tube so that spring force goes to zero.

Problem 2.6-1

2.0 in.

1.5 in.
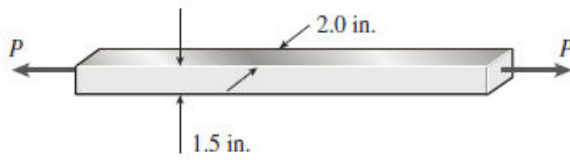
NUMERICAL DATA

$A = 3 \text{ in.}^2$ $\sigma_a = 14500 \text{ psi}$

$\tau_a = 7100 \text{ psi}$

MAXIMUM LOAD—TENSION

$P_{\text{max1}} = \sigma_a A$ $P_{\text{max1}} = 43500 \text{ lbs}$

MAXIMUM LOAD—SHEAR

$P_{\text{max2}} = 2\tau_a A$ $P_{\text{max2}} = 42{,}600 \text{ lbs}$

Because τ_{allow} is less than one-half of σ_{allow}, the shear stress governs.

Problem 2.6-2

P $P = 3.5$ kN

$$P_{max} = 2\tau_a\left(\frac{\pi}{4}d_{min}^2\right)$$

$$d_{min} = \sqrt{\frac{2}{\pi\tau_a}P}$$

NUMERICAL DATA $P = 3.5$ kN $\sigma_a = 118$ MPa
                           $\tau_a = 48$ MPa $d_{min} = 6.81$ mm   ←

Find P_{max} then rod diameter.
since τ_a is less than 1/2 of σ_a, shear governs.

Problem 2.6-3

8 in. 4 in. 2.5 in.
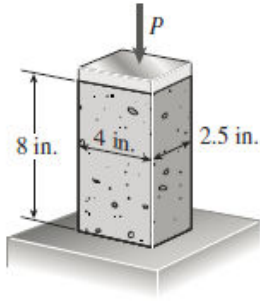
$A = 2.5$ in. $\times 4.0$ in. $= 10.0$ in.^{2}

Maximum normal stress:

$$\sigma_x = \frac{P}{A}$$

Maximum shear stress:

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$$

$\sigma_{ult} = 3600$ psi $\qquad \tau_{ult} = 1200$ psi

Because τ_{ult} is less than one-half of σ_{ult}, the shear stress governs.

$$\tau_{max} = \frac{P}{2A} \quad \text{or} \quad P_{max} = 2A\tau_{ult}$$

$$P_{max} = 2(10.0 \text{ in.}^2)(1200 \text{ psi}) = 24,000 \text{ lb} \quad \leftarrow$$

Problem 2.6-4
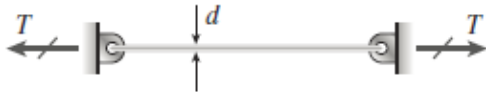
NUMERICAL DATA

$d = 2.42$ mm $T = 98$ N
$\alpha = 19.5 \ (10^{-6})/°C$ $E = 110$ GPa

(a) ΔT_{max} (DROP IN TEMPERATURE)

$$\sigma = \frac{T}{A} - (E\alpha \, \Delta T) \qquad \tau_{max} = \frac{\sigma}{2}$$

$$\tau_a = \frac{T}{2A} - \frac{E\alpha \, \Delta T}{2}$$

$$\tau_a = 60 \text{ MPa} \qquad A = \frac{\pi}{4} d^2$$

$$\Delta T_{max} = \frac{\dfrac{T}{A} - 2\tau_a}{E\alpha}$$

$$\Delta T_{max} = -46°C \text{ (drop)}$$

(b) ΔT AT WHICH WIRE GOES SLACK

Increase ΔT until $\sigma = 0$:

$$\Delta T = \frac{T}{E\alpha A}$$

$$\Delta T = 9.93°C \text{ (increase)}$$

Problem 2.6-5
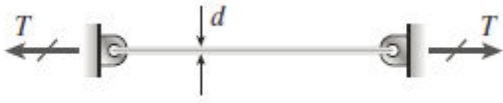
NUMERICAL DATA

$d = \dfrac{1}{16}$ in. $T = 37$ lb $\alpha = 10.6 \times (10^{-6})/°F$

$E = 15 \times (10^6)$ psi $\Delta T = -60°F$

$A = \dfrac{\pi}{4}d^2$

(a) τ_{max} (DUE TO DROP IN TEMPERATURE)

$$\tau_{max} = \dfrac{\sigma_x}{2} \qquad \tau_{max} = \dfrac{\dfrac{T}{A} - (E\alpha\Delta T)}{2}$$

$\tau_{max} = 10{,}800$ psi $\leftarrow$

(b) ΔT_{max} FOR ALLOWABLE SHEAR STRESS

$\tau_a = 10000$ psi

$$\Delta T_{max} = \dfrac{\dfrac{T}{A} - 2\tau_a}{E\alpha}$$

$\Delta T_{max} = -49.9°F$ $\leftarrow$

(c) ΔT AT WHICH WIRE GOES SLACK

Increase ΔT until $\sigma = 0$:

$$\Delta T = \dfrac{T}{E\alpha A}$$

$\Delta T = 75.9°F$ (increase) $\leftarrow$

Problem 2.6-6

(a) $d = 12$ mm $P = 9.5$ kN $A = \frac{\pi}{4}d^2 = 1.131 \times 10^{-4}$ m²

$$\sigma_x = \frac{P}{A} = 84 \text{ MPa}$$

(b) $\tau_{max} = \frac{\sigma_x}{2} = 42$ MPa   On plane stress element rotated 45°

(c) ROTATED STRESS ELEMENT (45°) HAS NORMAL TENSILE STRESS $\sigma_x/2$ ON ALL FACES, $-T_{max}$ (CW) ON $+x$-FACE, AND $+T_{max}$ (CCW) ON $+y$-FACE

$\tau_{xy1y1} = \tau_{max}$ $\sigma_{x1} = \frac{\sigma_x}{2}$ $\sigma_{y1} = \sigma_{x1}$

On rotated x-face: $\sigma_{x1} = 42$ MPa $\tau_{x1y1} = 42$ MPa

On rotated y-face: $\sigma_{y1} = 42$ MPa

(d) $\theta = 22.5°$ < CCW ROTATION OF ELEMENT

$\sigma_\theta = \sigma_x \cos(\theta)^2 = 71.7$ MPa   < on rotated x face $\sigma_y = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 = 12.3$ MPa   < on rotated y face

Eq. 2-29b $\tau_\theta = \frac{-\sigma_x}{2} \sin(2\theta) = -29.7$ MPa   < CW on rotated x-face

On rotated x-face: $\sigma_{x1} = 71.7$ MPa $\tau_{x1y1} = -29.7$ MPa

On rotated y-face: $\sigma_{y1} = 12.3$ MPa

Problem 2.6-7
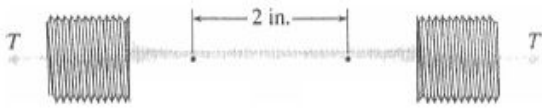
Elongation: $\delta = 0.00120$ in.
(2 in. gage length)

Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{0.00120 \text{ in.}}{2 \text{ in.}} = 0.00060$

Hooke's law: $\sigma_x = E\varepsilon = (30 \times 10^6 \text{ psi})(0.00060)$
$= 18,000$ psi

(a) MAXIMUM NORMAL STRESS

σ_x is the maximum normal stress.

$\sigma_{max} = 18,000$ psi ⟵

(b) MAXIMUM SHEAR STRESS
The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

$\tau_{max} = \dfrac{\sigma_x}{2} = 9,000$ psi ⟵

(c) STRESS ELEMENT AT $\theta = 45°$
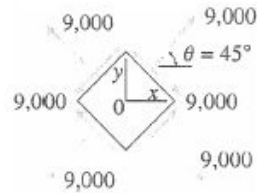
NOTE: All stresses have units of psi.

Problem 2.6-8

(a) $\alpha = 17.5\left(10^{-6}\right)$ $\Delta T = 50$ $E = 120\ \text{GPa}$

$\sigma_x = -E\alpha\,\Delta T = -105\ \text{MPa}$ $\tau_{max} = \dfrac{\sigma_x}{2} = -52.5\ \text{MPa}$ < at $\theta = 45°$

(compression)

Element A: $\sigma_x = 105\ \text{MPa}$ (compression);
Element B: $\tau_{max} = 52.5\ \text{MPa}$

(b) $\tau_\theta = 48\ \text{MPa}$

Eq. 2-29b $\tau_\theta = \dfrac{-\sigma_x}{2}\sin(2\theta)$

so $\theta = \dfrac{1}{2}\text{asin}\left(\dfrac{2\tau_\theta}{-\sigma_x}\right) = 33.1°$ < CCW rotation of element $\boxed{\theta = 33.1°}$

$\sigma_\theta = \sigma_x\cos(\theta)^2 = -73.8\ \text{MPa}$ < on rotated x face

$\sigma_y = \sigma_x\cos\left(\theta + \dfrac{\pi}{2}\right)^2 = -31.2\ \text{MPa}$ < on rotated y face

Problem 2.6-9

$P = 10\text{kip}$ $L = 3\text{ft}$ $A = 8 \cdot \text{in}^2$ $\theta = 35\text{deg}$

Normal compressive stress $\sigma_x = \dfrac{-P}{A} = -1.25 \cdot \text{ksi}$

Plane stress transformations $\sigma_\theta(\theta) = \sigma_x \cdot \cos(\theta)^2$ $\sigma_\theta(\theta) = -0.839 \cdot \text{ksi}$ $\sigma_\theta\left(\theta + \dfrac{\pi}{2}\right) = -0.411 \cdot \text{ksi}$

$\tau_\theta(\theta) = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta)$ $\tau_\theta(\theta) = 0.587 \cdot \text{ksi}$ $\tau_\theta\left(\theta + \dfrac{\pi}{2}\right) = -0.587 \cdot \text{ksi}$

Rotated stress element
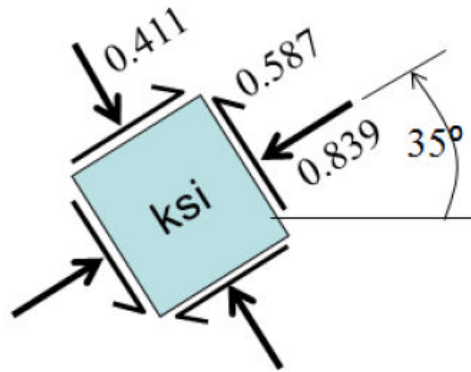
Problem 2.6-10

$$L = 1m \quad A = 1200mm^2 \quad \Delta T = 25 \quad \alpha = 12 \cdot \left(10^{-6}\right) \quad \theta = 45deg \quad E = 200GPa$$

Compressive thermal stress $\qquad \sigma_T = E \cdot \alpha \cdot \Delta T = 60 \cdot MPa$

Support reactions $\qquad R_A = \sigma_T \cdot A = 72 \cdot kN \qquad R_B = -R_A$

Plane stress transformations $\qquad \sigma_x = \dfrac{R_B}{A} = -60 \cdot MPa$

$$\sigma_\theta = \sigma_x \cdot \cos(\theta)^2 = -30 \cdot MPa \qquad \sigma_x \cdot \cos\left(\theta + \dfrac{\pi}{2}\right)^2 = -30 \cdot MPa \qquad \tau_\theta = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta) = 30 \cdot MPa$$

Rotated stress element
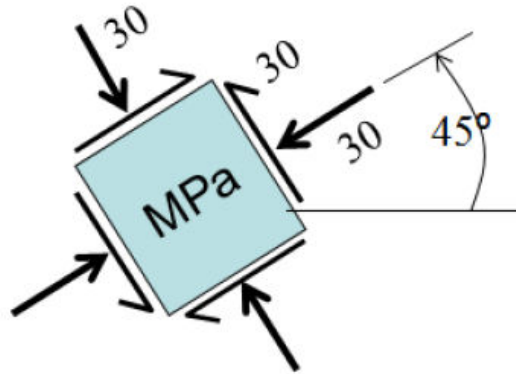
Problem 2.6-11

NUMERICAL DATA

$L = 10$ ft $b = 0.71\,L$ $P = 49$ k $\sigma_a = 14$ ksi $\tau_a = 7.5$ ksi $A = 5.87$ in.^{2}

(a) FOR LINEAR ANALYSIS, MEMBER FORCES ARE PROPORTIONAL TO LOADING

$$F_{AC} = \frac{P}{35}\,15.59 = 21.826\text{ k} \qquad F_{AB} = \frac{P}{35}\,62.2 = 87.08\text{ k}$$

(solution for $P = 35$ k)

$$F_{BC} = \frac{P}{35}(-78.9) \qquad F_{BC} = -110.46\text{ k}$$

Normal stresses in each member: $\sigma_{AC} = \dfrac{F_{AC}}{A} = 3.718$ ksi $\sigma_{AB} = \dfrac{F_{AB}}{A} = 14.835$ ksi

From Eq. 2-31: $\sigma_{BC} = \dfrac{F_{BC}}{A} = -18.818$ ksi

$$\boxed{\tau_{maxAC} = \frac{\sigma_{AC}}{2} = 1.859\text{ ksi}} \qquad \boxed{\tau_{maxAB} = \frac{\sigma_{AB}}{2} = 7.42\text{ ksi}}$$

$$\boxed{\tau_{maxBC} = \frac{\sigma_{BC}}{2} = -9.41\text{ ksi}}$$

(b) $\sigma_a < 2 \times T_a$ so normal stress will control; lowest value governs here

MEMBER AC: $P_{max\sigma} = \dfrac{P}{F_{AC}}(\sigma_a A) = 184.496$ k $P_{max\tau} = \dfrac{P}{F_{AC}}(2\tau_a A) = 197.675$ k

MEMBER AB: $P_{max\sigma} = \dfrac{P}{F_{AB}}(\sigma_a A) = 46.243$ k $P_{max\tau} = \dfrac{P}{F_{AB}}(2\tau_a A) = 49.546$ k

MEMBER BC: $\boxed{P_{max\sigma} = \left|\dfrac{P}{F_{BC}}\right|(\sigma_a A) = 36.5\text{ k}}$ $P_{max\tau} = \left|\dfrac{P}{F_{BC}}\right|(2\tau_a A) = 39.059$ k

Problem 2.6-12

NUMERICAL DATA

$d = 32$ mm $\qquad A = \dfrac{\pi}{4} d^2$

$P = 190$ N $\qquad A = 804.25$ mm^2

$a = 100$ mm

$b = 300$ mm

(a) STATICS—FIND COMPRESSIVE FORCE F AND STRESSES IN PLASTIC BAR

$F = \dfrac{P(a + b)}{a}$ $\qquad F = 760$ N

$\sigma_x = \dfrac{F}{A}$ $\qquad \sigma_x = 0.945$ MPa $\quad$ or $\quad \sigma_x = 945$ kPa

From (1), (2), and (3) below:

$\sigma_{max} = \sigma_x$ $\qquad \sigma_{max} = -945$ kPa

$\tau_{max} = 472$ kPa $\qquad \dfrac{\sigma_x}{2} = -472$ kPa

(1) $\theta = 0°$ $\qquad \sigma_x = -945$ kPa $\quad \leftarrow$

(2) $\theta = 22.50°$

On $+x$-face:

$\sigma_\theta = \sigma_x \cos(\theta)^2$
$\sigma_\theta = -807$ kPa $\quad \leftarrow$

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta)$
$\tau_\theta = 334$ kPa $\quad \leftarrow$

On $+y$-face: $\qquad \theta = \theta + \dfrac{\pi}{2}$

$\sigma_\theta = \sigma_x \cos(\theta)^2$
$\sigma_\theta = -138.39$ kPa

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta)$
$\tau_\theta = -334.1$ kPa

(3) $\theta = 45°$

On $+x$-face:

$\sigma_\theta = \sigma_x \cos(\theta)^2$
$\sigma_\theta = -472$ kPa $\quad \leftarrow$

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta)$
$\tau_\theta = 472$ kPa $\quad \leftarrow$

On $+y$-face: $\qquad \theta = \theta + \dfrac{\pi}{2}$

$\sigma_\theta = \sigma_x \cos(\theta)^2 \qquad \sigma_\theta = -472.49$ kPa

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) \qquad \tau_\theta = -472.49$ kPa

(b) ADD SPRING—FIND MAXIMUM NORMAL AND SHEAR STRESSES IN PLASTIC BAR
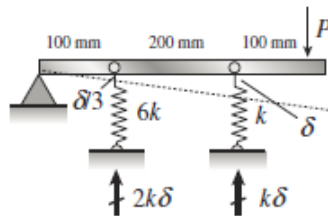
$\sum M_{pin} = 0$

$P(400) = [2k\delta(100) + k\delta(300)]$

$\delta = \dfrac{4}{5}\dfrac{P}{k}$

Force in plastic bar: $\qquad F = (2k)\left(\dfrac{4}{5}\dfrac{P}{k}\right)$

$F = \dfrac{8}{5} P \qquad F = 304$ N

Normal and shear stresses in plastic bar:

$\sigma_x = \dfrac{F}{A} \qquad \sigma_x = 0.38$

$\sigma_{max} = -378$ kPa $\quad \leftarrow$

$\tau_{max} = \dfrac{\sigma_x}{2} \qquad \tau_{max} = -189$ kPa $\quad \leftarrow$

Problem 2.6-13

$b = 1.5$ in. $h = 3$ in. $A = bh$ $\Delta T = (160 - 68)°F$

$\Delta T = 92°F$

$A = 4.5$ in.^{2} $\sigma_{pq} = -1700$ psi

$\alpha = 60 \times (10^{-6})/°F$

$E = 450 \times (10^3)$ psi

(a) SHEAR STRESS ON PLANE PQ
Statically indeterminate analysis gives,
for reaction at right support:

$R = -EA\alpha\Delta T$ $R = -11178$ lb

$\sigma_x = \dfrac{R}{A}$ $\sigma_x = -2484$ psi

Using $\sigma_\theta = \sigma_x \cos(\theta)^2$: $\cos(\theta)^2 = \dfrac{\sigma_{pq}}{\sigma_x}$

$\theta = a\cos\left(\sqrt{\dfrac{\sigma_{pq}}{\sigma_x}}\right)$ $\theta = 34.2°$

Now with θ, can find shear stress on plane pq:

$\tau_{pq} = -\sigma_x \sin(\theta)\cos(\theta)$ $\tau_{pq} = 1154$ psi   ←

$\sigma_{pq} = \sigma_x \cos(\theta)^2$ $\sigma_{pq} = -1700$ psi

Stresses at $\theta + \pi/2$ (y-face):

$\sigma_y = \sigma_x \cos\left(\theta + \dfrac{\pi}{2}\right)^2$ $\sigma_y = -784$ psi

---

(b) STRESS ELEMENT FOR PLANE PQ

Set $\tau_{max} = \tau_a$ and solve for P_{max1}:

$\tau_a = \dfrac{-E\alpha\Delta T}{2} + \dfrac{3P}{8A}$

$P_{max1} = \dfrac{4A}{3}(2\tau_a + E\alpha\Delta T)$

$P_{max1} = 34{,}704$ lb

$\tau_{max} = \dfrac{-E\alpha\Delta T}{2} + \dfrac{3P_{max1}}{8A}$

$\tau_{max} = 1650$ psi   < check

$\sigma_x = -E\alpha\Delta T + \dfrac{3P_{max1}}{4A}$

$\sigma_x = 3300$ psi   < less than σ_a
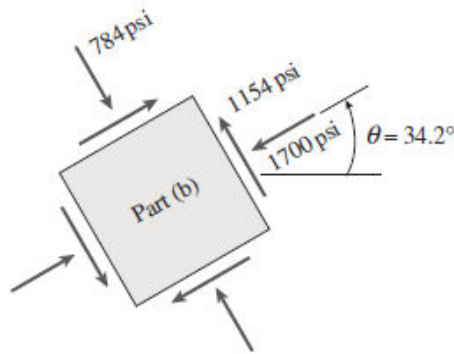
(c) MAXIMUM LOAD AT QUARTER POINT $\sigma_a = 3400$ psi

$\tau_a = 1650$ psi $2\tau_a = 3300$ < less than σ_a,
                                              so shear controls

Statically indeterminate analysis for P at $L/4$ gives
for reactions:

$R_{R2} = \dfrac{-P}{4}$ $R_{L2} = \dfrac{-3}{4}P$

(tension for 0 to $L/4$ and compression for rest of bar)

From part (a) (for temperature increase ΔT):

$R_{R1} = -EA\alpha\Delta T$ $R_{L1} = -EA\alpha\Delta T$

Stresses in bar (0 to $L/4$):

$\sigma_x = -E\alpha\Delta T + \dfrac{3P}{4A}$ $\tau_{max} = \dfrac{\sigma_x}{2}$

Stresses in bar ($L/4$ to L):

$\sigma_x = -E\alpha\Delta T - \dfrac{P}{4A}$ $\tau_{max} = \dfrac{\sigma_x}{2}$

Set $\tau_{max} = \tau_a$ and solve for P_{max2}:

$P_{max2} = -4A(-2\tau_a + E\alpha\Delta T)$

$P_{max2} = 14{,}688$ lb   ←   shear in segment ($L/4$
                                  to L) controls

$\tau_{max} = \dfrac{-E\alpha\Delta T}{2} - \dfrac{P_{max2}}{8A}$ $\tau_{max} = -1650$ psi

$\sigma_x = -E\alpha\Delta T - \dfrac{P_{max2}}{4A}$ $\sigma_x = -3300$ psi

Problem 2.6-14
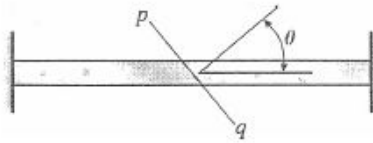
NUMERICAL DATA

$$\theta = 55\left(\frac{\pi}{180}\right) \text{ rad}$$

$b = 18 \text{ mm} \qquad h = 40 \text{ mm}$

$A = bh \qquad A = 720 \text{ mm}^2$

$\sigma_{pqa} = 60 \text{ MPa} \qquad \tau_{pqa} = 30 \text{ MPa}$

$\alpha = 17 \times (10^{-6})/°C \qquad E = 120 \text{ GPa}$

$\Delta T = 20°C \qquad P = 15 \text{ kN}$

(a) FIND ΔT_{max} BASED ON ALLOWABLE NORMAL AND SHEAR STRESS VALUES ON PLANE pq

$$\sigma_x = -E\alpha\Delta T_{max} \qquad \Delta T_{max} = \frac{-\sigma_x}{E\alpha}$$

$$\sigma_{pq} = \sigma_x\cos(\theta)^2 \qquad \tau_{pq} = -\sigma_x\sin(\theta)\cos(\theta)$$

Set each equal to corresponding allowable and solve for σ_x:

$$\sigma_{x1} = \frac{\sigma_{pqa}}{\cos(\theta)^2} \qquad \sigma_{x1} = 182.38 \text{ MPa}$$

$$\sigma_{x2} = \frac{\tau_{pqa}}{-\sin(\theta)\cos(\theta)} \qquad \sigma_{x2} = -63.85 \text{ MPa}$$

Lesser value controls, so allowable shear stress governs.

$$\Delta T_{max} = \frac{-\sigma_{x2}}{E\alpha} \qquad \Delta T_{max} = 31.3°C \quad \leftarrow$$

(b) STRESSES ON PLANE PQ FOR MAXIMUM TEMPERATURE

$$\sigma_x = -E\alpha\Delta T_{max} \qquad \sigma_x = -63.85 \text{ MPa}$$

$$\sigma_{pq} = \sigma_x\cos(\theta)^2 \qquad \sigma_{pq} = -21.0 \text{ MPa} \quad \leftarrow$$

$$\tau_{pq} = -\sigma_x\sin(\theta)\cos(\theta) \qquad \tau_{pq} = 30 \text{ MPa} \quad \leftarrow$$

(c) ADD LOAD P IN $+x$-DIRECTION TO TEMPERATURE CHANGE AND FIND LOCATION OF LOAD

$$\Delta T = 28°C$$

$P = 15$ kN from one-degree statically indeterminate analysis, reactions R_A and R_B due to load P:

$$R_A = -(1-\beta)P \qquad R_B = \beta P$$

Now add normal stresses due to P to thermal stresses due to ΔT (tension in segment 0 to βL, compression in segment βL to L).

Stresses in bar (0 to βL):

$$\sigma_x = -E\alpha\Delta T + \frac{R_A}{A} \qquad \tau_{max} = \frac{\sigma_x}{2}$$

Shear controls so set $\tau_{max} = \tau_a$ and solve for β:

$$2\tau_a = -E\alpha\Delta T + \frac{(1-\beta)P}{A}$$

$$\beta = 1 - \frac{A}{P}[2\tau_a + E\alpha\Delta T]$$

$$\beta = -5.1$$
Impossible so evaluate segment (βL to L):

Stresses in bar (βL to L):

$$\sigma_x = -E\alpha\Delta T - \frac{R_B}{A} \qquad \tau_{max} = \frac{\sigma_x}{2}$$

set $\tau_{max} = \tau_a$ and solve for P_{max2}

$$2\tau_a = -E\alpha\Delta T - \frac{\beta P}{A}$$

$$\beta = \frac{-A}{P}[-2\tau_a + E\alpha\Delta T]$$

$$\beta = 0.62 \quad \leftarrow$$

Problem 2.6-15

$P = 5 \text{ k} \qquad \alpha = 36° \qquad \sigma_a = 13.5 \text{ ksi}$

$\tau_a = 6.5 \text{ ksi}$

$\theta = \dfrac{\pi}{2} - \alpha \quad \theta = 54°$

$\sigma_{ja} = 6.0 \text{ ksi}$

$\tau_{ja} = 3.0 \text{ ksi}$

Tensile force N_{AC} using Method of Joints at C:

$N_{AC} = \dfrac{P}{\sin(60°)} \qquad \text{(tension)}$

$N_{AC} = 5.77 \text{ k} \qquad \leftarrow$

Minimum required diameter of bar AC:

(1) Check tension and shear in bars; $\tau_a < \sigma_a/2$ so shear controls $\tau_{\max} = \dfrac{\sigma_x}{2}$:

$2\tau_a = \dfrac{N_{AC}}{A} \qquad \sigma_x = 2\tau_a = 13 \text{ ksi}$

$A_{\text{reqd}} = \dfrac{N_{AC}}{2\tau_a} \qquad A_{\text{reqd}} = 0.44 \text{ in.}^2$

$d_{\min} = \sqrt{\dfrac{4}{\pi} A_{\text{reqd}}} \qquad d_{\min} = 0.75 \text{ in.}$

(2) Check tension and shear on brazed joint:

$\sigma_x = \dfrac{N_{AC}}{A} \qquad \sigma_x = \dfrac{N_{AC}}{\dfrac{\pi}{4} d^2} \qquad d_{\text{reqd}} = \sqrt{\dfrac{4}{\pi} \dfrac{N_{AC}}{\sigma_X}}$

Tension on brazed joint:

$\sigma_\theta = \sigma_x \cos(\theta)^2$

Set equal to σ_{ja} and solve for σ_x, then d_{reqd}:

$\sigma_x = \dfrac{\sigma_{ja}}{\cos(\theta)^2} \qquad \sigma_x = 17.37 \text{ ksi}$

$d_{\text{reqd}} = \sqrt{\dfrac{4}{\pi} \dfrac{N_{AC}}{\sigma_x}} \qquad d_{\text{reqd}} = 0.65 \text{ in.}$

Shear on brazed joint:

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta)$

$\sigma_x = \left| \dfrac{\tau_{ja}}{-(\sin(\theta)\cos(\theta))} \right| \qquad \sigma_x = -6.31 \text{ ksi}$

$d_{\text{reqd}} = \sqrt{\dfrac{4}{\pi} \dfrac{N_{AC}}{\sigma_X}} \qquad d_{\text{reqd}} = 1.08 \text{ in.} \qquad \leftarrow$

Problem 2.6-16
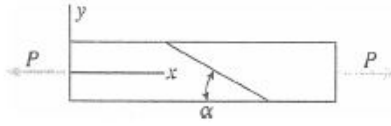
$10° \le \alpha \le 40°$

Due to load P: $\sigma_x = 4.9$ MPa

(a) STRESSES ON JOINT WHEN $\alpha = 20°$
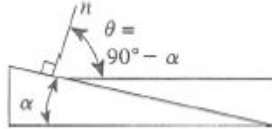
$\theta = 90° - \alpha = 70°$

$\sigma_\theta = \sigma_x \cos^2\theta = (4.9 \text{ MPa})(\cos 70°)^2$

$\quad = 0.57 \text{ MPa} \quad \leftarrow$

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$\quad = (-4.9 \text{ MPa})(\sin 70°)(\cos 70°)$

$\quad = -1.58 \text{ MPa} \quad \leftarrow$

(b) LARGEST ANGLE α IF $\tau_{\text{allow}} = 2.25$ MPa

$\tau_{\text{allow}} = -\sigma_x \sin\theta \cos\theta$

The shear stress on the joint has a negative sign. Its numerical value cannot exceed $\tau_{\text{allow}} = 2.25$ MPa. Therefore,

$-2.25 \text{ MPa} = -(4.9 \text{ MPa})(\sin\theta)(\cos\theta)$ or $\sin\theta \cos\theta = 0.4592$

From trigonometry: $\sin\theta\cos\theta = \dfrac{1}{2}\sin 2\theta$

Therefore: $\sin 2\theta = 2(0.4592) = 0.9184$

Solving: $2\theta = 66.69°$ or $113.31°$

$\theta = 33.34°$ or $56.66°$

$\alpha = 90° - \theta \quad \therefore \alpha = 56.66°$ or $33.34°$

Since α must be between $10°$ and $40°$, we select

$\alpha = 33.3° \quad \leftarrow$

NOTE: If α is between $10°$ and $33.3°$,

$|\tau_\theta| < 2.25$ MPa.

If α is between $33.3°$ and $40°$,

$|\tau_\theta| > 2.25$ MPa.

(c) WHAT IS α IF $\tau_\theta = 2\sigma_\theta$?

Numerical values only:

$|\tau_\theta| = \sigma_x \sin\theta\cos\theta \qquad |\sigma_\theta| = \sigma_x\cos^2\theta$

$\left|\dfrac{\tau_\theta}{\sigma_\theta}\right| = 2$

$\sigma_x \sin\theta\cos\theta = 2\sigma_x\cos^2\theta$

$\sin\theta = 2\cos\theta$ or $\tan\theta = 2$

$\theta = 63.43° \qquad \alpha = 90° - \theta$

$\alpha = 26.6° \quad \leftarrow$

NOTE: For $\alpha = 26.6°$ and $\theta = 63.4°$, we find $\sigma_\theta = 0.98$ MPa and $\tau_\theta = -1.96$ MPa.

Thus, $\left|\dfrac{\tau_\theta}{\sigma_\theta}\right| = 2$ as required.

Problem 2.6-17
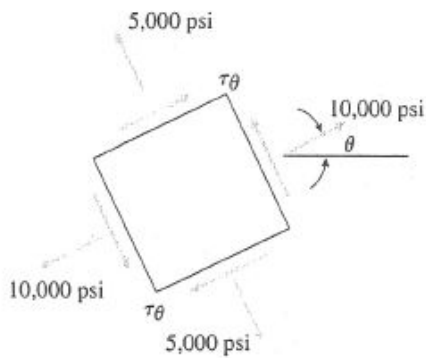
5,000 psi

τ_θ

10,000 psi

θ

10,000 psi

τ_θ

5,000 psi

(a) ANGLE θ AND SHEAR STRESS τ_θ

$$\sigma_\theta = \sigma_x \cos^2\theta$$

$$\sigma_\theta = 10{,}000 \text{ psi}$$

$$\sigma_x = \frac{\sigma_0}{\cos^2\theta} = \frac{10{,}000 \text{ psi}}{\cos^2\theta} \qquad (1)$$

PLANE AT ANGLE $\theta + 90°$

$$\sigma_\theta + 90° = \sigma_x[\cos(\theta + 90°)]^2 = \sigma_x[-\sin\theta]^2$$

$$= \sigma_x \sin^2\theta$$

$$\sigma_\theta + 90° = 5{,}000 \text{ psi}$$

$$\sigma_x = \frac{\sigma_{0+90°}}{\sin^2\theta} = \frac{5{,}000 \text{ psi}}{\sin^2\theta} \qquad (2)$$

Equate (1) and (2):

$$\frac{10{,}000 \text{ psi}}{\cos^2\theta} = \frac{5{,}000 \text{ psi}}{\sin^2\theta}$$

$$\tan^2\theta = \frac{1}{2} \quad \tan\theta = \frac{1}{\sqrt{2}} \quad \theta = 35.26° \quad \leftarrow$$

From Eq. (1) or (2):

$$\sigma_x = 15{,}000 \text{ psi}$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$

$$= (-15{,}000 \text{ psi})(\sin 35.26°)(\cos 35.26°)$$

$$= -7{,}070 \text{ psi} \quad \leftarrow$$

Minus sign means that τ_θ acts clockwise on the plane for which $\theta = 35.26°$.

5,000

10,000

$\theta = 35.26°$

7,070

y

0

x

7,070

10,000
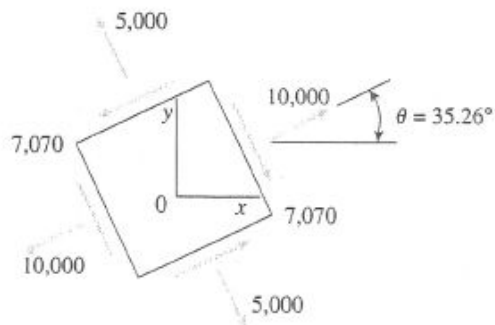
5,000

NOTE: All stresses have units of psi.

(b) MAXIMUM NORMAL AND SHEAR STRESSES

$$\sigma_{max} = \sigma_x = 15{,}000 \text{ psi} \quad \leftarrow$$

$$\tau_{max} = \frac{\sigma_x}{2} = 7{,}500 \text{ psi} \quad \leftarrow$$

Problem 2.6-18
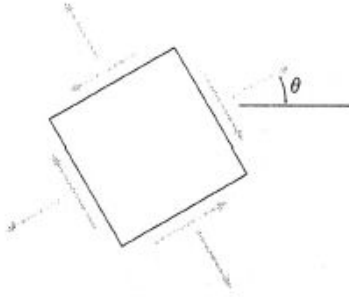
Find θ and σ_x for stress state shown in figure.

$\sigma_\theta = \sigma_x \cos(\theta)^2 \qquad \cos(\theta) = \sqrt{\dfrac{\sigma_\theta}{\sigma_x}}$

so $\quad \sin(\theta) = \sqrt{1 - \dfrac{\sigma_\theta}{\sigma_x}}$

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta)$

$\dfrac{\tau_\theta}{\sigma_x} = -\sqrt{1 - \dfrac{\sigma_\theta}{\sigma_x}}\sqrt{\dfrac{\sigma_\theta}{\sigma_x}}$

$\left(\dfrac{\tau_\theta}{\sigma_x}\right)^2 = \dfrac{\sigma_\theta}{\sigma_x} - \left(\dfrac{\sigma_\theta}{\sigma_x}\right)$

$\left(\dfrac{23}{\sigma_x}\right)^2 = \dfrac{65}{\sigma_x} - \left(\dfrac{65}{\sigma_x}\right)^2$

$\left(\dfrac{65}{\sigma_x}\right)^2 - \left(\dfrac{65}{\sigma_x}\right) + \left(\dfrac{23}{\sigma_x}\right)^2 = 0$

$\dfrac{-(-4754 + 65\sigma_x)}{\sigma_x^2} = 0$

$\sigma_x = \dfrac{4754}{65}$

$\sigma_x = 73.1 \text{ MPa} \qquad \sigma_\theta = 65 \text{ MPa}$

$\theta = a\cos\left(\sqrt{\dfrac{\sigma_\theta}{\sigma_x}}\right) \qquad \theta = 19.5°$
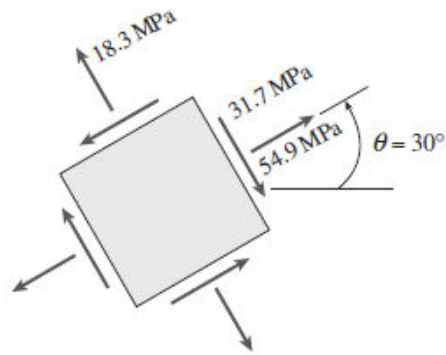
Now find σ_θ and τ_θ for $\theta = 30°$:

$\sigma_{\theta1} = \sigma_x \cos(\theta)^2 \qquad \sigma_{\theta1} = 54.9 \text{ MPa} \quad \leftarrow$

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) \qquad \tau_\theta = -31.7 \text{ MPa} \quad \leftarrow$

$\sigma_{\theta2} = \sigma_x \cos\left(\theta + \dfrac{\pi}{2}\right)^2 \qquad \sigma_{\theta2} = 18.3 \text{ MPa} \quad \leftarrow$

Problem 2.6-19
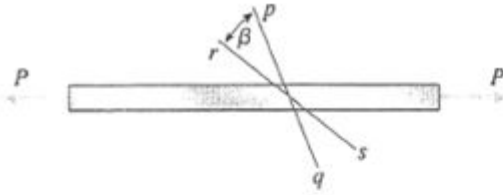
Eq. (2-29a)

$$\sigma_\theta = \sigma_x \cos^2\theta$$

$$\beta = 30°$$

PLANE pq: $\sigma_1 = \sigma_x\cos^2\theta_1$ $\qquad\qquad \sigma_1 = 7500$ psi

PLANE rs: $\sigma_2 = \sigma_x\cos^2(\theta_1 + \beta)$ $\qquad \sigma_2 = 2500$ psi

Equate σ_x from σ_1 and σ_2:

$$\sigma_x = \frac{\sigma_1}{\cos^2\theta_1} = \frac{\sigma_2}{\cos^2(\theta_1 + \beta)} \qquad\qquad \text{(Eq. 1)}$$

or

$$\frac{\cos^2\theta_1}{\cos^2(\theta_1 + \beta)} = \frac{\sigma_1}{\sigma_2}\frac{\cos\theta_1}{\cos(\theta_1 + \beta)} = \sqrt{\frac{\sigma_1}{\sigma_2}} \qquad \text{(Eq. 2)}$$

SUBSTITUTE NUMERICAL VALUES INTO EQ. (2):

$$\frac{\cos\theta_1}{\cos(\theta_1 + 30°)} = \sqrt{\frac{7500 \text{ psi}}{2500 \text{ psi}}} = \sqrt{3} = 1.7321$$

Solve by iteration or a computer program:

$$\theta_1 = 30°$$

MAXIMUM NORMAL STRESS (FROM EQ. 1)

$$\sigma_{max} = \sigma_x = \frac{\sigma_1}{\cos^2\theta_1} = \frac{7500 \text{ psi}}{\cos^2 30°}$$

$$= 10{,}000 \text{ psi} \quad \leftarrow$$

MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{\sigma_x}{2} = 5{,}000 \text{ psi} \quad \leftarrow$$

Problem 2.6-20
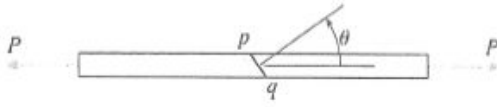
$25° < \theta < 45°$

$A = 225 \text{ mm}^2$

On glued joint: $\sigma_{allow} = 5.0 \text{ MPa}$

$\qquad\qquad\quad \tau_{allow} = 3.0 \text{ MPa}$

ALLOWABLE STRESS σ_x IN TENSION

$$\sigma_\theta = \sigma_x \cos^2\theta \qquad \sigma_x = \frac{\sigma_\theta}{\cos^2\theta} = \frac{5.0 \text{ MPa}}{\cos^2\theta} \qquad (1)$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$

Since the direction of τ_θ is immaterial, we can write:
$|\tau_\theta| = \sigma_x \sin\theta \cos\theta$

or

$$\sigma_x = \frac{|\tau_\theta|}{\sin\theta \cos\theta} = \frac{3.0 \text{ MPa}}{\sin\theta \cos\theta} \qquad (2)$$

GRAPH OF EQS. (1) AND (2)
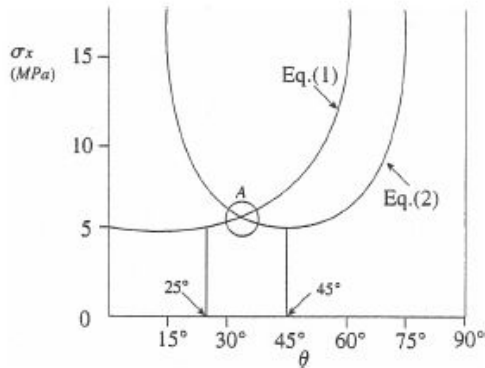
(a) DETERMINE ANGLE Θ FOR LARGEST LOAD

Point A gives the largest value of σ_x and hence the largest load. To determine the angle θ corresponding to point A, we equate Eqs. (1) and (2).

$$\frac{5.0 \text{ MPa}}{\cos^2\theta} = \frac{3.0 \text{ MPa}}{\sin\theta \cos\theta}$$

$$\tan\theta = \frac{3.0}{5.0} \qquad \theta = 30.96° \qquad \leftarrow$$

(b) DETERMINE THE MAXIMUM LOAD

From Eq. (1) or Eq. (2):

$$\sigma_x = \frac{5.0 \text{ MPa}}{\cos^2\theta} = \frac{3.0 \text{ MPa}}{\sin\theta \cos\theta} = 6.80 \text{ MPa}$$

$$P_{max} = \sigma_x A = (6.80 \text{ MPa})(225 \text{ mm}^2)$$

$$\qquad\quad = 1.53 \text{ kN} \qquad \leftarrow$$

Problem 2.6-21

$\alpha = 55(10^{-6})$ $E = 400$ ksi $L = 2$ ft $\Delta T = 100$ $k = 18$ k/in. $b = 0.75$ in. $h = 1.5$ in.

$\sigma_\theta = -950$ psi $\sigma_a = -1000$ psi $\tau_a = -560$ psi $L_\theta = 1.5$ ft $A = bh$ $f = \dfrac{1}{k} = 5.556 \times 10^{-5}$ in./lb

(a) FIND θ AND T_θ

$R_2 = \text{redundant}$ $R_2 = \dfrac{-\alpha \Delta TL}{\left(\dfrac{L}{EA}\right) + f} = -1.212 \times 10^3$ lb $\sigma_x = \dfrac{R_2}{A} = -1077.551$ psi $\sqrt{\dfrac{\sigma_\theta}{\sigma_x}} = 0.939$

$\theta = a\cos\left(\sqrt{\dfrac{\sigma_\theta}{\sigma_x}}\right) = 0.351$ $\cos(2\theta) = 0.763$ $\theta = 20.124°$

$\sigma_x \cos(\theta)^2 = -950$ psi   or $\dfrac{\sigma_x}{2}(1 + \cos(2\theta)) = -950$ psi $\sigma_y = \sigma_x \cos\left(\theta + \dfrac{\pi}{2}\right)^2 = -127.551$ psi

$\theta = 0.351$ $\theta = 20.124°$ $\sigma_x = -1077.551$ psi $2\theta = 0.702$

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) = 348.1$ psi   or $\tau_\theta = \dfrac{-\sigma_x}{2}\sin(2\theta) = 348.1$ psi

$\boxed{\tau_\theta = 348 \text{ psi}}$ $\boxed{\theta = 20.1°}$

(b) FIND σ_{x1} AND σ_{y1}

$\sigma_{x1} = \sigma_x \cos(\theta)^2$ $\sigma_{y1} = \sigma_x \cos\left(\theta + \dfrac{\pi}{2}\right)^2$

$\boxed{\sigma_{x1} = -950 \text{ psi}}$ $\boxed{\sigma_{y1} = -127.6 \text{ psi}}$

(c) GIVEN $L = 2$ ft, FIND k_{max}

$k_{max1} = \dfrac{\sigma_a A}{-\alpha \Delta TL - \sigma_a A \left(\dfrac{L}{EA}\right)} = 15625$ lb/in. < controls (based on σ_{allow})

or $k_{max2} = \dfrac{2\tau_a A}{-\alpha \Delta TL - 2\tau_a A \left(\dfrac{L}{EA}\right)} = 19444.444$ lb/in.   < based on allowable shear stress

$\boxed{k_{max} = 15625 \text{ lb/in.}}$

(d) GIVEN ALLOWABLE NORMAL AND SHEAR STRESSES, FIND L_{max}

$k = 18000$ lb/in.

$\sigma_x = \dfrac{R_2}{A}$ $\sigma_a A = \dfrac{-\alpha \Delta TL}{\left(\dfrac{L}{EA}\right) + f}$ $L_{max1} = \dfrac{\sigma_a A\,(f)}{-\left(\alpha \Delta T + \dfrac{\sigma_a}{E}\right)} = 1.736$ ft < controls (based on σ_{allow})

or $L_{max2} = \dfrac{2\tau_a A\,(f)}{-\left(\alpha \Delta T + \dfrac{2\tau_a}{E}\right)} = 2.16$ ft < based on T_{allow}

$\boxed{L_{max} = 1.736 \text{ ft}}$

(e) Find ΔT_{max} given L, k, and allowable stresses $k = 18000\,\text{lb/in.}$ $L = 2\,\text{ft}$ $\sigma_a = -1000\,\text{psi}$

$$\tau_a = -560\,\text{psi}$$

$$\Delta T_{max1} = \frac{\left(\dfrac{L}{EA} + f\right)\sigma_a A}{-\alpha L} = 92.803°\text{F} \qquad < \text{ based on } \sigma_{allow} \qquad \Delta T = 100$$

$$\Delta T_{max2} = \frac{\left(\dfrac{L}{EA} + f\right)2\tau_a A}{-\alpha L} = 103.939°\text{F} < \text{ based on } T_{allow}$$

$$\boxed{\Delta T_{max} = 92.8°\text{F}}$$

Problem 2.6-22

$b = 50\text{mm}$ $\alpha = 35\text{deg}$

$\sigma_a = 11.5\text{MPa}$ $\tau_a = 4.5\text{MPa}$

$\sigma_{ga} = 3.5\text{MPa}$ $\tau_{ga} = 1.25\text{MPa}$

Rotate stress element CW by angle θ to align with glue joint (see fig.)

$\theta = \alpha - 90\text{deg} = -55 \cdot \text{deg}$

Plane stress transformations $\sigma_x = \dfrac{P}{A}$ $A = b^2 = 2500 \cdot \text{mm}^2$

$\sigma_\theta = \sigma_x \cdot \cos(\theta)^2$ $\tau_\theta = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta)$

Equate σ_θ and τ_θ to allowable values and solve for P - min. P controls

$\sigma_{max} = \sigma_x$ $P_{max1} = \sigma_a \cdot A = 28.75 \cdot \text{kN}$

$\tau_{max} = -\left(\dfrac{P}{A}\right) \cdot \sin(45\text{deg}) \cdot \cos(45\text{deg})$ $P_{max2} = \dfrac{\tau_a}{2} \cdot A = 5.625 \cdot \text{kN}$ < shear in wood controls

$P_{max3} = \dfrac{\sigma_{ga} \cdot A}{\cos(\theta)^2} = 26.597 \cdot \text{kN}$

$P_{max4} = \dfrac{\tau_{ga} \cdot A}{-\sin(\theta) \cdot \cos(\theta)} = 6.651 \cdot \text{kN}$

Problem 2.7-1
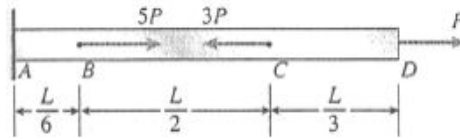
$P = 6\,k$

$L = 52\text{ in.}$

$E = 10.4 \times 10^6\text{ psi}$

$A = 2.76\text{ in.}^2$

INTERNAL AXIAL FORCES

$N_{AB} = 3P \qquad N_{BC} = -2P \qquad N_{CD} = P$

LENGTHS

$L_{AB} = \dfrac{L}{6} \qquad L_{BC} = \dfrac{L}{2} \qquad L_{CD} = \dfrac{L}{3}$

(a) STRAIN ENERGY OF THE BAR (EQ. 2-40)

$$U = \Sigma \frac{N_i^2 L_i}{2E_i A_i}$$

$$= \frac{1}{2EA}\left[(3P)^2\left(\frac{L}{6}\right) + (-2P)^2\left(\frac{L}{2}\right) + (P)^2\left(\frac{L}{3}\right)\right]$$

$$= \frac{P^2 L}{2EA}\left(\frac{23}{6}\right) = \frac{23P^2 L}{12EA} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES:

$$U = \frac{23(6\text{ k})^2(52\text{ in.})}{12(10.4 \times 10^6\text{ psi})(2.76\text{ in.}^2)}$$

$$= 125\text{ in.-lb} \quad \leftarrow$$

Problem 2.7-2
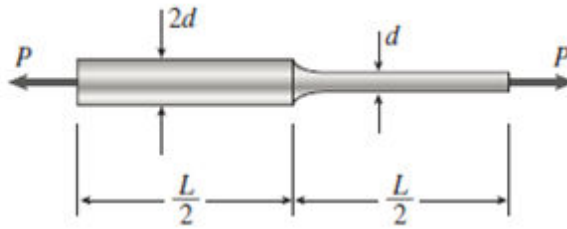
(a) STRAIN ENERGY OF THE BAR

Add the strain energies of the two segments of the bar (see Eq. 2-40).

$$U = \sum_{i=1}^{2} \frac{N_i^2 L_i}{2 E_i A_i} = \frac{P^2(L/2)}{2E}\left[\frac{1}{\frac{\pi}{4}(2d)^2} + \frac{1}{\frac{\pi}{4}(d^2)}\right]$$

$$= \frac{P^2 L}{\pi E}\left(\frac{1}{4d^2} + \frac{1}{d^2}\right) = \frac{5P^2 L}{4\pi E d^2} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES:

$P = 27$ kN $L = 600$ mm

$d = 40$ mm $E = 105$ GPa

$$U = \frac{5(27 \text{ kN}^2)(600 \text{ mm})}{4\pi(105 \text{ GPa})(40 \text{ mm})^2}$$

$$= 1.036 \text{ N} \cdot \text{m} = 1.036 \text{ J} \quad \leftarrow$$

Problem 2.7-3
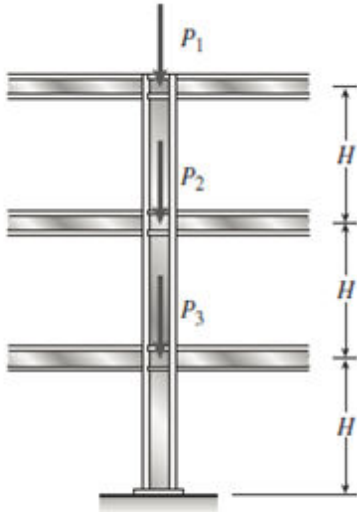
$H = 10.5 \text{ ft}$ $E = 30 \times 10^6 \text{ psi}$

$A = 15.5 \text{ in.}^2$ $P_1 = 40 \text{ k}$

$P_2 = P_3 = 60 \text{ k}$

To find the strain energy of the column, add the strain energies of the three segments (see Eq. 2-40).

Upper segment: $N_1 = -P_1$

Middle segment: $N_2 = -(P_1 + P_2)$

Lower segment: $N_3 = -(P_1 + P_2 + P_3)$

STRAIN ENERGY

$$U = \sum \frac{N_i^2 L_i}{2 E_i A_i}$$

$$= \frac{H}{2EA}[P_1^2 + (P_1 + P_2)^2 + (P_1 + P_2 + P_3)^2]$$

$$= \frac{H}{2EA}[Q]$$

$[Q] = (40 \text{ k})^2 + (100 \text{ k})^2 + (160 \text{ k})^2 = 37{,}200 \text{ k}^2$

$2EA = 2(30 \times 10^6 \text{ psi})(15.5 \text{ in.}^2) = 930 \times 10^6 \text{ lb}$

$$U = \frac{(10.5 \text{ ft})(12 \text{ in./ft})}{930 \times 10^6 \text{ lb}}[37{,}200 \text{ k}^2]$$

$= 5040 \text{ in.-lb} \quad \leftarrow$

Problem 2.7-4
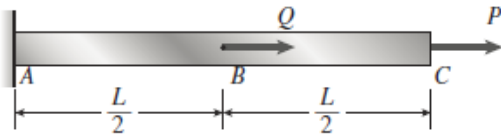
(a) FORCE P ACTS ALONE ($Q = 0$)

$$U_1 = \frac{P^2 L}{2EA} \quad \leftarrow$$

(b) FORCE Q ACTS ALONE ($P = 0$)

$$U_2 = \frac{Q^2(L/2)}{2EA} = \frac{Q^2 L}{4EA} \quad \leftarrow$$

(c) FORCES P AND Q ACT SIMULTANEOUSLY

Segment BC: $U_{BC} = \dfrac{P^2(L/2)}{2EA} = \dfrac{P^2 L}{4EA}$

Segment AB: $U_{AB} = \dfrac{(P+Q)^2(L/2)}{2EA}$

$$= \frac{P^2 L}{4EA} + \frac{PQL}{2EA} + \frac{Q^2 L}{4EA}$$

$$U_3 = U_{BC} + U_{AB} = \frac{P^2 L}{2EA} + \frac{PQL}{2EA} + \frac{Q^2 L}{4EA} \quad \leftarrow$$

(Note that U_3 is *not* equal to $U_1 + U_2$. In this case, $U_3 > U_1 + U_2$. However, if Q is reversed in direction, $U_3 < U_1 + U_2$. Thus, U_3 may be larger or smaller than $U_1 + U_2$.)

Problem 2.7-5

DATA:

Material	Weight density (lb/in.³)	Modulus of elasticity (ksi)	Proportional limit (psi)
Mild steel	0.284	30,000	36,000
Tool steel	0.284	30,000	75,000
Aluminum	0.0984	10,500	60,000
Rubber (soft)	0.0405	0.300	300

STRAIN ENERGY PER UNIT VOLUME

$$U = \frac{P^2 L}{2EA} \qquad \text{Volume } V = AL$$

$$\text{Stress } \sigma = \frac{P}{A}$$

$$u = \frac{U}{V} = \frac{\sigma_{PL}^2}{2E}$$

At the proportional limit:

$u = u_R$ = modulus of resistance

$$u_R = \frac{\sigma_{PL}^2}{2E} \qquad\qquad \text{(Eq. 1)}$$

STRAIN ENERGY PER UNIT WEIGHT

$$U = \frac{P^2 L}{2EA} \qquad \text{Weight } W = \gamma AL$$

γ = weight density

$$u_W = \frac{U}{W} = \frac{\sigma^2}{2\gamma E}$$

At the proportional limit:

$$u_W = \frac{\sigma_{PL}^2}{2\gamma E} \qquad\qquad \text{(Eq. 2)}$$

RESULTS

	u_R (psi)	u_w (in.)
Mild steel	22	76
Tool steel	94	330
Aluminum	171	1740
Rubber (soft)	150	3700

Problem 2.7-6
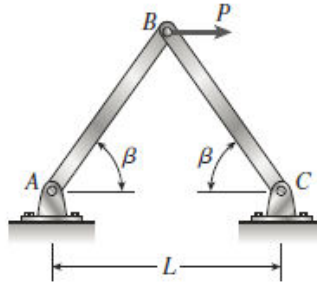
$\beta = 60°$

$L_{AB} = L_{BC} = L$

$\sin \beta = \sqrt{3}/2$

$\cos \beta = 1/2$

FREE-BODY DIAGRAM OF JOINT B
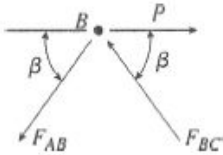
$\Sigma F_{vert} = 0 \quad \uparrow_+ \quad \downarrow^-$

$-F_{AB} \sin \beta + F_{BC} \sin \beta = 0$

$F_{AB} = F_{BC}$ (Eq. 1)

$\Sigma F_{horiz} = 0 \rightarrow \leftarrow$

$-F_{AB} \cos \beta - F_{BC} \cos \beta + P = 0$

$F_{AB} = F_{BC} = \dfrac{P}{2 \cos \beta} = \dfrac{P}{2(1/2)} = P$ (Eq. 2)

Axial forces: $N_{AB} = P$ (tension)

$\qquad\qquad N_{BC} = -P$ (compression)

(a) STRAIN ENERGY OF TRUSS (EQ. 2-40)

$U = \Sigma \dfrac{N_i^2 L_i}{2E_i A_i} = \dfrac{(N_{AB})^2 L}{2EA} + \dfrac{(N_{BC})^2 L}{2EA} = \dfrac{P^2 L}{EA} \quad \leftarrow$

(b) HORIZONTAL DISPLACEMENT OF JOINT B (EQ. 2-42)

$\delta_B = \dfrac{2U}{P} = \dfrac{2}{P}\left(\dfrac{P^2 L}{EA}\right) = \dfrac{2PL}{EA} \quad \leftarrow$

Problem 2.7-7
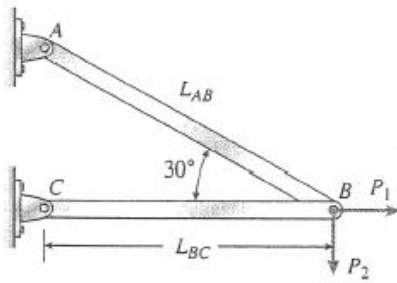
$P_1 = 300$ lb

$P_2 = 900$ lb

$A = 2.4$ in.^{2}

$E = 30 \times 10^6$ psi

$L_{BC} = 60$ in.

$\beta = 30°$

$\sin \beta = \sin 30° = \dfrac{1}{2}$

$\cos \beta = \cos 30° = \dfrac{\sqrt{3}}{2}$

$L_{AB} = \dfrac{L_{BC}}{\cos 30°} = \dfrac{120}{\sqrt{3}}$ in. $= 69.282$ in.

$2EA = 2(30 \times 10^6 \text{ psi})(2.4 \text{ in.}^2) = 144 \times 10^6$ lb

FORCES F_{AB} AND F_{BC} IN THE BARS

From equilibrium of joint B:

$F_{AB} = 2P_2 = 1800$ lb

$F_{BC} = P_1 - P_2\sqrt{3} = 300 \text{ lb} - 1558.8$ lb

Force	P_1 alone	P_2 alone	P_1 and P_2
F_{AB}	0	1800 lb	1800 lb
F_{BC}	300 lb	−1558.8 lb	−1258.8 lb

(a) LOAD P_1 ACTS ALONE

$U_1 = \dfrac{(F_{BC})^2 L_{BC}}{2EA} = \dfrac{(300 \text{ lb})^2(60 \text{ in.})}{144 \times 10^6 \text{ lb}}$

$= 0.0375$ in.-lb ←

(b) LOAD P_2 ACTS ALONE

$U_2 = \dfrac{1}{2EA}\left[(F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC}\right]$

$= \dfrac{1}{2EA}\left[(1800 \text{ lb})^2(69.282 \text{ in.})\right.$

$\left. + (-1558.8 \text{ lb})^2(60 \text{ in.})\right]$

$= \dfrac{370.265 \times 10^6 \text{ lb}^2\text{-in.}}{144 \times 10^6 \text{ lb}} = 2.57$ in.-lb ←

(c) LOADS P_1 AND P_2 ACT SIMULTANEOUSLY

$U_3 = \dfrac{1}{2EA}\left[(F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC}\right]$

$= \dfrac{1}{2EA}\left[(1800 \text{ lb})^2(69.282 \text{ in.})\right.$

$\left. + (-1258.8 \text{ lb})^2(60 \text{ in.})\right]$

$= \dfrac{319.548 \times 10^6 \text{ lb}^2\text{-in.}}{144 \times 10^6 \text{ lb}}$

$= 2.22$ in.-lb ←

NOTE: The strain energy U_3 is *not* equal to $U_1 + U_2$.

Problem 2.7-8

$k_1 = 3k$

$k_2 = 1.5k$

$k_3 = k$
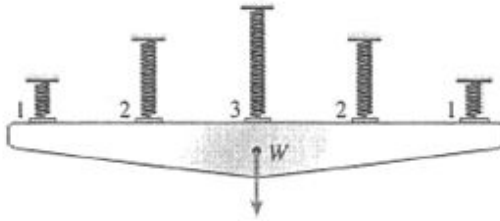
δ = downward displacement of rigid bar

For a spring: $U = \dfrac{k\delta^2}{2}$ Eq. (2-38b)

(a) STRAIN ENERGY U OF ALL SPRINGS

$$U = 2\left(\frac{3k\delta^2}{2}\right) + 2\left(\frac{1.5k\delta^2}{2}\right) + \frac{k\delta^2}{2} = 5k\delta^2 \;\; \leftarrow$$

(b) DISPLACEMENT δ

Work done by the weight W equals $\dfrac{W\delta}{2}$

Strain energy of the springs equals $5k\delta^2$

$\therefore \dfrac{W\delta}{2} = 5k\delta^2$ and $\delta = \dfrac{W}{10k}$ $\leftarrow$

(c) FORCES IN THE SPRINGS

$$F_1 = 3k\delta = \frac{3\,W}{10} \qquad F_2 = 1.5k\delta = \frac{3W}{20} \;\; \leftarrow$$

$$F_3 = k\delta = \frac{W}{10} \;\; \leftarrow$$

(d) NUMERICAL VALUES

$W = 600$ N $k = 7.5$ N/mm $= 7500$ N/mm

$$U = 5k\delta^2 = 5k\left(\frac{W}{10k}\right)^2 = \frac{W^2}{20k}$$

$$= 2.4 \text{ N·m} = 2.4 \text{ J} \;\; \leftarrow$$

$\delta = \dfrac{W}{10k} = 8.0$ mm $\leftarrow$

$F_1 = \dfrac{3W}{10} = 180$ N $\leftarrow$

$F_2 = \dfrac{3W}{20} = 90$ N $\leftarrow$

$F_3 = \dfrac{W}{10} = 60$ N $\leftarrow$

NOTE: $W = 2F_1 + 2F_2 + F_3 = 600$ N (Check)

Problem 2.7-9
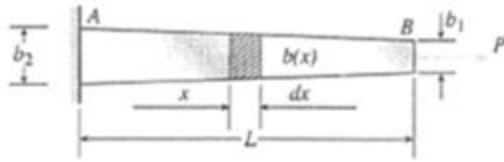
$$b(x) = b_2 - \frac{(b_2 - b_1)x}{L}$$

$$A(x) = tb(x)$$

$$= t\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]$$

(a) STRAIN ENERGY OF THE BAR

$$U = \int \frac{[N(x)]^2 dx}{2EA(x)} \quad \text{(Eq. 2-41)}$$

$$= \int_0^L \frac{P^2 dx}{2Etb(x)} = \frac{P^2}{2Et}\int_0^L \frac{dx}{b_2 - (b_2 - b_1)\frac{x}{L}} \quad (1)$$

From Appendix C: $\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$

Apply this integration formula to Eq. (1):

$$U = \frac{P^2}{2Et}\left[\frac{1}{-(b_2 - b_1)(\frac{1}{L})}\ln\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]\right]_0^L$$

$$= \frac{P^2}{2Et}\left[\frac{-L}{(b_2 - b_1)}\ln b_1 - \frac{-L}{(b_2 - b_1)}\ln b_2\right]$$

$$U = \frac{P^2 L}{2Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \quad \leftarrow$$

(b) ELONGATION OF THE BAR (EQ. 2-42)

$$\delta = \frac{2U}{P} = \frac{PL}{Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \quad \leftarrow$$

NOTE: This result agrees with the formula derived in Prob. 2.3-17.

Problem 2.7-10
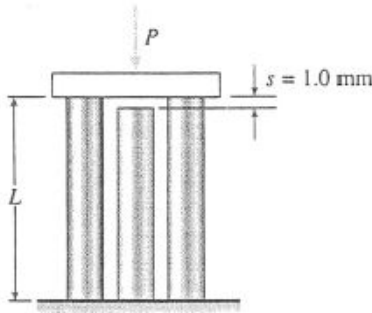
$s = 1.0$ mm

$L = 1.0$ m

For each bar:

$A = 3000$ mm^2

$E = 45$ GPa

$\dfrac{EA}{L} = 135 \times 10^6$ N/m

(a) LOAD P_1 REQUIRED TO CLOSE THE GAP

In general, $\delta = \dfrac{PL}{EA}$ and $P = \dfrac{EA\delta}{L}$

For two bars, we obtain:

$$P_1 = 2\left(\dfrac{EAs}{L}\right) = 2(135 \times 10^6 \text{ N/m})(1.0 \text{ mm})$$

$P_1 = 270$ kN   ←

(b) DISPLACEMENT δ FOR $P = 400$ kN

Since $P > P_1$, all three bars are compressed. The force P equals P_1 plus the additional force required to compress all three bars by the amount $\delta - s$.

$$P = P_1 + 3\left(\dfrac{EA}{L}\right)(\delta - s)$$

or 400 kN = 270 kN + 3(135 × 10^6 N/m)
(δ − 0.001 m)

Solving, we get $\delta = 1.321$ mm   ←

(c) STRAIN ENERGY U FOR $P = 400$ kN

$$U = \Sigma \dfrac{EA\delta^2}{2L}$$

Outer bars: $\delta = 1.321$ mm

Middle bar: $\delta = 1.321$ mm $- s$

$= 0.321$ mm

$$U = \dfrac{EA}{2L}[2(1.321 \text{ mm})^2 + (0.321 \text{ mm})^2]$$

$$= \dfrac{1}{2}(135 \times 10^6 \text{ N/m})(3.593 \text{ mm}^2)$$

$= 243$ N·m $= 243$ J   ←

(d) LOAD-DISPLACEMENT DIAGRAM

$U = 243$ J $= 243$ N·m

$$\dfrac{P\delta}{2} = \dfrac{1}{2}(400 \text{ kN})(1.321 \text{ mm}) = 264 \text{ N·m}$$

The strain energy U is *not* equal to $\dfrac{P\delta}{2} =$ because the load-displacement relation is not linear.
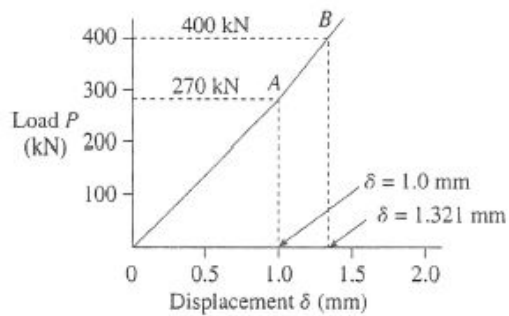
$U =$ area under line OAB.

$\dfrac{P\delta}{2} =$ area under a straight line from O to B, which is larger than U.

Problem 2.7-11
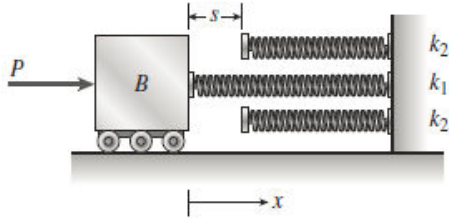
Force P_0 required to close the gap:

$$P_0 = k_1 s \tag{1}$$

FORCE-DISPLACEMENT RELATION BEFORE GAP IS CLOSED

$$P = k_1 x \qquad (0 \le x \le s)(0 \le P \le P_0) \tag{2}$$

FORCE-DISPLACEMENT RELATION AFTER GAP IS CLOSED

All three springs are compressed. Total stiffness equals $k_1 + 2k_2$. Additional displacement equals $x - s$. Force P equals P_0 plus the force required to compress all three springs by the amount $x - s$.

$$P = P_0 + (k_1 + 2k_2)(x - s)$$
$$= k_1 s + (k_1 + 2k_2)x - k_1 s - 2k_2 s$$
$$P = (k_1 + 2k_2)x - 2k_2 s \qquad (x \ge s); (P \ge P_0) \tag{3}$$

$P_1 = $ force P when $x = 2s$

Substitute $x = 2s$ into Eq. (3):

$$P_1 = 2(k_1 + k_2)s \tag{4}$$

(a) FORCE-DISPLACEMENT DIAGRAM
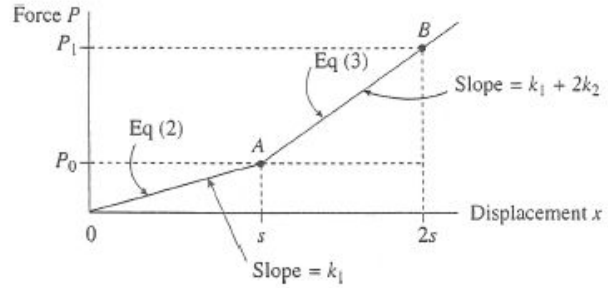
(b) STRAIN ENERGY U_1 WHEN $x = 2s$

$U_1 = $ Area below force-displacement curve

$$= \frac{1}{2}P_0 s + P_0 s + \frac{1}{2}(P_1 - P_0)s = P_0 s + \frac{1}{2}P_1 s$$
$$= k_1 s^2 + (k_1 + k_2)s^2$$
$$U_1 = (2k_1 + k_2)s^2 \quad \leftarrow \tag{5}$$

(c) STRAIN ENERGY U_1 IS NOT EQUAL TO $\dfrac{P\delta}{2}$

For $\delta = 2s$: $\dfrac{P\delta}{2} = \dfrac{1}{2}P_1(2s) = P_1 s = 2(k_1 + k_2)s^2$

(This quantity is greater than U_1.)

$U_1 = $ area under line OAB.

$\dfrac{P\delta}{2} = $ area under a straight line from O to B, which is larger than U_1.

Thus, $\dfrac{P\delta}{2}$ is *not* equal to the strain energy because the force-displacement relation is not linear.

Problem 2.7-12
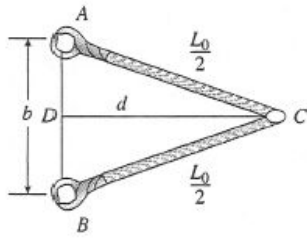
$L_0 = 760$ mm $\dfrac{L_0}{2} = 380$ mm

$b = 380$ mm

Bungee cord:
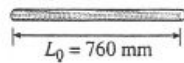
$k = 140$ N/m

$L_0 = 760$ mm

From triangle ACD:

$$d = \frac{1}{2}\sqrt{L_0^2 - b^2} = 329.09 \text{ mm} \qquad (1)$$

DIMENSIONS AFTER THE LOAD P IS APPLIED
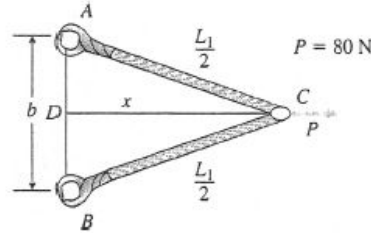
Let x = distance CD

Let L_1 = stretched length of bungee cord

From triangle ACD:

$$\frac{L_1}{2} = \sqrt{\left(\frac{b}{2}\right)^2 + x^2} \qquad (2)$$

$$L_1 = \sqrt{b^2 + 4x^2} \qquad (3)$$

EQUILIBRIUM AT POINT C

Let F = tensile force in bungee cord

$P = 80$ N
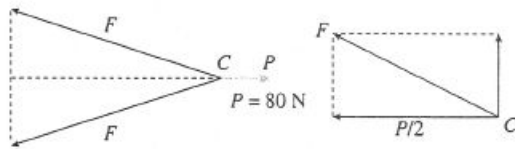
$$\frac{F}{P/2} = \frac{L_1/2}{x} \qquad F = \left(\frac{P}{2}\right)\left(\frac{L_1}{2}\right)\left(\frac{1}{x}\right)$$

$$= \frac{P}{2}\sqrt{1 + \left(\frac{b}{2x}\right)^2} \qquad (4)$$

ELONGATION OF BUNGEE CORD

Let δ = elongation of the entire bungee cord

$$\delta = \frac{F}{k} = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (5)$$

Final length of bungee cord = original length + δ

$$L_1 = L_0 + \delta = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (6)$$

SOLUTION OF EQUATIONS

Combine Eqs. (6) and (3):

$$L_1 = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = \sqrt{b^2 + 4x^2}$$

or $L_1 = L_0 + \dfrac{P}{4kx}\sqrt{b^2 + 4x^2} = \sqrt{b^2 + 4x^2}$

$$L_0 = \left(1 - \frac{P}{4kx}\right)\sqrt{b^2 + 4x^2} \qquad (7)$$

This equation can be solved for x.

SUBSTITUTE NUMERICAL VALUES INTO EQ. (7):

$$760 \text{ mm} = \left[1 - \frac{(80 \text{ N})(1000 \text{ mm/m})}{4(140 \text{ N/m})x}\right]$$

$$\times \sqrt{(380 \text{ mm})^2 + 4x^2} \qquad (8)$$

$$760 = \left(1 - \frac{142.857}{x}\right)\sqrt{144{,}400 + 4x^2} \qquad (9)$$

Units: x is in millimeters
Solve for x (Use trial-and-error or a computer program):

$x = 497.88$ mm

(a) STRAIN ENERGY U OF THE BUNGEE CORD

$$U = \frac{k\delta^2}{2} \qquad k = 140 \text{ N/m} \qquad P = 80 \text{ N}$$

From Eq. (5):

$$\delta = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = 305.81 \text{ mm}$$

$$U = \frac{1}{2}(140 \text{ N/m})(305.81 \text{ mm})^2 = 6.55 \text{ N·m}$$

$U = 6.55$ J $\leftarrow$

(b) DISPLACEMENT δ_C OF POINT C

$$\delta_C = x - d = 497.88 \text{ mm} - 329.09 \text{ mm}$$

$$= 168.8 \text{ mm} \qquad \leftarrow$$

(c) COMPARISON OF STRAIN ENERGY U WITH THE QUANTITY $P\delta_C/2$

$$U = 6.55 \text{ J}$$

$$\frac{P\delta_C}{2} = \frac{1}{2}(80 \text{ N})(168.8 \text{ mm}) = 6.75 \text{ J}$$

The two quantities are not the same. The work done by the load P is *not* equal to $P\delta_C/2$ because the load-displacement relation (see below) is non-linear when the displacements are large. (The *work* done by the load P is equal to the strain energy because the bungee cord behaves elastically and there are no energy losses.)

U = area OAB under the curve OA.

$\dfrac{P\delta_C}{2}$ = area of triangle OAB, which is greater than U.
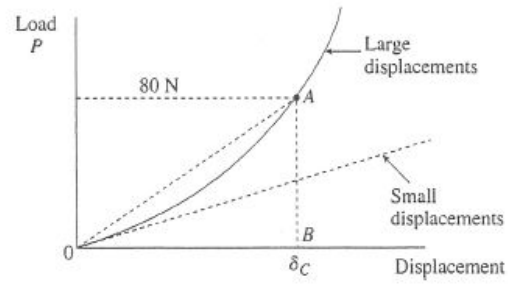
Problem 2.8-1
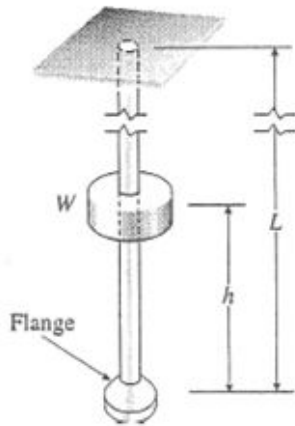
W = 150 lb

h = 2.0 in.   L = 4.0 ft = 48 in.

$E = 30 \times 10^6$ psi   A = 0.75 in.²

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.00032 \text{ in.}$$

Eq. (2-53):

$$\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.0361 \text{ in.} \quad \leftarrow$$

(b) MAXIMUM TENSILE STRESS (EQ. 2-55)

$$\sigma_{max} = \frac{E\delta_{max}}{L} = 22,600 \text{ psi} \quad \leftarrow$$

(c) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{0.0361 \text{ in.}}{0.00032 \text{ in.}}$$

$$= 113 \quad \leftarrow$$

Problem 2.8-2
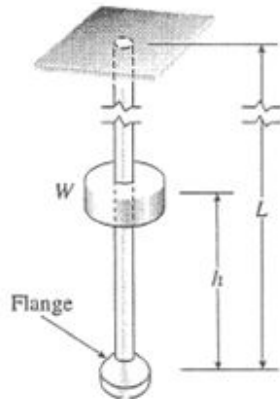
Flange

$M = 80$ kg

$W = Mg = (80 \text{ kg})(9.81 \text{ m/s}^2)$

$\quad = 784.8$ N

$h = 0.5$ m $\qquad L = 3.0$ m

$E = 170$ GPa $\qquad A = 350$ mm^2

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.03957 \text{ mm}$$

Eq. (2-53): $\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

$\quad = 6.33 \text{ mm} \quad \leftarrow$

(b) MAXIMUM TENSILE STRESS (EQ. 2-55)

$$\sigma_{max} = \frac{E\delta_{max}}{L} = 359 \text{ MPa} \quad \leftarrow$$

(c) IMPACT FACTOR (EQ. 2-61)

Impact factor $= \dfrac{\delta_{max}}{\delta_{st}} = \dfrac{6.33 \text{ mm}}{0.03957 \text{ mm}}$

$\qquad = 160 \quad \leftarrow$

Problem 2.8-3
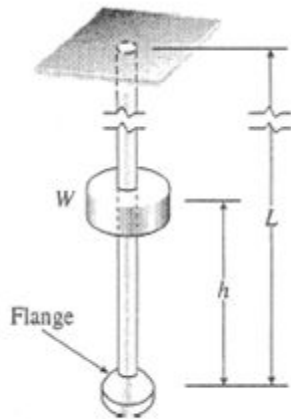
$W = 50$ lb $\qquad$ $h = 2.0$ in.

$L = 3.0$ ft $= 36$ in.

$E = 30{,}000$ psi $\qquad$ $A = 0.25$ in.^{2}

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.00024 \text{ in.}$$

Eq. (2-53): $\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

$= 0.0312$ in. ←

(b) MAXIMUM TENSILE STRESS (EQ. 2-55)

$$\sigma_{max} = \frac{E\delta_{max}}{L} = 26{,}000 \text{ psi} \quad \leftarrow$$

(c) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{0.0312 \text{ in.}}{0.00024 \text{ in.}}$$

$= 130$ ←
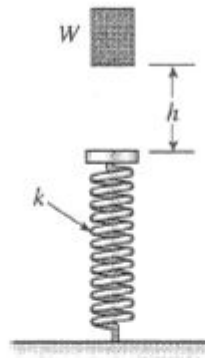
Problem 2.8-4

$W = 5.0\ \text{N}$ $h = 200\ \text{mm}$ $k = 90\ \text{N/m}$

(a) MAXIMUM SHORTENING OF THE SPRING

$$\delta_{st} = \frac{W}{k} = \frac{5.0\ \text{N}}{90\ \text{N/m}} = 55.56\ \text{mm}$$

Eq. (2-53): $\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

$$= 215\ \text{mm} \quad \leftarrow$$

(b) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{215\ \text{mm}}{55.56\ \text{mm}}$$

$$= 3.9 \quad \leftarrow$$
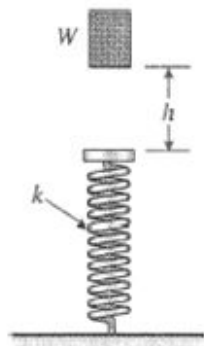
Problem 2.8-5

$W = 1.0$ lb $h = 12$ in. $k = 0.5$ lb/in.

(a) MAXIMUM SHORTENING OF THE SPRING

$$\delta_{st} = \frac{W}{k} = \frac{1.0 \text{ lb}}{0.5 \text{ lb/in.}} = 2.0 \text{ in.}$$

Eq. (2-53): $\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

 $= 9.21 \text{ in.}$ ←

(b) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{9.21 \text{ in.}}{2.0 \text{ in.}}$$

$\phantom{\text{Impact factor}}$ $= 4.6$ ←

Problem 2.8-6
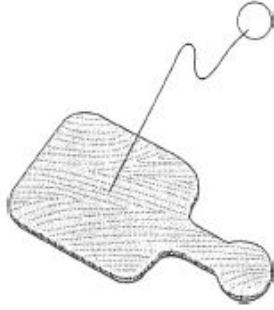
$g = 9.81 \text{ m/s}^2 \qquad E = 2.0 \text{ MPa}$

$A = 1.6 \text{ mm}^2 \qquad L_0 = 200 \text{ mm}$

$L_1 = 900 \text{ mm} \qquad W = 450 \text{ mN}$

WHEN THE BALL LEAVES THE PADDLE

$$KE = \frac{Wv^2}{2g}$$

WHEN THE RUBBER CORD IS FULLY STRETCHED:

$$U = \frac{EA\delta^2}{2L_0} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

CONSERVATION OF ENERGY

$$KE = U \qquad \frac{Wv^2}{2g} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

$$v^2 = \frac{gEA}{WL_0}(L_1 - L_0)^2$$

$$v = (L_1 - L_0)\sqrt{\frac{gEA}{WL_0}} \qquad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$v = (700 \text{ mm})\sqrt{\frac{(9.81 \text{ m/s}^2)(2.0 \text{ MPa})(1.6 \text{ mm}^2)}{(450 \text{ mN})(200 \text{ mm})}}$$

$$= 13.1 \text{ m/s} \qquad \leftarrow$$

Problem 2.8-7

$E = 1.6 \times 10^6$ psi

$\sigma_{allow} = 2500$ psi $(= \sigma_{max})$

Find h_{max}

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{4500 \text{ lb}}{113.10 \text{ in.}^2} = 39.79 \text{ psi}$$

MAXIMUM HEIGHT h_{max}

Eq. (2-59): $\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for h:

$$h = h_{max} = \frac{L\sigma_{max}}{2E}\left(\frac{\sigma_{max}}{\sigma_{st}} - 2\right) \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$h_{max} = \frac{(180 \text{ in.}) (2500 \text{ psi})}{2(1.6 \times 10^6 \text{ psi})}\left(\frac{2500 \text{ psi}}{39.79 \text{ psi}} - 2\right)$$

$$= 8.55 \text{ in.} \quad \leftarrow$$
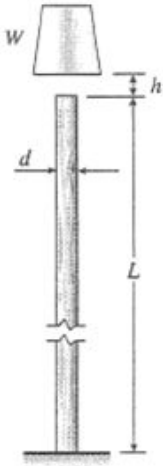
$W = 4500$ lb $\qquad d = 12$ in.

$L = 15$ ft $= 180$ in.

$A = \dfrac{\pi d^2}{4} = 113.10$ in.^{2}

Problem 2.8-8
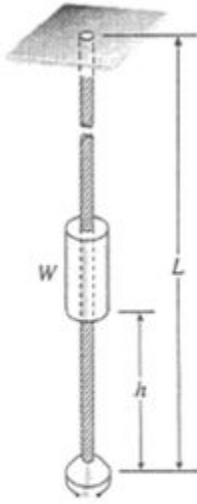
$W = Mg = (35\ kg)(9.81\ m/s^2) = 343.4\ N$

$A = 40\ mm^2 \qquad E = 130\ GPa$

$h = 1.0\ m \qquad \sigma_{allow} = \sigma_{max} = 500\ MPa$

Find minimum length L_{min}.

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{343.4\ N}{40\ mm^2} = 8.585\ MPa$$

MINIMUM LENGTH L_{min}

Eq. (2-59): $\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \qquad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$L_{min} = \frac{2(130\ GPa)\,(1.0\ m)\,(8.585\ MPa)}{(500\ MPa)\,[500\ MPa - 2(8.585\ MPa)]}$$

$$= 9.25\ m \qquad \leftarrow$$

Problem 2.8-9
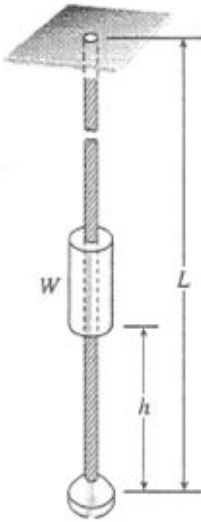
$W = 100$ lb

$A = 0.080$ in.^{2} $E = 21 \times 10^6$ psi

$h = 45$ in $\sigma_{allow} = \sigma_{max} = 70$ ksi

Find minimum length L_{min}.

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{100 \text{ lb}}{0.080 \text{ in.}^2} = 1250 \text{ psi}$$

MINIMUM LENGTH L_{min}

Eq. (2-59): $\quad \sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$L_{min} = \frac{2(21 \times 10^6 \text{ psi})(45 \text{ in.})(1250 \text{ psi})}{(70,000 \text{ psi})[70,000 \text{ psi} - 2(1250 \text{ psi})]}$$

$$= 500 \text{ in.} \quad \leftarrow$$

Problem 2.8-10
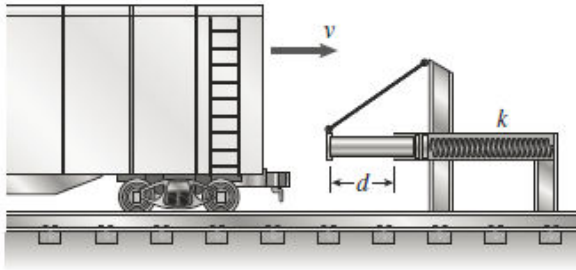
$k = 8.0$ MN/m $W = 545$ kN

d = maximum displacement of spring

$d = \delta_{max} = 450$ mm

Find v_{max}.

KINETIC ENERGY BEFORE IMPACT

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS COMPRESSED TO THE MAXIMUM ALLOWABLE AMOUNT

$$U = \frac{k\delta_{max}^2}{2} = \frac{kd^2}{2}$$

CONSERVATION OF ENERGY

$$KE = U \qquad \frac{Wv^2}{2g} = \frac{kd^2}{2} \qquad v^2 = \frac{kd^2}{W/g}$$

$$v = v_{max} = d\sqrt{\frac{k}{W/g}} \qquad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$v_{max} = (450 \text{ mm})\sqrt{\frac{8.0 \text{ MN/m}}{(545 \text{ kN})/(9.81 \text{ m/s}^2)}}$$

$$= 5400 \text{ mm/s} = 5.4 \text{ m/s} \qquad \leftarrow$$

Problem 2.8-11
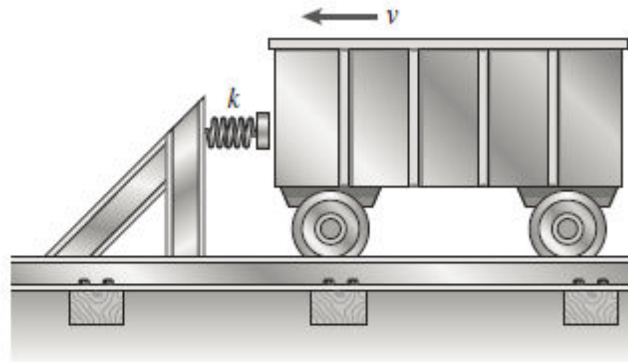
$k = 1120$ lb/in. $W = 3450$ lb

$v = 7$ mph $= 123.2$ in./sec

$g = 32.2$ ft/sec$^2 = 386.4$ in./sec^2

Find the shortening δ_{max} of the spring.

KINETIC ENERGY JUST BEFORE IMPACT

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS FULLY COMPRESSED

$$U = \frac{k\delta_{max}^2}{2}$$

Conservation of energy

$$KE = U \qquad \frac{Wv^2}{2g} = \frac{k\delta_{max}^2}{2}$$

Solve for δ_{max}: $\delta_{max} = \sqrt{\dfrac{Wv^2}{gk}}$ ←

SUBSTITUTE NUMERICAL VALUES:

$$\delta_{max} = \sqrt{\frac{(3450 \text{ lb}) (123.2 \text{ in./sec})^2}{(386.4 \text{ in./sec}^2) (1120 \text{ lb/in.})}}$$

$= 11.0$ in. ←

Problem 2.8-12
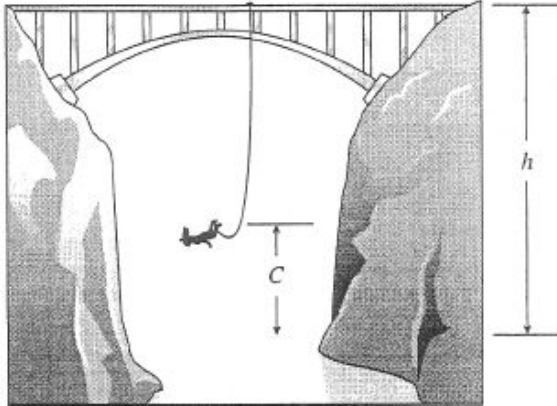
$W = Mg = (55 \text{ kg})(9.81 \text{ m/s}^2)$

$\qquad = 539.55 \text{ N}$

$EA = 2.3 \text{ kN}$

Height: $h = 60 \text{ m}$

Clearance: $C = 10 \text{ m}$

Find length L of the bungee cord.

P.E. = Potential energy of the jumper at the top of bridge (with respect to lowest position)

$\qquad = W(L + \delta_{max})$

U = strain energy of cord at lowest position

$\qquad = \dfrac{EA\delta_{max}^2}{2L}$

CONSERVATION OF ENERGY

P.E. = $U \qquad W(L + \delta_{max}) = \dfrac{EA\delta_{max}^2}{2L}$

or $\qquad \delta_{max}^2 - \dfrac{2WL}{EA}\delta_{max} - \dfrac{2WL^2}{EA} = 0$

SOLVE QUADRATIC EQUATION FOR δ_{max}:

$$\delta_{max} = \dfrac{WL}{EA} + \left[\left(\dfrac{WL}{EA}\right)^2 + 2L\left(\dfrac{WL}{EA}\right)\right]^{1/2}$$

$$= \dfrac{WL}{EA}\left[1 + \left(1 + \dfrac{2EA}{W}\right)^{1/2}\right]$$

VERTICAL HEIGHT

$$h = C + L + \delta_{max}$$

$$h - C = L + \dfrac{WL}{EA}\left[1 + \left(1 + \dfrac{2EA}{W}\right)^{1/2}\right]$$

SOLVE FOR L:

$$L = \dfrac{h - C}{1 + \dfrac{W}{EA}\left[1 + \left(1 + \dfrac{2EA}{W}\right)^{1/2}\right]} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\dfrac{W}{EA} = \dfrac{539.55 \text{ N}}{2.3 \text{ kN}} = 0.234587$

Numerator = $h - C = 60 \text{ m} - 10 \text{ m} = 50 \text{ m}$

Denominator = $1 + (0.234587)$

$$\times\left[1 + \left(1 + \dfrac{2}{0.234587}\right)^{1/2}\right]$$

$\qquad\qquad = 1.9586$

$L = \dfrac{50 \text{ m}}{1.9586} = 25.5 \text{ m} \quad \leftarrow$

Problem 2.8-13
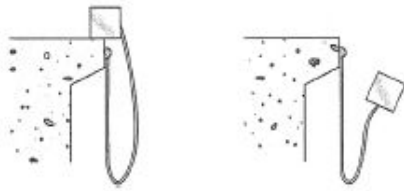
W = Weight

Properties of elastic cord:

E = modulus of elasticity

A = cross-sectional area

L = original length

δ_{max} = elongation of elastic cord

$P.E.$ = potential energy of weight before fall (with respect to lowest position)

$P.E. = W(L + \delta_{max})$

Let U = strain energy of cord at lowest position.

$$U = \frac{EA\delta_{max}^2}{2L}$$

CONSERVATION OF ENERGY

$$P.E. = U \qquad W(L + \delta_{max}) = \frac{EA\delta_{max}^2}{2L}$$

or $\quad \delta_{max}^2 - \dfrac{2WL}{EA}\delta_{max} - \dfrac{2WL^2}{EA} = 0$

SOLVE QUADRATIC EQUATION FOR δ_{max}:

$$\delta_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

STATIC ELONGATION

$$\delta_{st} = \frac{WL}{EA}$$

IMPACT FACTOR

$$\frac{\delta_{max}}{\delta_{st}} = 1 + \left[1 + \frac{2EA}{W}\right]^{1/2} \quad \leftarrow$$

NUMERICAL VALUES

$\delta_{st} = (2.5\%)(L) = 0.025L$

$\delta_{st} = \dfrac{WL}{EA} \qquad \dfrac{W}{EA} = 0.025 \qquad \dfrac{EA}{W} = 40$

Impact factor $= 1 + [1 + 2(40)]^{1/2} = 10 \quad \leftarrow$

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Problem 2.8-14
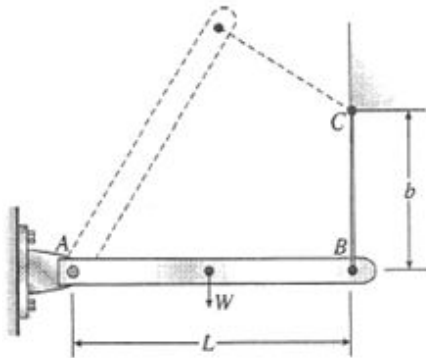
RIGID BAR:

$W = Mg = (1.0 \text{ kg})(9.81 \text{ m/s}^2)$

$\quad = 9.81 \text{ N}$

$L = 0.5 \text{ m}$

NYLON CORD:

$A = 30 \text{ mm}^2$

$b = 0.25 \text{ m}$

$E = 2.1 \text{ GPa}$

Find maximum stress σ_{max} in cord BC.
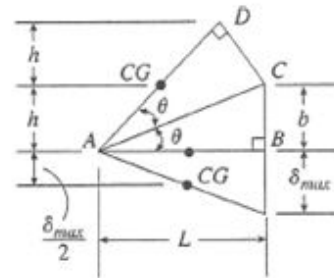
$\overline{CD} = \overline{CB} = b$

$\overline{AD} = \overline{AB} = L$

h = height of center of gravity of raised bar AD

δ_{max} = elongation of cord

From triangle ABC: $\sin \theta = \dfrac{b}{\sqrt{b^2 + L^2}}$

$\cos \theta = \dfrac{L}{\sqrt{b^2 + L^2}}$

From line AD: $\sin 2\theta = \dfrac{2h}{AD} = \dfrac{2h}{L}$

From Appendix C: $\sin 2\theta = 2 \sin \theta \cos \theta$

$\therefore \dfrac{2h}{L} = 2\left(\dfrac{b}{\sqrt{b^2 + L^2}}\right)\left(\dfrac{L}{\sqrt{b^2 + L^2}}\right) = \dfrac{2bL}{b^2 + L^2}$

and $\quad h = \dfrac{bL^2}{b^2 + L^2}$ \quad (Eq. 1)

CONSERVATION OF ENERGY

P.E. = potential energy of raised bar AD

$\quad = W\left(h + \dfrac{\delta_{max}}{2}\right)$

U = strain energy of stretched cord $= \dfrac{EA\delta_{max}^2}{2b}$

P.E. $= U \quad W\left(h + \dfrac{\delta_{max}}{2}\right) = \dfrac{EA\delta_{max}^2}{2b}$ \quad (Eq. 2)

For the cord: $\delta_{max} = \dfrac{\sigma_{max} b}{E}$

Substitute into Eq. (2) and rearrange:

$\sigma_{max}^2 - \dfrac{W}{A}\sigma_{max} - \dfrac{2WhE}{bA} = 0$ \quad (Eq. 3)

Substitute from Eq. (1) into Eq. (3):

$\sigma_{max}^2 - \dfrac{W}{A}\sigma_{max} - \dfrac{2WL^2E}{A(b^2 + L^2)} = 0$ \quad (Eq. 4)

SOLVE FOR σ_{max}:

$\sigma_{max} = \dfrac{W}{2A}\left[1 + \sqrt{1 + \dfrac{8L^2EA}{W(b^2 + L^2)}}\right]$ \quad ←

SUBSTITUTE NUMERICAL VALUES:

$\sigma_{max} = 33.3 \text{ MPa}$ \quad ←

Problem 2.10-1

(a)

(b)
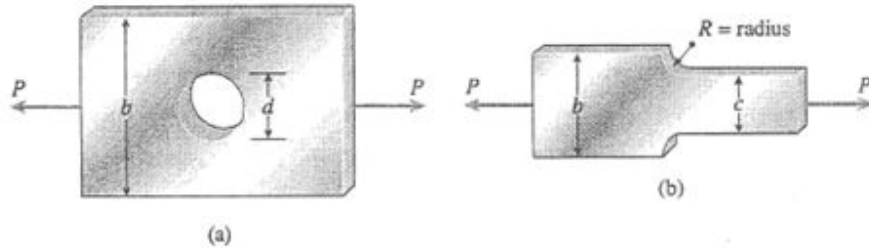
$P = 3.0\,k \qquad t = 0.25$ in.

(a) BAR WITH CIRCULAR HOLE ($b = 6$ in.)

Obtain K from Fig. 2-84

FOR $d = 1$ in.: $\qquad c = b - d = 5$ in.

$$\sigma_{nom} = \frac{P}{ct} = \frac{3.0\,k}{(5\text{ in.})\,(0.25\text{ in.})} = 2.40 \text{ ksi}$$

$d/b = \dfrac{1}{6} \quad K \approx 2.60$

$\sigma_{max} = k\sigma_{nom} \approx 6.2$ ksi $\quad\leftarrow$

FOR $d = 2$ in.: $c = b - d = 4$ in.

$$\sigma_{nom} = \frac{P}{ct} = \frac{3.0\,k}{(4\text{ in.})\,(0.25\text{ in.})} = 3.00 \text{ ksi}$$

$d/b = \dfrac{1}{3} \quad K \approx 2.31$

$\sigma_{max} = K\sigma_{nom} \approx 6.9$ ksi $\quad\leftarrow$

(b) STEPPED BAR WITH SHOULDER FILLETS

$b = 4.0$ in. $\qquad c = 2.5$ in.; Obtain k from Fig. 2-86

$$\sigma_{nom} = \frac{P}{ct} = \frac{3.0\,k}{(2.5\text{ in.})\,(0.25\text{ in.})} = 4.80 \text{ ksi}$$

FOR $R = 0.25$ in.: $R/c = 0.1 \qquad b/c = 1.60$

$k \approx 2.30 \quad \sigma_{max} = K\sigma_{nom} \approx 11.0$ ksi $\quad\leftarrow$

FOR $R = 0.5$ in.: $R/c = 0.2 \qquad b/c = 1.60$

$K \approx 1.87 \qquad \sigma_{max} = K\sigma_{nom} \approx 9.0$ ksi $\quad\leftarrow$

Problem 2.10-2

(a)

(b)
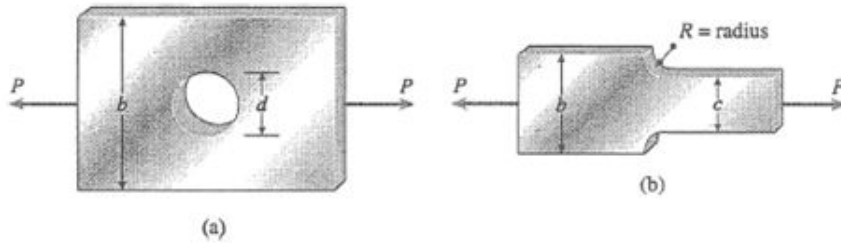
$P = 2.5$ kN $\quad t = 5.0$ mm

(a) BAR WITH CIRCULAR HOLE ($b = 60$ mm)

Obtain K from Fig. 2-84

FOR $d = 12$ mm: $c = b - d = 48$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(48 \text{ mm})(5 \text{ mm})} = 10.42 \text{ MPa}$$

$d/b = \dfrac{1}{5}$ $\quad K \approx 2.51$

$\sigma_{max} = K\sigma_{nom} \approx 26$ MPa $\quad \leftarrow$

FOR $d = 20$ mm: $c = b - d = 40$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(40 \text{ mm})(5 \text{ mm})} = 12.50 \text{ MPa}$$

$d/b = \dfrac{1}{3}$ $\quad K \approx 2.31$

$\sigma_{max} = K\sigma_{nom} \approx 29$ MPa $\quad \leftarrow$

(b) STEPPED BAR WITH SHOULDER FILLETS

$b = 60$ mm $\quad c = 40$ mm;

Obtain K from Fig 2-86

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(40 \text{ mm})(5 \text{ mm})} = 12.50 \text{ MPa}$$

FOR $R = 6$ mm: $R/c = 0.15$ $\quad b/c = 1.5$

$K \approx 2.00$ $\quad \sigma_{max} = K\sigma_{nom} \approx 25$ MPa $\quad \leftarrow$

FOR $R = 10$ mm: $R/c = 0.25$ $\quad b/c = 1.5$

$K \approx 1.75$ $\quad \sigma_{max} = K\sigma_{nom} \approx 22$ MPa $\quad \leftarrow$

Problem 2.10-3

t = thickness

σ_t = allowable tensile stress

Find P_{max}

Find K from Fig. 2-84

$$P_{max} = \sigma_{nom}\,ct = \frac{\sigma_{max}}{K}\,ct = \frac{\sigma_t}{K}(b - d)t$$

$$= \frac{\sigma_t}{K}\,bt\left(1 - \frac{d}{b}\right)$$

Because σ_t, b, and t are constants, we write:

$$P^* = \frac{P_{max}}{\sigma_t bt} = \frac{1}{K}\left(1 - \frac{d}{b}\right)$$

$\dfrac{d}{b}$	K	P^*
0	3.00	0.333
0.1	2.73	0.330
0.2	2.50	0.320
0.3	2.35	0.298
0.4	2.24	0.268

We observe that P_{max} decreases as d/b increases. Therefore, the maximum load occurs when the hole becomes very small.

$$\left(\frac{d}{b} \to 0 \quad \text{and} \quad K \to 3\right)$$

$$P_{max} = \frac{\sigma_t bt}{3} \quad \leftarrow$$

Problem 2.10-4

$d_2 = 26$ mm $d_1 = 20$ mm

P P

L_2 L_1 L_2
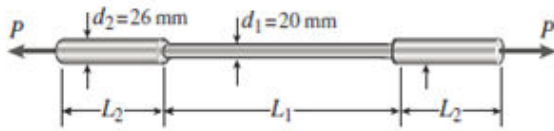
$E = 100$ GPa

$\delta = 0.12$ mm

$L_2 = 0.1$ m

$L_1 = 0.3$ m

$R = $ radius of fillets $= \dfrac{26 \text{ mm} - 20 \text{ mm}}{2} = 3$ mm

$\delta = 2\left(\dfrac{PL_2}{EA_2}\right) + \dfrac{PL_1}{EA_1}$

Solve for P: $P = \dfrac{\delta EA_1A_2}{2L_2A_1 + L_1A_2}$

Use Fig. 2-87 for the stress-concentration factor:

$$\sigma_{nom} = \dfrac{P}{A_1} = \dfrac{\delta EA_2}{2L_2A_1 + L_1A_2} = \dfrac{\delta E}{2L_2\left(\dfrac{A_1}{A_2}\right) + L_1}$$

$$= \dfrac{\delta E}{2L_2\left(\dfrac{d_1}{d_2}\right)^2 + L_1}$$

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_{nom} = \dfrac{(0.12 \text{ mm})(100 \text{ GPa})}{2(0.1 \text{ m})\left(\dfrac{20}{26}\right)^2 + 0.3 \text{ m}} = 28.68 \text{ MPa}$$

$\dfrac{R}{D_1} = \dfrac{3 \text{ mm}}{20 \text{ mm}} = 0.15$

Use the dashed curve in Fig. 2-87. $K \approx 1.6$

$\sigma_{max} = K\sigma_{nom} \approx (1.6)(28.68 \text{ MPa})$

≈ 46 MPa $\leftarrow$

Problem 2.10-5

$d_2 = 1.4$ in. $d_1 = 1.0$ in

$-L_2- \quad -L_1- \quad -L_2-$
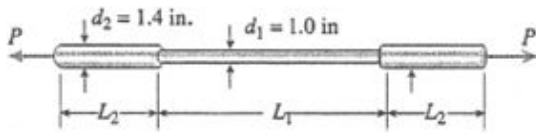
$E = 25 \times 10^6$ psi

$\delta = 0.0040$ in.

$L_1 = 20$ in.

$L_2 = 5$ in.

$R =$ radius of fillets $R = \dfrac{1.4 \text{ in.} - 1.0 \text{ in.}}{2}$

$= 0.2$ in.

$$\delta = 2\left(\frac{PL_2}{EA_2}\right) + \frac{PL_1}{EA_1}$$

Solve for P: $P = \dfrac{\delta EA_1 A_2}{2L_2 A_1 + L_1 A_2}$

Use Fig. 2-87 for the stress-concentration factor.

$$\sigma_{\text{nom}} = \frac{P}{A_1} = \frac{\delta EA_2}{2L_2 A_1 + L_1 A_2} = \frac{\delta E}{2L_2\left(\dfrac{A_1}{A_2}\right) + L_1}$$

$$= \frac{\delta E}{2L_2\left(\dfrac{d_1}{d_2}\right)^2 + L_1}$$

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_{\text{nom}} = \frac{(0.0040 \text{ in.})(25 \times 10^6 \text{ psi})}{2(5 \text{ in.})\left(\dfrac{1.0}{1.4}\right)^2 + 20 \text{ in.}} = 3{,}984 \text{ psi}$$

$$\frac{R}{D_1} = \frac{0.2 \text{ in.}}{1.0 \text{ in.}} = 0.2$$

Use the dashed curve in Fig. 2-87. $K \approx 1.53$

$\sigma_{\text{max}} = K\sigma_{\text{nom}} \approx (1.53)(3984 \text{ psi})$

$\approx 6100 \text{ psi} \quad \leftarrow$

Problem 2.10-6
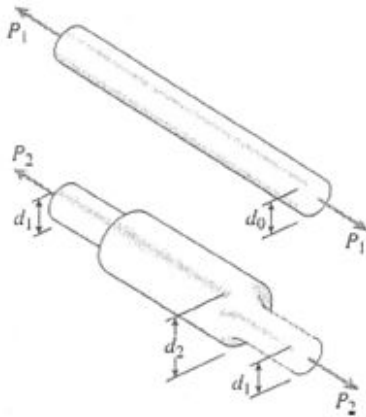
$d_0 = 20$ mm

$d_1 = 20$ mm

$d_2 = 25$ mm

Fillet radius: $R = 2$ mm

Allowable stress: $\sigma_t = 80$ MPa

(a) COMPARISON OF BARS

Prismatic bar: $P_1 = \sigma_t A_0 = \sigma_t \left(\dfrac{\pi d_0^2}{4} \right)$

$= (80 \text{ MPa})\left(\dfrac{\pi}{4} \right)(20 \text{mm})^2 = 25.1 \text{ kN}$ ←

Stepped bar: See Fig. 2-87 for the stress-concentration factor.

$R = 2.0$ mm	$D_1 = 20$ mm	$D_2 = 25$ mm
$R/D_1 = 0.10$	$D_2/D_1 = 1.25$	$K \approx 1.75$

$\sigma_{nom} = \dfrac{P_2}{\dfrac{\pi}{4}d_1^2} = \dfrac{P_2}{A_1}$ $\sigma_{nom} = \dfrac{\sigma_{max}}{K}$

$P_2 = \sigma_{nom} A_1 = \dfrac{\sigma_{max}}{K}A_1 = \dfrac{\sigma_t}{K}A_1$

$= \left(\dfrac{80 \text{ MPa}}{1.75} \right)\left(\dfrac{\pi}{4} \right)(20 \text{ mm})^2$

$\approx 14.4 \text{ kN}$ ←

Enlarging the bar makes it *weaker*, not stronger. The ratio of loads is $P_1/P_2 = K = 1.75$

(b) DIAMETER OF PRISMATIC BAR FOR THE SAME ALLOWABLE LOAD

$P_1 = P_2 \quad \sigma_t \left(\dfrac{\pi d_0^2}{4} \right) = \dfrac{\sigma_t}{K}\left(\dfrac{\pi d_1^2}{4} \right) \quad d_0^2 = \dfrac{d_1^2}{K}$

$d_0 = \dfrac{d_1}{\sqrt{K}} \approx \dfrac{20 \text{ mm}}{\sqrt{1.75}} \approx 15.1 \text{ mm}$ ←

Problem 2.10-7

$b = 2.4$ in.

$c = 1.6$ in.

Fillet radius: $R = 0.2$ in.

Find d_{max}

BASED UPON FILLETS (Use Fig. 2-86)

$b = 2.4$ in. $c = 1.6$ in. $R = 0.2$ in.

$R/c = 0.125$ $b/c = 1.5$ $K \approx 2.10$

$P_{max} = \sigma_{nom} ct = \dfrac{\sigma_{max}}{K} ct = \dfrac{\sigma_{max}}{K}\left(\dfrac{c}{b}\right)(bt)$

$\approx 0.317\, bt\, \sigma_{max}$

BASED UPON HOLE (Use Fig. 2-84)

$b = 2.4$ in. $d =$ diameter of the hole (in.)

$c_1 = b - d$

$P_{max} = \sigma_{nom}\, c_1 t = \dfrac{\sigma_{max}}{K}(b - d)t$

$= \dfrac{1}{K}\left(1 - \dfrac{d}{b}\right) bt\sigma_{max}$

d(in.)	d/b	K	$P_{max}/bt\sigma_{max}$
0.3	0.125	2.66	0.329
0.4	0.167	2.57	0.324
0.5	0.208	2.49	0.318
0.6	0.250	2.41	0.311
0.7	0.292	2.37	0.299
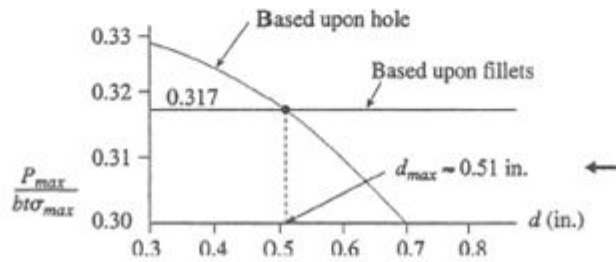
Problem 2.11-1
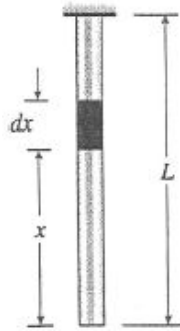
Let A = cross-sectional area

Let N = axial force at distance x

$N = \gamma A x$

$\sigma = \dfrac{N}{A} = \gamma x$

STRAIN AT DISTANCE x

$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma_0 \alpha}{E}\left(\frac{\sigma}{\sigma_0}\right)^m = \frac{\gamma x}{E} + \frac{\sigma_0}{\alpha E}\left(\frac{\gamma x}{\sigma_0}\right)^m$$

ELONGATION OF BAR

$$\delta = \int_0^L \varepsilon \, dx = \int_0^L \frac{\gamma x}{E} dx + \frac{\sigma_0 \alpha}{E}\int_0^L \left(\frac{\gamma x}{\sigma_0}\right)^m dx$$

$$= \frac{\gamma L^2}{2E} + \frac{\sigma_0 \alpha L}{(m+1)E}\left(\frac{\gamma L}{\sigma_0}\right)^m \qquad \text{Q.E.D.} \quad \leftarrow$$

Problem 2.11-2
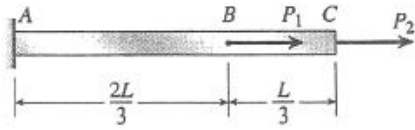
$L = 1.8$ m $\quad$ $A = 480$ mm^2

$P_1 = 30$ kN $\quad$ $P_2 = 60$ kN

Ramberg–Osgood equation:

$$\varepsilon = \frac{\sigma}{45{,}000} + \frac{1}{618}\left(\frac{\sigma}{170}\right)^{10} \quad (\sigma = \text{MPa})$$

Find displacement at end of bar.

(a) P_1 ACTS ALONE

$$AB: \sigma = \frac{P_1}{A} = \frac{30 \text{ kN}}{480 \text{ mm}^2} = 62.5 \text{ MPa}$$

$$\varepsilon = 0.001389$$

$$\delta_c = \varepsilon\left(\frac{2L}{3}\right) = 1.67 \text{ mm} \quad \leftarrow$$

(b) P_2 ACTS ALONE

$$ABC: \sigma = \frac{P_2}{A} = \frac{60 \text{ kN}}{480 \text{ mm}^2} = 125 \text{ MPa}$$

$$\varepsilon = 0.002853$$

$$\delta_c = \varepsilon L = 5.13 \text{ mm} \quad \leftarrow$$

(c) BOTH P_1 AND P_2 ARE ACTING

$$AB: \sigma = \frac{P_1 + P_2}{A} = \frac{90 \text{ kN}}{480 \text{ mm}^2} = 187.5 \text{ MPa}$$

$$\varepsilon = 0.008477$$

$$\delta_{AB} = \varepsilon\left(\frac{2L}{3}\right) = 10.17 \text{ mm}$$

$$BC: \sigma = \frac{P_2}{A} = \frac{60 \text{ kN}}{480 \text{ mm}^2} = 125 \text{ MPa}$$

$$\varepsilon = 0.002853$$

$$\delta_{BC} = \varepsilon\left(\frac{L}{3}\right) = 1.71 \text{ mm}$$

$$\delta_C = \delta_{AB} + \delta_{BC} = 11.88 \text{ mm} \quad \leftarrow$$

(Note that the displacement when both loads act simultaneously is *not* equal to the sum of the displacements when the loads act separately.)

Problem 2.11-3
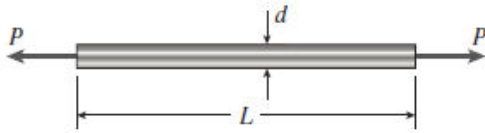
$L = 32$ in. $d = 0.75$ in.

$A = \dfrac{\pi d^2}{4} = 0.4418$ in.^{2}

(a) STRESS-STRAIN DIAGRAM

$$\sigma = \frac{18{,}000\varepsilon}{1 + 300\varepsilon} \quad 0 \le \varepsilon \le 0.03 \quad (\sigma = \text{ksi})$$
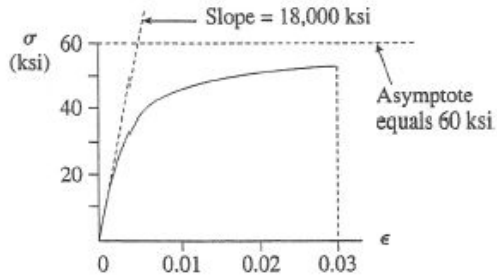
(b) ALLOWABLE LOAD P

Maximum elongation $\delta_{max} = 0.25$ in.

Maximum stress $\sigma_{max} = 40$ ksi

Based upon elongation:

$$\varepsilon_{max} = \frac{\delta_{max}}{L} = \frac{0.25 \text{ in.}}{32 \text{ in.}} = 0.007813$$

$$\sigma_{max} = \frac{18{,}000\varepsilon_{max}}{1 + 300\varepsilon_{max}} = 42.06 \text{ ksi}$$

BASED UPON STRESS:

$\sigma_{max} = 40$ ksi

Stress governs. $P = \sigma_{max} A = (40 \text{ ksi})(0.4418 \text{ in.}^2)$

$\qquad\qquad\qquad = 17.7 \text{ k} \quad \leftarrow$

Problem 2.11-4
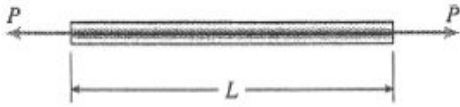
$L = 2.0$ m

$A = 249$ mm^2

STRESS-STRAIN DIAGRAM
(See the problem statement for the diagram)

LOAD-DISPLACEMENT DIAGRAM

P (kN)	$\sigma = P/A$ (MPa)	ε (from diagram)	$\delta = \varepsilon L$ (mm)
10	40	0.0009	1.8
20	80	0.0018	3.6
30	120	0.0031	6.2
40	161	0.0060	12.0
45	181	0.0081	16.2
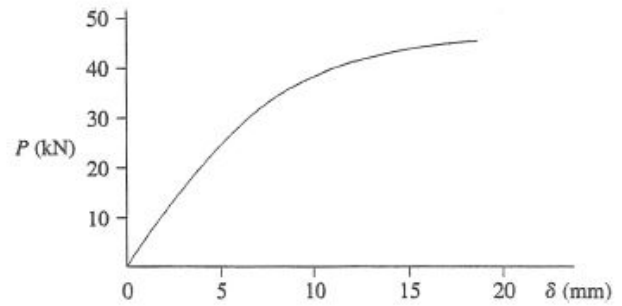
NOTE: The load-displacement curve has the same shape as the stress-strain curve.

Problem 2.11-5
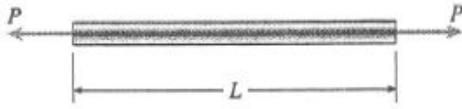
$L = 150$ in.

$A = 2.0$ in.^{2}

STRESS-STRAIN DIAGRAM

LOAD-DISPLACEMENT DIAGRAM

P (k)	$\sigma = P/A$ (psi)	ε (from Eq. 1 or Eq. 2)	$\delta = \varepsilon L$ (in.)
8	4,000	0.00040	0.060
16	8,000	0.00080	0.120
24	12,000	0.00120	0.180
32	16,000	0.00287	0.430
40	20,000	0.00453	0.680
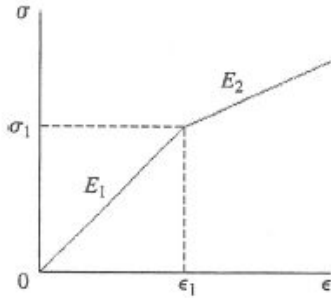
$E_1 = 10 \times 10^6$ psi

$E_2 = 2.4 \times 10^6$ psi

$\sigma_1 = 12,000$ psi

$$\varepsilon_1 = \frac{\sigma_1}{E_1} = \frac{12,000 \text{ psi}}{10 \times 10^6 \text{ psi}}$$

$$= 0.0012$$

For $0 \leq \sigma \leq \sigma_1$:

$$\varepsilon = \frac{\sigma}{E_1} = \frac{\sigma}{10 \times 10^6 \text{psi}} \quad (\sigma = \text{psi}) \quad \text{Eq. (1)}$$

For $\sigma \geq \sigma_1$:

$$\varepsilon = \varepsilon_1 + \frac{\sigma - \sigma_1}{E_2} = 0.0012 + \frac{\sigma - 12,000}{2.4 \times 10^6}$$

$$= \frac{\sigma}{2.4 \times 10^6} - 0.0038 \quad (\sigma = \text{psi}) \quad \text{Eq. (2)}$$
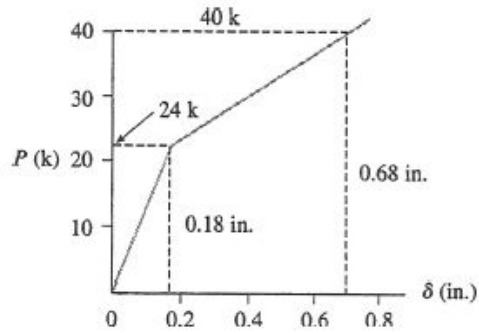
Problem 2.11-6
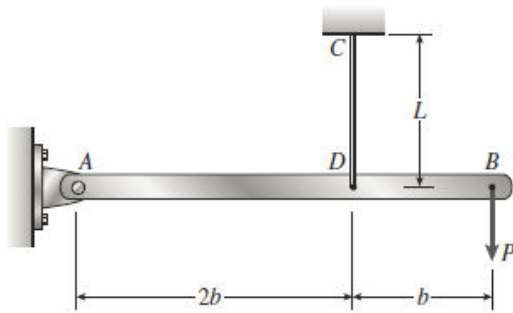
Wire: $E = 210$ GPa

$\sigma_Y = 820$ MPa

$L = 1.0$ m

$d = 3$ mm

$A = \dfrac{\pi d^2}{4} = 7.0686$ mm^2

STRESS-STRAIN DIAGRAM

$$\sigma = E\varepsilon \qquad (0 \leq \sigma \leq \sigma_Y) \qquad (1)$$

$$\sigma = \sigma_Y \left(\frac{E\varepsilon}{\sigma_Y}\right)^n \qquad (\sigma \geq \sigma_Y) \qquad (n = 0.2) \qquad (2)$$

(a) DISPLACEMENT δ_B AT END OF BAR

$$\delta = \text{elongation of wire } \delta_B = \frac{3}{2}\delta = \frac{3}{2}\varepsilon L \qquad (3)$$

Obtain ε from stress-strain equations:

From Eq. (1): $\varepsilon = \dfrac{\sigma E}{(0 \leq \sigma \leq \sigma_Y)}$ (4)

From Eq. (2): $\varepsilon = \dfrac{\sigma_Y}{E}\left(\dfrac{\sigma}{\sigma_Y}\right)^{1/n}$ (5)

Axial force in wire: $F = \dfrac{3P}{2}$

Stress in wire: $\sigma = \dfrac{F}{A} = \dfrac{3P}{2A}$ (6)

PROCEDURE: Assume a value of P
  Calculate σ from Eq. (6)
  Calculate ε from Eq. (4) or (5)
  Calculate δ_B from Eq. (3)

P (kN)	σ (MPa) Eq. (6)	ε Eq. (4) or (5)	δ_B (mm) Eq. (3)
2.4	509.3	0.002425	3.64
3.2	679.1	0.003234	4.85
4.0	848.8	0.004640	6.96
4.8	1018.6	0.01155	17.3
5.6	1188.4	0.02497	37.5

For $\sigma = \sigma_Y = 820$ MPa:

$\varepsilon = 0.0039048$ $P = 3.864$ kN $\delta_B = 5.86$ mm

(b) LOAD-DISPLACEMENT DIAGRAM
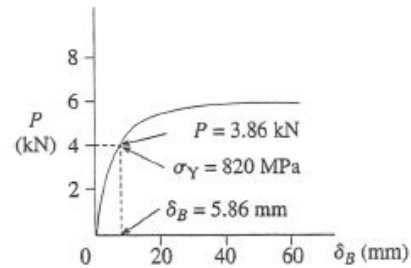
Problem 2.12-1
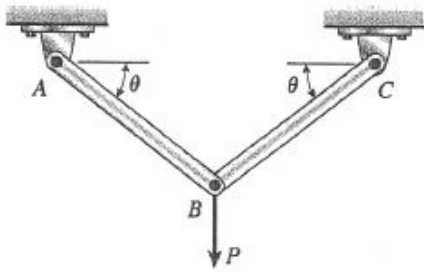
Structure is statically determinate. The yield load P_Y and the plastic lead P_P occur at the same time, namely, when both bars reach the yield stress.
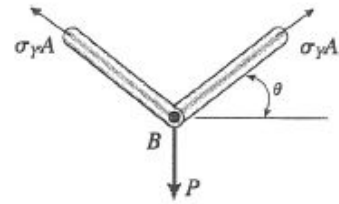
JOINT B

$\Sigma F_{\text{vert}} = 0$

$(2\sigma_Y A) \sin \theta = P$

$P_Y = P_P = 2\sigma_Y A \sin \theta$ ←

Problem 2.12-2
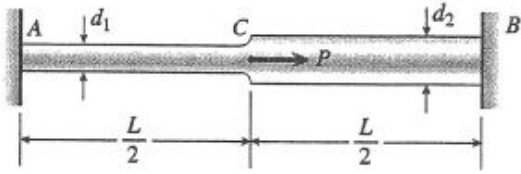
$d_1 = 20$ mm $d_2 = 25$ mm $\sigma_Y = 250$ MPa

DETERMINE THE PLASTIC LOAD P_P:

At the plastic load, all parts of the bar are stressed to the yield stress.

Point C:
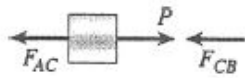
$F_{AC} = \sigma_Y A_1$ $F_{CB} = \sigma_Y A_2$

$$P = F_{AC} + F_{CB}$$

$$P_P = \sigma_Y A_1 + \sigma_Y A_2 = \sigma_Y (A_1 + A_2) \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$P_P = (250 \text{ MPa})\left(\frac{\pi}{4}\right)(d_1^2 + d_2^2)$$

$$= (250 \text{ MPa})\left(\frac{\pi}{4}\right)[(20 \text{ mm})^2 + (25 \text{ mm})^2]$$

$$= 201 \text{ kN} \quad \leftarrow$$

Problem 2.12-3
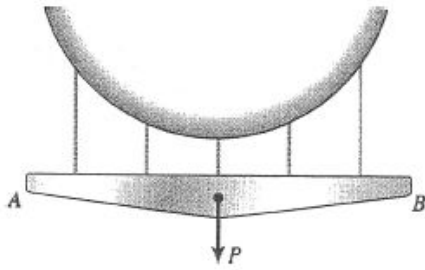
(a) Plastic load P_P
At the plastic load, each wire is stressed to the yield stress. $\therefore P_P = 5\sigma_Y A$ ←

$F = \sigma_Y A$
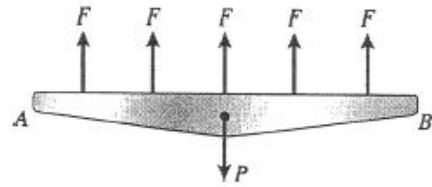
(b) Bar AB is flexible
At the plastic load, each wire is stressed to the yield stress, so the plastic load is not changed. ←

(c) Radius R is increased
Again, the forces in the wires are not changed, so the plastic load is not changed. ←

Problem 2.12-4
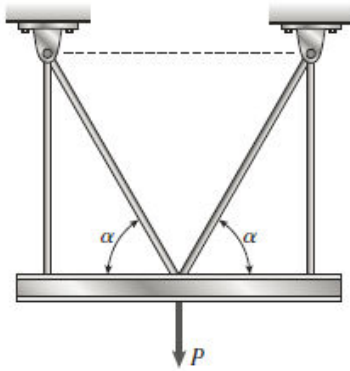
At the plastic load, all four rods are stressed to the yield stress.

$F = \sigma_Y A$
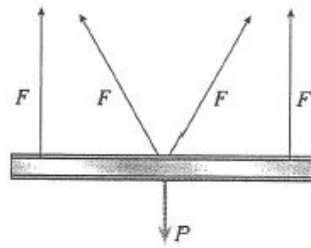
Sum forces in the vertical direction and solve for the load:

$P_P = 2F + 2F \sin \alpha$

$P_P = 2\sigma_Y A (1 + \sin \alpha) \quad \leftarrow$

Problem 2.12-5
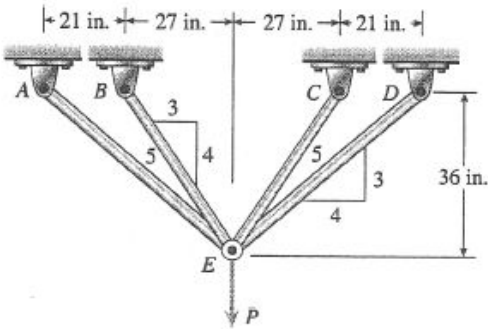
$L_{AE} = 60$ in. $L_{BE} = 45$ in.

JOINT E
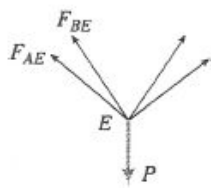
Equilibrium:

$$2F_{AE}\left(\frac{3}{5}\right) + 2F_{BE}\left(\frac{4}{5}\right) = P$$

or

$$P = \frac{6}{5}F_{AE} + \frac{8}{5}F_{BE}$$

PLASTIC LOAD P_P

At the plastic load, all bars are stressed to the yield stress.

$$F_{AE} = \sigma_Y A_{AE} \qquad F_{BE} = \sigma_Y A_{BE}$$

$$P_P = \frac{6}{5}\sigma_Y A_{AE} + \frac{8}{5}\sigma_Y A_{BE} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$A_{AE} = 0.307 \text{ in.}^2 \quad A_{BE} = 0.601 \text{ in.}^2$$

$$\sigma_Y = 36 \text{ ksi}$$

$$P_P = \frac{6}{5}(36 \text{ ksi})(0.307 \text{ in.}^2) + \frac{8}{5}(36 \text{ ksi})(0.601 \text{ in.}^2)$$

$$= 13.26 \text{ k} + 34.62 \text{ k} = 47.9 \text{ k} \quad \leftarrow$$

Problem 2.12-6
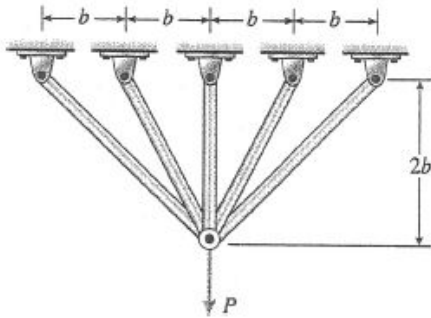
$d = 10$ mm

$A = \dfrac{\pi d^2}{4} = 78.54$ mm^2

$\sigma_Y = 250$ MPa
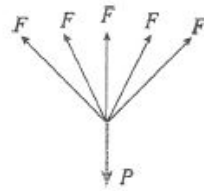
At the plastic load, all five bars are stressed to the yield stress

$F = \sigma_Y A$

Sum forces in the vertical direction and solve for the load:

$$P_P = 2F\left(\dfrac{1}{\sqrt{2}}\right) + 2F\left(\dfrac{2}{\sqrt{5}}\right) + F$$

$$= \dfrac{\sigma_Y A}{5}(5\sqrt{2} + 4\sqrt{5} + 5)$$

$$= 4.2031\sigma_Y A \quad \leftarrow$$

Substitute numerical values:

$$P_P = (4.2031)(250 \text{ MPa})(78.54 \text{ mm}^2)$$

$$= 82.5 \text{ kN} \quad \leftarrow$$

Problem 2.12-7
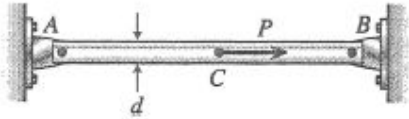
$d = 0.6$ in.

$\sigma_Y = 36$ ksi

Initial tensile stress $= 10$ ksi

(a) PLASTIC LOAD P_P

The presence of the initial tensile stress does not affect the plastic load. Both parts of the bar must yield in order to reach the plastic load.

POINT C:
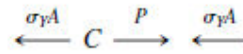
$$P_P = 2\sigma_Y A = (2)\,(36 \text{ ksi})\left(\frac{\pi}{4}\right)(0.60 \text{ in.})^2$$

$$= 20.4 \text{ k} \quad \leftarrow$$
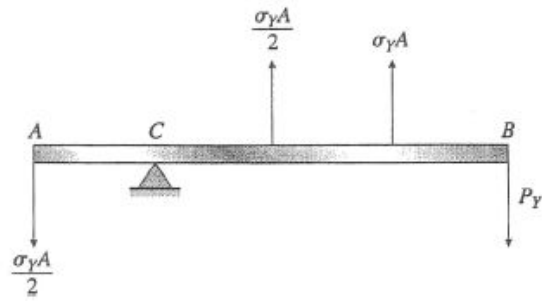
(B) INITIAL TENSILE STRESS IS DOUBLED

P_P is not changed. $\quad \leftarrow$

Problem 2.12-8
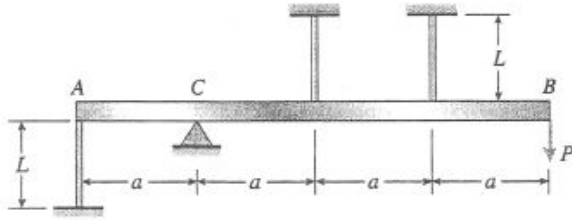
(a) YIELD LOAD P_Y

Yielding occurs when the most highly stressed wire reaches the yield stress σ_Y

$$\Sigma M_C = 0$$

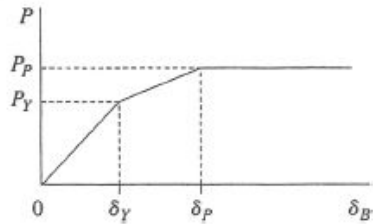$$P_Y = \sigma_Y A \quad \leftarrow$$

At point A:

$$\delta_A = \left(\frac{\sigma_Y A}{2}\right)\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{2E}$$

At point B:

$$\delta_B = 3\delta_A = \delta_Y = \frac{3\sigma_Y L}{2E} \quad \leftarrow$$

(b) PLASTIC LOAD P_P
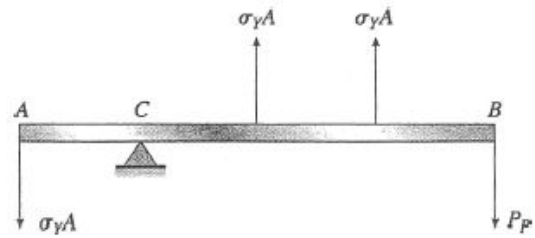
At the plastic load, all wires reach the yield stress.

$$\Sigma M_C = 0$$

$$P_P = \frac{4\sigma_Y A}{3} \quad \leftarrow$$

At point A:

$$\delta_A = (\sigma_Y A)\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{E}$$

At point B:

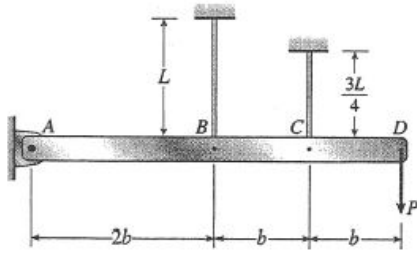$$\delta_B = 3\delta_A = \delta_P = \frac{3\sigma_Y L}{E} \quad \leftarrow$$

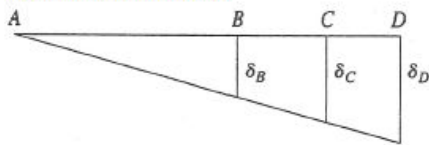(c) LOAD-DISPLACEMENT DIAGRAM

$$P_P = \frac{4}{3}P_Y$$

$$\delta_P = 2\delta_Y$$

Problem 2.12-9

A = cross-sectional area

σ_Y = yield stress
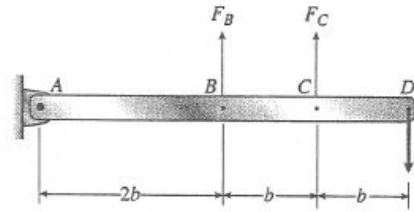
E = modulus of elasticity

DISPLACEMENT DIAGRAM

COMPATIBILITY:

$$\delta_C = \frac{3}{2}\delta_B \tag{1}$$

$$\delta_D = 2\delta_B \tag{2}$$

FREE-BODY DIAGRAM

EQUILIBRIUM:

$$\Sigma M_A = 0 \quad \curvearrowleft \quad F_B(2b) + F_C(3b) = P(4b)$$
$$2F_B + 3F_C = 4P \tag{3}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{F_B L}{EA} \qquad \delta_C = \frac{F_C\left(\frac{3}{4}L\right)}{EA} \tag{4, 5}$$

Substitute into Eq. (1):

$$\frac{3F_C L}{4EA} = \frac{3F_B L}{2EA}$$

$$F_C = 2F_B \tag{6}$$

STRESSES

$$\sigma_B = \frac{F_B}{A} \qquad \sigma_C = \frac{F_C}{A} \qquad \sigma_C = 2\sigma_B \tag{7}$$

Wire C has the larger stress. Therefore, it will yield first.

(a) YIELD LOAD

$$\sigma_C = \sigma_Y \qquad \sigma_B = \frac{\sigma_C}{2} = \frac{\sigma_Y}{2} \qquad \text{(From Eq. 7)}$$

$$F_C = \sigma_Y A \qquad F_B = \frac{1}{2}\sigma_Y A$$

From Eq. (3):

$$2\left(\frac{1}{2}\sigma_Y A\right) + 3(\sigma_Y A) = 4P$$

$$P = P_Y = \sigma_Y A \quad \leftarrow$$

From Eq. (4):

$$\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{2E}$$

From Eq. (2):

$$\delta_D = \delta_Y = 2\delta_B = \frac{\sigma_Y L}{E} \quad \leftarrow$$

(b) PLASTIC LOAD
 At the plastic load, both wires yield.

$$\sigma_B = \sigma_Y = \sigma_C \qquad F_B = F_C = \sigma_Y A$$

From Eq. (3):

$$2(\sigma_Y A) + 3(\sigma_Y A) = 4P$$

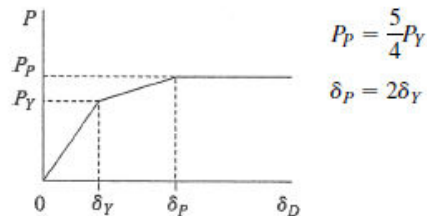$$P = P_P = \frac{5}{4}\sigma_Y A \quad \leftarrow$$

From Eq. (4):

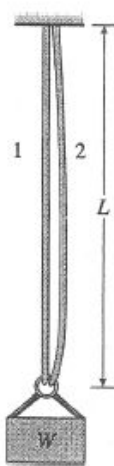$$\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{E}$$

From Eq. (2):

$$\delta_D = \delta_P = 2\delta_B = \frac{2\sigma_Y L}{E} \quad \leftarrow$$

(c) LOAD-DISPLACEMENT DIAGRAM

$$P_P = \frac{5}{4}P_Y$$

$$\delta_P = 2\delta_Y$$

Problem 2.12-10

$L = 40$ m $A = 48.0$ mm^2

$E = 160$ GPa

d = difference in length = 100 mm

$\sigma_Y = 500$ MPa

INITIAL STRETCHING OF CABLE 1

Initially, cable 1 supports all of the load. Let W_1 = load required to stretch cable 1 to the same length as cable 2

$$W_1 = \frac{EA}{L}d = 19.2 \text{ kN}$$

$\delta_1 = 100$ mm (elongation of cable 1)

$$\sigma_1 = \frac{W_1}{A} = \frac{Ed}{L} = 400 \text{ MPa} \ (\sigma_1 < \sigma_Y \therefore \ > \text{OK})$$

(a) YIELD LOAD W_Y

Cable 1 yields first. $F_1 = \sigma_Y A = 24$ kN

δ_{1Y} = total elongation of cable 1

δ_{1Y} = total elongation of cable 1

$$\delta_{1Y} = \frac{F_1 L}{EA} = \frac{\sigma_Y L}{E} = 0.125 \text{ m} = 125 \text{ mm}$$

$\delta_Y = \delta_{1Y} = 125$ mm   ←

δ_{2Y} = elongation of cable 2

$\quad = \delta_{1Y} - d = 25$ mm

$$F_2 = \frac{EA}{L}\delta_{2Y} = 4.8 \text{ kN}$$

$W_Y = F_1 + F_2 = 24 \text{ kN} + 4.8 \text{ kN}$

$\quad = 28.8$ kN   ←

(b) PLASTIC LOAD W_P

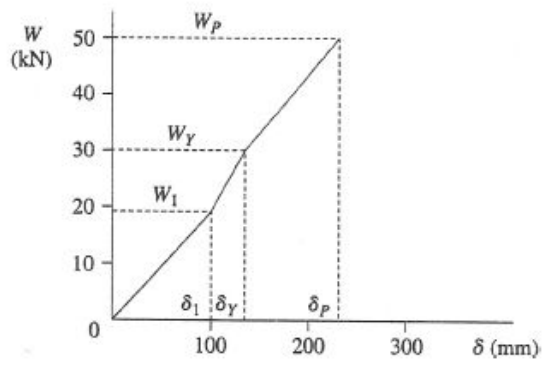$F_1 = \sigma_Y A$ $F_2 = \sigma_Y A$

$W_P = 2\sigma_Y A = 48$ kN   ←

δ_{2P} = elongation of cable 2

$$= F_2\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{E} = 0.125 \text{ mm} = 125 \text{ mm}$$

$\delta_{1P} = \delta_{2P} + d = 225$ mm

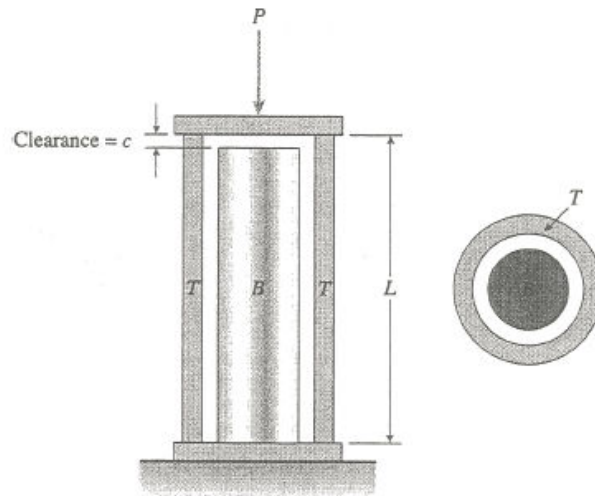$\delta_P = \delta_{1P} = 225$ mm   ←

(c) LOAD-DISPLACEMENT DIAGRAM

$\dfrac{W_Y}{W_1} = 1.5$ $\dfrac{\delta_Y}{\delta_1} = 1.25$

$\dfrac{W_P}{W_Y} = 1.667$ $\dfrac{\delta_P}{\delta_Y} = 1.8$

$0 < W < W_1$: slope = 192,000 N/m

$W_1 < W < W_Y$: slope = 384,000 N/m

$W_Y < W < W_P$: slope = 192,000 N/m

Problem 2.12-11

$L = 15$ in.

$c = 0.010$ in.

$E = 29 \times 10^3$ ksi

$\sigma_Y = 36$ ksi

TUBE:

$d_2 = 3.0$ in.

$d_1 = 2.75$ in.

$A_T = \dfrac{\pi}{4}(d_2^2 - d_1^2) = 1.1290$ in.^{2}

BAR

$d = 1.5$ in.

$A_B = \dfrac{\pi d^2}{4} = 1.7671$ in.^{2}

INITIAL SHORTENING OF TUBE T

Initially, the tube supports all of the load.

Let P_1 = load required to close the clearance

$P_1 = \dfrac{EA_T}{L} c = 21,827$ lb

Let δ_1 = shortening of tube $\delta_1 = c = 0.010$ in.

$\sigma_1 = \dfrac{P_1}{A_T} = 19,330$ psi   ($\sigma_1 < \sigma_Y$ ∴ OK)

(a) YIELD LOAD P_Y

Because the tube and bar are made of the same material, and because the strain in the tube is larger than the strain in the bar, the tube will yield first.

$F_T = \sigma_Y A_T = 40,644$ lb

δ_{TY} = shortening of tube at the yield stress

$\sigma_{TY} = \dfrac{F_T L}{EA_T} = \dfrac{\sigma_Y L}{E} = 0.018621$ in.

$\delta_Y = \delta_{TY} = 0.018621$ in.   ←

(b) PLASTIC LOAD P_P

$F_T = \sigma_Y A_T$ $F_B = \sigma_Y A_B$

$P_P = F_T + F_B = \sigma_Y(A_T + A_B)$

$\qquad = 104,300$ lb   ←

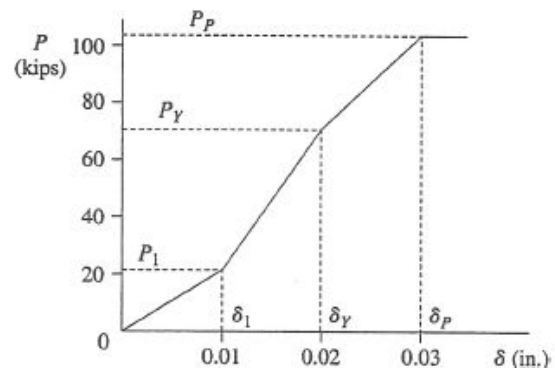δ_{BP} = shortening of bar

$\qquad = F_B\left(\dfrac{L}{EA_B}\right) = \dfrac{\sigma_Y L}{E} = 0.018621$ in.

$\delta_{TP} = \delta_{BP} + c = 0.028621$ in.

$\delta_P = \delta_{TP} = 0.02862$ in.   ←

(c) LOAD-DISPLACEMENT DIAGRAM

Part (c) continued

δ_{BY} = shortening of bar

$= \delta_{TY} - c = 0.008621$ in.

$F_B = \dfrac{EA_B}{L}\delta_{BY} = 29{,}453$ lb

$P_Y = F_T + F_B = 40{,}644$ lb $+ 29{,}453$ lb

$= 70{,}097$ lb

$P_Y = 70{,}100$ lb $\leftarrow$

$\dfrac{P_Y}{P_1} = 3.21 \quad \dfrac{\delta_Y}{\delta_1} = 1.86$

$\dfrac{P_P}{P_Y} = 1.49 \quad \dfrac{\delta_P}{\delta_Y} = 1.54$

$0 < P < P_1$: slope $= 2180$ k/in.

$P_1 < P < P_Y$: slope $= 5600$ k/in.

$P_Y < P < P_P$: slope $= 3420$ k/in.