3–1. A tension test was performed on a steel specimen having an original diameter of 0.503 in. and gauge length of 2.00 in. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the yield stress, the ultimate stress, and the rupture stress. Use a scale of 1 in. = 20 ksi and 1 in. = 0.05 in./in. Redraw the elastic region, using the same stress scale but a strain scale of 1 in. = 0.001 in./in.

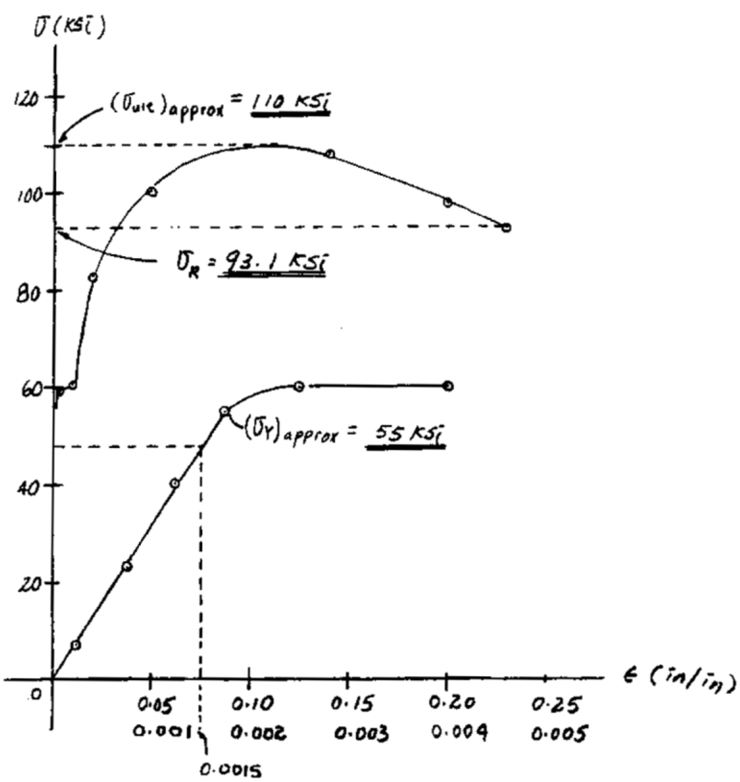
Load (kip)	Elongation (in.)
0	0
1.50	0.0005
4.60	0.0015
8.00	0.0025
11.00	0.0035
11.80	0.0050
11.80	0.0080
12.00	0.0200
16.60	0.0400
20.00	0.1000
21.50	0.2800
19.50	0.4000
18.50	0.4600

$$A = \frac{1}{4}\pi(0.503)^2 = 0.1987 \text{ in}^2$$

$L = 2.00$ in.

σ(ksi)	ε(in./in.)
0	0
7.55	0.00025
23.15	0.00075
40.26	0.00125
55.36	0.00175
59.38	0.0025
59.38	0.0040
60.39	0.010
83.54	0.020
100.65	0.050
108.20	0.140
98.13	0.200
93.10	0.230

$$E_{\text{approx}} = \frac{48}{0.0015} = 32.0(10^3) \text{ ksi}$$ **Ans.**



Ans:
$(\sigma_{\text{ult}})_{\text{approx}} = 110$ ksi, $(\sigma_R)_{\text{approx}} = 93.1$ ksi, $(\sigma_Y)_{\text{approx}} = 55$ ksi, $E_{\text{approx}} = 32.0(10^3)$ ksi

3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

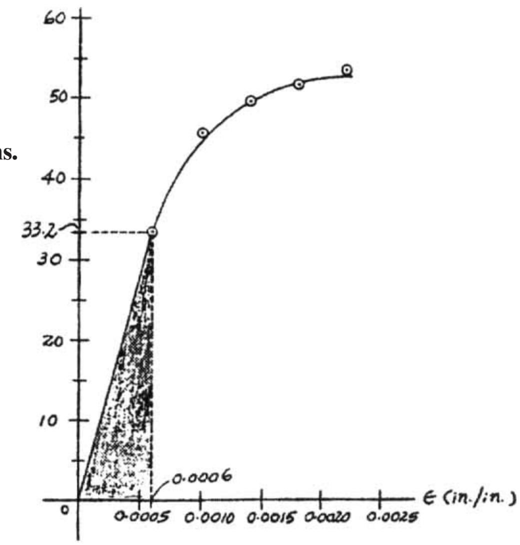
σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

Modulus of Elasticity: From the stress–strain diagram

$$E = \frac{33.2 - 0}{0.0006 - 0} = 55.3\left(10^3\right) \text{ ksi} \qquad \textbf{Ans.}$$

Modulus of Resilience: The modulus of resilience is equal to the area under the *linear portion* of the stress–strain diagram (shown shaded).

$$u_r = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.0006\,\frac{\text{in.}}{\text{in.}}\right) = 9.96\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



Ans:
$E = 55.3\left(10^3\right)$ ksi, $u_r = 9.96\,\dfrac{\text{in}\cdot\text{lb}}{\text{in}^3}$

3–3. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The rupture stress is $\sigma_r = 53.4$ ksi.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

Modulus of Toughness: The modulus of toughness is equal to the area under the stress-strain diagram (shown shaded).

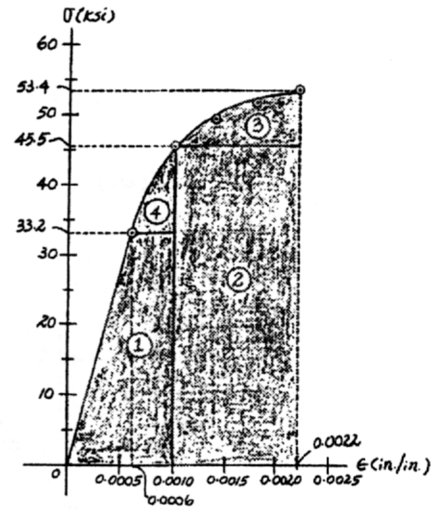
$$(u_t)_{\text{approx}} = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004 + 0.0010)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+\ 45.5\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+\ \frac{1}{2}(7.90)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+\ \frac{1}{2}(12.3)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$= 85.0\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



Ans:
$(u_t)_{\text{approx}} = 85.0\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3}$

***3–4.** A tension test was performed on a steel specimen having an original diameter of 0.503 in. and a gauge length of 2.00 in. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the ultimate stress, and the rupture stress. Use a scale of 1 in. = 15 ksi and 1 in. = 0.05 in./in. Redraw the linear-elastic region, using the same stress scale but a strain scale of 1 in. = 0.001 in.

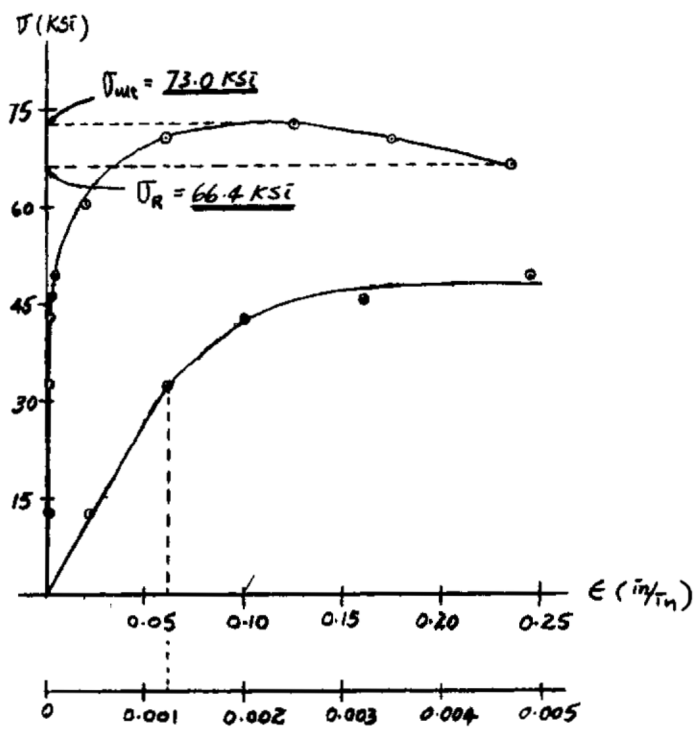
Load (kip)	Elongation (in.)
0	0
2.50	0.0009
6.50	0.0025
8.50	0.0040
9.20	0.0065
9.80	0.0098
12.0	0.0400
14.0	0.1200
14.5	0.2500
14.0	0.3500
13.2	0.4700

$$A = \frac{1}{4}\pi(0.503)^2 = 0.19871 \text{ in}^2$$

$$L = 2.00 \text{ in.}$$

$\sigma = \frac{P}{A}$(ksi)	$\epsilon = \frac{\Delta L}{L}$(in./in.)
0	0
12.58	0.00045
32.71	0.00125
42.78	0.0020
46.30	0.00325
49.32	0.0049
60.39	0.02
70.45	0.06
72.97	0.125
70.45	0.175
66.43	0.235

$$E_{\text{approx}} = \frac{32.71}{0.00125} = 26.2(10^3) \text{ ksi}$$ **Ans.**



3–5. A tension test was performed on a steel specimen having an original diameter of 0.503 in. and gauge length of 2.00 in. Using the data listed in the table, plot the stress–strain diagram and determine approximately the modulus of toughness.

Load (kip)	Elongation (in.)
0	0
2.50	0.0009
6.50	0.0025
8.50	0.0040
9.20	0.0065
9.80	0.0098
12.0	0.0400
14.0	0.1200
14.5	0.2500
14.0	0.3500
13.2	0.4700

Modulus of toughness (approx)

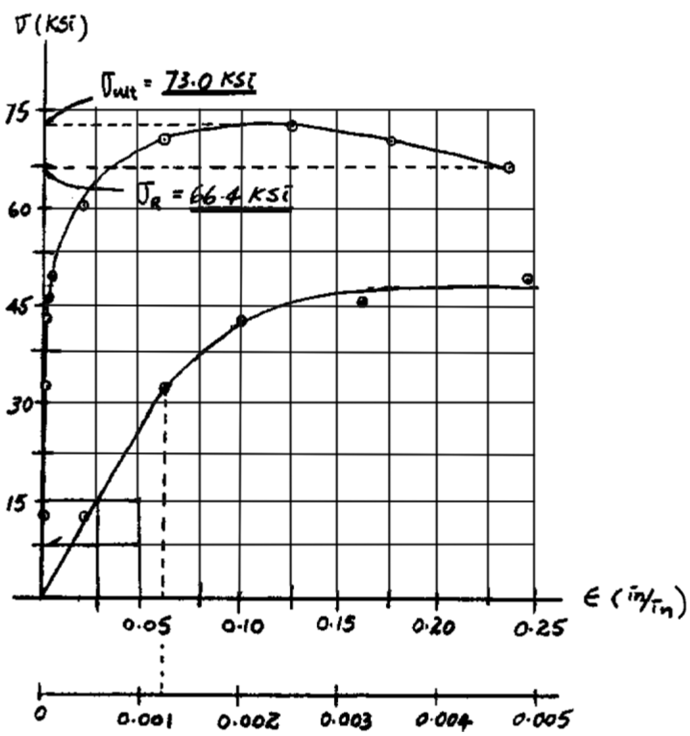
$u_t =$ total area under the curve

$$= 87\ (7.5)\ (0.025) \qquad (1)$$

$$= 16.3\ \frac{\text{in.} \cdot \text{kip}}{\text{in}^3} \qquad \textbf{Ans.}$$

In Eq.(1), 87 is the number of squares under the curve.

$\sigma = \frac{P}{A}$(ksi)	$\epsilon = \frac{\Delta L}{L}$(in./in.)
0	0
12.58	0.00045
32.71	0.00125
42.78	0.0020
46.30	0.00325
49.32	0.0049
60.39	0.02
70.45	0.06
72.97	0.125
70.45	0.175
66.43	0.235



Ans:
$u_t = 16.3\ \frac{\text{in.} \cdot \text{kip}}{\text{in}^3}$

3–6. A specimen is originally 1 ft long, has a diameter of 0.5 in., and is subjected to a force of 500 lb. When the force is increased from 500 lb to 1800 lb, the specimen elongates 0.009 in. Determine the modulus of elasticity for the material if it remains linear elastic.

Normal Stress and Strain: Applying $\sigma = \dfrac{P}{A}$ and $\varepsilon = \dfrac{\delta L}{L}$.

$$\sigma_1 = \frac{0.500}{\frac{\pi}{4}(0.5^2)} = 2.546 \text{ ksi}$$

$$\sigma_2 = \frac{1.80}{\frac{\pi}{4}(0.5^2)} = 9.167 \text{ ksi}$$

$$\Delta\epsilon = \frac{0.009}{12} = 0.000750 \text{ in./in.}$$

Modulus of Elasticity:

$$E = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{9.167 - 2.546}{0.000750} = 8.83\left(10^3\right) \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$E = 8.83\left(10^3\right)$ ksi

3–7. A structural member in a nuclear reactor is made of a zirconium alloy. If an axial load of 4 kip is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 relative to yielding. What is the load on the member if it is 3 ft long and its elongation is 0.02 in.? $E_{zr} = 14(10^3)$ ksi, $\sigma_Y = 57.5$ ksi. The material has elastic behavior.

Allowable Normal Stress:

$$\text{F.S.} = \frac{\sigma_y}{\sigma_{\text{allow}}}$$

$$3 = \frac{57.5}{\sigma_{\text{allow}}}$$

$$\sigma_{\text{allow}} = 19.17 \text{ ksi}$$

$$\sigma_{\text{allow}} = \frac{P}{A}$$

$$19.17 = \frac{4}{A}$$

$$A = 0.2087 \text{ in}^2 = 0.209 \text{ in}^2$$ **Ans.**

Stress–Strain Relationship: Applying Hooke's law with

$$\epsilon = \frac{\delta}{L} = \frac{0.02}{3\,(12)} = 0.000555 \text{ in./in.}$$

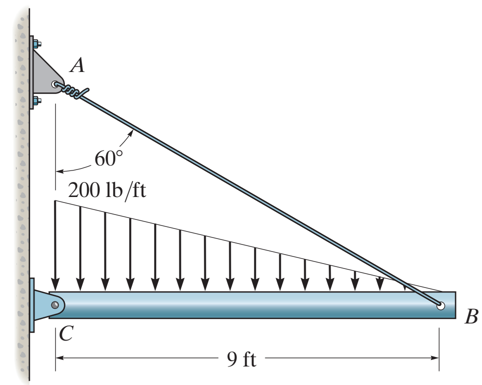
$$\sigma = E\epsilon = 14\left(10^3\right)(0.000555) = 7.778 \text{ ksi}$$

Normal Force: Applying equation $\sigma = \frac{P}{A}$.

$$P = \sigma A = 7.778\,(0.2087) = 1.62 \text{ kip}$$ **Ans.**

Ans:
$A = 0.209 \text{ in}^2$, $P = 1.62$ kip

***3–8.** The strut is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when the distributed load acts on the strut.



Here, we are only interested in determining the force in wire AB.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB} \cos 60°(9) - \frac{1}{2}(200)(9)(3) = 0 \qquad F_{AB} = 600 \text{ lb}$$

The normal stress the wire is

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{600}{\frac{\pi}{4}(0.2^2)} = 19.10(10^3) \text{ psi} = 19.10 \text{ ksi}$$

Since $\sigma_{AB} < \sigma_y = 36$ ksi, Hooke's Law can be applied to determine the strain in wire.

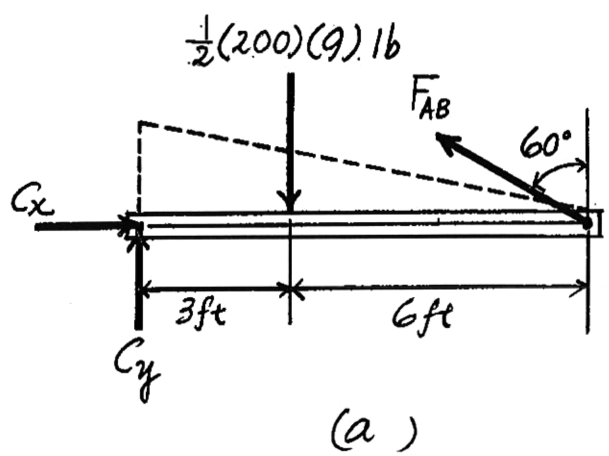
$$\sigma_{AB} = E\epsilon_{AB}; \qquad 19.10 = 29.0(10^3)\epsilon_{AB}$$

$$\epsilon_{AB} = 0.6586(10^{-3}) \text{ in/in}$$

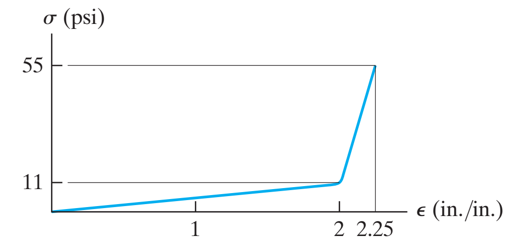
The unstretched length of the wire is $L_{AB} = \dfrac{9(12)}{\sin 60°} = 124.71$ in. Thus, the wire stretches

$$\delta_{AB} = \epsilon_{AB} L_{AB} = 0.6586(10^{-3})(124.71)$$

$$= 0.0821 \text{ in.} \qquad \textbf{Ans.}$$



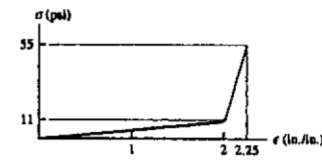
3–9. The $\sigma-\epsilon$ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.



$$E = \frac{11}{2} = 5.5 \text{ psi}$$ **Ans.**

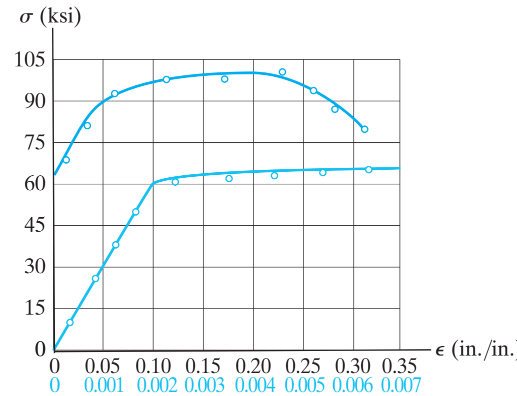
$$u_t = \frac{1}{2}(2)(11) + \frac{1}{2}(55 + 11)(2.25 - 2) = 19.25 \text{ psi}$$ **Ans.**

$$u_r = \frac{1}{2}(2)(11) = 11 \text{ psi}$$ **Ans.**



Ans:
$E = 5.5$ psi, $u_t = 19.25$ psi, $u_r = 11$ psi

3–10. The stress–strain diagram for a metal alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



From the stress–strain diagram, Fig. *a*,

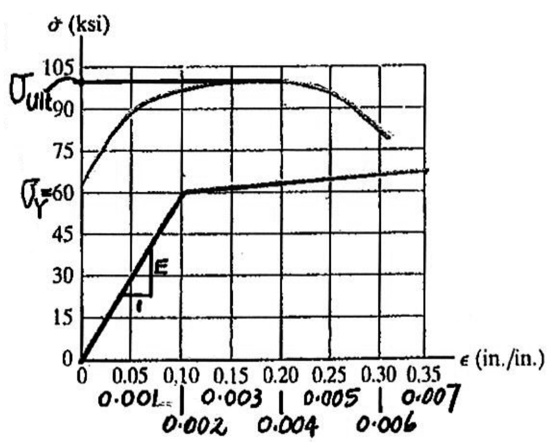
$$\frac{E}{1} = \frac{60 \text{ ksi} - 0}{0.002 - 0}; \qquad E = 30.0(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_y = 60 \text{ ksi} \qquad \sigma_{ult} = 100 \text{ ksi}$$

Thus,

$$P_Y = \sigma_Y A = 60\left[\frac{\pi}{4}(0.5^2)\right] = 11.78 \text{ kip} = 11.8 \text{ kip} \qquad \textbf{Ans.}$$

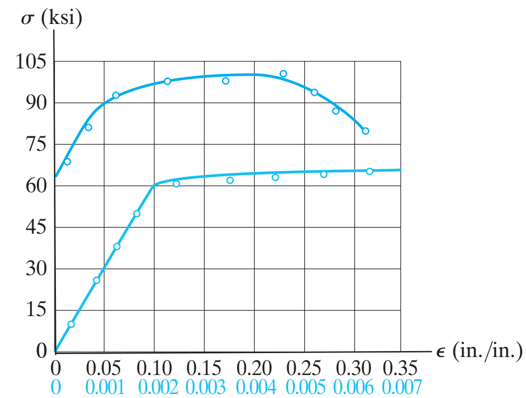
$$P_{ult} = \sigma_{ult} A = 100\left[\frac{\pi}{4}(0.5^2)\right] = 19.63 \text{ kip} = 19.6 \text{ kip} \qquad \textbf{Ans.}$$



(a)

Ans:
$E = 30.0(10^3)$ ksi, $P_Y = 11.8$ kip, $P_{ult} = 19.6$ kip

3–11. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. If the specimen is loaded until it is stressed to 90 ksi, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.



From the stress–strain diagram Fig. *a*, the modulus of elasticity for the steel alloy is

$$\frac{E}{1} = \frac{60\text{ ksi} - 0}{0.002 - 0}; \qquad E = 30.0(10^3)\text{ ksi}$$

when the specimen is unloaded, its normal strain recovers along line AB, Fig. *a*, which has a slope of E. Thus

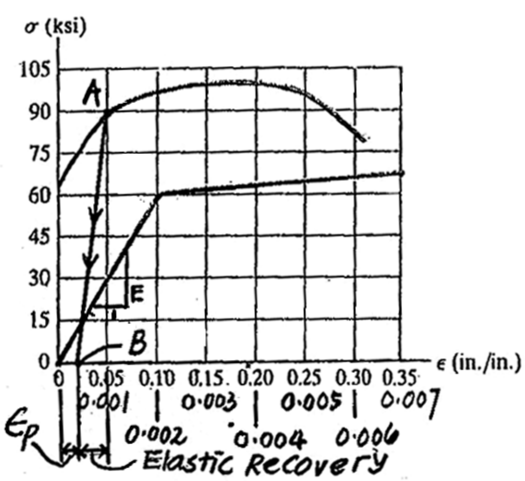
$$\textit{Elastic Recovery} = \frac{90}{E} = \frac{90\text{ ksi}}{30.0(10^3)\text{ ksi}} = 0.003\text{ in/in.} \qquad \textbf{Ans.}$$

Thus, the permanent set is

$$\epsilon_P = 0.05 - 0.003 = 0.047\text{ in/in.}$$

Then, the increase in gauge length is

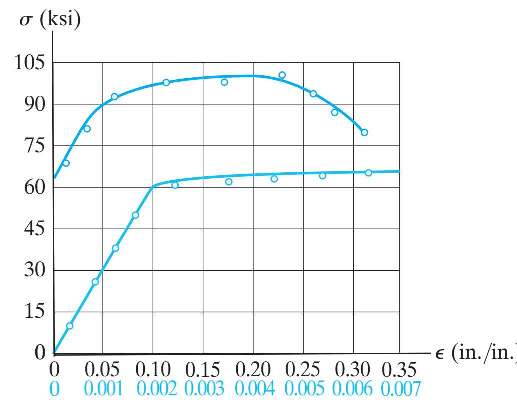
$$\Delta L = \epsilon_P L = 0.047(2) = 0.094\text{ in.} \qquad \textbf{Ans.}$$



(a)

Ans:
Elastic Recovery $= 0.003$ in./in., $\Delta L = 0.094$ in.

***3–12.** The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.



The Modulus of resilience is equal to the area under the stress–strain diagram up to the proportional limit.

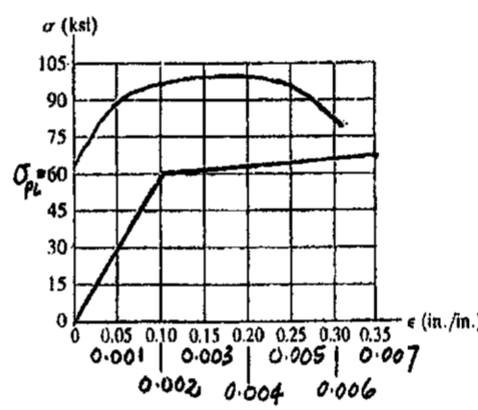
$$\sigma_{PL} = 60 \text{ ksi} \qquad \epsilon_{PL} = 0.002 \text{ in./in.}$$

Thus,

$$(u_i)_r = \frac{1}{2}\sigma_{PL}\epsilon_{PL} = \frac{1}{2}\left[60(10^3)\right](0.002) = 60.0\ \frac{\text{in.}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

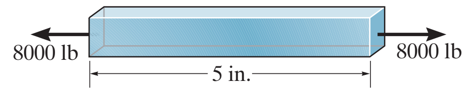
The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number is 38. Thus,

$$\left[(u_i)_t\right]_{\text{approx}} = 38\left[15(10^3)\ \frac{\text{lb}}{\text{in}^2}\right]\left(0.05\ \frac{\text{in.}}{\text{in.}}\right) = 28.5(10^3)\ \frac{\text{in.}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



(a)

3–13. A bar having a length of 5 in. and cross-sectional area of 0.7 in.^{2} is subjected to an axial force of 8000 lb. If the bar stretches 0.002 in., determine the modulus of elasticity of the material. The material has linear-elastic behavior.



Normal Stress and Strain:

$$\sigma = \frac{P}{A} = \frac{8.00}{0.7} = 11.43 \text{ ksi}$$

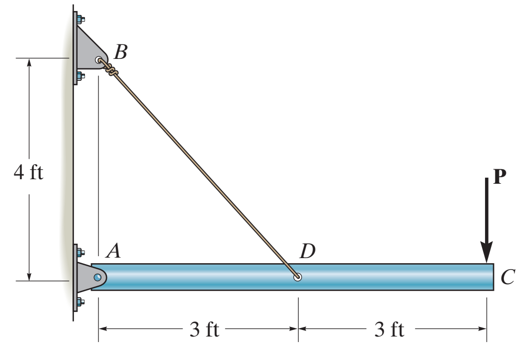
$$\epsilon = \frac{\delta}{L} = \frac{0.002}{5} = 0.000400 \text{ in./in.}$$

Modulus of Elasticity:

$$E = \frac{\sigma}{\epsilon} = \frac{11.43}{0.000400} = 28.6(10^3) \text{ ksi}$$ **Ans.**

Ans:
$E = 28.6(10^3)$ ksi

3–14. The rigid pipe is supported by a pin at A and an A-36 steel guy wire BD. If the wire has a diameter of 0.25 in., determine how much it stretches when a load of $P = 600$ lb acts on the pipe.



Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - 600(6) = 0 \qquad F_{BD} = 1500 \text{ lb}$$

The normal stress developed in the wire is

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{1500}{\frac{\pi}{4}(0.25^2)} = 30.56(10^3) \text{ psi} = 30.56 \text{ ksi}$$

Since $\sigma_{BD} < \sigma_y = 36$ ksi, Hooke's Law can be applied to determine the strain in the wire.

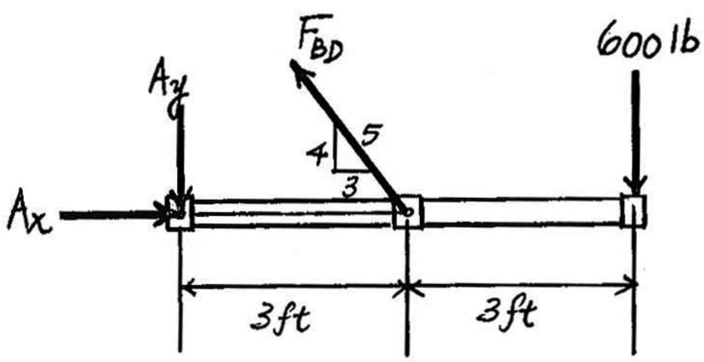
$$\sigma_{BD} = E\epsilon_{BD}; \qquad 30.56 = 29.0(10^3)\epsilon_{BD}$$

$$\epsilon_{BD} = 1.054(10^{-3}) \text{ in./in.}$$

The unstretched length of the wire is $L_{BD} = \sqrt{3^2 + 4^2} = 5 \text{ ft} = 60$ in. Thus, the wire stretches

$$\delta_{BD} = \epsilon_{BD} L_{BD} = 1.054(10^{-3})(60)$$

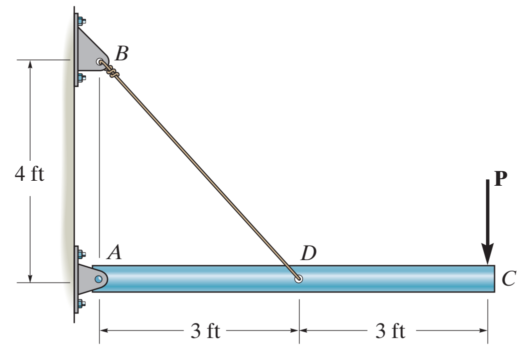
$$= 0.0632 \text{ in.}$$ **Ans.**



(a)

Ans:
$\delta_{BD} = 0.0632$ in.

3–15. The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 0.25 in., determine the load P if the end C is displaced 0.15 in. downward.



Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - P(6) = 0 \qquad F_{BD} = 2.50\,P$$

The unstretched length for wire BD is $L_{BD} = \sqrt{3^2 + 4^2} = 5\text{ ft} = 60\text{ in.}$ From the geometry shown in Fig. *b*, the stretched length of wire BD is

$$L_{BD'} = \sqrt{60^2 + 0.075^2 - 2(60)(0.075)\cos 143.13^\circ} = 60.060017$$

Thus, the normal strain is

$$\epsilon_{BD} = \frac{L_{BD'} - L_{BD}}{L_{BD}} = \frac{60.060017 - 60}{60} = 1.0003(10^{-3})\text{ in./in.}$$

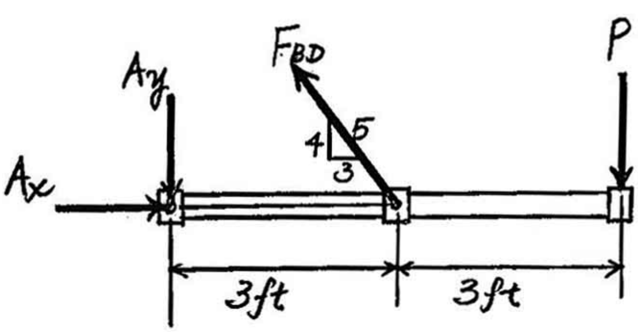
Then, the normal stress can be obtain by applying Hooke's Law.

$$\sigma_{BD} = E\epsilon_{BD} = 29(10^3)\left[1.0003(10^{-3})\right] = 29.01\text{ ksi}$$

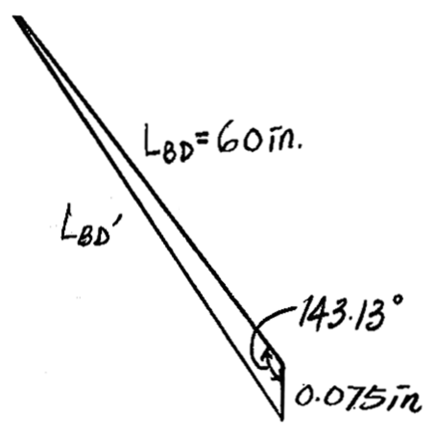
Since $\sigma_{BD} < \sigma_y = 36$ ksi, the result is valid.

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}}; \qquad 29.01(10^3) = \frac{2.50\,P}{\frac{\pi}{4}(0.25^2)}$$

$$P = 569.57\text{ lb} = 570\text{ lb} \qquad \textbf{Ans.}$$



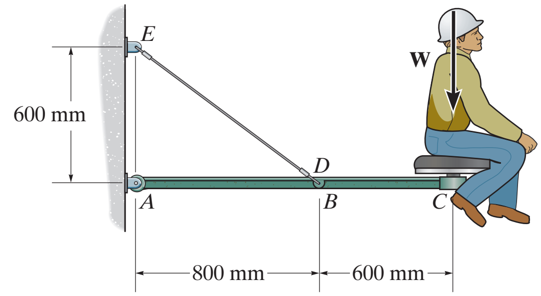
(a)



(b)

Ans:
$P = 570$ lb

***3–16.** The wire has a diameter of 5 mm and is made from A-36 steel. If a 80-kg man is sitting on seat C, determine the elongation of wire DE.



Equations of Equilibrium: The force developed in wire DE can be determined by writing the moment equation of equilibrium about A with reference to the free-body diagram shown in Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{DE}\left(\frac{3}{5}\right)(0.8) - 80(9.81)(1.4) = 0$$

$$F_{DE} = 2289 \text{ N}$$

Normal Stress and Strain:

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}} = \frac{2289}{\frac{\pi}{4}(0.005^2)} = 116.58 \text{ MPa}$$

Since $\sigma_{DE} < \sigma_Y$, Hooke's Law can be applied

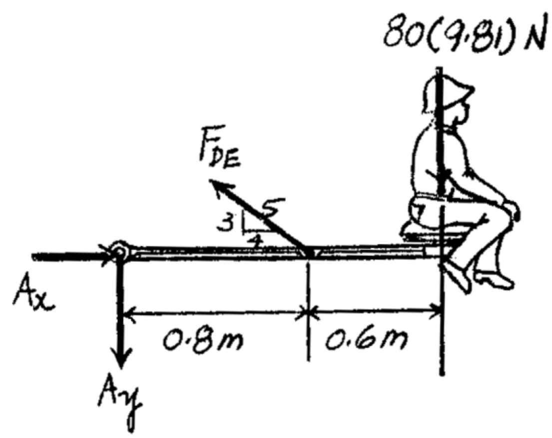
$$\sigma_{DE} = E\epsilon_{DE}$$

$$116.58(10^6) = 200(10^9)\epsilon_{DE}$$

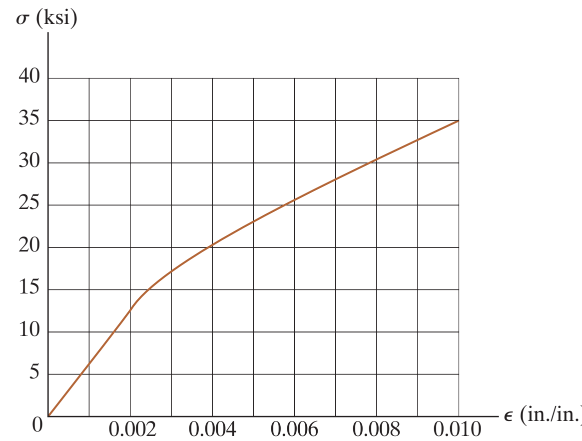
$$\epsilon_{DE} = 0.5829(10^{-3}) \text{ mm/mm}$$

The unstretched length of wire DE is $L_{DE} = \sqrt{600^2 + 800^2} = 1000$ mm. Thus, the elongation of this wire is given by

$$\delta_{DE} = \epsilon_{DE}L_{DE} = 0.5829(10^{-3})(1000) = 0.583 \text{ mm}$$ **Ans.**



3–17. A tension test was performed on a magnesium alloy specimen having a diameter 0.5 in. and gauge length 2 in. The resulting stress–strain diagram is shown in the figure. Determine the approximate modulus of elasticity and the yield strength of the alloy using the 0.2% strain offset method.

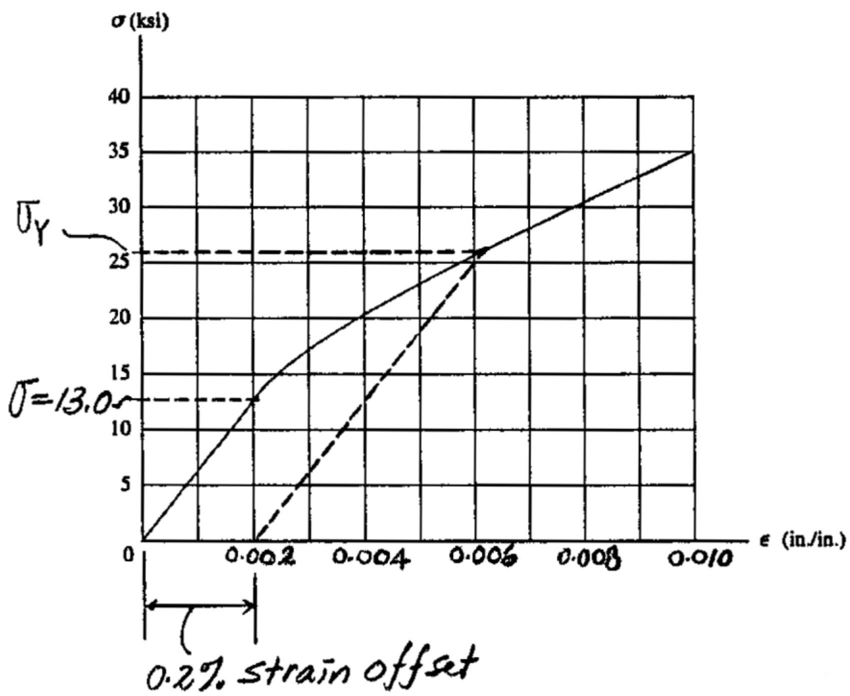


Modulus of Elasticity: From the stress–strain diagram, when $\epsilon = 0.002$ in./in., its corresponding stress is $\sigma = 13.0$ ksi. Thus,

$$E_{\text{approx}} = \frac{13.0 - 0}{0.002 - 0} = 6.50(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

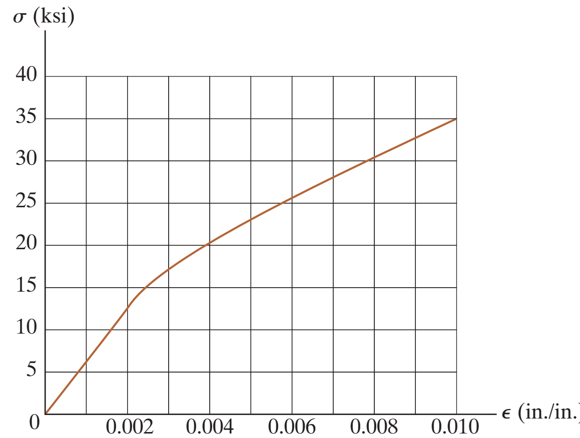
Yield Strength: The intersection point between the stress–strain diagram and the straight line drawn parallel to the initial straight portion of the stress–strain diagram from the offset strain of $\epsilon = 0.002$ in./in. is the yield strength of the alloy. From the stress–strain diagram,

$$\sigma_{YS} = 25.9 \text{ ksi} \qquad \textbf{Ans.}$$



Ans:
$E_{\text{approx}} = 6.50(10^3)$ ksi, $\sigma_{YS} = 25.9$ ksi

3–18. A tension test was performed on a magnesium alloy specimen having a diameter 0.5 in. and gauge length of 2 in. The resulting stress–strain diagram is shown in the figure. If the specimen is stressed to 30 ksi and unloaded, determine the permanent elongation of the specimen.

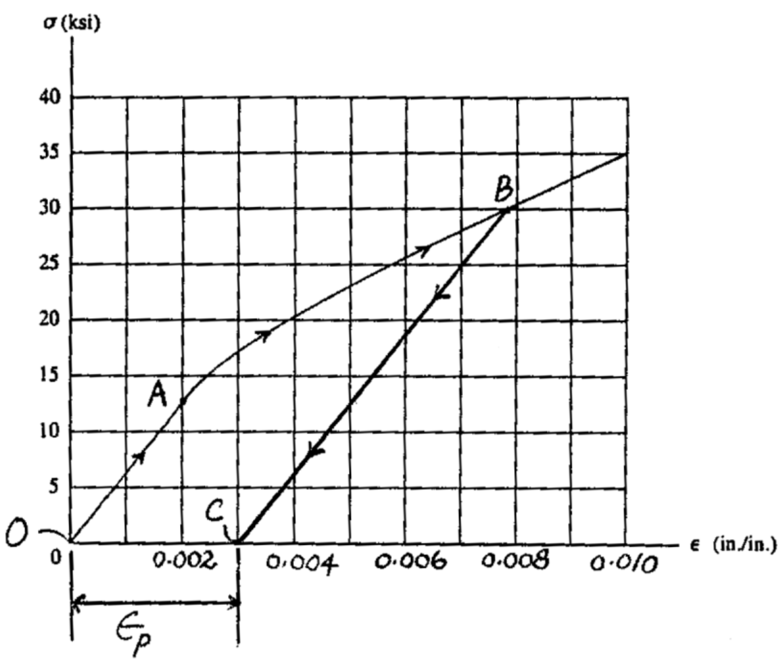


Permanent Elongation: From the stress–strain diagram, the strain recovered is along the straight line BC which is parallel to the straight line OA. Since $E_{\text{approx}} = \dfrac{13.0 - 0}{0.002 - 0} = 6.50(10^3)$ ksi, then the permanent set for the specimen is

$$\epsilon_P = 0.0078 - \frac{30(10^3)}{6.5(10^6)} = 0.00318 \text{ in./in.}$$

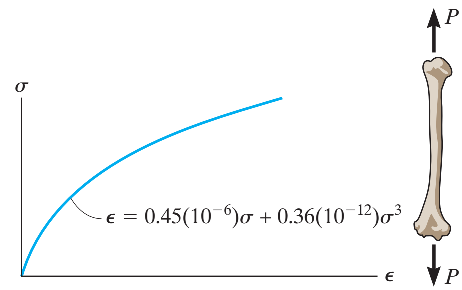
Thus,

$$\delta_P = \epsilon_P L = 0.00318(2) = 0.00637 \text{ in.} \qquad \textbf{Ans.}$$



Ans:
$\delta_P = 0.00637$ in.

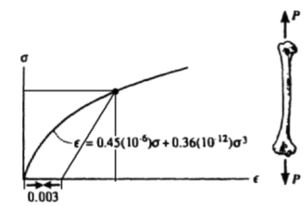
3–19. The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.



$$\epsilon = 0.45(10^{-6})\sigma + 0.36(10^{-12})\sigma^3,$$

$$d\epsilon = \left(0.45(10^{-6}) + 1.08(10^{-12})\,\sigma^2\right)d\sigma$$

$$E = \left.\frac{d\sigma}{d\epsilon}\right|_{\sigma=0} = \frac{1}{0.45(10^{-6})} = 2.22(10^6)\text{ kPa} = 2.22\text{ GPa}$$

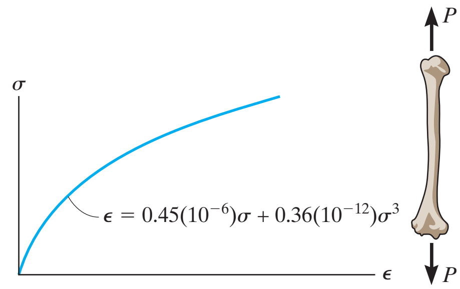


The equation for the recovery line is $\sigma = 2.22(10^6)(\epsilon - 0.003)$.

This line intersects the stress–strain curve at $\sigma_{YS} = 2027\text{ kPa} = 2.03\text{ MPa}$ **Ans.**

Ans:
$\sigma_{YS} = 2.03\text{ MPa}$

***3–20.** The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.



When $\epsilon = 0.12$

$$120(10^{-3}) = 0.45\,\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root:

$$\sigma = 6873.52 \text{ kPa}$$

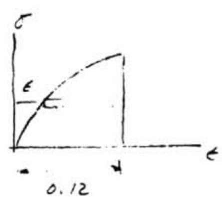
$$u_t = \int_A dA = \int_0^{6873.52} (0.12 - \epsilon)d\sigma$$

$$u_t = \int_0^{6873.52} (0.12 - 0.45(10^{-6})\sigma - 0.36(10^{-12})\sigma^3)d\sigma$$

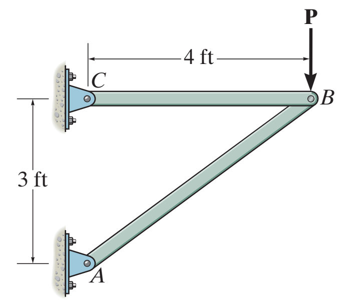
$$= 0.12\,\sigma - 0.225(10^{-6})\sigma^2 - 0.09(10^{-12})\sigma^4\Big|_0^{6873.52}$$

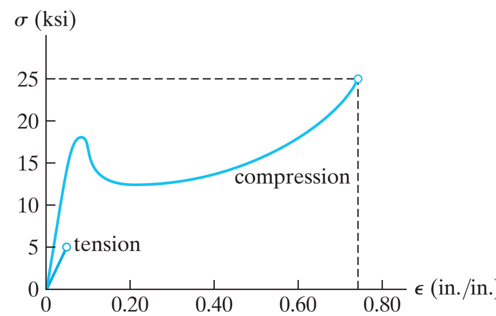
$$= 613 \text{ kJ/m}^3$$ **Ans.**

$$\delta = \epsilon L = 0.12(200) = 24 \text{ mm}$$ **Ans.**



3–21. The two bars are made of polystyrene, which has the stress–strain diagram shown. If the cross-sectional area of bar AB is 1.5 in^2 and BC is 4 in^2, determine the largest force P that can be supported before any member ruptures. Assume that buckling does not occur.





$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}F_{AB} - P = 0; \qquad F_{AB} = 1.6667\,P \qquad (1)$

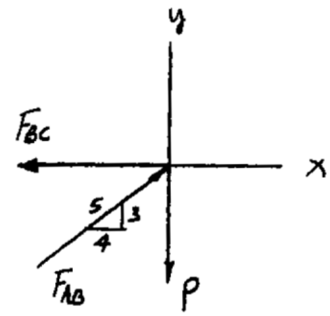
$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad F_{BC} - \frac{4}{5}(1.6667P) = 0; \qquad F_{BC} = 1.333\,P \qquad (2)$

Assuming failure of bar BC:

From the stress–strain diagram $(\sigma_R)_t = 5$ ksi

$\sigma = \frac{F_{BC}}{A_{BC}}; \qquad 5 = \frac{F_{BC}}{4}; \qquad F_{BC} = 20.0$ kip

From Eq. (2), $P = 15.0$ kip



Assuming failure of bar AB:

From stress–strain diagram $(\sigma_R)_c = 25.0$ ksi

$\sigma = \frac{F_{AB}}{A_{AB}}; \qquad 25.0 = \frac{F_{AB}}{1.5}; \qquad F_{AB} = 37.5$ kip

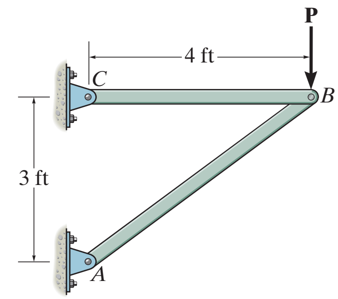
From Eq. (1), $P = 22.5$ kip

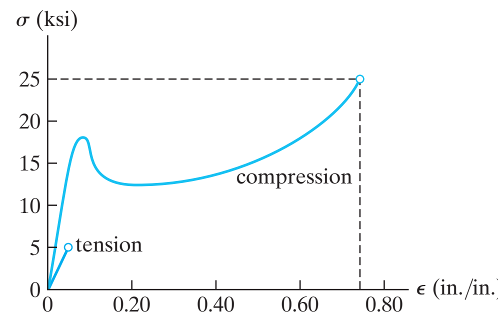
Choose the smallest value

$P = 15.0$ kip **Ans.**

Ans:
$P = 15.0$ kip

3–22. The two bars are made of polystyrene, which has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars rupture simultaneously when the load $P = 3$ kip. Assume that buckling does not occur.



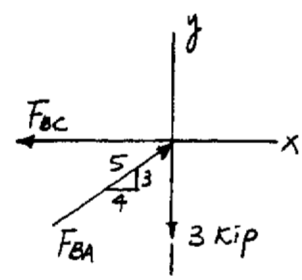


$+\uparrow \Sigma F_y = 0; \qquad F_{BA}\left(\frac{3}{5}\right) - 3 = 0; \qquad F_{BA} = 5 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -F_{BC} + 5\left(\frac{4}{5}\right) = 0; \qquad F_{BC} = 4 \text{ kip}$

For member BC:

$$(\sigma_{\max})_t = \frac{F_{BC}}{A_{BC}}; \quad A_{BC} = \frac{4 \text{ kip}}{5 \text{ ksi}} = 0.8 \text{ in}^2 \qquad \textbf{Ans.}$$



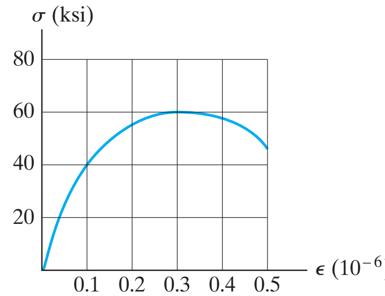
For member BA:

$$(\sigma_{\max})_c = \frac{F_{BA}}{A_{BA}}; \quad A_{BA} = \frac{5 \text{ kip}}{25 \text{ ksi}} = 0.2 \text{ in}^2 \qquad \textbf{Ans.}$$

Ans:

$A_{BC} = 0.8 \text{ in}^2, A_{BA} = 0.2 \text{ in}^2$

3–23. The stress–strain diagram for many metal alloys can be described analytically using the Ramberg-Osgood three parameter equation $\epsilon = \sigma/E + k\sigma^n$, where E, k, and n are determined from measurements taken from the diagram. Using the stress–strain diagram shown in the figure, take $E = 30(10^3)$ ksi and determine the other two parameters k and n and thereby obtain an analytical expression for the curve.



Choose,

$\sigma = 40$ ksi, $\varepsilon = 0.1$

$\sigma = 60$ ksi, $\varepsilon = 0.3$

$$0.1 = \frac{40}{30(10^3)} + k(40)^n$$

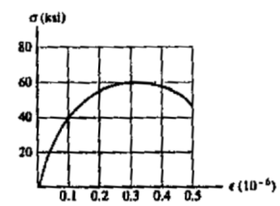
$$0.3 = \frac{60}{30(10^3)} + k(60)^n$$

$$0.098667 = k(40)^n$$

$$0.29800 = k(60)^n$$

$$0.3310962 = (0.6667)^n$$

$$\ln (0.3310962) = n \ln (0.6667)$$

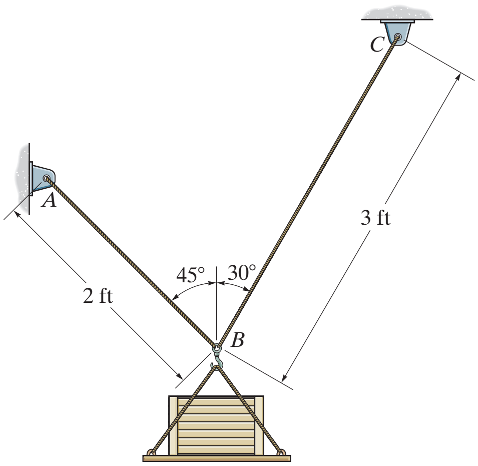


$n = 2.73$ **Ans.**

$k = 4.23(10^{-6})$ **Ans.**

Ans:
$n = 2.73, k = 4.23(10^{-6})$

3–24. The wires AB and BC have original lengths of 2 ft and 3 ft, and diameters of $\frac{1}{8}$ in. and $\frac{3}{16}$ in., respectively. If these wires are made of a material that has the approximate stress–strain diagram shown, determine the elongations of the wires after the 1500-lb load is placed on the platform.



Equations of Equilibrium: The forces developed in wires AB and BC can be determined by analyzing the equilibrium of joint B, Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BC}\sin 30° - F_{AB}\sin 45° = 0 \qquad (1)$$

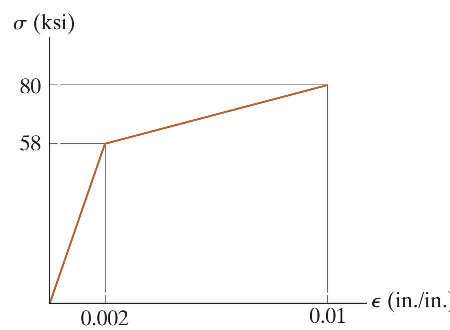
$$+\uparrow\Sigma F_y = 0; \qquad F_{BC}\cos 30° + F_{AB}\cos 45° = 1500 \qquad (2)$$

Solving Eqs. (1) and (2),

$$F_{AB} = 776.46 \text{ lb} \qquad F_{BC} = 1098.08 \text{ lb}$$

Normal Stress and Strain:

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{776.46}{\frac{\pi}{4}(1/8)^2} = 63.27 \text{ ksi}$$



$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{1098.08}{\frac{\pi}{4}(3/16)^2} = 39.77 \text{ ksi}$$

The corresponding normal strain can be determined from the stress–strain diagram, Fig. b.

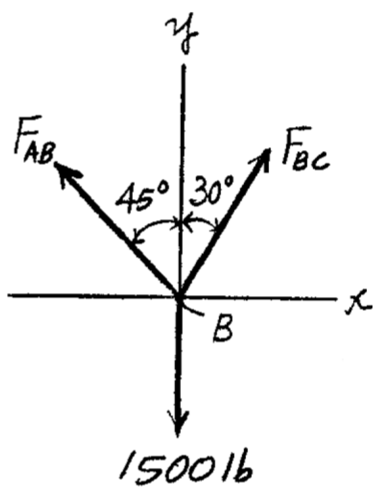
$$\frac{39.77}{\epsilon_{BC}} = \frac{58}{0.002}; \qquad \epsilon_{BC} = 0.001371 \text{ in./in.}$$

$$\frac{63.27 - 58}{\epsilon_{AB} - 0.002} = \frac{80 - 58}{0.01 - 0.002}; \qquad \epsilon_{AB} = 0.003917 \text{ in./in.}$$

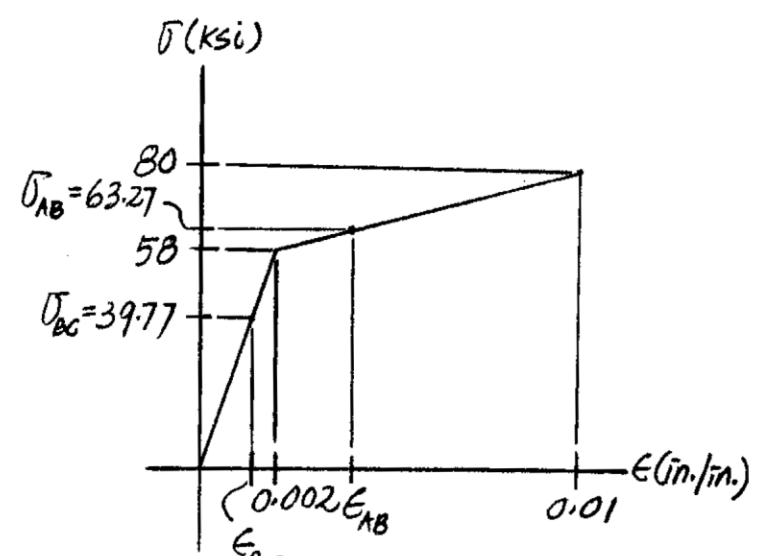
Thus, the elongations of wires AB and BC are

$$\delta_{AB} = \epsilon_{AB}L_{AB} = 0.003917(24) = 0.0940 \qquad \textbf{Ans.}$$

$$\delta_{BC} = \epsilon_{BC}L_{BC} = 0.001371(36) = 0.0494 \qquad \textbf{Ans.}$$

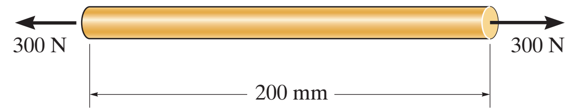


(a)



(b)

3–25. The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.678 \text{ MPa}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{1.678(10^6)}{2.70(10^9)} = 0.0006288$$

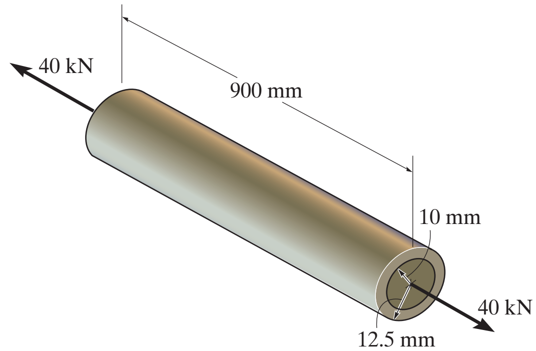
$$\delta = \epsilon_{\text{long}} L = 0.0006288\,(200) = 0.126 \text{ mm}$$ **Ans.**

$$\epsilon_{\text{lat}} = -\nu\epsilon_{\text{long}} = -0.4(0.0006288) = -0.0002515$$

$$\Delta d = \epsilon_{\text{lat}} d = -0.0002515\,(15) = -0.00377 \text{ mm}$$ **Ans.**

Ans:
$\delta = 0.126$ mm, $\Delta d = -0.00377$ mm

3–26. The thin-walled tube is subjected to an axial force of 40 kN. If the tube elongates 3 mm and its circumference decreases 0.09 mm, determine the modulus of elasticity, Poisson's ratio, and the shear modulus of the tube's material. The material behaves elastically.



Normal Stress and Strain:

$$\sigma = \frac{P}{A} = \frac{40(10^3)}{\pi(0.0125^2 - 0.01^2)} = 226.35 \text{ MPa}$$

$$\epsilon_a = \frac{\delta}{L} = \frac{3}{900} = 3.3333\ (10^{-3}) \text{ mm/mm}$$

Applying Hooke's law,

$$\sigma = E\epsilon_a; \quad 226.35(10^6) = E[3.3333(10^{-3})]$$

$$E = 67.91(10^6) \text{ Pa} = 67.9 \text{ GPa} \qquad \textbf{Ans.}$$

Poisson's Ratio: The circumference of the loaded tube is $2\pi(12.5) - 0.09 = 78.4498$ mm. Thus, the outer radius of the tube is

$$r = \frac{78.4498}{2\pi} = 12.4857 \text{ mm}$$

The lateral strain is

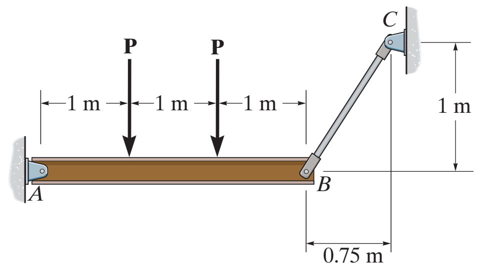
$$\epsilon_{\text{lat}} = \frac{r - r_0}{r_0} = \frac{12.4857 - 12.5}{12.5} = -1.1459(10^{-3}) \text{ mm/mm}$$

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_a} = -\left[\frac{-1.1459(10^{-3})}{3.3333(10^{-3})}\right] = 0.3438 = 0.344 \qquad \textbf{Ans.}$$

$$G = \frac{E}{2(1 + \nu)} = \frac{67.91(10^9)}{2(1 + 0.3438)} = 25.27(10^9) \text{ Pa} = 25.3 \text{ GPa} \qquad \textbf{Ans.}$$

Ans:
$E = 67.9$ GPa, $\nu = 0.344$, $G = 25.3$ GPa

3–27. When the two forces are placed on the beam, the diameter of the A-36 steel rod BC decreases from 40 mm to 39.99 mm. Determine the magnitude of each force **P**.



Equations of Equilibrium: The force developed in rod BC can be determined by writing the moment equation of equilibrium about A with reference to the free-body diagram of the beam shown in Fig. *a*.

$$\zeta+\Sigma M_A = 0; \qquad F_{BC}\left(\frac{4}{5}\right)(3) - P(2) - P(1) = 0 \qquad F_{BC} = 1.25P$$

Normal Stress and Strain: The lateral strain of rod BC is

$$\epsilon_{\text{lat}} = \frac{d - d_0}{d_0} = \frac{39.99 - 40}{40} = -0.25(10^{-3})\text{ mm/mm}$$

$$\epsilon_{\text{lat}} = -\nu\epsilon_a; \qquad -0.25(10^{-3}) = -(0.32)\epsilon_a$$

$$\epsilon_a = 0.78125(10^{-3})\text{ mm/mm}$$

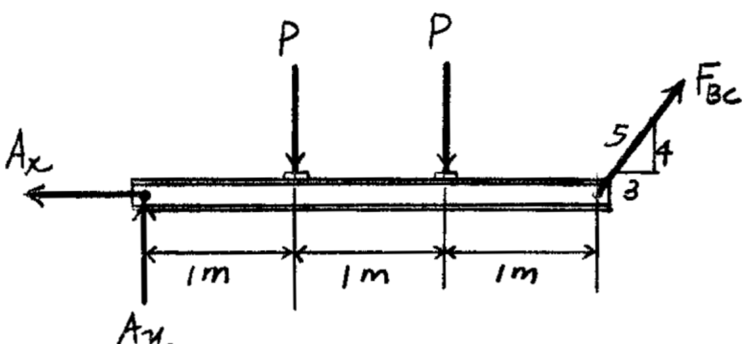
Assuming that Hooke's Law applies,

$$\sigma_{BC} = E\epsilon_a; \qquad \sigma_{BC} = 200(10^9)(0.78125)(10^{-3}) = 156.25\text{ MPa}$$

Since $\sigma < \sigma_Y$, the assumption is correct.

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}}; \qquad 156.25(10^6) = \frac{1.25P}{\frac{\pi}{4}\left(0.04^2\right)}$$

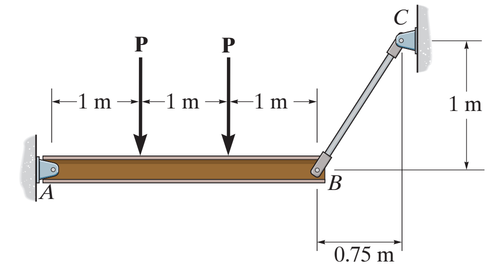
$$P = 157.08(10^3)\text{N} = 157\text{ kN} \qquad \textbf{Ans.}$$



(a)

Ans:
$P = 157$ kN

***3–28.** If $P = 150$ kN, determine the elastic elongation of rod BC and the decrease in its diameter. Rod BC is made of A-36 steel and has a diameter of 40 mm.



Equations of Equilibrium: The force developed in rod BC can be determined by writing the moment equation of equilibrium about A with reference to the free-body diagram of the beam shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{4}{5}\right)(3) - 150(2) - 150(1) = 0 \qquad F_{BC} = 187.5 \text{ kN}$$

Normal Stress and Strain: The lateral strain of rod BC is

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{187.5(10^3)}{\frac{\pi}{4}\left(0.04^2\right)} = 149.21 \text{ MPa}$$

Since $\sigma < \sigma_Y$, Hooke's Law can be applied. Thus,

$$\sigma_{BC} = E\epsilon_{BC}; \qquad 149.21(10^6) = 200(10^9)\epsilon_{BC}$$

$$\epsilon_{BC} = 0.7460(10^{-3}) \text{ mm/mm}$$

The unstretched length of rod BC is $L_{BC} = \sqrt{750^2 + 1000^2} = 1250$ mm. Thus the elongation of this rod is given by

$$\delta_{BC} = \epsilon_{BC}L_{BC} = 0.7460(10^{-3})(1250) = 0.933 \text{ mm} \qquad \textbf{Ans.}$$

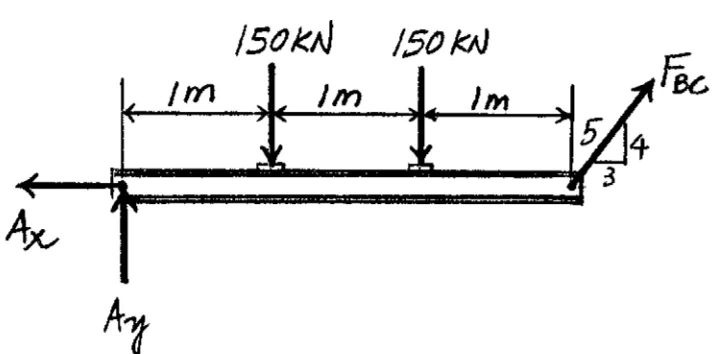
We obtain,

$$\epsilon_{\text{lat}} = -\nu\epsilon_a; \qquad \epsilon_{\text{lat}} = -(0.32)(0.7460)(10^{-3})$$

$$= -0.2387(10^{-3}) \text{ mm/mm}$$

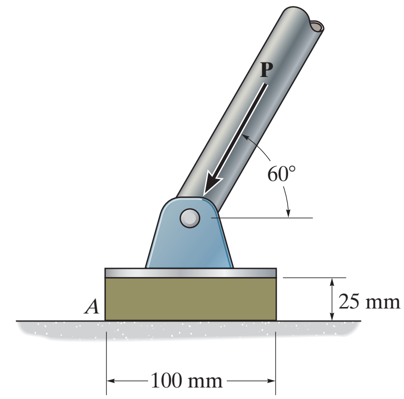
Thus,

$$\delta d = \epsilon_{\text{lat}}\, d_{BC} = -0.2387(10^{-3})(40) = -9.55(10^{-3}) \text{ mm} \qquad \textbf{Ans.}$$



(a)

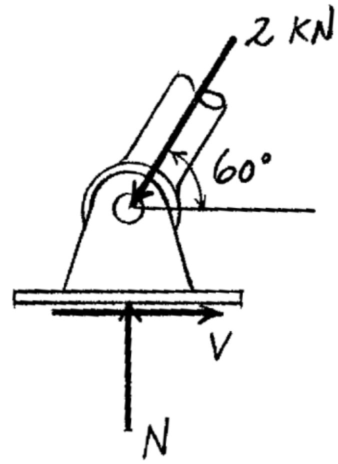
3–29. The friction pad A is used to support the member, which is subjected to an axial force of $P = 2$ kN. The pad is made from a material having a modulus of elasticity of $E = 4$ MPa and Poisson's ratio $\nu = 0.4$. If slipping does not occur, determine the normal and shear strains in the pad. The width is 50 mm. Assume that the material is linearly elastic. Also, neglect the effect of the moment acting on the pad.



Internal Loading: The normal force and shear force acting on the friction pad can be determined by considering the equilibrium of the pin shown in Fig. *a*.

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad V - 2\cos 60° = 0 \qquad V = 1\text{ kN}$

$+\uparrow \Sigma F_y = 0;\qquad N - 2\sin 60° = 0 \qquad N = 1.732\text{ kN}$



Normal and Shear Stress:

$$\tau = \frac{V}{A} = \frac{1(10^3)}{0.1(0.05)} = 200\text{ kPa}$$

$$\sigma = \frac{N}{A} = \frac{1.732(10^3)}{0.1(0.05)} = 346.41\text{ kPa}$$

Normal and Shear Strain: The shear modulus of the friction pad is

$$G = \frac{E}{2(1+\nu)} = \frac{4}{2(1+0.4)} = 1.429\text{ MPa}$$

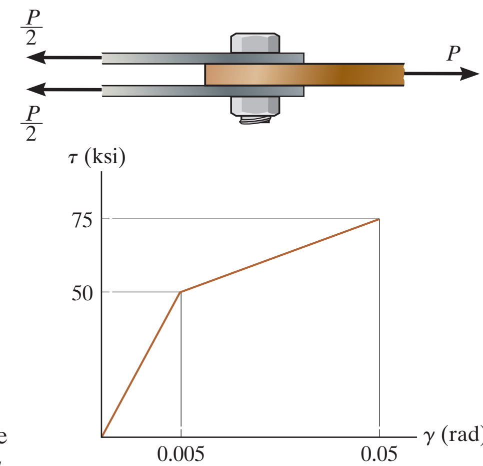
Applying Hooke's Law,

$\sigma = E\epsilon;\qquad 346.41(10^3) = 4(10^6)\epsilon \qquad \epsilon = 0.08660\text{ mm/mm}$ **Ans.**

$\tau = G\gamma;\qquad 200(10^3) = 1.429(10^6)\gamma \qquad \gamma = 0.140\text{ rad}$ **Ans.**

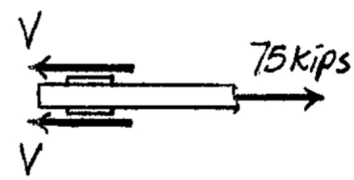
Ans:
$\epsilon = 0.08660$ mm/mm, $\gamma = 0.140$ rad

3–30. The lap joint is connected together using a 1.25 in. diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the shear strain developed in the shear plane of the bolt when $P = 75$ kip.



Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 75 - 2V = 0 \qquad V = 37.5 \text{ kip}$$



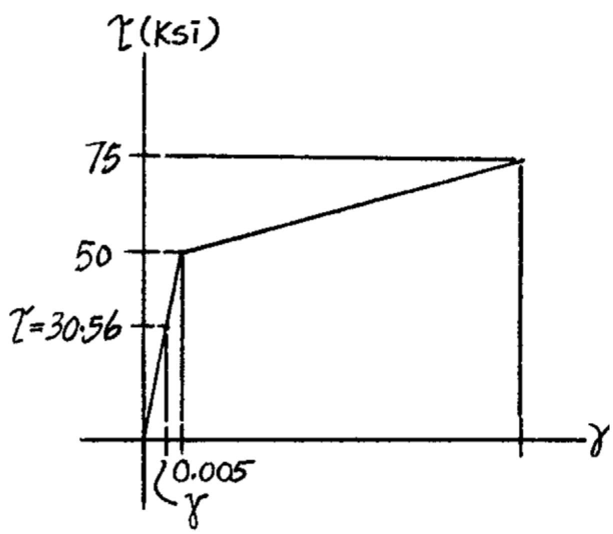
(a)

Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{37.5}{\frac{\pi}{4}\left(1.25^2\right)} = 30.56 \text{ ksi}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

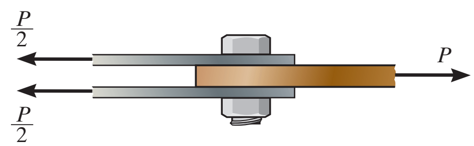
$$\frac{30.56}{\gamma} = \frac{50}{0.005}; \qquad \gamma = 3.06(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

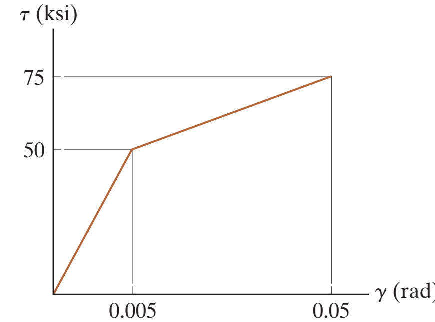


(b)

Ans:
$\gamma = 3.06(10^{-3})$ rad

3–31. The lap joint is connected together using a 1.25 in. diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the permanent shear strain in the shear plane of the bolt when the applied force $P = 150$ kip is removed.





Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 150 - 2V = 0 \qquad V = 75 \text{ kip}$$

Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{75}{\frac{\pi}{4}\left(1.25^2\right)} = 61.12 \text{ ksi}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

$$\frac{61.12 - 50}{\gamma - 0.005} = \frac{75 - 50}{0.05 - 0.005}; \qquad \gamma = 0.02501 \text{ rad}$$

When force **P** is removed, the shear strain recovers linearly along line *BC*, Fig. *b*, with a slope that is the same as line *OA*. This slope represents the shear modulus.

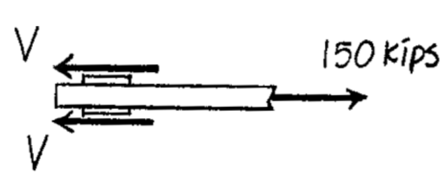
$$G = \frac{50}{0.005} = 10(10^3) \text{ ksi}$$

Thus, the elastic recovery of shear strain is

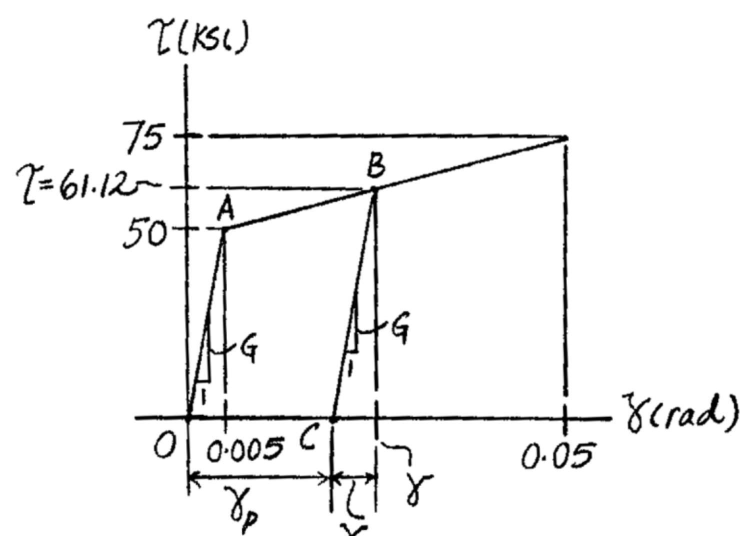
$$\tau = G\gamma_r; \qquad 61.12 = (10)(10^3)\gamma_r \qquad \gamma_r = 6.112(10^{-3}) \text{ rad}$$

And the permanent shear strain is

$$\gamma_P = \gamma - \gamma_r = 0.02501 - 6.112(10^{-3}) = 0.0189 \text{ rad}$$ **Ans.**



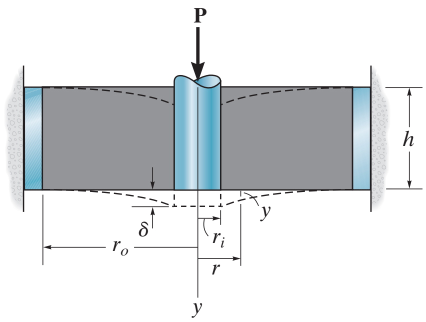
(a)



(b)

Ans:
$\gamma_P = 0.0189$ rad

***3–32.** A shear spring is made by bonding the rubber annulus to a rigid fixed ring and a plug. When an axial load **P** is placed on the plug, show that the slope at point y in the rubber is $dy/dr = -\tan\gamma = -\tan(P/(2\pi hGr))$. For small angles we can write $dy/dr = -P/(2\pi hGr)$. Integrate this expression and evaluate the constant of integration using the condition that $y = 0$ at $r = r_o$. From the result compute the deflection $y = \delta$ of the plug.



Shear Stress–Strain Relationship: Applying Hooke's law with $\tau_A = \dfrac{P}{2\pi\, r\, h}$.

$$\gamma = \frac{\tau_A}{G} = \frac{P}{2\pi\, h\, G\, r}$$

$$\frac{dy}{dr} = -\tan\gamma = -\tan\left(\frac{P}{2\pi\, h\, G\, r}\right) \qquad \text{(Q.E.D)}$$

If γ is small, then $\tan\gamma = \gamma$. Therefore,

$$\frac{dy}{dr} = -\frac{P}{2\pi\, h\, G\, r}$$

$$y = -\frac{P}{2\pi\, h\, G}\int\frac{dr}{r}$$

$$y = -\frac{P}{2\pi\, h\, G}\ln r + C$$

At $r = r_o$, $\quad y = 0$

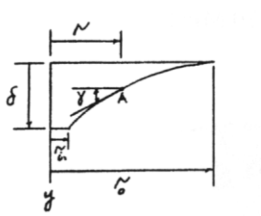
$$0 = -\frac{P}{2\pi\, h\, G}\ln r_o + C$$

$$C = \frac{P}{2\pi\, h\, G}\ln r_o$$

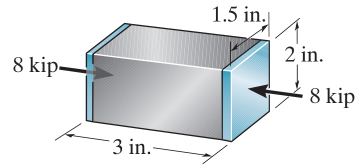
Then, $y = \dfrac{P}{2\pi\, h\, G}\ln\dfrac{r_o}{r}$

At $r = r_i$, $\quad y = \delta$

$$\delta = \frac{P}{2\pi\, h\, G}\ln\frac{r_o}{r_i} \qquad \textbf{Ans.}$$



3–33. The aluminum block has a rectangular cross section and is subjected to an axial compressive force of 8 kip. If the 1.5-in. side changed its length to 1.500132 in., determine Poisson's ratio and the new length of the 2-in. side. $E_{al} = 10(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{8}{(2)(1.5)} = 2.667 \text{ ksi}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{-2.667}{10(10^3)} = -0.0002667$$

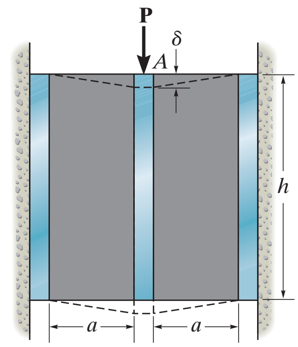
$$\epsilon_{\text{lat}} = \frac{1.500132 - 1.5}{1.5} = 0.0000880$$

$$\nu = \frac{-0.0000880}{-0.0002667} = 0.330$$ **Ans.**

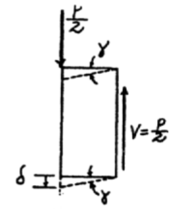
$$h' = 2 + 0.0000880(2) = 2.000176 \text{ in.}$$ **Ans.**

Ans:
$\nu = 0.330$, $h' = 2.000176$ in.

3–34. A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A if a vertical load $\mathbf{P}$ is applied to this plate. Assume that the displacement is small so that $\delta = a \tan \gamma \approx a\gamma$.



Average Shear Stress: The rubber block is subjected to a shear force of $V = \dfrac{P}{2}$.



$$\tau = \frac{V}{A} = \frac{\frac{P}{2}}{b\,h} = \frac{P}{2\,b\,h}$$

Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{\frac{P}{2\,b\,h}}{G} = \frac{P}{2\,b\,h\,G}$$

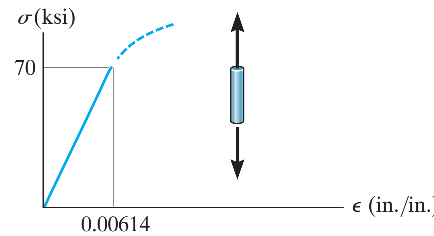
Thus,

$$\delta = a\,\gamma = \; = \frac{P\,a}{2\,b\,h\,G}$$ **Ans.**

Ans:

$$\delta = \frac{P\,a}{2\,b\,h\,G}$$

3–35. The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. When the applied load is 9 kip, the new diameter of the specimen is 0.49935 in. Compute the shear modulus G_{al} for the aluminum.



From the stress–strain diagram,

$$E_{al} = \frac{\sigma}{\epsilon} = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

When specimen is loaded with a 9 - kip load,

$$\sigma = \frac{P}{A} = \frac{9}{\frac{\pi}{4}(0.5)^2} = 45.84 \text{ ksi}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{45.84}{11400.65} = 0.0040205 \text{ in./in.}$$

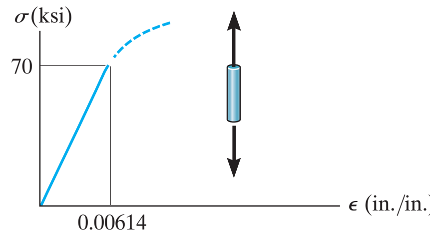
$$\epsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{0.49935 - 0.5}{0.5} = -0.0013 \text{ in./in.}$$

$$V = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} = -\frac{-0.0013}{0.0040205} = 0.32334$$

$$G_{al} = \frac{E_{at}}{2(1 + v)} = \frac{11.4(10^3)}{2(1 + 0.32334)} = 4.31(10^3) \text{ ksi}$$ **Ans.**

Ans:
$G_{al} = 4.31(10^3)$ ksi

***3–36.** The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip, determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.5)^2} = 50.9296 \text{ ksi}$$

From the stress–strain diagram

$$E = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{50.9296}{11400.65} = 0.0044673 \text{ in./in.}$$

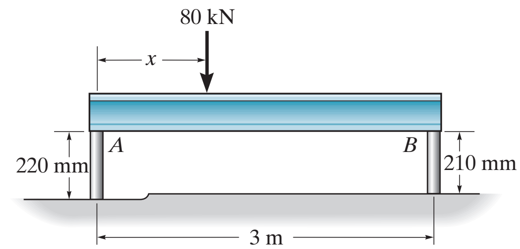
$$G = \frac{E}{2(1+\nu)}; \qquad 3.8(10^3) = \frac{11400.65}{2(1+\nu)}; \qquad \nu = 0.500$$

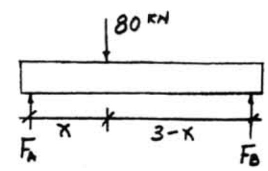
$$\epsilon_{\text{lat}} = -\nu\epsilon_{\text{long}} = -0.500(0.0044673) = -0.002234 \text{ in./in.}$$

$$\Delta d = \epsilon_{\text{lat}}\, d = -0.002234(0.5) = -0.001117 \text{ in.}$$

$$d' = d + \Delta d = 0.5 - 0.001117 = 0.4989 \text{ in.}$$ **Ans.**

3–37. The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm. determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.





$$\zeta+\Sigma M_A = 0; \qquad F_B(3) - 80(x) = 0; \qquad F_B = \frac{80x}{3} \qquad (1)$$

$$\zeta+\Sigma M_B = 0; \qquad -F_A(3) + 80(3 - x) = 0; \qquad F_A = \frac{80(3 - x)}{3} \qquad (2)$$

Since the beam is held horizontally, $\delta_A = \delta_B$

$$\sigma = \frac{P}{A}; \qquad \epsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$$

$$\delta = \epsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$$

$$\delta_A = \delta_B; \qquad \frac{\frac{80(3 - x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$$

$$80(3 - x)(220) = 80x(210)$$

$x = 1.53 \text{ m}$ **Ans.**

From Eq. (2),

$F_A = 39.07 \text{ kN}$

$$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$$

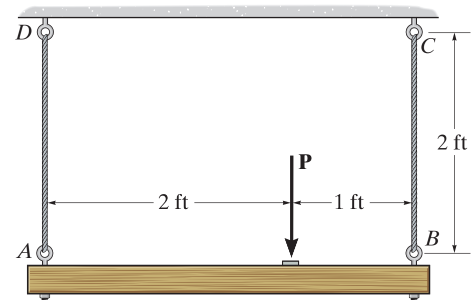
$$\epsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$$

$$\epsilon_{lat} = -\nu\epsilon_{long} = -0.35(-0.000756) = 0.0002646$$

$d'_A = d_A + d\,\epsilon_{lat} = 30 + 30(0.0002646) = 30.008 \text{ mm}$ **Ans.**

Ans:
$x = 1.53 \text{ m}, d'_A = 30.008 \text{ mm}$

3–38. The wires each have a diameter of $\frac{1}{2}$ in., length of 2 ft, and are made from 304 stainless steel. If $P = 6$ kip, determine the angle of tilt of the rigid beam AB.



Equations of Equilibrium: Referring to the free-body diagram of beam AB shown in Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BC}(3) - 6(2) = 0 \quad F_{BC} = 4 \text{ kip}$

$+\uparrow \Sigma M_B = 0;$ $\quad 6(1) - F_{AD}(3) = 0 \quad F_{AD} = 2 \text{ kip}$

Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{4(10^3)}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 20.37 \text{ ksi}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{2(10^3)}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 10.19 \text{ ksi}$$

Since $\sigma_{BC} < \sigma_Y$ and $\sigma_A < \sigma_Y$, Hooke's Law can be applied.

$\sigma_{BC} = E\epsilon_{BC};$ $\quad 20.37 = 28.0(10^3)\epsilon_{BC} \quad \epsilon_{BC} = 0.7276(10^{-3}) \text{ in./in.}$

$\sigma_{AD} = E\epsilon_{AD};$ $\quad 10.19 = 28.0(10^3)\epsilon_{AD} \quad \epsilon_{AD} = 0.3638(10^{-3}) \text{ in./in.}$

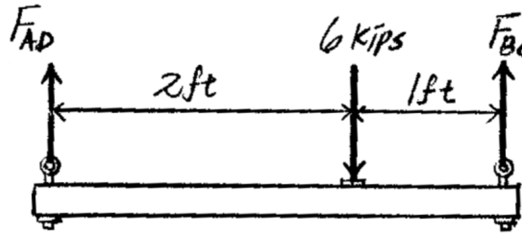
Thus, the elongation of cables BC and AD are given by

$$\delta_{BC} = \epsilon_{BC}L_{BC} = 0.7276(10^{-3})(24) = 0.017462 \text{ in.}$$

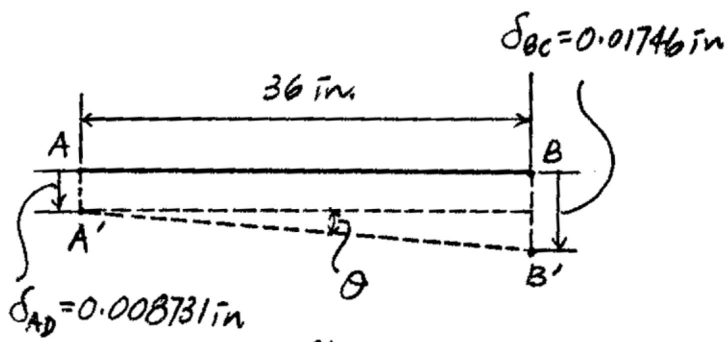
$$\delta_{AD} = \epsilon_{AD}L_{AD} = 0.3638(10^{-3})(24) = 0.008731 \text{ in.}$$

Referring to the geometry shown in Fig. b and using small angle analysis,

$$\theta = \frac{\delta_{BC} - \delta_{AD}}{36} = \frac{0.017462 - 0.008731}{36} = 0.2425(10^{-3}) \text{ rad}\left(\frac{180°}{\pi \text{rad}}\right) = 0.0139° \qquad \textbf{Ans.}$$



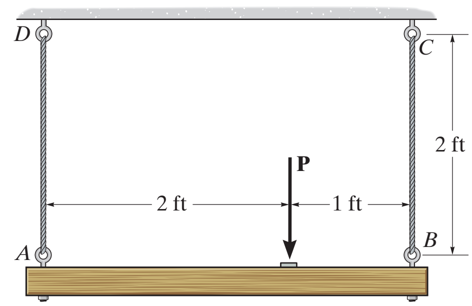
(a)



(b)

Ans:
$\theta = 0.0139°$

3–39. The wires each have a diameter of $\frac{1}{2}$ in., length of 2 ft, and are made from 304 stainless steel. Determine the magnitude of force **P** so that the rigid beam tilts 0.015°.



Equations of Equilibrium: Referring to the free-body diagram of beam AB shown in Fig. a,

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BC}(3) - P(2) = 0 \qquad F_{BC} = 0.6667P$$

$$+\uparrow \Sigma M_B = 0; \qquad P(1) - F_{AD}(3) = 0 \qquad F_{AD} = 0.3333P$$

Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{0.6667P}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 3.3953P$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{0.3333P}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 1.6977P$$

Assuming that $\sigma_{BC} < \sigma_Y$ and $\sigma_{AD} < \sigma_Y$ and applying Hooke's Law,

$$\sigma_{BC} = E\epsilon_{BC}; \qquad 3.3953P = 28.0(10^6)\epsilon_{BC} \qquad \epsilon_{BC} = 0.12126(10^{-6})P$$

$$\sigma_{AD} = E\epsilon_{AD}; \qquad 1.6977P = 28.0(10^6)\epsilon_{AD} \qquad \epsilon_{AD} = 60.6305(10^{-9})P$$

Thus, the elongation of cables BC and AD are given by

$$\delta_{BC} = \epsilon_{BC}L_{BC} = 0.12126(10^{-6})P(24) = 2.9103(10^{-6})P$$

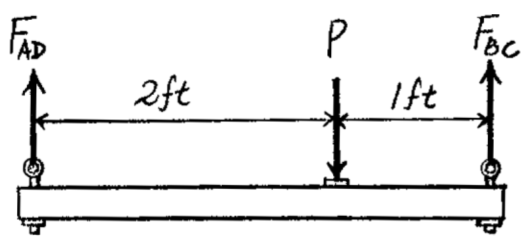
$$\delta_{AD} = \epsilon_{AD}L_{AD} = 60.6305(10^{-9})P(24) = 1.4551(10^{-6})P$$

Here, the angle of the tile is $\theta = 0.015°\left(\frac{\pi \text{ rad}}{180°}\right) = 0.2618(10^{-3})$ rad. Using small angle analysis,

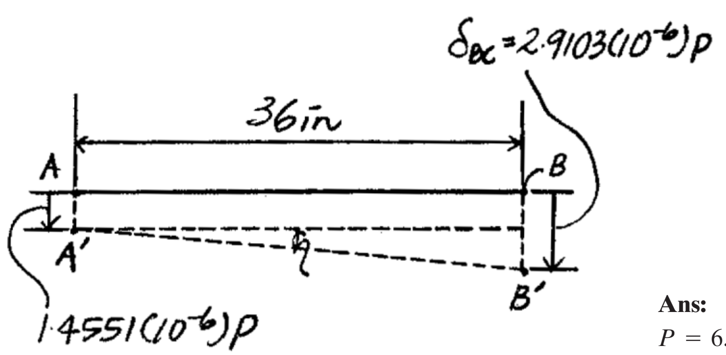
$$\theta = \frac{\delta_{BC} - \delta_{AD}}{36}; \qquad 0.2618(10^{-3}) = \frac{2.9103(10^{-6})P - 1.4551(10^{-6})P}{36}$$

$$P = 6476.93 \text{ lb} = 6.48 \text{ kip} \qquad \textbf{Ans.}$$

Since $\sigma_{BC} = 3.3953(6476.93) = 21.99 \text{ ksi} < \sigma_Y$ and $\sigma_{AD} = 1.6977(6476.93) = 11.00 \text{ ksi} < \sigma_Y$, the assumption is correct.

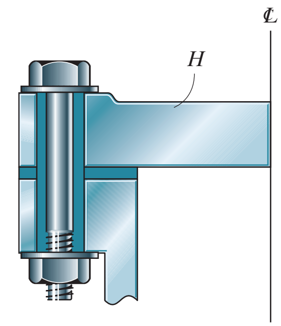


(a)



Ans:
$P = 6.48$ kip

***3–40.** The head H is connected to the cylinder of a compressor using six steel bolts. If the clamping force in each bolt is 800 lb, determine the normal strain in the bolts. Each bolt has a diameter of $\frac{3}{16}$ in. If $\sigma_Y = 40$ ksi and $E_{st} = 29(10^3)$ ksi, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



Normal Stress:

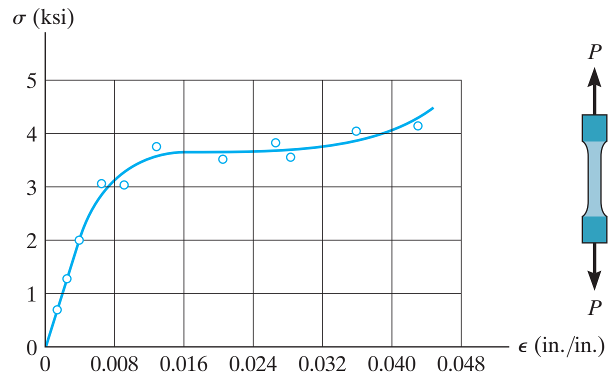
$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}\left(\frac{3}{16}\right)^2} = 28.97 \text{ ksi} < \sigma_\gamma = 40 \text{ ksi}$$

Normal Strain: Since $\sigma < \sigma_\gamma$, Hooke's law is still valid.

$$\epsilon = \frac{\sigma}{E} = \frac{28.97}{29(10^3)} = 0.000999 \text{ in./in.} \qquad \textbf{Ans.}$$

If the nut is unscrewed, the load is zero. Therefore, the strain $\epsilon = 0$ **Ans.**

3–41. The stress–strain diagram for polyethylene, which is used to sheath coaxial cables, is determined from testing a specimen that has a gauge length of 10 in. If a load P on the specimen develops a strain of $\epsilon = 0.024$ in./in., determine the approximate length of the specimen, measured between the gauge points, when the load is removed. Assume the specimen recovers elastically.



Modulus of Elasticity: From the stress–strain diagram, $\sigma = 2$ ksi when $\epsilon = 0.004$ in./in.

$$E = \frac{2 - 0}{0.004 - 0} = 0.500(10^3)\text{ ksi}$$

Elastic Recovery: From the stress–strain diagram, $\sigma = 3.70$ ksi when $\epsilon = 0.024$ in./in.

$$\text{Elastic recovery} = \frac{\sigma}{E} = \frac{3.70}{0.500(10^3)} = 0.00740\text{ in./in.}$$

Permanent Set:

$$\text{Permanent set} = 0.024 - 0.00740 = 0.0166\text{ in./in.}$$

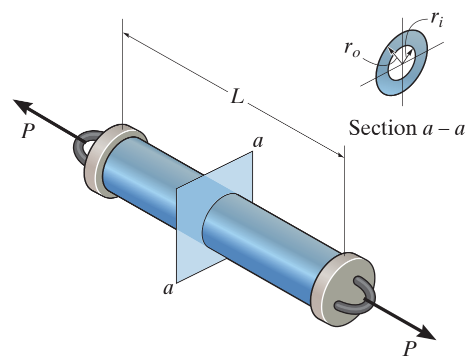
Thus,

$$\text{Permanent elongation} = 0.0166(10) = 0.166\text{ in.}$$

$$L = L_0 + \text{permanent elongation}$$
$$= 10 + 0.166$$
$$= 10.17\text{ in.} \qquad \textbf{Ans.}$$

Ans:
$L = 10.17$ in.

3–42. The pipe with two rigid caps attached to its ends is subjected to an axial force P. If the pipe is made from a material having a modulus of elasticity E and Poisson's ratio ν, determine the change in volume of the material.



Normal Stress: The rod is subjected to uniaxial loading. Thus, $\sigma_{\text{long}} = \dfrac{P}{A}$ and $\sigma_{\text{lat}} = 0$.

$$\delta V = A\delta L + 2\pi r L \delta r$$

$$= A\epsilon_{\text{long}} L + 2\pi r L \epsilon_{\text{lat}} r$$

Using Poisson's ratio and noting that $AL = \pi r^2 L = V$,

$$\delta V = \epsilon_{\text{long}} V - 2\nu\epsilon_{\text{long}} V$$

$$= \epsilon_{\text{long}}(1 - 2\nu)V$$

$$= \frac{\sigma_{\text{long}}}{E}(1 - 2\nu)V$$

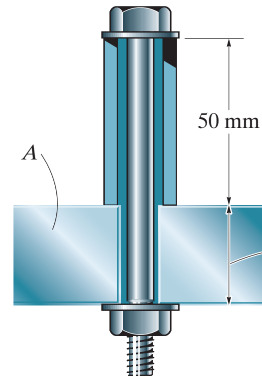
Since $\sigma_{\text{long}} = P/A$,

$$\delta V = \frac{P}{AE}(1 - 2\nu)AL$$

$$= \frac{PL}{E}(1 - 2\nu) \qquad \textbf{Ans.}$$

Ans:
$\delta V = \dfrac{PL}{E}(1 - 2\nu)$

3–43. The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



Normal Stress:

$$\sigma_b = \frac{P}{A_b} = \frac{8(10^3)}{\frac{\pi}{4}(0.008^2)} = 159.15 \text{ MPa}$$

$$\sigma_s = \frac{P}{A_s} = \frac{8(10^3)}{\frac{\pi}{4}(0.02^2 - 0.012^2)} = 39.79 \text{ MPa}$$

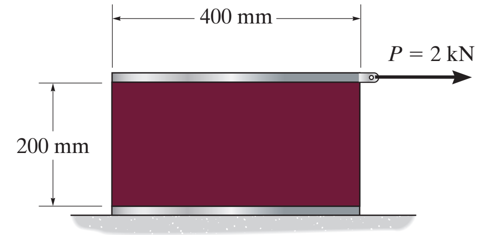
Normal Strain: Applying Hooke's Law

$$\epsilon_b = \frac{\sigma_b}{E_{al}} = \frac{159.15(10^6)}{70(10^9)} = 0.00227 \text{ mm/mm}$$ **Ans.**

$$\epsilon_s = \frac{\sigma_s}{E_{mg}} = \frac{39.79(10^6)}{45(10^9)} = 0.000884 \text{ mm/mm}$$ **Ans.**

Ans:
$\epsilon_b = 0.00227$ mm/mm, $\epsilon_s = 0.000884$ mm/mm

***3–44.** An acetal polymer block is fixed to the rigid plates at its top and bottom surfaces. If the top plate displaces 2 mm horizontally when it is subjected to a horizontal force $P = 2$ kN, determine the shear modulus of the polymer. The width of the block is 100 mm. Assume that the polymer is linearly elastic and use small angle analysis.

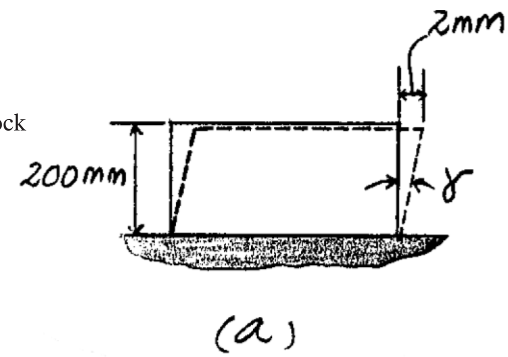


Normal and Shear Stress:

$$\tau = \frac{V}{A} = \frac{2(10^3)}{0.4(0.1)} = 50 \text{ kPa}$$

Referring to the geometry of the undeformed and deformed shape of the block shown in Fig. *a*,

$$\gamma = \frac{2}{200} = 0.01 \text{ rad}$$



Applying Hooke's Law,

$\tau = G\gamma;\quad 50(10^3) = G(0.01)$

$$G = 5 \text{ MPa} \qquad \textbf{Ans.}$$