P3.1 At the proportional limit, a 2 in. gage length of a 0.500 in. diameter alloy rod has elongated 0.0035in. and the diameter has been reduced by 0.0003 in. The total tension force on the rod was 5.45 kips. Determine the following properties of the material:
(a) the proportional limit.
(b) the modulus of elasticity.
(c) Poisson's ratio.

Solution

(a) Proportional Limit: The bar cross-sectional area is

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.500\text{ in.})^2 = 0.196350\text{ in.}^2$$

and thus, the normal stress corresponding to the 5.45 kip force is

$$\sigma = \frac{5.45\text{ kips}}{0.196350\text{ in.}^2} = 27.7566\text{ ksi}$$

Based on the problem statement, the stress in the bar is equal to the proportional limit; therefore,

$$\sigma_{PL} = \boxed{43.0\text{ ksi}}$$ **Ans.**

(b) Modulus of Elasticity: The longitudinal strain in the bar at the proportional limit is

$$\varepsilon_{\text{long}} = \frac{\delta}{L} = \frac{0.0035\text{ in.}}{2\text{ in.}} = 0.001750\text{ in./in.}$$

The modulus of elasticity is therefore

$$E = \frac{\sigma}{\varepsilon_{\text{long}}} = \frac{27.7566\text{ ksi}}{0.001750\text{ in./in.}} = \boxed{15{,}860\text{ ksi}}$$ **Ans.**

(c) Poisson's ratio: The longitudinal strain in the bar was calculated previously as

$$\varepsilon_{\text{long}} = 0.001750\text{ in./in.}$$

The lateral strain can be determined from the reduction of the diameter:

$$\varepsilon_{\text{lat}} = \frac{\Delta d}{d} = \frac{-0.0003\text{ in.}}{0.500\text{ in.}} = -0.000600\text{ in./in.}$$

Poisson's ratio for this specimen is therefore

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{-0.000600\text{ in./in.}}{0.001750\text{ in./in.}} = \boxed{0.343}$$ **Ans.**

P3.2 A solid circular rod with a diameter of d = 16 mm is shown in Figure P3.2. The rod is made of an aluminum alloy that has an elastic modulus of E = 72 GPa and Poisson's ratio of ν = 0.33. When subjected to the axial load P, the diameter of the rod decreases by 0.024 mm. Determine the magnitude of load P.

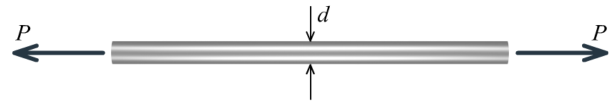


FIGURE P3.2

Solution

The lateral strain in the rod is

$$\varepsilon_{\text{lat}} = \frac{\Delta d}{d} = \frac{-0.024 \text{ mm}}{16 \text{ mm}} = -1{,}500 \times 10^{-6} \text{ mm/mm}$$

Using Poisson's ratio, compute the corresponding longitudinal strain:

$$\varepsilon_{\text{long}} = -\frac{\varepsilon_{\text{lat}}}{\nu} = -\frac{-1{,}500 \times 10^{-6} \text{ mm/mm}}{0.33} = 4{,}545.455 \times 10^{-6} \text{ mm/mm}$$

Use Hooke's law to calculate the stress in the rod:

$$\sigma = E\varepsilon_{\text{long}} = (72{,}000 \text{ MPa})(4{,}545.455 \times 10^{-6} \text{ mm/mm}) = 327.273 \text{ MPa}$$

The cross-sectional area of the rod is:

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(16 \text{ mm})^2 = 201.062 \text{ mm}^2$$

Consequently, the force P that acts on the rod must be

$$P = \sigma A = (327.273 \text{ MPa})(201.062 \text{ mm}^2) = 65{,}802.086 \text{ N} = \boxed{65.8 \text{ kN}}$$ **Ans.**

P3.3 The polymer bar shown in Figure P3.3 has a width of b = 50 mm, a depth of d = 100 mm, and a height of h = 270 mm. At a compressive load of P = 135 kN, the bar height contracts by Δh = –2.50 mm, and the bar depth elongates by Δd = 0.38 mm. At this load, the stress in the polymer bar is less than its proportional limit. Determine:
(a) the modulus of elasticity.
(b) Poisson's ratio
(c) the change in the bar width b.

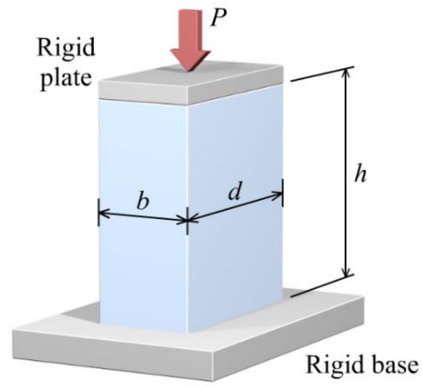


FIGURE P3.3

Solution

(a) Modulus of elasticity: The bar cross-sectional area is

$$A = (50\text{ mm})(100\text{ mm}) = 5{,}000\text{ mm}^2$$

and thus, the normal stress corresponding to the 135 kN axial load is

$$\sigma = \frac{(-135\text{ kN})(1{,}000\text{ N/kN})}{5{,}000\text{ mm}^2} = -27.0\text{ MPa}$$

The longitudinal strain in the bar is

$$\varepsilon_{\text{long}} = \frac{\delta}{L} = \frac{\Delta h}{h} = \frac{-2.5\text{ mm}}{270\text{ mm}} = -9{,}259.259 \times 10^{-6}\text{ mm/mm}$$

The modulus of elasticity is therefore

$$E = \frac{\sigma}{\varepsilon_{\text{long}}} = \frac{-27.0\text{ MPa}}{-9{,}259.259 \times 10^{-6}\text{ mm/mm}} = 2{,}916\text{ MPa} = \boxed{2.92\text{ GPa}}$$ **Ans.**

(b) Poisson's ratio: The lateral strain can be determined from the elongation of the bar depth:

$$\varepsilon_{\text{lat}} = \frac{\Delta d}{d} = \frac{0.380\text{ mm}}{100\text{ mm}} = 3{,}800 \times 10^{-6}\text{ mm/mm}$$

Poisson's ratio for this specimen is therefore

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{3{,}800 \times 10^{-6}\text{ mm/mm}}{-9{,}259.259 \times 10^{-6}\text{ mm/mm}} = \boxed{0.410}$$ **Ans.**

(c) Change in the bar width *b*: The change in bar width b can be found from the lateral strain:

$$\varepsilon_{\text{lat}} = \frac{\Delta b}{b}$$

$$\therefore \Delta b = \varepsilon_{\text{lat}} b = (3{,}800 \times 10^{-6}\text{ mm/mm})(50\text{ mm}) = \boxed{0.1900\text{ mm}}$$ **Ans.**

P3.4 A 0.625 in. thick rectangular alloy bar is subjected to a tensile load *P* by pins at *A* and *B* as shown in Figure P3.4. The width of the bar is *w* = 2.00 in. Strain gages bonded to the specimen measure the following strains in the longitudinal (*x*) and transverse (*y*) directions: ε_x = 1,140 με and ε_y = −315 με.

(a) Determine Poisson's ratio for this specimen.
(b) If the measured strains were produced by an axial load of *P* = 17.4 kips, what is the modulus of elasticity for this specimen?

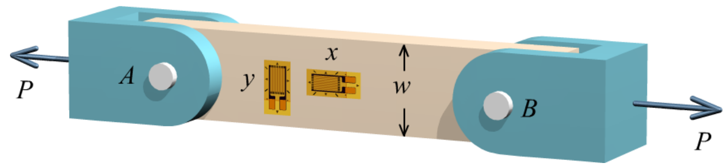


FIGURE P3.4

Solution

(a) Poisson's ratio for this specimen is

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{-315\ \mu\varepsilon}{1{,}140\ \mu\varepsilon} = \boxed{0.276}$$ **Ans.**

(b) The bar cross-sectional area is

$$A = (2.00\text{ in.})(0.625\text{ in.}) = 1.25\text{ in.}^2$$

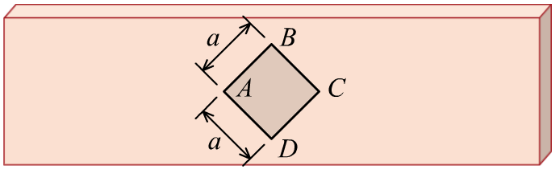
and so the normal stress for an axial load of *P* = 17.4 kips is

$$\sigma = \frac{17.4\text{ kips}}{1.25\text{ in.}^2} = 13.920\text{ ksi}$$

The modulus of elasticity is thus

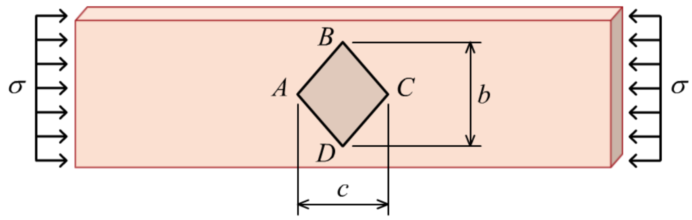
$$E = \frac{\sigma}{\varepsilon_{\text{long}}} = \frac{13.920\text{ ksi}}{1{,}140\times10^{-6}\text{ in./in.}} = 12{,}210.5\text{ ksi} = \boxed{12{,}200\text{ ksi}}$$ **Ans.**

P3.5 A 40 mm by 40 mm square *ABCD* (i.e., a = 40 mm) is drawn on a rectangular bar prior to loading (see Figure P3.5*a*). A uniform normal stress of σ = 54 MPa is then applied to the ends of the rectangular bar, and square *ABCD* is deformed into the shape of a rhombus, as shown in the Figure P3.5*b*. The dimensions of the rhombus after loading are b = 56.88 mm and c = 55.61 mm. Determine the modulus of elasticity for the material. Assume that the material behaves elastically for the applied stress.



Initial square drawn on bar before loading.

FIGURE P3.5*a*



Rhombus after bar is loaded by stress σ.

FIGURE P3.5*b*

Solution

The length of diagonal *AC* (and also diagonal *BD*) before deformation is

$$L_{AC} = L_{BD} = \sqrt{a^2 + a^2} = a\sqrt{2} = (40\,\text{mm})\sqrt{2} = 56.569\text{ mm}$$

After the normal stress is applied, the longitudinal strain is

$$\varepsilon_{\text{long}} = \frac{c - a\sqrt{2}}{a\sqrt{2}} = \frac{55.61\text{ mm} - 56.569\text{ mm}}{56.569\text{ mm}} = -16.9448\times10^{-3}\text{ mm/mm}$$

The modulus of elasticity is thus

$$E = \frac{\sigma}{\varepsilon_{\text{long}}} = \frac{-54\text{ MPa}}{-16.9448\times10^{-3}\text{ mm/mm}} = 3{,}186.8\text{ MPa} = \boxed{3.19\text{ GPa}}$$ **Ans.**

P3.6 A nylon [E = 2,500 MPa; ν = 0.4] bar is subjected to an axial load that produces a normal stress of σ. Before the load is applied, a line having a slope of 3:2 (i.e., 1.5) is marked on the bar as shown in Figure P3.6. Determine the slope of the line when σ = 105 MPa.

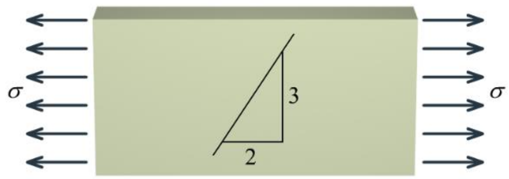


FIGURE P3.6

Solution

From the given stress and elastic modulus, compute the longitudinal strain in the bar:

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{105 \text{ MPa}}{2{,}500 \text{ MPa}} = 0.04200 \text{ mm/mm}$$

Use Poisson's ratio to calculate the lateral strain:

$$\varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -(0.4)(0.04200 \text{ mm/mm}) = -0.01680 \text{ mm/mm}$$

Before deformation, the slope of the line is

$$slope = \frac{3}{2} = 1.500$$

After deformation, the slope of the line is

$$slope = \frac{3(1-0.01680)}{2(1+0.04200)} = \frac{2.950}{2.084} = \boxed{1.415}$$ **Ans.**

P3.7 A nylon [E = 360 ksi; ν = 0.4] rod (1) having a diameter of d_1 = 2.50 in. is placed inside a steel [E = 29,000 ksi; ν = 0.29] tube (2) as shown in Figure P3.7. The inside diameter of the steel tube is d_2 = 2.52 in. An external load P is applied to the nylon rod, compressing it. At what value of P will the space between the nylon rod and the steel tube be closed?

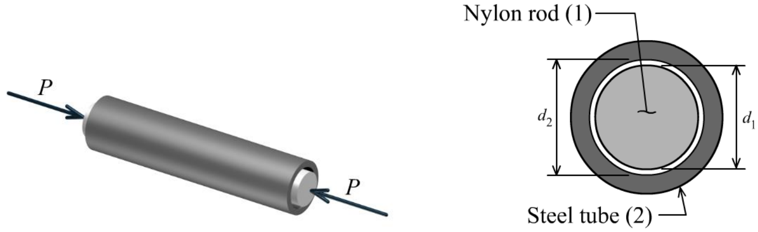


FIGURE P3.7

Solution

To close the space between the nylon rod and the steel tube, the lateral strain of the nylon rod must be:

$$\varepsilon_{\text{lat}} = \frac{2.52\text{ in.} - 2.50\text{ in.}}{2.50\text{ in.}} = 0.0080\text{ in./in.}$$

From the given Poisson's ratio, the longitudinal strain in the nylon rod must be

$$\varepsilon_{\text{long}} = -\frac{\varepsilon_{\text{lat}}}{\nu} = -\frac{0.0080\text{ in./in.}}{0.4} = -0.0200\text{ in./in.}$$

Use Hooke's law to calculate the corresponding normal stress:

$$\sigma = E\varepsilon_{\text{long}} = (360\text{ ksi})(-0.0200\text{ in./in.}) = -7.2000\text{ ksi}$$

The cross-sectional area of the nylon rod is

$$A = \frac{\pi}{4}(2.50\text{ in.})^2 = 4.9087\text{ in.}^2$$

Therefore, the force required to make the nylon rod touch the inner wall of the tube is

$$P = \sigma A = (-7.2000\text{ ksi})(4.9087\text{ in.}^2) = -35.3429\text{ kips} = \boxed{35.3\text{ kips (C)}}$$ **Ans.**

P3.8 A metal specimen with an original diameter of 0.500 in. and a gage length of 2.000 in. is tested in tension until fracture occurs. At the point of fracture, the diameter of the specimen is 0.260 in. and the fractured gage length is 3.08 in. Calculate the ductility in terms of percent elongation and percent reduction in area.

Solution

Percent elongation is simply the longitudinal strain at fracture:

$$\varepsilon = \frac{\delta}{L} = \frac{(3.08\text{ in.} - 2.000\text{ in.})}{2.000\text{ in.}} = \frac{1.08\text{ in.}}{2.000\text{ in.}} = 0.54\text{ in./in.}$$

$$\therefore \text{ percent elongation} = \boxed{54\%}$$ **Ans.**

The initial cross-sectional area of the specimen is

$$A_0 = \frac{\pi}{4}(0.500\text{ in.})^2 = 0.196350\text{ in.}^2$$

The final cross-sectional area at the fracture location is

$$A_f = \frac{\pi}{4}(0.260\text{ in.})^2 = 0.053093\text{ in.}^2$$

The percent reduction in area is

$$\text{percent reduction of area} = \frac{A_0 - A_f}{A_0}(100\%)$$

$$= \frac{(0.196350\text{ in.}^2 - 0.053093\text{ in.}^2)}{0.196350\text{ in.}^2}(100\%) = \boxed{73.0\%}$$ **Ans.**

P3.9 A portion of the stress-strain curve for a stainless steel alloy is shown in Figure P3.9. A 350-mm-long bar is loaded in tension until it elongates 2.0 mm and then the load is removed.
(a) What is the permanent set in the bar?
(b) What is the length of the unloaded bar?
(c) If the bar is reloaded, what will be the proportional limit?

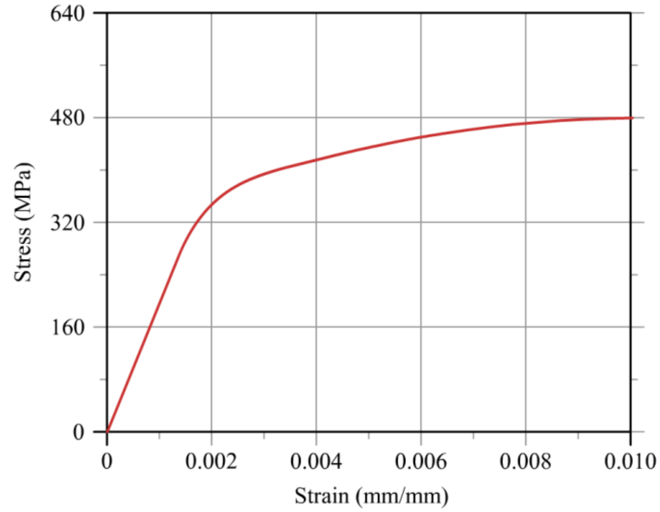
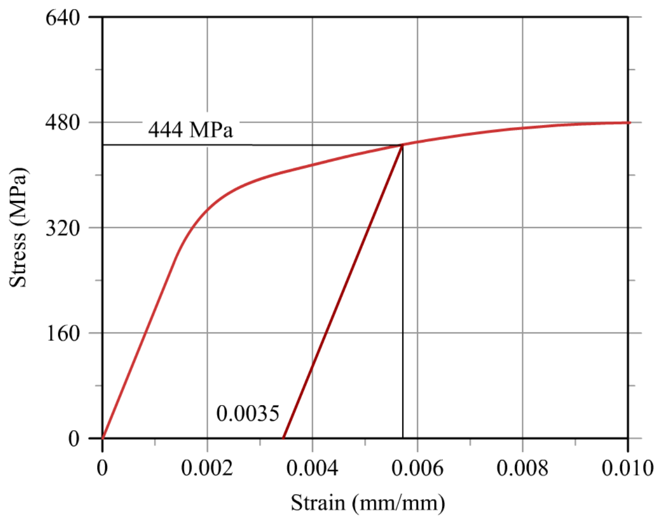


FIGURE P3.9

Solution



(a) The normal strain in the specimen is

$$\varepsilon = \frac{\delta}{L} = \frac{2.0 \text{ mm}}{350 \text{ mm}} = 0.005714 \text{ mm/mm}$$

Construct a line parallel to the elastic modulus line that passes through the data curve at a strain of $\varepsilon =$ 0.005714 mm/mm. The strain value at which this modulus line intersects the strain axis is the permanent set:

permanent set = $\boxed{0.0035 \text{ mm/mm}}$ **Ans.**

(b) The length of the unloaded bar is therefore:

$$\delta = \varepsilon L = (0.0035 \text{ mm/mm})(350 \text{ mm}) = 1.225 \text{ mm}$$

$$L_f = 350 \text{ mm} + 1.225 \text{ mm} = \boxed{351.225 \text{ mm}}$$ **Ans.**

(c) From the stress-strain curve, the reload proportional limit is $\boxed{444 \text{ MPa}}$. **Ans.**

P3.10 A plastic block is bonded to a fixed base and to a horizontal rigid plate as shown in Figure P3.10. The shear modulus of the plastic is G = 45,000 psi, and the block dimensions are a = 4.0 in., b = 2.0 in., and c = 1.50 in. A horizontal force of P = 8,500 lb is applied to the rigid plate. Determine the horizontal deflection of the rigid plate.

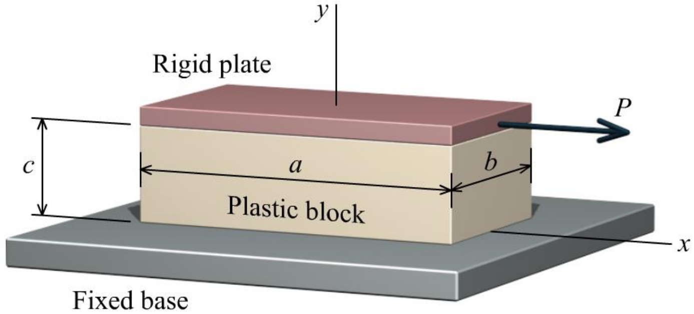


FIGURE P3.10

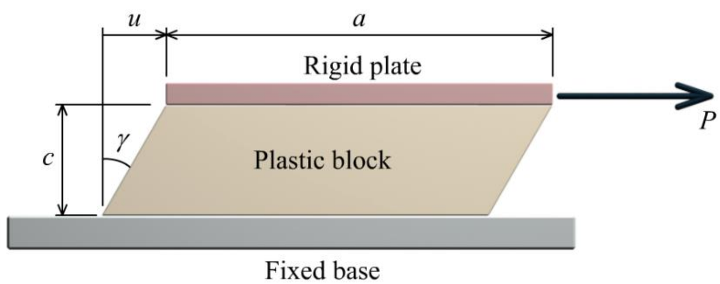
Solution

Determine the shear stress from the horizontal force P and the block area (i.e., the area of the y plane surface of the block). To determine the area, visualize the surface that is bonded to the fixed base. This area has dimensions of $a \times b$. The average shear stress acting on the block is therefore:

$$\tau = \frac{V}{A} = \frac{8,500 \text{ lb}}{(4.0 \text{ in.})(2.0 \text{ in.})} = 1,062.5 \text{ psi}$$

Since the shear modulus G is given, the shear strain can be calculated from Hooke's law for shear stress and shear strain:

$$\tau = G\gamma \qquad \therefore \gamma = \frac{\tau}{G} = \frac{1,062.5 \text{ psi}}{45,000 \text{ psi}} = 0.023611 \text{ rad}$$



Recall that shear strain is an angle. From the angle γ and the block thickness c, the horizontal deflection u of the rigid plate can be determined from:

$$\tan\gamma = \frac{u}{c} \qquad \therefore u = c\tan\gamma = (1.5 \text{ in.})\tan(0.023611 \text{ rad}) = 0.035423 \text{ in.} = \boxed{0.0354 \text{ in.}}$$

Ans.

P3.11 A 0.5 in. thick plastic panel is bonded to the pin-jointed steel frame shown in Figure P3.11. Determine the magnitude of the force P that would displace bar AB to the right by 0.8 in. Assume that a = 4.0 ft, b = 6.0 ft, and G = 70,000 psi for the plastic. Neglect the deformation of the steel frame.

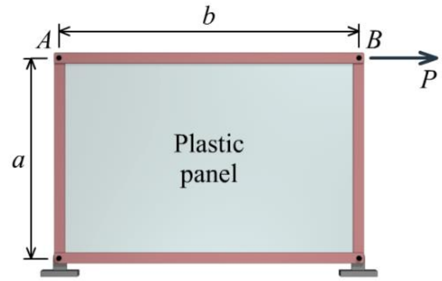


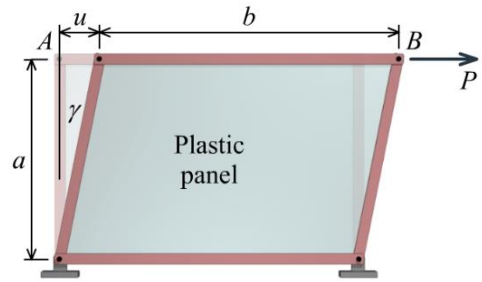
FIGURE P3.11

Solution

The horizontal displacement of bar AB is u = 0.8 in. This displacement creates a shear strain of

$$\tan\gamma = \frac{u}{a} = \frac{0.8\text{in.}}{(4.0\text{ ft})(12\text{ in./ft})} = 0.016667$$

$$\therefore \gamma = 0.016665\text{ rad}$$



From Hooke's law, determine the shear stress associated with this shear strain:

$$\tau = G\gamma = (70{,}000\text{ psi})(0.016665\text{ rad}) = 1{,}166.559\text{ psi}$$

Determine the horizontal force P from the shear area of the plastic panel. This area has dimensions of $b \times t$.

$$\tau = \frac{V}{A} = \frac{P}{bt}$$

$$\therefore P = \tau bt = (1{,}166.559\text{ psi})(6\text{ ft})(12\text{ in./ft})(0.5\text{ in.}) = \boxed{42{,}000\text{ lb}}$$ **Ans.**

P3.12 The complete stress–strain diagram for a particular stainless steel alloy is shown in Figure P3.12*a*/13*a*. This diagram has been enlarged in Figure P3.12*b*/13*b* to show in more detail the linear portion of the stress-strain diagram. A rod made from this material is initially 800 mm long at a temperature of 20°C. After a tension force is applied to the rod and the temperature is increased by 200°C, the length of the rod is 804 mm. Determine the stress in the rod, and state whether the elongation in the rod is elastic or inelastic. Assume the coefficient of thermal expansion for this material is 18×10^{-6}/°C.

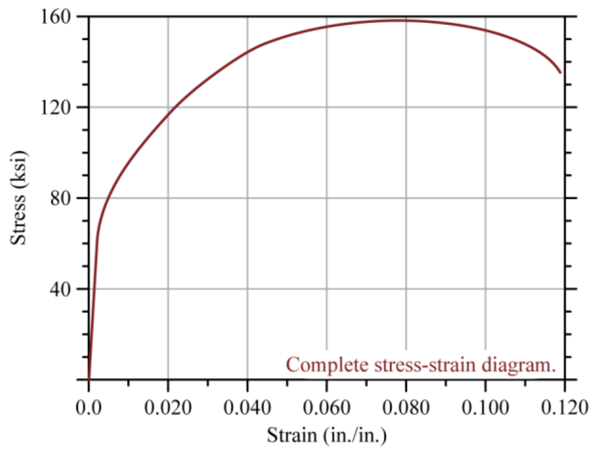


FIGURE P3.12*a*/13*a*

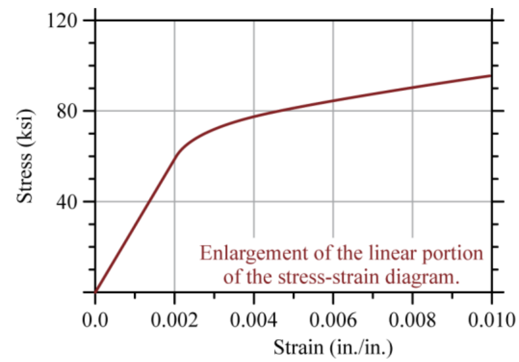


FIGURE P3.12*b*/13*b*

Solution

The 4 mm total elongation of the rod is due to a combination of load and temperature increase. The 200°C temperature increase causes a normal strain of:

$$\varepsilon_T = \alpha\,\Delta T = (18\times10^{-6}/°C)(200°C) = 0.003600 \text{ mm/mm}$$

which means that the rod elongates

$$\delta_T = \varepsilon_T L = (0.003600 \text{ mm/mm})(800 \text{ mm}) = 2.8800 \text{ mm}$$

The portion of the 4 mm total elongation due to load is therefore

$$\delta_\sigma = \delta - \delta_T = 4 \text{ mm} - 2.8800 \text{ mm} = 1.1200 \text{ mm}$$

The strain corresponding to this elongation is

$$\varepsilon_\sigma = \frac{\delta_\sigma}{L} = \frac{1.1200 \text{ mm}}{800 \text{ mm}} = 0.001400 \text{ mm/mm}$$

By inspection of the stress-strain curve, this strain is clearly in the linear region. Therefore, **the rod is elastic in this instance.**

For the linear region, the elastic modulus can be determined from the lower scale plot:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{(400 \text{ MPa} - 0)}{(0.002 \text{ mm/mm} - 0)} = 200{,}000 \text{ MPa}$$

Using Hooke's law (or directly from the σ–ε diagram), the stress corresponding to the 0.001400 mm/mm strain is

$$\sigma = E\varepsilon_\sigma = (200{,}000 \text{ MPa})(0.001400 \text{ mm/mm}) = \boxed{280 \text{ MPa}}$$ **Ans.**

P3.13 A tensile test specimen of stainless steel alloy having a diameter of 12.6 mm and a gage length of 50 mm was tested to fracture. The complete stress–strain diagram for this specimen is shown in Figure P3.12*a*/13*a*. This diagram has been enlarged in Figure P3.12*b*/13*b* to show in more detail the linear portion of the stress-strain diagram. Determine:

(a) the modulus of elasticity.
(b) the proportional limit.
(c) the ultimate strength.
(d) the yield strength (0.20% offset).
(e) the fracture stress.
(f) the true fracture stress if the final diameter of the specimen at the location of the fracture was 8.89 mm.

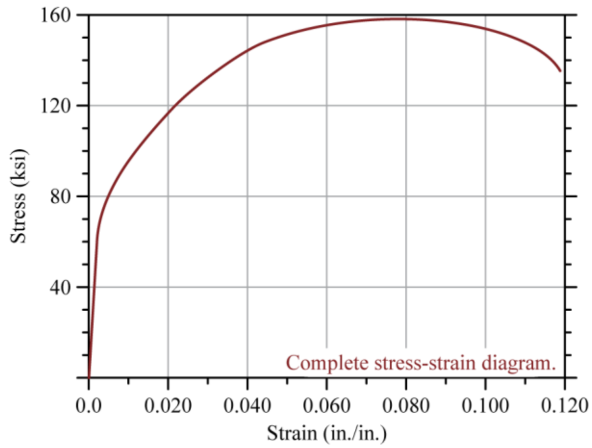


FIGURE P3.12*a*/13*a*

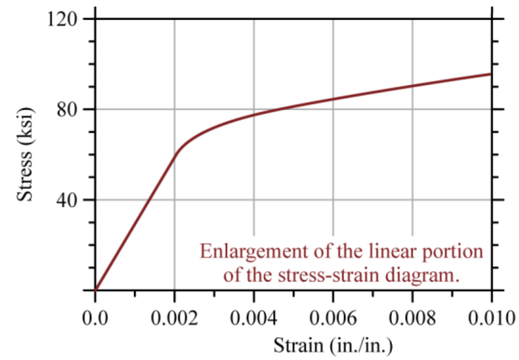


FIGURE P3.12*b*/13*b*

Solution

From the stress-strain curve, the proportional limit will be taken as $\sigma = 60$ ksi at a strain of $\varepsilon = 0.002$. (Obviously, there can be quite a bit of leeway in pulling numbers from such a limited plot.)

(a) The modulus of elasticity is

$$E = \frac{\sigma}{\varepsilon} = \frac{60 \text{ ksi}}{0.002 \text{ in./in.}} = \boxed{30{,}000 \text{ ksi}}$$ **Ans.**

(b) From the diagram, the proportional limit is taken as

$$\sigma_{PL} = \boxed{60 \text{ ksi}}$$ **Ans.**

(c) The ultimate strength is

$$\sigma_U = \boxed{159 \text{ ksi}}$$ **Ans.**

(d) The yield strength is

$$\sigma_Y = \boxed{80 \text{ ksi}}$$ **Ans.**

(e) The fracture stress is

$$\sigma_{\text{fracture}} = \boxed{135 \text{ ksi}}$$ **Ans.**

(f) The original cross-sectional area of the specimen is

$$A_0 = \frac{\pi}{4}(0.495 \text{ in.})^2 = 0.192442 \text{ in.}^2$$

The cross-sectional area of the specimen at the fracture location is

$$A_f = \frac{\pi}{4}(0.350\text{ in.})^2 = 0.096211\text{ in.}^2$$

The true fracture stress is therefore

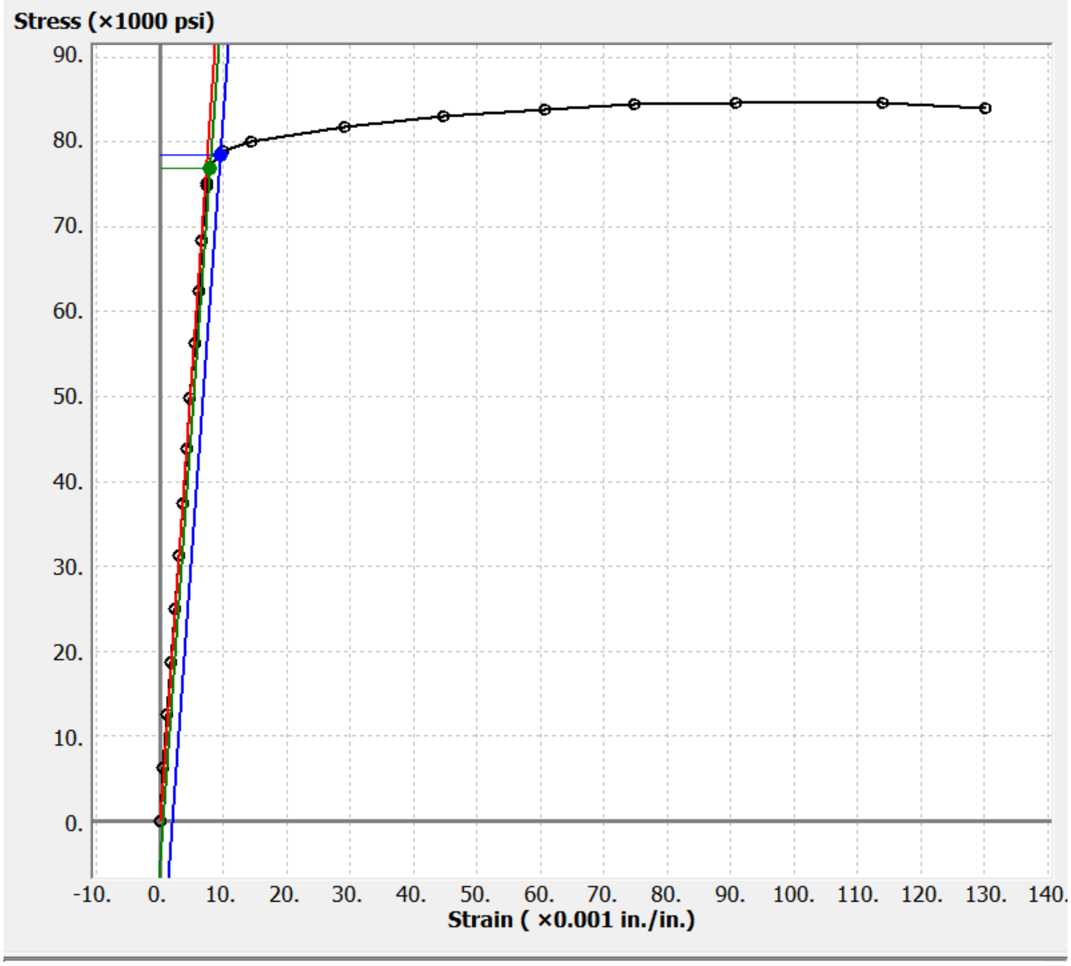
$$\text{true } \sigma_{\text{fracture}} = (135\text{ ksi})\frac{0.192442\text{ in.}^2}{0.096211\text{ in.}^2} = \boxed{270\text{ ksi}}$$ **Ans.**

P3.14 A 7075-T651 aluminum alloy specimen with a diameter of 0.500 in. and a 2.0-in. gage length was tested to fracture. Load and deformation data obtained during the test are given. Determine:
(a) the modulus of elasticity.
(b) the proportional limit.
(c) the yield strength (0.20% offset).
(d) the ultimate strength.
(e) the fracture stress.
(f) the true fracture stress if the final diameter of the specimen at the location of the fracture was 0.387 in.

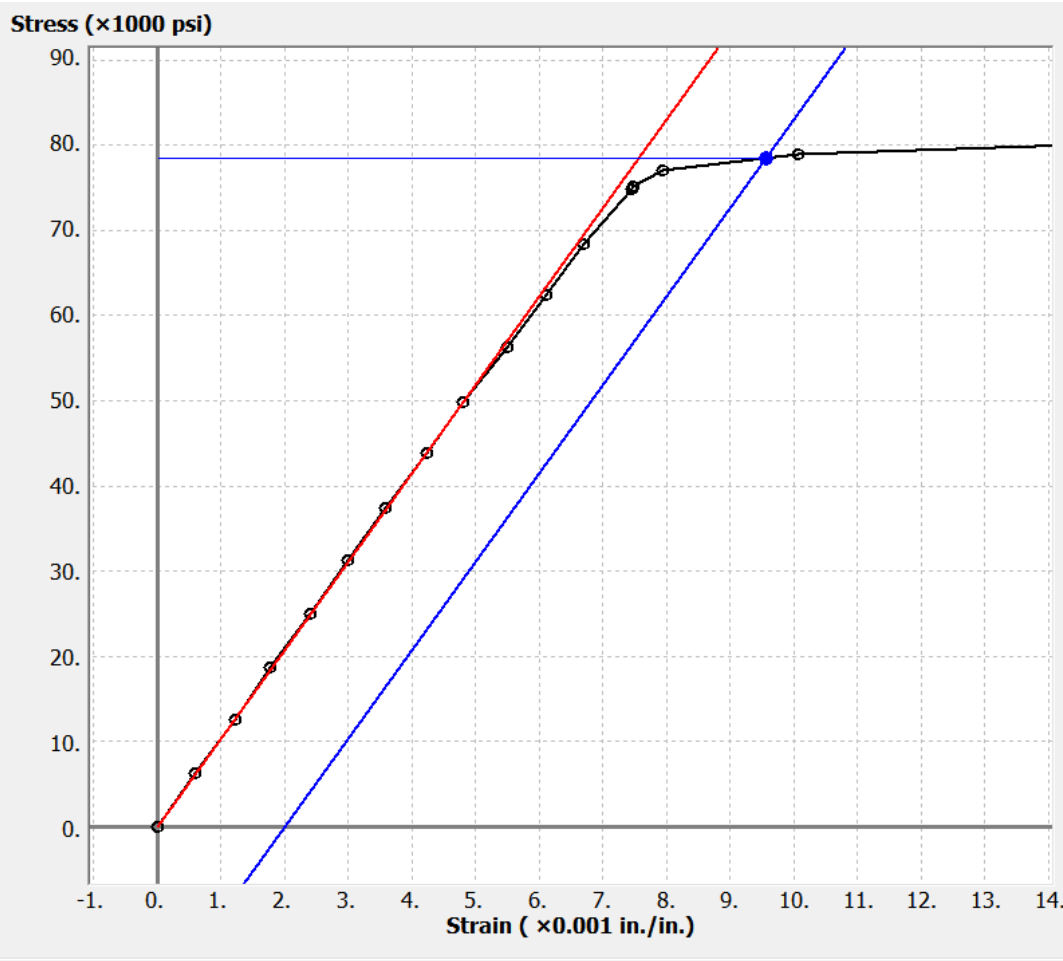
Load (lb)	Change in Length (in.)	Load (lb)	Change in Length (in.)
0	0	14,690	0.0149
1,221	0.0012	14,744	0.0150
2,479	0.0024	15,119	0.0159
3,667	0.0035	15,490	0.0202
4,903	0.0048	15,710	0.0288
6,138	0.0060	16,032	0.0581
7,356	0.0072	16,295	0.0895
8,596	0.0085	16,456	0.1214
9,783	0.0096	16,585	0.1496
11,050	0.0110	16,601	0.1817
12,247	0.0122	16,601	0.2278
13,434	0.0134	16,489	0.2605
		16,480	fracture

Solution

The plot of the stress-strain data is shown below.



The same plot is shown below, enlarged to better show the linear portion of the stress-strain curve.



The initial cross-sectional area of the specimen is

$$A = \frac{\pi}{4}(0.500 \text{ in.})^2 = 0.19635 \text{ in.}^2$$

(a) Using the data point for the 9,783 lb load and 0.0096 in. elongation, the modulus of elasticity can be calculated as

$$\sigma = \frac{9,783 \text{ lb}}{0.19635 \text{ in.}^2} = 49,824.3 \text{ psi}$$

$$\varepsilon = \frac{0.0096 \text{ in.}}{2 \text{ in.}} = 0.0048 \text{ in./in.}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{49,824.3 \text{ psi}}{0.0048 \text{ in./in.}} = \boxed{10,380,000 \text{ psi}}$$ **Ans.**

(b) Using the data point for the 9,783 lb load and 0.0096 in. elongation, the proportional limit is calculated as

$$\sigma_{PL} = \boxed{49,800 \text{ psi}}$$ **Ans.**

(c) The yield strength by the 0.20% offset method is

$$\sigma_Y = \boxed{78{,}500 \text{ psi}}$$ **Ans.**

(d) The ultimate strength is

$$\sigma_U = \frac{16{,}601 \text{ lb}}{0.19635 \text{ in.}^2} = \boxed{84{,}600 \text{ psi}}$$ **Ans.**

(e) The fracture stress is

$$\sigma_{\text{fracture}} = \frac{16{,}480 \text{ lb}}{0.19635 \text{ in.}^2} = \boxed{83{,}900 \text{ psi}}$$ **Ans.**

(f) The cross-sectional area of the specimen at the fracture location is

$$A_f = \frac{\pi}{4}(0.387 \text{ in.})^2 = 0.11763 \text{ in.}^2$$

The true fracture stress is therefore

$$\text{true } \sigma_{\text{fracture}} = \frac{16{,}480 \text{ lb}}{0.11763 \text{ in.}^2} = \boxed{140{,}100 \text{ psi}}$$ **Ans.**

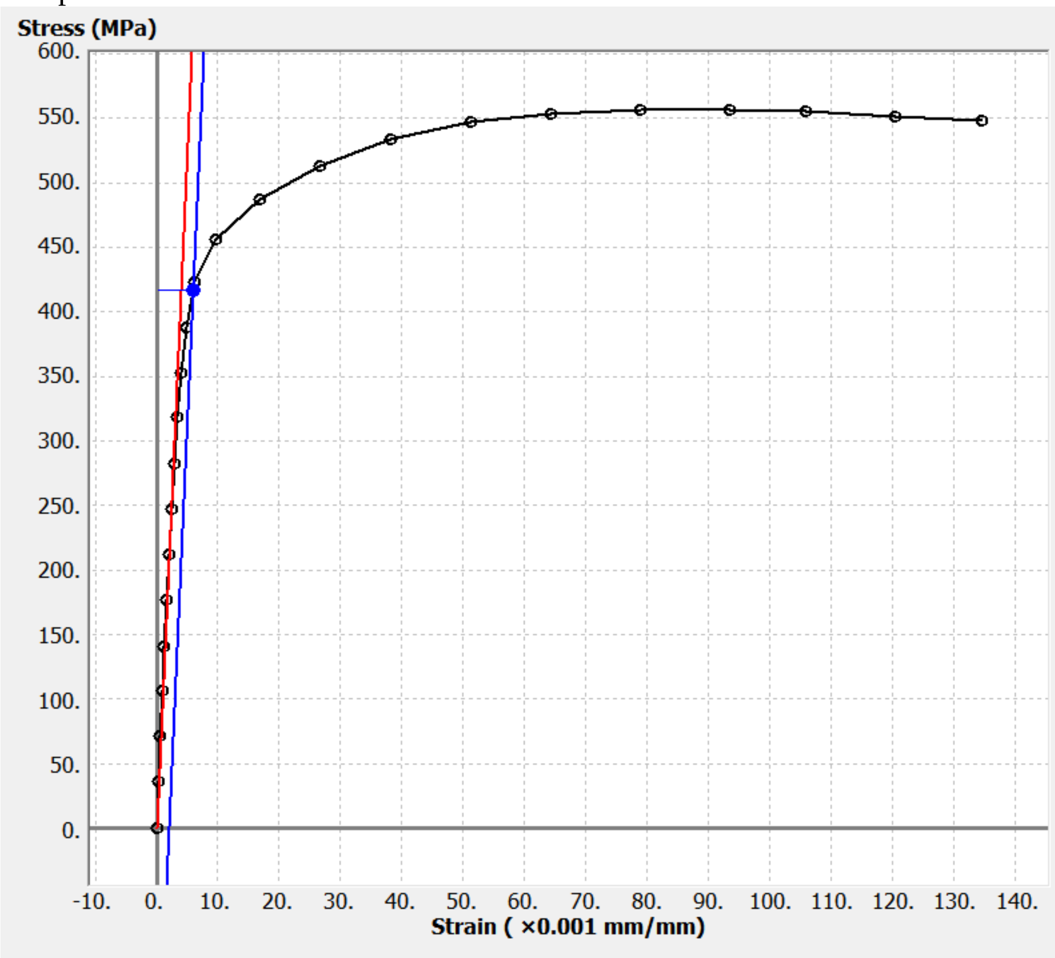
P3.15 A Grade 2 Titanium tension test specimen has a diameter of 12.60 mm and a gage length of 50 mm. In a test to fracture, the stress and strain data below were obtained. Determine:

(a) the modulus of elasticity.
(b) the proportional limit.
(c) the yield strength (0.20% offset).
(d) the ultimate strength.
(e) the fracture stress.
(f) the true fracture stress if the final diameter of the specimen at the location of the fracture was 9.77 mm.

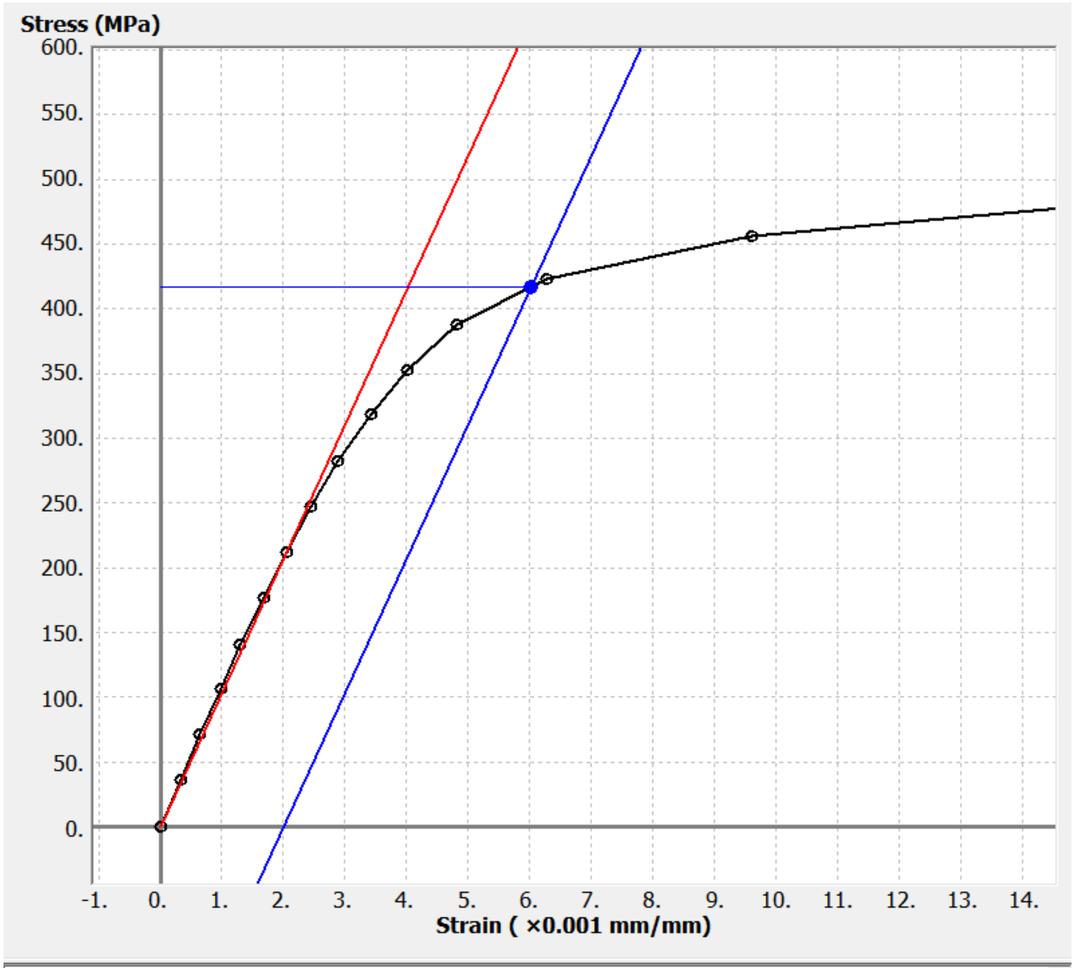
Load (kN)	Change in Length (mm)	Load (kN)	Change in Length (mm)
0.00	0.000	52.74	0.314
4.49	0.017	56.95	0.480
8.84	0.032	60.76	0.840
13.29	0.050	63.96	1.334
17.57	0.064	66.61	1.908
22.10	0.085	68.26	2.562
26.46	0.103	69.08	3.217
30.84	0.123	69.41	3.938
35.18	0.144	69.39	4.666
39.70	0.171	69.25	5.292
43.95	0.201	68.82	6.023
48.44	0.241	68.35	6.731
		68.17	fracture

Solution

The plot of the stress-strain data is shown below.



The same plot is shown below, enlarged to better show the linear portion of the stress-strain curve.



The initial cross-sectional area of the specimen is

$$A = \frac{\pi}{4}(12.60 \text{ mm})^2 = 124.690 \text{ mm}^2$$

(a) Using the data point for the 30.84 kN load and 0.123 mm elongation, the modulus of elasticity can be calculated as

$$\sigma = \frac{30,840 \text{ N}}{124.690 \text{ mm}^2} = 247.333 \text{ MPa}$$

$$\varepsilon = \frac{0.123 \text{ mm}}{50 \text{ mm}} = 0.002456 \text{ mm/mm}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{247.333 \text{ MPa}}{0.002456 \text{ mm/mm}} = 100,705.6 \text{ MPa} = \boxed{100.7 \text{ GPa}}$$ **Ans.**

(b) From the diagram, the proportional limit is taken as

$$\sigma_{PL} = \boxed{247 \text{ MPa}}$$ **Ans.**

(c) The yield strength by the 0.20% offset method is

$$\sigma_Y = \boxed{417 \text{ MPa}}$$ **Ans.**

(d) The ultimate strength is

$$\sigma_U = \frac{69,410 \text{ N}}{124.690 \text{ mm}^2} = \boxed{557 \text{ MPa}}$$ **Ans.**

(e) The fracture stress is

$$\sigma_{\text{fracture}} = \frac{68,170 \text{ N}}{124.690 \text{ mm}^2} = \boxed{547 \text{ MPa}}$$ **Ans.**

(f) The cross-sectional area of the specimen at the fracture location is

$$A_f = \frac{\pi}{4}(9.77 \text{ mm})^2 = 74.969 \text{ mm}^2$$

The true fracture stress is therefore

$$\text{true } \sigma_{\text{fracture}} = \frac{68,170 \text{ N}}{74.969 \text{ mm}^2} = \boxed{909 \text{ MPa}}$$ **Ans.**

P3.16 Compound axial member *ABC* has a uniform diameter of d = 1.50 in. Segment (1) is an aluminum [E_1 = 10,000 ksi] alloy rod with a length of L_1 = 90 in. Segment (2) is a copper [E_2 = 17,000 ksi] alloy rod with a length of L_2 = 130 in. When axial force P is applied, a strain gage attached to copper segment (2) measures a normal strain of ε_2 = 2,100 μin./in. in the longitudinal direction. What is the total elongation of member *ABC*?

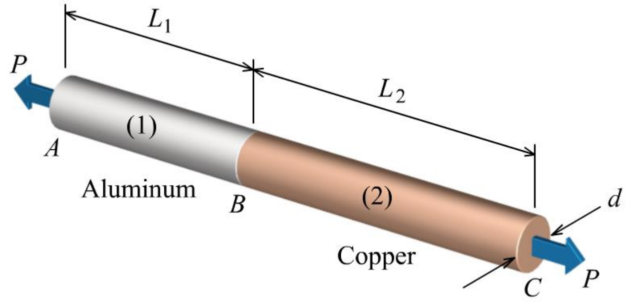


FIGURE P3.16

Solution

The strain in segment (2) is measured as 2,100 μin./in. From Hooke's law, the stress in segment (2) is:

$$\sigma_2 = E_2\varepsilon_2 = (17,000 \text{ ksi})(2,100\times10^{-6} \text{ in./in.}) = 35.70 \text{ ksi}$$

The deformation in segment (2) is:

$$\delta_2 = \varepsilon_2 L_2 = (2,100\times10^{-6} \text{ in./in.})(130 \text{ in.}) = 0.2730 \text{ in.}$$

The area of segment (2) is:

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(1.50 \text{ in.})^2 = 1.7671 \text{ in.}^2$$

The diameters of segments (1) and (2) are equal; thus, $A_1 = A_2$ = 1.7671 in.^{2}. The internal axial force in segments (1) and (2) is the same, and so are the normal stresses,

$$\sigma_1 = \sigma_2 = 35.70 \text{ ksi}$$

From Hooke's law, the strain in segment (1) must be

$$\sigma_1 = E_1\varepsilon_1$$

$$\therefore \varepsilon_1 = \frac{\sigma_1}{E_1} = \frac{35.70 \text{ ksi}}{10,000 \text{ ksi}} = 3,570\times10^{-6} \text{ in./in.}$$

Consequently, the deformation of segment (1) is

$$\delta_1 = \varepsilon_1 L_1 = (3,570\times10^{-6} \text{ in./in.})(90 \text{ in.}) = 0.3213 \text{ in.}$$

The total deformation of member *ABC* is equal to the sum of the deformations in segments (1) and (2):

$$\delta_{ABC} = \delta_1 + \delta_2$$

$$= 0.3213 \text{ in.} + 0.2730 \text{ in.}$$

$$= \boxed{0.594 \text{ in.}}$$ **Ans.**

P3.17 An aluminum alloy [E = 70 GPa; ν = 0.33; $\alpha = 23.0\times10^{-6}$/°C] plate is subjected to a tensile load P as shown in Figure P3.17. The plate has a depth of d = 260 mm, a cross-sectional area of A = 6,500 mm^2, and a length of L = 4.5 m. The initial longitudinal normal strain in the plate is zero. After load P is applied and the temperature of the plate has been increased by ΔT = 56°C, the longitudinal normal strain in the plate is found to be 2,950 με. Determine:
(a) the magnitude of load P.
(b) the change in plate depth Δd.

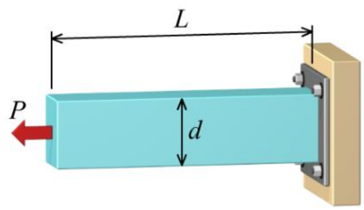


FIGURE P3.17

Solution

(a) Magnitude of load *P*: The total strain in the plate is measured as 2,950 με (elongation). Part of this strain is due to the stress acting in the bar and part of this strain is due to the temperature increase. The strain caused by the temperature change is

$$\varepsilon_T = \alpha\,\Delta T = \left(23.0\times10^{-6}\ /\ °\text{C}\right)\left(56°\text{C}\right) = 1,288\times10^{-6}\ \text{mm/mm}$$

Since the total strain is ε = 2,950 με = 0.002950 mm/mm, the strain caused by the normal stress in the bar must be:

$$\varepsilon_\sigma = \varepsilon - \varepsilon_T = 2,950\times10^{-6}\ \text{mm/mm} - 1,288\times10^{-6}\ \text{mm/mm} = 1,662\times10^{-6}\ \text{mm/mm}$$

From Hooke's law, the stress in the bar is therefore:

$$\sigma = E\varepsilon_\sigma = \left(70,000\ \text{MPa}\right)\left(1,662\times10^{-6}\ \text{mm/mm}\right) = 116.34\ \text{MPa}$$

and thus, the force in the bar is:

$$P = \sigma A = \left(116.34\ \text{N/mm}^2\right)\left(6,500\ \text{mm}^2\right) = 756,210\ \text{N} = \boxed{756\ \text{kN}}$$

(b) Change in plate depth Δ*d*: Due to the Poisson effect, the plate will contract in the lateral direction. However, the thermal strain will cause the width of the bar to increase. We will need to separately consider contract each of these strains.

The strain in the lateral direction due to the Poisson effect is calculated from the longitudinal strain due solely to the normal stress. This portion of the overall lateral strain is:

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}}$$

$$\therefore\ \varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -\left(0.33\right)\left(1,662\times10^{-6}\ \text{mm/mm}\right) = -548.46\ \times10^{-6}\ \text{mm/mm}$$

The thermal strain is $1{,}288\times10^{-6}$ mm/mm; and so, the total lateral strain is:

$$\begin{aligned}\varepsilon_{\text{lat total}} &= \varepsilon_{\text{lat Poisson}} + \varepsilon_T \\ &= -548.46\times10^{-6}\ \text{mm/mm} + 1,288\times10^{-6}\ \text{mm/mm} \\ &= 739.54\times10^{-6}\ \text{mm/mm}\end{aligned}$$

The change in plate width d is accordingly

$$\Delta d = \varepsilon_{\text{lat total}}\,d = \left(739.54\times10^{-6}\ \text{mm/mm}\right)\left(260\ \text{mm}\right) = \boxed{0.1923\ \text{mm}}$$ **Ans.**

P3.18 The rigid plate in Figure P3.18 is supported by bar (1) and by a double shear pin connection at *B*. Bar (1) has a length of L_1 = 60 in., a cross-sectional area of A_1 = 0.47 in.^{2}, an elastic modulus of *E* = 10,000 ksi, and a coefficient of thermal expansion of $\alpha = 13\times10^{-6}$/°F. The pin at *B* has a diameter of 0.438 in. After load *P* has been applied and the temperature of the entire assembly has been *decreased* by 30°F, the total strain in bar (1) is measured as 570 με (elongation). Assume dimensions of *a* = 12 in. and *b* = 20 in. Determine:
(a) the magnitude of load *P*.
(b) the average shear stress in pin *B*.

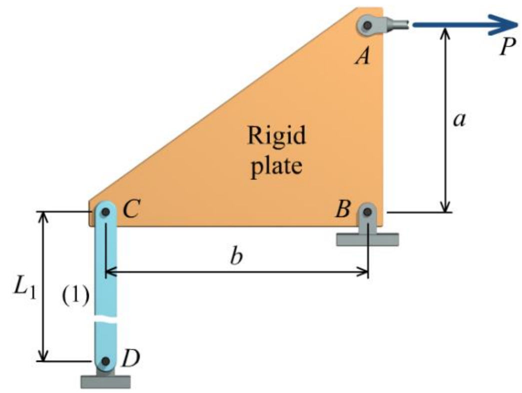


FIGURE P3.18

Solution

(a) Magnitude of load *P*: The total strain in bar (1) is caused partly by the axial force in the bar and partly by the decrease in temperature. The strain caused by the 30°F temperature decrease is:

$$\varepsilon_T = \alpha\,\Delta T = (13\times10^{-6}\ /°\text{F})(-30°\text{F}) = -0.000390\ \text{in./in.}$$

The strain caused by the axial force in the bar is thus:

$$\varepsilon_{1,\sigma} = \varepsilon_1 - \varepsilon_T = 0.000570\ \text{in./in.} - (-0.000390\ \text{in./in.}) = 0.000960\ \text{in./in.}$$

The stress in bar (1) is

$$\sigma_1 = E_1\varepsilon_{1,\sigma} = (10{,}000\ \text{ksi})(0.000960\ \text{in./in.}) = 9.6\ \text{ksi}$$

and the force in bar (1) is

$$F_1 = \sigma_1 A_1 = (9.6\ \text{ksi})(0.47\ \text{in.}^2) = 4.512\ \text{kips}$$

Next, consider an FBD of the rigid plate. Use a moment equilibrium equation to compute load *P*:

$$\Sigma M_B = F_1 b - Pa = 0$$

$$\therefore P = \frac{b}{a}F_1 = \frac{20\ \text{in.}}{12\ \text{in.}}(4.512\ \text{kips}) = \boxed{7.52\ \text{kips}}$$

Ans.

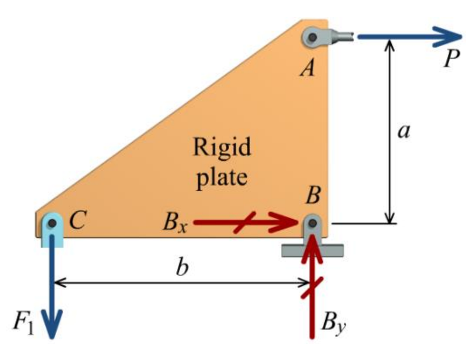
(b) Average shear stress in pin *B*: Use the following equilibrium equations for the rigid plate to calculate the reaction force at pin *B*,

$$\Sigma F_x = P + B_x = 0$$

$$\therefore B_x = -P = -7.520\ \text{kips}$$

$$\Sigma F_y = B_y - F_1 = 0$$

$$\therefore B_y = F_1 = 4.512\ \text{kips}$$



$$|B| = \sqrt{B_x^2 + B_y^2}$$

$$= \sqrt{(-7.520\ \text{kips})^2 + (4.512\ \text{kips})^2}$$

$$= 8.770\ \text{kips}$$

The cross-sectional area of pin *B* is

$$A = \frac{\pi}{4}(0.438 \text{ in.})^2 = 0.15067 \text{ in.}^2$$

Since the pin is supported in a double-shear connection, the area that carries shear stress is

$$A_V = 2(0.15067 \text{ in.}^2) = 0.30134 \text{ in.}^2$$

and thus, the shear stress in pin *B* is

$$\tau = \frac{|B|}{A_V} = \frac{8.770 \text{ kips}}{0.30134 \text{ in.}^2} = \boxed{29.1 \text{ ksi}}$$ **Ans.**

P3.19 Member (1) is an aluminum bar that has a cross-sectional area of A = 1.05 in.^{2}, an elastic modulus of E = 10,000 ksi, and a coefficient of thermal expansion of α = 12.5 × 10^{-6}/°F. After a load P of unknown magnitude is applied to the structure and the temperature is *increased* by 65°F, the normal strain in bar (1) is measured as –540 micros. Use dimensions of a = 24.6 ft, b = 11.7 ft, and c = 14.0 ft. Determine the magnitude of load P.

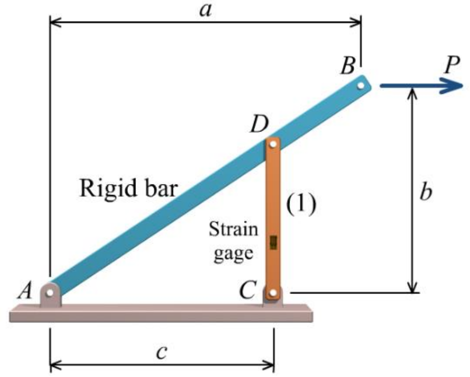


FIGURE P3.19

Solution

The total strain in bar (1) consists of thermal strain as well as normal strain caused by normal stress:

$$\varepsilon = \varepsilon_\sigma + \varepsilon_T$$

The normal strain due to the increase in temperature is:

$$\varepsilon_T = \alpha\,\Delta T = (12.5\times10^{-6}/°\text{F})(65°\text{F}) = 0.0008125 \text{ in./in.}$$

Therefore, the normal stress in bar (1) causes a normal strain of:

$$\varepsilon_\sigma = \varepsilon - \varepsilon_T = -0.000540 \text{ in./in.} - 0.0008125 \text{ in./in.} = -0.0013525 \text{ in./in.}$$

From Hooke's law, the normal stress in bar (1) can be calculated as:

$$\sigma_1 = E\varepsilon_\sigma = (10{,}000 \text{ ksi})(-0.0013525 \text{ in./in.}) = -13.525 \text{ ksi}$$

and thus the axial force in bar (1) must be:

$$F_1 = \sigma_1 A_1 = (-13.525 \text{ ksi})(1.05 \text{ in.}^2) = -14.201 \text{ kips}$$

Next, consider an FBD of the rigid bar. Use a moment equilibrium equation to compute load P:

$$\Sigma M_A = -F_1 c - Pb = 0$$

$$\therefore P = -\frac{c}{b}F_1 = -\frac{14.0 \text{ ft}}{11.7 \text{ ft}}(-14.201 \text{ kips}) = \boxed{16.99 \text{ kips}}$$ **Ans.**