3–1.

A tension test was performed on a steel specimen having an original diameter of 0.503 in. and gage length of 2.00 in. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the yield stress, the ultimate stress, and the fracture stress. Use a scale of 1 in. = 20 ksi and 1 in. = 0.05 in./in. Redraw the elastic region, using the same stress scale but a strain scale of 1 in. = 0.001 in./in.

Load (kip)	Elongation (in.)
0	0
1.50	0.0005
4.60	0.0015
8.00	0.0025
11.00	0.0035
11.80	0.0050
11.80	0.0080
12.00	0.0200
16.60	0.0400
20.00	0.1000
21.50	0.2800
19.50	0.4000
18.50	0.4600

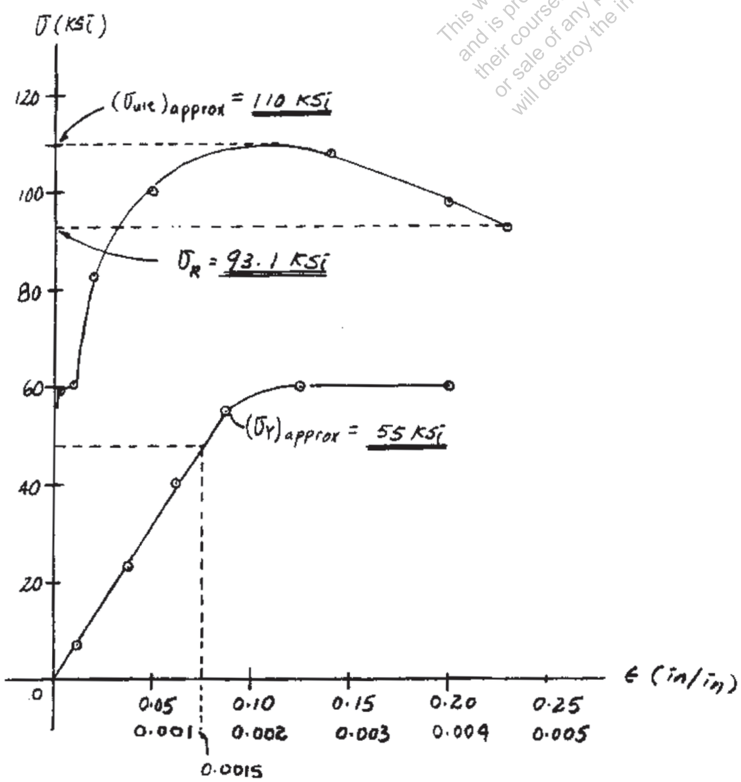
SOLUTION

$$A = \frac{1}{4}\pi(0.503)^2 = 0.1987\ \text{in}^2$$

$$L = 2.00\ \text{in.}$$

σ(ksi)	ϵ(in./in.)
0	0
7.55	0.00025
23.15	0.00075
40.26	0.00125
55.36	0.00175
59.38	0.0025
59.38	0.0040
60.39	0.010
83.54	0.020
100.65	0.050
108.20	0.140
98.13	0.200
93.10	0.230

$$E_{\text{approx}} = \frac{48}{0.0015} = 32.0(10^3)\ \text{ksi}$$ **Ans.**



Ans:
$(\sigma_u)_{\text{approx}} = 110$ ksi, $(\sigma_f)_{\text{approx}} = 93.1$ ksi,
$(\sigma_Y)_{\text{approx}} = 55$ ksi, $E_{\text{approx}} = 32.0(10^3)$ ksi

3–2.

Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

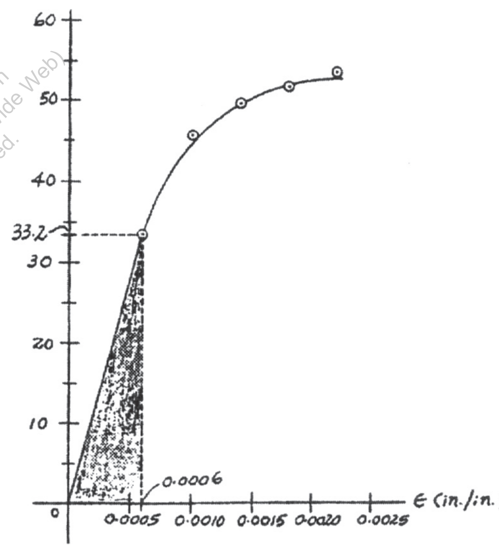
SOLUTION

Modulus of Elasticity: From the stress–strain diagram

$$E = \frac{33.2 - 0}{0.0006 - 0} = 55.3\left(10^3\right)\ \text{ksi} \qquad \textbf{Ans.}$$

Modulus of Resilience: The modulus of resilience is equal to the area under the *linear portion* of the stress–strain diagram (shown shaded).

$$u_r = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.0006\ \frac{\text{in.}}{\text{in.}}\right) = 9.96\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



Ans:
$E = 55.3\left(10^3\right)$ ksi, $u_r = 9.96\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3}$

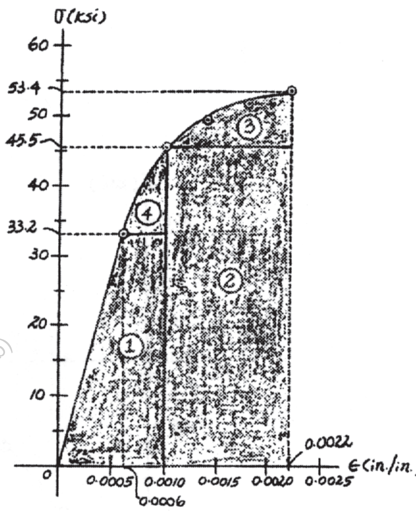
3–3.

Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The fracture stress is $\sigma_f = 53.4$ ksi.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

SOLUTION

Modulus of Toughness: The modulus of toughness is equal to the area under the stress–strain diagram (shown shaded).



$$(u_t)_{\text{approx}} = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004 + 0.0010)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+ 45.5\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+ \frac{1}{2}(7.90)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

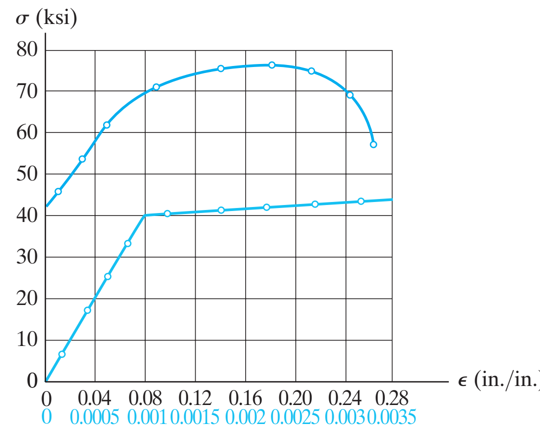
$$+ \frac{1}{2}(12.3)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$= 85.0\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3}$$ **Ans.**

Ans:
$(u_t)_{\text{approx}} = 85.0\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3}$

***3–4.**

The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gage length of 2 in. is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



SOLUTION

Modulus of Elasticity: From the stress–strain diagram, $\sigma = 40$ ksi when $\epsilon = 0.001$ in./in.

$$E_{\text{approx}} = \frac{40 - 0}{0.001 - 0} = 40.0\left(10^3\right) \text{ ksi}$$ **Ans.**

Yield Load: From the stress–strain diagram, $\sigma_Y = 40.0$ ksi.

$$P_Y = \sigma_Y A = 40.0\left[\left(\frac{\pi}{4}\right)(0.5^2)\right] = 7.85 \text{ kip}$$ **Ans.**

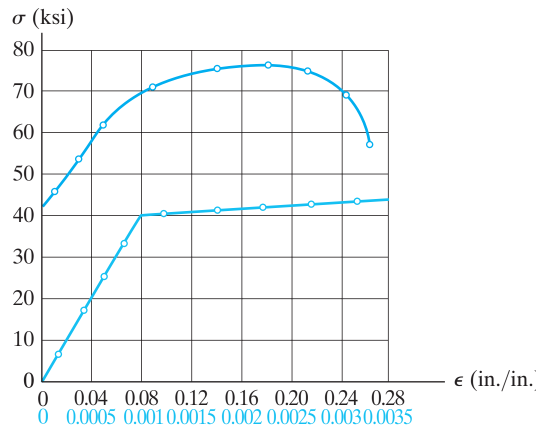
Ultimate Load: From the stress–strain diagram, $\sigma_u = 76.25$ ksi.

$$P_u = \sigma_u A = 76.25\left[\left(\frac{\pi}{4}\right)(0.5^2)\right] = 15.0 \text{ kip}$$ **Ans.**

Ans:
$E_{\text{approx}} = 40.0\left(10^3\right)$ ksi,
$P_Y = 7.85$ kip,
$P_u = 15.0$ kip

3–5.

The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gage length of 2 in. is given in the figure. If the specimen is loaded until it is stressed to 70 ksi, determine the approximate amount of elastic recovery and the increase in the gage length after it is unloaded.



SOLUTION

Modulus of Elasticity: From the stress–strain diagram, $\sigma = 40$ ksi when $\epsilon = 0.001$ in./in.

$$E = \frac{40 - 0}{0.001 - 0} = 40.0\left(10^3\right) \text{ ksi}$$

Elastic Recovery:

$$\text{Elastic recovery} = \frac{\sigma}{E} = \frac{70}{40.0\left(10^3\right)} = 0.00175 \text{ in./in.}$$

Thus,

$$\text{The amount of Elastic Recovery} = 0.00175(2) = 0.00350 \text{ in.}$$ **Ans.**

Permanent Set:

$$\text{Permanent set} = 0.08 - 0.00175 = 0.07825 \text{ in./in.}$$

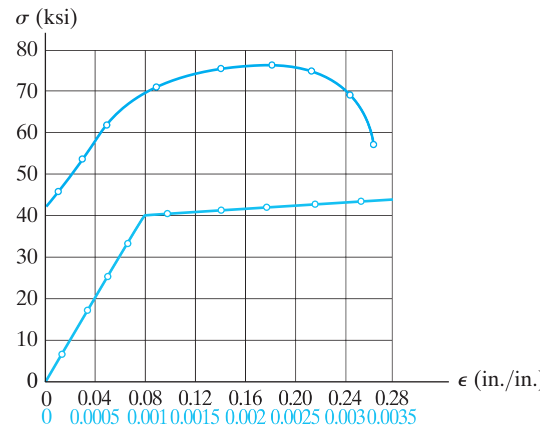
Thus,

$$\text{Permanent elongation} = 0.07825(2) = 0.1565 \text{ in.}$$ **Ans.**

Ans:
Elastic recovery = 0.00350 in.,
Permanent elongation = 0.1565 in.

3–6.

The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gage length of 2 in. is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.



SOLUTION

Modulus of Resilience: The modulus of resilience is equal to the area under the *linear portion* of the stress–strain diagram.

$$(u_r)_{\text{approx}} = \frac{1}{2}(40.0)(10^3)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.001\ \frac{\text{in.}}{\text{in.}}\right) = 20.0\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3}$$ **Ans.**

Modulus of Toughness: The modulus of toughness is equal to the total area under the stress–strain diagram and can be approximated by counting the number of squares. The total number of squares is 45.

$$(u_t)_{\text{approx}} = 45\left(10\ \frac{\text{kip}}{\text{in}^2}\right)\left(0.04\ \frac{\text{in.}}{\text{in.}}\right) = 18.0\ \frac{\text{in}\cdot\text{kip}}{\text{in}^3}$$ **Ans.**

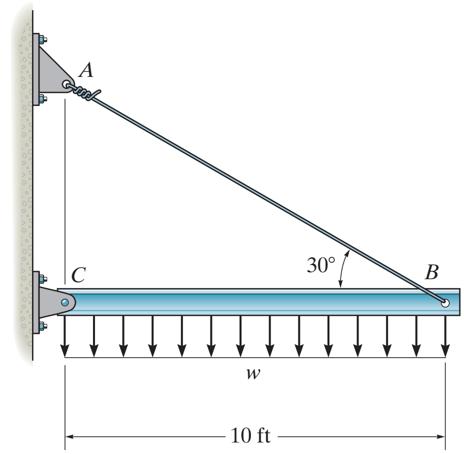
Ans:

$(u_r)_{\text{approx}} = 20.0\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3}$,

$(u_t)_{\text{approx}} = 18.0\ \frac{\text{in}\cdot\text{kip}}{\text{in}^3}$

3–7.

The rigid beam is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when a distributed load of $w = 100$ lb/ft acts on the beam. The material remains elastic.



SOLUTION

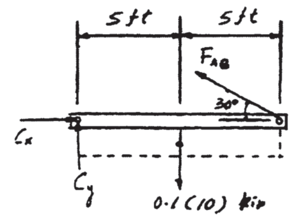
$$+\Sigma M_C = 0; \qquad F_{AB} \sin 30°(10) - 0.1(10)(5) = 0;$$

$$F_{AB} = 1.0 \text{ kip}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{1.0}{\frac{\pi}{4}(0.2)^2} = 31.83 \text{ ksi}$$

$$\sigma = E\epsilon; \quad 31.83 = 29(10^3)\,\epsilon_{AB}; \quad \epsilon_{AB} = 0.0010981 \text{ in./in.}$$

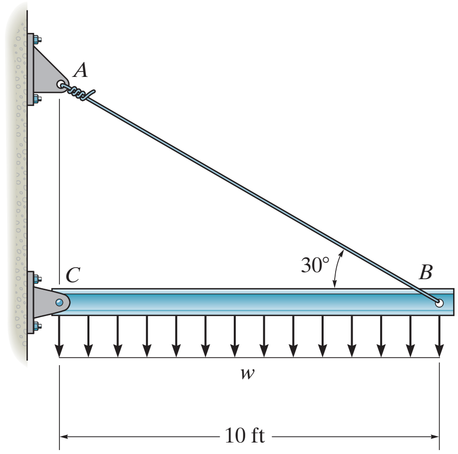
$$\delta_{AB} = \epsilon_{AB} L_{AB} = 0.0010981\left(\frac{120}{\cos 30°}\right) = 0.152 \text{ in.}$$ **Ans.**



Ans:
$\delta_{AB} = 0.152$ in.

***3–8.**

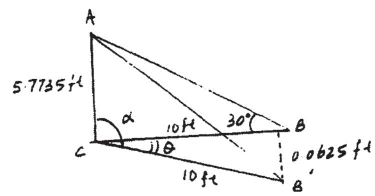
The rigid beam is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine the distributed load w if the end B is displaced 0.75 in. downward.



SOLUTION

$$\sin\theta = \frac{0.0625}{10}; \quad \theta = 0.3581°$$

$$\alpha = 90 + 0.3581° = 90.3581°$$



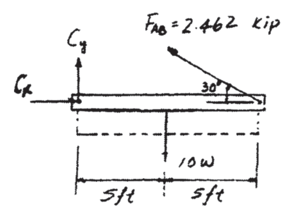
$$AB = \frac{10}{\cos 30°} = 11.5470\text{ ft}$$

$$AB' = \sqrt{10^2 + 5.7735^2 - 2(10)(5.7735)\cos 90.3581°}$$

$$= 11.5782\text{ ft}$$

$$\epsilon_{AB} = \frac{AB' - AB}{AB} = \frac{11.5782 - 11.5470}{11.5470} = 0.002703\text{ in./in.}$$

$$\sigma_{AB} = E\epsilon_{AB} = 29\left(10^3\right)(0.002703) = 78.38\text{ ksi}$$



$$F_{AB} = \sigma_{AB}A_{AB} = 78.38\left(\frac{\pi}{4}\right)(0.2)^2 = 2.462\text{ kip}$$

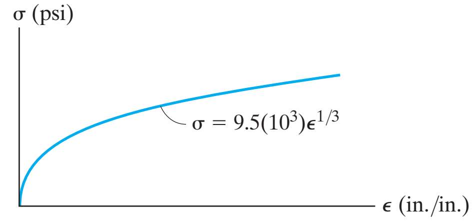
$$+\Sigma M_C = 0; \qquad 2.462\sin 30°(10) - 10w(5) = 0;$$

$w = 0.246$ kip/ft **Ans.**

Ans:
$w = 0.246$ kip/ft

3–9.

Acetal plastic has a stress–strain diagram as shown. If a bar of this material has a length of 3 ft and cross-sectional area of 0.875 in^2, and is subjected to an axial load of 2.5 kip, determine its elongation.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{2.5}{0.875} = 2.857 \text{ ksi}$$

$$\sigma = 9.5\left(10^3\right)\epsilon^{1/3}$$

$$2.857\left(10^3\right) = 9.5\left(10^3\right)\epsilon^{1/3}$$

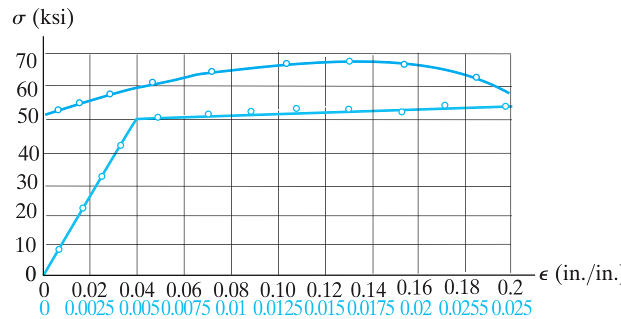
$$\epsilon = 0.0272 \text{ in./in.}$$

$$\delta = L\epsilon = 3(12)(0.0272) = 0.979 \text{ in.}$$ **Ans.**

Ans:
$\delta = 0.979$ in.

3–10.

The stress–strain diagram for an aluminum alloy specimen having an original diameter of 0.5 in. and a gage length of 2 in. is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



SOLUTION

Modulus of Elasticity: From the stress–strain diagram, $\sigma = 50.0$ ksi when $\epsilon = 0.005$ in./in. Thus

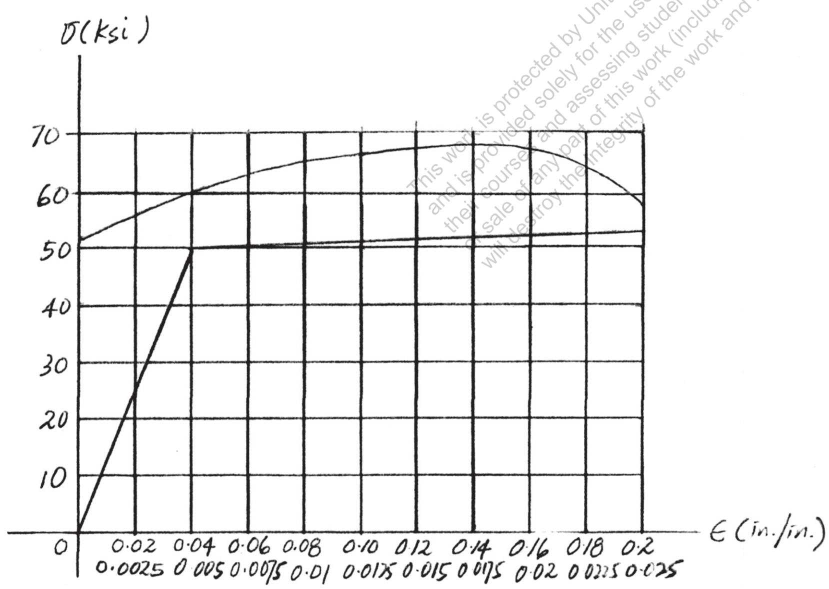
$$E_{\text{approx}} = \frac{50.0 - 0}{0.005 - 0} = 10.0(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

Yield Load: From the stress–strain diagram, $\sigma_y = 50.0$ ksi. Thus

$$P_Y = \sigma_y A = 50.0\left[\frac{\pi}{4}(0.5^2)\right] = 9.82 \text{ kip} \qquad \textbf{Ans.}$$

Ultimate Load: From the stress–strain diagram, $\sigma_u = 68.0$ ksi. Thus,

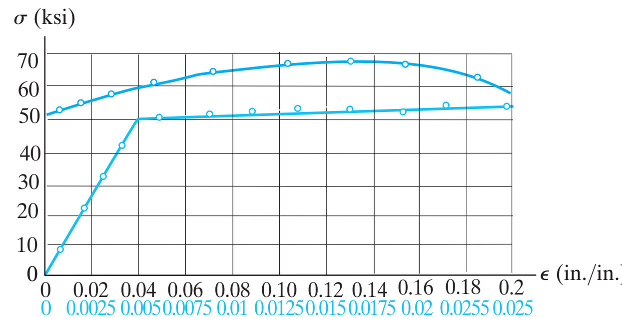
$$P_u = \sigma_u A = 68.0\left[\frac{\pi}{4}(0.5^2)\right] = 13.4 \text{ kip} \qquad \textbf{Ans.}$$



Ans:
$E_{\text{approx}} = 10.0(10^3)$ ksi,
$P_Y = 9.82$ kip,
$P_u = 13.4$ kip

3–11.

The stress–strain diagram for an aluminum alloy specimen having an original diameter of 0.5 in. and a gage length of 2 in. is given in the figure. If the specimen is loaded until it is stressed to 60 ksi, determine the approximate amount of elastic recovery and the increase in the gage length after it is unloaded.



SOLUTION

Modulus of Elasticity: From the stress–strain diagram, $\sigma = 50.0$ ksi when $\epsilon = 0.005$ in./in. Thus

$$E_{\text{approx}} = \frac{50.0 - 0}{0.005 - 0} = 10.0(10^3) \text{ ksi}$$

Elastic Recovery and Permanent Set: From the stress–strain diagram, $\sigma = 60$ ksi when $\epsilon = 0.04$ in./in. Thus

$$\frac{\sigma}{E} = \frac{60}{10.0(10^3)} = 0.006 \text{ in./in.}$$

Thus,

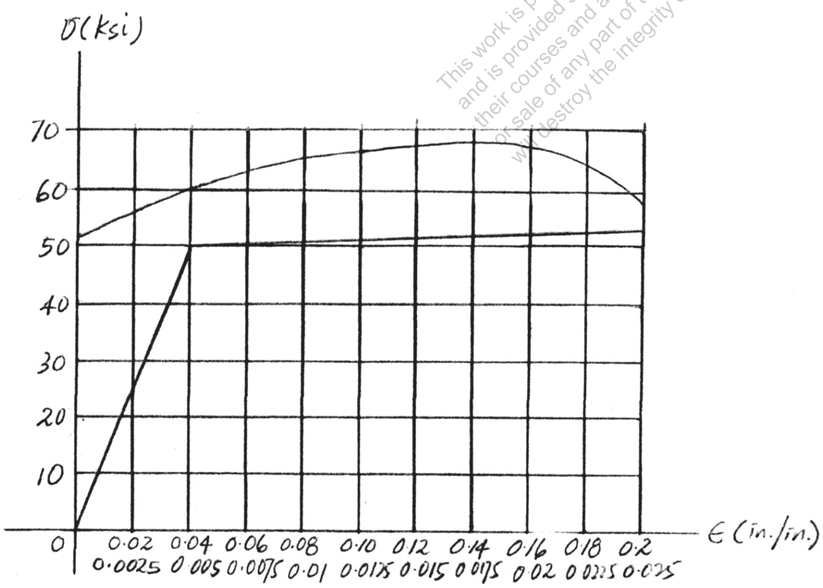
$$\text{Elastic Recovery} = (0.006)(2) = 0.012 \text{ in.}$$ **Ans.**

And

$$\text{Permanent set} = 0.04 - 0.006 = 0.034 \text{ in./in.}$$

Then

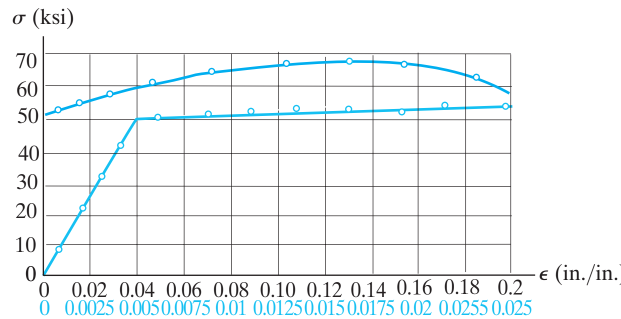
$$\text{Permanent elongation} = 0.034(2) = 0.0680 \text{ in.}$$ **Ans.**



Ans:
Elastic recovery = 0.012 in.,
Permanent elongation = 0.0680 in.

***3–12.**

The stress–strain diagram for an aluminum alloy specimen having an original diameter of 0.5 in. and a gage length of 2 in. is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.



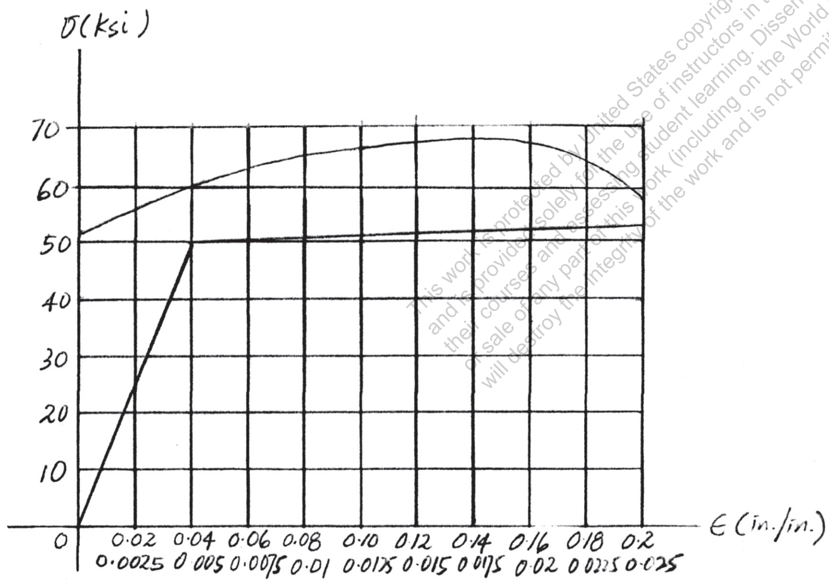
SOLUTION

Modulus of Resilience: The modulus of resilience is equal to the area under the linear portion of the stress–strain diagram.

$$(u_r)_{\text{approx}} = \frac{1}{2}\left[50.0\left(10^3\right)\text{lb/in}^2\right](0.005\text{ in./in.}) = 125\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

Modulus of Toughness: The modulus of toughness is equal to the entire area under the stress–strain diagram, which can be approximated by counting the number of squares. The total number of squares is 63.

$$(u_t)_{\text{approx}} = 63\left(10.0\text{ kip/in}^2\right)(0.02\text{ in./in.}) = 12.6\,\frac{\text{in}\cdot\text{kip}}{\text{in}^3} \qquad \textbf{Ans.}$$



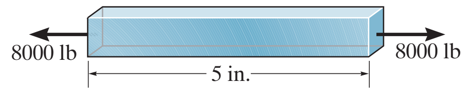
Ans:

$(u_r)_{\text{approx}} = 125\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3}$,

$(u_t)_{\text{approx}} = 12.6\,\frac{\text{in}\cdot\text{kip}}{\text{in}^3}$

3–13.

A bar having a length of 5 in. and cross-sectional area of 0.7 in.^{2} is subjected to an axial force of 8000 lb. If the bar stretches 0.002 in., determine the modulus of elasticity of the material. The material has linear elastic behavior.



SOLUTION

Normal Stress and Strain:

$$\sigma = \frac{P}{A} = \frac{8.00}{0.7} = 11.43 \text{ ksi}$$

$$\epsilon = \frac{\delta}{L} = \frac{0.002}{5} = 0.000400 \text{ in./in.}$$

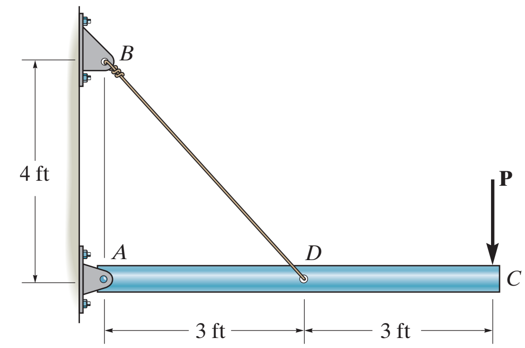
Modulus of Elasticity:

$$E = \frac{\sigma}{\epsilon} = \frac{11.43}{0.000400} = 28.6(10^3) \text{ ksi}$$ **Ans.**

Ans:
$E = 28.6(10^3)$ ksi

3–14.

The rigid pipe is supported by a pin at A and an A-36 steel guy wire BD. If the wire has a diameter of 0.25 in., determine how much it stretches when a load of $P = 600$ lb acts on the pipe.



SOLUTION

Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. a

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - 600(6) = 0 \qquad F_{BD} = 1500 \text{ lb}$$

The normal stress developed in the wire is

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{1500}{\frac{\pi}{4}(0.25^2)} = 30.56(10^3) \text{ psi} = 30.56 \text{ ksi}$$

Since $\sigma_{BD} < \sigma_y = 36$ ksi, Hooke's Law can be applied to determine the strain in the wire.

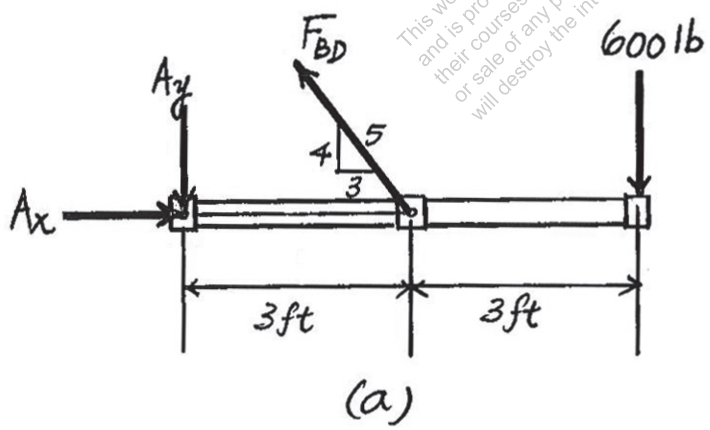
$$\sigma_{BD} = E\epsilon_{BD}; \qquad 30.56 = 29.0(10^3)\epsilon_{BD}$$

$$\epsilon_{BD} = 1.054(10^{-3}) \text{ in./in.}$$

The unstretched length of the wire is $L_{BD} = \sqrt{3^2 + 4^2} = 5 \text{ ft} = 60$ in. Thus, the wire stretches

$$\delta_{BD} = \epsilon_{BD} L_{BD} = 1.054(10^{-3})(60)$$

$$= 0.0632 \text{ in.}$$ **Ans.**

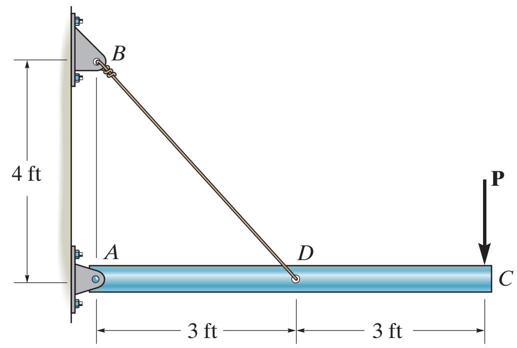


(a)

Ans:
$\delta_{BD} = 0.0632$ in.

3–15.

The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 0.25 in., determine the load P if the end C is displaced 0.075 in. downward.



SOLUTION

Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. a

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - P(6) = 0 \qquad F_{BD} = 2.50\,P$$

The unstretched length for wire BD is $L_{BD} = \sqrt{3^2 + 4^2} = 5\text{ ft} = 60\text{ in}$. From the geometry shown in Fig. b, the stretched length of wire BD is

$$L_{BD'} = \sqrt{60^2 + 0.075^2 - 2(60)(0.075)\cos 143.13°} = 60.060017$$

Thus, the normal strain is

$$\epsilon_{BD} = \frac{L_{BD'} - L_{BD}}{L_{BD}} = \frac{60.060017 - 60}{60} = 1.0003(10^{-3})\text{ in./in.}$$

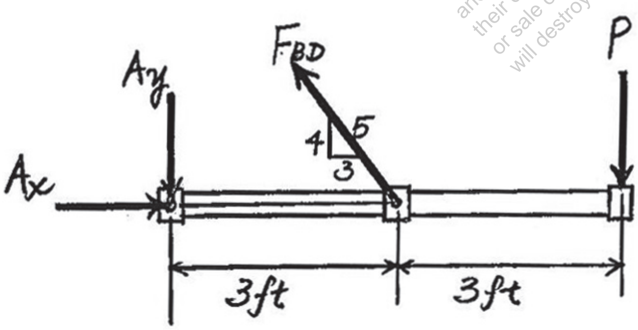
Then, the normal stress can be obtain by applying Hooke's Law.

$$\sigma_{BD} = E\epsilon_{BD} = 29(10^3)\left[1.0003(10^{-3})\right] = 29.01\text{ ksi}$$

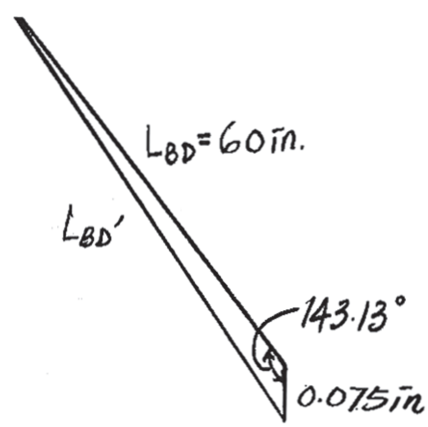
Since $\sigma_{BD} < \sigma_y = 36$ ksi, the result is valid.

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}}; \quad 29.01(10^3) = \frac{2.50\,P}{\frac{\pi}{4}(0.25^2)}$$

$$P = 569.57\text{ lb} = 570\text{ lb}$$ **Ans.**



(a)

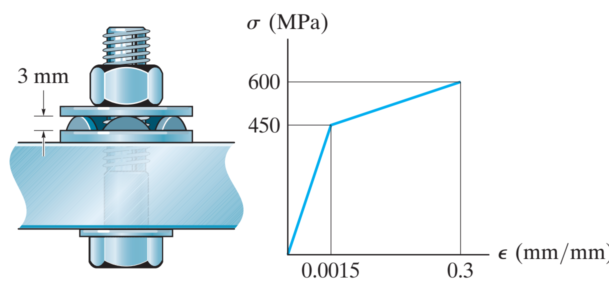


(b)

Ans:
$P = 570$ lb

***3–16.**

Direct tension indicators are sometimes used instead of torque wrenches to ensure that a bolt has a prescribed tension when used for connections. If a nut on the bolt is tightened so that the six 3-mm high heads of the indicator are strained 0.1 mm/mm, and leave a contact area on each head of 1.5 mm^2, determine the tension in the bolt shank. The material has the stress–strain diagram shown.



SOLUTION

Stress–Strain Relationship: From the stress–strain diagram with

$\epsilon = 0.1 \text{ mm/mm} > 0.0015 \text{ mm/mm}$

$$\frac{\sigma - 450}{0.1 - 0.0015} = \frac{600 - 450}{0.3 - 0.0015}$$

$$\sigma = 499.497 \text{ MPa}$$

Axial Force: For each head

$$P = \sigma A = 499.4971\left(10^6\right)(1.5)\left(10^{-6}\right) = 749.24 \text{ N}$$

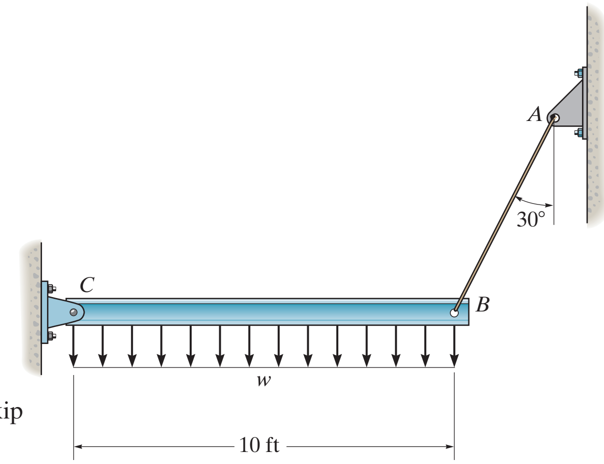
Thus, the tension in the bolt is

$$T = 6\,P = 6(749.24) = 4495 \text{ N} = 4.50 \text{ kN}$$ **Ans.**

Ans:
$T = 4.50 \text{ kN}$

3–17.

The rigid beam is supported by a pin at C and an A992 steel guy wire AB of length 6 ft. If the wire has a diameter of 0.2 in., determine how much it stretches when a distributed load of $w = 200$ lb/ft acts on the beam. The wire remains elastic.



SOLUTION

Support Reactions: Referring to the FBD of the beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB} \cos 30°(10) - 0.2(10)(5) = 0 \qquad F_{AB} = 1.1547 \text{ kip}$$

Normal Stress and Strain Relation: The normal stress is

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{1.1547}{\frac{\pi}{4}(0.2^2)} = 36.76 \text{ ksi}$$

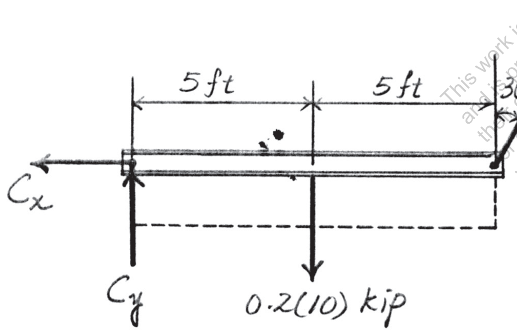
For structural A992 steel alloy, $E_{st} = 29.0(10^3)$ ksi.

$$\sigma = E\epsilon; \qquad 36.76 = 29.0(10^3)\epsilon_{AB}$$

$$\epsilon_{AB} = 0.001267 \text{ in./in.}$$

Thus,

$$\delta_{AB} = \epsilon_{AB} L_{AB} = (0.001267 \text{ in./in.})(72 \text{ in.}) = 0.0913 \text{ in.} \qquad \textbf{Ans.}$$

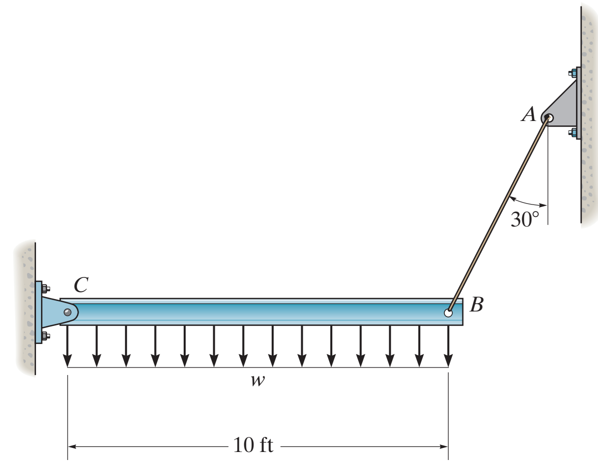


(a)

Ans:
$\delta_{AB} = 0.0913$ in.

3–18.

The rigid beam is supported by a pin at C and an A992 steel guy wire AB of length 6 ft. If the wire has a diameter of 0.2 in., determine the distributed load w if the end B is displaced 0.12 in. downward. The wire remains elastic.



SOLUTION

Support Reactions: Referring to the FBD of the beam, Fig. a,

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}\cos 30°(10) - 10w(5) = 0 \qquad F_{AB} = \frac{10\sqrt{3}}{3}w$$

Normal Stress and Strain Relation: Referring to the geometry shown in Fig. b, and applying cosine law and sine law,

$$L_{AC} = \sqrt{10^2 + 6^2 - 2(10)(6)\cos 120°} = 14 \text{ ft}$$

$$\frac{\sin\phi}{6} = \frac{\sin 120°}{14}; \qquad \phi = 21.7868°$$

$$\sin\theta = \frac{0.01}{10}; \qquad \theta = 0.05730°$$

Thus,

$$L_{AB}' = \sqrt{14^2 + 10^2 - 2(14)(10)\cos(21.7868° + 0.05730°)} = 6.008665 \text{ ft}$$

Then

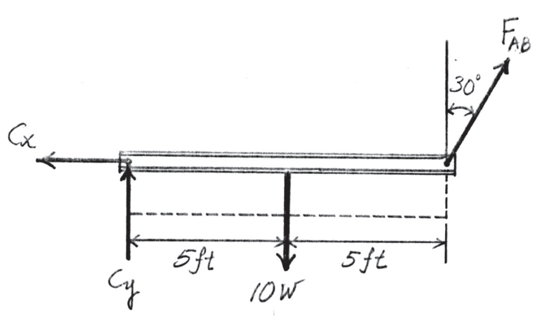
$$\epsilon_{AB} = \frac{L_{AB}' - L_{AB}}{L_{AB}} = \frac{6.008665 - 6}{6} = 0.001444 \text{ in./in.}$$

For structural A992 steel alloy, $E_{st} = 29.0(10^3)$ ksi. Thus,

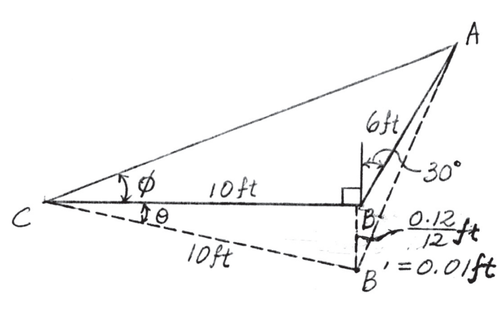
$$\sigma_{AB} = E\epsilon_{AB} = 29(10^3)(0.001444) = 41.88 \text{ ksi} < 50 \text{ ksi} \qquad \textbf{(O.K!)}$$

$$F_{AB} = \sigma_{AB}A_{AB}; \qquad \frac{10\sqrt{3}}{3}w = (41.88)\left[\frac{\pi}{4}(0.2^2)\right]$$

$$w = 0.2279 \text{ kip/ft} = 228 \text{ lb/ft} \qquad \textbf{Ans.}$$



(a)

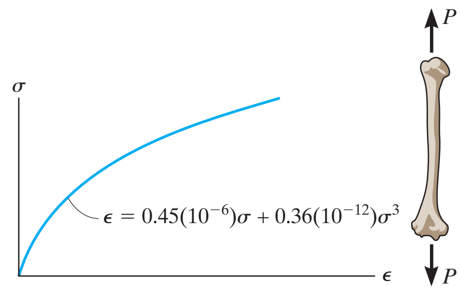


(b)

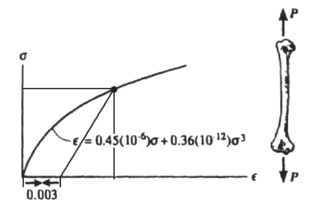
Ans:
$w = 228$ lb/ft

3–19.

The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.



SOLUTION



$$\epsilon = 0.45(10^{-6})\sigma + 0.36(10^{-12})\sigma^3,$$

$$d\epsilon = \left(0.45(10^{-6}) + 1.08(10^{-12})\,\sigma^2\right)d\sigma$$

$$E = \left.\frac{d\sigma}{d\epsilon}\right|_{\sigma=0} = \frac{1}{0.45(10^{-6})} = 2.22(10^6)\text{ kPa} = 2.22\text{ GPa}$$

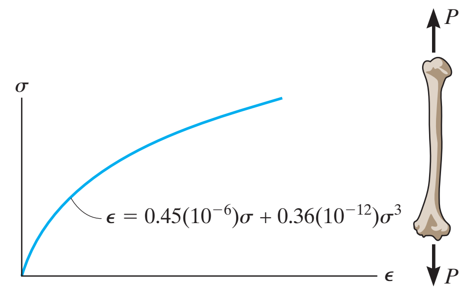
The equation for the recovery line is $\sigma = 2.22(10^6)(\epsilon - 0.003)$.

This line intersects the stress–strain curve at $\sigma_{YS} = 2027\text{ kPa} = 2.03\text{ MPa}$ **Ans.**

Ans:
$\sigma_{YS} = 2.03\text{ MPa}$

***3–20.**

The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.



SOLUTION

When $\epsilon = 0.12$

$$120(10^{-3}) = 0.45\,\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root:

$$\sigma = 6873.52 \text{ kPa}$$

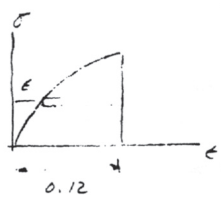
$$u_t = \int_A dA = \int_0^{6873.52} (0.12 - \epsilon)\,d\sigma$$

$$u_t = \int_0^{6873.52} (0.12 - 0.45(10^{-6})\sigma - 0.36(10^{-12})\sigma^3)\,d\sigma$$

$$= 0.12\,\sigma - 0.225(10^{-6})\sigma^2 - 0.09(10^{-12})\sigma^4 \Big|_0^{6873.52}$$

$$= 613 \text{ kJ/m}^3$$ **Ans.**

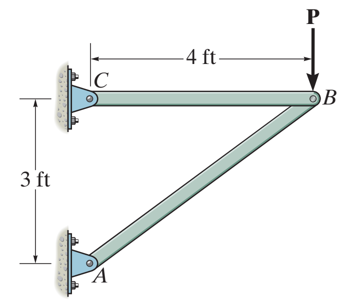
$$\delta = \epsilon L = 0.12(200) = 24 \text{ mm}$$ **Ans.**



Ans:
$u_t = 613 \text{ kJ/m}^3$,
$\delta = 24$ mm

3–21.

The two bars are made of a material that has the stress–strain diagram shown. If the cross-sectional area of bar AB is 1.5 in^2 and BC is 4 in^2, determine the largest force P that can be supported before any member fractures. Assume that buckling does not occur.



SOLUTION

$+\uparrow \Sigma F_y = 0;\quad \frac{3}{5}F_{AB} - P = 0;\qquad F_{AB} = 1.6667\,P \qquad \textbf{(1)}$

$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad F_{BC} - \frac{4}{5}(1.6667P) = 0;\qquad F_{BC} = 1.333\,P \qquad \textbf{(2)}$

Assuming failure of bar BC:

From the stress–strain diagram $(\sigma_R)_t = 5$ ksi

$\sigma = \frac{F_{BC}}{A_{BC}};\quad 5 = \frac{F_{BC}}{4};\qquad F_{BC} = 20.0$ kip

From Eq. (2), $P = 15.0$ kip

Assuming failure of bar AB:

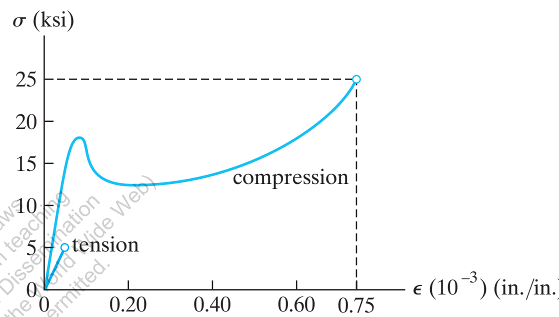
From stress–strain diagram $(\sigma_R)_c = 25.0$ ksi

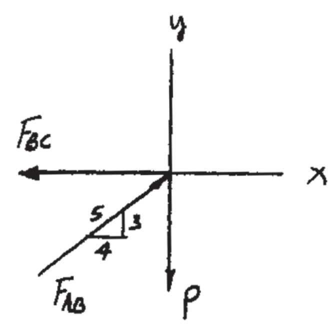
$\sigma = \frac{F_{AB}}{A_{AB}};\quad 25.0 = \frac{F_{AB}}{1.5};\qquad F_{AB} = 37.5$ kip

From Eq. (1), $P = 22.5$ kip

Choose the smallest value:

$P = 15.0$ kip **Ans.**

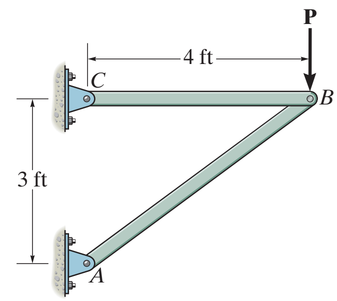


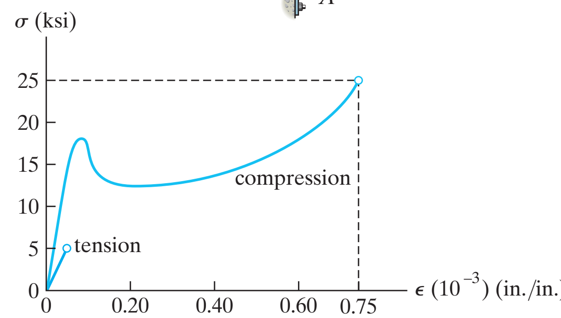


Ans:
$P = 15.0$ kip

3–22.

The two bars are made of a material that has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars fracture simultaneously when the load $P = 3$ kip. Assume that buckling does not occur.





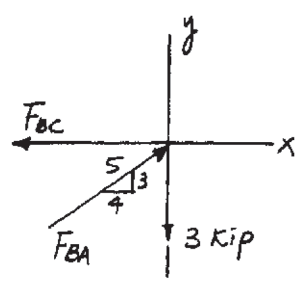
SOLUTION

$+\uparrow \Sigma F_y = 0;\qquad F_{BA}\left(\frac{3}{5}\right) - 3 = 0;\qquad F_{BA} = 5 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -F_{BC} + 5\left(\frac{4}{5}\right) = 0;\qquad F_{BC} = 4 \text{ kip}$

For member BC:

$$(\sigma_{\max})_t = \frac{F_{BC}}{A_{BC}};\quad A_{BC} = \frac{4 \text{ kip}}{5 \text{ ksi}} = 0.8 \text{ in}^2$$ **Ans.**



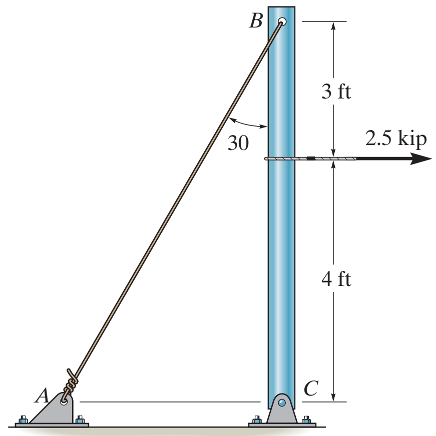
For member BA:

$$(\sigma_{\max})_c = \frac{F_{BA}}{A_{BA}};\quad A_{BA} = \frac{5 \text{ kip}}{25 \text{ ksi}} = 0.2 \text{ in}^2$$ **Ans.**

Ans:
$A_{BC} = 0.8 \text{ in}^2,\ A_{BA} = 0.2 \text{ in}^2$

3–23.

The pole is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when a horizontal force of 2.5 kip acts on the pole.



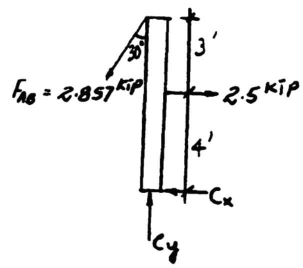
SOLUTION

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{2.857}{\frac{\pi}{4}(0.2^2)} = 90.94 \text{ ksi}$$

$$\epsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{90.94}{29(10^3)} = 0.003136$$

$$\delta_{AB} = \epsilon_{AB} L_{AB} = 0.003136\left(\frac{7(12)}{\cos 30°}\right)$$

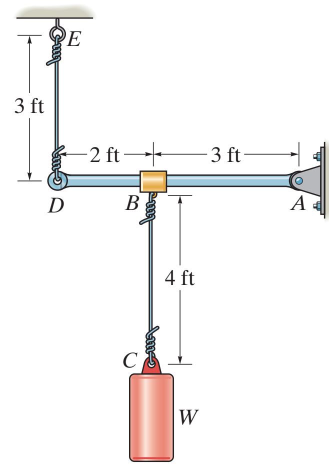
$$= 0.304 \text{ in.}$$ **Ans.**



Ans:
$\delta_{AB} = 0.304$ in.

***3–24.**

The bar DA is rigid and is originally held in the horizontal position when the weight W is supported from C. If the weight causes B to displace downward 0.025 in., determine the strain in wires DE and BC. Also, if the wires are made of A-36 steel and have a cross-sectional area of 0.002 in^2, determine the weight W.



SOLUTION

$$\frac{3}{0.025} = \frac{7}{\delta}$$

$$\delta = 0.0417 \text{ in.}$$

$$\epsilon_{DE} = \frac{\delta}{L} = \frac{0.0417}{3(12)} = 0.00116 \text{ in./in.} \qquad \textbf{Ans.}$$

$$\sigma_{DE} = E\epsilon_{DE} = 29(10^3)(0.00116) = 33.56 \text{ ksi}$$

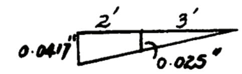
$$F_{DE} = \sigma_{DE}A_{DE} = 33.56(0.002) = 0.0672 \text{ kip}$$

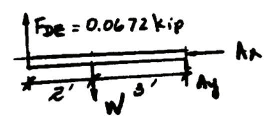
$$\circlearrowleft + \Sigma M_A = 0; \qquad -(0.0672)(5) + 3(W) = 0$$

$$W = 0.112 \text{ kip} = 112 \text{ lb} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{W}{A_{BC}} = \frac{0.112}{0.002} = 55.94 \text{ ksi}$$

$$\epsilon_{BC} = \frac{\sigma_{BC}}{E} = \frac{55.94}{29(10^3)} = 0.00193 \text{ in./in.} \qquad \textbf{Ans.}$$

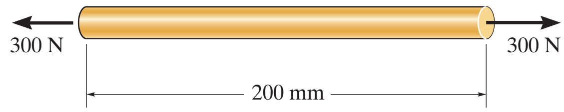




Ans:
$\epsilon_{DE} = 0.00116$ in./in.,
$W = 112$ lb,
$\epsilon_{BC} = 0.00193$ in./in.

3–25.

The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.678 \text{ MPa}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{1.678(10^6)}{2.70(10^9)} = 0.0006288$$

$$\delta = \epsilon_{\text{long}} L = 0.0006288(200) = 0.126 \text{ mm}$$ **Ans.**

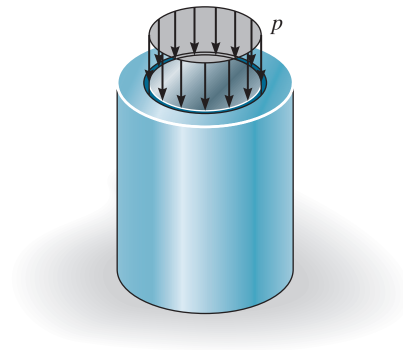
$$\epsilon_{\text{lat}} = -\nu\epsilon_{\text{long}} = -0.4(0.0006288) = -0.0002515$$

$$\Delta d = \epsilon_{\text{lat}} d = -0.0002515(15) = -0.00377 \text{ mm}$$ **Ans.**

Ans:
$\delta = 0.126$ mm, $\Delta d = -0.00377$ mm

3–26.

The plug has a diameter of 30 mm and fits within a rigid sleeve having an inner diameter of 32 mm. Both the plug and the sleeve are 50 mm long. Determine the axial pressure p that must be applied to the top of the plug to cause it to contact the sides of the sleeve. Also, how far must the plug be compressed downward in order to do this? The plug is made from a material for which $E = 5$ MPa, $\nu = 0.45$.



SOLUTION

$$\epsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{32 - 30}{30} = 0.06667 \text{ mm/mm}$$

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}; \qquad \epsilon_{\text{long}} = -\frac{\epsilon_{\text{lat}}}{\nu} = -\frac{0.06667}{0.45} = -0.1481 \text{ mm/mm}$$

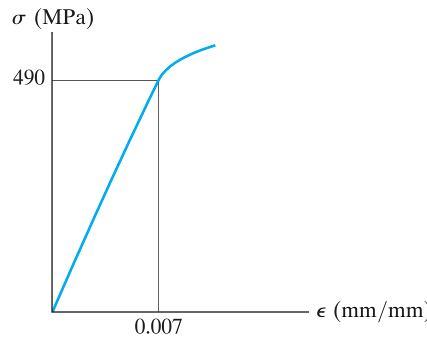
$$p = \sigma = E\,\epsilon_{\text{long}} = 5(10^6)(0.1481) = 741 \text{ kPa}$$ **Ans.**

$$\delta = |\epsilon_{\text{long}} L| = |-0.1481(50)| = 7.41 \text{ mm}$$ **Ans.**

Ans:
$p = 741$ kPa,
$\delta = 7.41$ mm

3–27.

The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. When the applied load on the specimen is 50 kN, the diameter is 12.67494 mm. Determine Poisson's ratio for the material.



SOLUTION

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{50\left(10^3\right)}{\frac{\pi}{4}\left(0.0127^2\right)} = 394.71\left(10^6\right) \text{ Pa} = 394.71 \text{ MPa}$$

Average Normal Strain: Referring to the stress–strain diagram, the modulus of elasticity is $E = \dfrac{490\left(10^6\right)}{0.007} = 70.0\left(10^9\right) \text{ Pa} = 70.0 \text{ GPa}$.

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{394.71\left(10^6\right)}{70.0\left(10^9\right)} = 0.0056386 \text{ mm/mm}$$

$$\epsilon_{\text{lat}} = \frac{d - d_0}{d_0} = \frac{12.67494 - 12.7}{12.7} = -0.0019732$$

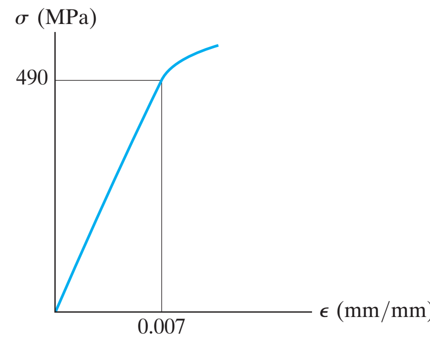
Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio, that is

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} = -\frac{(-0.019732)}{0.0056386} = 0.350$$ **Ans.**

Ans:
$\nu = 0.350$

***3–28.**

The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. If a load of $P = 60$ kN is applied to the specimen, determine its new diameter and length. Take $\nu = 0.35$.



SOLUTION

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{60\left(10^3\right)}{\frac{\pi}{4}\left(0.0127^2\right)} = 473.65\left(10^6\right)\text{ Pa} = 473.65\text{ MPa}$$

Average Normal Strain: Referring to the stress–strain diagram, the modulus of elasticity is $E = \frac{490\left(10^6\right)}{0.007} = 70.0\left(10^9\right)\text{ Pa} = 70.0\text{ GPa}$.

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{473.65\left(10^6\right)}{70.0\left(10^9\right)} = 0.0067664\text{ mm/mm}$$

Thus,

$$\delta L = \epsilon_{\text{long}} L_0 = 0.0067664(50.8) = 0.34373\text{ mm}$$

Then

$$L = L_0 + \delta L = 50.8 + 0.34373 = 51.1437\text{ mm} \qquad \textbf{Ans.}$$

Poisson's Ratio: The lateral strain can be related to the longitudinal strain using Poisson's ratio.

$$\epsilon_{\text{lat}} = -\nu\epsilon_{\text{long}} = -0.35(0.0067664) = -0.0023682\text{ mm/mm}$$

Thus,

$$\delta d = \epsilon_{\text{lat}}\, d = -0.0023682(12.7) = -0.030077\text{ mm}$$

Then

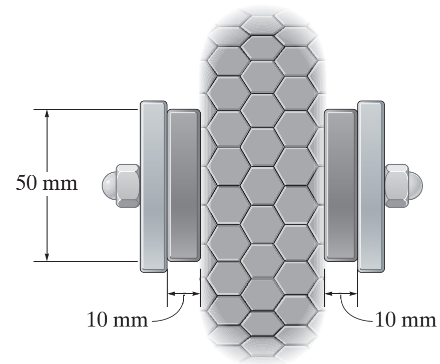
$$d = d_0 + \delta d = 12.7 + (-0.030077) = 12.66992\text{ mm}$$

$$= 12.67\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$L = 51.1437$ mm,
$d = 12.67$ mm

3–29.

The brake pads for a bicycle tire are made of rubber. If a frictional force of 50 N is applied to each side of the tires, determine the average shear strain in the rubber. Each pad has cross-sectional dimensions of 20 mm and 50 mm. $G_r = 0.20$ MPa.



SOLUTION

Average Shear Stress: The shear force is $V = 50$ N.

$$\tau = \frac{V}{A} = \frac{50}{0.02(0.05)} = 50.0 \text{ kPa}$$

Shear-Stress – Strain Relationship: Applying Hooke's law for shear

$$\tau = G\gamma$$

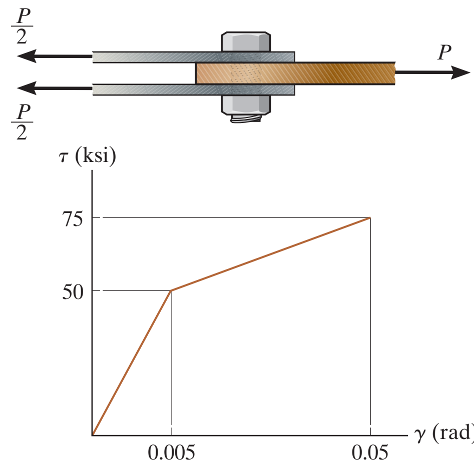
$$50.0(10^3) = 0.2(10^6)\,\gamma$$

$$\gamma = 0.250 \text{ rad}$$ **Ans.**

Ans:
$\gamma = 0.250$ rad

3–30.

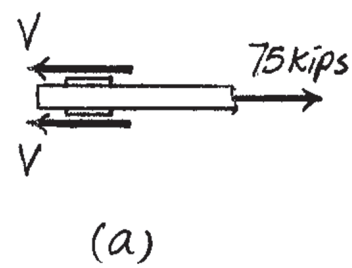
The lap joint is connected together using a 1.25 in. diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the shear strain developed in the shear plane of the bolt when $P = 75$ kip.



SOLUTION

Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$\xrightarrow{+} \Sigma F_x = 0; \qquad 75 - 2V = 0 \qquad V = 37.5 \text{ kip}$

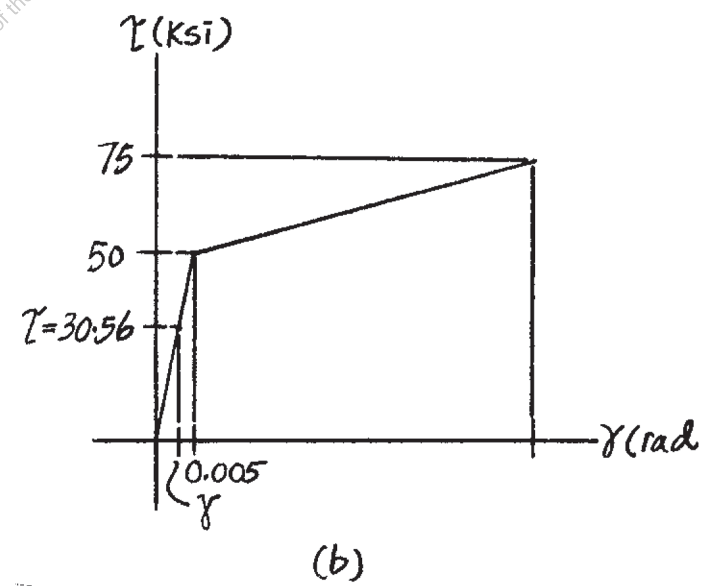


Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{37.5}{\frac{\pi}{4}\left(1.25^2\right)} = 30.56 \text{ ksi}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

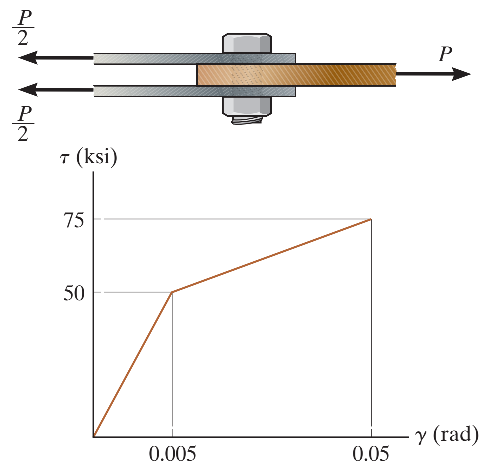
$$\frac{30.56}{\gamma} = \frac{50}{0.005}; \qquad \gamma = 3.06(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$



Ans:
$\gamma = 3.06(10^{-3})$ rad

3–31.

The lap joint is connected together using a 1.25 in. diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the permanent shear strain in the shear plane of the bolt when the applied force $P = 150$ kip is removed.



SOLUTION

Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. a.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 150 - 2V = 0 \qquad V = 75 \text{ kip}$$

Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{75}{\frac{\pi}{4}\left(1.25^2\right)} = 61.12 \text{ ksi}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

$$\frac{61.12 - 50}{\gamma - 0.005} = \frac{75 - 50}{0.05 - 0.005}; \qquad \gamma = 0.02501 \text{ rad}$$

When force **P** is removed, the shear strain recovers linearly along line *BC*, Fig. *b*, with a slope that is the same as line *OA*. This slope represents the shear modulus.

$$G = \frac{50}{0.005} = 10(10^3) \text{ ksi}$$

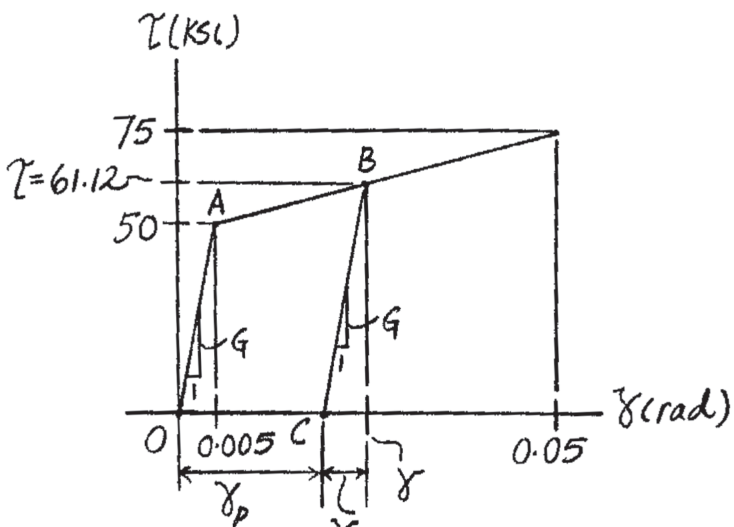
Thus, the elastic recovery of shear strain is

$$\tau = G\gamma_r; \qquad 61.12 = (10)(10^3)\gamma_r \qquad \gamma_r = 6.112(10^{-3}) \text{ rad}$$

And the permanent shear strain is

$$\gamma_P = \gamma - \gamma_r = 0.02501 - 6.112(10^{-3}) = 0.0189 \text{ rad} \qquad \textbf{Ans.}$$

(a)

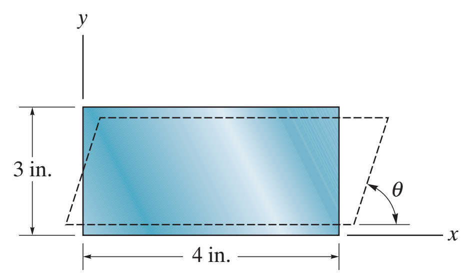


(b)

Ans:
$\gamma_P = 0.0189$ rad

***3–32.**

The rubber block is subjected to an elongation of 0.03 in. along the x axis, and its vertical faces are given a tilt so that $\theta = 89.3°$. Determine the strains ϵ_x, ϵ_y and γ_{xy}. Take $\nu_r = 0.5$.



SOLUTION

$$\epsilon_x = \frac{\delta L}{L} = \frac{0.03}{4} = 0.0075 \text{ in./in.}$$ **Ans.**

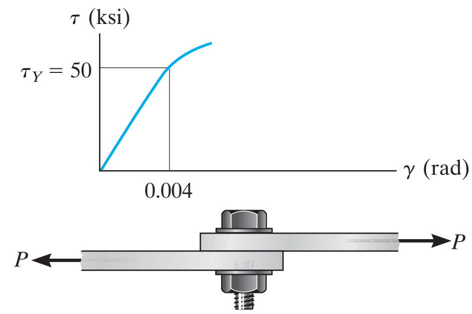
$$\epsilon_y = -\nu\epsilon_x = -0.5(0.0075) = -0.00375 \text{ in./in.}$$ **Ans.**

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 89.3°\left(\frac{\pi}{180°}\right) = 0.0122 \text{ rad}$$ **Ans.**

Ans:
$\epsilon_x = 0.0075$ in./in.,
$\epsilon_y = -0.00375$ in./in.,
$\gamma_{xy} = 0.0122$ rad

3–33.

The shear stress–strain diagram for an alloy is shown in the figure. If a bolt having a diameter of 0.25 in. is made of this material and used in the lap joint, determine the modulus of elasticity E and the force P required to cause the material to yield. Take $\nu = 0.3$.



SOLUTION

Modulus of Rigidity: From the stress–strain diagram,

$$G = \frac{50}{0.004} = 12.5\left(10^3\right) \text{ ksi}$$

Modulus of Elasticity:

$$G = \frac{E}{2(1+\nu)}$$

$$12.5\left(10^3\right) = \frac{E}{2(1+0.3)}$$

$$E = 32.5\left(10^3\right) \text{ ksi} \qquad \textbf{Ans.}$$

Yielding Shear: The bolt is subjected to a yielding shear of $V_\gamma = P$. From the stress–strain diagram, $\tau_\gamma = 50$ ksi

$$\tau_\gamma = \frac{V_\gamma}{A}$$

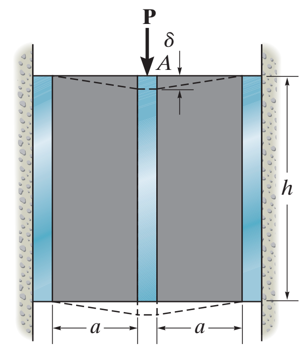
$$50 = \frac{P}{\frac{\pi}{4}\left(0.25^2\right)}$$

$$P = 2.45 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$E = 32.5\left(10^3\right)$ ksi,
$P = 2.45$ kip

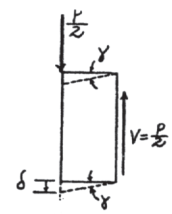
3–34.

A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A when the vertical load **P** is applied. Assume that the displacement is small so that $\delta = a \tan \gamma \approx a\gamma$.



SOLUTION

Average Shear Stress: The rubber block is subjected to a shear force of $V = \dfrac{P}{2}$.



$$\tau = \frac{V}{A} = \frac{\frac{P}{2}}{bh} = \frac{P}{2bh}$$

Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{\frac{P}{2bh}}{G} = \frac{P}{2bhG}$$

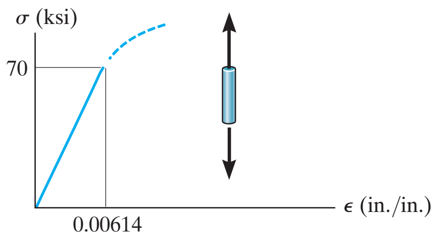
Thus,

$$\delta = a\gamma = \frac{Pa}{2bhG}$$ **Ans.**

Ans:
$\delta = \dfrac{Pa}{2bhG}$

R3–1.

The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gage length of 2 in. and a diameter of 0.5 in. When the applied load is 9 kip, the new diameter of the specimen is 0.49935 in. Calculate the shear modulus G_{al} for the aluminum.



SOLUTION

From the stress–strain diagram,

$$E_{al} = \frac{\sigma}{\epsilon} = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

When specimen is loaded with a 9 - kip load,

$$\sigma = \frac{P}{A} = \frac{9}{\frac{\pi}{4}(0.5)^2} = 45.84 \text{ ksi}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{45.84}{11400.65} = 0.0040205 \text{ in./in.}$$

$$\epsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{0.49935 - 0.5}{0.5} = -0.0013 \text{ in./in.}$$

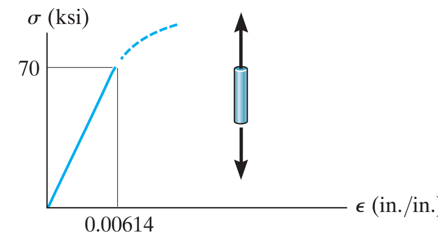
$$V = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} = -\frac{-0.0013}{0.0040205} = 0.32334$$

$$G_{\text{al}} = \frac{E_{at}}{2(1 + \nu)} = \frac{11.4(10^3)}{2(1 + 0.32334)} = 4.31(10^3) \text{ ksi}$$ **Ans.**

Ans:
$G_{\text{al}} = 4.31(10^3)$ ksi

R3–2.

The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gage length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip, determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.5)^2} = 50.9296 \text{ ksi}$$

From the stress–strain diagram

$$E = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

$$\epsilon_{long} = \frac{\sigma}{E} = \frac{50.9296}{11400.65} = 0.0044673 \text{ in./in.}$$

$$G = \frac{E}{2(1+\nu)}; \qquad 3.8(10^3) = \frac{11400.65}{2(1+\nu)}; \qquad \nu = 0.500$$

$$\epsilon_{lat} = -\nu\epsilon_{long} = -0.500(0.0044673) = -0.002234 \text{ in./in.}$$

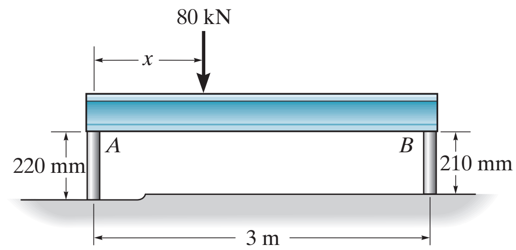
$$\Delta d = \epsilon_{lat}\, d = -0.002234(0.5) = -0.001117 \text{ in.}$$

$$d' = d + \Delta d = 0.5 - 0.001117 = 0.4989 \text{ in.}$$ **Ans.**

Ans:
$d' = 0.4989$ in.

R3–3.

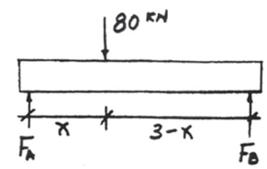
The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad F_B(3) - 80(x) = 0; \quad F_B = \dfrac{80x}{3}$ **(1)**

$\circlearrowleft + \Sigma M_B = 0; \quad -F_A(3) + 80(3 - x) = 0; \quad F_A = \dfrac{80(3 - x)}{3}$ **(2)**



Since the beam is held horizontally, $\delta_A = \delta_B$

$$\sigma = \frac{P}{A}; \quad \epsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$$

$$\delta = \epsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$$

$$\delta_A = \delta_B; \quad \frac{\frac{80(3 - x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$$

$$80(3 - x)(220) = 80x(210)$$

$x = 1.53$ m **Ans.**

From Eq. (2),

$$F_A = 39.07 \text{ kN}$$

$$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$$

$$\epsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$$

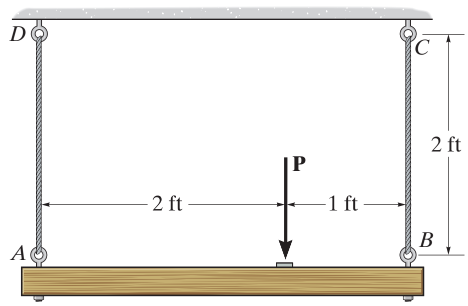
$$\epsilon_{lat} = -\nu\epsilon_{long} = -0.35(-0.000756) = 0.0002646$$

$d'_A = d_A + d\,\epsilon_{lat} = 30 + 30(0.0002646) = 30.008$ mm **Ans.**

Ans:
$x = 1.53$ m, $d'_A = 30.008$ mm

*R3–4.

The wires each have a diameter of $\frac{1}{2}$ in., length of 2 ft, and are made from 304 stainless steel. If $P = 6$ kip, determine the angle of tilt of the rigid beam AB.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of beam AB shown in Fig. a,

$+\Sigma M_A = 0; \qquad F_{BC}(3) - 6(2) = 0 \qquad F_{BC} = 4 \text{ kip}$

$+\uparrow\Sigma M_B = 0; \qquad 6(1) - F_{AD}(3) = 0 \qquad F_{AD} = 2 \text{ kip}$

Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{4(10^3)}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 20.37 \text{ ksi}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{2(10^3)}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 10.19 \text{ ksi}$$

Since $\sigma_{BC} < \sigma_Y$ and $\sigma_A < \sigma_Y$, Hooke's Law can be applied.

$\sigma_{BC} = E\epsilon_{BC}; \qquad 20.37 = 28.0(10^3)\epsilon_{BC} \qquad \epsilon_{BC} = 0.7276(10^{-3}) \text{ in./in.}$

$\sigma_{AD} = E\epsilon_{AD}; \qquad 10.19 = 28.0(10^3)\epsilon_{AD} \qquad \epsilon_{AD} = 0.3638(10^{-3}) \text{ in./in.}$

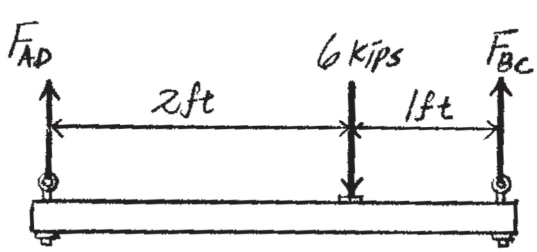
Thus, the elongation of cables BC and AD are given by

$$\delta_{BC} = \epsilon_{BC}L_{BC} = 0.7276(10^{-3})(24) = 0.017462 \text{ in.}$$

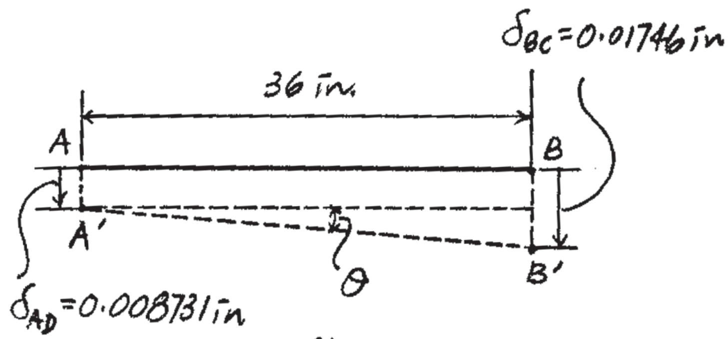
$$\delta_{AD} = \epsilon_{AD}L_{AD} = 0.3638(10^{-3})(24) = 0.008731 \text{ in.}$$

Referring to the geometry shown in Fig. b and using small angle analysis,

$$\theta = \frac{\delta_{BC} - \delta_{AD}}{36} = \frac{0.017462 - 0.008731}{36} = 0.2425(10^{-3}) \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 0.0139° \quad \textbf{Ans.}$$



(a)

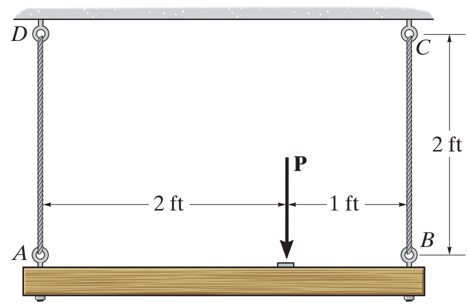


(b)

Ans:
$\theta = 0.0139°$

R3–5.

The wires each have a diameter of $\frac{1}{2}$ in., length of 2 ft, and are made from 304 stainless steel. Determine the magnitude of force **P** so that the rigid beam tilts 0.015°.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of beam AB shown in Fig. a,

$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}(3) - P(2) = 0 \qquad F_{BC} = 0.6667P$

$+\uparrow \Sigma M_B = 0; \qquad P(1) - F_{AD}(3) = 0 \qquad F_{AD} = 0.3333P$

Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{0.6667P}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 3.3953P$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{0.3333P}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2} = 1.6977P$$

Assuming that $\sigma_{BC} < \sigma_Y$ and $\sigma_{AD} < \sigma_Y$ and applying Hooke's Law,

$\sigma_{BC} = E\epsilon_{BC}; \qquad 3.3953P = 28.0(10^6)\epsilon_{BC} \qquad \epsilon_{BC} = 0.12126(10^{-6})P$

$\sigma_{AD} = E\epsilon_{AD}; \qquad 1.6977P = 28.0(10^6)\epsilon_{AD} \qquad \epsilon_{AD} = 60.6305(10^{-9})P$

Thus, the elongation of cables BC and AD are given by

$$\delta_{BC} = \epsilon_{BC}L_{BC} = 0.12126(10^{-6})P(24) = 2.9103(10^{-6})P$$

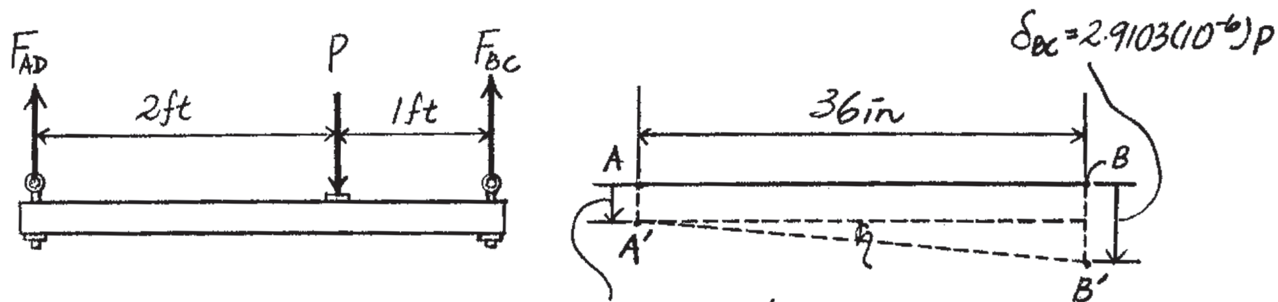
$$\delta_{AD} = \epsilon_{AD}L_{AD} = 60.6305(10^{-9})P(24) = 1.4551(10^{-6})P$$

Here, the angle of the tile is $\theta = 0.015°\left(\frac{\pi \text{rad}}{180°}\right) = 0.2618(10^{-3})$ rad. Using small angle analysis,

$$\theta = \frac{\delta_{BC} - \delta_{AD}}{36}; \qquad 0.2618(10^{-3}) = \frac{2.9103(10^{-6})P - 1.4551(10^{-6})P}{36}$$

$$P = 6476.93 \text{ lb} = 6.48 \text{ kip} \qquad \textbf{Ans.}$$

Since $\sigma_{BC} = 3.3953(6476.93) = 21.99$ ksi $< \sigma_Y$ and $\sigma_{AD} = 1.6977(6476.93) = 11.00$ ksi $< \sigma_Y$, the assumption is correct.

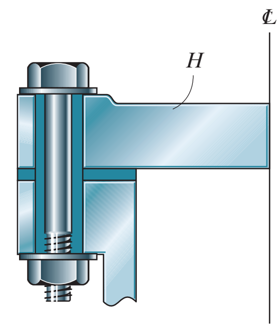


(a)

Ans:
$P = 6.48$ kip

R3–6.

The head H is connected to the cylinder of a compressor using six $\frac{3}{16}$-in. diameter steel bolts. If the clamping force in each bolt is 800 lb, determine the normal strain in the bolts. If $\sigma_Y = 40$ ksi and $E_{st} = 29(10^3)$ ksi, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



SOLUTION

Normal Stress:

$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}\left(\frac{3}{16}\right)^2} = 28.97 \text{ ksi} < \sigma_\gamma = 40 \text{ ksi}$$

Normal Strain: Since $\sigma < \sigma_\gamma$, Hooke's law is still valid.

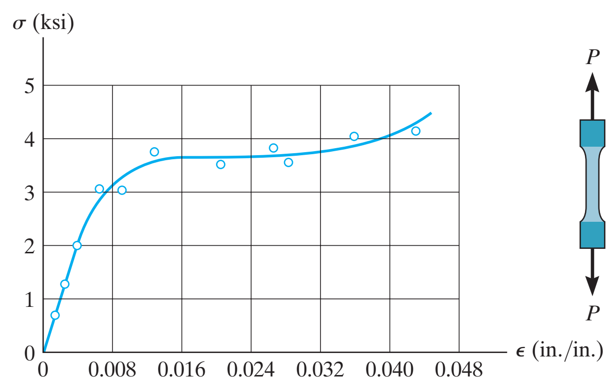
$$\epsilon = \frac{\sigma}{E} = \frac{28.97}{29(10^3)} = 0.000999 \text{ in./in.} \qquad \textbf{Ans.}$$

If the nut is unscrewed, the load is zero. Therefore, the strain $\epsilon = 0$ **Ans.**

Ans:
$\epsilon = 0.000999$ in./in.,
$\epsilon = 0$

R3–7.

The stress–strain diagram for polyethylene, which is used to sheath coaxial cables, is determined from testing a specimen that has a gage length of 10 in. If a load P on the specimen develops a strain of $\epsilon = 0.024$ in./in., determine the approximate length of the specimen, measured between the gage points, when the load is removed. Assume the specimen recovers elastically.



SOLUTION

Modulus of Elasticity: From the stress–strain diagram, $\sigma = 2$ ksi when $\epsilon = 0.004$ in./in.

$$E = \frac{2 - 0}{0.004 - 0} = 0.500(10^3) \text{ ksi}$$

Elastic Recovery: From the stress–strain diagram, $\sigma = 3.70$ ksi when $\epsilon = 0.024$ in./in.

$$\text{Elastic recovery} = \frac{\sigma}{E} = \frac{3.70}{0.500(10^3)} = 0.00740 \text{ in./in.}$$

Permanent Set:

$$\text{Permanent set} = 0.024 - 0.00740 = 0.0166 \text{ in./in.}$$

Thus,

$$\text{Permanent elongation} = 0.0166(10) = 0.166 \text{ in.}$$

$$L = L_0 + \text{permanent elongation}$$

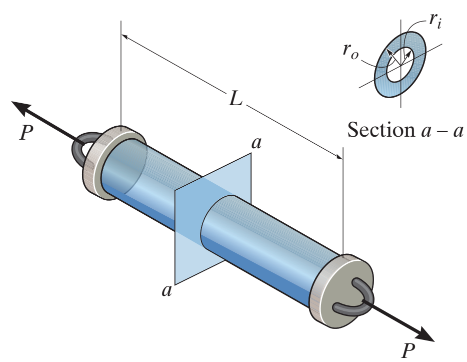
$$= 10 + 0.166$$

$$= 10.17 \text{ in.}$$ **Ans.**

Ans:
$L = 10.17$ in.

*R3–8.

The pipe with two rigid caps attached to its ends is subjected to an axial force P. If the pipe is made from a material having a modulus of elasticity E and Poisson's ratio ν, determine the change in volume of the material.



SOLUTION

Normal Stress: The rod is subjected to uniaxial loading. Thus, $\sigma_{\text{long}} = \dfrac{P}{A}$ and $\sigma_{\text{lat}} = 0$.

$$\delta V = A\delta L + 2\pi r L \delta r$$

$$= A\epsilon_{\text{long}} L + 2\pi r L \epsilon_{\text{lat}} r$$

Using Poisson's ratio and noting that $AL = \pi r^2 L = V$,

$$\delta V = \epsilon_{\text{long}} V - 2\nu \epsilon_{\text{long}} V$$

$$= \epsilon_{\text{long}} (1 - 2\nu) V$$

$$= \frac{\sigma_{\text{long}}}{E} (1 - 2\nu) V$$

Since $\sigma_{\text{long}} = P/A$,

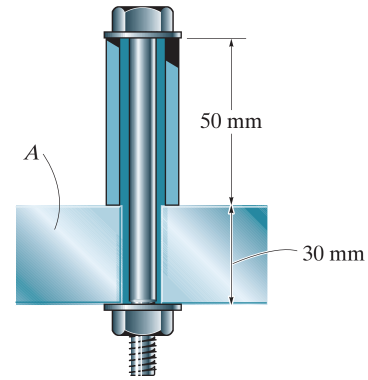
$$\delta V = \frac{P}{AE} (1 - 2\nu) AL$$

$$= \frac{PL}{E} (1 - 2\nu)$$ **Ans.**

Ans:
$\delta V = \dfrac{PL}{E}(1 - 2\nu)$

R3–9.

The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



SOLUTION

Normal Stress:

$$\sigma_b = \frac{P}{A_b} = \frac{8(10^3)}{\frac{\pi}{4}(0.008^2)} = 159.15 \text{ MPa}$$

$$\sigma_s = \frac{P}{A_s} = \frac{8(10^3)}{\frac{\pi}{4}(0.02^2 - 0.012^2)} = 39.79 \text{ MPa}$$

Normal Strain: Applying Hooke's Law

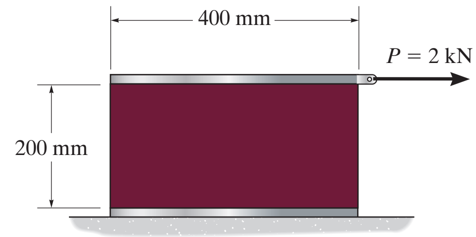
$$\epsilon_b = \frac{\sigma_b}{E_{al}} = \frac{159.15(10^6)}{70(10^9)} = 0.00227 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_s = \frac{\sigma_s}{E_{mg}} = \frac{39.79(10^6)}{45(10^9)} = 0.000884 \text{ mm/mm} \qquad \textbf{Ans.}$$

Ans:
$\epsilon_b = 0.00227$ mm/mm, $\epsilon_s = 0.000884$ mm/mm

R3–10.

An acetal polymer block is fixed to the rigid plates at its top and bottom surfaces. If the top plate displaces 2 mm horizontally when it is subjected to a horizontal force $P = 2$ kN, determine the shear modulus of the polymer. The width of the block is 100 mm. Assume that the polymer is linearly elastic and use small angle analysis.



SOLUTION

Normal and Shear Stress:

$$\tau = \frac{V}{A} = \frac{2(10^3)}{0.4(0.1)} = 50\text{ kPa}$$

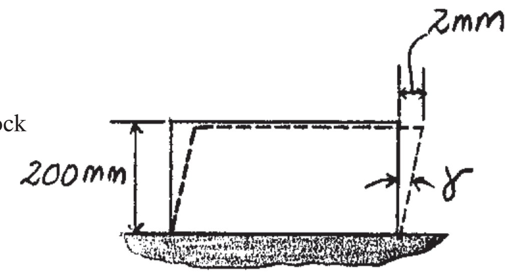
Referring to the geometry of the undeformed and deformed shape of the block shown in Fig. *a*,

$$\gamma = \frac{2}{200} = 0.01\text{ rad}$$

Applying Hooke's Law,

$$\tau = G\gamma; \quad 50(10^3) = G(0.01)$$

$$G = 5\text{ MPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$G = 5$ MPa