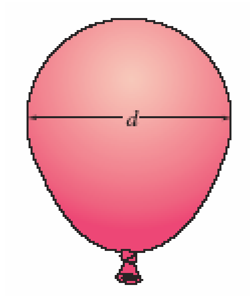
2-1 The center portion of the rubber balloon has a diameter of $d = 100$ mm. If the air pressure within it causes the balloon's diameter to become $d = 125$ mm, determine the average normal strain in the rubber.



Given: $d_0 := 100\text{mm}$ $\quad d := 125\text{mm}$

Solution:

$$\varepsilon := \frac{\pi d - \pi d_0}{\pi d_0}$$

$$\varepsilon = 0.2500 \frac{\text{mm}}{\text{mm}}$$ **Ans**

Ans:
$\epsilon_{CE} = 0.00250 \text{ mm/mm}, \ \epsilon_{BD} = 0.00107 \text{ mm/mm}$

2–2. A thin strip of rubber has an unstretched length of 375 mm. If it is stretched around a pipe having an outer diameter of 125 mm, determine the average normal strain in the strip.

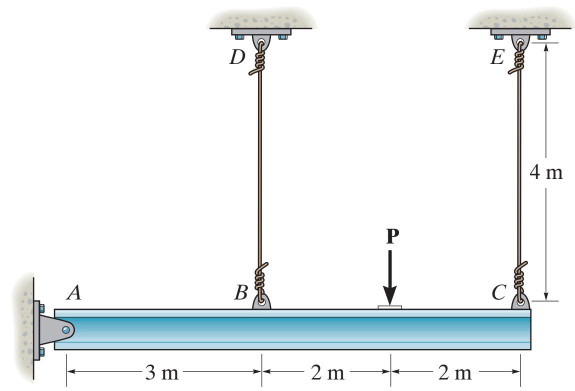
$L_0 = 375$ mm

$L = \pi(125 \text{ mm})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{125\pi - 375}{375} = 0.0472 \text{ mm/mm}$$ **Ans.**

Ans:
$\epsilon = 0.0472$ mm/mm

2–3. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.

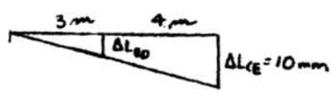


$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

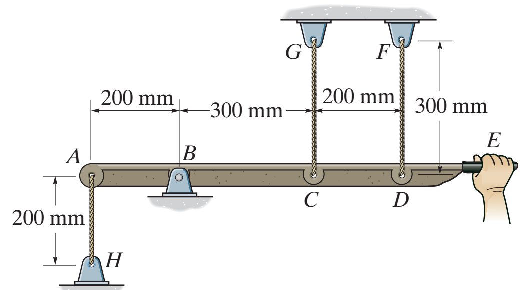
$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_{CE} = 0.00250$ mm/mm, $\epsilon_{BD} = 0.00107$ mm/mm

***2–4.** The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin B through an angle of 2°. Determine the average normal strain developed in each wire. The wires are unstretched when the lever is in the horizontal position.



Geometry: The lever arm rotates through an angle of $\theta = \left(\frac{2°}{180}\right)\pi \text{ rad} = 0.03491$ rad. Since θ is small, the displacements of points A, C, and D can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

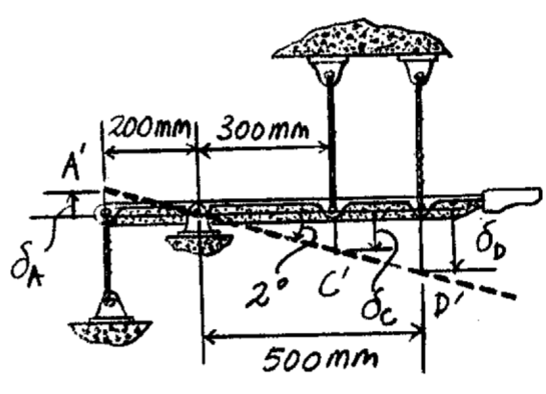
$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

Average Normal Strain: The unstretched length of wires AH, CG, and DF are $L_{AH} = 200$ mm, $L_{CG} = 300$ mm, and $L_{DF} = 300$ mm. We obtain

$$(\epsilon_{avg})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm}$$ **Ans.**

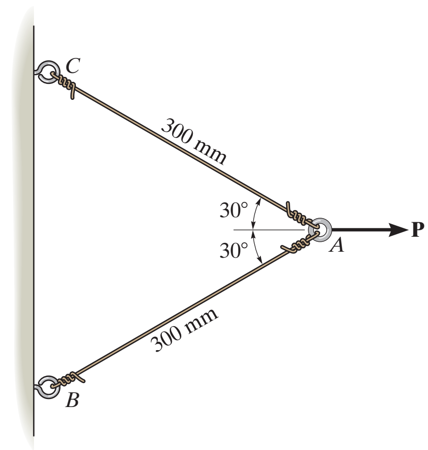
$$(\epsilon_{avg})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm}$$ **Ans.**

$$(\epsilon_{avg})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm}$$ **Ans.**



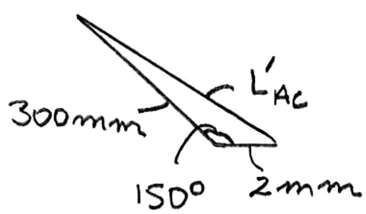
(a)

2–5. The two wires are connected together at A. If the force $\mathbf{P}$ causes point A to be displaced horizontally 2 mm, determine the normal strain developed in each wire.



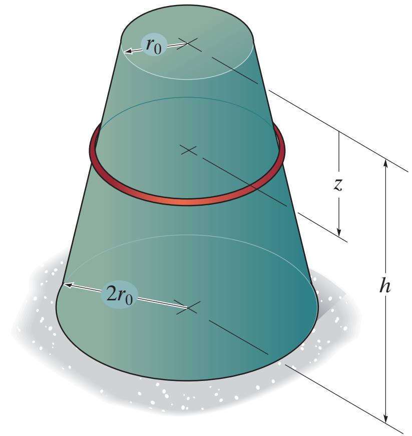
$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150^\circ} = 301.734 \text{ mm}$$

$$\epsilon_{AC} = \epsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578 \text{ mm/mm} \qquad \textbf{Ans.}$$



Ans:
$\epsilon_{AC} = \epsilon_{AB} = 0.00578$ mm/mm

2–6. The rubber band of unstretched length $2r_0$ is forced down the frustum of the cone. Determine the average normal strain in the band as a function of z.

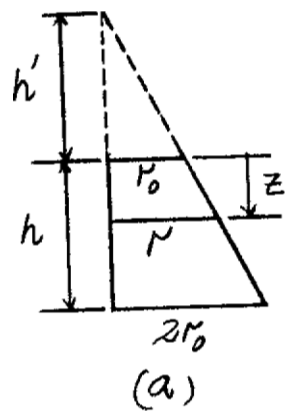


Geometry: Using similar triangles shown in Fig. *a*,

$$\frac{h'}{r_0} = \frac{h' + h}{2r_0}; \qquad h' = h$$

Subsequently, using the result of h'

$$\frac{r}{z + h} = \frac{r_0}{h}; \qquad r = \frac{r_0}{h}(z + h)$$



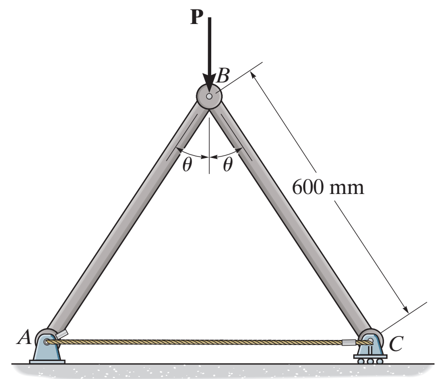
Average Normal Strain: The length of the rubber band as a function of z is $L = 2\pi r = \dfrac{2\pi r_0}{h}(z+h)$. With $L_0 = 2r_0$, we have

$$\epsilon_{avg} = \frac{L - L_0}{L_0} = \frac{\dfrac{2\pi r_0}{h}(z + h) - 2r_0}{2r_0} = \frac{\pi}{h}(z + h) - 1 \qquad \textbf{Ans.}$$

Ans:

$$\epsilon_{avg} = \frac{\pi}{h}(z + h) - 1$$

2–7. The pin-connected rigid rods AB and BC are inclined at $\theta = 30°$ when they are unloaded. When the force $\mathbf{P}$ is applied θ becomes $30.2°$. Determine the average normal strain developed in wire AC.



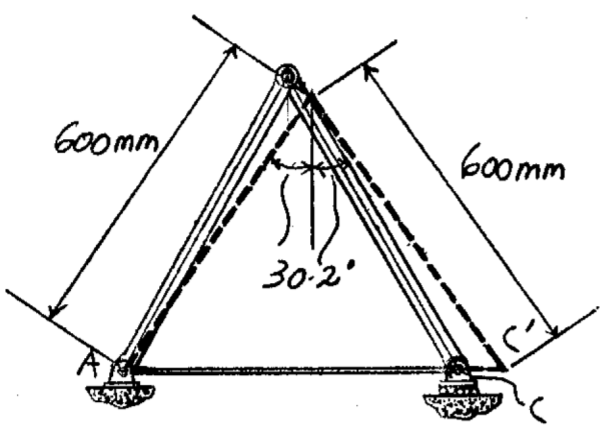
Geometry: Referring to Fig. *a*, the unstretched and stretched lengths of wire AD are

$$L_{AC} = 2(600 \sin 30°) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2°) = 603.6239 \text{ mm}$$

Average Normal Strain:

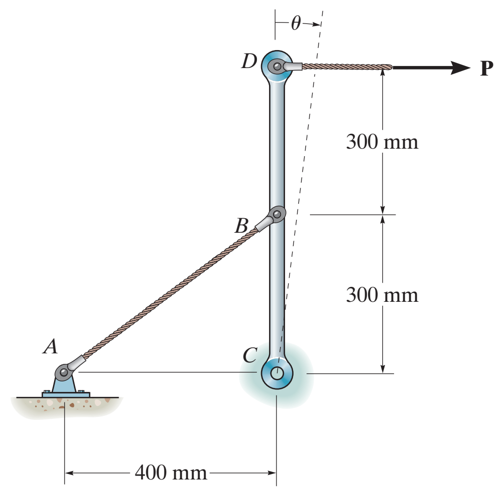
$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3})\,\text{mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3})\,\text{mm/mm}$

***2–8.** Part of a control linkage for an airplane consists of a rigid member CBD and a flexible cable AB. If a force is applied to the end D of the member and causes it to rotate by $\theta = 0.3°$, determine the normal strain in the cable. Originally the cable is unstretched.



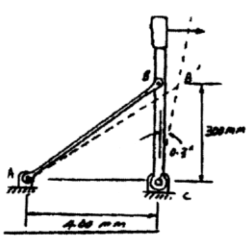
$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3°}$$

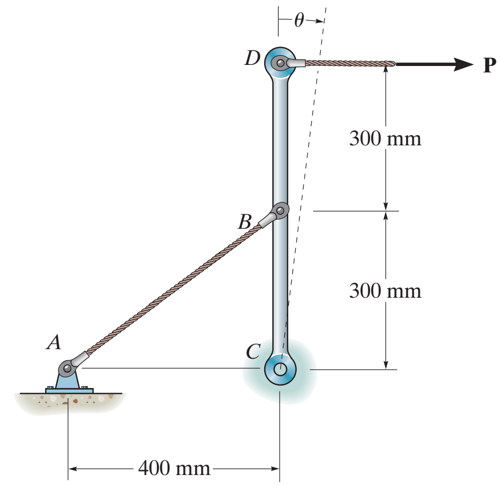
$$= 501.255 \text{ mm}$$

$$\epsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm}$$ **Ans.**



2–9. Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point *D*. Originally the cable is unstretched.



$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$AB' = AB + \varepsilon_{AB} AB$$

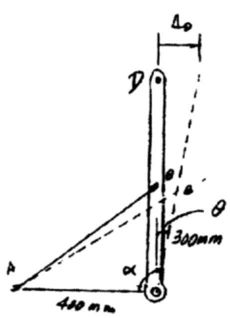
$$= 500 + 0.0035(500) = 501.75 \text{ mm}$$

$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

$$\alpha = 90.4185^\circ$$

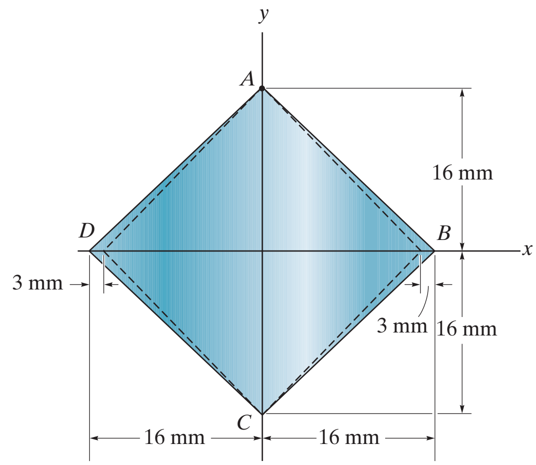
$$\theta = 90.4185^\circ - 90^\circ = 0.4185^\circ = \frac{\pi}{180^\circ}(0.4185) \text{ rad}$$

$$\Delta_D = 600(\theta) = 600\left(\frac{\pi}{180^\circ}\right)(0.4185) = 4.38 \text{ mm}$$ **Ans.**



Ans:
$\Delta_D = 4.38$ mm

2–10. The corners of the square plate are given the displacements indicated. Determine the shear strain along the edges of the plate at A and B.



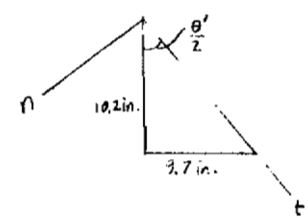
At A:

$$\frac{\theta'}{2} = \tan^{-1}\left(\frac{9.7}{10.2}\right) = 43.561°$$

$$\theta' = 1.52056 \text{ rad}$$

$$(\gamma_A)_{nt} = \frac{\pi}{2} - 1.52056$$

$$= 0.0502 \text{ rad}$$ **Ans.**



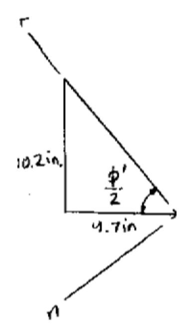
At B:

$$\frac{\phi'}{2} = \tan^{-1}\left(\frac{10.2}{9.7}\right) = 46.439°$$

$$\phi' = 1.62104 \text{ rad}$$

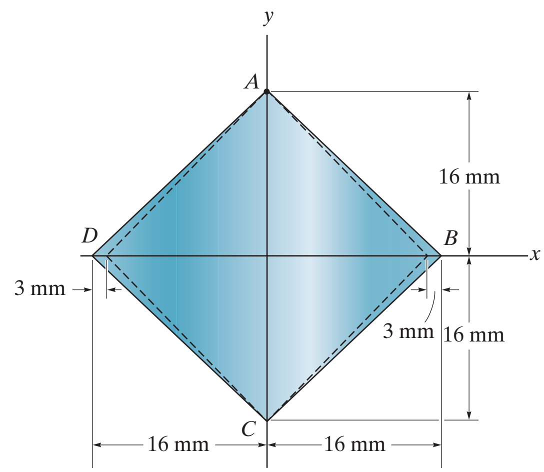
$$(\gamma_B)_{nt} = \frac{\pi}{2} - 1.62104$$

$$= -0.0502 \text{ rad}$$ **Ans.**



Ans:
$(\gamma_A)_{nt} = 0.0502$ rad, $(\gamma_B)_{nt} = -0.0502$ rad

2–11. The corners B and D of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonal DB.



Referring to Fig. a,

$$L_{AB} = \sqrt{16^2 + 16^2} = \sqrt{512}\ \text{mm}$$

$$L_{AB'} = \sqrt{16^2 + 13^2} = \sqrt{425}\ \text{mm}$$

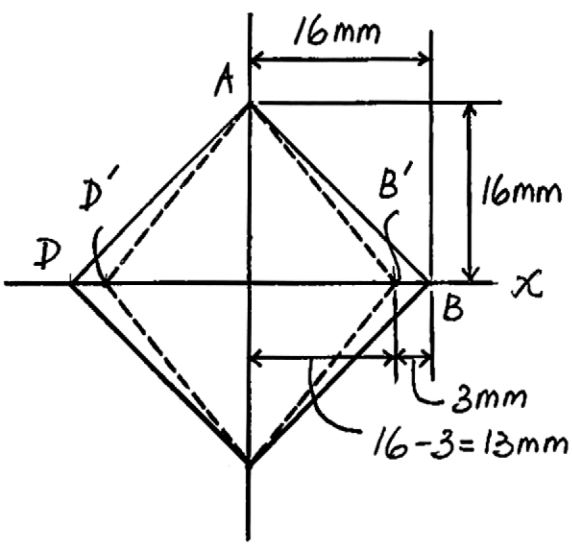
$$L_{BD} = 16 + 16 = 32\ \text{mm}$$

$$L_{B'D'} = 13 + 13 = 26\ \text{mm}$$

Thus,

$$\left(\varepsilon_{\text{avg}}\right)_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{\sqrt{425} - \sqrt{512}}{\sqrt{512}} = -0.0889\ \text{mm/mm}$$ **Ans.**

$$\left(\varepsilon_{\text{avg}}\right)_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{26 - 32}{32} = -0.1875\ \text{mm/mm}$$ **Ans.**



(a)

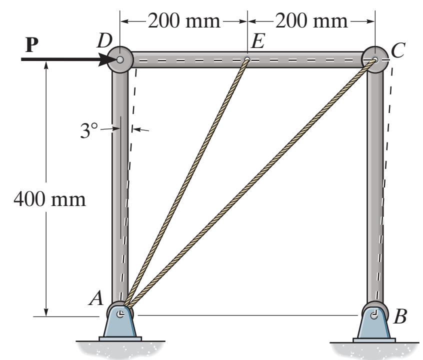
2–12

2–13

Ans:

$\epsilon_{DB} = \epsilon_{AB} \cos^2 \theta + \epsilon_{CB} \sin^2 \theta$

2–14. The force **P** applied at joint D of the square frame causes the frame to sway and form the dashed rhombus. Determine the average normal strain developed in wire AC. Assume the three rods are rigid.



Geometry: Referring to Fig. *a*, the stretched length of $L_{AC'}$ of wire AC' can be determined using the cosine law.

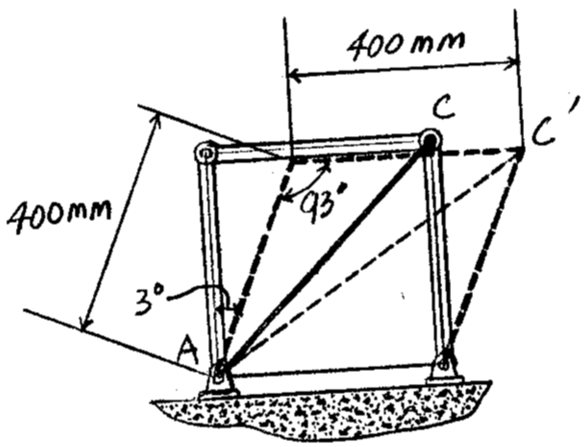
$$L_{AC'} = \sqrt{400^2 + 400^2 - 2(400)(400)\cos 93^\circ} = 580.30 \text{ mm}$$

The unstretched length of wire AC is

$$L_{AC} = \sqrt{400^2 + 400^2} = 565.69 \text{ mm}$$

Average Normal Strain:

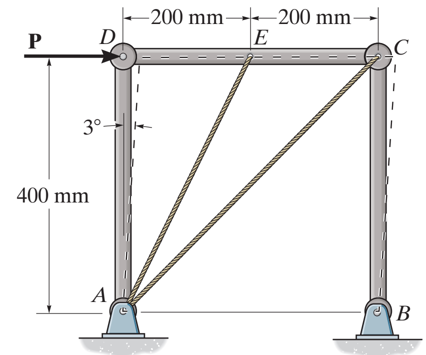
$$(\epsilon_{avg})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{580.30 - 565.69}{565.69} = 0.0258 \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{avg})_{AC} = 0.0258$ mm/mm

2–15. The force **P** applied at joint D of the square frame causes the frame to sway and form the dashed rhombus. Determine the average normal strain developed in wire AE. Assume the three rods are rigid.



Geometry: Referring to Fig. a, the stretched length of $L_{AE'}$ of wire AE can be determined using the cosine law.

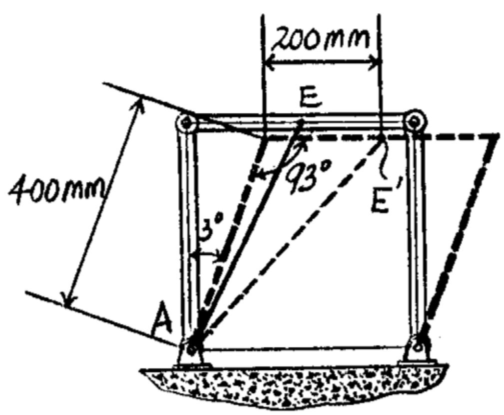
$$L_{AE'} = \sqrt{400^2 + 200^2 - 2(400)(200)\cos 93^\circ} = 456.48 \text{ mm}$$

The unstretched length of wire AE is

$$L_{AE} = \sqrt{400^2 + 200^2} = 447.21 \text{ mm}$$

Average Normal Strain:

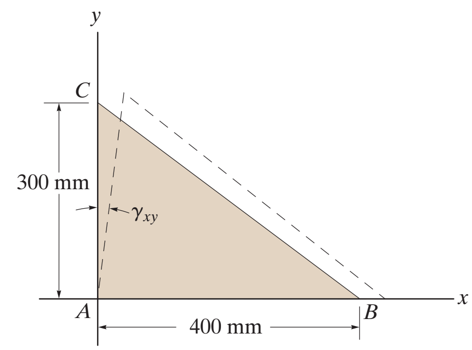
$$(\epsilon_{\text{avg}})_{AE} = \frac{L_{AE'} - L_{AE}}{L_{AE}} = \frac{456.48 - 447.21}{447.21} = 0.0207 \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{AE} = 0.0207 \text{ mm/mm}$

***2–16.** The triangular plate ABC is deformed into the shape shown by the dashed lines. If at A, $\varepsilon_{AB} = 0.0075$, $\epsilon_{AC} = 0.01$ and $\gamma_{xy} = 0.005$ rad, determine the average normal strain along edge BC.



Average Normal Strain: The stretched length of sides AB and AC are

$$L_{AC'} = (1 + \varepsilon_y)L_{AC} = (1 + 0.01)(300) = 303 \text{ mm}$$

$$L_{AB'} = (1 + \varepsilon_x)L_{AB} = (1 + 0.0075)(400) = 403 \text{ mm}$$

Also,

$$\theta = \frac{\pi}{2} - 0.005 = 1.5658 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 89.7135°$$

The unstretched length of edge BC is

$$L_{BC} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

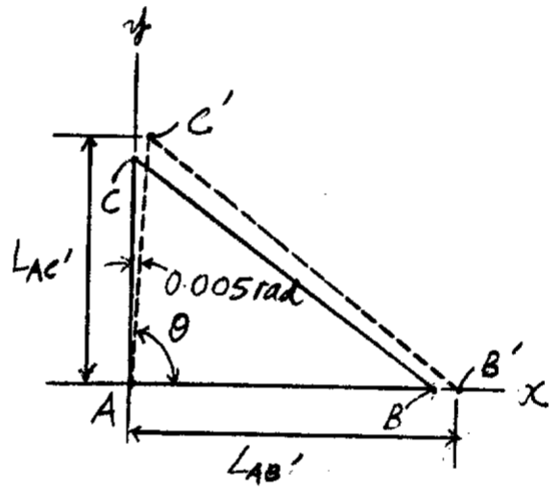
and the stretched length of this edge is

$$L_{B'C'} = \sqrt{303^2 + 403^2 - 2(303)(403)\cos 89.7135°}$$

$$= 502.9880 \text{ mm}$$

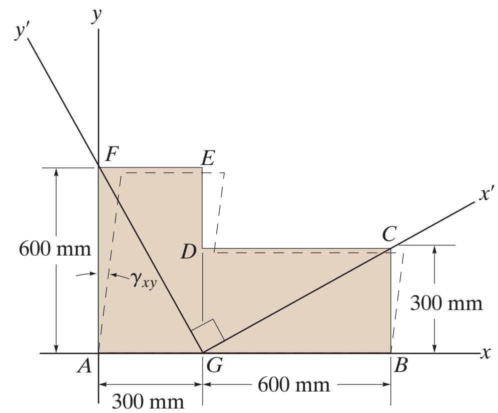
We obtain,

$$\epsilon_{BC} = \frac{L_{B'C'} - L_{BC}}{L_{BC}} = \frac{502.9880 - 500}{500} = 5.98(10^{-3}) \text{ mm/mm}$$ **Ans.**



(a)

2–17. The plate is deformed uniformly into the shape shown by the dashed lines. If at A, $\gamma_{xy} = 0.0075$ rad., while $\epsilon_{AB} = \epsilon_{AF} = 0$, determine the average shear strain at point G with respect to the x' and y' axes.



Geometry: Here, $\gamma_{xy} = 0.0075 \text{ rad}\left(\dfrac{180°}{\pi \text{ rad}}\right) = 0.4297°$. Thus,

$$\psi = 90° - 0.4297° = 89.5703° \qquad \beta = 90° + 0.4297° = 90.4297°$$

Subsequently, applying the cosine law to triangles AGF' and GBC', Fig. a,

$$L_{GF'} = \sqrt{600^2 + 300^2 - 2(600)(300)\cos 89.5703°} = 668.8049 \text{ mm}$$

$$L_{GC'} = \sqrt{600^2 + 300^2 - 2(600)(300)\cos 90.4297°} = 672.8298 \text{ mm}$$

Then, applying the sine law to the same triangles,

$$\frac{\sin \phi}{600} = \frac{\sin 89.5703°}{668.8049}; \qquad \phi = 63.7791°$$

$$\frac{\sin \alpha}{300} = \frac{\sin 90.4297°}{672.8298}; \qquad \alpha = 26.4787°$$

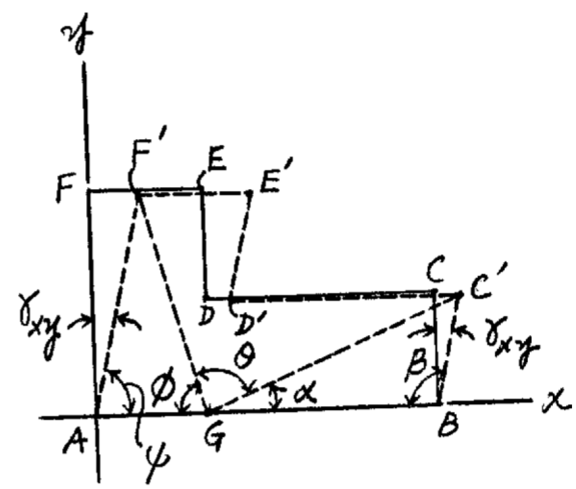
Thus,

$$\theta = 180° - \phi - \alpha = 180° - 63.7791° - 26.4787°$$

$$= 89.7422°\left(\frac{\pi \text{ rad}}{180°}\right) = 1.5663 \text{ rad}$$

Shear Strain:

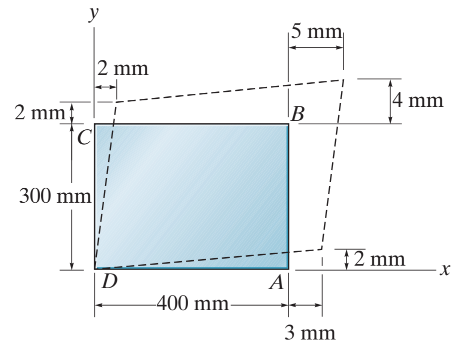
$$(\gamma_G)_{x'y'} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.5663 = 4.50(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\gamma_G)_{x'y'} = 4.50(10^{-3})$ rad

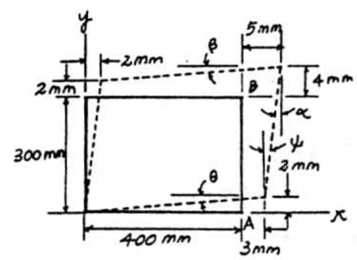
2–18. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain:

$$(\gamma_B)_{xy} = \alpha + \beta$$

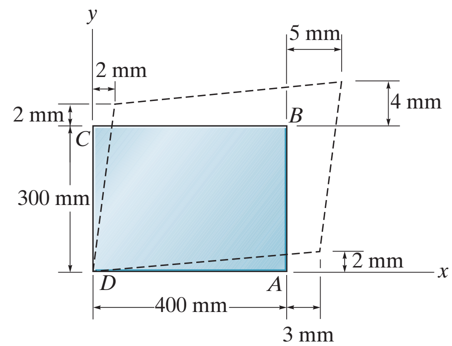
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_A)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_B)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_A)_{xy} = 11.6(10^{-3})$ rad

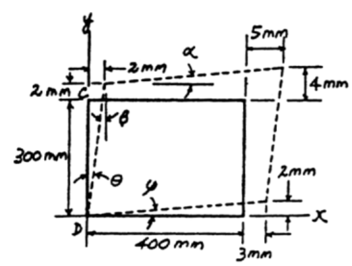
2–19. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain:

$$(\gamma_C)_{xy} = \alpha + \beta$$

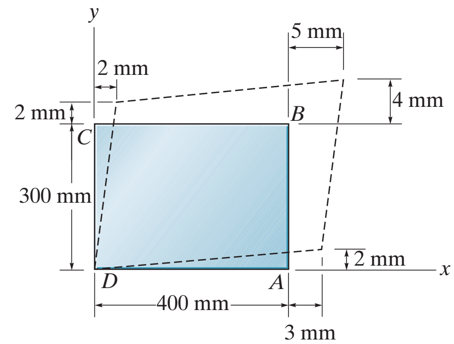
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_C)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_D)_{xy} = 11.6(10^{-3})$ rad

***2–20.** The piece of plastic is originally rectangular. Determine the average normal strain that occurs along the diagonals *AC* and *DB*.

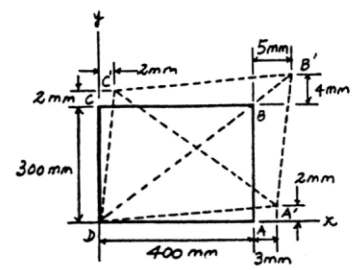


Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$



Average Normal Strain:

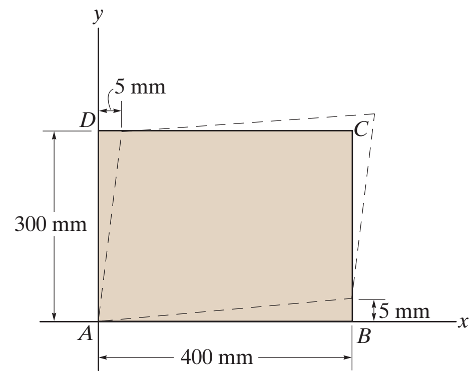
$$\epsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

$$\epsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

2–21. The rectangular plate is deformed into the shape of a parallelogram shown by the dashed lines. Determine the average shear strain γ_{xy} at corners A and B.



Geometry: Referring to Fig. *a* and using small angle analysis,

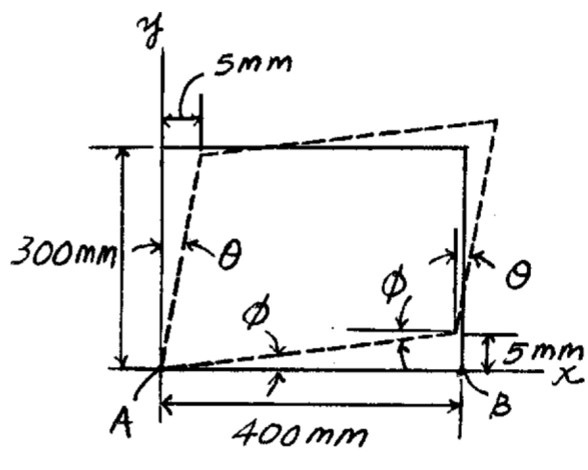
$$\theta = \frac{5}{300} = 0.01667 \text{ rad}$$

$$\phi = \frac{5}{400} = 0.0125 \text{ rad}$$

Shear Strain: Referring to Fig. *a*,

$$(\gamma_A)_{xy} = \theta + \phi = 0.01667 + 0.0125 = 0.0292 \text{ rad} \qquad \textbf{Ans.}$$

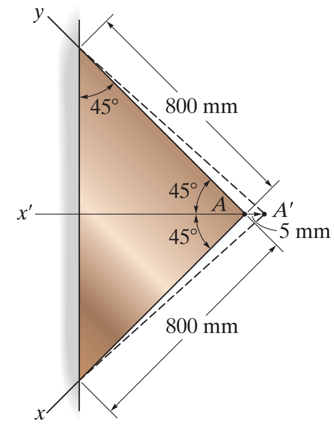
$$(\gamma_B)_{xy} = \theta + \phi = 0.01667 + 0.0125 = 0.0292 \text{ rad} \qquad \textbf{Ans.}$$



(a)

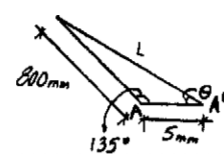
Ans:
$(\gamma_A)_{xy} = 0.0292$ rad, $(\gamma_B)_{xy} = 0.0292$ rad

2–22. The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain, γ_{xy}, at A.



$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\frac{\sin 135^\circ}{803.54} = \frac{\sin\theta}{800}; \qquad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

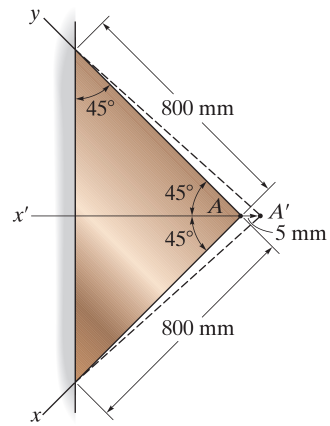


$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

$$= 0.00880 \text{ rad}$$ **Ans.**

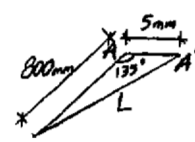
Ans:
$\gamma_{xy} = 0.00880$ rad

2–23. The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.



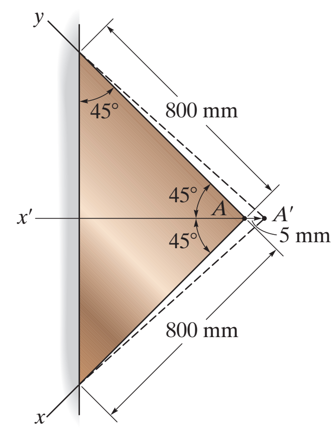
$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm} \qquad \textbf{Ans.}$$



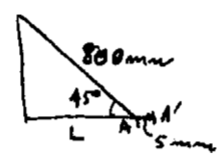
Ans:
$\epsilon_x = 0.00443$ mm/mm

***2–24.** The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.

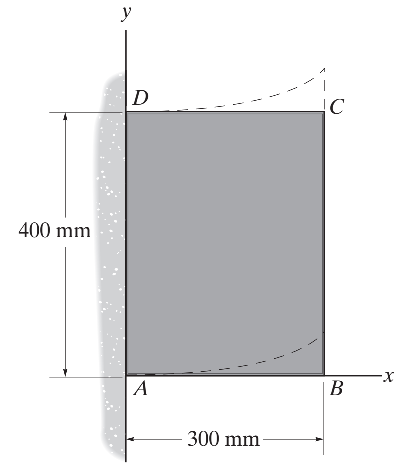


$$L = 800 \cos 45° = 565.69 \text{ mm}$$

$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm} \qquad \textbf{Ans.}$$



2–25. The square rubber block is subjected to a shear strain of $\gamma_{xy} = 40(10^{-6})x + 20(10^{-6})y$, where x and y are in mm. This deformation is in the shape shown by the dashed lines, where all the lines parallel to the y axis remain vertical after the deformation. Determine the normal strain along edge BC.



Shear Strain: Along edge DC, $y = 400$ mm. Thus, $(\gamma_{xy})_{DC} = 40(10^{-6})x + 0.008$. Here, $\dfrac{dy}{dx} = \tan(\gamma_{xy})_{DC} = \tan[40(10^{-6})x + 0.008]$. Then,

$$\int_0^{\delta_c} dy = \int_0^{300\text{ mm}} \tan[40(10^{-6})x + 0.008]dx$$

$$\delta_c = -\frac{1}{40(10^{-6})}\left\{\ln\cos\left[40(10^{-6})x + 0.008\right]\right\}\Bigg|_0^{300\text{ mm}}$$

$$= 4.2003 \text{ mm}$$

Along edge AB, $y = 0$. Thus, $(\gamma_{xy})_{AB} = 40(10^{-6})x$. Here, $\dfrac{dy}{dx} = \tan(\gamma_{xy})_{AB} = \tan[40(10^{-6})x]$. Then,

$$\int_0^{\delta_B} dy = \int_0^{300\text{ mm}} \tan[40(10^{-6})x]dx$$

$$\delta_B = -\frac{1}{40(10^{-6})}\left\{\ln\cos\left[40(10^{-6})x\right]\right\}\Bigg|_0^{300\text{ mm}}$$

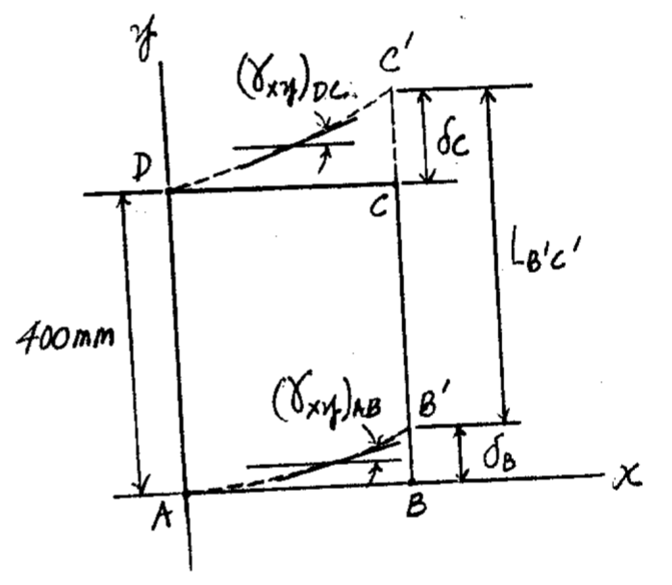
$$= 1.8000 \text{ mm}$$

Average Normal Strain: The stretched length of edge BC is

$$L_{B'C'} = 400 + 4.2003 - 1.8000 = 402.4003 \text{ mm}$$

We obtain,

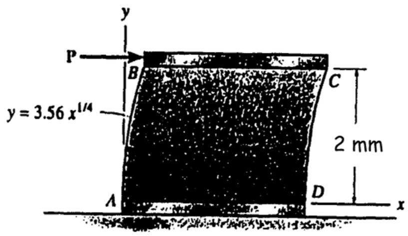
$$(\epsilon_{\text{avg}})_{BC} = \frac{L_{B'C'} - L_{BC}}{L_{BC}} = \frac{402.4003 - 400}{400} = 6.00(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{BC} = 6.00(10^{-3})$ mm/mm

2–26. The Polysulfone block is glued at its top and bottom to the rigid plates. If a tangential force, applied to the top plate, causes the material to deform so that its sides are described by the equation $y = 3.56x^{1/4}$, determine the shear strain in the material at its corners A and B.



Prob. 2–33

$$y = 3.56\,x^{1/4}$$

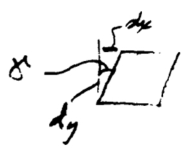
$$\frac{dy}{dx} = 0.890\,x^{-3/4}$$

$$\frac{dx}{dy} = 1.123\,x^{3/4}$$

At A, $x = 0$

$$\gamma_A = \frac{dx}{dy} = 0 \qquad \textbf{Ans}$$

At B,



$$2 = 3.56\,x^{1/4}$$

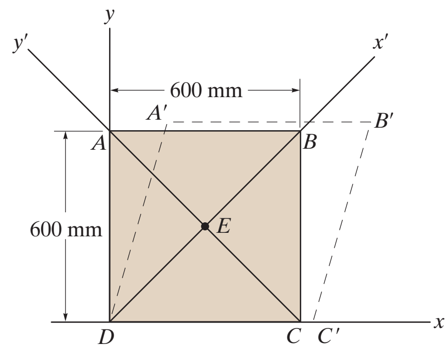
$$x = 0.0996 \text{ mm}$$

$$\gamma_B = \frac{dx}{dy} = 1.123(0.0996)^{3/4} = 0.199 \text{ rad} \qquad \textbf{Ans}$$

Ans:

$(\epsilon_{\text{avg}})_{CA} = -5.59(10^{-3})$ mm/mm

2–27. The square plate $ABCD$ is deformed into the shape shown by the dashed lines. If DC has a normal strain $\epsilon_x = 0.004$, DA has a normal strain $\epsilon_y = 0.005$ and at D, $\gamma_{xy} = 0.02$ rad, determine the shear strain at point E with respect to the x' and y' axes.



Average Normal Strain: The stretched length of sides DC and BC are

$$L_{DC'} = (1 + \epsilon_x)L_{DC} = (1 + 0.004)(600) = 602.4 \text{ mm}$$

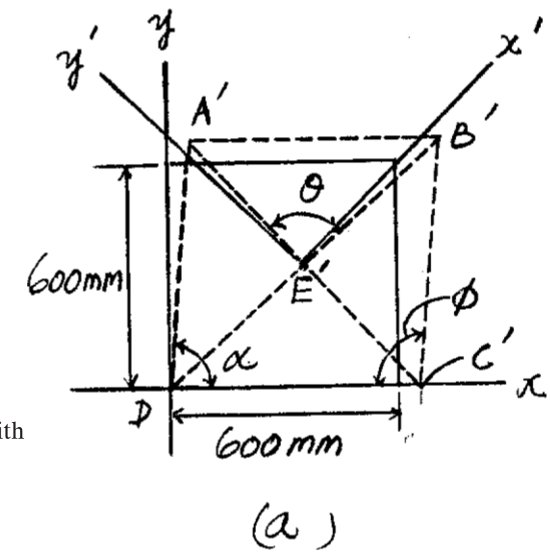
$$L_{B'C'} = (1 + \epsilon_y)L_{BC} = (1 + 0.005)(600) = 603 \text{ mm}$$

Also,

$$\alpha = \frac{\pi}{2} - 0.02 = 1.5508 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 88.854°$$

$$\phi = \frac{\pi}{2} + 0.02 = 1.5908 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 91.146°$$

Thus, the length of $C'A'$ and DB' can be determined using the cosine law with reference to Fig. a.



$$L_{C'A'} = \sqrt{602.4^2 + 603^2 - 2(602.4)(603)\cos 88.854°} = 843.7807 \text{ mm}$$

$$L_{DB'} = \sqrt{602.4^2 + 603^2 - 2(602.4)(603)\cos 91.146°} = 860.8273 \text{ mm}$$

Thus,

$$L_{E'A'} = \frac{L_{C'A'}}{2} = 421.8903 \text{ mm} \qquad L_{E'B'} = \frac{L_{DB'}}{2} = 430.4137 \text{ mm}$$

Using this result and applying the cosine law to the triangle $A'E'B'$, Fig. a,

$$602.4^2 = 421.8903^2 + 430.4137^2 - 2(421.8903)(430.4137)\cos\theta$$

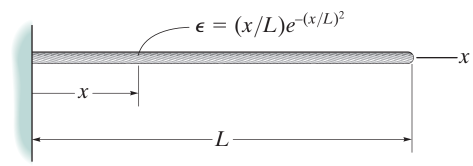
$$\theta = 89.9429°\left(\frac{\pi \text{ rad}}{180°}\right) = 1.5698 \text{ rad}$$

Shear Strain:

$$(\gamma_E)_{x'y'} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.5698 = 0.996(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_E)_{x'y'} = 0.996(10^{-3})$ rad

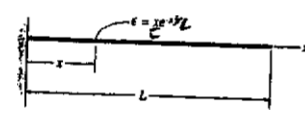
***2–28.** The wire is subjected to a normal strain that is defined by $\epsilon = (x/L)e^{-(x/L)^2}$. If the wire has an initial length L, determine the increase in its length.



$$\Delta L = \frac{1}{L}\int_0^L xe^{-(x/L)^2}dx$$

$$= -L\left[\frac{e^{-(x/L)^2}}{2}\right]_0^L = \frac{L}{2}[1 - (1/e)]$$

$$= \frac{L}{2e}[e - 1]$$ **Ans.**

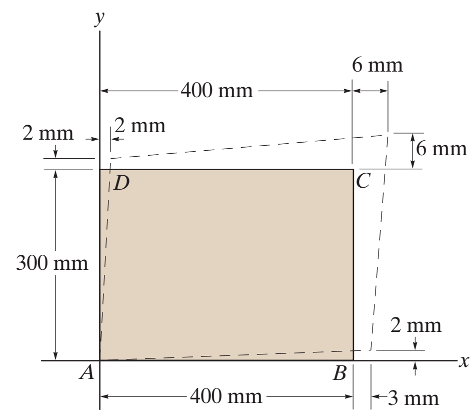


2–29.

Ans:

$(\epsilon_{\text{avg}})_{AC} = 0.0168 \text{ mm/mm},$
$(\gamma_A)_{xy} = 0.0116 \text{ rad}$

2–30. The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal BD, and the average shear strain at corner B.



Geometry: The unstretched length of diagonal BD is

$$L_{BD} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. a, the stretched length of diagonal BD is

$$L_{B'D'} = \sqrt{(300 + 2 - 2)^2 + (400 + 3 - 2)^2} = 500.8004 \text{ mm}$$

Referring to Fig. a and using small angle analysis,

$$\phi = \frac{2}{403} = 0.004963 \text{ rad}$$

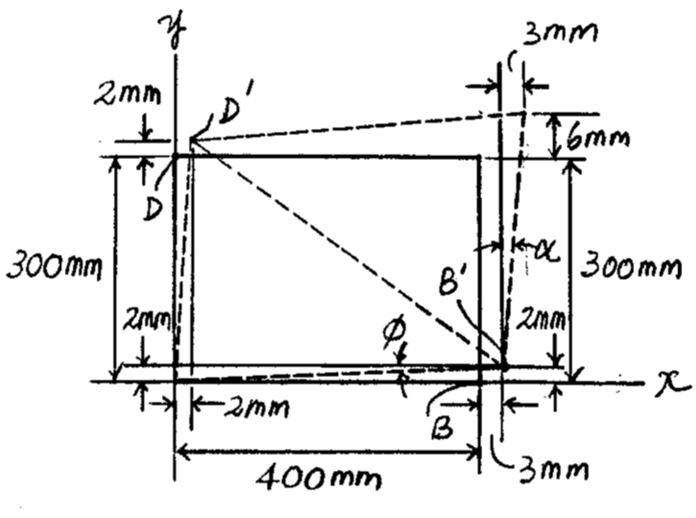
$$\alpha = \frac{3}{300 + 6 - 2} = 0.009868 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{avg})_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{500.8004 - 500}{500} = 1.60(10^{-3}) \text{ mm/mm} \quad \textbf{Ans.}$$

Shear Strain: Referring to Fig. a,

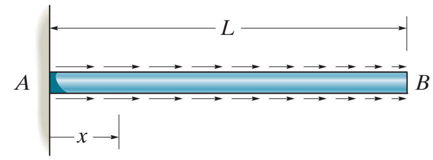
$$(\gamma_B)_{xy} = \phi + \alpha = 0.004963 + 0.009868 = 0.0148 \text{ rad} \quad \textbf{Ans.}$$



(a)

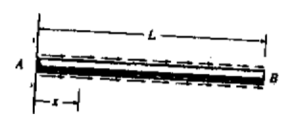
Ans:
$(\epsilon_{avg})_{BD} = 1.60(10^{-3})$ mm/mm,
$(\gamma_B)_{xy} = 0.0148$ rad

2–31. The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = kx^2$, where k is a constant. Determine the displacement of the end B. Also, what is the average normal strain in the rod?



$$\frac{d(\Delta x)}{dx} = \epsilon_x = kx^2$$

$$(\Delta x)_B = \int_0^L kx^2 = \frac{kL^3}{3}$$ **Ans.**

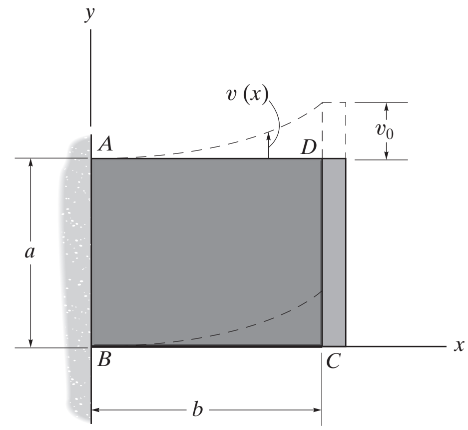


$$(\epsilon_x)_{\text{avg}} = \frac{(\Delta x)_B}{L} = \frac{\frac{kL^3}{3}}{L} = \frac{kL^2}{3}$$ **Ans.**

Ans:

$(\Delta x)_B = \dfrac{kL^3}{3}$, $(\epsilon_x)_{\text{avg}} = \dfrac{kL^2}{3}$

***2–32** The rubber block is fixed along edge AB, and edge CD is moved so that the vertical displacement of any point in the block is given by $v(x) = (v_0/b^3)x^3$. Determine the shear strain γ_{xy} at points $(b/2, a/2)$ and (b, a).



Shear Strain: From Fig. *a*,

$$\frac{dv}{dx} = \tan\gamma_{xy}$$

$$\frac{3v_0}{b^3}x^2 = \tan\gamma_{xy}$$

$$\gamma_{xy} = \tan^{-1}\left(\frac{3v_0}{b^3}x^2\right)$$

Thus, at point $(b/2, a/2)$,

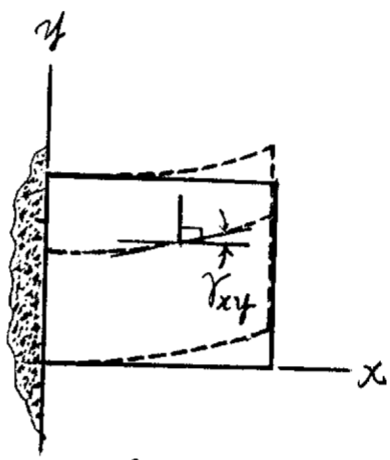
$$\gamma_{xy} = \tan^{-1}\left[\frac{3v_0}{b^3}\left(\frac{b}{2}\right)^2\right]$$

$$= \tan^{-1}\left[\frac{3}{4}\left(\frac{v_0}{b}\right)\right] \qquad \textbf{Ans.}$$

and at point (b, a),

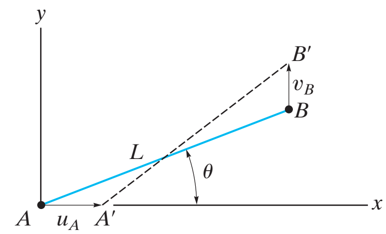
$$\gamma_{xy} = \tan^{-1}\left[\frac{3v_0}{b^3}(b^2)\right]$$

$$= \tan^{-1}\left[3\left(\frac{v_0}{b}\right)\right] \qquad \textbf{Ans.}$$

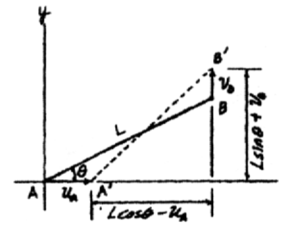


(a)

2–33. The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B, respectively, determine the normal strain in the fiber when it is in position $A'B'$.



Geometry:



$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

Neglecting higher terms u_A^2 and v_B^2

$$\epsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\epsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$

$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$ **Ans.**

Ans.

$$\epsilon_{AB} = \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$

2–34. If the normal strain is defined in reference to the final length, that is,

$$\epsilon_n' = \lim_{p \to p'} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2 , show that the difference in these strains is represented as a second-order term, namely, $\epsilon_n - \epsilon_n' = \epsilon_n \epsilon_n'$.

$$\epsilon_B = \frac{\Delta S' - \Delta S}{\Delta S}$$

$$\epsilon_B - \epsilon_A' = \frac{\Delta S' - \Delta S}{\Delta S} - \frac{\Delta S' - \Delta S}{\Delta S'}$$

$$= \frac{\Delta S'^2 - \Delta S \Delta S' - \Delta S' \Delta S + \Delta S^2}{\Delta S \Delta S'}$$

$$= \frac{\Delta S'^2 + \Delta S^2 - 2\Delta S' \Delta S}{\Delta S \Delta S'}$$

$$= \frac{(\Delta S' - \Delta S)^2}{\Delta S \Delta S'} = \left(\frac{\Delta S' - \Delta S}{\Delta S} \right) \left(\frac{\Delta S' - \Delta S}{\Delta S'} \right)$$

$$= \epsilon_A \epsilon_B' \text{ (Q.E.D)}$$